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A value of prediction model to estimate optimal response time to threats for accident prevention

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A R T I C L E I N F O

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A B S T R A C T

This paper presents a novel value of (imperfect) prediction (VoP) model to estimate optimal response time to a threat that may result in an accident. The proposed VoP model is based on information value theory and considers both prediction accuracy and action failure probability over time. The optimal response time is dependent on parameters: the ratio between the accident cost and response action cost, accident probability, action failure probability, prediction performance, and response strategy (a series of sequential responses or a single response). A case study of iceberg management is presented to demonstrate the proposed approach; a sensitivity study is done to evaluate how optimal response time changes with those parameters. The case study show that it is reasonable to respond as early as possible if the threat can lead to a serious accident, while the response can be postponed when the potential consequence is moderate. In addition, the proposed VoP model is proven able to calculate accuracy requirements, thresholds for tolerating risk and acting precautionarily, and maximum investment in accident prevention. Imperfect prediction can lower risk acceptance threshold and higher the threshold of being precautionary; and it is reasonable to increase action cost.

1. Introduction

1.1. Background

When an iceberg is approaching an offshore installation, it is not clear whether a collision will happen. For such a threat that has a potential to damage an asset of interest, the decision-maker needs to decide how to respond to avoid accidents with the lowest cost. For example, one can tow the iceberg away [1] immediately after the iceberg is observed, to avoid any risks, but it is unnecessary in most cases since the chance of collision is very low. Thus, after receiving the first signals of a threat (e.g., seeing an iceberg), the decision-maker does not need to respond until she/he has higher confidence that the threat will make a damage. Situation changes rapidly, and validity of prediction is also time-varying [2, 3]. When prediction is made early, the signal for anticipating either an accident or no accident may be very weak [4], or there is too much noise and too many possible future scenarios. The prediction accuracy may be insufficient to support decision-making. Short-term prediction tends to be more accurate, and it may be worthwhile to postpone the response. However, relying on short-term prediction leaves little time for action implementation; a failed action may lead to a disastrous result.

Therefore, after observing a threat, a question is raised: when is the optimal time to respond? In other words, what is the optimal prediction horizon to use? We call this as the “optimal responding problem”. It is a question to both decision-makers and the academics because a responding decision must be made under partial predictability where it is possible to take observations and predict the behavior of the threat (e.g., the iceberg) but only to a certain degree. Here, the response time t_r is a time interval and it is defined in relation to time t_t when the hazardous situation terminates as illustrated by Fig. 1. The time point of response t_{pr} is the time to make & accept a prediction and implement a given response action to prevent accident. By default, a response includes making a prediction based on the information available, decision-making based on the prediction result, and a potential following action to reduce risks. An actual action will follow the decision-making if an accident is predicted to occur while no actual action is implemented if no accident is predicted.

Determining the optimal time to respond is a generic problem and is valuable for the planning of time-critical and expensive response

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actions, risk prediction and control during operation (e.g., flood [5], ship collision [6, 7], vehicle collision [8]), design of predictive and intelligent alarms [9], threat management, emergency response [10–12] (such as when we stop watching a hurricane and start evacuation), system health monitoring and response [13–15]. Ambiguity in when to respond can lead to action delay and eventually cause incident or accident [16]. Solving this problem also bring benefits to action planning in autonomous systems which can take in and process information from the environment to make decisions themselves.

In practices, the problem of when to respond to a threat is resolved by risk monitoring, experience, or regulation rules. An example is response to ships on collision course with an offshore installation in the North Sea, where there are fixed time limits for various response actions (alarm, mustering, evacuation). As for how rules are set, the rationale is often not very clear. When it comes to risk monitoring, a risk threshold is usually assumed to exist to trigger an response action [17]. Such a threshold is like an alarm trip point. If risk exceeds the threshold, action should be taken, otherwise not. To formulate a good rationale to determine when to respond to threat, how far in the future we are predicting/monitoring the risk, how soon the threat will propagate into an accident (which is called the process safety time in the process industries [18]), or the availability of actions when the threshold is breached [19] should also have been considered. Overall, rationale for when to respond and approach to derive optimal response time are needed so that risk can be further minimized, and system safety can be maintained at a better cost-efficient manner.

1.2. Time of response

In the field of psychology and neuroscience, speed-accuracy tradeoff [20, 21] is used to describe the complex relationship between an individual's willingness to respond slowly and make relatively fewer errors compared to the willingness to respond quickly and make relatively more errors. The time interval until the decision-maker decide from the presentation of stimulus is called the choice reaction time [22–24]. From the information theory and behavioral decision-making points of view, there is a cost in spending time for information accumulation [25, 26]. A decision-maker tends to respond later if accuracy is emphasized and tends to decide early if speed is emphasized. If high discriminability can be reached early, the decision-maker is able to decide early [27].

Following the speed-accuracy tradeoff, this paper proposes a novel rationale, and a mathematical problem formulation to determine when to respond to threat. The key to determine whether to respond immediately or to postpone is whether future updated information will give a more reliable prediction to warrant the postponing; what should be managed is the tradeoff between decreased cost by increased prediction accuracy, and increased cost by acting late, as illustrated in Fig. 2. The optimal time to respond is when the marginal expected gain of prediction is equivalent or smaller than the marginal cost of postponing action.

Information value theory has been used to quantify the added value from information and support rational decision making under uncertainty [28–34]. Value of information (VoI) is recommended for sensitivity analysis for reliability analysis [33], probabilistic safety assessment [29] and safety management [35]; here, a sensitivity analysis is to evaluate which input variables one should collect more information to reduce their uncertainty. VoI is also used for making testing/inspection and maintenance decision [14, 32]. Prediction is to provide information. The value of prediction can be evaluated in a similar manner of information. A Value of (imperfect) Prediction (VoP) method based on information value theory is formulated to calculate the optimal response time quantitatively. The gain from an improved prediction can be evaluated by the cost reduction when the predicted result is used in decision-making. The cost of acting later can be modelled by the increased probability of failure of the action due to reduced time available. The optimal time to respond can be found by minimizing the total cost as the combination of gain from improved prediction by postponing and loss from acting later. This paper does the following:

- Develops a utility model of the tradeoff phenomenon.
- Proposes a solution for the "optimal responding problem" by modeling the tradeoff.
- Formulates the "optimal responding problem" and its solution through probabilistic mathematics. Such formulation provides a unified manner to describe and solve the problem and enables a general application of the solution.

1.3. Potential contributions of this research

This research provides a novel rationale to determine when to respond provided in this paper can reduce ambiguity in the timing of decision-making, thus, lower the possibility of decision error [16]. The methodological novel contribution of this research can be expected from the formulation of the "optimal responding problem" and the VoP model as a solution. Thus, the optimal response time can be derived.

In addition, the method proposed in this paper can also be used to answer the following questions:

- Whether can we rely on our prediction?
- How good must the prediction be to be usable or to provide additional value?
- How much can we invest in the response action?
- What is the risk threshold between making decisions based on prediction and making decisions without prediction (such as taking a precautionary approach or simply tolerating the risk)?

In the remainders of this paper, Section 2 describes the problem and its assumptions, mathematical representation, and proposed solution. Section 3 presents additional theoretical applications of the proposed VoP model. Section 4 describes a case study and result obtained by using the proposed method. Section 5 includes implications and discussions; and finally, Section 6 concludes the study.

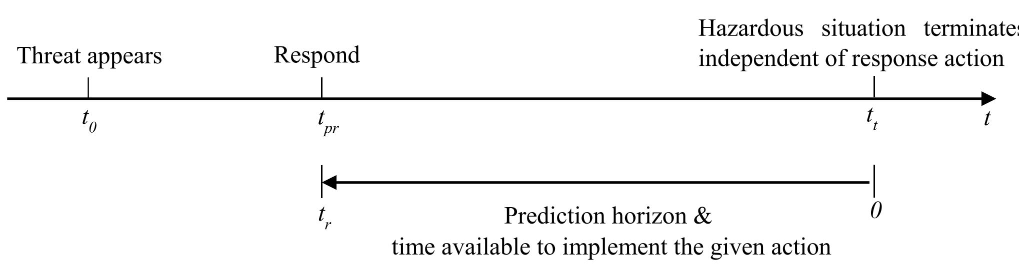

Fig. 1. Response time (time is not represented to scale).

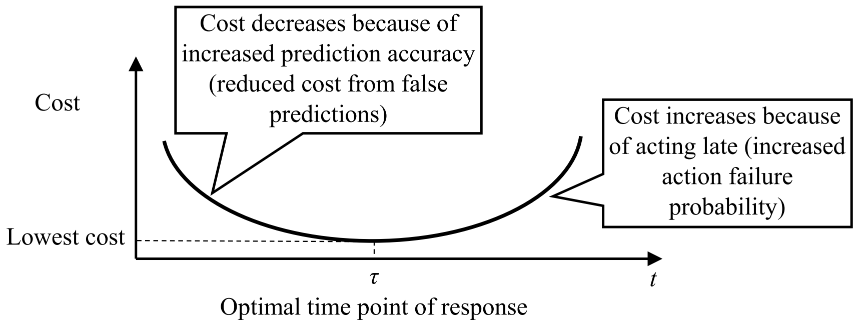

Fig. 2. Illustration of trade-off between prediction accuracy and action failure probability over time.

2. Mathematical description of the problem and solution

2.1. The optimal responding problem

To find the optimal response time, we can have the following general assumptions:

1. The threatening event terminates at time t_t, either because an accident occurs, or the threat is not relevant anymore.
2. The system state in the future is unknown. It is not clear whether an accident will happen or not until the last moment. However, the (statistical) probability of occurrence can be described.
3. The system state in the future is partially predictable, which means that the prediction of future system state is imperfect, and accuracy will be not 100%.
4. Predictability of system state improves when prediction horizon (how far ahead we predict) is shorter. Namely, we are more confident to discriminate "no accident" and "accident" when the threat is approaching.
5. Costs and chances of failure (fail to eliminate the threat under the period of time t_r available) of potential actions are known.
6. Under a certain prediction horizon t_r, the prediction model predicts whether the accident will happen or not (the prediction output is binary: "yes" or "no") at time t_r in the future. For instance, if the prediction horizon is 3 h, the model predicts whether the accident will occur or not in 3 h.
7. The decision-maker will respond immediately according to the prediction if the decision-maker trusts the prediction, meaning no time gap between prediction and response action.
8. The decision-maker only uses the result of one prediction to determine whether to implement the response action or not. Since predictions always are made based on accumulated and updated information, earlier predictions can never be more reliable than the later one. Using multiple (redundant) predictions will not improve accuracy either.
9. It is not possible to change the response after a decision has been made.

The termination time t_t may be known or not known. In the cases when we are only interested in a specific time, for example a deadline, or the execution time of a planned activity, the termination time t_t is considered known. For continuous operation, we are concerned with the whole operation period. We do not know when an accident will occur, and the termination time t_t is considered unknown.

- For the first scenario when the termination time t_t is known, the solution could tell us the optimal time is to respond (t before the specific termination time point t_t).
- For the second scenario when the termination time t_t is not known, a monitoring process is introduced assuming that the termination time is t_r time period ahead of current time in the future. The response action will only be taken when the prediction predicts "accident". The solution tells us what the best prediction horizon is for the monitoring.

When the prediction horizon **T** is a list of discrete values, the following elements are considered:

- **S** is a set of states, preliminarily expressed by ["accident", "no accident"], corresponding to accidental scenario and safe scenario. The cost associated with each state is expressed by C_{acc} and $C_{no\ acc}$ respectively.
- **R** is a set of response action alternatives [r_1, r_2, …, r_n], and their costs of implementation.
- $\mathbf{Pr_i}$ is a set of failure probabilities of action alternative r_i under various periods of time available.
- **P** is a set of prediction performances for different prediction horizons. True positive rate (TPR) and false positive rate (FPR) are used as prediction performance measurements. TPR is known as sensitivity, recall or probability of detection, while FPR is known as probability of false alarm. TPR and FPR can be derived from the prediction error matrix (also called confusion matrix) [36].
- **T** is a set of time (prediction horizon) alternatives.
- p_{acc} is the occurrence probability of an accident at termination time t_t conditional on threat presence; correspondingly, 1-p_{acc} is the occurrence probability of no accident.

The system starts in a safe state but with a probability p_{acc} to end up in an accidental scenario at termination time t_t. The decision-maker receives a result from the prediction model with a prediction horizon $t_r \in \mathbf{T}$, which tells whether the accident would occur or not at the future time point t_t (t_r time period into the future after prediction is made). If the prediction model predicts "accident", the decision-maker needs to take an action $r \in \mathbf{R}$ to prevent the system moving into the accident state. The failure probability of the chosen action is $p_{t_r,r}$. If the prediction model predicts "no accident", the decision-maker will not do anything. As shown in Fig. 2, the predicting and responding time will affect the cost, and the decision-maker needs to find an optimal response time t_r before the termination time t_t to minimize the expected cost of such threatening scenario. If there are multiple response actions available, the final solution of this optimal responding problem is a pair of t_r and r since the response time t_r is dependent on the response action r also. If t_r variable is continuous instead of discrete in the problem formulation, then p_r can be expressed as a function of t_r for response action r. *TPR* and *FPR* also can be expressed as functions of t_r. The notations used for the mathematical formulation are presented in Table 1.

2.2. Value of prediction (VoP) model

Information has properties like degree of precision, quality, and utility [37]. The same applies for prediction models (not limited to computational models). The Value of Information (VoI) [28] is usually

Table 1
Notations.

Notations	Descriptions
VoP	Value of (imperfect) prediction
C_{acc}	Cost of accident which is used to describe the consequence of accidental scenario.
$C_{no\ acc}$	Cost when there is no accident. $C_{no\ acc}$ is assumed to be 0 in this paper.
p_{acc}	Occurrence probability of the accident.
t_r	Response time; it is equivalent to prediction horizon and time available for action implementation.
r	Response action.
p_r	Failure probability of the response action r.
$p_{t_r,r}$	Failure probability of response action r when there is time interval t_r is available for implementation.
C_r	The cost of response action r. C_A, C_B represent cost of response action A, B respectively.
TPR	True positive rate.
TPR_{t_r}	True positive rate of the predictive model with prediction horizon t_r.
FPR	False positive rate.
FPR_{t_r}	False positive rate of the predictive model with prediction horizon t_r.
$LR+$	Positive likelihood ratio of the predictive model. It is the ratio between TPR and FPR.
$C(\text{with prediction}\|t_r, r)$	Expected cost of the threat response activity if response action r is the candidate response action and prediction made with prediction horizon t_r.
$C(\text{be precautionary}\|t_r, r)$	Expected cost of being precautionary and respond with action r at time t_{pr} with t_r time period available for action implementation.
$C(\text{ignore the risk})$	Expected cost if the risk is neglected.

calculated by the following formula:

VoI = "the value of decision situation with additional information" - "the value of current decision situation"

In this paper, since the information comes from prediction and the value is provided by cost reduction, we propose the concept of VoP based on VoI as:

VoP = "the cost of current decision situation without prediction" - "the cost of decision situation with prediction"

The optimal time to respond arrives when VoP does not increase anymore, in another word, when the marginal VoP is 0 if the marginal VoP is a monotonic function with time, where:

Marginal VoP (ΔVoP) = "the cost of current decision situation with prediction" - "the cost of decision situation with new prediction with a unit time shorter prediction horizon"

When prediction model is not used, the decision maker has the option of being precautionary and taking action immediately or the option of tolerating the risk so that no action is taken, as illustrated in Fig. 3. Application of a precautionary approach [38, 39] may lead to waste of resources. When the risk is too low for a response action, ignoring the risk is preferred. The cost of ignoring the risk is expressed by Eq. (1). The cost of being precautionary is expressed by Eq. (2). Failure probability of the response action can be assumed to be 0 when the response action is taken early. Note that respond precautionarily does not mean that the response is early enough for the action failure probability to be 0.

$$C(\text{ignore the risk}) = (1 - p_{acc}) * C_{no\ acc} + p_{acc} * C_{acc} = p_{acc} * C_{acc} \tag{1}$$

$$C(\text{be precautionary}|t_r, r) = (1 - p_{acc}) * C_{no\ acc} + p_{acc} * p_{\ t_r,r} * C_{acc} + C_r = p_{acc} * p_{t_r,r} * C_{acc} + C_r \tag{2}$$

The optimal response should be the option with the minimum cost among $C(\text{be precautionary}|t_r, r)$, $C(\text{ignore the risk})$, and $\min(C(\text{with prediction}|t_r, r))$. The objective of the optimal responding problem is to find the $\mathbf{t_r}$ for a given action r which minimize expected cost as shown by Eq. (3).

$$\text{Optimal } C(t_r, r) = \min\left\{\begin{array}{c} C(be\ precautionary|t_r, r) \\ C(\text{ignore the risk }) \\ min(C(\text{with prediction}|t_r, r)) \end{array}\right\} \tag{3}$$

For a two-states ("accident, "no accident") system, there are four kinds of scenarios considering the prediction model is imperfect. The four kinds of scenarios are:

- True Positive (TP): actual "accident" state which is correctly predicted as "accident".
- False Negative (FN): actual "accident" state which is wrongly predicted as "no accident".
- False Positive (FP): actual "no accident" state which is wrongly predicted as "accident".
- Ture Negative (TN): actual "no accident" state which is correctly predicted as "no accident".

The cost of each scenario is not same. Fig. 4 illustrates that the decision process and all the possible consequences after a response decision is made based on a prediction with prediction horizon $\mathbf{t_r}$. Both the distribution of the four prediction scenarios and the distribution between successful & unsuccessful action are dependent on $\mathbf{t_r}$. The expected cost C(with prediction) can be expressed conditionally on taking the response action $\mathbf{r}$ at $\mathbf{t_r}$ time ahead of $\mathbf{t_t}$ as shown by Eq. (4).

$$C(\text{with prediction}|t_r, r) = Expected\left(C^{TN}_{t_r} + C^{FN}_{t_r} + C^{FP}_{t_r,r} + C^{TP,S}_{t_r,r} + C^{TP,F}_{t_r,r}\right) \tag{4}$$

In Eq. (4), $C^{TN}_{t_r}$ is the cost of true negative (TN) prediction with cost $C_{no\ acc}$; $C_{no\ acc}$ can assumed to be 0 because there is no accident; $C^{FP}_{t_r,r}$ represents the cost of false positive (FP) prediction taken with a response time t_r; $C^{FN}_{t_r}$ is the cost of false negative (FN) prediction; $C^{TP,S}_{t_r,r}$ represents the cost of true positive (TP) prediction and action r is successful while $C^{TP,F}_{t_r,r}$ represents the cost of true positive (TP) prediction but the action r fails. To calculate the expected cost, the percentages of the four prediction results can be obtained from the confusion matrix [36]. Each cell in the confusion matrix shows the number of false positives, false negatives, true positives, and true negatives like in Table 2. Prediction performance measurements true positive rate (TPR, see Eq. (5)), false

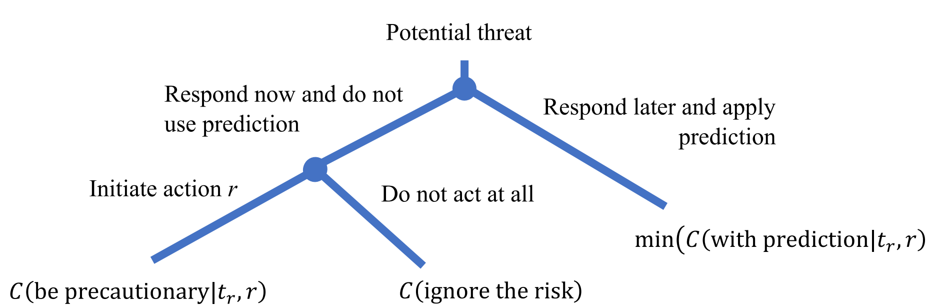

Fig. 3. Illustration of decision tree for determining whether to apply prediction and which prediction horizon to use.

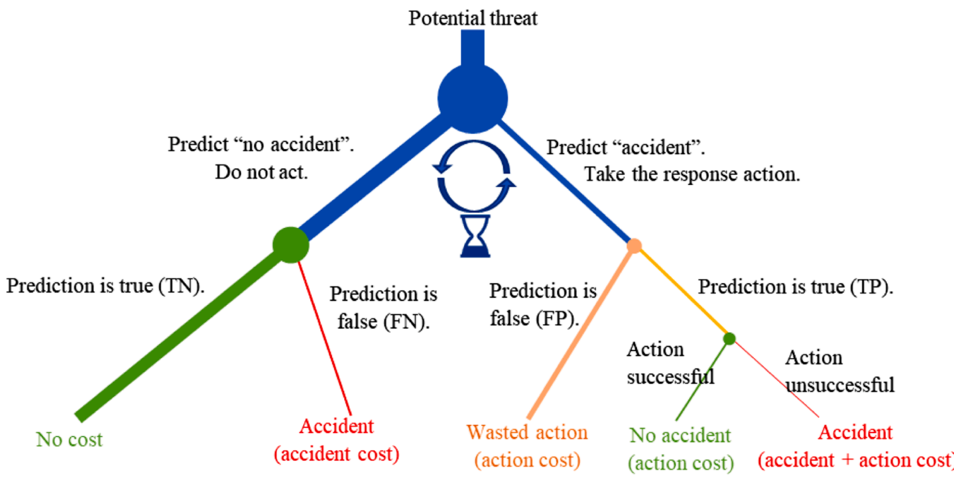

Fig. 4. Decision tree diagram involving prediction for threat management.

Table 2
Confusion matrix.

		Actual	
		Accident (positive)	No accident (negative)
Predicted	Accident (positive)	# True Positive (TP)	# False Positive (FP)
	No accident (negative)	# False Negative (FN)	# True Negative (TN)
	Sum	# Positive (P)	# Negative (N)

positive rate (FPR, see Eq. (6)) and prediction accuracy (ACC, see Eq. (8)) can be derived from the confusion matrix. Positive likelihood ratio (LR+) (see Eq. (7)) which is used to assess the usefulness of a prediction [40] can also be derived.

True positive rate:

$$TPR = \frac{TP}{TP+FN} = \frac{TP}{P} \tag{5}$$

False positive rate:

$$FPR = \frac{FP}{FP+TN} = \frac{FP}{N} \tag{6}$$

Positive likelihood ratio (LR+):

$$LR+ = \frac{TPR}{FPR} \tag{7}$$

Accuracy:

$$ACC = \frac{TN+TP}{FN+TN+TP+FP} = \frac{TN+TP}{P+N} = p_{acc} * TPR + (1-p_{acc}) * (1-FPR)$$
$$\text{where } p_{acc} = \frac{P}{P+N}; \textit{it is also called prior accident probability} \tag{8}$$

To include prediction performance measurement, the cost function can be altered into:

$$\begin{aligned}
C\ (\text{with prediction}) &= Expected\left(C^{TN} + C^{FP} + C^{FN} + C^{TP,S} + C^{TP,F}\right) \\
&= \frac{TN}{P+N} * C_{no\ acc} + \frac{FN}{P+N} * C_{acc} + \frac{FP}{P+N} * C_r + \frac{TP}{P+N} * [(1-p_r) * C_r + p_r * (C_r + C_{acc})] \\
&= \frac{1}{P+N}\{TN * C_{no\ acc} + FN * C_{acc} + FP * C_r + TP * [C_r + p_r * C_{acc}]\} \\
&= (1-p_{acc}) * \left[\frac{TN}{TN+FP} * C_{no\ acc} + \frac{FP}{TN+FP} * C_r\right] + p_{acc} * \left\{\frac{FN}{TP+FN} * C_{acc} + \frac{TP}{TP+FN} * [C_r + p_r * C_{acc}]\right\} \\
&= (1-p_{acc}) * [(1-FPR) * C_{no\ acc} + FPR * C_r] + p_{acc} * \{(1-TPR) * C_{acc} + TPR * [C_r + p_r * C_{acc}]\} \\
&= (1-p_{acc}) * [C_{no\ acc} + FPR * (C_r - C_{no\ acc})] + p_{acc} * [C_{acc} + TPR * (C_r + (p_r - 1) * C_{acc})] \\
&= (1-p_{acc}) * C_{no\ acc} + p_{acc} * C_{acc} + (1-p_{acc}) * FPR * (C_r - C_{no\ acc}) + p_{acc} * TPR * (C_r + (p_r - 1) * C_{acc})
\end{aligned} \tag{9}$$

$C_{no\ acc}$ is assumed to be 0. The cost function in Eq. (9) is a linear function with 6 parameters: p_{acc}, C_{acc}, FPR, TPR, p_r, C_r; p_{acc} and C_{acc} are the probability and cost of accident respectively. p_r and C_r are the failure probability and cost of response action r. The derivative of the expected cost of each input parameter is constant but associated with other input parameters meaning that the importance of each parameter to the expected cost is dependent on other parameters. For example,

$$\Delta C(\text{with prediction})/\Delta FPR = (1-p_{acc}) * (C_r - C_{no\ acc}) = (1-p_{acc}) * C_r$$

The importance of FPR to the expected cost C (with prediction) is dependent on the value of p_{acc} and C_r

Since FPR and TPR are dependent on the response time t_r, and p_r is dependent on the response time t_r and the chosen response action, the cost function C can be altered into $C(\text{with prediction}|t,r)$ in Eq. (10).

$$C(\text{with prediction}|t,r) = p_{acc} * C_{acc} + (1-p_{acc}) * FPR_{t_r} * C_r + p_{acc} * TPR_{t_r} * \left(C_r - \left(1-p_{t_r,r}\right) * C_{acc}\right) \tag{10}$$

When only t_r and its corresponding FPR_{t_r}, TPR_{t_r}, *and* $p_{t_r,r}$ are variants, minimizing $C(\text{with prediction}|t_r,r)$ is equivalent to minimize:

$$FPR_{t_r} + TPR_{t_r} * \frac{p_{acc}}{1-p_{acc}} * \left(1 - \left(1-p_{t_r,r}\right) * \frac{C_{acc}}{C_r}\right)$$

FPR_{t_r} and TPR_{t_r} are conditional on the prediction horizon and model used for prediction. They can be expressed as a function of time and prediction model, as shown by Eq. (11) and (12) correspondingly. According to the overall assumption, the prediction performance improves with shorter prediction horizon. The prediction improvement can be a result of multisensory integration and information accumulation cross over time. Therefore, FPR_{t_r} should decrease, while TPR_{t_r} should increase with shorter prediction horizon t_r.

$$FPR_{t_r} = f(t_r, model), \text{ with constraints } 0 \leq FPR_{t_r} \leq 1. \tag{11}$$

$$TPR_{t_r} = g(t_r,\ model), \text{ with constraints } 0 \leq TPR_{t_r} \leq 1. \tag{12}$$

The failure probability of a response action is dependent on the time available for implementation. Shorter time available implies higher failure probability. The relation between time and probability may be linear, but more complex relationships are also possible. A logarithmic function and cumulative gamma distribution allow flexibility to model different relations by varying their parameters. In Eq. (13), a logarithmic function is used to express the relation between failure probability and time available. The Gamma distribution automatically constrains the failure probability between [0,1]. Eq. (14) express the action failure probability using the Gamma distribution.

$$p_{t_r,r} = \begin{cases} 0, & if\ t_r \geq c \\ b - d\log_x(t_r - a + 1), & if a \leq t_r < c \\ 1, & if\ t_r < a \end{cases} \tag{13}$$

or

$$p_{t_r,r} = 1 - \frac{1}{\Gamma(\mathrm{k})}\gamma\left(k,\ \frac{(t_r - a)}{\theta}\right) \tag{14}$$

The failure probability function and its parameters can be estimated from historical data, experimental data or from a physical understanding of the phenomenon. The failure probability of a response action can be dependent on factors such as equipment and manpower availability and threat condition. Conditional failure probability functions on other factors can be derived for real application.

When prediction is perfect ($TPR = 1$, $FPR = 0$), there will be no wasted actions (false positive prediction) and no undetected accident cost (false negative prediction). The only risk is from failure of response action. The potential value of prediction meaning cost reduction is maximized. The Value to Invest (VtI) in developing and improving the prediction model can be determined by the potential cost reduction through perfect prediction and number of applications. The expected cost of perfect prediction is expressed in Eq. (15).

$$C(\text{perfect prediction}|t_r, r) = p_{acc} * \left(C_r + C_{acc} * p_{t_r,r}\right) \tag{15}$$

2.3. *Cost function for a series of responses (a series of predictions and actions)*

An underlying assumption in the previous section is that there is only one response, namely only one prediction will be made, and one action will be taken. Such an assumption is often challenged. In many cases, the second action may be possible to be taken if the first action fails, formulated as strategy 1 below. An accident will occur only if both the first action and the remedial action fail. In addition, we can also make a second prediction if the first action fails and proceed further based on the second prediction with a shorter prediction horizon (higher accuracy because of new information available) to reduce unnecessary response action in case the first prediction is false positive (predicted “accident” but actual “no accident”), which is formulated as the strategy 2. New information can be e.g., that we see that the first action does not have the effect that we expected, other characteristics of the situation change in unanticipated ways. Before an accident occurs, further information may tell the decision-maker that the first prediction was false negative (predicted “no accident” but actual “accident”) and allow another response. For severe accidents with high consequence, the cost of false negative is very high. To reduce the possibility of false negative, we can take a second prediction to double check and make decision based on the second prediction, which is formulated as strategy 3. The time interval between the first and the second prediction should be long enough that possible new information will be available.

The response can be extended to a series of responses, as shown in Fig. 5. Five new cost functions are obtained as shown below. In this paper, we only consider a series of two actions, no further action C, action D etc. can be initiated after action B fails. The principle will be the same if more response actions are considered thus not discussed in this paper. To take action B, we assume that we can observe action A fails after some time so that it is possible to start action B. When it comes to prediction, the second prediction can be made after some time if the first prediction predicts “no accident” or can be made when action A fails or in either situation.

Strategy 1: Take action B without taking a second prediction after action A fails.

$$\begin{aligned} C(\text{strategy 1}) = {} & (1 - p_{acc}) * FPR_{t_{r1}} * \left(C_A + p_{t_{r1},a} * C_B\right) \\ & + p_{acc} * \left\{(1 - TPR_{t_{r1}}) * C_{acc} + TPR_{t_{r1}} * \left[C_A + p_{t_{r1},a} * C_B + C_{acc} * p_{t_{r1},a} * p_{t_{r2},b}\right]\right\} \end{aligned}$$

Strategy 2: Take action based on the second prediction after action A fails.

$$\begin{aligned} C(\text{strategy 3}) = {} & (1 - p_{acc}) * \left[FPR_{t_{r1}} * \left(C_A + p_{t_{r1},a} * FPR_{t_{r2}} * C_B\right)\right] \\ & + p_{acc} * \left\{(1 - TPR_{t_{r1}}) * C_{acc} + TPR_{t_{r1}} * \left[C_A + p_{t_{r1},a} * \left((1 - TPR_{t_{r2}}) * C_{acc} + TPR_{t_{r2}} * \left(C_B + p_{t_{r2},b} * C_{acc}\right)\right)\right]\right\} \end{aligned}$$

Strategy 3: Predict for the second time if the first prediction predicts “no accident” and respond based on the second prediction.

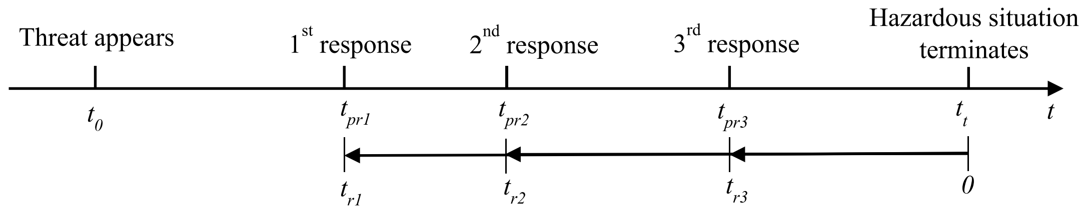

Fig. 5. A series of responses.

$$\begin{aligned} C(\text{strategy 2}) = {} & (1-p_{acc})*[(1-FPR_{t_{r1}})*FPR_{t_{r2}}*C_B+FPR_{t_{r1}}*C_A] \\ & +p_{acc}*\left\{(1-TPR_{t_{r1}})*\left[(1-TPR_{t_{r2}})*C_{acc}+TPR_{t_{r2}}*\left(C_B+p_{t_{r2},b}*C_{acc}\right)\right]+TPR_{t_{r1}}*\left[C_A+p_{t_{r1},a}*C_{acc}\right]\right\} \end{aligned}$$

Strategy 4: Predict for the second time if action A fails or the first prediction predicts "no accident".

$$\begin{aligned} C(\text{strategy 4}) = {} & (1-p_{acc})*\left[(1-FPR_{t_{r1}})*FPR_{t_{r3}}*C_C+FPR_{t_{r1}}*\left(C_A+p_{t_{r1},a}*FPR_{t_{r2}}*C_B\right)\right] \\ & +p_{acc}*\left\{(1-TPR_{t_{r1}})*\left[\left(1-TPR_{t_{r3}}\right)*C_{acc}+TPR_{t_{r3}}*\left(C_{A/B}+p_{t_{r3},a/b}*C_{acc}\right)\right]\right. \\ & \left.+TPR_{t_{r1}}*\left[C_A+p_{t_{r1},a}*\left(\left(1-TPR_{t_{r2}}\right)*C_{acc}+TPR_{t_{r2}}*\left(C_B+p_{t_{r2},b}*C_{acc}\right)\right)\right]\right\} \end{aligned}$$

Strategy 5: Be precautionary for the first response action and use prediction later if the first action fails.

$$\begin{aligned} C(\text{strategy 5}) = {} & C_A+p_{t_{r1},a} \\ & *\left\{(1-p_{acc})*\left[(1-FPR_{t_{r2}})*FPR_{t_{r3}}*C_B+FPR_{t_{r2}}*C_B\right]+p_{acc}\right. \\ & \left.*\left\{(1-TPR_{t_{r2}})*\left[(1-TPR_{t_{r3}})*C_{acc}+TPR_{t_{r3}}*\left(C_B+p_{t_{r3},b}*C_{acc}\right)\right]+TPR_{t_{r2}}*\left[C_B+p_{t_{r2},b}*C_{acc}\right]\right\}\right\} \end{aligned}$$

With action B available, the cost of being precautionary is also different. The new cost function is:

$$C(\text{ being precautionary } \mid \text{two actions}) = (1-p_{acc})*C_{no\ acc}+p_{acc}*p_{t_{r1},\ a}*p_{t_{r2},b}*C_{acc}+p_{t_{r1},a}*C_B+C_A \quad (16)$$

In the extended cost functions, we use:

- C_A is the cost of first response action A.
- $p_{t_{r1},a}$ is the failure probability of action A with t_{r1} time period available.
- C_B is the cost of second response action B if the response action fails.
- $p_{t_{r2},b}$ is the failure probability of action B with t_{r2} time period available.
- $C_{A/B}$ is the cost of either response action A or B.
- $p_{t_{r3},a/b}$ is the failure probability of action A or action B with t_{r3} time period available.
- $TPR_{t_{r1}}$, $FPR_{t_{r1}}$, $TPR_{t_{r2}}$, $FPR_{t_{r2}}$, $TPR_{t_{r3}}$, $FPR_{t_{r3}}$ are the prediction performance with prediction horizon t_{r1}, t_{r2}, t_{r3}.

With strategy 4, the first prediction is the originally assumed one; the second prediction with prediction horizon t_{r2} is made if action A fails or is made with a prediction horizon of t_{r3} if the first prediction predicts "no accident". Consequently, t_{r2} and t_{r3} must be smaller than t_{r1}. With strategy 5, the first response is taken precautionarily without prediction. Therefore, there is no $TPR_{t_{r1}}$, $FPR_{t_{r1}}$ in the cost function. When considering a series of responses, the optimal time for the earlier response is dependent on the planned later responses except when a precautionary approach is taken. An optimization of the later prediction and action is needed to obtain an overall optimal response. To calculate the best time to decide, a multivariate optimization algorithm, greedy searching method, or dynamic programming is needed.

3. Additional applications of the VoP model

3.1. Whether we should rely on prediction

Even though we know that the prediction is imperfect, we may still use it because it provides additional value or is better than not using it. Therefore, the key question is whether the prediction can improve our decisions. From Eq. (17), it can be found that to make the prediction provide additional value compared with simply ignoring the risk or being precautionary, the VoP should be at least greater than 0 or greater than the cost of applying the prediction model. If VoP > 0, then we should rely on the prediction, otherwise, not.

$$\text{VoP} = \min(\text{C(ignore the risk)},\ \text{C(be precuationary)}) - \text{C(with prediction)} \quad (17)$$

Positive likelihood ratio LR+ is one measurement of the usefulness of prediction, as calculated by Eq. (7) in Section 2.2. However, positive likelihood ratio LR+ does not consider the consequences associated with

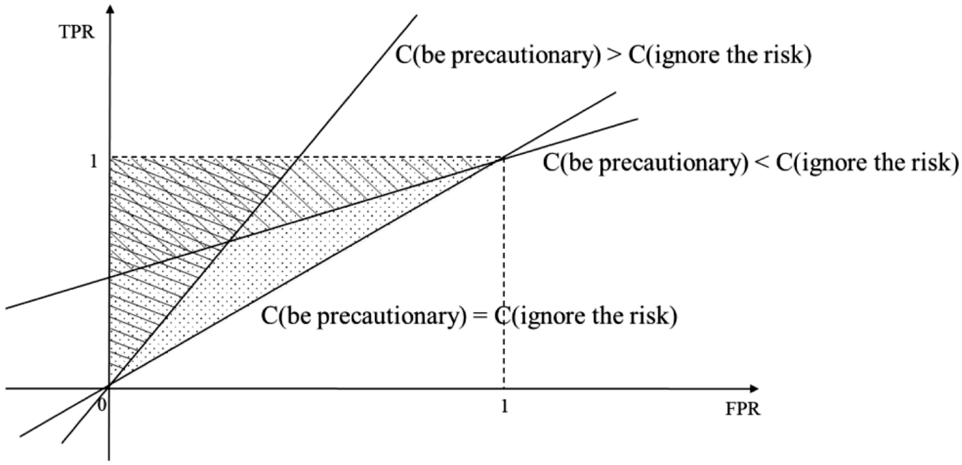

Fig. 6. Feasible regions of TPR & FPR which makes VoP >0.

different prediction results or the possibility of not relying on prediction. We therefore suggest prediction effectiveness as formulated by Eq. (18) to measure the usefulness of a prediction model. If the prediction effectiveness is larger than 1, the result from prediction is useful. Prediction effectiveness (PE) potentially can be used to evaluate the quality of diagnostic techniques.

$$\mathrm{PE} = \frac{\min(C(\text{ignore the risk}),\ \mathrm{C}(\text{be precautionary}))}{\mathrm{C}(\text{with prediction})} \tag{18}$$

The constraint function VoP > 0 can be also used to calculate prediction performance requirement (Section 3.2), the risk thresholds (Section 3.3), and the maximum investment in a response action (how much we can spend to prevent an accident with a known consequence) (Section 3.4).

3.2. Prediction performance requirement

At any time for a known threat and given response action, the minimal prediction performance requirement is that the FPR & TPR pair makes VoP > 0. The FPR &TPR requirement is dependent on three factors: accident probability, action failure probability, and the ratio between accident cost and action cost, as shown in Eq. (19).

Constraint function:

$$VoP > 0$$

$$\xrightarrow{yields}$$

$$\min(p_{acc} * C_{acc},\quad p_{acc} * p_r * C_{acc} + \ C_r) - [p_{acc} * C_{acc} + (1 - p_{acc}) * FPR * C_r + p_{acc} * TPR * (C_r - (1 - p_r) * C_{acc})] \rangle 0$$

$$\xrightarrow{yields}$$

$$\min\left(p_{acc} * \frac{C_{acc}}{C_r},\ p_{acc} * p_r * \frac{C_{acc}}{C_r} + 1\right) - \left[p_{acc} * \frac{C_{acc}}{C_r} + (1 - p_{acc}) * FPR + p_{acc} * TPR * \left(1 - (1 - p_r) * \frac{C_{acc}}{C_r}\right)\right] \Big\rangle 0$$

$$\xrightarrow{yields}$$

$$c > \ a * FPR + b * TPR \tag{19}$$

where $a = 1 - p_{acc}$, $b = p_{acc} * \left(1 - (1 - p_r) * \frac{C_{acc}}{C_r}\right)$, $c = \min\left(p_{acc} * \frac{C_{acc}}{C_r}, p_{acc} * p_r * \frac{C_{acc}}{C_r} + 1\right) - p_{acc} * \frac{C_{acc}}{C_r}$

The constraint function is linear, and the gradient of the constraint function is a constant. With additional constraints that $0 \le FPR \le 1$ and $0 \le TPR \le 1$, the feasible regions which make VoP >0 can be established. The feasible regions of FPR and TPR are different dependent on relative costs between being precautionary and ignoring risk. The shaded area with a distinguishable pattern is the corresponding feasible region for each, as shown in Fig. 6.

3.3. Risk thresholds

For a decision maker, a common situation is to determine which threats require a response and which threats can be tolerated. A threat should be responded if there is reasonably practical measure to reduce the risk even lower. Usually, we look at the potential consequence of the threat (accident cost), probability of the accidental scenario, or the combination, i.e., risk, to determine how to respond, as we do by using risk matrices.

Since the occurrence of accident is uncertain, being precautionary is a main principle to deal with the risk that we cannot afford to ignore [39]. The boundary condition between being precautionary and ignoring the risk can be derived from:

$$C(\text{be precautionary}) = C(\text{ignore the risk}) \tag{20}$$

Meaning:

$$p_{acc} * p_r * C_{acc} + \ C_r = p_{acc} * C_{acc}$$

According to Eq. (20), boundary values for accident consequence, probability, and risk between being precautionary and tolerate the risk can be obtained and they are:

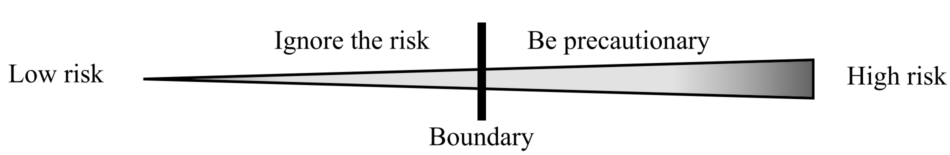

Fig. 7. Illustration of risk threshold between being precautionary and ignoring the risk.

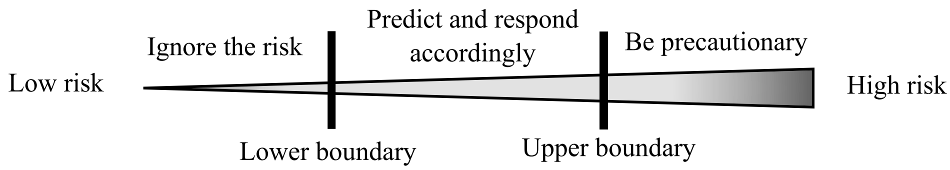

Fig. 8. New boundaries when there is partial predictability.

$$C_{acc} = \frac{C_r}{(1-p_r)*p_{acc}}$$

$$p_{acc} = \frac{C_r}{(1-p_r)*C_{acc}}$$

$$p_{acc}*C_{acc} = \frac{C_r}{(1-p_r)}$$

Decision maker should be precautionary and respond if p_{acc}, C_{acc}, or $p_{acc}*C_{acc}$ exceed the boundary value, which means being precautionary and respond is able to obtain a lower residual risk; otherwise, the risk can be tolerated and ignored as illustrated by Fig. 7. For example, consider the extreme case where the response action has 100% success probability, C_{acc}/C_r must be larger than $1/p_{acc}$ to make the response action rational and worth taken.

With prediction which provides positive value is involved, two new response boundaries are established, as illustrated by Fig. 8. One is upper boundary which is between being precautionary and trusting prediction; the other one is the lower boundary which is between trusting prediction and ignoring risk.

For the lower boundary, following the principle that C (with prediction) > C (ignore the risk), we get:

$$p_{acc}*C_{acc} + (1-p_{acc})*FPR*C_r + p_{acc}*TPR*(C_r-(1-p_r)*C_{acc})\rangle p_{acc} * C_{acc}$$

For scenario with potential accident consequence, the condition to tolerate the risk is:

$$C_{acc} < C_r * \left[\frac{(1-P_{acc})*FPR}{(1-p_r)*P_{acc}*TPR} + \frac{1}{(1-p_r)}\right] = C_r * \left[\frac{(1-P_{acc})}{(1-p_r)*P_{acc}*LR+} + \frac{1}{(1-p_r)}\right]$$

or

$$\frac{C_{acc}}{C_r} < \frac{1}{(1-p_r)} * \left[\frac{(1-p_{acc})*FPR}{p_{acc}*TPR} + 1\right] = \frac{1}{p_{acc}*(1-p_r)} * \left[\frac{(1-p_{acc})}{LR+} + p_{acc}\right]$$

When it comes accident probability for a known consequence, the condition to ignore the risk is:

$$p_{acc} < \frac{1}{1+\left[(1-p_r)*\frac{C_{acc}}{C_r}-1\right]*LR+} = \frac{C_r}{(1-p_r)*C_{acc}} * \frac{(1-p_r)*C_{acc}}{C_r+[(1-p_r)*C_{acc}-C_r]*LR+}$$

When it comes to risk (probability*consequence), the condition to ignore the risk is:

$$p_{acc}*C_{acc} < \frac{C_r}{1-p_r} * \left[p_{acc} + \frac{(1-p_{acc})*FPR}{TPR}\right] = \frac{C_r}{1-p_r} * \left[p_{acc} + \frac{1-p_{acc}}{LR+}\right]$$

When prediction is involved, the minimum accident cost, minimum accident probability, and risk to initiate the response has changed. The availability of partial predictability pushes down the risk acceptance level. When prediction is involved and $LR+ \; > 1$, the risk tolerance threshold is reduced for both probability and consequence. $LR+ \; > 1$ is not difficult to achieve when VoP>0. The consequence tolerance threshold is reduced by a ratio of $\left[\frac{(1-p_{acc})}{LR+} + p_{acc}\right]$, and the probability tolerance threshold is reduced by a ratio of $\frac{(1-p_r)*C_{acc}}{C_r+[(1-p_r)*C_{acc}-C_r]*LR+}$; the risk tolerance threshold is reduced by $\left[p_{acc} + \frac{1-p_{acc}}{LR+}\right]$. The reduction ratio is in contrary with LR+, which means the better prediction, the high reduction of the risk tolerance threshold. Therefore, when there is no prediction, the risk tolerance threshold is quite high in order to initiate the response action; the consequence of the accident must be quite high compared with the action cost. It is not rational to initiate a costly response action for a threat with low risk, which meets the common sense.

For the upper boundary, following the principle that C (with prediction) >C (be precautionary), we get:

$$p_{acc}*\frac{C_{acc}}{C_r} + (1-p_{acc})*FPR + p_{acc}*TPR*\left(1-(1-p_r)*\frac{C_{acc}}{C_r}\right)\rangle p_{acc}*p_r * \frac{C_{acc}}{C_r} + 1$$

For a threat with potential accident consequence, the condition to be precautionary is:

$$C_{acc} > \frac{C_r}{1-p_r} * \frac{1-p_{acc}*TPR-(1-p_{acc})*FPR}{(1-TPR)*p_{acc}} = \frac{C_r}{1-p_r} * \frac{\text{Predicted negative (FN + TN)}}{\text{False negative(FN)}}$$

When it comes accident probability with a known consequence, the condition to be precautionary is:

$$p_{acc} > \frac{1-FPR}{\frac{C_{acc}}{C_r}(1-TPR)*(1-p_r)+TPR-FPR} = \frac{C_r}{(1-p_r)*C_{acc}} * \frac{C_{acc}*(1-FPR)*(1-p_r)}{C_{acc}(1-TPR)*(1-p_r)+C_r(TPR-FPR)}$$

When it comes to risk (probability*consequence), the condition to be precautionary is:

$$p_{acc}*C_{acc} > \frac{C_r}{(1-p_r)} * \left[\frac{1-p_{acc}*TPR-(1-p_{acc})*FPR}{1-TPR}\right] = \frac{C_r}{(1-p_r)} * \frac{\text{Predicted negative(FN + TN)}}{\text{False negative(FN)} + \frac{1-p_{acc}}{p_{acc}}}$$

When prediction which provides positive value is involved, the thresholds of accident consequence, probability, and risk to be precautionary is pushed up. For accident consequence, the threshold increases by a rate of $\frac{1-p_{acc}*TPR-(1-p_{acc})*FPR}{(1-TPR)}$; the probability threshold is increased by a ratio of $\frac{C_{acc}*(1-FPR)*(1-p_r)}{C_{acc}(1-TPR)*(1-p_r)+C_r(TPR-FPR)}$; the threshold of risk level increases by a rate of $\left[\frac{1-p_{acc}*TPR-(1-p_{acc})*FPR}{1-TPR}\right]$. Therefore, it may not be necessary to be precautionary anymore if there is partial predictability that the decision maker can rely on for the same threat.

Risk thresholds established above only consider a single time point, namely the current time. The potentially improved predictability in the future is not considered. The established risk thresholds tell us "Should

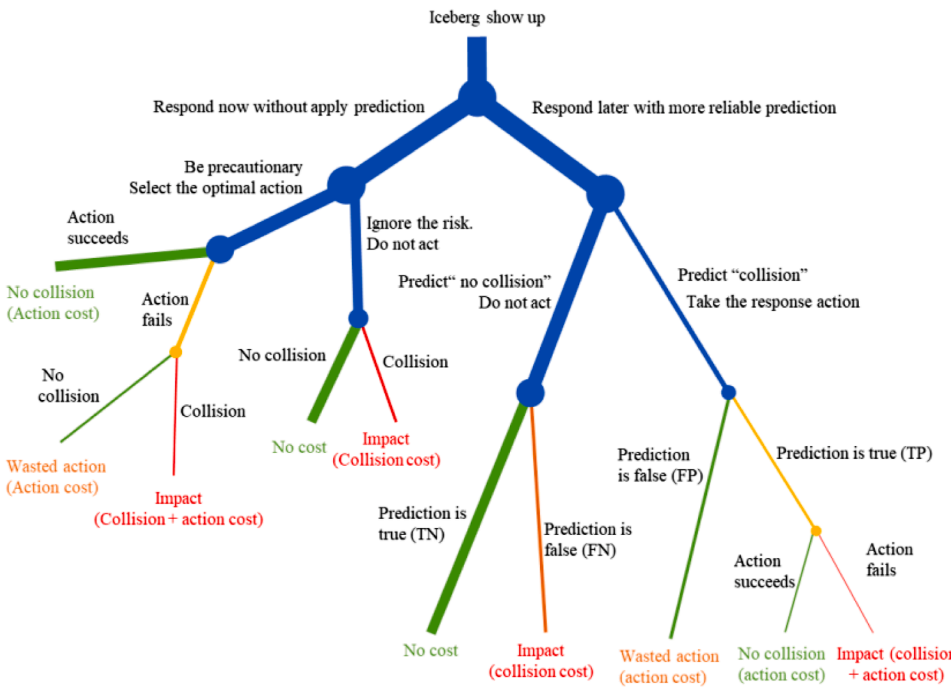

Fig. 9. Decision tree diagram involving prediction for iceberg management.

we ignore the risk, or respond based on prediction, or just be precautionary and take action immediately?" without considering the option of postponing decision. In the same way, risk thresholds between responding now and responding later considering the whole-time spectrum can be established in a similar way.

3.4. Maximum response investment

Another situation that the decision maker needs to determine how much cost to invest in response action for accident prevention of a known threat. For a threat with a potential accident consequence of C_{acc} and occurrence probability of p_{acc}, what is the maximum cost that should be spent on accident prevention? In this case, we should compare the cost of taking action based on imperfect prediction (or the cost of being precautionary) with the cost of ignoring the risk.

When partial predictability is not considered, the condition to respond is:

$$C(\text{be precautionary}) < C(\text{ignore the risk})$$

Meaning:

$$C_r < (1 - p_r) * p_{acc} * C_{acc}$$

The maximum cost we should spend in accident prevention should be $(1 - p_r) * p_{acc}$ times of the damage that the accident could cause, which is much smaller than the potential accident consequence. Assuming $p_{acc} = 0.1$ and $p_r = 0$, the maximum cost should be invested for accident prevention is only 10% of the accident consequence.

When the decision is made under partial predictability, the condition to respond is:

$$C(\text{with prediction}) < C(\text{ignore the risk})$$

Meaning:

$$C_r < C_{acc} * (1 - p_r) * \left[\frac{p_{acc} * TPR}{p_{acc} * TPR + (1 - p_{acc}) * FPR}\right] = (1 - p_r) * p_{acc} * C_{acc} * \left[\frac{LR+}{1 - p_{acc} + p_{acc} * LR+}\right]$$

The maximum cost we can spend in accident prevention is $C_{acc} * (1 - p_r) * p_{acc} * \left[\frac{LR+}{1 - p_{acc} + p_{acc} * LR+}\right]$. It is $\left[\frac{LR+}{1 - p_{acc} + p_{acc} * LR+}\right]$ times higher than the maximum responding cost when predictability is not considered. If the prediction is near perfect, the cost we can spend for accident prevention is slightly smaller than the damage of the accident. For accidents with a small probability, the investment in response action is much smaller than it would be if we do not consider the capability of prediction. The investment in response action increases after prediction capability is considered. Low response action investment based on risk (expected cost) is not reasonable anymore. Hence, predictability should be taken into account.

Table 3
Costs of accident and response.

Prior collision probability (p_{acc})	Collision severity	Cost factor (C_{acc})	Action type	Cost factor (C_r)
0.1	No collision	0	Disconnection	50
	Light collision	10	Towing	1
	Medium collision	300	–	–
	Severe collision	4000	–	–

4. Case study on iceberg management

Iceberg management for collision avoidance is a time-critical task. If an iceberg is detected and approaching a Floating Production Storage and Offloading unit (FPSO), the offshore installation manager (OIM) needs to decide whether and when to respond. Iceberg collisions can be extreme events, depending on the iceberg mass and the drifting speed. Iceberg drifting is observable physical process that depends on weather and ocean dynamics. This process can only be partially understood and modelled. Therefore, whether collision will occur or not is only partially predictable. There have been many studies of iceberg trajectory prediction for iceberg management to prevent iceberg-offshore structure collision [3, 41]. To handle threatening icebergs and prevent collision, different measures can be taken dependent on the size of iceberg,

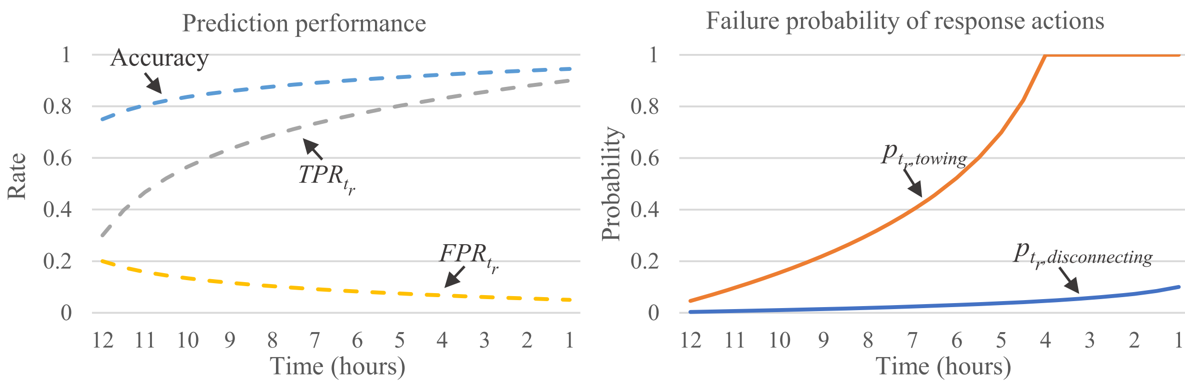

Fig. 10. Assumptions about prediction performance and towing failure probability in relation to time (prediction horizon).

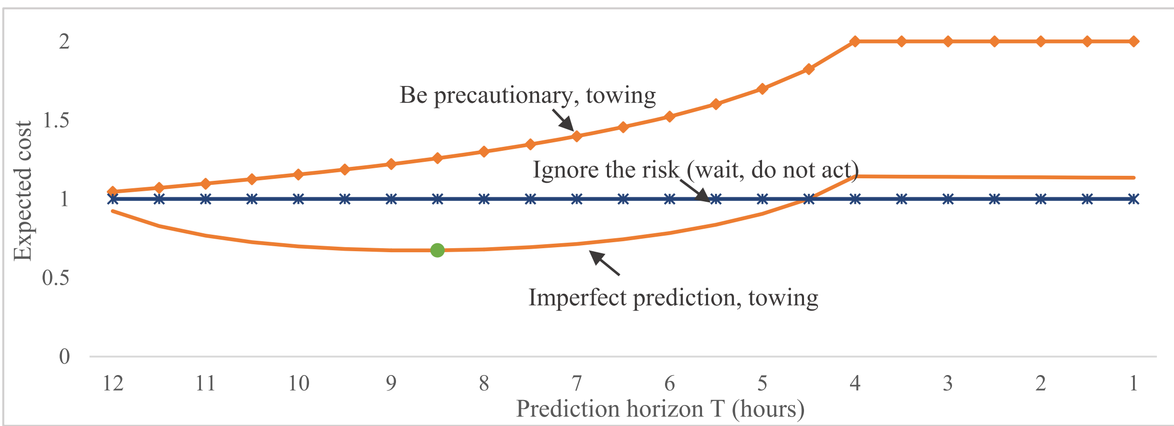

Fig. 11. Expected cost of response actions with prediction horizons for light collision. (Note: for light collision, the costs of disconnecting action are much higher than the costs of towing action and ignoring the risk. Therefore, disconnecting is not shown in this figure.).

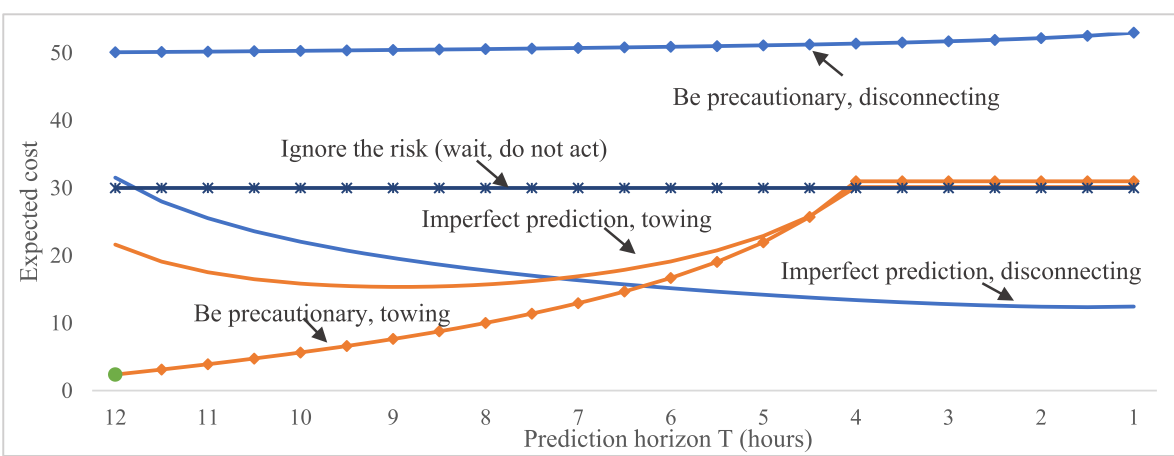

Fig. 12. Expected cost of response actions with prediction horizons for medium collision.

weather, and time available etc. [1, 42] It is common to monitor the iceberg and evaluate the risk of collision when the iceberg is far away [43, 44]. When the iceberg moves closer, and a collision is more likely to happen, an actual action is needed. For big and threatening icebergs, towing is the common response action. For small icebergs, water cannon and propeller washing are more commonly used. Disconnecting the FPSO is usually the last option, because of the high cost of disconnection [45]. Only towing and disconnection are considered in this case study.

4.1. *Preliminary analysis*

The decision-maker is to find the optimal response time and action to minimize cost. Fig. 9 illustrates the decision tree diagram for iceberg management. Being precautionary and ignoring the risk are included as options in the decision tree diagram.

4.1.1. *List of assumptions and input data*

The following assumptions in addition to those listed in Section 2.1 are needed in this case:

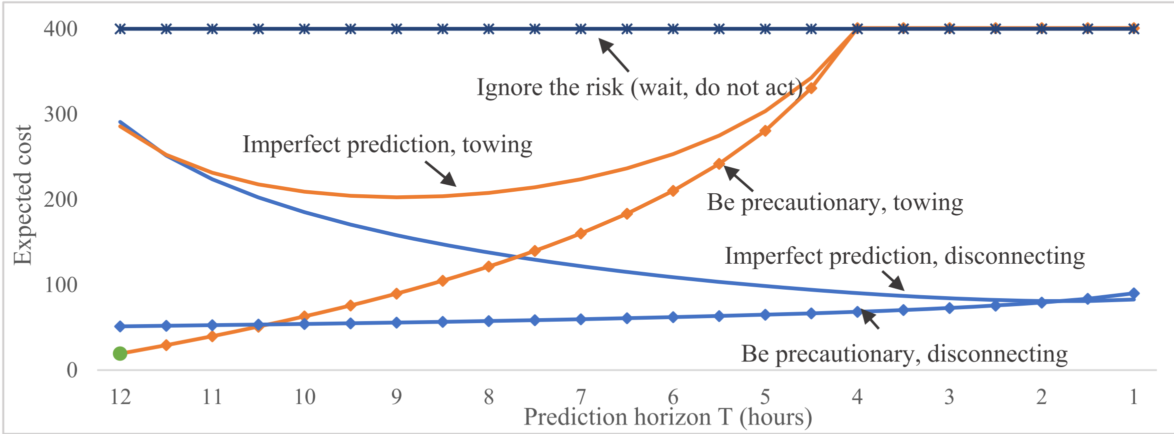

Fig. 13. Expected cost of response actions with prediction horizons for severe collision.

Table 4
Optimal prediction horison and action.

Collision severity	Optimal prediction horizon and action	Cost
Light	Towing the iceberg based on 8.5 hours' prediction	0.67
Medium	Being precautionary and towing as early as possible	2.4
Severe	Being precautionary and towing as early as possible	19.3

- The prediction performances are the same for the different types of icebergs.
- The prior collision probability p_{acc} is the same for all types of icebergs.
- Termination time t_t is unknown.
- Action costs are constant.
- The collision cost is dependent on the collision severity.
- The collision severity can be estimated with a satisfactory accuracy and is considered as known when an iceberg is detected far away and far earlier than the time to consider whether a response action is required. The collision severity is usually determined by the kinetic energy of the iceberg, which in turn is determined by the mass and velocity of the iceberg on the point of contact. However, the mass of the iceberg will not change much for 12 h because the deterioration rate is very small in cold region. The same applies to velocity. So, it is reasonable to assume the kinetic energy that the iceberg carries on the point of contact is the same as when it was observed.

Assumed prior collision probability, costs for different collision severities and response actions are presented in Table 3. The assumptions are made based on the authors' judgment, therefore may not be accurate. For the preliminary analysis, relative costs (unitless) are used instead of the absolute costs of actions and accidents. This will not influence the results.

Assumptions about the prediction performance and action failure probability relative to time are shown in Fig. 10. The longest prediction horizon considered is 12 h. The minimum prediction horizon considered is 1 hour because the minimal time required for disconnecting is around 40–50 min [45]; and the minimal time required for towing is 4 h. Eqs. (21) and (22) represent TPR_{t_r} and FPR_{t_r} respectively, based on assumptions. Eq. (23) expresses the $p_{t_r,towing}$. The assumption about $p_{t_r,towing}$ is made based on judgement in combination with industrial data from Rudkin, Young [1] and Randell, Freeman [43]. Disconnecting is a costly but reliable response action and therefore has a lower failure probability than towing [45]. Eq. (24) expresses the assumed $p_{t_r,\ disconnecting}$.

$$TPR_{t_r} = 0.3 + 0.6\log_{12}(13 - t_r),\ \ 1 \le t_r \le 12 \tag{21}$$

$$FPR_{t_r} = 0.2 - 0.15\log_{12}(13 - t_r),\ \ 1 \le t_r \le 12 \tag{22}$$

$$p_{t_r,towing} = \begin{cases} 1 - \log_{10}(9), & if\ t_r \ge 12 \\ 1 - \log_{10}(t_r - 3), & if\ 4 \le t_r < 12 \\ 1, & if\ t_r < 4 \end{cases} \tag{23}$$

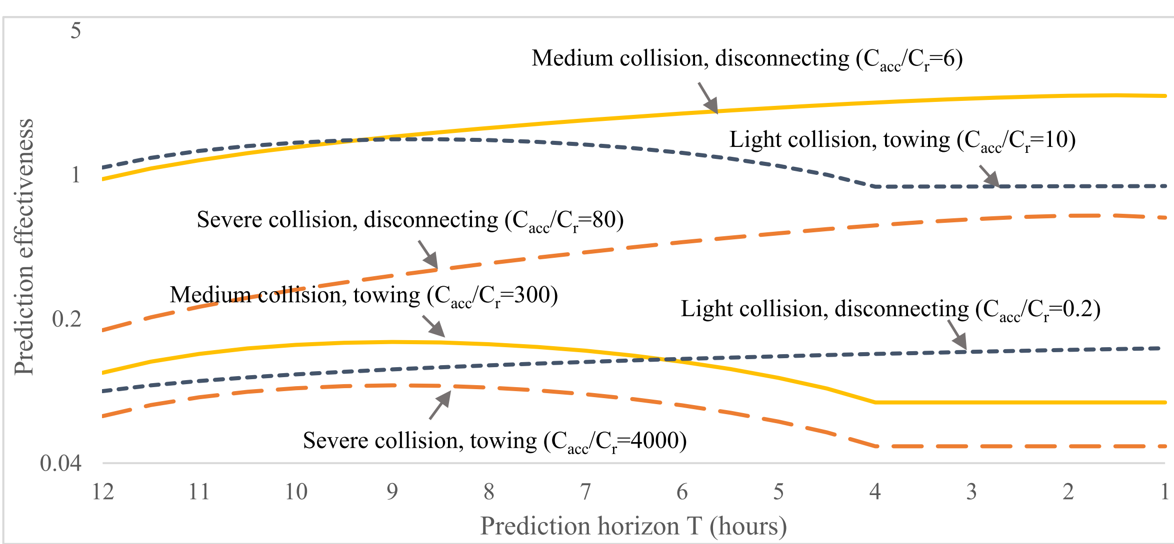

Fig. 14. Prediction effectiveness measured by the ratio between min (be precautionary, ignore the risk) and expected cost.

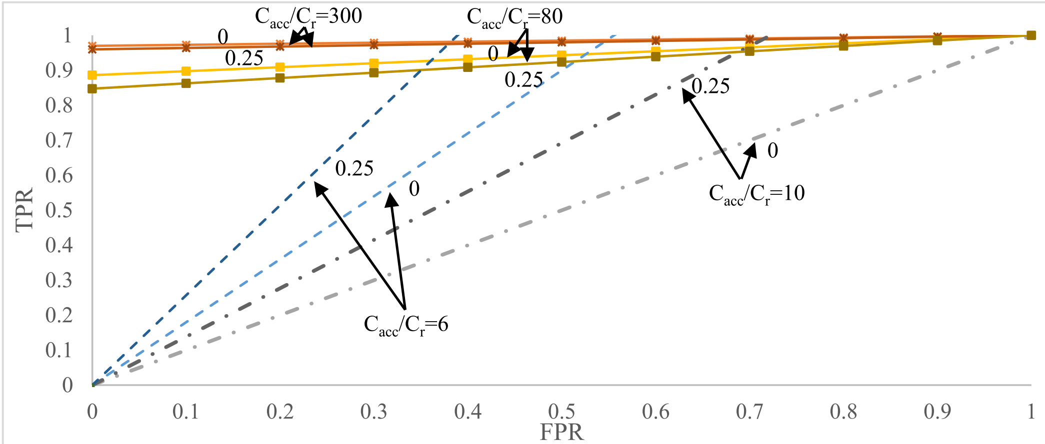

Fig. 15. Boundaries of TPR and FPR to enable positive VoP.

$$p_{t_r,\ disconnecting} = \begin{cases} 0.1 * (1 - \log_{13} 12), & if\ t_r > 12 \\ 0.1 * (1 - \log_{13} t_r), & if\ 1 \le t_r < 12 \end{cases} \quad (24)$$

4.1.2. Results from the preliminary analysis

In this section, we describe the cost picture for 3 collision scenarios in a rank of severity/consequence: light, medium, severe collision. The cost pictures in relation to prediction horizon *t* for those 3 collision severities are shown in Fig. 11, Fig. 12 and Fig. 13 respectively. In each figure, the expected cost of imperfect prediction with two types of response action, the expected cost of ignoring risk, and the expected cost of being precautionary (lines with square markers) are presented. The expected cost of ignoring risk is shown by the dark blue line with double cross markers. The orange lines are for the towing action, while the blue lines are for the disconnecting action. By comparing the expected cost of ignoring risk, being precautionary and trusting imperfect prediction for the two actions, the optimal response time can be found, which is highlighted by a green dot in each figure.

If the assumptions match the real situation, the decision-maker can respond according to the calculated results by selecting the right severity level. Table 4 summarizes the optimal prediction horizon and action including its cost for each collision scenario. For light collision scenario, the best option is towing the iceberg based on 8.5 hours' prediction. For a medium collision scenario, the expected collision cost is 30, which is between towing cost (1) and disconnecting cost (50). The best option is being precautionary and towing the iceberg away as early as possible if there is more than 7.5 h available. The suboptimal option is disconnecting based on imperfect prediction with 1.5 h prediction horizon. For severe collision, the best action at the best time is towing the iceberg anyway whenever detected if there is more than 10.5 h available. If there is less than 10.5 h available, the decision maker should be precautionary and initiate the disconnecting action. Postponing response and waiting for a better prediction is more costly than being precautionary in Fig. 13.

With the assumption that the response action failure probability is dependent on time available, and the prediction performance with different prediction horizon presented by Fig. 10, the decision cost of towing based on imperfect prediction matches the trade-off phenomenon, as illustrated by Fig. 2. Cost decreases with the increased prediction accuracy by using shorter prediction horizon in the beginning but increases if even shorter prediction horizon is used. This is because the increased failure probability of the towing action overweighs the benefit from improved predictability in the right part of the tradeoff curve. For disconnecting, the decision cost of disconnecting matches the left part of the trade-off curve because the failure probability of disconnecting does not increase as fast as the predictability.

It is better to be precautionary than to trust an imperfect prediction if the ratio of accidental cost and action cost is very high when considering the three collision scenarios. If the ratio between accident cost and action cost is small, it is better to ignore the risk than trust imperfect prediction. Applying and trust imperfect prediction is mainly useful for the case when accident cost and response cost are within a medium range, where the imperfect prediction to a large degree can provide added value (prediction effectiveness >1), as shown in Fig. 14.

The prediction performance requirement should vary for different accidental scenarios and response actions. Fig. 15 shows the boundary line of TPR and FPR which is calculated according to Eq. (16). For the

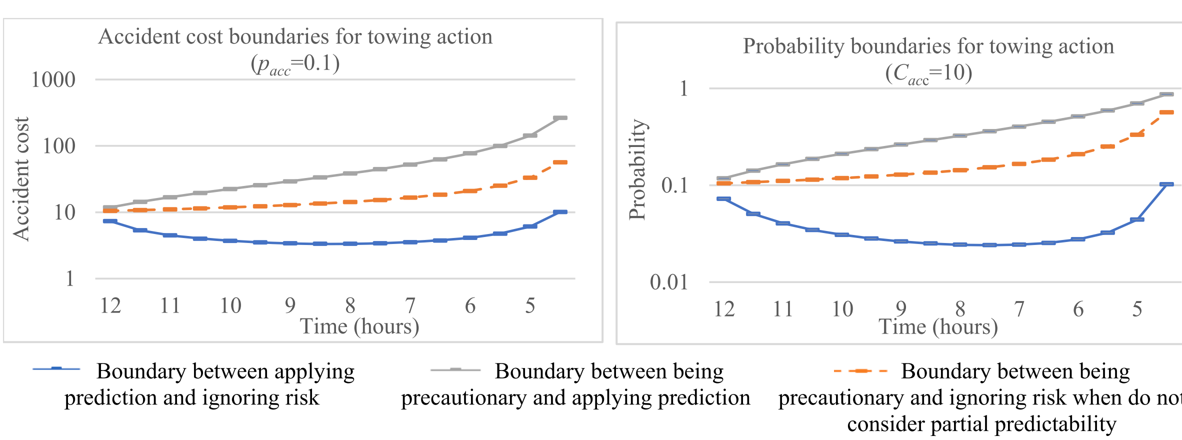

Fig. 16. Threshold of accident consequence and probability for towing action.

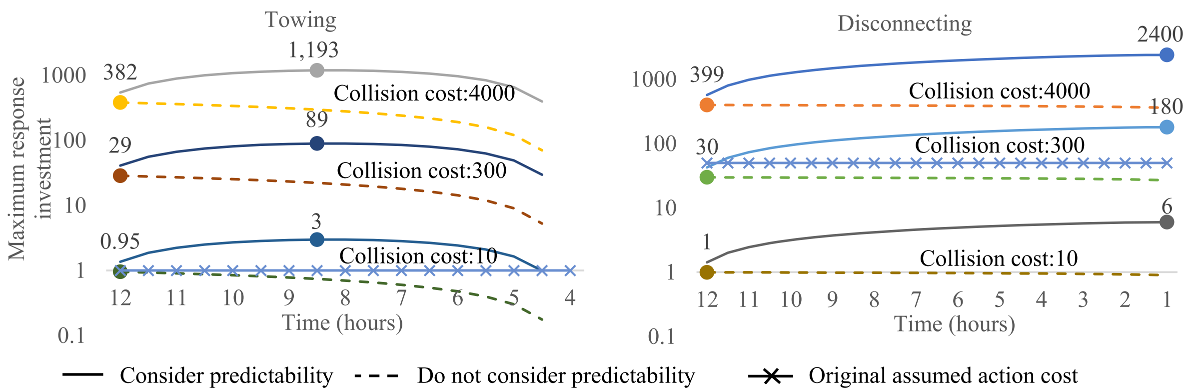

Fig. 17. Maximum response cost with/without considering predictability (the dot marks the maximum response investment).

Table 5
Optimal prediction horizon for optimal action with relation to collision cost.

Collision cost region	Response action (optimal time)
(0, 3.4)	Ignoring the risk
[3.4, 12.2)	Towing based on the optimal prediction horizon (8.5 h)
[12.2, 18.1)	Towing based on the optimal prediction horizon (9 h)
[18.1, 11,493)	Being precautionary and towing as early as possible when the iceberg is observed
[11,493, +∞)	Being precautionary and disconnect as early as possible

case with accident cost 10 and disconnecting cost 50, there is no valid FPR & TPR which enable disconnecting response because disconnecting has a higher cost. The left region above each line is the feasible region of TPR and FPR for each specific severity scenario and response action (the ratio of accident cost to response action cost) with respect to 0 and 0.25 failure probability. For a high ratio between accident cost and response action cost, there is a high requirement towards TPR (true positive rate). This is because a failure to predict a severe accident will lead to a huge loss. On the other hand, for a low cost ratio, there is a high requirement towards FPR (false positive rate). Each false positive prediction (false alarm) will lead to big loss because of the wasted action. When the cost ratios are 6 and 10, FPR requirement become stricter when the failure

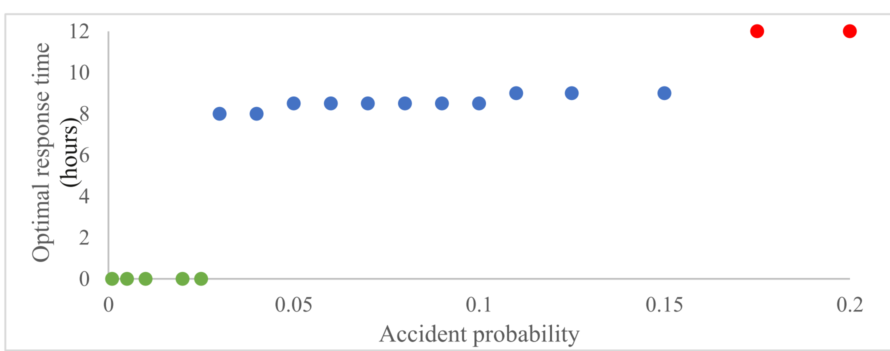

Fig. 18. Optimal response time of towing in relation to accident probability of light collision (collision cost is 10).

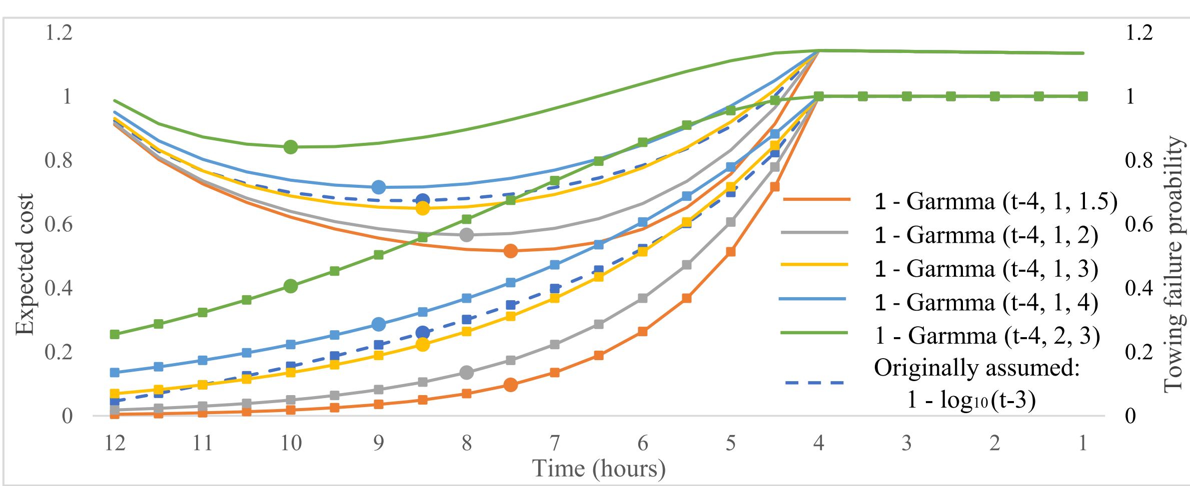

Fig. 19. Expected cost and towing action failure probability (the points with dot markers are the best responding time; the lines with markers are failure probability and the lines in the same color without markers are the corresponding expected cost).

probability increases. When the cost ratios are 80 and 300, TPR requirement becomes less strict when the response failure probability increases.

When prediction is not involved, taking action is the rational decision only if the cost ratio between accident and action is equivalent to or larger than 10 with assumed 10% chance of collision and 0% response failure probability. However, the cost ratio can be decreased when prediction is involved. With the assumed prediction performance, the accident cost threshold to disconnect the FPSO decreases to 84 (ratio =1.68) in the lowest (reduction factor of 4). For towing action, the accident cost threshold decreases to 3.4 in the lowest (reduction factor of 6.7). Fig. 16 presents the risk thresholds of towing action for ignoring risk and being precautionary when partial predictability is considered and when partial predictability is not considered. The thresholds of probability and accident consequence for ignoring risk is reduced while the thresholds for being precautionary is raised if partial predictability is considered.

At the same time, it is rational if more investment is required for the response actions with the assumed prediction performance, and response failure probability, as shown in Fig. 17. In Fig. 17, the maximum rational response investment is marked by dot. For light collision with a cost of 10, the maximum response investment is 0.95 which is lower than the towing action cost. Therefore, it is rational to ignore the risk when predictability is not considered. However, when partial predictability is considered, the maximum response investment increases to 3 which is higher than the towing cost. Then it is irresponsible to ignore the risk. The same occurs to disconnecting. For medium collision severity, it is better not to take disconnecting action when predictability is not considered since the maximum response investment is 30 and below the disconnecting cost 50. However, it worth to invest up to 180 for disconnecting when considering the predictability. The calculation provides rationale to judge whether the investment for accident prevention is reasonable or not.

4.2. Varying assumptions

Based on the results in 4.1.2, we can conclude that the solution provided by the VoP model is useful to find the optimal decision time point/prediction horizon for the planned response action. At the same time, the results show that the optimal decision time point is conditional on a number of factors, such as the ratio of accident cost to response cost, response failure probability, prediction performance, and accident probability. It is useful to know how the results change with different input data.

As the case study results, the optimal response time varies when it comes to different collision cost. Table 5 shows collision cost region and their optimal prediction horizons for the optimal action. It is rational to ignore the risk if when the collision cost is lower than 3.4 times of the towing cost. Otherwise, if the collision cost is equivalent or higher than 18.1 times the towing cost, it is rational to be precautionary and take action as early as possible rather than use prediction. In the middle range, waiting for time with better prediction available gives better output. This means that the prediction will not always provide additional value and be useful. The higher the accident cost, the earlier response the action should be taken, and a more reliable response action should be chosen.

The prior accident probability also has an impact on the response time. Fig. 18 shows the optimal response time in relation to accident probability of light collision given towing as the response action. It is better to ignore the risk and not respond if the probability is below and equal to 0.025 (the optimal response time is 0). Further, it is better to be precautionary and respond as early as possible when the accident probability is above and equal to 0.173 (the optimal response time is 12). The prediction horizon should be longer which means that response decision should be made earlier if there is a higher accident probability when the accident probability is between the two boundary values. However, the optimal response time does not change dramatically and is anyway between 8 and 9 h under the current assumptions. The result here is only valid when the prediction performance is not impacted by the accident probability. Whether accident occur or not is skewed and not evenly distributed and this has an impact on the prediction performance; it is harder to obtain the same prediction performance for a small probability event; the FPR will increase to achieve the same TPR.

It is originally assumed that towing failure probability is close to the average performance of towing. However, whether towing can be successful is also dependent on ocean conditions such as wave condition, visibility and iceberg size. Fig. 19 shows the costs with five additional towing failure probability-time functions for the light collision severity scenario. New optimal response time is obtained which is marked by the large dot on each curve. The optimal response time is sensitive to the increase in action failure probability with decreased time available.

So far, there is no data available about prediction performance of any collision prediction model. There is considerable uncertainty about the future prediction performance. It would be necessary to see the impact of different prediction performance on the optimal response time and cost to obtain some indication of the uncertainty. Fig. 20 and Fig. 21 show the optimal response time and it's cost in relation to prediction performance. Basically, there will be a lower cost and at the same time it is better to respond earlier if the prediction is improved; TPR and FPR do not have the same degree of impact on the cost and response time.

In the preliminary analysis, only one response (one prediction & one action) is considered meaning no further action will be taken when the chosen action fails. In the case of an approaching iceberg, we may still be able to disconnect the FPSO if towing fails. Therefore, the expected collision cost may not be as high as only considering a single action available. The same applies to the false negative (wrongly predicting "no

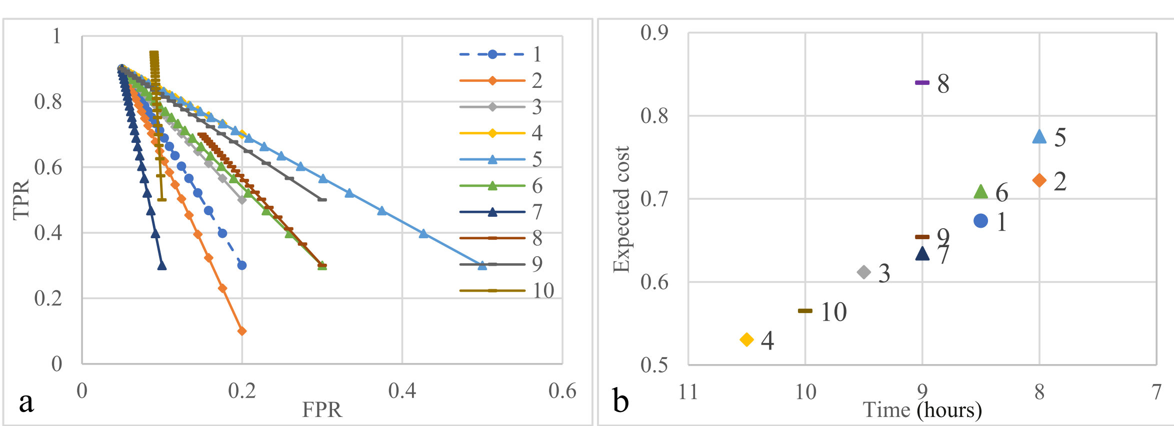

Fig. 20. Optimal prediction horizons for varied prediction performances (1 is the originally assumed).

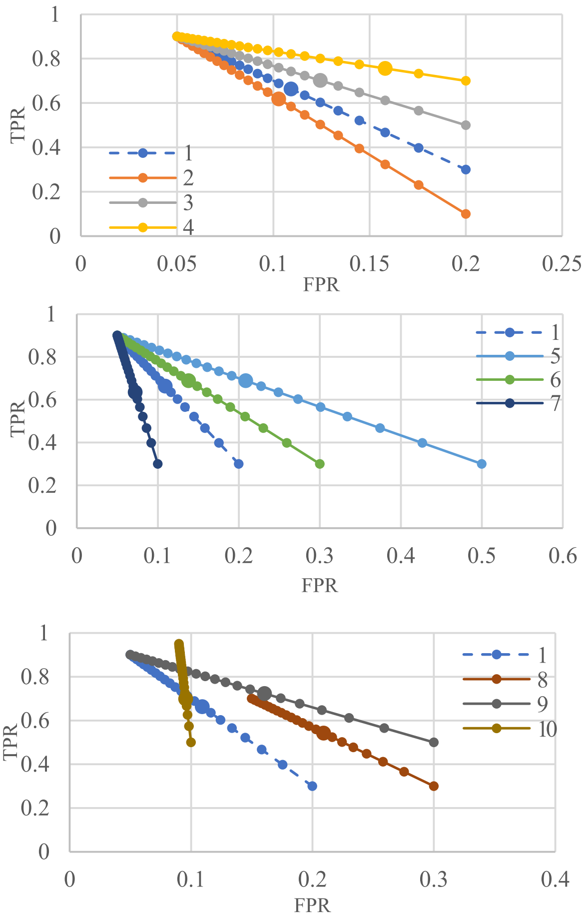

Fig. 21. Cost distribution and optimal prediction horizon in relation to prediction performance (the large dot markers indicate the optimal prediction horizon and 1 is the originally assumed prediction performance).

Table 6
Optimal response strategy and the expected costs.

Collision severity	Optimal strategy	Optimal prediction horizons and the sequence of actions	Cost	Lowest cost with single response
Light	Single prediction	Towing based on **8.5** hours' prediction horizon.	0.67	–
Medium	Strategy 5	Towing first, disconnecting based on **3** hours' prediction horizon. Take a second prediction with **1** hour' prediction horizon if the first prediction predicts "no collision".	1.6	2.4
Severe	Strategy 5	Towing first, disconnecting based on **5.5** hours' prediction horizon. Take a second prediction with **1** hour' prediction horizon if the first prediction predicts "no collision".	2.7	19.3

accident"). It is interesting to see the result if a "series of responses" (multiple predictions and actions) strategy is taken. A second prediction can be implemented with a new optimal prediction horizon, assuming that whether the first implemented action is successful or not can be observed after it has been implemented for 2 h. Table 6 summarizes the new optimal responses when consider multiple prediction and actions. Comparing with single prediction and action, taking a series of responses can make a large cost reduction for severe collision and the optimal response time changes also.

With the assumed data, for a light collision, making a single prediction with 8.5 hours' prediction horizon and towing based on the predicted results is the best option. For medium and severe collision scenario, it is optimal to take "strategy 5″ described in Section 2.3, which is to initiate the towing action immediately and take prediction for the disconnecting and take a second prediction if the first prediction predicts "no accident" to compensate for errors in the first prediction. However, the prediction horizons for the disconnecting actions are not the same. For medium collision, a 3 hours' prediction horizon should be used and take the disconnecting action if the prediction predicts "collision". The expected cost of this strategy is 1.6 and less than the precautionary approach with an expected cost of 3.3. When a severe collision is expected, the best option is to use 5.5 hours' prediction horizon and to determine whether to take the disconnecting action. The expected cost of this strategy is 2.7 and less than the precautionary approach with an expected cost of 3.5.

When the collision cost is as high as 106, it is beneficial to consider a combination of two responses comparing with a single response with towing as the response action. If the collision cost is higher than 7078, being precautionary and take the series of actions as early as possible without any prediction is the best. The towing cost is 1/10 of the light collision cost, 1/300 of the medium collision cost, 1/4000 of the severe collision cost. The towing action is very cheap, so prediction is not

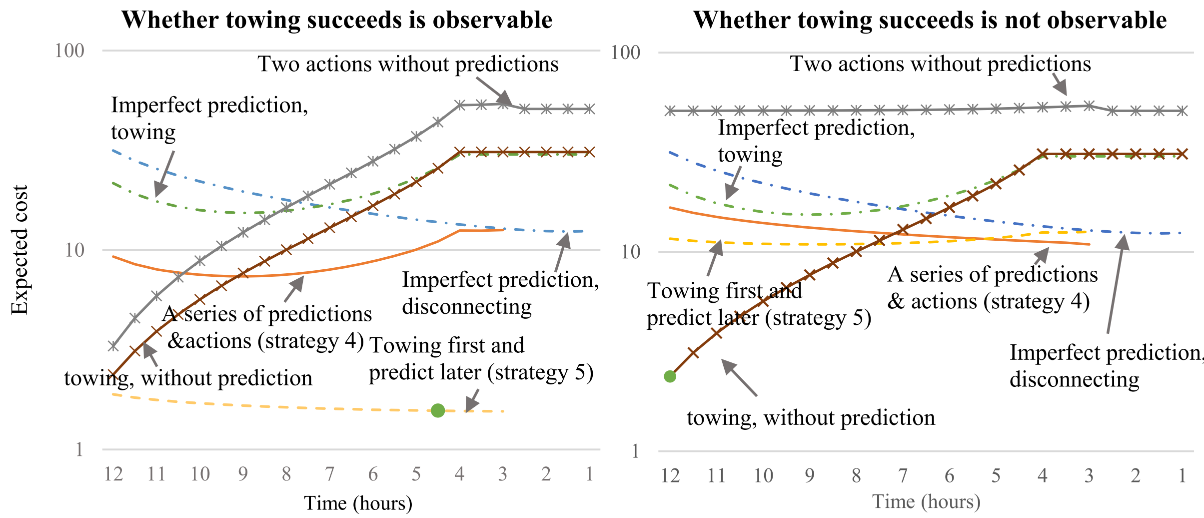

Fig. 22. Expected cost when the statues of towing action is observable and is not observable for the medium severity collision (Note: optimal response time is marked by the green dot.).

necessary to determine whether to tow for medium and severe collision. When a series of responses are planned, towing without prediction and only predict for the expensive disconnecting action and have a second prediction to compensate for wrong predictions. Prediction will be beneficial for the first towing action if the cost of the towing action is as high as 9.

Taking a series of responses can reduce the expected cost to a large degree when the accident consequence is severe, and the expected cost is much lower than a single response. For accident with severe consequences, expected cost of taking a series of actions without any prediction is much lower than the minimized cost when prediction is involved for a single action, which means that a low expected cost can be achieved by introducing a series of actions. Applying prediction and optimizing response time can reduce the cost even further. However, the expected cost will be higher as shown by Fig. 22 if the status (failure or success) of towing is not observable. Fig. 22 compares the cost pictures when the status of response action towing is observable and not observable for medium collision severity scenario. Comparison shows the importance of information related to the status of response action and cost reduction from the status information.

5. Implications and discussions

5.1. Implications for iceberg management

Considering an iceberg which is detected early and will cause a light collision, it is optimal to postpone the response and determine whether to tow the iceberg or not based on prediction made under 8–9 h prediction horizon. For a more severe collision, a series of responses would keep a low residual risk; towing should be initiated early. If the towing action fails, the decision whether to disconnect should be made based on 3–5 h prediction. To mitigate errors in the previous prediction, the last hour prediction can be made to determine whether disconnecting should be conducted if the previous prediction predicts “no collision”. If the observed iceberg is very large, the consequence of collision is extreme and disconnecting the FPSO immediately is the best option.

Overall, prediction with good performance can reduce uncertainty and save unnecessary response actions, which eventually reduces the overall cost. If high prediction performance can be achieved for a longer prediction horizon, response decision can be made earlier, and cost can also be reduced. Prediction models should therefore be further improved. The decision can be postponed if there are ways to make the response action effective faster and more reliable. When a series of responses is considered, the information about response action status is critical and can reduce cost.

The result in the preliminary analysis is strongly influenced by our input assumptions. In the analysis, we compared three scenarios of collision in a range of cost with the assumption that the action failure probability and action cost is independent of the accident severity (accident cost). This assumption may not be reasonable in all situations. For larger icebergs, the towing action failure probability with the same amount of time available may be increased so that the response decision should be made earlier. A new failure probability function conditional on iceberg size and weather condition etc. can be used to calculate a more realistic response time.

One assumption is that iceberg collision can happen in any future unknown time point with a probability. There is also a deterministic model made to estimate time to collide or time to closest point of approach (CPA). Therefore, the time length of interest can be reduced if the deterministic model is used in combination with the prediction model. A waiting period that does not require monitoring can be calculated.

5.2. Implications for generic scenarios

The method in this paper answers the question when a decision should be made, and it can be used further for automated decision-making such as in autonomous systems. It is not always beneficial to postpone response and wait for more information except when the cost ratio between accident and action is in a moderate range. Therefore, decision making is actually easy when it comes to severe consequences or small losses. Following the basic principles of “being precautionary” or “ignoring risk” is good enough. Optimization of when to respond is not necessary. The middle part is where the decision maker tends to hesitate and may miss the optimal chance so that calculating the optimal response time is helpful. Risk boundaries between respond now or later can be established accordingly.

The prediction performance of different prediction horizons has an impact on the optimal response time. However, this impact is limited and dependent on how sensitive the prediction performance is over prediction horizon and how fast the response reliability decreases with time available. The response strategies will be in contradiction with each other for the two extreme cases 1) prediction performance does not increase for shorter prediction while failure probability increases fast, 2) prediction performance increases quickly while failure probability does not increase. For the first case, it is better to respond as early as possible if the risk from the threat exceeds the tolerance threshold. For the second case, it is better to postpone and respond late as long as there is enough time available for the response action.

Partial predictability should be considered. When partial predictability is involved and provide a positive value, risk is reduced, the minimum accident cost (risk tolerance threshold) for taking the response action is also reduced, the maximum response investment is raised. During operation, it is reasonable and possible that some predictability about major accident can be obtained than the design phase. Therefore, it is rational if more resources are required for accident prevention during operation. The risk tolerance threshold should be lower during operation than the threshold used in the design phase for the same threat. As claimed by the ALARP principle, risk should be reduced to a level as low as reasonably practicable. At the same time, the threshold to be precautionary is raised. There are both pros and cons for the increased threshold for being precautionary. The pros are that unnecessary action is avoided, and the saved resources can be used to create other values. The con is the lower down safeguards because reduced expected cost is not the same as increased safety. A careful verification and justification of the predictability is necessary.

In addition, the cost function which models the trade-off can be extended to optimize other parameters instead of time, such as:

- Distance, to calculate the optimal safe distance to take action (such as brake, give way) or not.
- Degradation degree, to calculate the optimal degradation degree to conduct preventive repairs or replace components.
- Information input (accuracy-processing load tradeoff, accuracy-noise tradeoff), to calculate the optimal information input to form a judgement and initiate an action.
- Cutoff probability for risk monitoring. Many dynamic risk analyses are aimed at monitoring the accident probability and taking action when the probability increases to an unacceptable level. A cutoff probability needs to be defined to guarantee the best output (lowest cost).

The VoP model can be used to elicit prediction accuracy requirements for model development. In model development, it is a challenge to tune and select the optimal model to use. We foresee that the developed VoP model can be used for model development and a measurement of prediction performance (weighted performance indicator). The best model is the one that provides highest value of information instead of the highest accuracy. Because the costs of false positive and false negative predictions are different and "no accident"/ "accident" distribution is skewed, optimizing accuracy does not necessary obtain the least cost.

When a prediction model is used in a monitoring matter, a question we can ask is what the prediction frequency should be? In general, low prediction frequency (long monitor interval) can be used when an accident progresses slowly. In the case that the accident progresses faster, there should be a higher prediction frequency (shorter monitor interval). The VoP model perhaps can be used to optimize prediction frequency for accident monitoring also. If the resources required to obtain a prediction are high, this may be beneficial.

5.3. Information dynamics and use of information for accident prevention

New information does not necessary arrive by waiting. In addition, information does not necessarily enhance prediction performance. An important factor is how much predictability improves by waiting. Therefore, different possibilities and consequences should be included to determine whether waiting is beneficial in average. The method proposed calculates a statistical or Bayesian optimal response time which balances different possibilities.

In this paper, we discussed the increased prediction accuracy by waiting. Information about potential accident severity, weather conditions, barrier status etc. which influence response efficiency, can be collected also. Therefore, we may also include the prediction model and its error matrixes of a series of prediction horizons for potential accident severity and response efficiency etc. in the optimization model to obtain a refined optimal response time and action.

The value of information implicitly means the value of new information, but old information also plays a role in determining the value of new information. This implies that the time when information becomes available matters. In a real situation, we actively search for information and passively receive information from the environment. The understanding of importance of information dynamics may contribute to the design of an artificial information environment.

5.4. Limitations

This article excludes the situations where a threat is observed too late, or the accident occurs simultaneously with detection. In those cases, optimizing the time to respond is irrelevant because it is already too late. For the relevant and applicable situations, the challenges to the proposed method come from 1) input availability, 2) input accuracy and 3) assumptions. To calculate the optimal prediction horizon, time to decide, input data about the cost of accident, cost of action, action failure probability and prediction performance over time is required. Those inputs may not be easy to obtain. For example, we need information about the accidental scenario to know what action is suitable and resources we have to determine what action is feasible. The predictability-time function estimation needs a set of prediction error matrixes across all prediction horizons. For application, the function can be updated with new data from expert judgment, and/or experiments. Often, the cost of accident is difficult to estimate. However, the result will not change when the accident cost is extremely large (being precautionary is the best) or small (ignore the risk is the best). Accuracy in cost estimation is only required in the middle range.

In the article, we assumed that the prediction cost is negligible compared with accident cost and action cost. This may not be true in some cases, such as when it is expensive to obtain the inputs or expensive to construct a useful model. A prediction cost can be added to the calculation. In addition, accident cost and response costs are assumed to be known and constant. The model can be extended further considering costs as variants instead of as invariants. Varied prediction performance for each accident severity level can be included.

6. Conclusion

The observations made in this paper have wide application. For any risk control, response action must be taken ahead before accident occurs. When to respond, is a problem which needs to be answered for time-critical risk-control tasks. In this article, a Value of Prediction (VoP) model based on information value theory is proposed to calculate the optimal response time considering the trade-off between prediction performance and action failure probability over time. The optimal response time is dependent on the ratio between the accident cost and response action cost, accident probability, action failure probability, prediction performance and response strategy. An optimization of response strategy and time can maintain a low risk and high efficiency level. The results show that prediction does not always provide added value in accident prevention. When the accident consequence is extremely high, it is better to be precautionary and act as early as possible when the lowest failure probability is guaranteed, rather than trusting the prediction. When the consequence is comparable with the cost of response action, then it is rational to ignore the risk. This confirms common sense. Calculating optimal response time based on VoP is supportive for threat which pose a moderate risk. Other important implications of imperfect prediction are that it can push down the threshold of risk acceptance and raise up the threshold of being precautionary and maximum response investment. Therefore, ignoring partial predictability is not proper. The VoP model can also be used to derive the prediction performance requirement for prediction model development.

CRediT authorship contribution statement

Tiantian Zhu: Conceptualization, Funding acquisition, Methodology, Writing – original draft, Writing – review & editing. **Stein Haugen:** Conceptualization, Funding acquisition, Project administration, Supervision, Writing – review & editing. **Yiliu Liu:** Funding acquisition, Supervision, Writing – review & editing. **Xue Yang:** Supervision, Writing – review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data Availability

No data was used for the research described in the article.

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