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Sequential production of two oil fields with an option to switch

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ABSTRACT

Re-using existing infrastructure has become a standard approach for the development of marginal offshore oil fields. Many small discoveries are, however, located in remote areas or require unreasonably high costs to be tied back to already installed production facilities. Feasible solutions must be found to develop such prospects in a cost-efficient way. In this paper, we analyze whether sequential development of two stand-alone small fields (A and B) using the same production facilities can be an economical strategy compared to the investment in parallel development. We apply and compare two different approaches to evaluate such a sequential strategy. The first considers that the decision maker maximizes the recovery factor of Field A first and then invests in Field B disregarding the information about uncertain factors (“myopic” approach). The second approach allows accounting for the decision maker’s ability to optimize the development strategy given it can learn about uncertainties over time and initiate switching between two fields when it is optimal, or leave Field B undeveloped (“options” approach). We account for several sources of uncertainty affecting the project value: Fields A and B reservoir uncertainty that is replicated by a benchmark reservoir model, oil price, operational expenditure (OPEX), and capital expenditure (CAPEX) to switch between two fields. Our findings are threefold: (1) sequential investment can be the preferred development concept for small stand-alone discoveries; (2) the sequential development strategy allows the downside risks of the investment to be partly hedged, including the reservoir, oil price, and cost risks; (3) the “options” approach is needed to capture the additional monetary value of such a strategy and is considered to be a superior method to assess the value of the sequential production as opposed to a “myopic” approach.

1. Introduction

The average size of new hydrocarbon discoveries in mature production areas such as the Norwegian continental shelf (NCS) has been steadily decreasing in the last few decades (NPD, 2019). Developing smaller fields is associated not only with lower profits, but also with more risks (Dias, 2004). Field development decisions have to be made facing relatively more subsurface uncertainty because of the high cost of drilling appraisal wells compared to the expected revenues of small prospects.

A standard way to develop a small offshore field is to connect it to existing infrastructure (NPD, 2020). This can significantly reduce capital expenditures needed to develop a satellite field and increase revenues of already installed infrastructure due to tariffs paid to the facility owner. However, in some cases it is not possible to use a tie-back solution. This is the case if a discovery is located too far from already installed production facilities. Other reasons could be some capacity constraints and technological challenges that make the tie-back too costly (Lei et al., 2021).

In this paper, we develop a method to evaluate the opportunity to use a single production unit to develop several small offshore fields sequentially. We analyze sequential development as a cost-efficient solution allowing exploration and production (E&P) companies to manage uncertainty in a small field development process and capture the value of flexibility. We analyze a project case where an oil company holds two independent licenses, A and B, that both represent small oil fields located relatively far away from the existing production infrastructure. If implemented, the development solution must provide a basis for the stand-alone production of each discovery. The idea is that a movable platform or a floating production and operation unit is used to develop the two fields sequentially.

We first analyze whether sequential production is more beneficial than the parallel development of Fields A and B using two production units. For the sequential investment, two different approaches can be used to estimate the project value under uncertainty. The first one implies that the decision maker disregards the opportunity to learn

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about the development of such factors as the oil price, expected production rate of Field B and CAPEX to switch between two fields during the production phase of Field A and maximizes the recovery factor of Field A first, independent of the potential of Field B. Once Field A is depleted (or starts generating a negative cash flow), the field is decommissioned and facilities are moved to develop Field B. We refer to this approach as ''myopic''. The second approach allows optimizing the switching decision due to learning about the uncertain factors and, if optimal, switch from Field A to Field B at a different time than suggested by the ''myopic'' approach. We refer to the second approach for the valuation as the ''options'' approach as it is based on real options analysis. Overall, four types of uncertainty are considered to have an impact on the switching decision: oil price, production level, OPEX and CAPEX to switch. Although the same factors are used as input for the valuation under the ''myopic'' approach, the main difference between the two approaches is that within the ''options'' approach the valuation procedure accounts for the decision maker's ability to change the course of the project based on the updated knowledge. The switching decision can be triggered by, for example, a low level of drilling CAPEX that is needed to start developing Field B. If this CAPEX is expected to increase, the optimal strategy might be to switch as soon as possible. In case information reveals an unfavorable outcome, i.e. low expected recoverable reserves of Field B, the operator can also decide to leave Field B undeveloped. The main problem that we consider in this case is the project valuation using the switching decision optimization based on updated information about the uncertain factors.

In order to allow for a fair comparison of the sequential and parallel development strategies, using both the ''myopic'' and ''options'' approaches, we use the same inputs to simulate project cash flows. We use the production optimization based an oriented workflow to address the reservoir uncertainty and generate 50 probable realizations of the oil production profiles of each of the two fields. The underlying reservoir model represents the benchmark case, Olympus, introduced by Fonseca et al. (2018). The production optimization based on the Olympus case also yields realizations of the water injection and water production rates that impact the level of OPEX. In order to replicate the development of future oil price, we use a two-factor stochastic process presented in Schwartz and Smith (2000). The CAPEX components are modeled as geometric Brownian motion (GBM) processes correlated with the long-term oil price parameter, which allows the avoidance of bias in the valuation procedure. For the valuation of the parallel development and sequential development using the ''myopic'' approach, a rather simple discounted cash flow (DCF) analysis based on Monte Carlo simulation is used. For the ''options'' approach, we apply the least-squares Monte Carlo approach (Longstaff and Schwartz, 2001) to evaluate the project based on switching time optimization under uncertainty.

As Cortazar et al. (2021) point out, the solution to the switching problem is twofold: ''the maximized expected profit given by the flexible operation, and the underlying decision policy, which is defined in terms of a set of switching boundaries triggering regime transitions''. Applying the ''options'' approach allows us to provide insight on both considerations making our results relevant for real-world problem settings. Our results show that for a specific case study, the sequential production is a preferred option compared to the parallel production. We also demonstrate that the ''options'' approach is an important tool to capture the value of flexibility when the sequential production strategy is considered. By accounting for the opportunity to tune the switching strategy depending on the information revelation, the decision maker can significantly increase the expected project value.

In this paper, we contribute to the literature focusing on petroleum investment under uncertainty and applications of the real options approach (ROA) focusing on switching options in oil and gas field development. The switching option is one of the classic types of real options that finds various applications in natural resource production problems. A number of contributions study end of the lifetime decisions for fields and focus on the oil-to-gas production switching option. Hahn and Dyer (2008) evaluate a switching option from only oil production to combined oil and gas production in the North Slope of Alaska. Hahn and Dyer (2008) use two correlated one-factor mean reverting processes to model oil and gas prices to determine the optimal course of action considering the switching decision. They derive the value of the asset with flexibility using a binomial lattice method and deterministic production model. Hem et al. (2011) and Thomas and Bratvold (2015) also evaluate the switching from oil production to gas production in a depleting field, assuming that oil and gas prices follow two-factor stochastic processes (Schwartz and Smith, 2000). Hem et al. (2011) use a zero-dimensional reservoir model to replicate the production uncertainty, whereas Thomas and Bratvold (2015) use deterministic input from a material balance simulator. Both studies employ the least-squares Monte Carlo (LSM) method to optimize the switching timing and obtain the option values. Hem et al. (2011) and Thomas and Bratvold (2015) also compare results based on ROA and simplistic DCF analysis where the switching decision is made ignoring the opportunity to learn about the uncertainty in the future. Hong et al. (2019) use the LSM algorithm to analyze the optimal switch time from primary recovery method to the improved oil recovery. The geological uncertainty, described by a two-factor production model, is considered to be the only source of uncertainty for the switching decision. Hong et al. (2019) compare the project value under the state-of-the-art reservoir management approach and the proposed LSM method considering the impact of future information on switching time.

We also contribute to the literature that presents work on the value of flexibility in oil field development. Contributions in this field include Dias (2004), Suslick and Schiozer (2004), Lin et al. (2013), Santos et al. (2018, 2021), among others. In this paper, we analyze the economic effect of a field development strategy that can be highly relevant for stand-alone small discoveries. To the best of our knowledge, the sequential production strategy has not been evaluated before. We also contribute to previous research methodologically by accounting for four types of uncertainty that affect the optimal decision: geological, oil price, operational and capital expenditure. Moreover, compared to the existing contributions, we provide a much more practice-oriented and realistic modeling approach to reservoir uncertainty by means of the production optimization based on the Olympus case. This makes our method more relevant for real-life problem settings. We demonstrate how to utilize the results of production optimization in the real options valuation procedure and decision-making process. Consequently, we extend the above-mentioned contributions, which mostly use simplistic approaches for the reservoir uncertainty modeling and account only for at most two types of uncertainty. Our analysis shows that the LSM approach is able to handle such an amount of uncertain factors within the valuation procedure. At the same time, the results that the LSM approach provides, are still intuitive enough to be used for the decision support. Despite the complexity of the problem, we obtain a tractable decision rule for making a switch between two licenses. We also contribute to the literature by modeling the CAPEX and the oil price as two correlated stochastic processes. Cardenas et al. (2018) argue that the effect of the correlation between oil price and capital costs on the project valuation is not thoroughly studied in the literature.

The remainder of this paper is organized as follows. In Section 2, we describe the investment problem that an oil company faces when considering a sequential investment in the development of two hydrocarbon fields. In Section 3, we formulate and develop the modeling approach for the valuation of the project under both parallel and sequential development. For the ''options'' approach we introduce the optimization algorithm to evaluate the option to switch between two licenses. In Section 4, we introduce a case study, providing a detailed description of the Olympus reservoir case and assumptions regarding the oil price and cost uncertainty modeling. Section 5 presents the results and sensitivity analysis. Section 6 concludes the paper.

2. Problem statement

An oil company is evaluating whether to invest in the development of two oil field licenses offshore Norway. We assume that the two fields are identical in terms of the reservoir characteristics. Therefore, we disregard the assessment of which field should be developed first to maximize the overall value. This assumption can be straightforwardly relaxed by adding a different underlying reservoir model that can represent the range of reservoir uncertainty in Field B. This would also require to choose the optimal development sequence, identifying which field should be developed first. However, in most of the cases, it will be optimal to develop a field with higher expected initial oil in place.

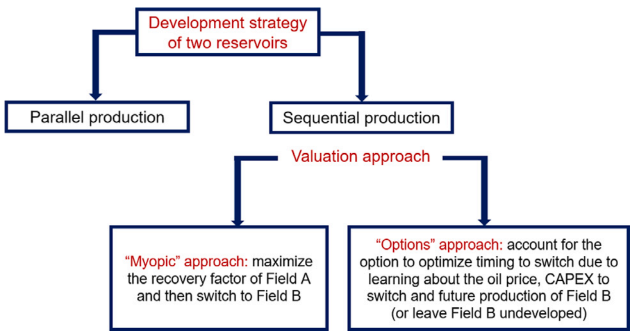


Fig. 1. The field development strategies and valuation choices.

Due to the remoteness from existing infrastructure, only stand alone development can be considered in both cases. A conventional approach using two different production units to develop Fields A and B in parallel might be both costly and risky due to prominent reservoir uncertainty and the small size of the individual discoveries. As illustrated in Fig. 1, we analyze whether a sequential development of the two licenses with a single production facility can improve their revenues and decrease the downside risk of the project sufficiently to make the development economically viable. For the sequential development, a flexible production and operation unit that can be moved from the Field A to the Field B location must be considered. This strategy significantly reduces the risk of occurrence of stranded assets if both or one of the fields' reservoir performance proves to be much lower than expected, making the production unit unemployed. Therewith, apart from saving capital costs, the sequential strategy can contribute to the implementation of the policies aiming to reduce the environmental impact of petroleum production.[1] The central question in terms of the sequential development strategy is determining the time when Field A should be abandoned and the production unit moved to develop Field B. Having the flexibility to leave Field B undeveloped can be also highly valuable if additional information turns out to eventually render the investment uneconomical.

The first approach that can be used is to maximize the recovery factor of the Field A and decommission it once it has depleted enough to be economically non-viable to be developed further. We refer to this strategy as the "myopic" approach. Fig. 2 illustrates the project timeline under the "myopic" approach.

The second approach implies that the oil company maximizes the value of the overall project rather than maximizing only the revenue generated by Field A. We refer to this as the "options" approach. Intuitively, the optimal decision in some scenarios might be to switch to Field B production at a different time than suggested by the first

[1] This includes the UN sustainability goals, the European Green Deal, and Norwegian climate action plan in particular. Delaying Field B production can also allow using novel solutions for carbon capture and implement production facility electrification using offshore wind to decrease the carbon footprint from operations.

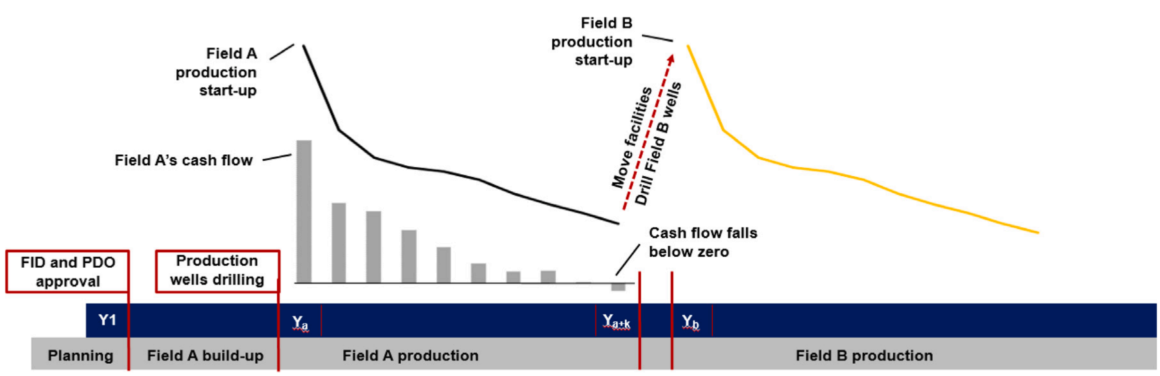


Fig. 2. Project timeline under the "myopic" approach.

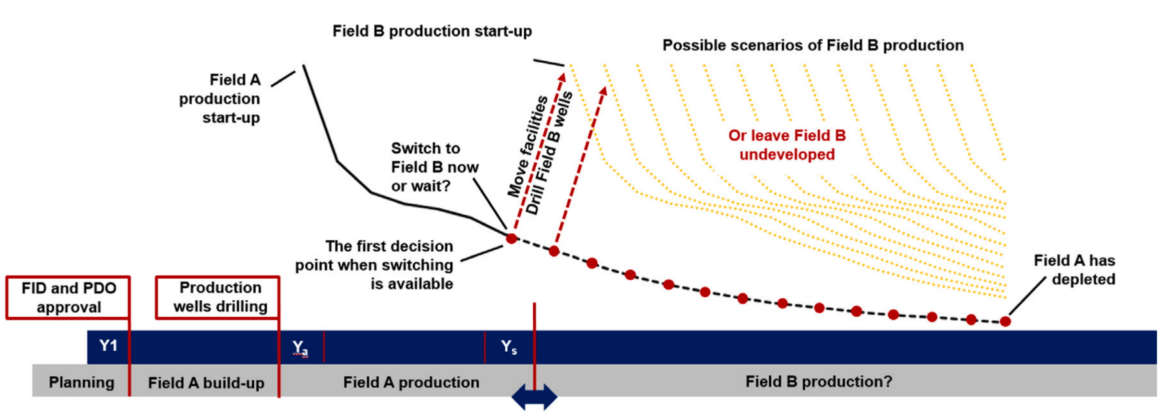


Fig. 3. Project timeline under the "options" approach.

approach. This decision can be driven by the state of several factors, including the oil price, the expected remaining reserves of Field A, expected production of Field B, the level of OPEX of Field A and CAPEX needed to switch from Field A to B. The possibility to learn about these factors over time can also allow assessing whether the investment in Field B is profitable. Fig. 3 illustrates that starting from year Y_s, the earliest point when the regulator can approve the abandonment of Field A upon reaching a certain recovery factor, the field operator decides whether it is optimal to "exercise" the option to switch from Field A to Field B or to continue developing Field A and reevaluate the decision at the next decision point. If the expected values of both decisions are negative; i.e., prolonging the production of Field A and switching to Field B, the field operator can decommission Field A and leave Field B undeveloped. Our goal is to identify whether accounting for the switching flexibility can affect the results of the project valuation based on the "myopic" approach and impact the choice between the "parallel" and "sequential" development strategies.

We use the Olympus benchmark case as an underlying reservoir model for both fields to capture the effect of production uncertainty on the decision-making process and project valuation. Both fields are considered to be small discoveries with estimated recoverable reserves ranging from 51.6 to 107.5 mmbbl throughout 50 equally probable realizations of the reservoir uncertainty (mean of 82.6 mmbbl).

3. Methodology

This section describes the key components of the proposed valuation of the investment in two oil fields. The valuation is based on a simulation approach. Fig. 4 illustrates the main building blocks and states the output produced by each block as well as the information flow between individual blocks.

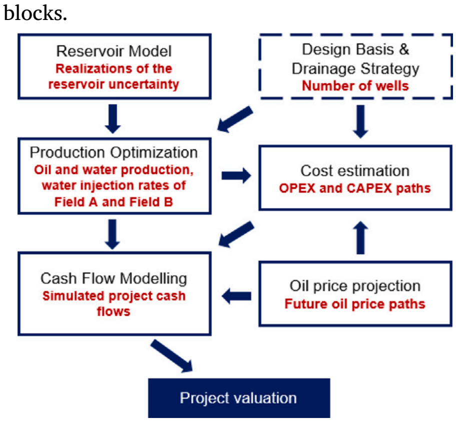


Fig. 4. The valuation procedure (uncertainty modeling blocks, output produced by each block and information flow between blocks).

In this paper, we use an ensemble of 50 realizations of the Olympus benchmark reservoir model[2] to estimate the possible outcomes of the hydrocarbon production. Based on the benchmark case data and drainage strategy described in Fonseca et al. (2018), we perform the production optimization (see Section 3.1) that yields 50 realizations of oil and water production and water injection rates of Field A that maximize the economic value of the field development under static conditions (fixed oil price and control parameters). In order to make our results less case-dependent and decrease the complexity of the production optimization task, we assume that Field A and Field B are identical in terms of reservoir properties. That means that we use the same 50 realizations to replicate the reservoir uncertainty of Field B.

Then we proceed to the valuation of the parallel and sequential production strategies, where we account for several additional uncertain factors that can affect the decision making process. Section 3.2 describes the oil price simulation process. In the case of the parallel development, the oil price level affects only the abandonment time of Fields A and B. In the sequential development, the decision maker would optimize the switching timing between the two fields (within the "options" approach) taking into account the oil price level, CAPEX and information regarding expected production of Field B. The simulated production profiles and oil prices also impact the level of OPEX and CAPEX. The estimation process is described in Section 3.3. Combining the output of the modeling blocks, we simulate the project cash flows and perform the project valuation under parallel and sequential production. For the latter case, cash flows for both the "myopic" and "options" approaches are simulated. Within the "options" approach, these cash flows are used for the switching time optimization (see Section 3.4) and valuation of the project with flexibility.

3.1. Production optimization

One of the main factors driving risk in small field development is the reservoir uncertainty. In this paper we use the Olympus benchmark case presented in Fonseca et al. (2018) as an underlying reservoir model in order to build the oil production forecast for Field A and Field B. Clearly, considering two different underlying reservoir models in order to relax the assumption regarding the fact that Fields A and B are identical, would increase the production optimization task. The performance and run time of the economic valuation model presented in Section 3.4, however, is not expected to be affected by that.

The uncertainty about the reservoir properties in the Olympus case allows us to account for reservoir uncertainty when evaluating field development strategies. Probabilistic approaches have been widely used in the literature in recent years in order to capture the full uncertainty range by generating a set of geological realizations (Correia et al., 2015; Santos et al., 2018; Mahjour et al., 2019, 2022). In addition to the oil production profiles, we derive good estimates for the level of water production and water injection, which directly impact the level of OPEX. In this section we discuss the proposed production optimization algorithm. More details on the Olympus case and specific parameter values used for the optimization procedure are provided in Section 4.1, where we introduce the case study, which is used to illustrate the application of the proposed valuation methodology.

In order to estimate the production parameters, we use well control optimization, optimizing the bottom hole pressure (BHP) of the producer and injector wells when implementing water flooding (WF). WF is chosen as a default recovery strategy by Fonseca et al. (2018) who originally introduced the Olympus challenge. The well control optimization is considered as one of the most attractive approaches for increasing the recovery factors and the associated profits while developing both green and mature reservoirs (Mohaghegh et al., 2012; Amar et al., 2018; Amar and Zeraibi, 2020). However, with a high number of wells and in the presence of various kinds of uncertainties, such as the geological data, the degree of complexity in the task of identifying suitable BHP values increases significantly. Therefore, an oriented workflow was considered in this study for optimizing the BHP values of the producer and injector wells.

WF is a rigorous technique for improving oil recovery, which demands relatively low costs to implement, but can significantly improve and extend oil displacement efficiency. However, the above-mentioned benefits of WF are dependent on an adequate design and wise choice of its main control parameters. There are several recent contributions that proposed relevant workflows for optimizing the control parameters of WF by combining some numerical, statistical, and optimization approaches. Among them is Mohaghegh (2011), who establishes an

[2] Benchmark reservoir models are developed by the reservoir optimization community to test methods for field development optimization under geological uncertainty. Using the benchmark cases allows the standardization of the set of models, problem definition and objective function calculation. This makes the comparison of results proposed by different researchers fair. It also provides consensus on best practices for field development optimization.

artificial intelligence based proxy for investigating suitable wells for relaxing their rates in a WF case after performing an initial number of simulations using a commercial software. Alenezi and Mohaghegh (2017) developed a smart proxy model for mimicking the outputs of a commercial simulator during WF optimization of the SACROC case based on series of artificial neural networks (ANN). Artun (2017) demonstrated the higher performance of ANN compared with Capacitance Resistance Model (CRM) for modeling interwell connectivity in WF cases. Hourfar et al. (2019) introduced an reinforcement learning approach to optimize a WF project. Their workflow was applied on the Egg-model. Ng et al. (2021) proposed a hybrid model combining an ANN and two nature-inspired algorithms, particle swarm optimization and grey wolf optimization, for optimizing the control parameters of producer and injector wells during the WF phase. Xue et al. (2022) propose a divide-and-conquer optimization paradigm to decompose a large scale WF production optimization problem into a number of simpler data-driven surrogates. Wang et al. (2022) propose a novel self-adaptive multi-fidelity surrogate-assisted multi-objective production optimization algorithm to reduce the computational burden and enhance the accuracy of the surrogate model.

The BHP values of the wells are chosen with the objective to maximize the net present value (NPV) and ensure safe and efficient delivery of hydrocarbons from the well to the collection facilities (Leporini et al., 2019). Possible problems related to flow assurance occurring during the transportation (such as liquid loading, wax deposition, sand deposition, etc.) (Dall'Acqua et al., 2017) or across particular equipment (such as multiphase valves) (Giacchetta et al., 2014) are addressed. The NPV is defined as follows (Bellout, 2014):

$$NPV = \sum_{k=1}^{N_s} \left(\sum_{j=1}^{N_p} P_o q_o^{j,k} \Delta t_k - \sum_{j=1}^{N_p} C_{wp} q_{wp}^{j,k} \Delta t_k - \sum_{j=1}^{N_i} C_{wi} q_{wi}^{j,k} \Delta t_k \right) / (1+r)^t. \quad (1)$$

In the above-equation, N_s is the number of time-steps Δt , N_p and N_i denote the number of producer and injector wells, respectively. $q_o^{j,k}$ and $q_{wp}^{j,k}$ denote the flow rates of produced oil and water from producers during time step t, respectively. $q_{wi}^{j,k}$ represents the rate of the injected water during time step Δt. P_o, C_{wp}, and C_{wi} stand for the oil price, the cost of water produced and injected, respectively, r is the annual discount rate, and t is the total number of years. The simulation runs were performed using the commercial simulator Eclipse. In order to reduce the computational time of the realizations, a Matlab code was written for running the cases in Eclipse and extracting their results automatically. Notably, the simulation runs and the calculations were done using an Intel® Xeon® Gold 6248 2.50 GHz and 64 Gb of RAM, and an Intel® CoreTM i7-7700HQ 2.80 GHz and 16 Gb of RAM. The main steps of the workflow applied for each realization of the reservoir model are summarized as follows:

- Step 1: Generate a predefined number of runs. In this step, the Latin Hypercube Design was applied for generating the BHP values of "n" runs for each realization of the reservoir uncertainty. This step aimed at ensuring a good distribution of the runs around the search space and it allows the identification of the regions susceptible to involve high NPV values. Based on the length of the BHP intervals and the computational time of each realization, it was judged suitable to assume n=60 runs.
- Step 2: Simulate the cases using Eclipse and the written Matlab code.
- Step 3: Sort the runs with respect to their calculated NPV values.
- Step 4: Select the best 5 runs having the highest NPV among the 60 runs.
- Step 5: Apply a grid search technique on the selected 5 runs by dividing the neighbor regions of the decision variables of these runs into sub-intervals and simulate the new cases using Eclipse and the written Matlab code.
- Step 6: Find the best run with the highest NPV value.

The optimization yields oil production, water production and water injection rates that maximize the NPV in each realization. We present the results of the production optimization in Section 4.1. Our primary focus is on the comparison between the project values resulting from parallel and sequential development strategies that are based on the same production profiles. Therefore, issues that do not add much difference between the two strategies, such as well placement optimization, are not considered in this study. However, the well control optimization problem presented in the paper can be extended by considering the optimal well placement. If the well placement is considered as a decision variable in addition to the procedure that we discuss, the number of runs needed to achieve the optimal NPV within the presented well control optimization has to be increased. We refer to several contributions including Silva et al. (2020), Sayyafzadeh and Alrashdi (2020) and Kristoffersen et al. (2021) that perform well placement optimization based on the Olympus case.

3.2. Oil price modeling

We consider that the future oil price evolves over the project time as a two-factor stochastic process as proposed by Schwartz and Smith (2000). Earlier contributions including Jafarizadeh et al. (2012) and Fedorov et al. (2021), demonstrated that this price model presents a good balance between the ability to replicate the range of uncertainty in future oil prices and ease of calibration process. However, different price models such as geometric Brownian motion (GBM) or mean-reversion process can be used to reflect uncertainty in future oil prices. We refer to Al-Harthy (2007), Xu et al. (2012) and Fedorov et al. (2021) who perform a comparison between the above-mentioned price models in petroleum projects valuation.

The Schwartz and Smith (2000)'s model assumes that the commodity price is driven by two stochastic factors: the long-term factor ξ_t, which follows a Brownian motion as given by

$$d\xi_t = \mu_\xi dt + \sigma_\xi dz_\xi, \quad (2)$$

where μ_ξ denotes the drift rate and σ_ξ is the volatility; and the short-term factor χ_t, which is modeled as a mean reverting Ornstein–Uhlenbeck process as given by

$$d\chi_t = -\kappa \chi_t dt + \sigma_\chi dz_\chi, \quad (3)$$

where κ is the mean-reversion coefficient and σ_χ denotes the volatility.

As we simulate the development of project cash flows in discrete time with time steps of one year and apply the risk-neutral valuation technique, we discretize the risk-neutral versions of both price process components as given by

$$\xi_t^* = \xi_{t-1}^* + \mu_\xi^* \Delta t + \sigma_\xi \varepsilon_\xi \sqrt{\Delta t}, \quad (4)$$

$$\chi_t^* = \chi_{t-1}^* e^{-\kappa \Delta t} - (1 - e^{-\kappa \Delta t}) \frac{\lambda_\chi}{\kappa} + \sigma_\chi \varepsilon_\chi \sqrt{\frac{(1 - e^{-2\kappa \Delta t})}{2\kappa}}, \quad (5)$$

where ε_ξ and ε_χ in Eqs. (4) and (5) are standard normal random variables and are correlated in each time period with the correlation coefficient $\rho_{\xi\chi}$.

We use the Kalman filter to calibrate the parameters of Schwartz and Smith (2000)'s price process. We estimate seven coefficients used in Eqs. (4) and (5) based on historical spot and futures prices by maximizing the log-likelihood score. Further details regarding the calibration process using the Kalman filter are discussed by Thomas and Bratvold (2015) and Fedorov et al. (2021). The results of the calibration are presented in Section 4.2.

3.3. Cost uncertainty modeling

The yearly operating costs $OPEX_t$ of both licenses are assumed to consist of a fixed (FO) and several variable parameters that depend on the yearly production rate of the field q_{t_n}, the average annual oil price P_{t_n}, water injection q_{wit_n} and water production q_{wpt_n} rates:

$$OPEX_{t_n} = FO + aP_{t_n} + bq_{t_n} + cq_{wi_{t_n}} + dq_{wp_{t_n}}, \quad (6)$$

where a, b, c and d are the coefficients determining the contribution of each parameter to the total cost.

In this study, we also consider capital expenditure (CAPEX) uncertainty. The amount of CAPEX in our case is mainly dictated by the cost of construction and installation of the production facilities, drilling costs and other associated costs. One of the main factors affecting the switching decision between Fields A and B is the compound CAPEX to switch that consists of the abandonment cost of Field A, modernization of the production unit and the drilling of wells in Field B.

Following the approach in Cardenas et al. (2018), we model each year's individual CAPEX as GBM processes with an identical drift rate μ_θ and volatility σ_θ, described by the differential equation:

$$d\theta_t = \mu_\theta \theta_t dt + \sigma_\theta \theta_t dz_\theta, \tag{7}$$

where θ_t denotes the CAPEX in year t and dz_θ represents the Brownian increment. The discretized version of Eq. (7) can be written as

$$\theta_{t+1} = \theta_t \exp^{[(\mu_\theta - 0.5\sigma_\theta^2)\Delta t + \sigma_\theta \varepsilon_\theta \sqrt{\Delta t}]}. \tag{8}$$

Brandão et al. (2005) and Smith (2005) argue that in order to include cost uncertainty into the risk-neutral valuation procedure in a correct (from the methodological point of view) way, it can be modeled using subjective knowledge, but must be directly correlated with market uncertainty (i.e. the oil price in our case). In the context of how the risks must be treated in the valuation procedure, the cost uncertainty falls somewhere between the notion of private and market risks. Correlating the cost uncertainty with the market parameters allows bias to be avoided where the valuation based on simulation paths with high oil prices and low CAPEX can lead to overestimation of the real option value.

Willigers (2009) provides evidence of a strong correlation between oil prices and oil rig rental rates. His results indicated that the correlation coefficient between these two parameters in the North Sea and the Gulf of Mexico was 0.87 with one year's delay. Using this argument, we consider that all capital expenditure in time step t are correlated with the long-term component of the oil price in time step $t-1$.

We correlate the normal random variables of the oil price and CAPEX stochastic processes as described by Wiersema (2008) and applied by Cardenas et al. (2018) in the following way:

$$\varepsilon_{\theta_t} = \varepsilon_{\xi_{t-1}} \rho_{\theta\xi} + \varepsilon \sqrt{1 - \rho_{\theta\xi}^2}. \tag{9}$$

The approach that we take by correlating the CAPEX process and oil price process allows us to follow the recommendations of Smith (2005) regarding treatment of different types of risks within a single risk-neutral valuation procedure. In Section 5.3, we provide sensitivity analysis on the correlation factor, proving that disregarding the correlation between the CAPEX to switch and the oil price, can lead to biased valuation results.

3.4. Project valuation and real options approach

In order to estimate the investment value, we use simulated project cash flows that result from combining the production profiles of Fields A and B, oil price and cost trajectories (as illustrated in Fig. 4). We first estimate the project value under the parallel development by means of Monte Carlo simulation. Then, we apply two alternative approaches to evaluate the sequential development, a "myopic" approach and an "options" approach. In case of the "myopic" approach, we use a simple Monte Carlo simulation method to generate realizations of the cash flow in Field A. The decision rule for abandonment of Field A is based on cash flow and production levels that indicate the depletion of Field A. Once Field A is abandoned in year t_m, the facility is moved to develop Field B, whose cash flow is modeled in the same manner as in Field A. We assume that it takes k years between Field A being abandoned and Field B starting production. Therewith, we disregard the flexibility to wait to invest in Field B after the Field A was abandoned. Considering this flexibility would require valuation of a compound option including both the option to abandon/switch and the option to wait for Field B investment. This is expected to further increase the additional value captured by the "options" approach.

In order to account for the switching flexibility in case of the "options" approach, we apply a real options analysis. We identify the optimal time to switch between two fields accounting for the fact that in order to make a decision to initiate switching in year t_o, the oil company can use information resolved by the end of year $t_o - 1$. If the decision to switch is made, production of Field A is ceased at time step t_o and facilities are moved to Field B, which starts production in year $t_o + k$. In order to allow for a fair comparison between the "myopic" approach and the "options" approach, we use the same simulated cash flow paths. The key difference between the two valuation approaches is the time when Field A is abandoned (t_m and t_o) and the opportunity to leave Field B undeveloped in case of the "options" approach. In some simulation cases, however, t_m and t_o can be the same. By exploiting the flexibility to leave Field B undeveloped, the oil company can additionally hedge against the downside risk if the information regarding the oil price, expected recoverable reserves of Field B and CAPEX to switch indicates that investing in Field B is sub-optimal. In this case, production facilities that were used for the development of Field A can be sold or used for a different license.

For the "options" approach, we use the LSM method introduced by Longstaff and Schwartz (2001) to identify the optimal time to switch and whether Field B should be developed in each simulation case. The LSM has been applied in several recent contributions studying investment cases in oil and gas under uncertainty (see for example Willigers and Bratvold (2010), Fleten et al. (2011), Jafarizadeh et al. (2012), Thomas and Bratvold (2015), Jafarizadeh and Bratvold (2015) and Hong et al. (2019)). The main advantage of using the LSM in our case is its ability to handle multiple sources of uncertainties in problems with downstream decisions in a computationally effective manner.

In order to optimize the switching decision, the LSM algorithm works in a backward fashion starting from the last decision point when switching is possible. In the last decision point, the option to switch is exercised if the expected value of the immediate exercise, denoted by $\Pi_{t_n}(t_n, P_{t_n}, Q_{B\,t_n}, CAPEX_{t_n})$, is larger than zero. In every time step t between the last and the first decision point when the switching is possible, the LSM algorithm compares the expected value of the immediate exercise of the option to switch, and the continuation value, associated with the decision to continue production of Field A at least until the next decision point, expressed as $\Phi_{t_n}(t_n, P_{t_n}, q_{A\,t_n}, OPEX_{A\,t_n}, Q_{B\,t_n})$. Π_{t_n} is conditional on the oil price level P_{t_n}, expected recoverable reserves of Field B $Q_{B\,t_n}$ and CAPEX to switch $CAPEX_{t_n}$, while Φ_{t_n} is conditional on the oil price P_{t_n}, the then-current oil production rate of Field A $q_{A\,t_n}$, the level of operational expenses of Field A $OPEX_{A\,t_n}$, the expected recoverable reserves of Field B if the switching is delayed by one year $Q_{B\,t+1_n}$ and the expected CAPEX to switch if the switching is delayed by one year $CAPEX_{t+1_n}$. The maximum of the two values indicates the optimal switching policy at each point in time.

The optimal value function F at time step t_n can be formulated using the following Bellman equation (Rodrigues and Rocha Armada, 2006):

$$\begin{aligned} F_{t_n} = max \Big\{ & \mathbb{E}^*_{t_n} \left[\Pi_{t_n}(t_n, P_{t_n}, Q_{B\,t_n}, CAPEX_{t_n}) \right], \\ & \mathbb{E}^*_{t_n} \left[\Phi_{t_n}(t_n, P_{t_n}, q_{A\,t_n}, OPEX_{A\,t_n}, Q_{B\,t_n}) \right] \Big\}, \end{aligned} \tag{10}$$

Notably, we must make assumptions regarding the decision maker's knowledge regarding the expected recoverable reserves of Field B $Q_{B\,t_n}$, $Q_{B\,t+1_n}$ and the expected CAPEX to switch if the switching is delayed by one year, i.e. $CAPEX_{t+1_n}$, at time step t_n. We assume that the field operator has perfect information on both parameters when the decision to switch is being made. While for the capital expenditure this assumption is fully realistic, as oil companies can conclude contracts with suppliers in advance, leaving little room for deviations in the final cost, the assumption regarding the accurate knowledge of how much oil can be produced by Field B before drilling production wells is considered to be strong, but allows us to significantly simplify the optimization algorithm, which accounts for five uncertain factors, and makes the valuation procedure more transparent. In case of perfect information, $Q_{B\,t_n}$ is equal to the simulated sum of the future annual production rates from Field B. In Section 5.3, we test this assumption, changing the quality of information regarding the expected recoverable reserves of Field B that the decision maker has when optimizing the switching timing.

At the decision point t_n both Π_{t_n} and Φ_{t_n} are unknown due to the fact that they depend on future oil prices and production rates, which are not observable when the decision is being made. However, we can estimate the expected value of Π_{t_n} and Φ_{t_n} conditional on then-current information regarding the oil price, productions rates and costs. We take the approach originally suggested by Longstaff and Schwartz (2001) and use a linear regression to perform this estimation as given by

$$\begin{aligned}
&\mathbb{E}^*_{t_n}\left[(\Pi_{t_n}(t_n, P_{t_n}, Q_{B\,t_n}, CAPEX_{t_n}))\right] = \beta_1 P_{t_n} \\
&+\beta_2 Q_{B\,t_n} + \beta_3 CAPEX_{t_n} + \beta_4 P_{t_n}^2 + \beta_5 Q_{B\,t_n}^2 + \beta_6 CAPEX_{t_n}^2 \\
&+\beta_7 P_{t_n} Q_{B\,t_n} + \beta_8 P_{t_n} CAPEX_{t_n} + \beta_9 Q_{B\,t_n} CAPEX_{t_n} \\
&+\beta_{10} P_{t_n} Q_{B\,t_n} CAPEX_{t_n},
\end{aligned} \tag{11}$$

$$\begin{aligned}
&\mathbb{E}^*_{t_n}\left[(\Phi_{t_n}(t_n, P_{t_n}, q_{A\,t_n}, OPEX_{A\,t_n}, Q_{B\,t+1_n}, CAPEX_{t+1_n}))\right] = \alpha_1 P_{t_n} \\
&+\alpha_2 q_{A\,t_n} + \alpha_3 OPEX_{A\,t_n} + \alpha_4 Q_{B\,t+1_n} \\
&+\alpha_5 CAPEX_{t+1_n} + \alpha_6 P_{t_n}^2 + \alpha_7 q_{A\,t_n}^2 + \alpha_8 OPEX_{A\,t_n}^2 \\
&+\alpha_9 Q_{B\,t+1_n}^2 + \alpha_{10} CAPEX_{t+1_n}^2 + \alpha_{11} P_{t_n} q_{A\,t_n} \\
&+\alpha_{12} P_{t_n} OPEX_{A\,t_n} + \alpha_{13} P_{t_n} Q_{B\,t+1_n} + \alpha_{14} P_{t_n} CAPEX_{t+1_n} \\
&+\alpha_{15} q_{A\,t_n} OPEX_{A\,t_n} + \alpha_{16} q_{A\,t_n} Q_{B\,t+1_n} \\
&+\alpha_{17} q_{A\,t_n} CAPEX_{t+1_n} + \alpha_{18} OPEX_{A\,t_n} Q_{B\,t+1_n} \\
&+\alpha_{19} OPEX_{A\,t_n} CAPEX_{t+1_n} + \alpha_{20} Q_{B\,t+1_n} CAPEX_{t+1_n} \\
&+\alpha_{21} P_{t_n} q_{A\,t_n} OPEX_{A\,t_n} Q_{B\,t+1_n} CAPEX_{t+1_n},
\end{aligned} \tag{12}$$

where $\alpha_{1\ldots21}$ and $\beta_{1\ldots10}$ denote the regression coefficients.

At each decision point where the option to switch is available, we take the optimal course of action in each simulation case and calculate the project value under the "options" approach, discounting cash flows at a risk-free rate within a risk-neutral valuation routine.

The option to leave Field B undeveloped should be exercised when both the immediate exercise of the option to switch Π_{t_n} and continuation of the Field A development Φ_{t_n} have negative expected values. Therefore, we identify all simulation cases with negative value function F at time of optimal switching t_{sw_n}, found on the previous step of the procedure. The oil company is then considered to abandon Field A at t_{sw_n} and sell the production unit that still has a substantial residual value.

The main requirement for implementing the LSM approach for such an optimization, is having a large enough set of realizations of each factor included in the regression function. A potential bottleneck is the number of needed production profile paths. In our case, 50 realizations of the reservoir uncertainty enabled performing the regression analysis. However, if the decision maker has, for example, only three production profile paths, this might result in invalidity of the regression analysis. Fedorov et al. (2020) reach model validity by considering five realizations of the Olympus case.

4. Case study

4.1. Field development case and production optimization

The considered reservoir model, Olympus case, is a synthetic reservoir that has been proposed as a benchmark model for studies aiming at establishing field development plans under geological uncertainties Fonseca et al. (2017), Fonseca et al. (2018) and Chaturvedi (2021). The reservoir involves 16 wells (10 producers and 6 water-injectors) (see Fig. 5) operating under BHP control mode. In this study, the producer wells are denoted by P1, P2, …, P10, respectively, while the injector wells are designated as I1, I2, …, I6, respectively. The area of the field is 9 km ×3 km and is bounded on one side by a fault. The average thickness of the reservoir is 50 m. The model consists of 341,728 grid cells (118 × 181×16), among which 192,750 are active. The 3D dimensions of grids are approximately 50m×50m×3 m. The facies types of the reservoir include Channel Sand (in the upper zone), Shale (in the upper zone and barrier), and Coarse Sand, Medium Sand, and Fine Sand (in the lower zone). From the perspective of geological uncertainties, the Olympus reservoir includes 50 different model cases with different distributions of net-to-gross, porosity, and permeability. More details about the Olympus model can be found in Fonseca et al. (2018) and Chaturvedi (2021).

Table 1 states the main parameters used for calculating the NPV that serves as an objective function for the production optimization workflow.[3]

[3] All parameters except the oil price are taken from the original publication of Fonseca et al. (2018). The oil price is revised from 45 \$/bbl to 60 \$/bbl to reflect the price levels observed in the first half of 2021. The changes, however, do not affect the production optimization results.

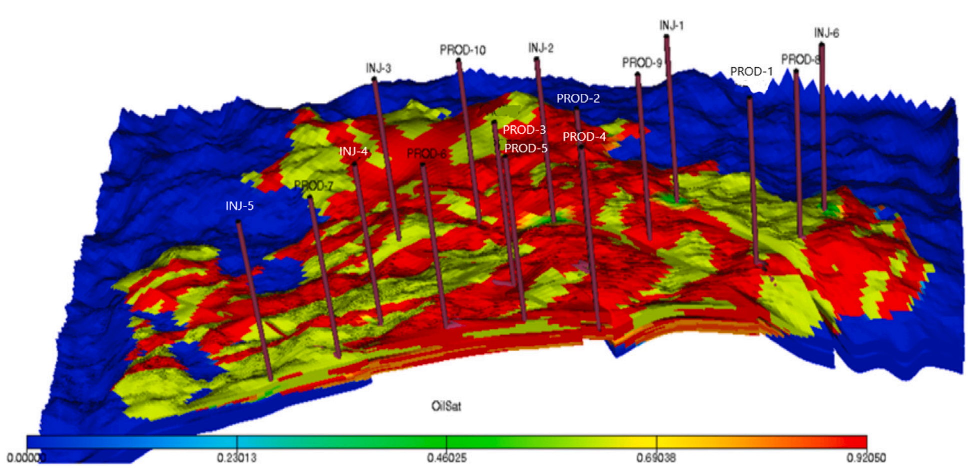


Fig. 5. Olympus case well placement.

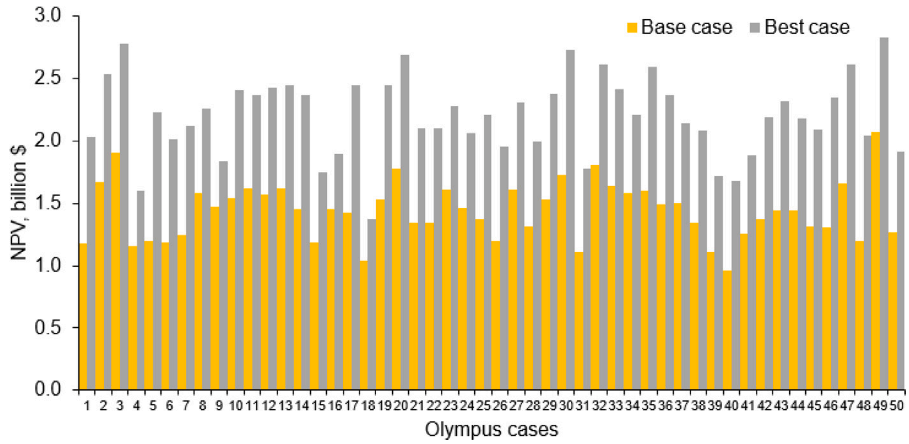


Fig. 6. Comparison between the resulting NPV values before (base case) and after (best case) the optimization for 50 realizations of the Olympus case.

Table 1
The main economic parameters used for NPV calculation.

Parameter	Unit	Value
P_o	$/bbl	60
C_{wp}	$/bbl	6
C_{wi}	$/bbl	2
d	fraction	0.08
t	years	20

Before proceeding to the optimization steps for each case of the ensemble, some preliminary runs were performed in order to set the maximum and minimum BHP values supported by the producers and injectors. Table 2 reports the BHP intervals considered for the wells of the Olympus model in this study.

Table 2
The BHP intervals considered for the wells of Olympus model in this study.

Wells' type	Wells	BHP (bar)	
		Min	Max
Producers	P1, P2, P3, P5, and P6	75	130
	P4	75	100
	P7	75	155
	P8	75	165
	P9	75	125
	P10	75	160
Injectors	I1, …, I6	220	260

The comparison between the NPV results before (base case) and after (best case) the optimization for the different Olympus cases is reported in Fig. 6. As can be seen, significant improvements were made in the NPV values for all cases after implementing the optimization approach.

The build-up phase is considered to take four years, after which Field A delivers its first oil in Year 5. The optimization yields the forecast for the oil production, water injection and water production rates for the first 20 years of the field lifetime. As the production phase lasts for more than 20 years in many simulation cases, we had to extrapolate the respective rates for an additional 10 years. As the production rate between 20 and 30 years in rather low, the extrapolation is not expected to provide significant discrepancies with the case if the production optimization was used for the whole period of 30 years. The resulting oil production, water injection and water production rates for three example cases (Olympus 18, 6 and 49), representing the "worst", "medium" and "best" case of the reservoir performance, are illustrated in Fig. 7. This figure demonstrates the range of uncertainty built in the Olympus reservoir case.

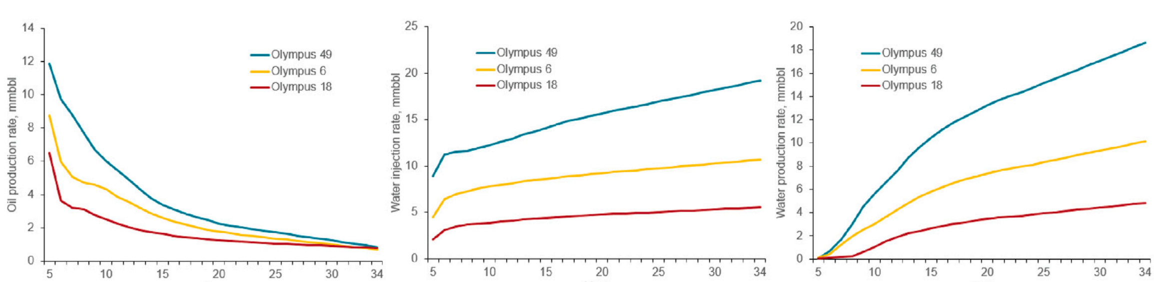
4.2. Oil price simulation

We calibrate the oil price process parameters based on the historical market data by using the Kalman filter as demonstrated by Fedorov et al. (2021). We use the Refinitiv Eikon® data on the ICE Brent historical futures contracts and Dated Brent spot prices from March 2006 to June 2021.

Table 3
Calibrated parameter values used for the Schwartz–Smith two-factor price process simulation.

Parameter	Value	Std error	Parameter	Value	Std error
ξ_0	4.07	–	χ_0	0.1	–
σ_ξ	11.5%	0.005	σ_χ	56%	0.023
μ_ξ^*	−0.45%	0.001	$\rho_{\xi\chi}$	0.12	0.036
κ	0.45	0.006	λ_χ	10.9%	0.011

The resulting oil price process parameters are reported in Table 3. Fig. 8 illustrates examples of the simulated price paths and confidence bands based on 2,500 simulated cases.



(a) Oil production rates (b) Water injection rates (c) Water production rates

Fig. 7. Results of the production optimization for three realizations of the Olympus case.

Table 4
Parameters for the GBM process modeling the CAPEX.

Parameter	Value	Description
$\theta_{0_{AY2}}$	\$100 million	CAPEX Field A Year 2 - initial installment for the production facility
$\theta_{0_{AY3}}$	\$400 million	CAPEX Field A Year 3 - second installment for the production facility, drilling template
$\theta_{0_{AY4}}$	\$1300 million	CAPEX Field A Year 4 - final installment for the production facility, drilling costs
$\theta_{0_{Bt}}$	\$100 million	CAPEX Field B Year 1 - Field A decommissioning, production facility modernization
$\theta_{0_{Bt+1}}$	\$550 million	CAPEX Field B Year 2 - drilling costs, production facility modernization
μ_θ	2%	
σ_θ	10%	

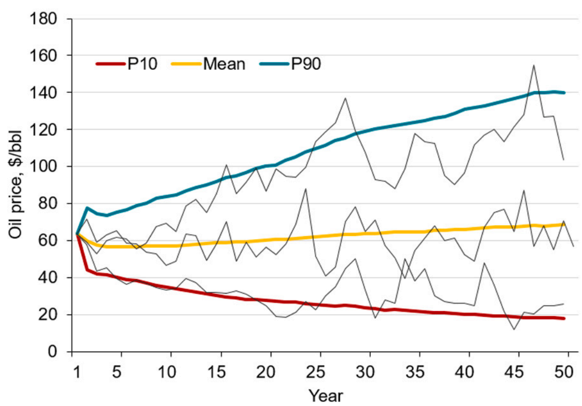


Fig. 8. Oil price simulation results, confidence bands and example price paths.

4.3. Costs

During the build-up phase of Field A, the operator incurs capital costs in Year 2, Year 3 and Year 4, which mainly consist of the facility cost and the cost of production drilling. Since we assume that Fields A and B are identical, the cost structure for the two fields for the parallel development case is the same. In case of the sequential production, in order to switch to Field B, the operator has to modernize the production unit in the first year after the production in Field A is shut down, i.e. year t, and perform drilling in the second year $t+1$ in order to switch to Field B. The production of Field B starts in year $t+2$, i.e. the parameter k that we introduced in Section 3.4 equals 2.

As mentioned in Section 3.3, we model individual components of CAPEX as GBM processes correlated with the oil price. We split total CAPEX into several components that the field operator is expected to incur in each year (Y2, Y3, Y4 as well as the CAPEX in the first and second years after Field A is abandoned, t and $t+1$) that reflect different types of costs, which are independent from each other. We use identical parameters for the annual drift rate μ_θ of 2% and volatility σ_θ of 10% for GBM processes in Eq. (8). However, each CAPEX process has its own initial value reflecting an existing estimate as of now for each component. Initial values for each component and description of types of cost components are reported in Table 4. We take the same cost simulation paths for the valuation of the project under the parallel and sequential development.

Additionally, we must decide on the correlation level between the CAPEX and the oil price. Willigers (2009), who studies the correlation between the rig rental rates in the North Sea and oil prices, found that the correlation coefficient equals 0.87. However, the field development case that we analyze also includes the cost of components (such as the production unit) and operations that are less dependent on the developments in the oil market than the rig rates studied by Willigers (2009). Therefore, we decided to slightly downgrade the correlation coefficient used by Willigers (2009) to 0.8. We also include a sensitivity analysis for the correlation factor in Section 5.3.

Fig. 9 illustrates example simulation paths for the oil price and CAPEX components, respectively. As can be seen, accounting for the correlation between the oil price and CAPEX allows us to avoid unrealistic simulation scenarios, where oil prices are low, whereas CAPEX is high. Therefore, we can decrease bias when analyzing the switching option.

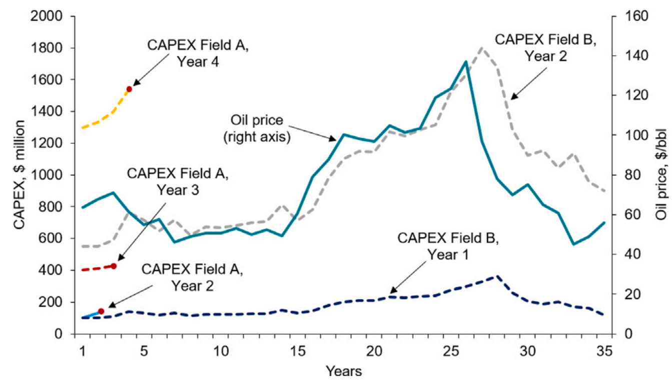


Fig. 9. Example path of the oil price and the individual components of CAPEX.

The following parameters are used for Eq. (6) to relate the annual OPEX (in million \$) with the field production rate q_{t_n}, the oil price P_{t_n}, water injection q_{wit_n} and water production q_{wpt_n}: $FO = 66.3$, $a = 0.36$, $b = 0.36$, $c = 1.2$, $d = 3.6$. This makes the Eq. (6) to take the following form:

$$OPEX_{t_n} = 66.3 + 0.36P_{t_n} + 0.36q_{t_n} + 1.2q_{wi_{t_n}} + 3.6q_{wp_{t_n}}, \tag{13}$$

4.4. Abandonment

Under the parallel development and the "myopic" approach, Field A is abandoned as soon the cash flow that it generates approaches negative values. Based on the discussion with our industry partner and our preliminary Monte Carlo simulation results, we concluded that it is more beneficial to decommission Field A earlier than at the moment when it starts generating negative cash flows under certain conditions. Firstly, the annual cash flow must be just slightly above zero in year t_n. Secondly, the oil production rate must be rather low. Our results show that in most of the simulation cases, the cash flow from Field A is likely to fall below zero. Therefore, decommissioning Field A already in year t_n+1 avoids operating Field A under a negative cash flow for one year. Our results show that this approach allows an increase in expected project value under the "myopic" approach compared to the case where a simple "decommission only if the cash is negative" rule is used. Therefore, when we compare the "myopic" and the "options" approaches, the percentage difference between project values is more conservative.

The decision rules that we used to identify the optimal abandonment time of Field A under the parallel development and the "myopic" approach are reported in Table 5. Thereby, the abandonment is triggered, for example, if the annual cash flow generated by the field is below \$20 million and the production rate is lower than 1.5 mmbbl/year. If the cash flow is lower than \$10 million, it is worth sacrificing an even higher production rate (below 2.5 mmbbl/year) to decommission the field as soon as possible. The field is abandoned in

Table 5
Decision rule for the abandonment of Field A under the "myopic" approach.

Annual cash flow, CF, $, million	Production rate, q (mmbbl/year)
10 < CF ≤ 20	q ≤1.5
0 < CF ≤10	q ≤ 2.5
CF ≤0	q ≤ 3.5

every case where the cash flow falls below zero and the production rate is lower than 3.5 mmbbl/year.

The salvage value that the production unit is sold for after Field A is abandoned is set to $200 million in those cases where Field B is left undeveloped. In cases where both fields are developed, however, the residual value of the production unit equals $100 million.

4.5. Switching option

Given that the decision to switch is made at time step t, the production of Field B starts in year $t+2$. Fig. 10 illustrates the project cash flow for an example simulation path where switching is initiated in Year 15.

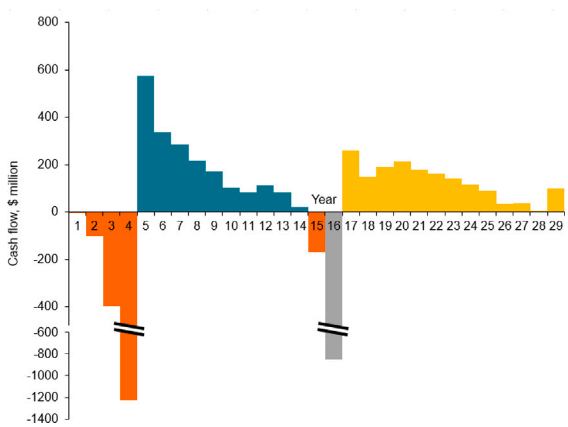


Fig. 10. Example of the project cash flow.

The LSM algorithm starts evaluating the switching decision from the last year when Field A can produce oil, Year 34 in our case (taking into account information revealed by the end of that year). At that point of time, the operator has only two alternatives: to initiate switching to Field B in Year 35 or to leave Field B undeveloped. Therefore, Year 34 serves as the upper time limit for considering the switching option (see Fig. 11). Due to the fact that the continuation value equals zero in Year 34, the option to switch is exercised in those simulated cases where the expected value of immediate exercise is above zero. Then, at each decision point between Year 33 and the first year when the decision to switch can be made (depends on the level of cash flow from Field A and the recovery factor in each simulated case), the algorithm defines the optimal course of action. It evaluates whether to continue production from Field A for at least one more year or to initiate switching to Field B already next year. Year 10 is assumed to be the lower time limit for considering the switching option for all simulated cases. We use a risk-free rate of 2.5% to discount cash flows.

We also consider the fact that the regulator's approval is needed to abandon Field A.[4] One consequence is that the field operator is obliged to reach a sufficient recovery factor to be able to decommission the field. In order to reflect this requirement in the modeling, only those simulation cases that generate annual cash flow below $70 million and deliver a production rate below 3.6 mmbbl per year are considered for a switching decision.

In some simulation cases the LSM optimization results in unreasonably late switching time between Fields A and B, letting Field A operate with a negative cash flow for several years before initiating the investment in Field B. This happens, for example, when the expected value of switching at a later time is higher than the immediate exercise value due to a high level of CAPEX to switch immediately. In this case, the algorithm captures the value of waiting to invest in Field B. If we had accounted for the possibility to abandon Field A first and then wait to invest in Field B, this value of waiting would have been addressed more accurately. However, in this paper, we explicitly disregard such analysis, accounting only for the option to switch with a fixed time between the abandonment of Field A and the production start-up for Field B. Therefore, we set another boundary condition, implying that the switching time suggested by the "myopic" approach for each simulation case serves as the latest point to switch for the "options" approach. It means that if the LSM optimization yields a switching time that is later than the one suggested by the "myopic" approach, we take the latter value as optimal. This avoids operating Field A under negative cash flow for more than one year. Therefore, the "options" approach can only result in a switching time that is either equal to the one resulting from the "myopic" approach or earlier. If the expected value of switching to Field B in year t_{sw_n} suggested by the "myopic" approach has a negative value, the optimal decision for the "options" approach is to leave Field B undeveloped. In fact, accounting for the option to wait to invest in Field B after Field A is abandoned can be considered as a potential extension for future research. The approach that we take in this paper can be regarded as relatively conservative as the additional flexibility to wait to invest in Field B is likely to add more value to the project.

5. Results

5.1. Project valuation

The primary goal of our analysis is to analyze whether sequential production results in a higher expected project value compared to parallel development.

Based on the procedure discussed in Section 3, we first evaluate the value of developing two licenses in parallel, which requires a

[4] For example, the Norwegian Petroleum Directorate requires that every petroleum project on the NCS contributes optimally to overall social benefit by extracting as much economical reserves as possible (OG21, 2021).

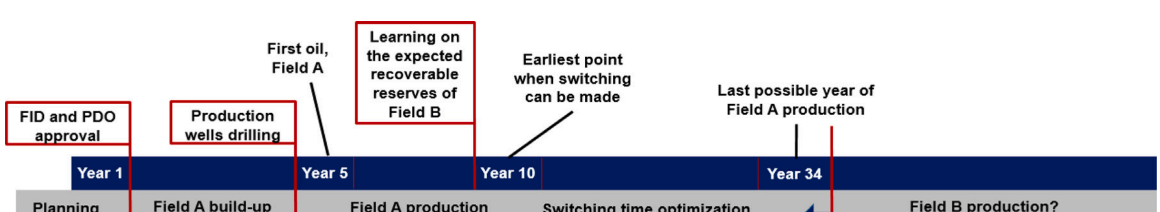


Fig. 11. Project timeline.

Table 6
Confidence bands of the pre-tax values of the project under the parallel and sequential production (“myopic” approach and “options” approach), $, million.

	P10	P50	Expected value (mean)	P90
Parallel production	−1731.6	−250.8	105.5	2337.7
Sequential production (“myopic” approach)	−937.3	236.3	625.9	2623.7
Sequential production (“options” approach)	−812.9	279.8	700.2	2675.3

construction of two production units. The resulting expected value of parallel development is equal to $151 million.

In case of the sequential development, we use the “myopic” and “options” approaches for the valuation, respectively. As mentioned in Section 3.4, the main difference between the “myopic” and “options” approaches is the switching time between Fields A and B and the opportunity to leave Field B undeveloped in the case of the “options” approach. For the “myopic” approach, the decision to switch is made as soon as the cash flow from Field A approaches negative values. The valuation approach is based on a simple Monte Carlo simulation. For the “options” approach, however, we need to know how to optimally operate the assets in order to be able to evaluate them correctly, i.e. we need to identify the optimal time to abandon Field A and switch to Field B (or to leave it undeveloped) for each simulated case. The fact that the field operator can make optimal decisions over time is addressed within the “options” approach. Fig. 12 demonstrates how the abandonment time of Field A changes depending on which approach is used. Applying the real options analysis results in earlier abandonment of Field A in 56.9% of the total simulated cases. This makes the distribution of the optimal abandonment time for Field A under the “options” approach shift to the left compared to the “myopic” approach.

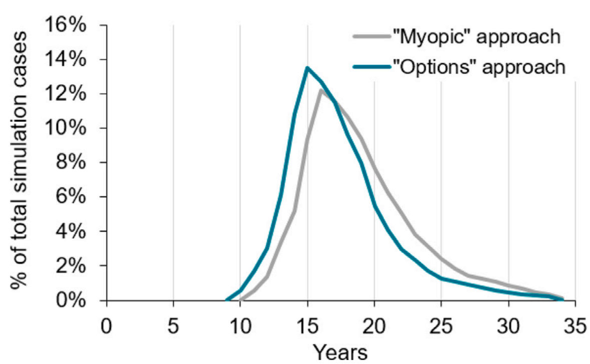


Fig. 12. Field A’s optimal abandonment time distribution, “options” approach vs. “myopic” approach.

We should note that the LSM algorithm does not always capture the optimal course of action,[5] triggering the switching earlier or later than optimal[6] in some simulation cases. This is due to the fact that the regression function based on the current state of the decision variables is not a perfect estimate for the future cash flows. Therefore, in some simulated cases, following the policy suggested by the “options” approach results in lower value than those of the “myopic” approach. In our case, in 20.4% of the simulation cases the project value under the “options” approach is lower compared to the value resulting from the “myopic” approach, while in 21.7% of cases the resulting values are equal as the switching time is the same. However, in 57.9% of the cases, the “options” approach identifies a strategy that increases the overall project value compared to the “myopic” approach. This strategy may be either to switch earlier or to leave Field B undeveloped.

In fact, our results showed that in 16.4% of the simulated cases the development of Field B has a negative NPV. Due to the fact that the license owner can learn about the uncertain factors included in the regression functions in Eqs. (11) and (12) before making the switching decision, a significant part of the downside risk can be mitigated. Due to capturing those simulation cases where both the continuation and immediate exercise values are negative, the number of cases where the field operator invests in Field B that eventually would turn out to be unprofitable (based on true NPVs), decreases to 4.5%.

This leads to the fact that the expected project value under the “options” approach is 11.9% larger compared to the value resulting from the “myopic” approach. Table 6 summarizes the results for the project valuation under the sequential production using these two approaches and the parallel development. The sequential production is clearly the optimal solution for the investment, resulting in the highest expected value. If the option to leave Field B undeveloped is not accounted for in the “options” approach, which means that the operator is considered to be obliged to invest in Field B irrespective of the information revealed and we only use the switching timing optimization, the resulting project value is only 6.5% larger compared to the value resulting from the “myopic” approach.

5.2. Validation

The advantage of our approach is that it allows the modeling of the effect of several types of uncertainty on the decision to switch. It allows us to capture the value of learning about the uncertain factors within the valuation procedure. However, the fact that we included five uncertain factors in the regression analysis described in Section 3.4 (Eqs. (11) and (12)), makes it difficult to derive tractable option exercise boundaries, illustrating which state of the uncertain factors triggers the option exercise. Exercise boundaries in real options problems often serve as validation instruments, enabling the demonstration that the results are intuitive and, for example, the option exercise is triggered by a certain combination of the oil price, production level and/or CAPEX.

In this section, we perform such a validation and demonstrate that the LSM algorithm addresses the decision maker’s ability to use new information in order to optimize decisions over time and account for this within the investment valuation procedure. We identify conditions of uncertain parameters that result in the decision to switch immediately, continue developing Field A or to abandon Field A, or leave Field B undeveloped.

Fig. 13 illustrates parameters of each simulated case in Year 14 where the switching option is still available. The decision whether to switch from Field A to Field B in Year 15 is made based on parameters observed at the end of Year 14. Cases where the optimal decision according to the LSM algorithm was to switch in Year 15 are illustrated in Fig. 13 by blue dots, while the cases where the optimal course of action was to continue Field A production are illustrated by black dots. Red dots reflect those cases where the optimal decision is to abandon Field A in Year 15 and leave Field B undeveloped. The x-axis refers to the annual cash flow from Field A in Year 14, y-axis is the CAPEX to switch from Field A to Field B in Year 15 (sum of the CAPEX in Year 15 and Year 16), z-axis is the expected recoverable reserves of Field B if switching is initiated in Year 15. The annual cash flow from Field A reflects three parameters included in the regression in Eq. (12) at the

[5] This problem is also discussed by Jafarizadeh and Bratvold (2009) and Hong et al. (2019).

[6] Optimal switching time refers to the time that maximizes the true NPV that is unobservable for the decision maker when the decision to switch is made, but can be estimated using simulated cash flow paths.

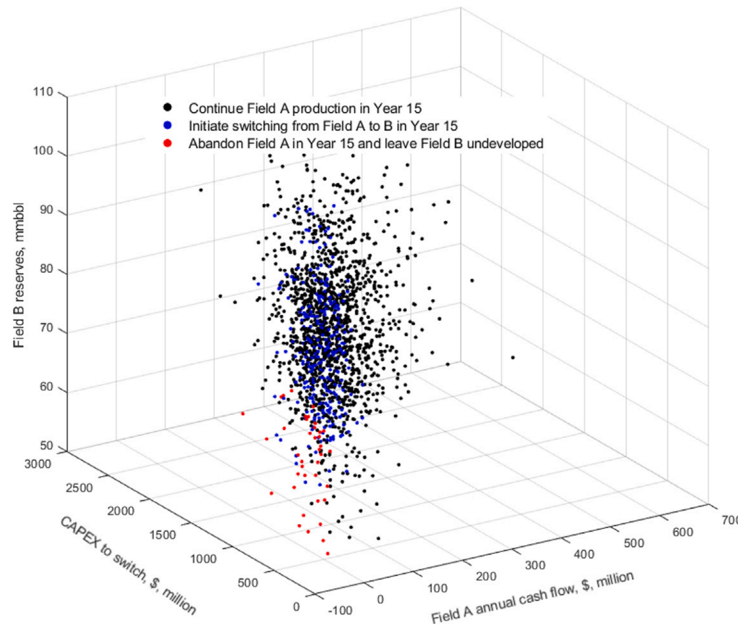


Fig. 13. Optimal option policy in Year 15 in all simulated cases where option to switch is available (3 components).

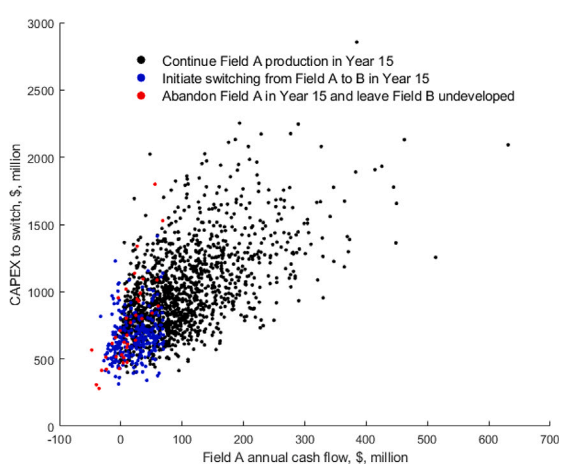


Fig. 14. Optimal option policy in Year 15 in all simulated cases where option to switch is available (2 components, Field B reserves excluded).

same time: the oil production rate from Field A, the oil price and the OPEX. The data in Fig. 13 reflects the boundary condition that Field A cannot be decommissioned as long as it generates an annual cash flow exceeding $70 million. The switching decision is triggered not only when the expected reserves of Field B are high, but in the whole range of the z-axis. Fig. 13 also shows an intuitive result that the decision maker is most likely to refrain from the investment in Field B if the information indicates that its recoverable reserves are low.

Slicing Fig. 13 into a two-dimensional chart, we can analyze how the relation between the cash flow from Field A indicated in the x-axis and CAPEX to switch indicated in the y-axis impact the decision to switch. As can be seen in Fig. 14, the option to switch immediately is exercised only in those simulated cases where the CAPEX to switch is relatively low.[7] Our results show that in those simulation cases where the LSM algorithm initiates the switching decision relatively early (before Year 15), it captures those cases in which CAPEX are rather low. Fig. 15 illustrates the 95-th percentile of CAPEX of those simulated cases, in which the optimal decision is to switch to Field B immediately. It means that only 5% of simulated cases that were exercised have a higher CAPEX to switch in a specific year. Thus, if the simulated CAPEX to switch is above the threshold, it is unlikely that immediate switching is optimal. After Year 15 the factor of low CAPEX to initiate switching becomes less significant as Field A reaches a maximum recovery factor and has to be decommissioned in an increasing number of cases. In these conditions, the investment in Field B is triggered if the expected investment value is positive, no matter how high the CAPEX to switch is.

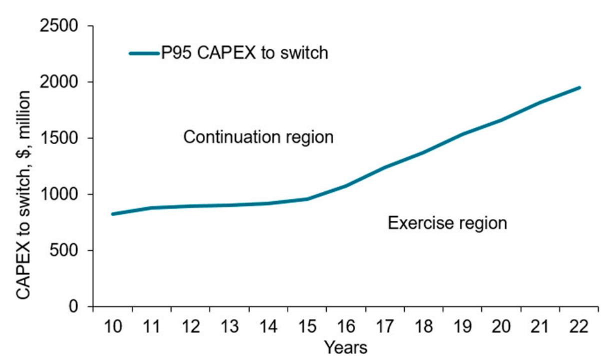


Fig. 15. 95th percentile of CAPEX for all simulated cases where switching is optimal in a specific year.

Overall, this validates that the LSM algorithm captures the optimal course of action in a significant number of simulated cases. This is done due to the possibility to rely the knowledge that the decision maker has when considering the switching option with the future cash flows that are unobservable when the decision is made. However, the accuracy of the switching time optimization depends strongly on the "quality" of the information the decision maker can obtain regarding, for example, the recoverable reserves in Field B. In the following section we discuss how the accuracy of the LSM optimization can change if different assumptions about the information "quality" are used.

5.3. Sensitivity analysis

Our assumption that the field operator obtains perfect information regarding expected recoverable reserves of Field B before they can

[7] The fact that Field A's annual cash flow falls below $70 million is often caused by an oil price decline, which causes that the simulated CAPEX to switch also decrease due to the correlation with the oil price.

make the decision to switch enabled reaching high accuracy in the LSM algorithm. The regression function describes the future cash flow that is unavailable to the decision maker when the switching decision is being made, quite well. This captured the optimal course of action (based on true NPVs) in most of the simulation cases. However, the assumption about the perfect information can be considered rather strong as hydrocarbon reservoirs are exposed to high amount of uncertainty, especially small ones. Performing an extensive appraisal program in such reservoirs might be too costly.

In this section we test different assumptions regarding the decision maker's knowledge about the reservoir uncertainty for Field B. We compare the percentage of project value increase created by the "options" approach compared to the "myopic" approach when the "quality" of the information about the expected recoverable reserves of Field B is reduced from 1 (perfect information) to 0.1 – 0.95. If there is perfect information about the reservoir parameters, $Q_{B t_n}$, which is used in the regression analysis in Eqs. (11) and (12), is equal to the simulated sum of the future annual production rates from Field B. This provides the decision maker with the actual value of the production potential of Field B. We now assume that the "quality" of the reservoir information is reduced from 1 to 0.9. It means that the information regarding the recoverable reserves can deviate from the true value within a range of +/−10%. The degree of deviation is determined by a random parameter k generated from a uniform distribution $k \sim U(0.9, 1.1)$. For each simulation case, the actual sum of the future annual production rates from Field B is multiplied by k. In this way we model that $Q_{B t_n}$ is an imperfect estimate of the production potential of Field B. When the reservoir information "quality" is further decreased to 0.1, $k \sim U(0.1, 1.9)$, $Q_{B t_n}$ can differ from the actual recoverable reserves within a range of +/−90%, which reduces the accuracy of the LSM algorithm as the regression function describes future cash flows with much less precision. This can be analyzed through the difference between expected project values resulting from the "options" approach and the "myopic" approach. The expected project value under the "options" approach when perfect information is used, is an upper boundary that provides a maximal difference between project values under the two approaches.

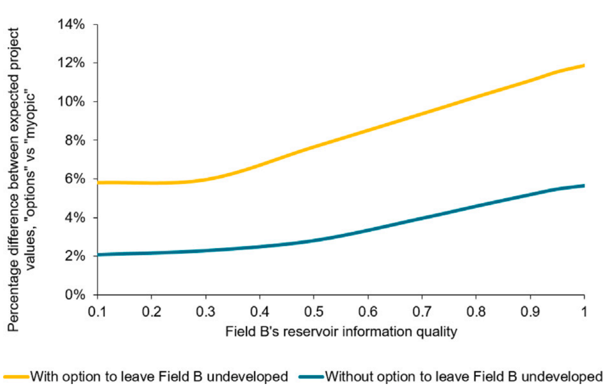


Fig. 16. Sensitivity of the percentage difference between expected project values under the "options" and "myopic" approaches to changes in the "quality" of the reservoir information for Field B.

Fig. 16 illustrates that the percentage difference between the respective values decreases from 11.9% in case of the perfect information to 5.8% when the information "quality" equals 0.1. If the possibility to leave Field B undeveloped is not accounted for, the respective value reduces from 5.7% to 2.1%. As already mentioned above, several existing contributions point out that the LSM approach becomes less reliable when factors included in the regression analysis are not able to describe the future cash flow. In our case, when the information "quality" from Field B is low (equals 0.1), the $Q_{B t_n}$ parameter becomes statistically insignificant for the regression function. This results in the "options" approach increasing the project value only in 44.3% of the simulated cases compared to the "myopic" approach (vs 57.9% when the perfect information is used). In this situation, the parameter $Q_{B t_n}$ can be excluded from the regression functions. This makes the LSM algorithm take into account only learning about the oil price, OPEX and CAPEX levels, which still improves the expected project value compared to the "myopic" approach.

As already mentioned, the only contribution in the petroleum literature studying the effect of uncertain future CAPEX on optimal field development decisions and a real option value, points out that considering correlation between the CAPEX and the oil price is very important for the project valuation results (Cardenas et al., 2018). Our results confirm this finding. Fig. 17 illustrates the percentage difference between the expected project value under the "options" and the "myopic" approaches. In case the correlation between the CAPEX and the oil price is disregarded, the valuation can result in an overestimated option value. This results from the fact that simulation paths with high oil prices and low switching cost (as well as the opposite case) in particular time steps can be frequently found. This is not in line with the market reality, and is considered to be a bias that affects optimal option policy. As the correlation factor is increased, these biased estimations cease, leading to more realistic project value estimations. The chosen correlation factor of 0.8 allowed us to realistically address the value of the option to switch. Overall, we can conclude that accounting for the correlation between oil prices and the level of CAPEX to switch is of high importance for valuation of such investment problems.

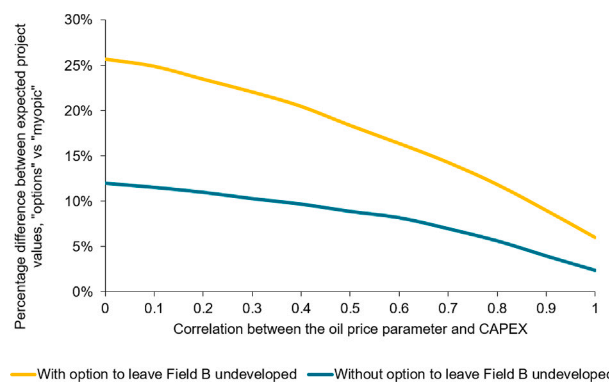


Fig. 17. Sensitivity of the percentage difference between expected project values under the "options" and "myopic" approaches to changes in correlation factor between the oil price parameter and CAPEX.

6. Conclusion

This paper analyzes the economic potential of a sequential production strategy using the same production facilities for two oil fields. This strategy is considered as a cost-efficient solution for the development of marginal fields that cannot be tied back to existing infrastructure. By investing sequentially, the operator can gather additional information regarding uncertain parameters: the expected production rates of both oil fields, the oil price, OPEX of the first field and CAPEX to switch. We perform the valuation of the sequential production using two approaches: the "myopic" and "options". In the "myopic" approach, the switching is initiated only when Field A is depleted fully (reaches a negative cash flow). The "options" approach allows for early switching if it maximizes the overall value of the project due to opportunity to learn about uncertain factors and hedges the downside risk of developing Field B.

We use the Olympus benchmark case as an underlying reservoir model for both fields and apply an oriented workflow to address the full range of the technical uncertainty embedded in the Olympus case. We also model the oil price and CAPEX to switch as two correlated stochastic processes.

For the "options" approach, we implement a real options valuation procedure. We consider the decision to switch between two licenses as an optimization problem that is solved by the LSM algorithm. The optimization yields the optimal timing to switch between two licenses for each simulation case.

Our results suggest that sequential production and accounting for the option to switch earlier or leave the second field undeveloped can add substantial value that might affect the final investment decision. Our analysis also supports the findings by Cardenas et al. (2018) regarding the importance of the correlation between capital expenditure and the oil price for valuation of the flexibility in petroleum projects where CAPEX is one of the decision variables. Overall, our findings allow to conclude that the sequential production is a viable strategy for development of marginal stand-alone fields and can be considered to be used in practice. Applying a real options analysis is then needed to quantify the economic effect of flexibility embedded in this strategy.

Future research may be aimed at: (1) analyzing the value of additional flexibility to delay investment in subsequent field(s) after the first field had been decommissioned by considering the compound option; (2) addressing the opportunity to invest in appraisal of future field(s) in a more robust manner by means of history matching and Bayesian updating; (3) solving the optimal development order problem that we omitted by assuming that the two oil fields are identical.

CRediT authorship contribution statement

Semyon Fedorov: Conceptualization, Methodology, Software, Data curation, Visualization, Writing – original draft, Writing – review & editing. **Menad Nait Amar:** Conceptualization, Methodology, Software, Writing – original draft. **Verena Hagspiel:** Conceptualization, Methodology, Supervision, Reviewing and editing, Data curation. **Thomas Lerdahl:** Conceptualization, Supervision, Reviewing and editing, Data curation.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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