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Transforming dis-embodied mathematical representations into embodied representations, and vice versa: a two-way mechanism for understanding mathematics

Omid Khatin-Zadeh[1], Danyal Farsani[2]* and Babak Yazdani-Fazlabadi[3]

Abstract: Since formal mathematics is discussed in terms of abstract symbols, many students face difficulties to acquire a clear understanding of mathematical concepts and ideas. Transforming abstract or dis-embodied representations of mathematical concepts and ideas into embodied representations is a strategy to make mathematics more tangible and understandable. This representational transformation allows us to actively employ our sensorimotor resources in the process of mathematical understanding. However, a one-way transformation of dis-embodied mathematical representations into embodied representations cannot always be the best strategy to understand mathematics. In order to omit or suppress contextually irrelevant information, we may need to transform an embodied representation into the dis-embodied representation. In other words, a dynamic mechanism in which the individual shifts between the two types of representational transformation could be the best way to acquire a better understanding of mathematical concepts/ideas and solve mathematical problems.

Subjects: Nonverbal Communication; Visual Communication; Adult Education and Lifelong Learning

Keywords: abstract concepts; dis-embodied representation; embodied representation; mathematical concepts; representational transformation

1. Introduction

Embodying abstract concepts in terms of concrete ones has been the subject of a large body of empirical and theoretical works. Among various types of abstract concepts that have been examined in said studies, mathematical concepts occupy a unique place (for a review, see Abrahamson et al., 2020). Embodying and understanding abstract mathematical concepts in terms of concrete concepts or representations is especially interesting because such an inherently metaphorical process is a strategy to acquire an easier understanding of mathematics. When a mathematical idea is expressed in terms of abstract symbols, it may be difficult for us to grasp as we may have no idea of what these symbols refer to. But, when the same idea is expressed in terms of concretely-perceivable objects, it may be much easier for us to capture it and as such, this can be a way for making abstract mathematics visible and tangible. Here, an abstract mathematical idea is embodied as a concrete representation. This process is a transformation of a dis-embodied representation of an idea into an embodied representation of that idea. The two representations of the idea are not necessarily different. They are the same but are expressed in two forms: one form is more abstract or dis-embodied; and the other one is more concrete and embodied. In this paper, we intend to discuss the functions of dis-embodied and embodied representations of concepts and how a shift between dis-embodied and embodied representations of concepts can help us deepen our understanding of concepts.

2. Dis-embodied vs. embodied representations of mathematical concepts

Many mathematical concepts such as numbers, arithmetic operations, function, limit, and continuity are formally discussed in terms of abstract symbols in mathematics textbooks. However, they are understood in terms of strongly embodied representations at a conceptual level. For example, three types of observation support the idea that numbers are conceptualized as strongly embodied representations: numerical distance effect, size effect, and spatial-numerical association (Fischer & Shaki, 2018). Numerical distance effect refers to the ability of individuals to make a quicker and more correct comparison between the magnitudes of two numbers when the distance between them is larger. For example, magnitude comparison between 7 and 10 is easier than magnitude comparison between 7 and 8. This effect has been supported by magnitude comparison tasks (e.g., Moyer & Landauer, 1967), same-different judgments (e.g., Van Opstal & Verguts, 2011), and priming tasks (e.g., Sasanguie et al., 2011). Size effect refers to the ability of individuals to make faster and more correct magnitude comparisons between two numbers when those numbers are smaller. This is the case when the numerical distance is the same for two pairs of numbers. For example, magnitude comparison between 3 and 5 is easier than magnitude comparison between 8 and 9. This effect has been supported by some empirical evidence (e.g., Buckley & Gillman, 1974; Gallistel & Gelman, 1992; Lyons & Ansari, 2015). Spatial numerical association refers to the tendency to associate smaller numbers with left space and larger ones with right space (Daar & Pratt, 2008). There is also some evidence that suggests abstract arithmetic operations are conceptualized as embodied representations. Masson and Pesenti (2014) found that processing addition and subtraction problems induce attentional shift to right space and left space, respectively (see, also Pinhas & Fischer, 2008). This suggests that addition and subtraction are embodied as rightward and leftward movements, respectively. Function, limit, and continuity are also abstract mathematical concepts that are understood in terms of motions (Khatin-Zadeh et al., 2021; Lakoff & Núñez, 2000; Marghetis & Núñez, 2013), which are spatial and motoric representations of concepts. This metaphorical understanding may allow us to actively employ sensorimotor systems to understand these concepts (Khatin-Zadeh, 2021; Khatin-Zadeh et al., 2021).

3. Functions of dis-embodied and embodied representations of mathematical concepts

As previously mentioned, any given mathematical concept or idea may have one dis-embodied representation and one (or more than one) embodied representation (Rosa & Farsani, 2021; Farsani et al., 2022). Each representation has a specific function. In the early stages of learning a concept and when a student is faced with the dis-embodied representation of the concept in mathematics textbooks, it is usually difficult to acquire a full and deep understanding of that concept. At this stage, transforming the dis-embodied representation into an embodied representation could be really helpful and facilitating in the process of knowledge acquisition. This transformation of representation helps the student utilize extra cognitive resources in the process of understanding the concept (Khatin-Zadeh, 2021). In this process, the cognitive resources involved in the understanding of the embodied representation of the concept could be actively employed to understand dis-embodied representation. This inherently metaphorical mechanism allows us to use the embodied representation as a metaphorical tool. Here, the embodied representation functions as the base of the metaphor and the dis-embodied representation functions as the target of the metaphor. Based on the strong version of embodiment, sensorimotor networks that are activated/recruited during the processing of the embodied representation of the mathematical concept are activated/recruited during the processing of the dis-embodied representation of that concept when the metaphor is understood. Transforming the dis-embodied representation into the embodied representation can be done via concrete examples of a mathematical concept or idea. These examples can tangibly show how an abstract mathematical concept or idea is concretely realized.

While transforming the dis-embodied representation into an embodied representation can help students acquire a better understanding of a concept in an early stage of learning a concept, transforming the embodied representation into the dis-embodied representation is helpful in more advanced levels of learning. Transforming the embodied representation into dis-embodied representation is a way to discover general rules or general patterns among concrete representations. Since the dis-embodied representation of a mathematical concept is expressed in terms of abstract symbols, the process of transforming the embodied representation into the dis-embodied representation has been called "symbolic schematization" (Khatin-Zadeh, 2021). Symbolic schematization enables us to discover the general features of a concept and make knowledge of that concept explicit. This is a very high level of understanding a concept. At this level, the key features of the concept are expressed in terms of abstract symbols, while the rest of features, which may be contextually irrelevant or non-useful, are omitted from thinking processes. Although transforming the embodied representation into the dis-embodied representation makes the process of understanding more difficult, it helps the individuals to focus their attention on the most relevant parts of the information. This is done by representing the most relevant parts of information in terms of abstract symbols and omitting the rest of information.

Since each one of these transformations has a specific function, the individual may shift between them when s/he deals with a mathematical problem. In one specific part of a problem, the individual may need to transform the dis-embodied representation into an embodied representation. Depending on the nature of the problem and her/his background knowledge, the individual may need to shift between these two types of transformation to acquire a better understanding of the problem and find a solution for the problem. This is a highly dynamic process. One type of transformation helps the individual to employ a wider range of sensorimotor resources to make the problem more tangible. Another type of transformation helps her/him to omit unnecessary information and focus on a specific part of information. In each type of transformation, a certain aspect of the problem can be dealt with. Depending on the problem and previous knowledge of the individual, optimal dynamic shifts between the two types of representational transformations are the best way to understand a problem and find a solution for it.

4. Conclusion

Although dis-embodied-to-embodied and embodied-to-dis-embodied transformations of mathematical representations are directionally opposite, they could have complementary functions. While the first type of representational transformation helps an individual utilize a wider range of sensorimotor resources to acquire a tangible understanding of a concept or an idea, the second type of representational transformation helps her/him omit irrelevant contextual information and focus on the most relevant parts of information during thinking processes. A dynamic shift between these two opposite types of representational transformation allows us to deal with various aspects of a problem or an idea. Therefore, the use of a one-way representational transformation cannot always be the best way for understanding a mathematical idea or solving a mathematical problem. What is important is to employ each representational transformation in its appropriate place to deal with an appropriate part of a mathematical idea or problem.

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