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An optimisation Model for minimising Totes exchange in VLM and SBS/RS integrated System

Jakob Marolt* Goran Đukić Fabio Sgarbossa*** Tone Lerher******

**University of Maribor, Faculty of Logistics, Celje, Slovenia; (e-mail: jakob.marolt@um.si)*

***University of Zagreb, Faculty of Mechanical Engineering and Naval Architecture, Zagreb, Croatia; (e-mail: Goran.Dukic@fsb.hr)*

**** Norwegian University of Science and Technology, Department of Mechanical and Industrial Engineering, Norway; (e-mail: fabio.sgarbossa@ntnu.no)*

*****University of Maribor, Faculty of Mechanical Engineering, Maribor, Slovenia; (e-mail: tone.lerher@um.si)*

Abstract:
Warehouses and distribution centres are based on integrated material handling solutions. Most of the research papers in intralogistics are only concerned with one system to analyse and optimise the addressed system's performance, such as maximum throughput performance, minimum total cost, maximum energy efficiency, etc. This paper presents a concept to justify the proposed VLM and SBS/RS integrated system and a binary integer programming (BIP) model that minimises totes exchange between the two storage systems. The results show that the minimal number of exchanges between VLM and SBS/RS was achieved in a case study with a longer order list (T = 1600) and a highly dependent SKU's Pareto relationship (20-50).

Keywords: Automated warehousing, intralogistics, VLM and SBS/RS, integrated system, optimisation, performance analysis.

1. INTRODUCTION

Logistics proves to be very important nowadays, especially the part of logistics that is related to internal transport and warehousing, called intralogistics. One of the important segments of intralogistics is warehouses and distribution centres, as their number is continually rising worldwide.

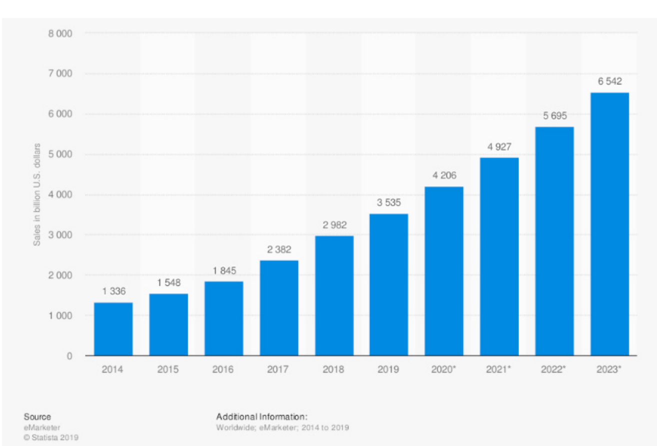

Figure 1: Retail e-commerce sales worldwide from 2014 to 2023

The driving force for the increased number of warehouses and distribution centres is (among other factors) e-commerce, which is expressively grown over the last years (see Figure 1).

With the beginning of e-commerce, new generations of warehouses have emerged that specialise in the specific needs of online Business-to-Consumer (B2C) retailers. Special attention is given to the warehouse process called "Order-picking", as this process accounts for more than 40% of the total costs of the warehouse.

Over the last decade, warehouse automation and robotisation have evolved rapidly. Suppose AS/RS for pallets and totes represented the "state-of-the-art" warehouse technology decades ago. In that case, autonomous mobile robots (AMR) and collaborative robots (COBOTs) are replacing the existing technology in warehouses, which has an expressive impact on a high level of productivity of order-picking systems. Another significant trend in warehouses is the application of integrated systems, where two or more (individual) systems are interconnected to improve throughput performance and reduce investment costs.

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This paper will address Vertical Lift Modules (VLM) and Shuttle-Based Storage and Retrieval Systems (SBS/RS) as an integrated system.

2. RELATED WORKS

In recent years researchers have studied both VLM and SBS/RS intensively. Academics analysed the storage system performance with different constraints and provided methods to determine the storage system's cycle times and, consequently, estimate the storage system's throughput performance. The analytical models are usually supported with simulation studies to show their accuracy and repeatability.

SBS/RS is a special design of an automated warehouse, which is assembled with an elevator with a lifting table, tier-captive shuttle carriers and buffer positions in each tier and the storage rack. Lerher et al. studied single-deep SBS/RS and considered the elevator lifting table's and shuttle vehicle's kinematic properties (acceleration, deceleration and maximum velocity) (Lerher *et al.*, 2015). In the following year (2016) Lerher published a study considering double-deep SBS/RS (Lerher, 2016). Because of the second depth, the analytical model needs different expressions, as a relocation cycle emerged whenever another SKU blocked the retrieving SKU. Typical throughput of SBS/RS is approximately $\lambda_{SBS/RS}$ = 700 totes/hour (double lifting table).

NOTE: Actual throughput performance of SBS/RS depends on multiple factors such as kinematic properties of the lift and shuttle carriers, SBS/RS dimensions, storage policy, etc.).

Similarly, the VLM storage system was studied by Dukic et al. They provided guidelines for throughput estimation for single dual-tray VLM (Dukic, Opetuk and Lerher, 2015). VLMs are typically used for small parts in totes that are placed on trays. The trays are stored on shelves in the VLM, while the lift in the VLM operates in a vertical motion to store and retrieve trays according to the customer's order. Typical throughput of VLM is λ_{VLM} = 130 trays/hour (dual-tray VLM).

NOTE: Actual throughput performance of VLM depends on multiple factors such as kinematic properties of the lift, VLM dimensions, storage policy, etc.).

More recently, the authors Kumar et al. presented a review of warehouse research. They emphasised that the warehousing research transitioned from the traditional storeroom analysis to studies of automated and integrated warehousing systems that are technology-driven (Kumar, Narkhede and Jain, 2021). This claim can also be confirmed with a review work of Azadeh, De Koster and Roy. The flexibility and endurance of the robotic handling systems give them an advantage for the demanding e-commerce operations compared to traditional warehousing (Azadeh, De Koster and Roy, 2019).

The goal of this paper is to present an integrated "VLM - SBS/RS" storage system where the totes are exchanged between both systems (Figure 2). The VLM is utilised for the order-picking activity. Meanwhile, the SBS/RS is used as the buffer system.

Figure 2: Integrated "VLM - SBS/RS" system

To the best of the authors' knowledge, no work has yet been published that studied an integrated VLM and SBS/RS storage systems and examined the exchange policy of totes between them.

3. PROBLEM DESCRIPTION

At first sight, one would say that if the throughput performance of VLM is lover compared to SBS/RS ($\lambda_{VLM} < \lambda_{SBS/RS}$), this integrated system does not make any sense, which is, by the way, correct. If we want the proposed integrated system "VLM - SBS/RS" to be efficiently utilised, then the opposite throughput performance of VLM must be higher compared to SBS/RS ($\lambda_{VLM} > \lambda_{SBS/RS}$). Note that in the proposed "VLM - SBS/RS" integrated system, an SBS/RS is meant to be a buffer system, not a high throughput performance system. This can be achieved by applying a unique design of an SBS/RS, as shown in Figure 3.

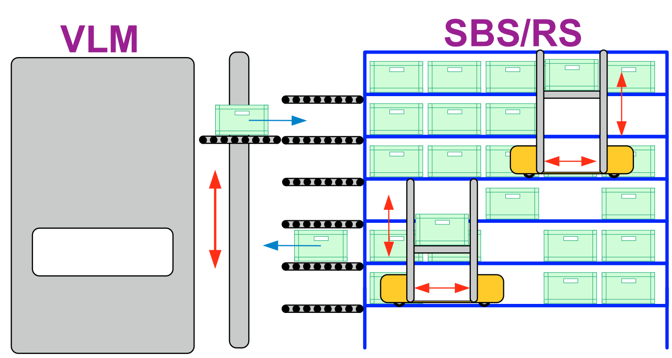

Figure 3: SBS/RS (multi-tier-captive shuttle carriers)

To justify the application of the proposed "VLM - SBS/RS" integrated system, we will give a straightforward example.

Suppose we have a warehouse volume of Q = 2.500 totes filled with items. If one (1) VLM can hold Q_{VLM} = 500 totes, we need another four (4) VLMs to store the remaining totes. In total, we have five (5) VLMs with one (1) operator (order-picker) that can serve VLMs.

Let's assume that we store Q = 500 totes in VLM and the rest of the Q = 2.000 totes in a buffer (SBS/RS) at the back of the VLM. As mentioned before, the proposed SBS/RS is not a classic SBS/RS with a single (tier-captive) shuttle carrier in each tier of the storage rack but a special design of SBS/RS (see Figure 3).

According to a simple calculation, the investment cost for warehouse volume Q = 2.500 totes would be the following:

(*i*) Investment in VLM, only

It is assumed that the cost of one VLM with Q_{VLM} = 500 totes is 70.000 EUR. The total (estimated) investment cost for five VLMs with Q_{VLM} = 500 totes is equal to 350.000 EUR:

- 5 VLMs · 70.000 EUR = 350.000 EUR.

(*ii*) Investment in "VLM - SBS/RS" integrated system

It is assumed that the cost of one multi-tier shuttle carrier (see Figure 5) is 30.000 EUR, the cost of the lift is 60.000 EUR, and the cost of the storage rack is 30 EUR per storage location. The total estimated investment cost for "VLM - SBS/RS" integrated system is equal to 250.000 EUR:

- 2 multi-tier shuttle carriers · 30.000 EUR = 60.000 EUR;
- 1 lift · 60.000 EUR = 60.000 EUR;
- 2.000 storage locations · 30 EUR/location = 60.000 EUR;
- 1 VLM · 70.000 EUR = 70.000 EUR.

NOTE: Both calculations exclude labour costs, the cost of the warehouse building, information systems cost and other related costs.

Based on the above calculation, we can see that the application of "VLM - SBS/RS" integrated system would be competitive compared to the VLM system alone.

From an operational point of view, it is also very important how these two systems, VLM and SBS/RS, would work together. The objective function would be the minimum number of tote exchanges between VLM and SBS/RS, which depends heavily on the orders. In the continuation of this paper, an optimisation model for minimising totes exchange between VLM and SBS/RS integrating system will be presented and discussed.

4. METHOD

The order-picking process is carried out by order-picker picking products from totes in front of the Vertical lift module (VLM) storage system. However, because the entire assortment of items (SKUs) is not inside the VLM, the SKUs have to be exchanged with the SBS/RS if an order contains an item not included in the VLM. Each exchange is twofold. An inbound SKU leaves the SBS/RS and enters the VLM, and an outbound SKU leaves the VLM and enters the SBS/RS. The selection of an outbound SKU has to be carefully selected to prevent the necessity for future exchanges between the VLM and the SBS/RS. We can define a binary integer programming (BIP) model that finds a perfect exchange policy of SKUs between the VLM and the SBS/RS by minimising the required exchanges between the two storage systems.

Assumptions:

- There is no repetition of items in different SKUs. Each SKU holds unique items.
- Each order consists of one item.
- The order sequence is fixed, meaning that there is no order batching.
- The initial configuration of VLM consists of the most popular SKUs (A products).

4.1 Model formulation

The binary integer programming formulation is given by:

Table 1: Notation of the BIP model

Sets	
$y = \{1, 2, \dots Y)$	Set of SKUs.
$t = \{1, 2, \dots T)$	Set of orders.
$t' = \{2, 3, \dots T)$	Set of orders excluding 1.
Parameters	
V^{init}	1-D matrix of initial VLM configuration. 1, if an SKU (j) is present, otherwise 0.
S^{init}	1-D matrix of initial SBS/RS configuration. 1, if an SKU (j) is present, otherwise 0.
$O_{i,j}$	2-D matrix representing order sequence. 1, if at order (i) an SKU (j) is requested, otherwise 0.
L_{VLM}	Maximum number of SKUs in the VLM.
$L_{SBS/RS}$	Maximum number of SKUs in the SBS/RS.
Y	Number of SKUs.
T	Number of Orders.
Decision variables	
$X_{i,j}$	2-D matrix of exchanges. 1, if at order (i), an SKU (j) has to be exchanged from either SBS or VLM.
$V_{i,j}$	2-D matrix of VLM configuration. 1, if at order (i) an SKU (j) is present, otherwise 0
$S_{i,j}$	2-D matrix of SBS/RS configuration. 1, if at order (i) an SKU (j) is present, otherwise 0

$$\text{Min } e = \frac{\left(\sum_{i=1}^{T}\sum_{j=1}^{Y} X_{i,j}\right)}{2} \tag{1}$$

$$\sum_{j=1}^{Y} V_{i,j} = L_{VLM} \qquad \forall i \in t \tag{2}$$

$$\sum_{j=1}^{Y} S_{i,j} = L_{SBS/RS} \qquad \forall i \in t \tag{3}$$

$$(S_{i,j} + V_{i,j}) = 1 \qquad \forall i \in t;\ \forall j \in y \tag{4}$$

$$(O_{i,j} - V_{i,j}) \leq 0 \qquad \forall i \in t;\ \forall j \in y \tag{5}$$

$$V_{i,j} - V_{i-1,j} \leq X_{i,j} \qquad \forall i \in t';\ \forall j \in y \tag{6}$$

$$-V_{i,j} + V_{i-1,j} \leq X_{i,j} \qquad \forall i \in t';\ \forall j \in y \tag{7}$$

$$V_{1,j} = V_j^{init} \qquad \forall j \in y \tag{8}$$

$$S_{1,j} = S_j^{init} \qquad \forall j \in y \tag{9}$$

The objective function (1) minimises the number of exchanges between VLM and SBS/RS. Constraints (2) and (3) bound the capacity of VLM to the parameter L_{VLM} and SBS/RS to the parameter $L_{SBS/RS}$, respectively. Constraint (4) ensures that each SKU has to be present either in the VLM or in the SBS/RS. Further, it prohibits an SKU from appearing in both storage systems at once. Constraints (5) ensure that the VLM holds the SKU in the current order. Constraints (6) and (7) count VLM and SBS/RS exchanges. And constraints (8) and (9) give the initial configuration to VLM and SBS/RS.

4.2 Order data modelling

We hypothesised that the number of required exchanges between VLM and SBS/RS is strongly dependent on the orders list (O). The picking frequency of SKUs in a storage system is usually distributed in some form of a Pareto distribution. Some SKUs are higher in demand and therefore emerge more frequently in the orders (A products). While others are not that popular and rarely appear in the orders (C products). We were interested in this phenomenon and how it affects the number of required exchanges between the two storage systems. That is why we created four different scenarios of orders based on the exponential distribution (9).

$$G(j) = j^{\left(\frac{2s}{s+1}\right)} = j^{s'} \tag{9}$$

In the first scenario (RND), each SKU had an equal probability of being in the order ($s' = 1$). While in the other three scenarios, some SKUs had a higher probability of being selected for the given order. In the second scenario (20-30), 20% of the most frequent SKUs occur in 30% of all orders ($s' = 0.748$), in the third scenario (20-40), 20% of most frequent SKUs occur in 40% of all orders ($s' = 0.569$), and in the fourth scenario (20-50), 20% of the most popular SKUs occur in 50% of all orders ($s' = 0.431$). The cumulative distribution functions (CDF) for all four scenarios are plotted in Figure 4.

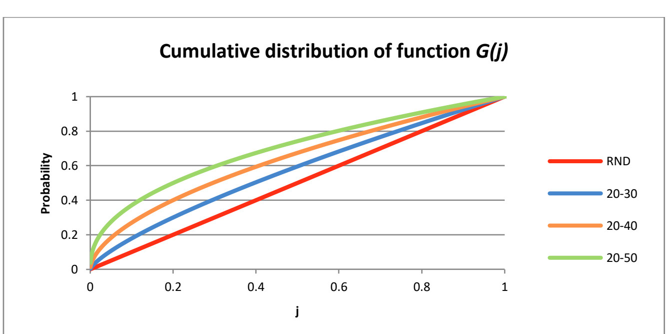

Figure 4: The CDF of G(j), based on which the orders were generated

5. EXPERIMENTAL RESULTS

The BIP model was modelled in Python. We utilised Gurobi v9.5, mathematical optimisation solver, on a standard PC with an Intel i9 @3.6 GHz processor and 64 GB of RAM.

We analysed various case studies of the VLM and SBS/RS integration by altering the number of SKUs in two systems $Y \in \{1000, 1500, 2000\}$ and the length of the order list (O) $T \in \{1000, 1200, 1400, 1600\}$.

The VLM had the maximum number of SKUs set to ($L_{VLM} = 500$), representing VLM with 50 trays (25 on each side) and 10 SKUs per tray. The maximum number of SKUs in SBS/RS was calculated with equation 10.

$$L_{SBS/RS} = Y - L_{VLM} \tag{10}$$

The results of a first case study with 1000 SKUs are presented in Figure 5. The ratio of capacity between VLM and SBS/RS was 1:1 ($L_{SBS/RS} = 500$).

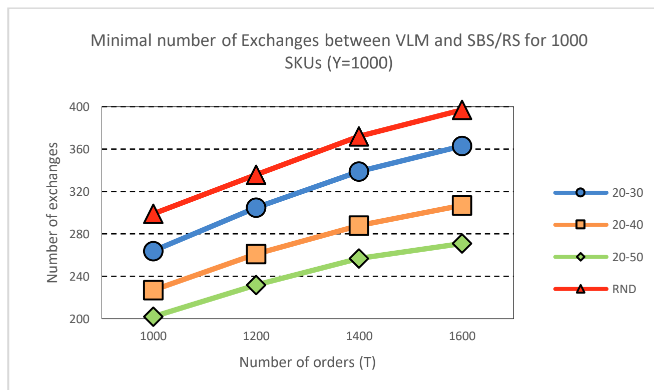

Figure 5: Results of the BIP model for a case study with a ratio of 1:1 (L_{VLM}: $L_{SBS/RS}$)

The second case study with the ratio of 1:2, in which the SBS/RS holds 1000 SKUs ($L_{SBS/RS} = 1000$) is presented in Figure 6.

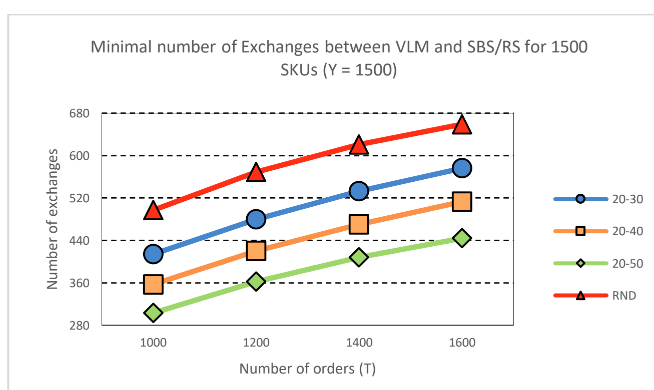

Figure 6: Results of the BIP model for a case study with a ratio of 1:2 (L_{VLM}: $L_{SBS/RS}$)

The last case study with the ratio of 1:3 ($L_{SBS/RS} = 1500$) is presented in Figure 7.

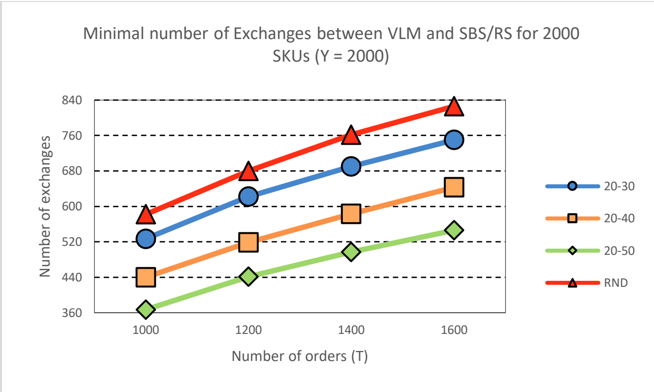

Figure 7: Results of the BIP model for a case study with a ratio of 1:3 (L_{VLM}: $L_{SBS/RS}$)

In all three figures representing the results of the BIP model (Figures 5, 6 and 7), it is evident that the minimal number of exchanges between VLM and SBS/RS is strongly affected by the CDF utilised to generate orders. The smallest number of exchanges between two storage systems were required in the scenario "20-50", where the 20% of most frequent SKUs appear in 50% of all orders, and the highest number of exchanges were required in the case of "RND" scenario, where every SKU had an equal probability of being present in the order.
It is also apparent that the number of exchanges diminishingly rises with the higher number of orders (T). We can hypothesise that the plotted lines will converge as the number of orders (T) rises even further.
The numerical values of the results for all three case studies are presented in Table 2.

Table 2: Results of the BIP model for the minimal number of exchanges between VLM and SBS/RS for all three ratios (1:1, 1:2, 1:3)

	Minimal number of exchanges (*e*) between VLM and SBS/RS			
Number of orders (*T*)	1000	1200	1400	1600
***Y* = 1000**				
20-30	264	305	339	363
20-40	227	261	288	307
20-50	202	232	257	271
RND	299	336	372	397
***Y* = 1500**				
20-30	414	480	533	576
20-40	357	420	470	513
20-50	303	362	408	444
RND	497	569	621	659
***Y* = 2000**				
20-30	527	622	690	750
20-40	440	519	583	643
20-50	367	441	497	546
RND	582	680	762	826

The measurements of the BIP model computation time are given in Table 3. Because the longest computation time for finding an optimal solution was less than one (1) hour, we find our approach with potential for application in practice.

Table 3: Computation time of the BIP model for every scenario presented

	The computation time of the BIP model in seconds			
Number of orders (*T*)	1000	1200	1400	1600
***Y* = 1000**				
20-30	415.5	588.8	810.2	1063.8
20-40	399.1	543.4	726.1	939.5
20-50	404.6	547.2	754.0	937.9
RND	429.1	646.8	867.3	1069.8
***Y* = 1500**				
20-30	720.2	1057.6	1425.9	1936.6
20-40	684.3	1095.6	1435.5	1898.4
20-50	634.3	949.5	1280.3	1611.5
RND	814.1	1225.7	1580.7	2078.6
***Y* = 2000**				
20-30	1105.6	1612.9	2221.4	2932.4
20-40	892.1	1392.9	1972.3	2684.3
20-50	860.7	1333.3	1869.0	2445.4
RND	1096.4	1766.0	2390.4	3416.8

6. CONCLUSIONS

In recent years researchers have studied VLM and SBS/RS intensively. This paper upgrades their research with the proposed "VLM - SBS/RS" integrated system focusing on the operation between both systems in terms of totes exchange. The proposed model for minimising the totes exchange between VLM and SBS/RS considers various configurations of the "VLM - SBS/RS" integrated system (Y = 1000, 1500, 2000). According to the performance comparison, the minimal number of the exchanged totes between VLM and SBS/RS depends on the length of the order list (T = 1000, 1200, 1400, 1600) and the item's popularity in terms of the Pareto relationship (20-30, 20-40, 20-50). This relationship stands for all selected configurations of the "VLM - SBS/RS" integrated system (Y = 1000, 1500, 2000). Generally, the best results are achieved by using a long order list (T = 1600) and a highly dependent Pareto relationship (20-50). In future, we are planning to analyse a detailed technical design and cycle time calculation of "VLM and SBS/RS" integrating system. Another idea is to use the proposed research to develop software for warehouse planners and managers to plan the warehouse layout and calculate the picking process's effectiveness.

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