Fourth graders explore a computational thinking task using Robot Emil: A multimodal analysis of pupils' thinking

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Abstract

This paper presents an analysis of two pairs of fourth graders' exploration of a computational thinking task (*related to optimization*) *within a multimodal paradigm perspective in the Robot Emil context. The data included video recordings of the pupils' work and the analysis was guided by the theoretical lens of the Action, Production and Communication space's semiotic bundle. According to findings, the semiotic bundle regarding solving an optimization task through the Robot Emil context had three major components: verbal signs, tool use, and gestures. Pencil and finger gestures mediated an interpretative link between the tool use and the discourse. Further, they were acting as a strong communication tool when pupils described the trials and the conjectures to each other and also when they evaluated the obtained results. However, we underline that the Robot Emil context does not provide feedback, and therefore the teacher's role is crucial to orchestrate collective classroom discussions in order to compare and consolidate pupils' strategies and optimized solutions.*

1. Introduction

The notion of computational thinking goes back to a seminal work by Papert [13]. It is considered as one of the fundamental skills for the 21st century [15] because such an overarching skillset includes various important and sophisticated practices. For example, Weintrop and colleagues [18] elaborated a four-category taxonomy regarding computational thinking in mathematics and science, which are "data practices, modeling and simulation practices, computational problem-solving practices, and systems thinking practices" [p. 135]. An overall description was provided by Wing [19, p. 33], where computational thinking "involves solving problems, designing systems, and understanding human behavior, by drawing on the concepts fundamental to computer science", and it includes abstraction and decomposition with heuristic reasoning. Wing's [19] definition and Weintrop et al.'s [18] taxonomy share a number of important components of mathematical thinking and practices, like problem-solving, abstraction, and reasoning. Therefore, the link between computational thinking and mathematics has received particular attention by researchers, for example [5], [7], and [8] referring to certain programming activities. In a recent study, Kallia and colleagues [11] elaborated three main aspects regarding characterizing computational thinking in mathematics education [11, pp. 179–180]:

- *problem-solving* (like understanding the problem, developing a solution strategy, performing the strategy),
- *cognitive processes* (like abstraction, decomposition, pattern recognition, algorithmic thinking, modeling, logical and analytical thinking, generalization and evaluation of solutions and strategies),

- *transposition* (phrasing the solution of a mathematical problem in such a way that it can be transferred/outsourced to another person or a machine).

Regarding the link between computational thinking and mathematics, Bråting and Kilhamn [7] explored the intersection between algebraic thinking and computational thinking by analyzing how syntax and programming languages are in line or far away from algebraic symbols. As a result of three examples, they underlined the differences and similarities between algebraic notation and programming languages, and they pointed out the importance of moving between different representations.

Programming practices need different but interrelated reasoning types like abstracting, decomposing, and algorithmic thinking [15]. Bearing this in mind, Benton and her colleagues [5] presented the ScratchMaths project that included various curriculum materials and professional development support for teaching mathematics through programming activities, where Scratch (http://scratch.mit.edu) was the main tool. A randomized control trial with over 100 primary schools (pupils aged 9-11 years) across England participated, and results indicated positive student engagement and a better understanding of mathematics. In addition to Scratch, there are available resources where young students can work with programming, such as CS Unplugged (https://csunplugged.org) and micro:bit (https://microbit.org/). In our pilot study, we utilized Robot Emil (https://www.robotemil.com) [10], which is a resource developed for programming instruction, thus it is, not focused only on learning “traditional” mathematical topics. In the Robot Emil context, pupils are expected to work in pairs and collaborate, communicate, and explore various situations together.

Computational thinking and programming have a significant role in different countries’ curricula [5, 7]. A new curriculum [16] was started to be implemented in Norwegian schools in 2020. Now, programming has become an integral part of mathematics in schools from the 3rd grade. Computational thinking is mentioned only briefly in the new curriculum. Its main component is to develop strategies and procedures to solve problems, especially by breaking a problem down into subproblems that can be solved systematically, with or without digital tools [16]. Faced with challenges of programming and computational thinking in the new curriculum, Norwegian schools and teachers were required to find ways how to incorporate these topics into regular mathematics lessons. With that, a need for research-supported instructional practices arose.

While thinking computationally, even though thinking conveys what to do, the focus is often on “tools”. First, one considers the affordances and constraints, and then one learns how to transform and use the same tool in different situations. This process of developing utilization schemes is called instrumental genesis [1]. However, collaborative instrumented activities [14] could also be heuristic tools to exchange ideas and discuss the functions of the tool. In light of this, one potential practice to work with programming tools in schools could be pair programming. This means students work in pairs, which in many cases can be advantageous [9]. Students’ working in pairs could also be communicative, tool-based, and discussion-oriented when they work within a digital environment as a mediator. Generally speaking, thinking in or with a digital environment may be considered to be an intersection of our bodily functions with digital tool use. The synergy between tool use and one’s thinking is substantially coordinated with the task that one is trying to accomplish, while tapping, clicking, or moving. One may intentionally or unintentionally shape their thinking as a consequence of these bodily functions. The sensory-perceptual and motor functions have a central role in thinking, communication, and learning [4]. Therefore, thinking with digital tools can be interlaced with gestures, mimics, specific tapping, sketches, and so on. In other words, thinking in or with digital tools can be considered as a *multimodal* process [2]. For example, Kopcha, Ocak and Qian [12] presented a methodological framework to analyze the embodied interaction with technology and applied this framework to explore computational thinking of two fifth graders. They revealed that "computational thinking was extended to include both the participant's bodies and structures in the environment" ([12], p. 1995).

In our pilot study, we approached (digital) computational thinking tools with a multimodal lens and analyze the elaboration of two pairs of Norwegian fourth graders of an optimization task in the Robot Emil context.

2. Theoretical Framework

Semiotics, the science of signs, is regarded as a powerful tool used to analyze the observed phenomena in the classroom where actual learning takes place. In the classroom, students learn in a shared environment, which has a number of major components such as communication with peers and teachers, tool use, and thinking [4]. Therefore, bodily functions are dialectically intertwined with thinking and learning and produce a group of signs as a multimodal process. In order to frame the dynamic and evolving interaction of the bodily functions with thinking and learning, Arzarello [2, p. 162] introduced the cognitive space of Action, Production and Communication (APC-space), which is built on three main components: "the body", "the physical world", and "the cultural environment". In other words, APC-space has a Vygotskian and multimodal paradigm view and therefore frames various multimodal resources in the classroom such as discourse, gestures, mimics, sketches, written productions, and so on, while the students (as well as the teacher) explore and discuss mathematical situations. Following this perspective, Arzarello and his colleagues [2, 3] elaborated on the notion of the *semiotic bundle*, which "... is a system of signs, with Peirce's comprehensive notion of sign, that is, produced by one or more interacting subjects and that evolves over time" ([3], p. 100).

The semiotic bundle has a two-dimensional analysis tool to look at learning phenomena that occur in a socio-cultural context; a synchronic analysis and a diachronic analysis [2, 3, 4]. The first refers to an analysis of different emerging signs that are "simultaneously activated by the subjects at a certain moment" ([3], p. 100). The latter refers to an evolution of signs associated with the proposed mathematical content in successive moments [4]. Through such a two-dimensional view, the semiotic bundle enables researchers and educational designers to understand the role of tools and functions of the digital environment in student thinking and argumentation, for example regarding the notion of function [17]. Currently, research has not examined multimodal resources attached to student thinking in computational activities within the cognitive space of APC.

3. Research Question and Methods

In this pilot study, we focused on the multimodal resources that facilitate pupils' thinking while they explore an optimization task in the Robot Emil context. We focused on the following research question: *How do pairs of fourth-graders explore an optimization task within a multimodal paradigm perspective through Robot Emil?*

3.1. The Robot Emil Context and the Task

The programming environment with Robot Emil for years 3 and 4 is a new method of learning the basics of programming and computing [6, 10]. In our study, we worked with Emil for Year 3, which consists of three "worlds". Each world contains an ordered progression of units of tasks – a string of eight letters A, B, C … to H. In the first world, Emil collects various things, such as fruits, coins, or letters, in a tray or on a shelf, according to the instructions (an exemplary task could be found here:

https://www.robotemil.com/home/play/). Pupils control Emil by clicking on or touching the squares on the stage. The activity encourages collaboration and exploration, as it is expected that pupils always work in pairs with limited help from the teacher. Moreover, Emil does not give feedback or tell the pupils whether their solutions are right or wrong. It is intended that pupils should give feedback to each other. The method of programming with Emil includes "combined" tasks when the pupils need to transfer their attention/communication between the (digital) screen, their partner, and workbooks several times. The teacher would also lead a whole class group discussion after the completion of several tasks. The focus of these facilitated discussions is on comparison and analysis of solutions, argumentation, and explanations [10].

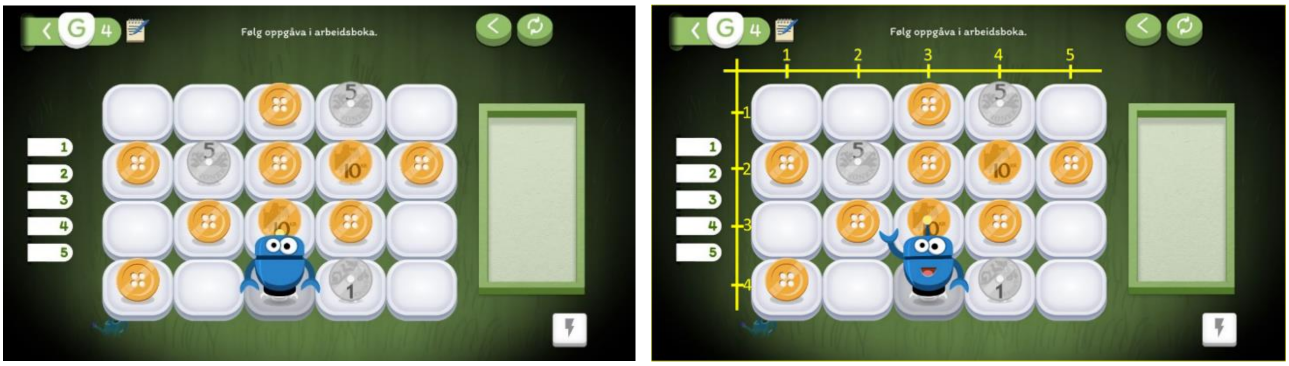
Our data derived from unit G, and specifically the fourth task (G4). The instruction in task G4 is described as follows [6, p. 11]:

Imagine that Emil has to pay for the buttons he puts in the tray. Pick up the buttons and pick up what they cost. Help Emil to buy:

1. *Two buttons for 10 kroner*[1].
2. *Three buttons for 6 kroner*.
3. *Four buttons for 11 kroner*.

Make two similar tasks for your learning partner.

In previous units E and F, the pupils became familiar with the constraint concerning the limited number of moves (for example, "With only four clicks collect as much as thing you can"). Now, this constraint is an integral part of the assignment; the pupils find that the maximum number of moves available for each question of the task, is expressed by bookmarks with numbers 1 to 5 on the left side of Emil's world (Figure 1a). We note that the three questions are independent from each other.



(a) (b)

Figure 1. (a) *The initial look of Emil's world in G4* (*screenshot from the app*), (b) *the added axes by the authors*

The tasks in unit G require careful thinking, planning of the steps and optimization, since the initial and the clicked positions cannot be marked as Emil's destination again. The stage here (in Figure 1a) is composed of 4x5 squares. Coins (with numbers 1, 5 or 10 corresponding to their values) or buttons are placed on only some squares. Every time Robot Emil walks over a square with a button or a coin, these are collected into the rectangular tray on the right-hand side of the screen.

[1]The *krone*, plural *kroner* is the Norwegian currency, with coins of value 1, 5, 10 or 20. "10-kroning" is a usual way to name the 10-kroner coin.

To describe Emil's positions and movements within the context of this paper, we use a coordinate system where the first number represents the row and the second represents the column. For example, in Figure 1b, Robot Emil is located at [4,3]. From the starting position, Robot Emil can only move to another position in a given row or column (i.e., horizontally or vertically). For example, if a pupil clicks or taps [1,3], then Robot Emil moves to [1,3] and everything on his way will be placed into the tray. In this case, it will be 10 kroner and two buttons. By this movement, the first bookmark will be placed on [1,3] and only four bookmarks (i.e., four moves) will be available for completing the task. However, for a pupil to move to [1,4] from Emil's starting location, they would need at least two moves. For example, a pupil could click [4,4] and collect 1 krone in the tray. Bookmark 1 would drop at this location. Next, they would click [1,4] and collect an additional 15 kroner and 1 button in the tray. A less optimized solution in terms of the number of movements would be, for example, clicking on [4,5], then [1,5] and finally [1,4]. By performing these movements, there would be 6 kroner and 1 button collected in the tray. If one clicks the refresh button (top right in Figure 1a), one obtains the initial appearance of Robot Emil environment as in Figure 1a.

3.2. Participants

The participants of the research were eighteen grade-four pupils in a Norwegian primary school, which had been using the context of Robot Emil. They had worked with Robot Emil since their third grade, approximately once every three weeks, as a part of their mathematics lessons. The data was collected in November 2020. During the lesson, the pupils worked in pairs on a Chromebook that they shared, and the work was combined with tasks that they completed in their individual Emil workbooks. In this pilot study, two pairs of grade-four pupils were selected based on their cooperation and communication skills.

3.3. The Data and Analysis

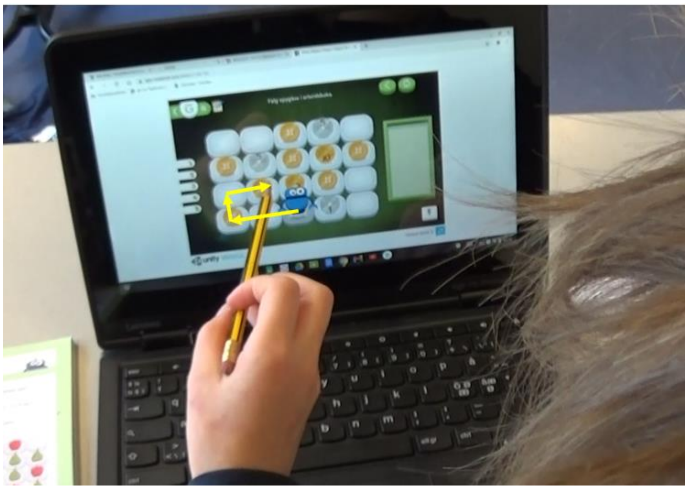
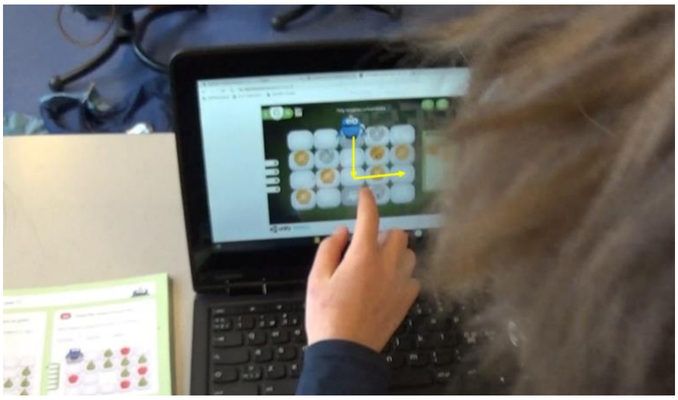
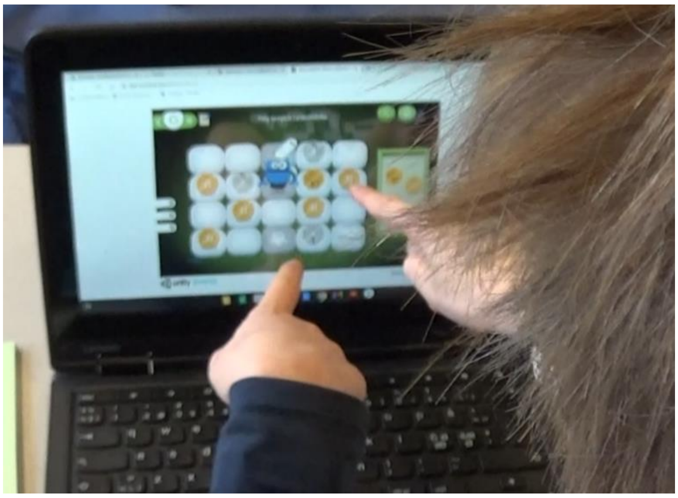
The participants were videotaped while they were involved with activities in unit G. The filming was conducted from behind so that the researchers were able to see what was happening on the screen, where the pupils pointed and what they instructed Emil to do. The videos were later transcribed and translated from Norwegian into English by two researchers. Then, a particular emphasis was given to synchronous moments including different signs (gestures, tapping, clicking, written descriptions, and discourse) to understand pupils' interaction with Robot Emil in-depth. We tried to address the role of these specific signs in pupils' thinking while they were solving the task described above. Regarding diachronic analysis, we showed how pupils' thinking (with different signs as attachments) evolved over time.

4. Findings

Considering APC-space and its analysis tool, we organize the synchronous analysis through a table including verbal signs (discourse), the manner of tool use (like tapping, clicking, moving etc.), and gestures. After this, we summarize the diachronic analysis regarding each part of the task. We presented two pairs' (pseudonyms are Per and Ane, and Ida and Mia) cases in different subsections.

4.1. Synchronic Analysis of Per and Ane's Case

After the teacher reminded the pupils how the workbook and digital environment should be used, the pupils focused on the Chromebooks in front of them. After they read the given details of the task in the workbook, they started to explore. Table 1 summarizes the synchronic (multimodal) resources of Per and Ane's exploration and optimization work.

Verbal Signs and Tool Use	*Gestures*
3 Ane (2:45): *… now I understand ... if we should have two buttons…* /points with a pencil [3,2] and then [4,1]/ *… if we have two buttons there, it costs 10 kroner*/ points at 10 kr coin [3,3]/. *Then we have … /* shows with a pencil [4,3]→[4,1]→[3,1]→[3,2]/ *then we have to have one "10-kroning" since two buttons cost 10 kroner.*	
11 Ane (3:30): /points at the screen and shows a sequence of moves (i.e., a path) starting from Emil's actual position: [1,3]→[3,3]→[3,5]/ *if we get to…*	
12 Per (3:32): *Wait. We might be able to get in here* /clicks on [2,3] and points [2,5] without clicking/	

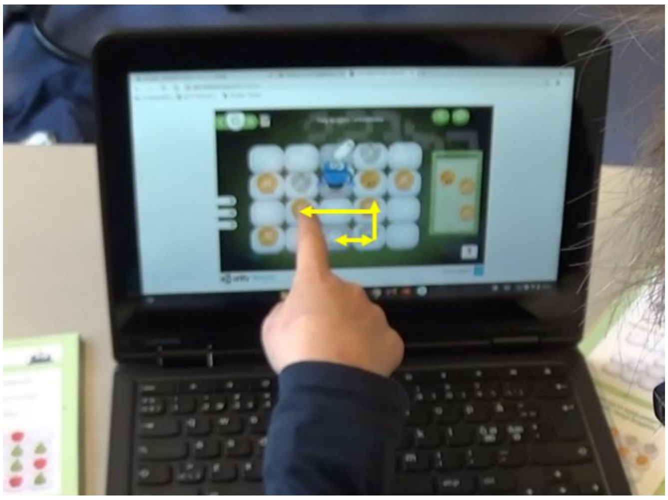
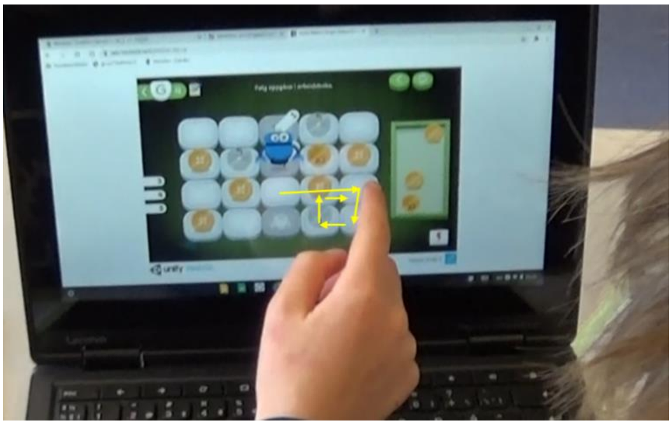
13 <u>Ane</u> (3:35): *Yes, but we move like this* /simulates a path with the finger: [4,4]→[4,3]→[4,4]/ *and then like this* /points at [3,4]/ *and so* /points at [3,2]/ … *maybe*?	
17 <u>Ane</u> (4:13): *But if we go all the way down again* /clicks on [4,3]/ *oh yeah, it does not work, no.* /thinks and clicks on [2,3] and moves her finger along the stage while thinking/	

Table 1: *A summary of Per and Ane's elaboration of the task*

In #3, Ane proposed one solution. However, Per found a more optimized solution where he clicked only on [1,3], and later said "*It is 10 kroner and two buttons*". This produced his optimized solution. Because Per and Ane already had two buttons for 10 kroner, they skipped to question 3 of the task in order to collect two more buttons and 1 krone. Therefore, Ane explored what to do with the remaining moves. She noticed the bookmarks with numbers. They explored and exchanged their views for a number of cases (#11–13). Ane had a plan to accomplish the third question of the task (#13) and refreshed the screen. Consequently, Ane tried to perform this plan, but she realized that they could not mark a position where Robot Emil initially was (#17). After this, Per tried and reasoned mathematically, "*Wait! If two buttons are 10 kroner, aren't then four buttons 20 kroner*" (after #17). Ane considered the context of the requested task and most likely she thought that it was impossible or too difficult to continue. Then, a few seconds later Ane refreshed the screen, and they focused on the second question of the task (i.e. three buttons for 6 kroner). Ane suggested to move Emil up and later performed a path by clicking [4,5]→[1,5]→[1,3] (which gave two buttons with 6 kroner), then Per said that it was little. After this, Ane clicked on [2,3], and they completed the second question.

The synchronic signs show that pupils understood the given task and started to optimize. Pupils mostly gestured (i.e., finger traces to show the estimated path) to exchange ideas when they explored the task situations. Gestures were accompanied by verbal descriptions regarding the hypothesized solutions.

4.2. Diachronic Analysis of Per and Ane's Case

The semiotic bundle regarding the case of Ane and Per consists of three (interrelated) elements: verbal signs, tool use, and gestures for explaining/describing the screen. Verbal signs indicated that Per and Ane exchanged their views regarding how they needed to move Emil to collect a given number of buttons and the amount of money, where gestures and tool use primarily mediated their discussion (like #3, #12, and #17). For example, Ane's gestures with a pencil and tracing with her finger acted as pre-explanation and pre-performing tool to share her conjectures in finding the steps that would solve the given task. The tool use not only worked to test conjectures and reflected Ane's thinking, but also it functioned to remind her of the characteristic properties of the environment (#17).

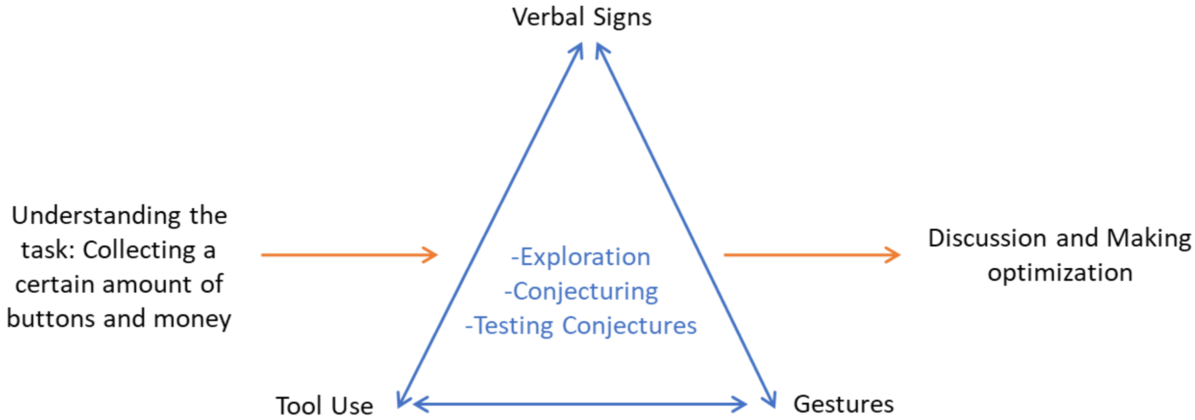
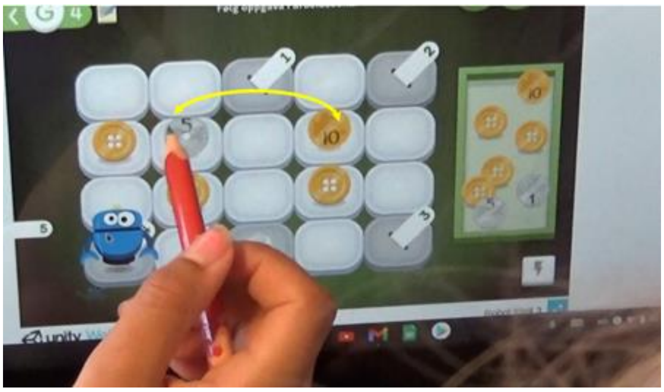


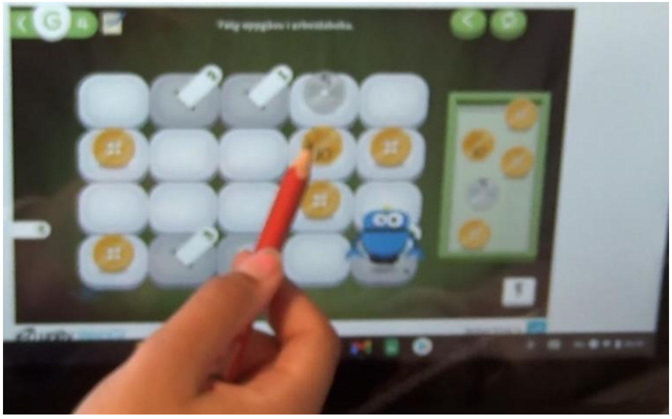
Figure 2: *Per and Ane's exploration of the task*

This exploration of the task by the pair of students is presented in Figure 2. The pupils started with a goal, collecting buttons and kroner synchronously and later considering the bookmarks with numbers. Through the function of digital context, they established certain conjectures (#3, #17) and tested them. Per and Ane managed to discuss optimized ways by thinking computationally (#12), rather than performing a collection of trial-and-error strategies.

4.3. Synchronic Analysis of Ida and Mia's Case

Ida and Mia started from question 3 as question 1 and 2 of the task were marked as "done" in their workbook. The pupils tried a number of steps for Emil. One used the Chromebook, and the other oriented or described her thinking and/or vice versa. Table 2 overviews synchronic resources regarding Ida and Mia's case when they tried to accomplish the task.

Verbal Signs and Tool Use	*Gestures*
7 Ida (6:02): *and so we need 11 kroner but we have 15* /points first 10-kroner at [2,4] and then 5-kroner at [2,2]/ … *5 here … and this will be 15.*	

<table>
<tr><td>17 <u>Ida</u> (6:38): Go there! /points repeatedly at [2,4] with a pencil/</td><td></td></tr>
<tr><td>18 <u>Mia</u> (6:40): Do you think it is possible to go there when we only have one move left?!</td><td>No Gesture</td></tr>
<tr><td>20 <u>Ida</u> (6:45): I have an idea! Because if you had gone here /points [4,4] with a pencil /we could have gone like this / shows a path [4,4]→[2,4]→[2,5]/</td><td>
</td></tr>
<tr><td>24 <u>Mia</u> (6:58): So then we go here and then here and here? /shows a path [1,3]→[1,2]→[4,2]→[4,4]→[1,4]/</td><td>
</td></tr>
<tr><td>34 <u>Mia</u> (7:31): We have ten, twenty /points 10-kroner(s) in the tray with the index finger and counts/ thirty… /points 5-kroner in the tray/</td><td></td></tr>
</table>

35 <u>Ida</u> (7:35): *Look here! Eleven.* /points 10-kroner and 1-kroner, back and forth three times/ *… and so we also have 11 kroner. I think we managed it.*	

Table 2: *An overview of Ida and Mia's case*

As seen from Table 2, at the beginning, Ida and Mia tried to collect four buttons, and later Ida expressed that there are 15 (as 10+5) kroner on the screen (#7). Mia then refreshed the screen, and they tried several steps to obtain four buttons, where they considered the number of moves stating "*then we only have one turn left*". After this, Ida proposed a way (#17), however, Mia questioned going to [2,4] as a fifth click (#18).

Then Ida presented a (partial) plan (#20), which was aimed for collecting 11 kroner. Mia refreshed the screen again, stating, "*Okay, fine, I understand*.", and Ida emphasized her belief again that the plan was going to work. However, Mia suggested another path (#24), which would give four buttons and 31 kroner. They explored a few steps more, then Mia counted what they had in the tray (#34). Ida confirmed that they had 11 kroner in the tray, and they managed to complete the task (#35). They wrote their findings into the workbook, they tried to draw the path they followed, and they counted the buttons and kroner in the tray.

The synchronic analysis here shows the interpretative link between the gestures and pupils' thinking. They exchanged ideas, where gestures functioned as a strong communication tool. First, they focused only on the stage to collect a certain amount of money while neglecting already collected coins and buttons in the tray. Later, they realized the function of bookmarks and focused on "four buttons" and the number of moves but they neglected how much money they would collect.

4.4. Diachronic Analysis of Ida and Mia's Case

As in the case of Per and Ane, there are three interrelated multimodal resources: verbal signs, tool use, and gestures. Verbal expressions show how they expressed a few proposals (such as #7, #20, and #24) to solve the task, while the gestures were frequent and dominant in the process to describe potential steps. In this case, both pupils gestured frequently and then performed the thought plans. This means that the tool use was frequent, which is possibly due to several employed trial-and-error steps.

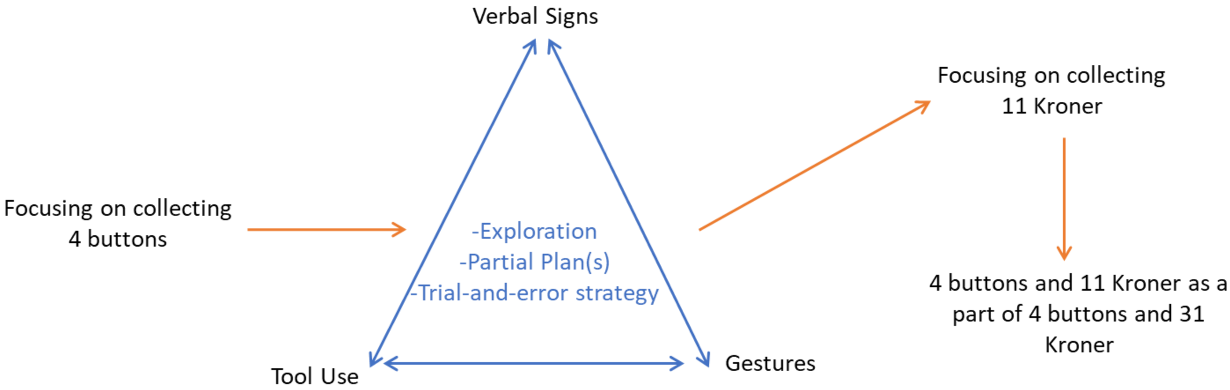


Figure 3: *Ida and Mia's exploration of the task* (The double arrows show the dialectics between signs and pupils' steps)

Figure 3 suggests the exploration path used by the second pair of students. Ida and Mia first thought analytically, focusing on a part of the given task, the *four buttons*. Ida and Mia focused only on collecting four buttons mostly through a trial-and-error strategy, putting aside the case of 11 kroner. After this, the pupils focused on collecting 11 kroner (as 10 + 1 kroner) rather than exploring and thinking in different ways (like 5+5+1 kroner) of optimizing. Their final solution suggests that they did not fully grasp the expectation of the task, which was to collect buttons and kroner synchronously while considering the number of bookmarks. Later, the pupils recorded the path of Emil in the workbook, recalling it from memory even though the steps were visible on the screen through bookmarks.

5. Conclusions and Discussion

In this paper, we focused on multimodal resources that are connected to fourth-graders' exploration for solving an optimization task with Robot Emil. Our analysis suggested that the semiotic bundle regarding solving an optimization task consisted of three main components: verbal signs, tool use and gestures. We observed that pencil and/or finger gestures were strong communication tools while exploring different situations. In other words, pencil and finger gestures mediated an interpretative link between the tool use and the discourse. We interpreted that gesturing was an easier way to exchange ideas related to planning, performing/discussing, and evaluating, which is in line with the findings of Kopcha et al. [12]. Verbal signs showed that the pairing pupils strategy [9] for the exploration of an optimization task worked well, where the pupils exchanged their ideas and discussed possible strategies to solve the task. In sum, our pilot study showed how gesturing and the tool use conveyed the development of computational thinking and the discourse.

However, we should not neglect the fact that the role of feedback is crucial here. This is because the Emil context does not provide feedback regarding the completion of the parts of the task. In our case, even though pupils gave feedback to each other), one pair of pupils thought that "4 buttons" and "11 kroner" could be separately collected, so they found 11 kroner as a part of 31 kroner in the tray. Therefore, we underline the critical role of collective classroom discussion orchestrated by the teacher regarding optimization tasks to compare and consolidate pupils' findings. As emphasized by Kalaš [10], it is a good strategy for teachers to listen to each pair, discuss possibilities, make an agreement on the most optimized solution. In this way, pupils will be able to reconsider the correctness of their solution.

We underline that the pair of girls focused on their workbook to record what they did, while the Emil environment showed bookmarks with numbers (where the movements took place). In other words, in order to write the steps in the workbook, the pupils could have recalled the path that they programmed Emil to perform by bookmarks on the screen. Instead, they reconstructed what they did from memory. This shows us a weak link and dysfunctional synergy between the task and the tool. Furthermore, we conclude that this was also an indicator that for learning all affordances and constraints of functions of Robot Emil, many practices would be necessary. Therefore, an *instrumental genesis* perspective [1, 14] regarding tool use is important. In this study, the pupils did not take advantage of the environment to draw Emil's path. Therefore, we plan to focus on instrumentation of the functions of the Robot Emil context for successive tasks where it is possible to program Emil to perform a certain task.

Finally, we reflect that an APC perspective does not function (fully) to explain everything that happened in exploring an optimization task. In our pilot study, certain *transposition* (i.e. forgetting the Emil context and redoing something on the paper) exists as underlined by Kallia et al. [11]. Therefore, we believe that there is a need to analyze the computational thinking and optimization phenomenon by adopting two lenses (e.g., APC and instrumental genesis) in future settings.

Further research may seek to focus on the Robot Emil context with additional groups of pupils over a longer period with successive tasks, in order to analyze multimodal resources and dialectics between body involvement and computational thinking. One can also focus on the teacher's orchestration of classroom situations regarding unplugged/plugged activities in the Robot Emil environment. At that moment, the teacher's practices and the whole class argumentation would be another direction that could be investigated.

Acknowledgement

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