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Astrid Junker[1]

[1]Norwegian University of Science and Technology, Department of Teacher Education, Trondheim, Norway; astrid.junker@ntnu.no

This paper explores metacognition to Andrews & Sayers' (2015) number sense framework in descriptions of systematic counting competence, using Kilpatrick et al.'s (2001) components of mathematical competence as guides. A case study design was provided to explore a mathematically competent 6-year-old girl's counting and counting strategies when entering the first grade. She is referred to as Agnes. Qualitative investigations of video recordings of a systematic counting interview and a digital number sense assessment were done. Agnes met some of Andrews & Sayers' (2015) systematic counting criteria. Regarding Kilpatrick et al.'s (2001) competence framework, Agnes demonstrated fluency and flexibility in counting, and outstandingly showed adaptive reasoning and productive disposition. Including aspects of Kilpatrick et al.'s (2001) mathematical competence framework seemed to supply appropriate additional information about counting strategies.

Keywords: Counting strategies, number sense, mathematical competence, 1st grade students.

Introduction

Number sense predicts later mathematical competence and is the ability to flexibly work with numbers and quantities (Andrews & Sayers, 2015; Dehaene, 2011). Despite differing definitions of the term, counting and arithmetic are central components of number sense. Counting is important for arithmetic competence and supports the development of a mental number line (Morrissey et al., 2020). Verbally counting numbers is found to help cardinal understanding, while finger-counting helps develop number concepts and simple arithmetic (Geary et al., 2018; Morrissey et al., 2020). Children at risk of developing mathematical difficulties are said to exhibit inflexible and inefficient counting and calculation strategies (Geary et al., 2018).

The way children count is certainly interrelated with number sense. Although flexibility is essential in number sense definitions, flexible counting has been sparsely described beyond fluently mastering counting strategies and successively shifting between them (Andrews & Sayers, 2015). Andrews & Sayers (2015) defined systematic counting as counting upwards and backwards between zero and twenty, including understanding ordinality and being able to start from an arbitrary starting point between zero and twenty. Beyond this, systematic counting abilities are primarily described in relation to competence in other number sense components. Thus, additional descriptions of counting and counting strategies used by mathematically competent students entering 1st grade is sought. Extended knowledge about counting strategies used by competent students will help expand knowledge about what strategies to expect and better teach students who show a slower or different development. Such knowledge can explain the typical and expected complexity and flexibility of counting strategies. In this way, delayed or deviating strategy use will be more easily discovered, enabling the identification and possible prevention of students with mathematical learning difficulties, as it will recommend counting concept contents and strategies to teach them.

Kilpatrick et al. (2001) defined mathematical competence or proficiency as an interplay between conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. This paper attempts to add metacognition to Andrews & Sayers' (2015) number sense framework. This is done by exploring a mathematically competent six-year-old girl's counting strategies as she enters 1st grade through an interview based on Andrews & Sayers' (2015) definition of systematic counting. Systematic counting strategies were explored using the mentioned interplay in Kilpatrick et al.'s (2001) definition of mathematical competence. This paper has the following research questions:

What counting strategies does Agnes use? Does she show flexibility? What relations are observed between her counting strategies and number sense?

This paper reports a preliminary analysis from an ongoing Ph.D. project studying 75 Norwegian 1st grade student's number sense variation and its relations to verbal and nonverbal aspects of cognition. The study includes several measures of number sense and cognition. Here, data from a task-based systematic counting interview is used, in addition to data from a digital number sense assessment that a Ph.D. student and colleague of mine is developing. A girl kept under the name of Agnes was considered mathematically competent as she mastered most tasks on the digital number sense assessment conducted 6 weeks after entering 1st grade. The systematic counting interview was conducted two weeks later.

Previous research

Andrews & Sayers created a number sense framework consisting of nine components, including systematic counting (Andrews & Sayers, 2015; Sayers et al., 2016). These are described in more detail in the "Assessments" section. Previous research have explored interrelations between these number sense components and suggested that understanding numbers and counting systematically relies on estimation abilities and being able to compare small quantities without counting (Dehaene, 2011). Subitising is further assumed as essential for verbal counting skills and arithmetic (Sayers et al., 2016).

In addition, students' counting strategies are found to reflect counting knowledge and performance in counting procedures (Morrissey et al., 2020). Embodied activities like jumping on a number line or finger-counting are found to contribute to developing counting skills and counting strategies (Morrissey et al., 2020). Counting is obviously part of the cognitive complex. Depending on the students' counting strategy, visual, verbal, and tactile information is processed to create or recall mental representations of factual, conceptual, and procedural knowledge of numbers and arithmetical operations (Hiebert, 2009).

Backup strategies such as count-all strategies and retrieval strategies such as count-on strategies reflect this complexity. They serve as expressions for what and how students understand the complexity. Compared to at-risk students that are found to have immature and inefficient counting strategies and have problems with shifting to more advanced strategies (Koponen et al., 2018), competent students are expected to flexibly master this.

The aforementioned research appears to represent the components of mathematical competence that Kilpatrick et al. (2001) defined as conceptual understanding, which is understanding mathematical concepts, operations, and relations. Kilpatrick et al. (2001) claimed that strategic competence is the ability to formulate, represent, and solve mathematical problems.

Together with this, procedural fluency seems to cover systematic counting strategies, being defined as the ability to conduct procedures accurately, flexibly, efficiently, and appropriately, including both conscious and unconscious implicit and explicit cognitive actions involved in doing procedures fluently. Any case of logical thought, reflection, explanation, and justification is covered by Kilpatrick et al.'s (2001) adaptive reasoning. They claimed that adaptive reasoning required students to recognise repeated patterns, often observed via language. I also searched for signs reflecting productive disposition in the interview, that is, if Agnes viewed mathematics as useful and saw herself as diligent and efficient.

Even though components interplay and are likely inseparable, I looked for components of Kilpatrick et al.'s (2001) mathematical competence in the systematic counting interview to add descriptions of metacognition and flexibility to Andrews & Sayers' (2001) definition of mastery to count systematically.

Methods

Video recordings of the student's systematic counting strategies and digital number sense tasks were qualitatively investigated.

Participant

After obtaining informed parental consent, a girl aged 5 years and 11 months was recruited from a typical neighborhood school in mid-Norway. She is referred to Agnes in this study.

Assessments

Systematic counting in the digital number sense assessment

A number sense assessment consistent with Kilpatrick et al.'s (2001) mathematical competence framework and Andrews & Sayers' (2015) definition of number sense was conducted (Saksvik-Raanes & Solstad, personal communication, 2020). Table 1 describes the number sense tasks that assessed eight of the nine components. Representing numbers was not included in the assessment due to feasibility consideration of the tasks for all 75 students. Each task included either verbal or visual instruction, or both. The student tapped, dragged, dropped, and organised objects on the screen.

Table 1: Number sense component and a description of each component

Category	Content
Number identification	Recognise number symbols, vocabulary, and meaning.
Systematic counting	Includes ordinality. Count to twenty and back (also from an arbitrary starting point).

Number and quantity	Cardinality. 1:1 correspondence between symbol and quantity.
Quantity discrimination	Compare quantities. Vocabulary: larger, smaller, more/less than.
Estimation	Estimate the size of a set and the position on a number line.
Arithmetic competence	Transform small sets by using addition or subtraction.
Number patterns	Continue or complete a number sequence.
Subitising	Perceive quantity without counting. Perceptual/conceptual. Timed.

Andrews & Sayers (2015) defined systematic counting as described (Saksvik-Raanes & Solstad, pers. comm., 2020).

A digital version of some of the systematic counting tasks were also investigated in the interview. Task designs provided insight into visual, verbal, and embodied contribution in systematic counting and scaffolding. The eight tasks that made up the systematic counting component were developed into the three following task designs.

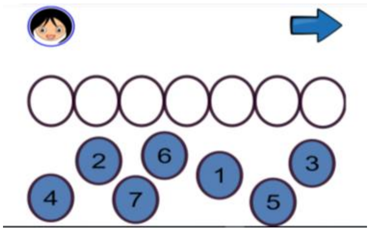

Figure 1. A digital task from the systematic counting component

The task gave the verbal instruction: *"Put the numbers in order"*

Systematic counting interview

A task-based interview was developed based on Andrews & Sayers' (2015) definition of systematic counting but excluding the ordinality aspect. Backup and retrieval strategies were observed. A frog named Mr. Minus asked ten questions to facilitate Agnes' counting, allowing her to demonstrate her counting strategies. If needed, Mr. Minus modelled counting strategies. The interview lasted for 10 minutes and was administered by the author. No time limits were given for the exploration of time-consuming strategies, self-correction, and Agnes's verbally expressed reflections and consciousness.

Analytical procedures

The systematic counting interview was videotaped and qualitatively explored based on Andrews & Sayers' (2015) definition of systematic counting and using Kilpatrick et al.'s (2001) components as guides. Digital number sense tasks were registered as correct or incorrect. They were likewise qualitatively investigated regarding how they provided information to the student.

Results - findings of mathematical competence

Agnes's systematic counting strategies in the digital number sense assessment and in the interview was explored using Kilpatrick et al.'s (2001) definition of mathematical competence. Their descriptions of competence functioned as a lens for determining and describing Agnes's systematic counting strategies.

Conceptual understanding

Agnes demonstrated conceptual understanding by identifying numbers and by meeting the requirement to count upwards from an arbitrary starting point between 0 and 20. She was also able to count backwards from an arbitrary starting point between 0 and 10. The interview did not test her understanding of ordinality and cardinality, but the digital number sense assessment confirmed her understanding in this regard.

As counting upwards was easy for her (she counted to 100), she began counting backwards from 99 to 80. As shown here, her fluency or prolonged pronunciation of 95 and 91 could indicate that transitioning from one set of 10 to the next required thinking and was not a completely automatic process for her. She also finger-counted and whispered every 10th number, starting at 10 and continuing upwards when counting backwards was difficult:

Agnes:	I manage to count from 100 to 80, I think. 99-98-97-96-95-94-93-92-91-80-89-88-87-86-85-84-83-82-81.
Mr. Minus:	What comes next, after 81?
Agnes:	70!

Agnes finger-counted backwards from 20 by combining a retrieval strategy where the thumb represented the number 10, followed by a backup strategy where adjacent fingers represented consecutive numbers. She counted upwards from 10 and stopped when she saw there was one less finger on her hand compared with the number of fingers she remembered seeing on the preceding answer. Agnes combined adding and subtracting in her counting procedures. An unstable one-to-one correspondence was observed as she provided two incorrect answers.

These results suggest that her mental number line is not completely stable. Agnes mastered all the digital number sense tasks except one half of the estimation tasks and one third of the arithmetic tasks. Her limited conceptual understanding and her lack of a stable mental number line may relate to her performance on the estimation and arithmetic tasks.

According to Kilpatrick et al. (2001), conceptual understanding involves understanding procedures as well as concepts; thus, I considered Agnes's counting strategies an aspect of her conceptual understanding. I also interpreted her strategies to reflect procedural fluency. These findings underline the interplay between the aspects of mathematical competence.

Strategic competence

Agnes was able to formulate, represent, and solve mathematical problems when she both successfully and unsuccessfully used backup and retrieval strategies—sometimes combining the strategies. For instance, she understood the systematic counting system when continuing a number sequence that started with 2-4-6. She automatically answered with 8-10-12. Interestingly, she then began counting using another pattern (i.e., 3-6-9). She could not automatically provide the following number, but she used her fingers to solve the problem and continue the pattern.

Procedural fluency

That Agnes self-corrected when counting, that she used both retrieval and backup strategies, and that she combined addition and subtraction in the same procedure illustrate her ability to consciously and perhaps subconsciously follow procedures flexibly, fluently, and appropriately. That Agnes wanted to share having counted from 10 to 100 by counting only the 10s illustrates that she was familiar with efficient strategies and could be flexible when counting to 100.

However, these findings are inconsistent. For instance, Agnes did not self-correct after having incorrectly counted from 99 to 80, and she mastered only two or three sequences of systematic counting. Kilpatrick et al. (2001) also regarded accuracy as indicative of procedural fluency, and Agnes was inconsistent with regard to accuracy; she sometimes did not count with a one-to-one correspondence. This may further indicate that her mental number line is not stable, as was already suggested. That Agnes finger-counted between 20 and 10 and stopped doing so when counting from 10 to 0, may also indicate an unstable mental number line. Her lack of fluency indicates perhaps that transitioning from one set of 10 to the next required thinking and was not a completely automatic process for her.

Adaptive reasoning

Agnes's expression when counting from 10 to 100 (i.e., "Now I counted only the 10s"), that she verbally argued she thought it was possible to count from 7 to 20 because that was almost like counting from 1 to 20, and that she explained she counted by 2s when she correctly continued the number sequence 2-4-6 were interpreted as expressions of her logical thought, reflection, explanation, and justification. She was able to recognise patterns and reason adaptively.

Productive disposition

Many examples indicate that Agnes saw herself as diligent and efficient, and there are some indications that she found mathematics useful. In general, she was positive and seemed eager to discover new ways of counting, or to demonstrate different ways of counting that she had mastered:

Agnes: I also manage to count to 100 like this: 10-20-30-40-50-60-70-80-90-100. Now I counted only the 10s.

She also had the confidence to try counting in ways she that she was at first unsure of. She modelled finger-counting and moved her lips as she pronounced 1-2-3-4-5.

Agnes: I do not think I know much about counting backwards. Sometimes I need to do like this, and maybe I need to do now.

Agnes also experimented with finger-counting, and systematically counting in a way she was not fully capable of mastering. Here is an example when, with a one-to-one correspondence between fingers and numbers, she tried to count by 3s:

Agnes I know this thing: 3-6-9 (pause)-12-15-18-20-23-26-29.

Discussion

Results from the interview suggest that Agnes, mathematically competent upon entering the first grade, met some of Andrews & Sayer's (2015) systematic counting criteria. She was able to count upwards to 20 but did not count correctly backwards from 20 to 0. Agnes used both backup and retrieval strategies, sometimes in combination. She did so both successfully and unsuccessfully, as when her counting lacked a one-to-one-correspondence. Agnes demonstrated both mastery and a lack of competence when using advanced and basic counting strategies. This brief analysis suggests that backup and retrieval counting strategies are interrelated; Agnes is perhaps now developing backup strategies exclusively, before developing retrieval strategies. Further study is needed with regard to mathematically competent students' counting strategies.

Overall, Agnes demonstrated fluency and flexibility in counting. She showed that finger counting is used as a mental support among highly competent students, enabling them to develop their mental number lines. Most notably, she demonstrated an outstanding ability to reason about and justify her strategies, engage in the counting tasks, and believe in herself.

Her strong subitising abilities, demonstrated during the digital number sense assessment, perhaps played a role when her thumb represented 10, and when she counted by 2s (Sayers et al., 2016). Mastering only one half of the estimation tasks may have affected her backwards counting because estimation may be important for mentally representing the number line (Dehaene, 2011).

In sum, operationalising Andrews & Sayers's (2015) definition of number sense provides us with a good theoretical starting point. However, their definition may be insufficient when describing flexibility in counting. Including aspects of Kilpatrick et al.'s (2001) mathematical competence framework seemed to supply appropriate additional information about counting strategies. Agnes, a mathematically competent first grade student, demonstrated outstanding mastery with regard to adaptive reasoning and productive disposition. A more thorough exploration of such metacognitive aspect when describing the flexibility in counting and counting strategies may provide desired insight.

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