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EFFICIENT LONG-TERM EXTREME RESPONSE ANALYSIS OF FLOATING BRIDGES USING MULTIPLE TIMESCALE SPECTRAL ANALYSIS

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ABSTRACT

Floating pontoon bridges offer viable alternatives to cable-supported bridges when crossing long straits. Design of such large floating structures requires estimation of long-term load effects (e.g. 100 years) under stochastic wave loading. The long-term extreme response of such floating structures can be estimated using the so-called full-long term method, where the structural response is calculated for all possible environmental conditions over a long-term period. Such a procedure is naturally computationally demanding and depends highly on the computational efficiency of the short-term response calculation. Although the procedure is rather efficient in frequency domain compared to time domain, the computational demand can still be considerable when the number of environmental variables is large. In such cases, the computational burden is usually reduced using approximate methods or surrogate modeling, e.g. based on machine learning. Either approach requires compromise of accuracy to some extent and training of the surrogate models can be just as computationally demanding, depending on the problem. Here, we shall explore the possibility of carrying out the short-term response calculations using multiple timescale spectral analysis, which allows analytical approximations of the short-term response statistics. Through comparisons with the more time-consuming numerical solution, the results are discussed in terms of computational efficiency and accuracy.

Keywords: floating bridge; long-term extreme; wave loading; multiple timescale spectral analysis, frequency domain

1. INTRODUCTION

Floating bridges are often exposed to dynamic wave loads, which can cause large dynamic excitations of these structures and thus are critical for structural design. For reliable design of any floating structure that is exposed to wave actions, maximum dynamic load effects in a long-term period (i.e. 100 years, service life of the structure) needs to be estimated considering the variability in the sea state. This can most accurately be achieved by using the full long-term analysis, where the probability distribution of the selected extreme response in the long-term is obtained through integration of short-term responses under all possible sea states weighted by their probability of occurrence [1]. Therefore, in order to carry out such analyses, the short-term extreme response (i.e. maximum response in a short-term stationary interval such as 1-3 hours) must be calculated for many sea states. Even though in the cases where the nonlinearities in the system can be neglected and the calculations can be done in the frequency domain, the procedure is still computationally demanding owing to the large number of analyses that need to be carried out.

Over the years, many approaches were proposed to reduce the computational burden of carrying out a full long-term analysis when estimating long-term extreme response of a variety of marine structures such as floating offshore wind turbines [2] or floating bridges [3]. Recently, applications in wind engineering that account for uncertain turbulence fields were also published [4]. Approximate methods such as first and second order reliability methods [5] or inverse reliability methods can be used [6] to estimate the extreme responses. However, such approximations are only accurate if certain conditions are satisfied concerning the limit state functions and require verification with a full long-term analysis. Alternatively, an environmental contour approach can be adopted [7], which is widely used in the marine engineering industry. Such an approach neglects the variability in the short-term extreme and commonly used along a correction factor, which is structure dependent. In addition to the reliability theory based approximate methods, other studies exist where the full long-term integral is solved in a more efficient manner, using for instance surrogate modeling [5] or active learning algorithms based on machine learning [8].

Another obvious approach to reduce the computational burden of the full long-term analysis is to replace the function calls that evaluate the dynamic response by closed-form expressions. Closed-form solutions, however, are not commonly available for most applications, especially complex structures. In such cases, approximate analytical solutions are possible through exploiting the separation between timescales of the response (the so-called resonant and background components) as shown by Davenport [9] for buffeting responses of structures. This approach was extended and generalized by Denoël [10] to the framework of multiple timescale spectral analysis (MTSA). If successful, such an approach is fast and efficient and offers obvious advantages when a full long-term analysis is carried out where a large number of response calculations are required.

The MTSA framework will be used here to calculate the long-term extreme response of a large floating bridge under several simplifying assumptions to explore the potential of the approach. The formulations are briefly presented referring to the necessary sources. The analyses results are compared with the conventional spectral density approach. Finally, the results are discussed with emphasis on the potential application in long-term extreme response assessment of floating bridges.

2. METHODOLOGY

2.1 Long-term extreme analysis of floating bridges under wave actions

Considering a floating bridge that is supported by discrete pontoons, the stochastic action of the waves acts on the pontoons of the bridge, which results into dynamic motion of the structure. For structural design purposes, we seek the largest load effect (e.g. displacement, stress or section force) that is caused by the stochastic wave loading during the service life of the structure (i.e. long-term, typically 100 years). To formulate this response, we first start with the statistics of the so-called short-term response. In a short-term interval where the loading and the dynamic response can both be assumed stationary and Gaussian, the cumulative probability distribution (CDF) of the short-term extreme response can be written as

$$F_{\tilde{X}|\omega}(x\,|\,\boldsymbol{\omega}) = \exp\left\{\frac{1}{2\pi}\frac{\sigma_{\dot{x}}(\boldsymbol{\omega})}{\sigma_x(\boldsymbol{\omega})}T_{short}\exp\left(-\frac{x^2}{2\sigma_x(\boldsymbol{\omega})^2}\right)\right\} \quad (1)$$

Here, $\tilde{X}$ denotes the maximum value of the response process X(t) in the short-term interval T_{short}. Moreover, $\tilde{X}\,|\,\mathbf{W}$ is the extreme response given a sea state $\mathbf{W} = \{H_s, T_p\}$ which is described by two environmental parameters: significant wave height Hs, and the peak period Tp. σ_x and $\sigma_{\dot{x}}$ are the standard deviations of the response process X(t) and its time derivative, respectively. The long-term is simply composed of many consecutive short-term intervals, and the CDF of the long-term extreme response can be written through ergodic averaging as:

$$F_{\tilde{X}}(x) = \exp\left\{\int_{\boldsymbol{\omega}}\left(\ln F_{\tilde{X}|\mathbf{W}}(x\,|\,\boldsymbol{\omega})\right) f_{\mathbf{W}}(\boldsymbol{\omega})d\boldsymbol{\omega}\right\} \quad (2)$$

where $f_{\mathbf{W}}(\boldsymbol{\omega})$ is the joint probability distribution function (PDF) of the environmental parameters that define the sea state. As seen from the equation, the quantity inside the integral should be evaluated for all the considered sea states, which requires calculation of the short-term response statistics given in Eqn(1). In the next two sections, we will see how these can be calculated for a floating bridge in the frequency domain using the classical spectral density method and the multiple timescale spectral analysis.

2.2 Stochastic dynamic analysis in the frequency domain

Stochastic dynamic analyses of a floating bridge under wave loading can be conveniently carried out in the frequency domain using the framework introduced by Kvåle et al. [11]. Using the spectral density approach, the frequency response function of the system can be written in generalized coordinates of the dry modes shapes, including the hydrodynamic effects as:

$$\tilde{\mathbf{H}}(\omega) = \left[-\omega^2\left(\tilde{\mathbf{M}}_\mathbf{s} + \tilde{\mathbf{M}}_\mathbf{h}(\omega)\right) + i\omega\left(\tilde{\mathbf{C}}_\mathbf{s} + \tilde{\mathbf{C}}_\mathbf{h}(\omega)\right) + \left(\tilde{\mathbf{K}}_\mathbf{s}\right)\right]^{-1} \quad (3)$$

where $\tilde{\mathbf{M}}_\mathbf{s}, \tilde{\mathbf{C}}_\mathbf{s}$, and $\tilde{\mathbf{K}}_\mathbf{s}$ are structural mass, damping and stiffness matrices in generalized coordinates and $\tilde{\mathbf{M}}_\mathbf{h}(\omega)$ and $\tilde{\mathbf{C}}_\mathbf{h}(\omega)$ are frequency-dependent added mass and potential damping matrices that arise due to fluid-structure interaction, also written in generalized coordinates. Under stochastic loading, the spectral density matrix of the response can thus be written in generalized coordinates as:

$$\tilde{\mathbf{S}}_\mathbf{x}(\omega) = \tilde{\mathbf{H}}^*(\omega)\tilde{\mathbf{S}}_\mathbf{p}(\omega)\tilde{\mathbf{H}}(\omega)^T \quad (4)$$

Here, $\tilde{\mathbf{S}}_\mathbf{p}(\omega)$ denotes the spectral density matrix of the wave loading and $\tilde{\mathbf{S}}_\mathbf{r}(\omega)$ of the response, both given in generalized coordinates. The spectral density matrix can also be transformed to the nodal coordinates of the finite element model using the mode shape matrix:

$$\mathbf{S_x}(\omega) = \mathbf{\Phi_p} \tilde{\mathbf{S}}_\mathbf{x}(\omega) \mathbf{\Phi_p}^T \tag{5}$$

Where the $\mathbf{\Phi_p}$ denotes the dry mode shape matrix at the pontoon degrees of freedom. The variances and covariances of the response process can then be obtained through numerical integration:

$$\boldsymbol{\sigma}_\mathbf{x}^2 = \int_\omega \mathbf{S_x}(\omega) d\omega \tag{6}$$

where $\boldsymbol{\sigma}_\mathbf{x}^2$ is the covariance matrix of the response process in bridge degrees-of-freedom. Note that the long-term statistics can be applied for a selected quantity in the matrix using Eqns. (1) and (2).

2.3 Stochastic dynamic analysis using MTSA

Although the methodology presented in the previous section is rather efficient, especially compared to time-domain alternatives, calculation of the covariances of the response is still time-consuming due to tedious numerical integration which often requires a fine frequency grid. Alternatively, multiple timescale spectral analysis (MTSA) can be used to obtain semi-analytical approximations of the covariances of the response [10]. Notice that the equation of motion, the frequency response function of which is given in Eqn.(3) can also be formulated in state-space form:

$$\left[\mathbf{A} + i\omega \mathbf{B} \mathbf{y}\right](\omega) = \mathbf{g}(\omega) \tag{7}$$

In which the matrices read:

$$\mathbf{A} = \begin{bmatrix} \mathbf{K} & \mathbf{0} \\ \mathbf{0} & -\mathbf{M} \end{bmatrix}, \mathbf{B} = \begin{bmatrix} \mathbf{C} & \mathbf{M} \\ \mathbf{M} & \mathbf{0} \end{bmatrix}, \quad \mathbf{y}(\omega) = \begin{bmatrix} \mathbf{x}(\omega) \\ i\omega\mathbf{x}(\omega) \end{bmatrix}, \mathbf{g}(\omega) = \begin{bmatrix} \mathbf{p}(\omega) \\ \mathbf{0} \end{bmatrix} \tag{8}$$

The matrices $\mathbf{x}(\omega)$ and $\mathbf{p}(\omega)$ represent the Fourier transforms of the response and the wave load processes, respectively. It should also be noted that the frequency dependency of the mass and damping matrices is disregarded in the formulation. The following eigenproblem can then be solved to obtain the natural frequencies and mode shapes of the system:

$$i\mathbf{A}\boldsymbol{\theta} = \mathbf{B}\boldsymbol{\theta}\boldsymbol{\lambda} \tag{9}$$

where $\boldsymbol{\theta}$ denotes the matrix of complex mode shapes and $\boldsymbol{\lambda}$ is a vector of complex eigenvalues. The normalization of the mode shapes follows:

$$\boldsymbol{\theta}^T \mathbf{A} \boldsymbol{\theta} = \boldsymbol{\lambda} \mathbf{D}^{-1}, \ \boldsymbol{\theta}^T \mathbf{B} \boldsymbol{\theta} = i\mathbf{D}^{-1} \tag{10}$$

where the maximum absolute value of the elements the mode shape matrix is forced to unity by means of the matrix $\mathbf{D}$.

The spectral density matrix of the modal state forces can also be written transforming from the spectral density matrix of the wave loading in the nodal coordinates of the system $\mathbf{S_p}(\omega)$:

$$\hat{\mathbf{S}}_\mathbf{p}(\omega) = \boldsymbol{\theta}^T \mathbf{S_p}(\omega) \boldsymbol{\theta} \tag{11}$$

Further, when the characteristic frequency of the loading ω_p can be considered different from the natural frequencies of the considered modal responses, the covariances of the modal state response can be written as the combination of two contributions as:

$$\hat{\boldsymbol{\sigma}}_x^2 = \hat{\boldsymbol{\sigma}}_r^2 + \hat{\boldsymbol{\sigma}}_l^2 \tag{12}$$

Where the covariance matrices $\hat{\boldsymbol{\sigma}}_r^2$ and $\hat{\boldsymbol{\sigma}}_l^2$ represent the resonant and the loading components of the modal state responses, respectively. The former arise due to the peaks in the transfer function of the structural system where the latter arises due to the peaks in the loading spectrum in the cases where the peak frequency of the loading spectrum lies far away from the resonant frequencies. In the context of the current paper the loading component is generally small and will be neglected, however, should be included in a general setting. The resonant component on the other hand can be written as [10]:

$$\left(\hat{\boldsymbol{\sigma}}_r^2\right)_{mn} = i\pi \frac{D_m D_n}{\lambda_m - \lambda_n^*} \left[\hat{\mathbf{S}}_{\mathbf{p},mn}(\psi_m) + \hat{\mathbf{S}}_{\mathbf{p},mn}(\psi_n)\right] \tag{13}$$

For the m^th^ and n^th^ modal state responses. The matrices $\boldsymbol{\lambda}$ and $\mathbf{D}$ are given above, the operator (*) denotes conjugate of the complex number, and

$$\psi_i = real(\lambda_i) \tag{14}$$

The covariance matrix of the response in the nodal finite element (FE) coordinates of the system can then be obtained through coordinate transform:

$$\boldsymbol{\sigma}_{x,MTSA}^2 = \boldsymbol{\theta} \hat{\boldsymbol{\sigma}}_r^2 \boldsymbol{\theta}^T \tag{15}$$

3. CASE STUDY: BJØRNAFJORD FLOATING BRIDGE

3.1 General

A floating pontoon arch bridge that spans the 5km wide Bjørnafjord in Norway is currently in its design phase. The bridge will be used here as a case study where we will calculate its long-term extreme response using the methodology described above. The layout of the bridge is shown in Figure 1. The bridge is approximately 5000 meters long and rests of 46 steel pontoons, the geometry of which are assumed identical (58 x 10 meters). The bridge deck is a streamlined steel box girder, which is 31 meters wide and 3.5 meters wide. The radius of curvature of the arch is 5000 meters. For further details on the bridge, the reader is referred to published articles or reports.

The bridge was modeled in ABAQUS finite element (FE) software, where the girder, tower and the pontoon columns are modeled using beam elements (Figure 2). The structural matrices are assembled using the FE model and the modal analysis is carried out in the absence of hydrodynamic effects. A few first dry modes of the structure are given in Table 1 to provide a glimpse into the dynamic properties of the complex structure.

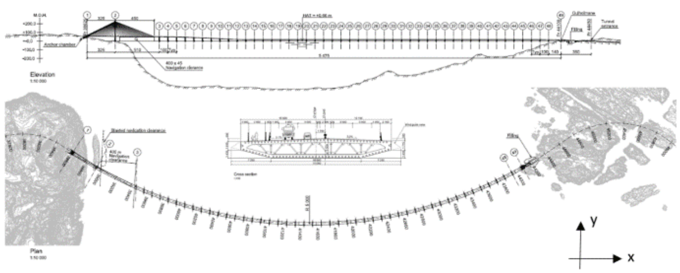


FIGURE 1 : LAYOUT OF THE BRIDGE AND THE GLOBAL COORDINATE SYSTEM

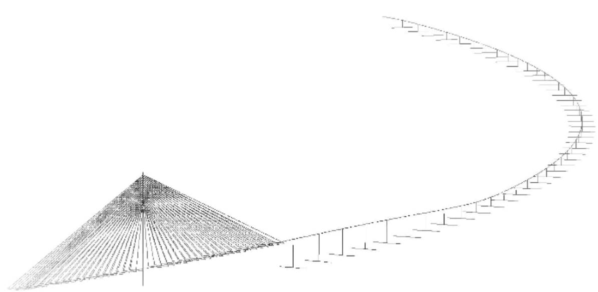


FIGURE 2 : FINITE ELEMENT MODEL OF THE BRIDGE IN ABAQUS

TABLE 1: DRY MODES OF THE BRIDGE

Mode No	*f* (Hz)	*T* (s)	Motion*
1	0.009223	108.43	H
2	0.017115	58.427	H
3	0.030574	32.708	H
4	0.043776	22.843	H
5	0.063739	15.689	H
6	0.075672	13.215	H
7	0.092569	10.803	H
8	0.10827	9.2359	H
9	0.12236	8.1728	H
10	0.13517	7.3979	H

3.2 Hydrodynamic loads

The wave excitation forces acting on the pontoons of the bridge can be written in terms of a cross-spectral density matrix, also mentioned in the methodology section. The part of the spectral density matrix for pontoons n and m will read:

$$\mathbf{S}_{p_n p_m}(\omega) = \int_{\alpha} \mathbf{W}_n(\omega,\alpha) S_{\eta_n \eta_m}(\omega,\alpha) \mathbf{W}_m{}^H(\omega,\alpha) d\alpha \tag{16}$$

where $\mathbf{W}_m{}^H(\omega,\theta)$ is the Hermitian transpose of the wave excitation transfer function, which is obtained through potential flow analyses using WADAM software. $S_{\eta_n \eta_m}$ denote the wave elevation cross-spectral density.

When the hydrodynamic radiation effects are considered, the total system mass and damping matrices can be written approximately as:

$$\mathbf{M} = \mathbf{M_s} + \mathbf{M}_h(\omega_t),\ \mathbf{C} = \mathbf{C_s} + \mathbf{C}_h(\omega_t) \tag{17}$$

Disregarding the frequency-dependency of the added mass and potential damping matrices. ω_t is selected as a predominant response frequency. The frequency-dependent added mass and potential damping matrices were obtained using potential flow theory just together with the hydrodynamic transfer functions and the matrices corresponding to the predominant frequency were used in the analyses.

3.3 Joint probability modeling of the sea-states

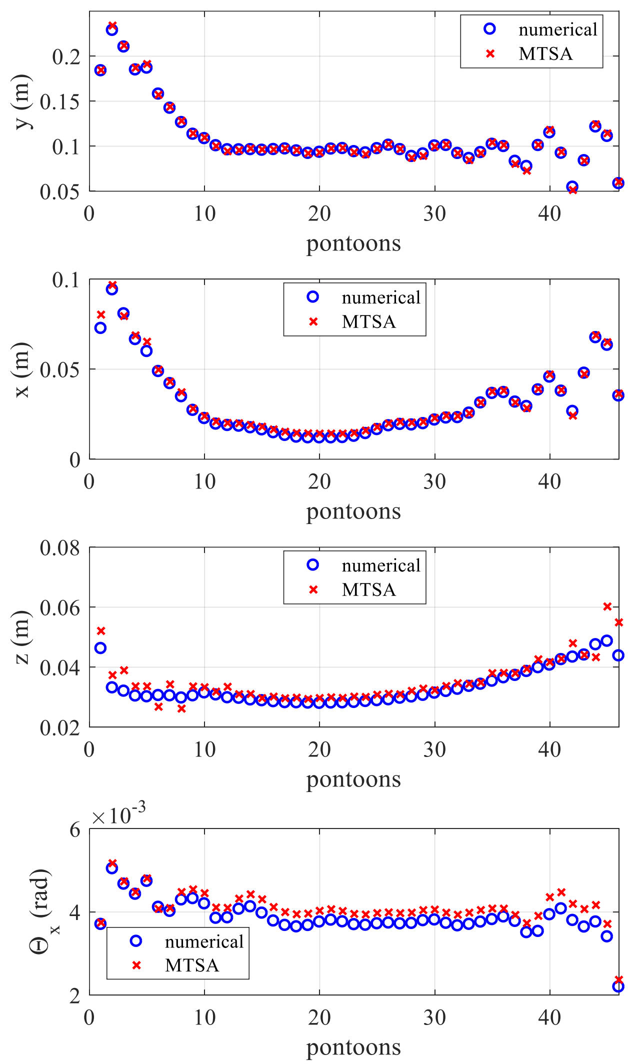
The calculation of the long-term extreme response of the bridge using Eqn.(2) requires the joint probability distribution of environmental variables that show randomness in the long-term. Investigations of the wave conditions at the site were carried out by the Norwegian Public Roads Administration using both numerical simulations and site measurements. The JONSWAP spectrum is used to model the one-dimensional wave spectrum, which relies on two well-known parameters: significant wave height (H_s) and peak period (T_p). Using the data available, Cheng et al. [12] proposed joint probability modeling of the parameters using a marginal PDF of H_s and a conditional PDF of T_p that depends on the H_s. A short-term stationary interval of 1 hour was considered to obtain the statistical properties. The probabilistic model is summarized in Table 2.

TABLE 2: JOINT PROBABILITY DISTRIBUTION OF WAVE PARAMETERS IN THE LONG-TERM

PDF	Type	parameters
$f_{H_s}(h)$	Weibull	$\alpha = 1.256$ (*shape*) $\beta = 0.261$ (*scale*)
$f_{T_p\|H_s}(t\|h)$	Lognormal	$\mu = -6.727(h)^{-0.06} + 8.088$ $\sigma = 0.002(h)^{-1.397} + 0.002$

4. RESULTS

Stochastic dynamic response of the bridge was calculated through the methods described in section 2. The system matrices and the spectral wave load matrix were assembled using the finite element model of the bridge. The frequency dependent added mass and damping matrices were taken as constant using their values at a predominant frequency of 0.93 rad/s. Analyses were repeated using the spectral density approach and the multiple spectral timescale analysis, where the latter was significantly faster. The analyses were then repeated for different sea states by altering the parameters Hs and Tp. Dynamic displacement responses of the pontoons are given for two different sea states in FiguresFigure 3 &Figure 4. The plots indicate good agreement between the responses obtained through the classical spectral density approach and the approximation by MTSA.



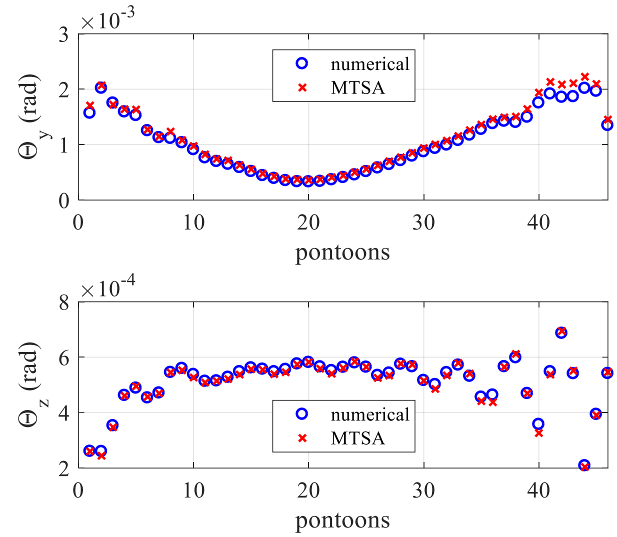
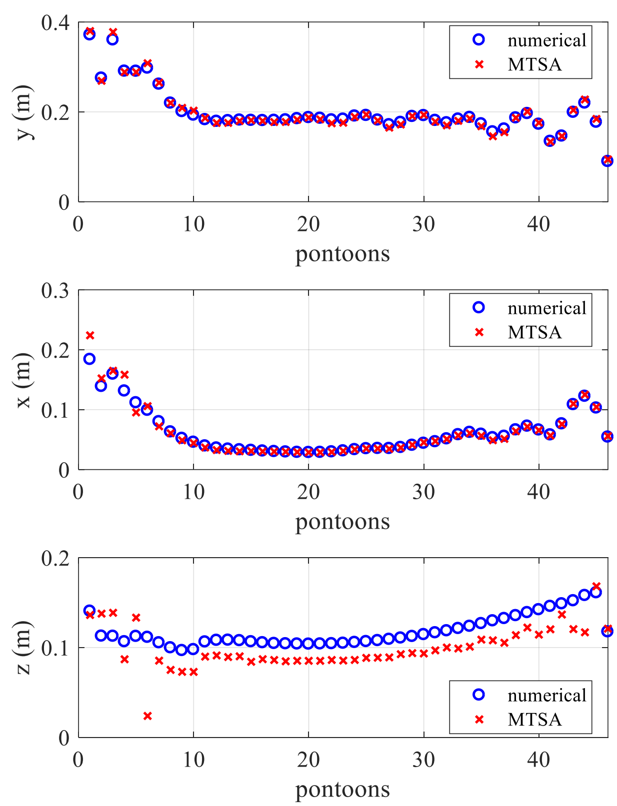


FIGURE 3 : RMS DISPLACEMENT OF BRIDGE PONTOONS IN GLOBAL COORDINATES (SEA STATE: H_S= 2 M, T_S = 4.5 S)



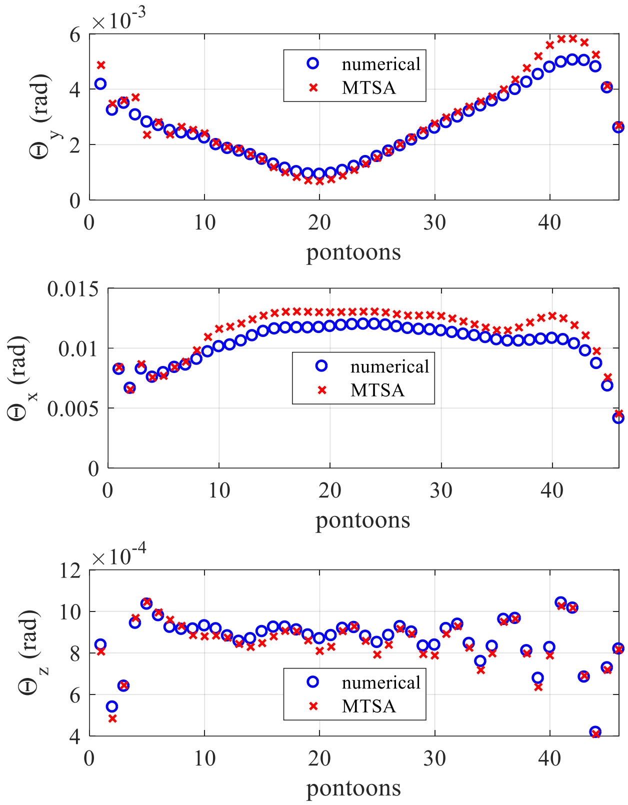


FIGURE 4 : RMS DISPLACEMENT OF BRIDGE PONTOONS IN GLOBAL COORDINATES (SEA STATE: Hs= 2.4 M, Ts = 5.9 S)

It can be observed from the comparison plots that although in general a good agreement is achieved between the two responses, discrepancies are also present, which manifest itself in particular response components and sea states. The accuracy of the method depends on the response spectra, therefore, will be affected by the frequency content of the response and the loading. To illustrate the overall performance, the relative discrepancy between the responses obtained by the two methods for the range of considered sea states is presented in Figure 5. For the sake of brevity, the results are presented for the sway response (x) of the middle pontoon (pontoon 23 in Figs. Figure 3Figure 4). It is observed that the error is sensitive to changes in the peak period but not in the significant wave height. For the most important range of the sea states the discrepancy was below 10%.

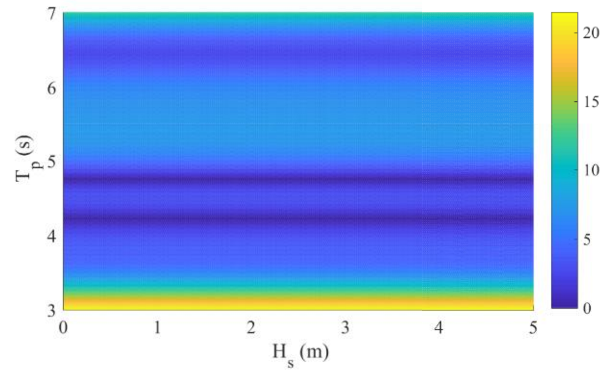


FIGURE 5 : RELATIVE DISCREPANCY (%) IN THE RMS RESPONSE (DISPLACEMENT X IN PONTOON 23) FOR DIFFERENT SEA STATES

Then, for the considered displacement response in the middle pontoon, a long-term extreme response analysis was carried out. The full log-term method that was described in section 2 was used along with the joint environmental described in section 3.3. The integral in Eqn.(2) was evaluated numerically. Same numerical integration scheme was used for both methods, where the only difference was the routines for the calculation of the short-term responses. The results for the long-term periods of 1, 50 and 100 years are presented in Table 1. A reasonable agreement is observed between the two methods. It is also important to assess which sea states are the most important contributors to the integral in Eqn.(2) since the accuracy of the final estimate hinges greatly on the accuracy of the short-term analysis given those environmental conditions. The relative contributions (normalized so that the largest value is unity) to the long-term integral is presented in Figure 5 accordingly. The two methods exhibit very similar behavior, and the contribution is concentrated at the sea states highlighted in the plots.

TABLE 3: COMPARISON OF THE LONG-TERM EXTREME RESPONSE ESTIMATES

Long-term	numerical	MTSA
1 year	0.062 m	0.059 m
50 years	0.095 m	0.089 m
100 years	0.101 m	0.095 m

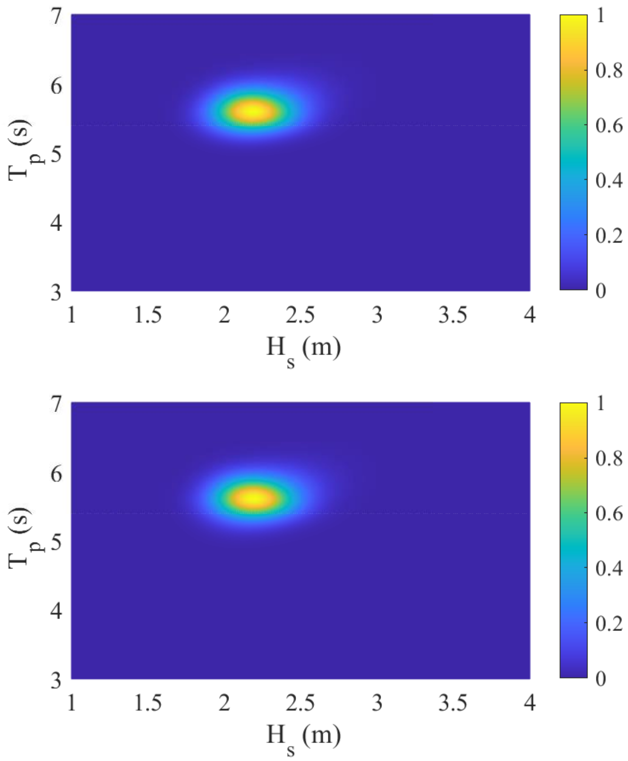


FIGURE 6 : RELATIVE CONTRIBUTION OF EACH SEA STATE TO THE LONG-TERM INTEGRAL (TOP: NUMERICAL, BOTTOM: MTSA)

5. DISCUSSION

When estimating the long-term responses of floating structures under random wave loading, many simulations must be carried out due to the uncertain environmental conditions. This is a rather tedious and time-consuming process, even in the frequency domain, as the response spectra must be numerically integrated using a fine frequency grid. Therefore, replacing the response calculations by analytical closed-form solutions is very attractive given that the accuracy is preserved. Here, in a preliminary study, we showed that reasonable estimations of the long-term extreme responses can be obtained using the much less time-consuming multiple timescale spectral analysis. The application gave promising results under rather crude assumptions that could further be improved, such as neglecting the loading component. In continuation of this work, the effect of frequency dependency of hydrodynamic matrices will certainly be accounted for. Although the results show great potential for further improving the accuracy, there are also inherent limitations of the approximation resulting from the fundamental assumptions that the theory is based on. It should be noted that in such cases (sea states) where the assumptions are not quite satisfied, the calculations could still be carried out using numerical integration. Given the large amount of calculations, even if the short-term response calculations are partially replaced by MTSA, the total computation time will still be reduced significantly, without loss of accuracy.

6. CONCLUDING REMARKS

In this paper, the possibility of estimating long-term extreme responses of floating bridges by the use of multiple timescale spectral analysis is investigated through a case study. The selected case study was a super-long floating pontoon bridge that is planned to be built in Norway. Using a probabilistic model of the wave conditions at the site based on site investigations, long-term extreme response of the bridge was estimated using the full long-term method. The analyses were repeated using the classical spectral density method and MTSA and the results are compared. It was shown that very reasonable estimates were reached using MTSA with a significant reduction of the computation time. The results, although preliminary in nature, encourage pursuing the methodology further.

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