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Understanding the dis-embodied representation of continuity in terms of a strongly embodied representation

Omid Khatin-Zadeh¹, Danyal Farsani^{2*} and Babak Yazdani-Fazlabadi³

Abstract: In this article, we discuss the process of understanding continuity, which is one of the most fundamental concepts in mathematics. The continuity of mathematical functions is formally defined in terms of abstract symbols and operations. This representation of continuity is very abstract or dis-embodied. Therefore, it is difficult to acquire a clear understanding of it. To facilitate the process of understanding this concept, it is transformed into a strongly embodied motion-based representation which enables us to employ a wider range of sensorimotor networks to understand the concept of continuity. This is one of the most important cases of understanding a highly abstract and difficult-to-define mathematical concept as a strongly embodied motion concept.

Subjects: Nonverbal Communication; Study of Higher Education; Theories of Learning

Keywords: continuity; formal representation; embodied representation; representational transformation

1. Introduction

Formal representations of mathematical concepts and ideas are often abstract. Mathematical concepts are defined in terms of formal symbols. We use abstract symbols to refer broadly to our mathematical ideas; but these symbols are very abstract or dis-embodied. They are represented by simple written/oral formats where absolutely no other property is attached to them. Therefore, many people find mathematics detached from the real perceivable world. They see mathematics as a difficult subject matter because formal mathematical symbols, which constitute the basis of formal mathematics, cannot be perceived through sensorimotor systems. Fortunately, formal symbol-based representations of mathematical concepts and ideas, which are dis-embodied and detached from the real perceivable world, can be converted into strongly embodied representations. This way, we can talk and think about the abstract/disembodied representation of a concept in terms of a concrete/embodied representation of that concept provided that the two representations are isomorphic to one another. This makes mathematics much more real, visible, tangible and, therefore, understandable. It is precisely for this reason that embodied mathematical learning has garnered an increasing amount of attention in recent years (Gordon & Ramani, 2021; Macedonia, 2019, for a review, see Abrahamson et al., 2020).

Within the framework of embodied learning, movement-based mathematics learning is perhaps one of the most interesting areas. Some fundamental mathematical concepts such as function, limit, continuity, series, and infinity are metaphorically described in terms of motion (e.g., Lakoff & Núñez, 2000; Marghetis & Núñez, 2013; Núñez & Lakoff, 1998). Although these concepts are formally defined in terms of abstract symbols and abstract concepts (e.g., number, absolute value, addition,

subtraction), they are metaphorically described as movements in mathematics discourse. These metaphoric descriptions of abstract mathematical concepts are representational transformations (e.g., Farsani et al., 2022; Khatin-Zadeh, 2021; Khatin-Zadeh et al., 2019, 2022; Rosa & Farsani, 2021; Rosa et al., 2020). That is, a formal abstract representation, which is detached from sensorimotor experiences, is transformed into a strongly embodied representation. In this paper, we specifically focus on the metaphorical understanding of function continuity in terms of motion. This is one of the most important cases of understanding a highly abstract and difficult-to-define mathematical concept as a strongly embodied motion concept. The aim is to offer a picture of cognitive mechanisms through which the concept of continuity is understood as a motion.

2. Understanding continuity of function in terms of motion

The notion of continuity is defined on the basis of the concept of limit. Limit itself does not have a formal definition and according to this definition, limit of the function $f(x)$ at c equals to L ($\lim_{x \rightarrow c} f(x) = L$) if and only if:

$$\forall \varepsilon > 0, \exists \delta > 0, \text{ such that if } 0 < |x - c| < \delta, \text{ then } |f(x) - L| < \varepsilon$$

This formal definition or formal representation has a metaphorical equivalent which is an embodied representation of the concept of limit. According to this metaphorical representation, when the moving point x approaches the fixed point c , the moving point $f(x)$ approaches the fixed point L , and the distance between $f(x)$ and L becomes smaller than any small distance. This representation of limit is much easier to understand because the elements involved are embodied and much more perceivable, compared to elements involved in the formal definition of limit. Based on these two definitions of limit, the concept of continuity is defined and understood in two ways:

- 1) abstract formal definition/representation
- 2) embodied definition/representation

According to the formal definition, the function $f(x)$ is continuous at c if three conditions are satisfied: 1) $f(x)$ must be defined at c ; 2) $f(x)$ must have a limit at c ; 3) this limit must be equal to $f(c)$. Since this definition of continuity is entirely based on formal abstract symbols, it may be difficult to acquire a clear understanding of it. The embodied definition or embodied representation of continuity, which is expressed in terms of a motion, is much easier to understand. Based on this metaphorical definition, continuity is understood as the trace of a moving object in the space. When a mathematical function is represented in terms of a graph in the Cartesian coordinate system, the function itself is understood as the trace of a fictive motion. In other words, the graph of a function, which is one representation of the function, is understood as the trace of the fictive motion of a point in the Cartesian coordinate system. If the trace of movement passes C (with an x value of c) and covers a neighborhood around C , the function is continuous at C . Therefore, continuity is understood as the trace of a movement in the space. Compared to dis-embodied representation of continuity, this representation is highly embodied and has strong sensorimotor associations. From the strong version of embodiment perspective (Gallese & Lakoff, 2005), in order to understand this representation of continuity, our sensorimotor systems are actively employed. The strong version of embodiment holds that the same sensorimotor networks that are activated during the understanding of the base of a metaphor are also activated during the understanding of the target of the metaphor (Lakoff, 2008, 2014; for a review, see Khatin-Zadeh et al., 2021). One implication of this view is that understanding continuity in terms of motion (base domain of the metaphor) involves the use of sensorimotor networks that are employed during the processing of the motion. This embodied motion-based representation of continuity is visual and motoric. As such, our visual system and motor system can support us to acquire a much clearer understanding of what continuity is.

3. Strong version of embodiment and representational transformation in mathematics

From the perspective of the strong version of embodiment (Gallese & Lakoff, 2005), representational transformation of mathematical concepts, which is essentially a metaphor, allows us to employ a wider range of cognitive resources to understand abstract concepts (e.g., Khatin-Zadeh, 2021). Within the framework of the strong version of embodiment, the sensorimotor resources that are utilized to process the base of a metaphor are also employed to process the target of the metaphor. Therefore, when an abstract mathematical concept is understood as a motion, the motor system can be actively employed to process that abstract concept (Khatin-Zadeh et al., 2021). If the abstract concept is metaphorically understood as a human body movement, the sensorimotor networks that are involved in these body movements are activated during the processing of the abstract concept. If the abstract concept is understood as the motion of a non-human object, the sensorimotor networks could be recruited because the movement of that object can be described as a body gesture. It is for this reason that gestures can be a powerful tool in the learning process. Gestures are concrete realizations of a mental process that takes place in the mind. As Hostetter and Alibali (2019) argue, gestures are embodied simulations of sensorimotor processes. In other words, when we talk or think about a concept, our gestures concretely show what simulation takes place in our mind. In this way, gestures can support and strengthen the embodiment process. This is the case when an abstract concept is metaphorically described in terms of a movement.

4. Conclusion

Continuity of mathematical functions is an abstract concept. When it is defined in terms of abstract symbols and abstract operations, it is difficult to acquire a clear understanding of it. Therefore, we transform its formal abstract representation into a metaphorical concrete representation to make it more perceivable, and as such, more understandable. This representational transformation gives body or shape to a dis-embodied representation and makes it embodied. Representational transformation does not change the essence of a concept or an idea but rather just transforms it into another representation. The key point is that the two representations share some key similarities at a conceptual level. These deep conceptual similarities can explain why the two representations share some key features. The final point that should be noted is that transforming abstract representations of mathematical concepts and ideas into motion-based representations is used in the understanding of many mathematical concepts, including function, limit, derivative, integral, etc. This emphasizes the fact that embodied approaches to mathematics learning with a focus on body movement could be an effective way to enhance our understanding of mathematics.

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Data sharing not applicable to this article as no datasets were generated or analyzed during the current study.

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