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Locating hydrogen production in Norway under uncertainty *

Šárka Štádlerová[0000−0002−4636−573X], Trygve Magnus Aglen, Andreas Hofstad, and Peter Schütz[0000−0002−9466−0354]

Department of Industrial Economics and Technology Management,
Norwegian University of Science and Technology, NO-7491 Trondheim, Norway
https://www.ntnu.edu/
{sarka.stadlerova}@ntnu.no

Abstract. In this paper, we study a two-stage stochastic multi-period facility location and capacity expansion problem. The problem is motivated by the real-world problem of locating facilities for green hydrogen in Norway. We formulate a model with modular capacities. Investment in a facility and expansion costs represents long-term costs. For each capacity, we define a convex short-term production cost function which enables to capture economies of scale in investment as well as in production. The objective is to minimize the total expected investment, expansion, production and distribution costs while satisfying demand in each scenario. We solve the problem using sample average approximation. The results from solving the problem show that the stochastic problem leads to lower installed capacity in the opening decisions than the expected value problem.

Keywords: Stochastic Facility Location · Capacity Expansion · Hydrogen supply chain

1 Introduction

In February 2020, Norway adopted more ambitious emission reduction targets than agreed upon in the Paris Agreement. The new target is to reduce greenhouse gas (GHG) emissions by at least 50% towards 2030, compared to the 1990 level [35]. To achieve this goal, the emissions from the transport sector also need to be halved. With a share of more than 30%, the transportation sector is an important contributor to total GHG emissions [37].

One of the key instruments for achieving the emission reduction targets is to use green hydrogen as a zero-emission energy carrier [37]. Only hydrogen coming from a CO_2-free production process can be considered a green zero-emission fuel. Electrolysis (EL) using energy from renewable sources is the most mature

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technology for green hydrogen production [16]. EL is a quite flexible production technology and can produce in a range of $20 - 100\%$ of installed capacity [27]. The production costs are subject to economies of scale as higher production quantities result in lower average unit costs [15].

In order to start the transition towards hydrogen in Norway, municipalities can require the usage of hydrogen as fuel when public transport contracts for ferries, high-speed passenger vessels, and coastal routes are renewed. Hydrogen is also a promising energy carrier for long-distance buses and heavy trucks [11]. The Norwegian government is also working on designing possible low- and zero-emission requirements for offshore supply vessels [31]. The conversion potential to zero-emission energy carrier of the offshore fleet with respect to the fleet composition and future demand is presented in [32]. Future hydrogen demand is highly uncertain because the market share of hydrogen vehicles in the road traffic sector and the future energy carrier in the offshore sector are also subject to uncertainty.

In this paper, we study the problem of locating hydrogen production facilities in Norway under uncertain demand. We formulate our problem as a two-stage stochastic multi-period facility location problem with capacity expansion. We consider modular capacities in order to model economies of scale. The goal is to minimize expected investment, expansion, production and distribution costs of satisfying the customer demand. We distinguish between long-term investment costs and short-term operational costs to capture economies of scale in investment and production. This approach also enables the modelling of different utilization of the installed capacity. The problem is solved using sample average approximation (SAA). We compare the first-stage solution of the stochastic problem (SP) and the expected value problem (EVP) and discuss the value of the stochastic solution. We analyse the hydrogen production infrastructure and provide a managerial insight into the investment capacity of new facilities.

The remainder of this paper is structured as follows: we first provide an overview of related work to deterministic and stochastic facility location and capacity expansion problems in Section 2. We formulate the mathematical model for the stochastic two-stage multi-period facility location problem in Section 3. The solution approach is presented in Section 4. Case study and Computational results are discussed in Section 5 and 6, respectively. We conclude in Section 7.

2 Related work

We structure the related work into three main parts. First, we focus on literature related to deterministic facility location and capacity expansion problems before we continue with two-stage facility location and supply chain design problems. Finally, we present literature related to SAA.

Deterministic multi-period facility location and capacity expansion problems with modular capacities are studied in [41], [10]. In these papers, both capacity expansion and capacity reduction are allowed. Expansion is modelled as new-building of another facility at a given location while capacity reduction means closing some or all of the facilities at a given location. An approach where capacity expansion is modelled as a modification of an existing facility is presented in [18], [19], [20], [43]. In [43], the number of capacity expansions is limited, and capacity reduction is not allowed. In [18], capacity expansion and reduction are allowed multiple times. An extended version of the model from [18] for multiple commodities is presented in [19], [20]. See also the review [26], [28] for an overview over multi-period facility location problems.

Uncertainty in demand in two-stage stochastic problems is more commonly found in single-period facility location problems. The first-stage decisions usually refer to the opening of facilities and determining their capacities, while the second-stage decisions are related to distribution and demand satisfaction. A model with random demand and non-linear cost function to model economies of scale is discussed in [4], [39]. The problem in [39] is solved using Lagrangian relaxation. A two-stage facility location problem with depots is presented in [24] and also solved by Lagrangian relaxation. The model presented in [24] can be solved by an effective genetic algorithm as shown in [12]. A two-stage multi-period facility location model with a capacity expansion is studied in [7]. The authors compare two model formulations: In the first model, capacity expansion is a part of the first-stage decisions while in the second model, capacity expansion is a second-stage decision. A multi-stage formulation of a multi-period stochastic problem is discussed in [3].

Supply chain network design problems are similar to facility location problems and have received lots of attention. A study on designing the hydrogen supply chain under uncertain demand with a similar decision structure to [4], [39] is presented in [21], [30]. The first-stage decisions correspond to investing in production and storage capacity during the planning horizon while the second-stage decisions correspond to the distribution plan. A two-stage stochastic programming model for minimizing the total daily costs of the hydrogen supply chain with uncertain demand is presented in [9]. Compared to previous work in the hydrogen supply chain, the authors provide emission, energy consumption and risk costs. An early literature review on dynamic facility location and supply chain problems with stochastic data can be found in [34]. A review on facility location problems under uncertainty is provided in [42] and a recent summary on facility location problems under uncertainty is presented in [14], [6].

The SAA algorithm allows for solving large two-stage stochastic problems with a binary first stage. See [25] and [22] for the details on methodology. The application of SAA to a facility location problem where the availability of opened facilities is uncertain is presented in [13]. A similar problem with facility disruptions is discussed in [23]. The authors combine SAA with a scenario decomposition algorithm to solve the problem. A combined solution approach of SAA and Benders decomposition for a supply chain design problem with uncertain demand is studied in [38]. A supply chain design problem with a model that captures short-term as well as long-term demand uncertainty is discussed in [40]. In order to increase the number of scenarios in the sample, SAA combined with dual decomposition is applied to solve the problem.

3 The mathematical programming model

We study a stochastic two-stage multi-period facility location and capacity expansion problem with uncertain demand. The objective is to minimize the total expected costs.

3.1 Problem description

We formulate our problem as a two-stage stochastic multi-period facility location and capacity expansion problem. The goal is to minimize the sum of expected discounted investment, expansion, production and distribution costs while satisfying demand in each scenario. The decisions when and where to open and which capacity to invest in are taken before the uncertainty is disclosed. In the second stage, decisions covering capacity expansion, production, and distribution are taken. Capacity expansion is allowed only once in each scenario and only in the sense of increasing the capacity level. Once a facility is opened, it cannot be closed.

We consider a set of candidate locations and a set of customers. For each facility-customer combination, we have specific unit distribution costs. However, not all customers can be served from all facilities. The investment costs are given by the installed capacity while the production costs depend both on installed capacity and production quantity. Note that investment and production costs can depend on location. The production quantities can vary from the installed capacity. However, there is a lower and an upper limit. The lower limit is given by the minimum production quantities for each capacity. The installed capacity represents the upper limit for production. This upper limit can be extended by expansion.

We model the investment and capacity decision as a discrete choice from a set of modular capacities. Expansion is then modelled as a jump between available capacities. We consider opening a small facility and expanding it as a more expensive alternative to opening a large facility right away. These extra costs are modelled as a one-time payment when expanding. However, the short-term production costs are independent of whether the capacity results from expansion or from opening the facility right away.

For each available capacity, we provide a piecewise linear convex short-term production cost function which enables a variation in production quantities. This approach enables to capture the economies of scale in investment as well as in production. Figure 3.1 shows our long-term (dashed line) and short-term (solid line) production costs. The capacity index of installed modular capacity is denoted k and Q_k is the appropriate quantity. The total costs for production at installed capacity k are denoted C_k. For each capacity k, we define a short-term production costs function $f_k(q)$ that enables production in a range between minimum and maximum limit. However, higher utilization of installed capacity leads to lower unit costs. This approach of modelling investment and production costs is similar to the one in [39].

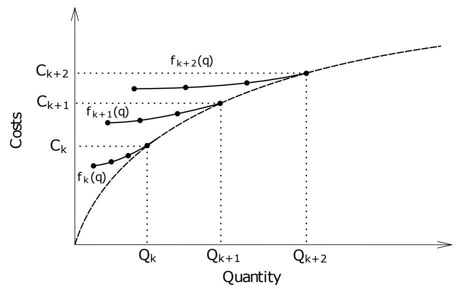


Fig. 3.1: Long-term and short-term production costs

3.2 Mathematical formulation

Let us first introduce the following notation:

Sets

$\mathscr{B}$	Set of breakpoints of the short-term cost function
$\mathscr{I}$	Set of possible facility locations
$\mathscr{J}$	Set of customer ports
$\mathscr{K}$	Set of available discrete capacities
$\mathscr{S}$	Set of scenarios
$\mathscr{T}$	Set of time periods
$\mathscr{T}_1$	Set of time periods corresponding to the first-stage, $\mathscr{T}_1 \subset \mathscr{T}$

Parameters and coefficients

C_{ik}	investment costs at location i for capacity point k;
D^s_{jt}	demand at customer j in period t, and scenario s;
E_{ikl}	costs of expansion at location i from capacity in point k to capacity in point l;
F_{ibk}	costs at location i at breakpoint b of the short-term cost function of capacity k;
L_{ij}	1 if demand at location j can be served from facility i, 0 otherwise;
Q_{bk}	production volume at breakpoint b of the short-term cost function, for capacity point k;
T_{ij}	distribution costs from facility i to customer j;
y_{ikl0}	initial facility variable;
δ_t	discount factor in period t;
p^s	probability of scenario s;

Decision variables

x^s_{ijt} amount of customer demand at customer location j satisfied from facility i in period t in scenarios s;
y^s_{iklt} 1 if facility is operated in location i in period t, with originally installed capacity k, and operated capacity l in scenario s, 0 otherwise;
μ^s_{bilt} weight of breakpoint b at location i for capacity point k in period t and scenario s.

We present a two-stage stochastic multi-period model. The model is given as:

$$\min \sum_{s\in\mathscr{S}} p^s \Bigg[\sum_{i\in\mathscr{I}}\sum_{k\in\mathscr{K}}\sum_{t\in\mathscr{T}} \delta_t C_{ik}\left(y^s_{ikkt} - y^s_{ikk(t-1)}\right) + \sum_{i\in\mathscr{I}}\sum_{k\in\mathscr{K}}\sum_{l\in\mathscr{K}:l>k}\sum_{t\in\mathscr{T}} \delta_t E_{ikl}\left(y^s_{iklt} - y^s_{ikl(t-1)}\right) + \sum_{b\in\mathscr{B}}\sum_{i\in\mathscr{I}}\sum_{l\in\mathscr{K}}\sum_{t\in\mathscr{T}} \delta_t F_{ibl}\mu^s_{bilt} + \sum_{i\in\mathscr{I}}\sum_{j\in\mathscr{J}}\sum_{t\in\mathscr{T}} \delta_t T_{ij} x^s_{ijt}\Bigg], \quad (1)$$

subject to:

$$\sum_{k\in\mathscr{K}}\sum_{l\in\mathscr{K}:l\geq k} y^s_{iklt} \leq 1, \qquad i\in\mathscr{I}, t\in\mathscr{T}, s\in\mathscr{S}, \quad (2)$$

$$\sum_{k\in\mathscr{K}}\sum_{l\in\mathscr{K}:l>k} y^s_{iklt} = 0, \qquad i\in\mathscr{I}, t\in\mathscr{T}_1, s\in\mathscr{S}, \quad (3)$$

$$\sum_{t'=1}^{t-1} y^s_{ikkt'} \geq \sum_{l\in\mathscr{K}:l>k} y^s_{iklt}, \qquad i\in\mathscr{I}, k\in\mathscr{K}, t\in\mathscr{T}, s\in\mathscr{S}, \quad (4)$$

$$\sum_{l\in\mathscr{K}:l\geq k} y^s_{iklt} \geq \sum_{l\in\mathscr{K}:l\geq k} y^s_{ikl(t-1)}, \qquad i\in\mathscr{I}, k\in\mathscr{K}, t\in\mathscr{T}, s\in\mathscr{S} \quad (5)$$

$$y^s_{iklt} - y^s_{ikl(t-1)} \geq 0, \qquad i\in\mathscr{I}, k\in\mathscr{K}, l\in\mathcal{K}: l>k, t\in\mathscr{T}, s\in\mathscr{S}, \quad (6)$$

$$\sum_{b\in\mathscr{B}} \mu^s_{bilt} = \sum_{k\in\mathscr{K}} y^s_{iklt}, \qquad i\in\mathscr{I}, l\in\mathscr{K}, t\in\mathscr{T}, s\in\mathscr{S}, \quad (7)$$

$$\sum_{j\in\mathscr{J}} x^s_{ijt} = \sum_{b\in\mathscr{B}}\sum_{l\in\mathscr{K}} Q_{bl}\mu^s_{bilt}, \qquad i\in\mathscr{I}, t\in\mathscr{T}, s\in\mathscr{S}, \quad (8)$$

$$\sum_{i\in\mathscr{I}} x^s_{ijt} = D^s_{jt}, \qquad j\in\mathscr{J}, t\in\mathscr{T}, s\in\mathscr{S}, \quad (9)$$

$$x^s_{ijp} \leq L_{ij} D^s_{jt}, \qquad i\in\mathscr{I}, j\in\mathscr{J}, t\in\mathscr{T}, s\in\mathscr{S}, \quad (10)$$

$$\frac{1}{|\mathscr{S}|}\sum_{s'\in\mathscr{S}}\sum_{l\in\mathscr{K}:l\geq k} y^{s'}_{iklt} = \sum_{l\in\mathscr{K}:l\geq k} y^{s}_{iklt}, \qquad i\in\mathscr{I}, k\in\mathscr{K}, t\in\mathscr{T}, s\in\mathscr{S}, \quad (11)$$

$$y^{s}_{iklt}\in\{0,1\}, \qquad i\in\mathscr{I}, k\in\mathscr{K}, l\in\mathscr{K}: l\geq k, t\in\mathscr{T}, s\in\mathscr{S}, \quad (12)$$

$$x^{s}_{ijt}\geq 0, \qquad i\in\mathscr{I}, j\in\mathscr{J}, t\in\mathscr{T}, s\in\mathscr{S} \quad (13)$$

$$\mu^{s}_{bilt}\geq 0, \qquad b\in\mathscr{B}, i\in\mathscr{I}, k\in\mathscr{K}, t\in\mathscr{T}, s\in\mathscr{S}. \quad (14)$$

The objective function (1) is equal to the expected discounted costs of investment, expansion, production and distribution costs. Restrictions (2) guarantee that only one facility is opened at a given location and that this facility is operated at only one capacity at a time. Constraints (3) ensure that we are allowed to open facilities in the first stage, but not to expand them. Restrictions (4) make sure that only previously opened facilities can be expanded and constraints (5) ensure that a facility can be expanded but cannot be closed. Capacity expansion is allowed only once during the planning horizon in each scenario. The variable y^{s}_{iklt} contains information about the initially installed capacity k as well as the capacity l at which it is currently operated. After expansion, the operated capacity l is higher than the installed capacity k. Inequalities (6) ensure that capacity index l can change only once. Equations (7) guarantee that production is allocated only to opened facilities and that the short-term production cost function depends on the operated capacity. Equations (8) express the requirement that the whole production has to be distributed to customers. Equations (9) ensure demand satisfaction in each scenario, while constraints (10) specify if customer j can be served from facility i.

Constraints (11) are the non-anticipativity constraints (see e.g. [36]) that ensure that the opening capacity k is the same in all scenarios. Once a facility has been opened with capacity k in a given scenario s, it has to be operated at a capacity $l\geq k$. Hence, the right-hand side, $\sum_{l\in\mathscr{K}:l\geq k} y^{s}_{iklt}$, is equal to 1. The left-hand side then ensures that the facility is opened with capacity k in all scenarios, even though it might be operated at different capacities l in different scenarios.

Restrictions (12)–(14) are the binary and non-negativity requirements for the decision variables. The variables are defined for each scenario. However, investment decisions must be taken before the uncertainty is disclosed.

4 Solution approach

We use the SAA algorithm [25], [22] to solve our two-stage stochastic multi-period model with binary variables. A description of the algorithm can also be found in [38] and [40], but we summarize it here for the sake of completeness. Using the SAA approach, the problem is repeatedly solved with a smaller set of scenarios. First, a random sample $\xi^1, ..., \xi^n$ with a size N is generated. Then the expectation $\mathbb{E}[Q(y,\xi)]$ is approximated by the sample average function $\frac{1}{N}\sum_{n=1}^{N} Q(y,\xi^n)$. We approximate our problem with the following SAA problem:

$$\min \left\{ \hat{g}(y) = c^T y + \frac{1}{N} \sum_{n=1}^{N} Q(y, \xi^n) \right\} \tag{15}$$

With increasing sample size, the optimal solution of (15), $\hat{y}_N$ converges to the optimal solution of the original problem with probability one. In practical implementations, the sample size is often chosen with respect to the computational effort. As we have issues solving our model with more than 10 scenarios, we follow the approach from [38]. The authors show that a higher number of samples can be more efficient than increasing the number of scenarios.

Let M be the number of independent samples and v_N^m the optimal objective function of a problem for $m = 1, ..., M$. The average objective function value is then computed as:

$$\overline{v}_{N,M} = \frac{1}{M} \sum_{m=1}^{M} v_N^m \tag{16}$$

Equation (16) represents a statistical lower bound (LB) on the objective function value for the original problem [25], [29].

Let $N' >> N$ be the reference sample representing the true uncertainty in the problem and $\overline{y}$ a feasible first-stage solution. Then, the objective function of the original problem for a given solution $\overline{y}$ can be calculated as:

$$\widetilde{g}_{N'}(\overline{y}) = c^T \overline{y} + \frac{1}{N'} \sum_{n=1}^{N'} Q(\overline{y}, \xi^n) \tag{17}$$

Equation (17) provides an upper bound (UB) on the optimal objective function value. Having the lower and upper bound estimates, we can compute the estimated optimality gap as:

$$gap_{N,M,N'}(\overline{y}) = \widetilde{g}_{N'}(\overline{y}) - \overline{v}_N^m. \tag{18}$$

5 Case study

In this section, we provide the real-world input data used for solving the problem of locating hydrogen production in Norway under uncertainty.

5.1 Facilities and production

We consider 17 candidate locations for the opening of new facilities on the Norwegian west coast. The candidate locations are taken from [33]. We approximate the facility capacity by 8 discrete points and provide the investment and production costs at full capacity utilization for EL in Table 5.1.

There are minimum production requirements for electrolysis, as the production rate can decrease towards 20% of the installed capacity. We approximate

Discrete capacity	1	2	3	4	5	6	7	8
Capacity [tonnes/day]	0.6	3.1	6.2	12.2	30.3	61.0	151.5	304.9
Investment EL [mill. €]	1.4	6.0	11.2	20.5	46.5	87.2	197.7	371.5
Production EL [€/kg]	1.95	1.61	1.53	1.45	1.43	1.42	1.40	1.38

Table 5.1: Investment and production costs at full capacity utilization for EL [43]

the short-term production costs by a convex piecewise linear function with three linepieces. We define four breakpoints at $20\%, 50\%, 80\%$, and 100% of installed production quantity. The 20% breakpoint represents the minimum production requirement based on the technical specifications for electrolysis, and the 100% breakpoint represents full utilization of installed capacity. Each breakpoint is characterized by a specific production quantity and production costs. We can produce arbitrary quantities from the range between $20-100\%$ of the installed capacity by a linear combination of two neighbourhood breakpoints. The short-term costs at a breakpoint are calculated based the a model provided in [17]. We assume that the investment and production costs are independent of facility location.

We calculate the expansion costs E_{ikl} as: $E_{ikl} = (C_{il} - C_{ik}) \cdot (100 + \alpha)\%$. The expansion costs are equal to the difference between investment costs of opening a facility with capacity l and a facility with capacity k, where $k < l$, plus an additional mark-up α. In our case, the mark-up α is 10%

Distance [km]	1-50	51-100	101-200	201-400	401-800	801-1000
Costs	0.00498	0.00426	0.00390	0.00372	0.00363	0.00360

Table 5.2: Hydrogen distribution costs in [€/km/kg H_2] [8]

We use the distribution costs for compressed hydrogen provided by [8]. We consider demand points that aggregate customer demand from the whole municipality, and if a demand point is located in the same municipality as a facility, we assume zero distribution costs. The reason is that the starting point for our case study is the production of hydrogen for maritime transportation. The demand points for this sector are limited to ports. For locations along the Norwegian coastline, we assume that hydrogen production will take place in port or close to the port with negligible distribution costs. This assumption has then been extended to municipalities producing hydrogen for other sectors than maritime for reasons of consistency. We set the distance limit between a production facility and a customer to 1000 km. See Table 5.2 for the distribution costs for compressed hydrogen. The production cost and distribution cost data for our case are identical to the from [43]. For simplification, we assume that the discount factor is equal to one in each period.

5.2 Demand

We consider three main demand components. In the maritime sector, the hydrogen demand estimations are based on current ferry routes and the assumption that the new public contracts will require hydrogen as an energy carrier [32], [1]. The demand estimations in the land-based sector from [11] are based on the emission reduction goal within 2030 stated in [37]. In the offshore sector, we use the hydrogen demand estimations from [2]. These estimations are based on the medium penetration scenario from [32] which calculates the energy consumption for ammonia. However, hydrogen fuel alternative is just as likely to occur [45]. These different demand components are shown in Figure 5.1 together with the expected demand level and the maximum potential hydrogen demand consisting of all three components. The maritime demand is quite certain. Thus, it represents the minimum demand level and is present in all demand scenarios.

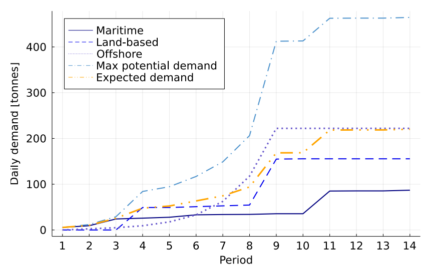


Fig. 5.1: Demand development

We aggregate individual customer demand into 70 demand points located in Norway. These demand points consist of 51 ports that are relevant for the maritime and the offshore sector and 19 municipalities with the highest road traffic volumes according to the statistic provided in [44]. Based on the traffic volumes statistic [44], we divide the road traffic demand among the different municipalities. We remove municipalities with demand lower than 3.65 tonnes H_2/year. However, not all customers, respectively demand points, have demand in all scenarios.

Our planning horizon is 14 periods. Demand is non-decreasing during the whole planning horizon in all considered sectors. In the maritime sector, demand is slightly increasing until period 10 and there is a jump in period 11 when the coastal route Bergen-Kirkenes is to be operated on hydrogen fuels. The jumps in the land-based sector correspond to the strategic government plan to start with the transition towards hydrogen for buses and trucks. The offshore sector will not start the transition towards hydrogen before period 4.

The market share of hydrogen vehicles and hydrogen-driven offshore supply vessels is highly uncertain. We consider demand in the land-based sector and offshore sector to represent a conversion potential and assume that the probability of reaching the maximal potential demand is low. Therefore, we assume that our demand scenarios are not evenly distributed between the minimum and the maximum potential demand. We assume the expected value to be a weighted average of minimum and maximum demand with coefficients 0.65 and 0.35, respectively.

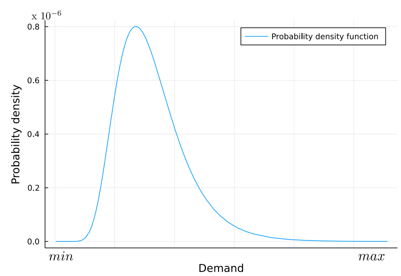


Fig. 5.2: Probability density function for hydrogen demand

We expect that scenarios with lower demand consisting of maritime demand and a share of the land-based and offshore sector are more likely to occur than very optimistic hydrogen scenarios with very high demand. Thus, we need a left-skewed distribution with a low probability of extreme values to sample the scenarios from. We therefore assume a log-normal distribution, $D \sim Lognormal(\mu, \sigma^2)$. The expected value $E(D)$ is given by the previously computed expected demand level and we assume the standard deviation to be $\sigma = 0.3$ as this value still allows some of the high demand scenarios to occur. The probability density function of our log-normal distribution is shown in Figure 5.2.

6 Computational results

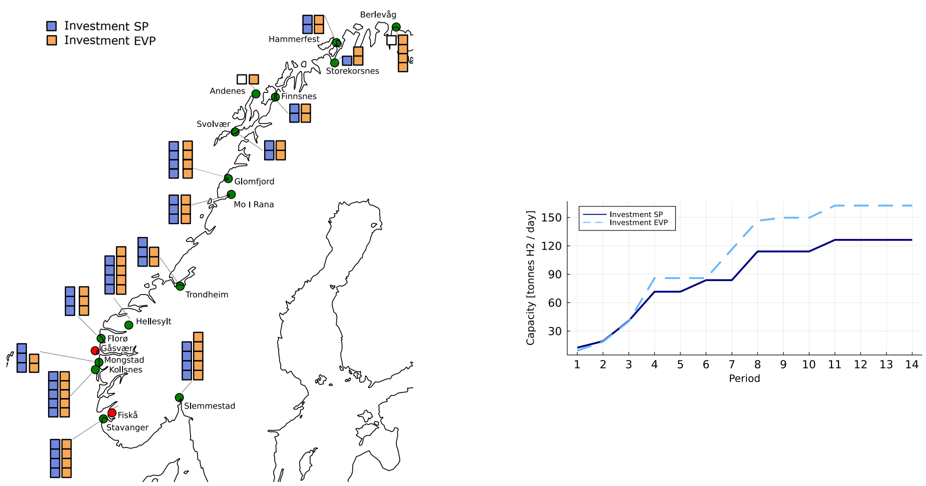
The model is implemented in Julia 1.6.5 and solved using Gurobi Optimizer version 9.5. All calculations have been run on a computer with two 3.6 GHz Intel Xeon Gold 6244 CPU (8 core) processors and 384 GB RAM.

The problem (15) is solved for $M = 50$ SAA problems where each of the problems has a sample size of $N = 10$. The reference sample size is $N' = 1000$ and we evaluate the performance on the reference sample for each of the 50 SAA solutions. We choose to solve the problems (15) with relative optimality gap $\gamma' < 2\%$.

Problem	LB [x10^6 €]	UB [x10^6 €]	$gap_{N,M,N'}(\overline{y})[\%]$
SP	1381.2	1455.2	5.36
EEV	-	∞	-

Table 6.1: Evaluation of the SP and the EEV

We show the best statistical lower and upper bound of the SP in Table 6.1 and compare the results with the EVP. We calculate the expected value of the EVP solution (EEV) and compare the results with the SP. The value of the stochastic solution is: $VSS = EEV - SP$ [5]. The results show that the EVP solution is infeasible. Thus, the VSS goes to infinity. This shows that even if the EVP problem is easier to solve and we can find an optimal solution, it is important to consider the uncertainty in our problem.



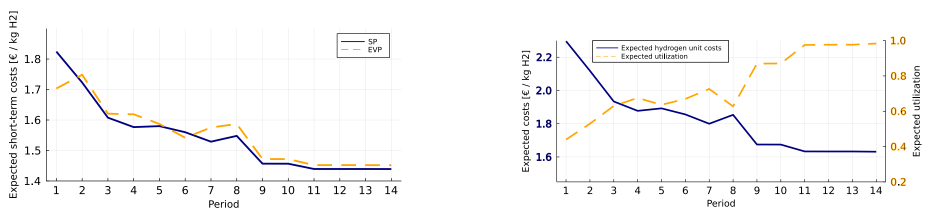
(a) Location and opening capacity of hydrogen facilities in the SP and the EVP

(b) Opening capacity in the SP and the EVP

Fig. 6.1: First-stage decisions: Investment in the SP and the EVP

To analyze the first stage decisions, we study the opening decisions in the SP and the EVP. Figure 6.1a illustrates the facility locations and the opening size of facilities before expansion. When comparing the number of opened facilities, we open 13 facilities in the SP and 15 facilities in the EVP. However, in general, the differences between the SP and the EVP are very small. The main differences can be seen in the northern part of Norway where we do not open a facility in Berlevåg and Andenes in the SP. Thus, we install more capacity in the EVP in comparison to SP. However, the infeasibility comes from the south-western part of Norway even if the number of opened facilities is equal. Please note that the difference between capacity 2 and 3 is only 3.1 tonnes daily while the difference between capacity 4 and 5 is 18.1 tonnes daily. Thus, we install more capacity in the EVP as we open two large facilities in Hellesylt and Slemmestad. Most of the land-based demand is located in the south-western part of Norway and this area is also affected a lot by the offshore demand. Thus, here, we observe the highest differences between the scenarios and the large capacities installed for the EVP cause infeasibility for scenarios with low demand. In the EVP, we cannot fulfil the minimum production requirements for scenarios with low demand due to the large facilities in Hellesylt and Slemmestad.

The development of installed capacity in the first stage in the SP and the EVP solution can be seen in Figure 6.1b. The installed capacity is almost the same in the first three periods because the differences between scenarios are low until period three. Then, both lines indicate growing capacity. However, the installed capacity in SP is considerably lower. The solution of the SP leads to more conservative investment decisions and additional capacity is installed in the expansion step. The expected demand level is considerably higher than the minimum demand so the EVP problem leads to more extensive investments than the SP which is also the reason for the infeasibility of the EVP.



(a) Expected unit short-term costs in the SP and the EVP

(b) Expected unit costs and expected capacity utilization in the SP

Fig. 6.2: Expected hydrogen costs

For illustration, we show the expected unit short-term costs in the SP and the EVP in Figure 6.2a. Please note that we show results for a feasible subset of scenarios in the EVP. The EVP provides lower costs in the first period due to the lower installed capacity (see Figure 6.1b) resulting in higher utilization. In the following periods, the costs in the SP are, in general, lower. However, the costs are very similar because expansion in the second stage provides a lot of flexibility to adjust the infrastructure as a reaction to growing demand. Expected unit hydrogen costs and expected utilization for the SP are shown in Figure 6.2b. The expected unit hydrogen costs have a decreasing tendency that is in line with the growing capacity (see Figure 6.1b) and increasing utilization. The unit production costs have two peaks in period 5 and 8 that are related to a decrease in capacity utilization as lower utilization results in higher unit costs. In expectation, unit production costs are decreasing together with increasing capacity and its utilization which indicates the presence of economies of scale in hydrogen production.

7 Conclusion

We study the optimal hydrogen production infrastructure under uncertain demand in Norway. We present a model for a two-stage stochastic multi-period facility location problem with capacity expansion. The problem is hard to solve and using commercial software, we can solve it with 10 scenarios. Therefore, we use SAA to solve the problem. This approach provides good solutions with an estimated gap between the lower and the upper bound of 5.36%.

The quality of the solution is limited by the number of scenarios we can solve the problem with. Implementing an efficient solution method in order solve the problem with more scenarios and thus improve the solution quality is a natural extension of this work.

Another extension of this work is to study how the investment structure will change when we modify the underlying demand distribution.

Expansion in the second stage provides a lot of flexibility in terms of reaction to growing demand. It is worth considering, how the investment decisions will change for different models. We can consider a multi-stage model, or a more restrictive model where expansion is the first-stage decision and only decisions regarding demand allocation are taken in the second stage. In future work, uncertainty in costs might be considered as well.

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