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Assessment of the steady glide phase in ski jumping

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ABSTRACT

The purpose of this investigation was to compare how key variables of the steady glide phase relate to performance in the two hill sizes used in World Cup and Olympic competitions, i.e, normal and large hills. In this study, 38 and 33 jumps of elite ski jumpers were measured with a differential global navigation satellite system (dGNSS) on a normal (HS106) and large hill (HS140), respectively. For the steady glide phase, the average aerodynamic forces, lift-to-drag-ratio (LD-ratio), vertical and horizontal acceleration and velocity were measured and related to the jump distance as a performance outcome. The aerial time difference between the two hill sizes was 1.1 s, explained by the time spent in the steady glide phase. The results for HS106 were in line with the assumptions in recent literature, which propose that the performance is largely determined by the take-off and glide preparation. Hence for normal hills, skiers should aim to reduce vertical acceleration through high aerodynamic forces during the glide phase. Also, no correlation was observed between the LD-ratio and jump length. The data from the large hill indicate that the performance during the steady glide is very important for performance; hence clear differences were found compared to the normal hill. On a large hill, the aim should be to minimize the horizontal deceleration by reducing the aerodynamic drag. A high LD-ratio was correlated to jump length for HS140 and seen to be one of the most important performance factors.

1. Introduction

The aerial phase of a ski jump is the most eye-catching phase, which fascinates spectators and makes ski jumping one of the most attractive Olympic winter sports. It is also the longest phase of a ski jump and can be divided into three sub-phases: glide preparation, steady glide and landing preparation. The glide preparation, also known as early flight, is the phase from the release instant from the take-off table until the time point at which the relationship between the lift (F_L) and drag force (F_D), known as the lift-to-drag-ratio (LD-ratio) has stabilized. The steady glide phase lasts for as long as the LD-ratio is in a steady state, and the landing preparation ranges from where the steady glide ends until the ski jumper reengages with the ground (Elfmark et al., 2022). The take-off and glide preparation are considered the most important parts of a ski jump because the initial conditions for the steady glide phase are created here and these conditions are assumed impossible to correct at a later stage (Schwameder, 2008; Virmavirta et al., 2009; Ettema et al., 2020; Elfmark and Ettema, 2021; Virmavirta, 2016). Thus, there is some consensus that while a ski jumper cannot win with a good glide phase alone, a competition can be lost due to a bad one. It is worth noting that this assumption is made on a general basis from research mainly performed on normal hills (hill size (HS) 85–109 m). The average aerial time in ski flying (HS ≥185 m) can almost be 3 times as long as on a normal hill. Jung et al. (2014) found, in a simulation study, that the aerodynamic strategy of the ski jumper was the main component in ski flying and also had a leading role on a large hill (HS110-145 m), indicating that the aforementioned assumption may not hold for all hill sizes.

Research on the steady glide phase is scarce compared to the take-off and glide preparation, one reason being the former assumption of the importance of the steady glide and another the difficulties of establishing valid methods to collect data on ski jumpers moving through volumes as large as the entire aerial phase, at high speed. In the absence of field measurements, researchers have used computer simulations simulations and wind tunnel measurements to investigate how a glide posture can influence F_L, F_D and the LD-ratio and thus get more

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insight into the gliding phase (Gardan et al., 2017; Lee et al., 2012; Virmavirta and Kivekäs, 2019; Schmölzer and Müller, 2002, 2005; Müller et al., 1996; Jung et al., 2014, 2019). A well-suited, and recently used, method to provide accurate field measurements from the steady glide phase is differential global navigation satellite system (dGNSS) technology. By using a geodetic high-end dGNSS, it is possible measure the trajectory of a ski jump with ± 0.05 m global position accuracy, and to derive both velocity and forces from the collected data (Gilgien et al., 2014, 2013; Elfmark et al., 2022, 2021a). This has made it possible to investigate the steady glide phase from a physics perspective, rather than an athlete-action-centered perspective.

Given the limited research on the steady glide phase, the purpose of this study was to compare how key characteristics of the steady glide phase relate to performance in the two hill sizes used in World Cup and Olympic competitions, i.e., normal and large hills. This was done by using dGNSS measurements and a method to determine the start and end of the steady glide phase, recently proposed by Elfmark et al. (2022). Two data collections were conducted, one on a normal hill (HS106) and one on a large hill (HS140), where 38 and 33 jumps by ski jumpers normally competing in the World Cup (WC) and Continental Cup (COC) (next highest level) were measured, respectively. The data collections were used to compare how variables in the steady glide phase related to performance on the two hill sizes. We expected that the performance in the steady glide phase would be of greater importance on a large hill due to the longer time spent in this phase.

2. Methods

The data collections were carried out in Midstubakken (HS106, K-point: 95 m) in Oslo, Norway, and Lysgaardsbakken (HS140, K-point: 123 m) in Lillehammer, Norway. The shapes of ski jumping hills are regulated by the Federation Internationale de Ski (FIS) (2022) and therefore modern hills are very similar, including the hills used in this study that represent standard normal and large ski jumping hills. For HS106, 38 jumps were measured using dGNSS for 8 male ski jumpers, 4 at WC and 4 at COC level. The data collection was conducted over 4 separate training sessions over a period of 3 days. For HS140, 33 jumps were measured using dGNSS for 3 male ski jumpers, 2 at WC and 1 at COC level. The data collection was conducted over 4 separate training sessions over a period of 2 days. The study was conducted in accordance with the Declaration of Helsinki (WMA, 2001), and was approved by the Norwegian Centre for Research Data and the ethical committee of the Norwegian School of Sport Sciences.

2.1. dGNSS measurement

The athletes' head trajectories were captured using a dGNSS with a receiver carried in a backpack, an antenna mounted on the helmet and a base station positioned adjacent to the outrun of the hill. The dGNSS setup, together with the forces acting on the athlete in the steady glide phase, are illustrated in Fig. 1. GNSS signal reception through the GNSS antenna requires a direct line of sight to the satellites and is therefore constrained to being mounted on the head of the ski jumper.

Point mass kinematics and kinetics were derived for the entire ski jump from the dGNSS measurement. The antenna mounting point can be considered a reasonable representation of the athlete as a point mass as soon as the ski jumper has reached a steady posture (Elfmark et al., 2021a). For detailed information about the dGNSS measurement equipment, setup and calculation of glide variables the reader is referred to Elfmark et al. (2021a). The raw dGNSS positions were filtered with a weighted cubic spline filter, where position error estimates from the geodetic dGNSS proceedings were applied as weights (Skaloud and Limpach, 2003; Wägli, 2009). The cut-off frequency for position, first derivative and second derivative were set to 2 Hz, 3 Hz and 2 Hz, respectively. The proposed definition by Elfmark et al. (2022) was used to define the steady glide phase as the phase where the variations in the rate-of-change in the LD-ratio are within a bandwidth (τ) of $0.01\ \mathrm{s}^{-1}$ (Elfmark et al., 2022). The steady glide phase in each jump was found using an algorithm created in Matlab R2019b, where a search for the rate-of-change in the LD-ratio started at 40 m after take-off, and the start and end were defined as the first points before and after 40 m where the LD-ratio exceeded τ, respectively. For more information about the determination of the steady glide phase and filter settings the reader is referred to Elfmark et al. (2022).

2.2. Definition of performance variables

Average performance variables of the steady glide phase for each jump were related to the horizontal jump length. The horizontal jump length was chosen as the performance outcome as this was accurately measured by the dGNSS. The total jump length was also measured from video annotation as in a FIS competition. The lift and drag forces were defined as the forces acting perpendicular and opposite to the direction of motion (DoM) (Anderson Jr., 2010). The lift and drag forces were defined as

$$F_{L,D} = ma_{L,D} = \frac{1}{2}\rho v^2 C_{L,D} A, \tag{1}$$

where $a_{L,D}$ are the lift and drag acceleration, respectively, ρ is the air density, v the velocity of the ski jumper relative to the air stream, A the frontal area of the ski jumper and $C_{L,D}$ the lift and drag coefficients. This study used lift and drag acceleration, rather than force, to produce results that are independent of the athlete and equipment mass. The lift and drag acceleration were calculated from the horizontal (a_x) and vertical (a_y) acceleration measured by the dGNSS, as explained by Elfmark et al. (2022). As seen in (1), the aerodynamic acceleration is highly influenced by v. The velocity of a ski jumper increases through the latter part of the aerial phase, thus jump length and relative velocity are correlated factors due to gravity (Elfmark et al., 2021a). The lift and drag, extended from variables, defined as

$$\Omega C_{L,D} A = \frac{a_{L,D}}{v^2} \tag{2}$$

where $\Omega = \frac{\rho}{2m}$, were introduced to investigate the aerodynamic properties not influenced by velocity. Lift area ($C_L A$) and drag area ($C_L A$) are commonly used quantities in aerodynamic investigations (Müller et al., 1996; Schmölzer and Müller, 2002; Elfmark et al., 2021a). However, because air density and mass were unknown for our data, the exact $C_{L,D}A$ values could not be ascertained. The LD-ratio is the ratio between the lift and drag coefficients (C_L/C_D), as all the other variables from Eq. (1) are identical. The aerodynamic accelerations, $\Omega C_{L,D}A$ and the LD-ratio were investigated to test the hypotheses about hill size and the role of the steady glide phase.

As shown in Fig. 1, the direction of the aerodynamic forces is dependent on the DoM, which rapidly changes during the aerial phase (Schmölzer and Müller, 2005; Elfmark et al., 2021a). Hence, the way in which these forces and their interactions influence performance might be more complex than in sports such as alpine skiing and speed skating, where the sole aim is to reduce aerodynamic drag (Elfmark et al., 2021b). The horizontal and vertical acceleration component in Fig. 1 can be described as

$$a_x = -a_D\cos(\varphi) + a_L\sin(\varphi) \tag{3}$$

and

$$a_y = a_D\sin(\varphi) + a_L\cos(\varphi) - a_g, \tag{4}$$

where a_g is the gravitational acceleration. The angle φ indicates the angle between DoM and the horizontal plane and is not to be confused with the angle of attack (α), used to describe the angle between body posture and the horizontal plane (Müller et al., 1996; Gardan et al., 2017). The angle φ increases through the aerial phase, thus drag will have an increasingly positive contribution to a_y and lift to a_x (Eqs. (3) and (4)) throughout the glide phase. For further explanation, the reader

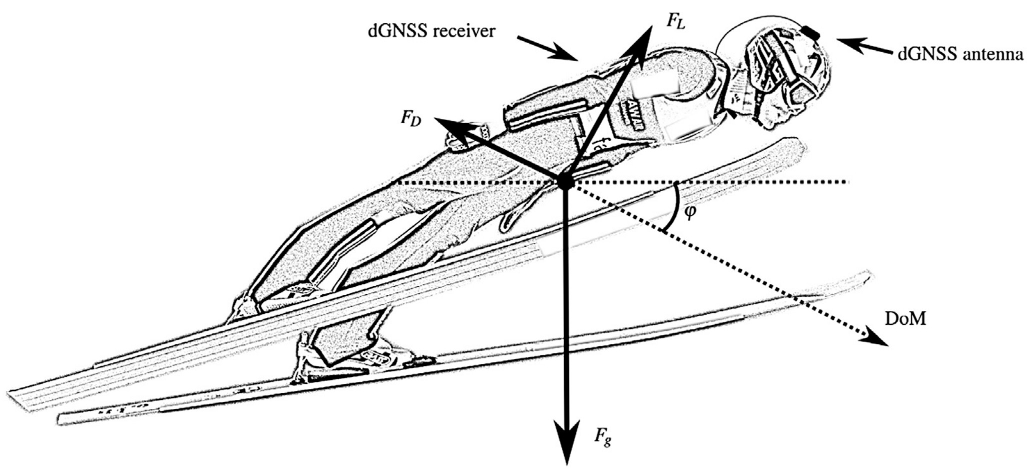

Fig. 1. Illustration of a ski jumper in the steady glide phase with the dGNSS antenna mounted on the helmet and the receiver in a backpack, carried under the ski jumping suit. The angle φ indicates the angle between the direction of motion (DoM) and the horizontal plane, F_g being the gravitational force and the lift (F_L) and drag force (F_D) acting perpendicular and opposite to the DoM, respectively.

is referred to Fig. A.6 in Appendix A, where the relationship between the aerodynamic forces and φ is illustrated, together with the average trajectory over all ski jumpers and φ through the aerial phase for both hills. This effect that gravitational force has on vertical (and resultant) velocity is regulated by the laws of physics and increases with jump duration; thus it may have an effect on jump length. To separate variables not naturally influenced by the jump length, special attention was paid to the horizontal velocity component, presented in the results section, while the vertical component and the resultant velocities are presented in Appendix B.

Table 1
Average time distribution of the aerial phase and horizontal jump distance for HS106 and HS140, presented as mean±SD.

	HS106	HS140
Total jump distance [m]	92.88 ± 5.95	124.04 ± 9.53**
Horizontal jump distance [m]	75.91 ± 4.91	102.94 ± 6.36**
Aerial time [s]	3.04 ± 0.20	4.12 ± 0.25**
Glide preparation [s]	0.76 ± 0.07	0.80 ± 0.05
Steady glide [s]	1.80 ± 0.22	2.87 ± 0.28**
Landing preparation [s]	0.49 ± 0.07	0.45 ± 0.05

**Indicates a statistically significant difference ($p < 0.001$).

2.3. Statistical analysis

Unpaired-samples t-tests were used to compare the average values over all ski jumpers of the glide characteristics between the hills. Statistical significance was set at an alpha level of 0.05. To assess differences for glide variables and jump length between the hills, linear regression analysis was performed. The regression lines of the two hills were compared using the regression on the pooled data of both hills with an F-test as described by Crowder and Hand (Crowder and Hand, 2017). The strengths (r values) of the relationships were compared between hills using the z-transformation. Where the F-test showed differences between hills, the regression slopes were compared using t-tests.

3. Results

Jumping distance and time distribution for both hills are displayed in Table 1. On average, the aerial time for HS140 was in average 1.1 s; significantly longer than for HS106. Significant differences were also found for the total and horizontal jump distances. Interestingly, between-HS differences were not found for glide- and landing-preparation: and hence the aerial time difference was solely explained by a significantly longer time spent in the steady glide phase.

The correlations between the horizontal jump length and average aerodynamic variables through the steady glide show dissimilar trends for the two hill sizes, as shown in Fig. 2.

For HS106, both a_L and a_D correlated positively with jump length. A correlation was also observed between a_L and jump length in HS140, but there was no correlation between drag and jump length. For $\Omega C_{L,D}A$, a similar but weaker relationship was observed for HS106. For HS140, no relationship was observed between jump length and $\Omega C_L A$ and there was a negative relationship (opposite to HS106) for $\Omega C_D A$. For all variables, significant differences were found between the hills.

The LD-ratio shows a correlation with jump length only in HS140 and the difference between the hills is significant (Fig. 3).

The absence of an LD-ratio relationship with jump length for HS106 is explained by the positive relationship of both $\Omega C_L A$ and $\Omega C_D A$. These drag and lift relationships were significantly different for HS140. The significant difference in LD-ratio between the hills, and the strong relationships for HS140, summarize and highlight how different the aerodynamic variables in the steady glide phase are and how differently they relate to performance for the two hills.

In Fig. 4, the dissimilarities in how the aerodynamic variables relate to performance are expressed by the horizontal and vertical acceleration components.

While a positive correlation between a_y and jump length was observed for HS106, a positive correlation was found for a_x for HS140. Interestingly, the ski jumpers could maintain an average $a_x \sim 0\,\mathrm{m\,s^{-2}}$ in the longest jumps in HS140, meaning that the mean velocity in the horizontal direction was maintained (or even increased). Significant differences were found between the hills for both variables.

While the correlation between the horizontal velocity component and jump length was maintained throughout the aerial phase in HS140, it diminished after the onset of the steady glide phase in HS106. The relationships between jump length and horizontal velocity at take-off, start, average and end of the steady glide phase are displayed in Fig. 5.

The correlations between horizontal jump length and velocity at take-off and start (Fig. 5) were similar and no significant differences were found between the hills for those variables. The correlation between horizontal velocity and jump length diminished for HS106 after the start of the steady glide phase, while it was sustained for in HS140. A significant difference between the hills was found for both the average horizontal velocity and the end velocity, supporting the findings from the horizontal acceleration measurements (Fig. 4).

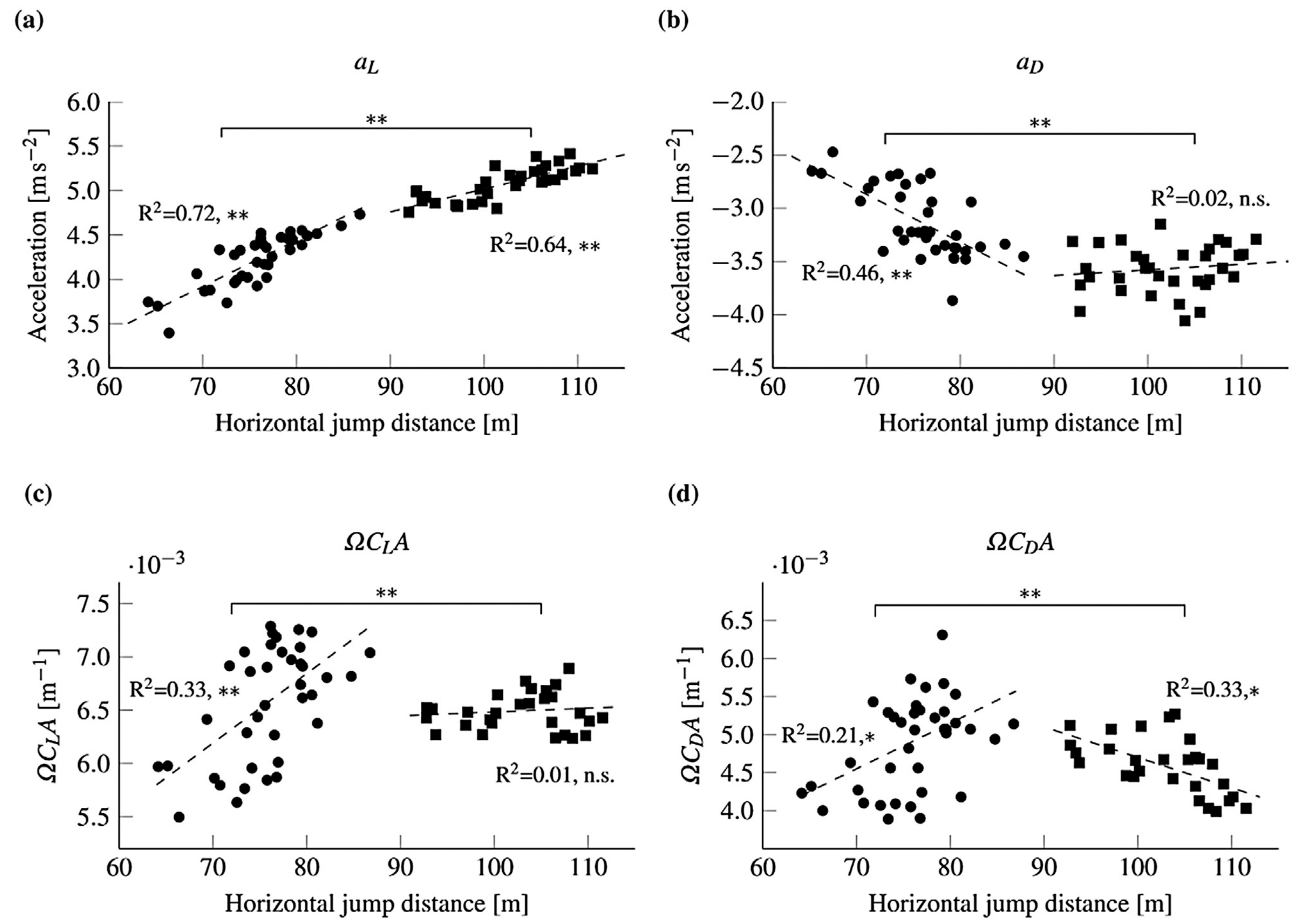

Fig. 2. Relationship between the average aerodynamic variables and horizontal jump length. **(a)** lift acceleration (a_L), **(b)** drag acceleration (a_D), **(c)** $\Omega C_L A$ and **(d)** $\Omega C_D A$. The measurements for HS106 are indicated with circles and for HS140 with squares. The linear relationships together with the coefficient of determination (R^2) are displayed for both hills. $*$ and $**$ indicate statistically significant differences ($p < 0.01$ and $p < 0.001$, respectively).

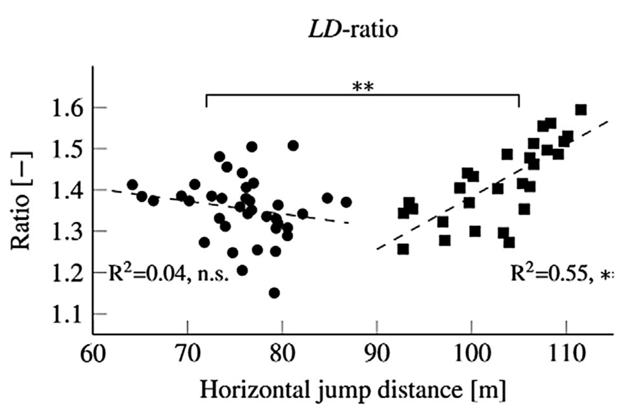

Fig. 3. Relationship between the lift-to-drag ratio (LD-ratio) and horizontal jump length. The measurements for HS106 are indicated with circles and for HS140 with squares. The linear relationships together with the coefficient of determination (R^2) are displayed for both hills and $**$ indicates statistically significant difference ($p < 0.001$).

4. Discussion

The purpose of this study was to compare how the key characteristics of the steady glide phase relate to performance for the two hill sizes used in World Cup and Olympic competitions. The aerial time difference between the two hill sizes was in average 1.1 s, explained by the steady glide phase. Clear differences in how the glide variables related to performance were observed between the hills.

The velocity increase throughout the steady glide phase, due to gravity and the aerodynamic forces that are proportional to the squared product of velocity. Hence, the natural assumption would be that the average aerodynamic forces through the phase increase with increasing jump length. However, this was only found for HS106 and the aerodynamic variables related very differently to performance for the two hill sizes. Exact $C_{L,D}A$ values could not be obtained, but using realistic values for m (66 kg) and ρ (1.25 kg m^{-3}) to estimate Ω (Elfmark and Ettema, 2021), the current results indicate $C_L A$ = [0.58, 0.79] and $C_D A$ = [0.42, 0.69], which resemble values from CFD simulations and wind tunnel measurements (Gardan et al., 2017; Müller et al., 1996; Schmölzer and Müller, 2005). This strengthens the validity of the study but the absolute values may not be directly comparable, as the lift and drag coefficients are dependent on Reynolds numbers. However, the important finding of this study is not the absolute $C_{L,D}A$ values, but how these variables relate to performance.

Both Müller et al. (1996), through wind tunnel measurements, and Gardan et al. (2017), through CFD-simulations, highlighted the relationship between the α and $C_{L,D}A$, where a high α was associated with high $C_{L,D}A$. $C_D A$ had a more rapid increase than $C_L A$, with an observed stall around $\alpha \geq 30°$ (Müller et al., 1996; Gardan et al., 2017). In this study, LD-ratios between 1.1–1.6 were measured, corresponding well with the existing literature (Gardan et al., 2017; Müller et al., 1996; Schmölzer and Müller, 2005; Lee et al., 2012). There is a consensus that a high LD-ratio is beneficial for performance; however, such a correlation was only found for HS140. An explanation for this might be that the steady glide phase on a normal hill is too short to be of importance in the overall performance, which strengthens

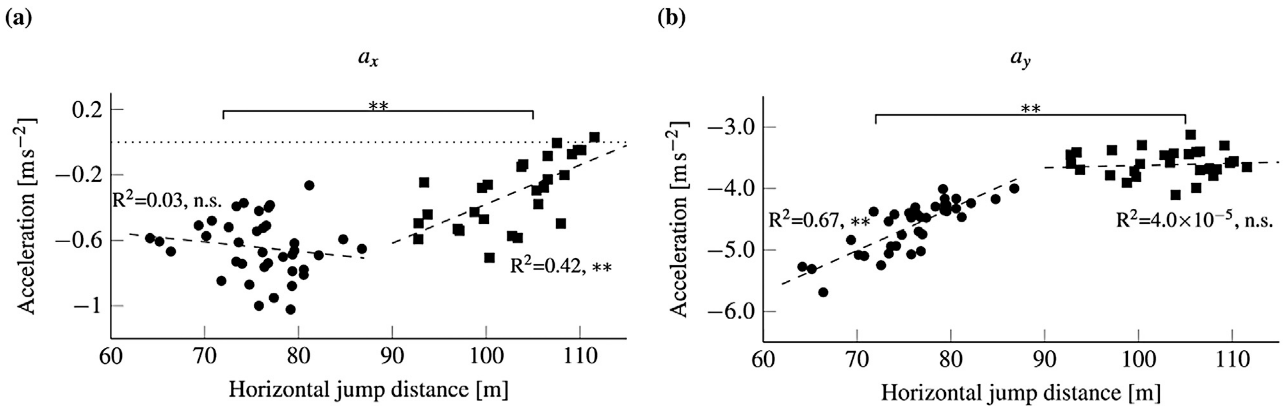

Fig. 4. Relationship between acceleration components and jump length. **(a)** displays the horizontal acceleration (a_x) and **(b)** the vertical acceleration (a_y). The measurements for HS106 are indicated with circles and for HS140 with squares. The linear relationships together with the coefficients of determination are displayed for both hills and ∗∗ indicate statistically significant differences ($p < 0.001$). The dotted line in **(a)** indicate $0\,\mathrm{m\,s^{-2}}$.

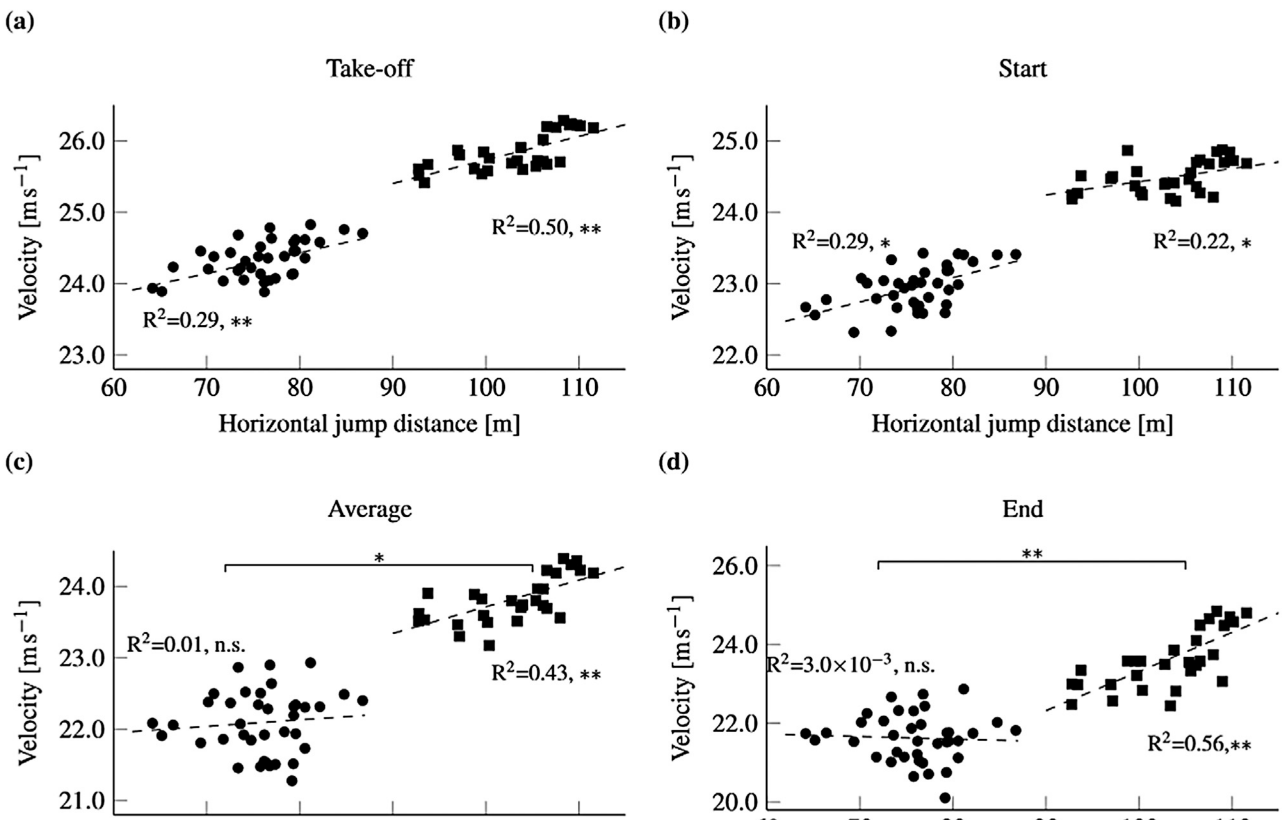

Fig. 5. Relationship between horizontal velocity and jump length at **(a)** take-off, **(b)** start, **(c)** average and **(d)** end of the steady glide phase. The measurements for HS106 are indicated with circles and for HS140 with squares. The linear relationships and the coefficients of determination are displayed for both hills. ∗ and ∗∗ indicate statistically significant differences ($p < 0.01$ and $p < 0.001$, respectively).

the previous findings that the take-off and glide preparation are the most important phases here. High values for both $C_{L,D}A$ are correlated to performance for HS106 (Fig. 2), hence, no relationship was found between the LD-ratio and jump length for HS106. While the steady glide phase is of minor importance on a normal hill, it was found to be of greater importance for HS140, where a high LD-ratio was related to performance. This indicates that the aerodynamic variables during the steady glide phase are highly important on a large hill, even if the duration of the phase is only 1.1 s longer. If the differences between these hills can be extrapolated to ski-flying, one may hypothesize that the characteristics of the steady glide will dominate the performance. The high LD-ratio in HS140 is obtained by reducing $\Omega C_D A$ while maintaining $\Omega C_L A$. Based on these simulations, an LD-ratio of 1.2–1.6 is reached when α is in the region of 30–40°, a region where $C_L A$ has stalled (Gardan et al., 2017). Hence, a reduction in $C_D A$ whilst maintaining $C_L A$, as observed in this study, may be explained by a reduction in α.

During the glide, DoM changes gradually (from around −10° to −40°) and thus the aerodynamic variables also change global direction and properties (Fig. A.6). To illuminate these mechanisms, results are

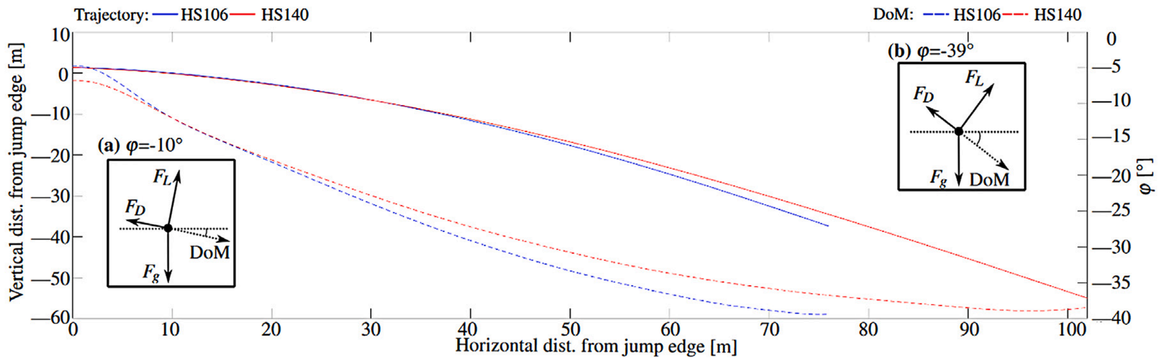

Fig. A.6. Relationship between the direction of motion (DoM) and aerodynamic forces, together with the average trajectory for the two hill sizes. HS106 is indicated in blue and HS140 in red. The left vertical axis displays the vertical distance from the jump edge and the fully drawn line represents the corresponding average trajectory. The right axis displays the angle between the DoM and the horizontal plane (φ) and the dashed line shows the corresponding average DoM for the two hill sizes. Horizontal jump distance is shown on the x-axis. The relationship between the direction of the aerodynamic forces and φ at the start of the phase is displayed in **(a)** and at the end in **(b)** with $\varphi = -10°$ and $-39°$, respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

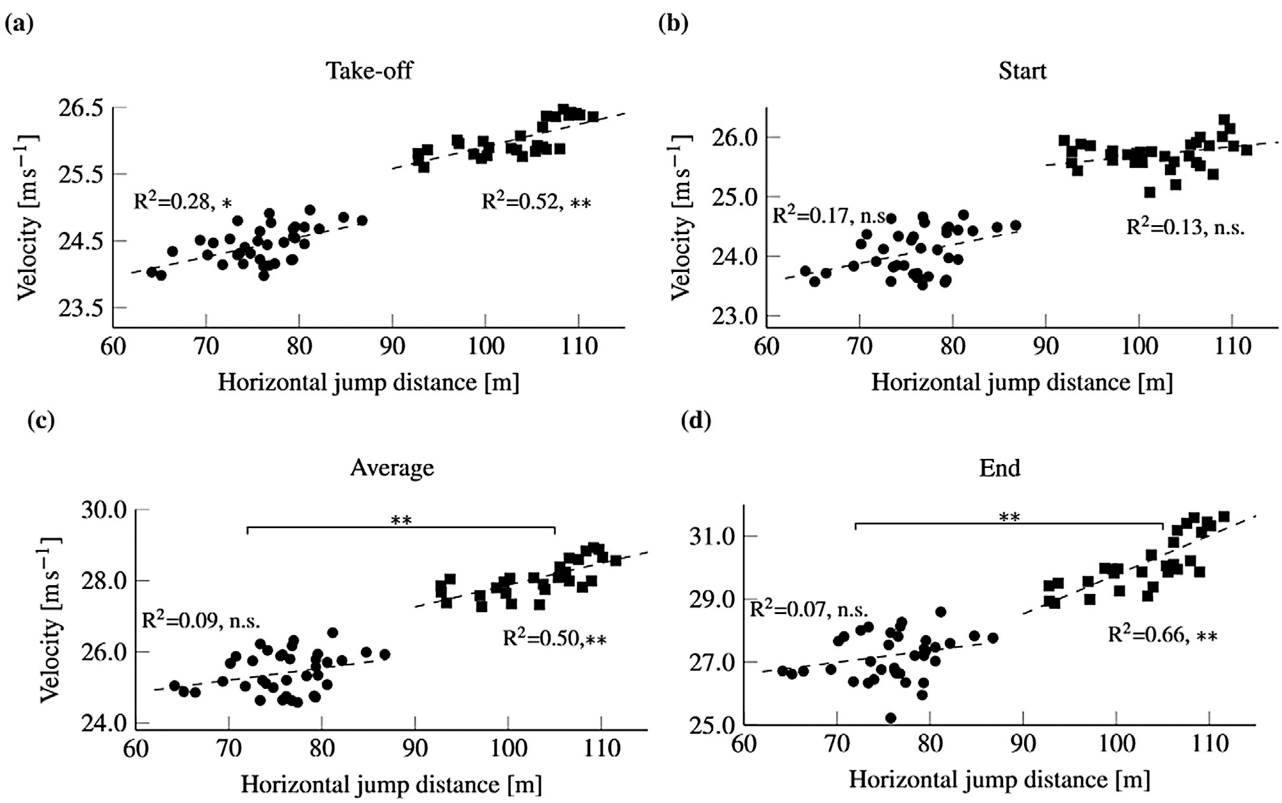

Fig. B.7. Relationship between resultant velocity and jump length at **(a)** take-off, **(b)** start, **(c)** average and **(d)** end of the steady glide phase. The measurements for HS106 are indicated with circles and for HS140 with squares. The linear relationship and the coefficient of determination are displayed for both hills. $*$ and $**$ indicate statistically significant differences ($p < 0.01$ and $p < 0.001$).

presented in a global coordinate system. In general, to jump as far as possible, a skier aims to I: Generate the highest possible vertical component of the aerodynamic forces counteracting the gravitational force (opposing vertical velocity), II: reach the highest possible horizontal component of aerodynamic forces (maximize horizontal velocity). However, the vertical (I) and horizontal (II) aims do, to some extent, contradict each other, which may be an explanation for the differences in performance variables observed between hill sizes. Because of the oblique DoM (downward and forward), both a_L and a_D have a positive component to a_y (Eq. (4)). The vertical requirement (I) is fulfilled by having high aerodynamic forces. Requirement (II) is achieved by having the greatest difference between lift and drag (Eq. (3)). This is obtained for low values of α (Gardan et al., 2017; Müller et al., 1996), where the absolute value of the forces are low. This constitutes the difference between a normal and a large hill, where the most important requirement on a normal hill is (I), whereas (II) is most important on a large hill. As shown in Fig. 4, downward acceleration is related to jump length in HS106 and is explained by the ski jumper having high aerodynamic forces throughout the steady glide phase. In HS140, high horizontal acceleration is related to jump length and is explained by the ski jumper having a low drag area. Interestingly, in some of the longest jumps in HS140, the ski jumpers achieve $a_x \sim 0\,\mathrm{m\,s^{-2}}$ on average through the phase.

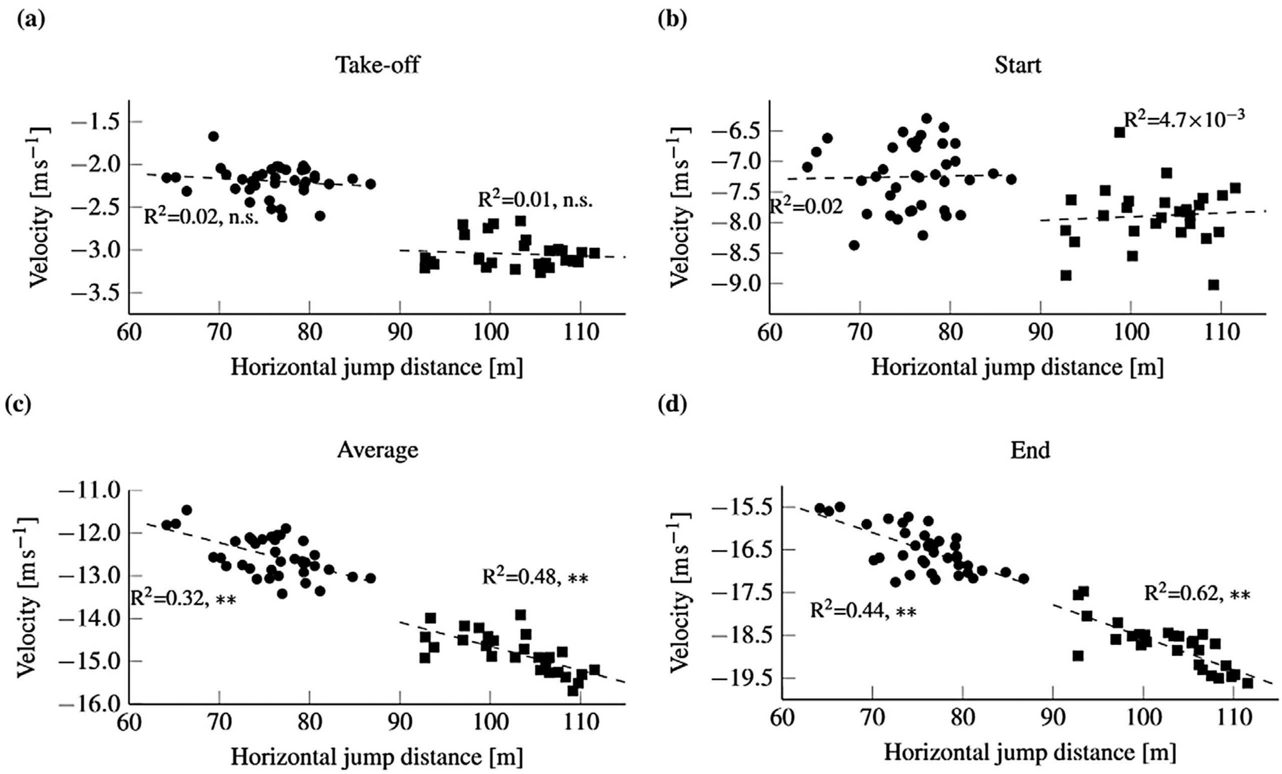

Fig. B.8. Relationship between vertical velocity and jump length at **(a)** take-off, **(b)** start, **(c)** average and **(d)** end of the steady glide phase. The measurements for HS106 are indicated with circles and HS140 with squares. The linear relationship and the coefficient of determination are displayed for both hills. $*$ and $**$ indicate statistically significant differences ($p < 0.01$ and $p < 0.001$).

The importance of the glide phase on a large hill is also apparent from the relationship between jump length and velocity (Fig. 5) that is maintained throughout the glide. The velocity at the end of the phase is as important as the take-off velocity and athletes increase their velocity during the phase. The velocity measures also strengthen the findings from previous studies where the take-off was found to have the highest importance on a normal hill, since the velocity relationship diminishes after the start of the steady glide phase. Both hills show clear correlations between take-off velocity and performance, which in for hills strengthen the horizontal requirement without influencing the vertical requirement.

This study has shown clear differences in the kinematic and kinetic characteristics of the steady glide phase between normal and large hills. The findings for the normal hill are in line with recent literature on ski jumping performance, where the main part of performance can be explained before entering the steady glide phase. The data from the large hill, however, do not support the same assumption. An increase in LD-ratio is related to an increase in jump length, and the horizontal velocity at the end of the steady glide phase is as important as the take-off velocity. While skiers should aim to reduce the vertical acceleration on a normal hill, they should aim to minimize horizontal braking acceleration on a large hill. However, these factors contradict each other to some extent. Thus, the athlete may have to optimize rather than minimize/maximize drag and lift to enhance performance, in line with computer simulations by Jung et al. (2014). Hence, it may, for example, not be beneficial to reduce vertical acceleration on a normal hill too much, if the cost in the horizontal direction is too high.

5. Conclusion

This study has investigated the steady glide phase in ski jumping and compared how key characteristic variables of the phase relate to performance in the two hill sizes used in World Cup and Olympic competitions; i.e, normal and large hills. The aerial time difference for the two hills used in this study was 1.1 s. The time difference was explained by the time spent in the steady glide phase and clear differences in how the glide variables relate to performance were found. The performance was largest influenced by the phase prior to entering the steady glide phase in the normal hill, which is in accordance with recent literature. For this hill size, the most important factor in the glide phase is to oppose as much of the vertical acceleration as possible, through high aerodynamic forces. No correlation was observed between the LD-ratio and jump length. For the large hill, the steady glide phase was found to be as important as the take-off and glide preparation. The most important factor in the large hill size was minimizing the horizontal deceleration by reducing the aerodynamic drag, related to the fact that a high LD-ratio was also correlated with jump length.

CRediT authorship contribution statement

Ola Elfmark: Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Project administration, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Gertjan Ettema:** Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Resources, Project administration, Conceptualization. **Matthias Gilgien:** Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Software, Resources, Project administration, Methodology, Investigation, Funding acquisition, Data curation, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Direction of motion and aerodynamic forces

Fig. A.6 displays the average trajectory and φ during the aerial phase for HS106 and HS140, and illustrates the relationship between the direction of the aerodynamic forces and φ at the start and end of the aerial phase.

In the start of the aerial phase (Fig. A.6 **(a)**), the main part of the drag force acts in the horizontal plane and the lift force acts in the vertical plane. As the ski jumper moves through the phase φ increases, and stabilizes at $\sim -39°$ (Fig. A.6 **(b)**). Here, the lift counteracts the drag force in the horizontal plane and both of the aerodynamic forces have a large positive vertical component.

Appendix B. Resultant and vertical velocity

The relationships between jump length and resultant velocity at take-off, start of steady glide, average of steady glide and end of steady glide are displayed in Fig. B.7.

Similar trends were observed as for horizontal velocity in Fig. 5. While the correlation between resultant velocity and jump length was maintained through the aerial phase for HS140, it diminished after the onset of the steady glide phase for HS106. A significant difference between the hills was found for the average and end velocity, while similar trends were observed for the take-off and start velocity. The relationships between jump length and vertical velocity at take-off, start of steady glide, average of steady glide and end of steady glide are displayed in Fig. B.8.

No significant differences were found between the trends in the two hill sizes for the vertical velocity. No correlations between jump length and vertical velocity were found for the take-off and start. Similar correlations were found for the average and end velocity in both hills, explained by the gravitational force acting for a longer period in the longer jumps.

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