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## What Can We Learn from the Different Understandings of Mathematical Literacy?

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# What Can We Learn from the Different Understandings of Mathematical Literacy?

## Abstract

Mathematical literacy, quantitative literacy, numeracy, matheracy, disciplinary literacy, and content-area literacy are among a plethora of terms used to link mathematics and literacy. In addition to this abundance of terms, the content of the terms is not precise, and in some cases the terms are used interchangeably. I delve into this landscape and dissect the meaning of each of them. From such an analysis of mathematical literacy, numeracy, and quantitative literacy, we learn about the importance of quantitative practices and elementary mathematics in society, including both how mathematics is necessary for everyday life but also how it can be used to change the world. By discussing content-area literacy in mathematics, we learn about text in mathematics and the shortcomings of general text-competence approaches are revealed. By analyzing disciplinary literacy in mathematics, we learn to appreciate mathematics as a subject with its own language, tools, techniques, and ways of thinking that emphasize inquiry and experimentation following the heritage from Euclid, Archimedes, Newton and Euler. Having so many competing terms and approaches has a negative effect on the academic discourse but also on the communication with the world outside of academia.

## Keywords

mathematical literacy, numeracy, quantitative literacy, disciplinary literacy, content area literacy, critical mathematics education

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## Cover Page Footnote

Svein Arne Sikko is Associate Professor in Mathematics Education at NTNU, Norwegian University of Science and Technology. He has a doctor degree in pure mathematics within the field of representation theory of algebras. Since 1998, he has been working with mathematics teacher education. His main areas of research interest are mathematics teacher education, teacher professional development, lesson design studies, inquiry-based learning, mathematical literacy, and issues related to the transition between educational levels.

## Introduction

Widely conceived, mathematical literacy is not well defined, and several concepts pertaining to it are considered more or less synonymous, most prominently numeracy and quantitative literacy (Geiger et al. 2015b; Vacher 2017). Furthermore, a fundamental difference in the use and concept of mathematical literacy exists based on the professional tradition to which one belongs. Within mathematics education research, mathematical literacy is largely related to the use of mathematics in everyday life and life as a citizen, but this is not unambiguous, and involves the use of several different words and phrases. In other areas of literacy research, there is a distinction between disciplinary literacy and content area literacy, and none of these approaches to mathematical literacy is the same as what we find in mathematics education literature. In this article I analyze the different aspects and content of the literacy concept and study what those of us interested in quantitative literacy can learn by considering the different perspectives.

There are two problems to solve. One concerns the use of the concepts of mathematical literacy, quantitative literacy, and numeracy within mathematics education research: there is a need for clarification regarding both the terms used and the content of the concepts. In addition to the existence of different terms, there is also a need for a clarification of the meaning of the terms. The second problem to be investigated concerns the different uses of the term *literacy*. I want to investigate how the term *mathematical literacy* is used within mathematics education research compared to the terms *disciplinary literacy* and *content area literacy* when these latter terms are used with mathematics as the discipline or content area. In doing so, I want to develop a deeper understanding of how the concept of literacy is connected to and used in connection with mathematics, quantitative practices and mathematics teaching. This investigation should be of interest to researchers not only in mathematical or quantitative literacy or literacy scholars but can also contribute to discussions in mathematics education more broadly. An interrogation of the literacy concept should also be of interest beyond academic circles, given the big impact from international surveys like PISA (OECD 2018), where mathematical and scientific literacy are tested, influencing national curricula in many countries, not least those countries where the results are found to be disappointing.

This article is structured as follows: I first write about mathematical literacy in mathematics education research. This section includes a subsection where I consider whether mathematical literacy, numeracy, and quantitative literacy are to be seen as synonymous, and a subsection where I discuss whether there is a hierarchy of terms. In the section following thereafter I look at mathematical literacy in a critical-mathematical perspective. I then proceed to discuss

mathematics in content area and disciplinary literacy. I provide a separate discussion section before the conclusion.

## Mathematical Literacy in Mathematics Education Research

Within mathematics education research there is no agreement on the content of the concept of mathematical literacy. As someone who is trained in mathematics, I find this situation quite intriguing. Profke (2014) proclaims, “There is no standard definition of ‘mathematical literacy’” (327). In the discussion document for the 24<sup>th</sup> ICMI Study we read that “the word literacy itself is ambiguous with multiple meanings, and trying to translate it into different languages and cultures is a difficult, if not sometimes impossible task” (ICMI 2017, 4). In the *Encyclopedia of Mathematics Education* Niss and Jablonka (2014) point to the resistance toward using the term mathematical literacy stemming from the fact that there are no translations of the term into languages such as German or the Scandinavian languages. Jurdak (2016, 44) likewise teaches us that there is no word in Arabic covering literacy, but that there is a term for the negation, illiteracy, viz. *ummiyyah*. In the *Second International Handbook of Mathematics Education*, Jablonka (2003) also makes it clear that in several languages it is not possible to find direct translations of mathematical literacy or numeracy, and the same goes for quantitative literacy. The meaning of such terms therefore must be paraphrased. The language issue is probably more problematic outside of a mostly English-language dominated academia. For example, having no native language term that can be used may cause communication problems with teachers and students, something I myself, being a non-native English speaker, struggle with. It may also be seen as problematic not to have a native language term in light of social practice views of connecting mathematics and quantitative knowledge and skills to everyday life.

Let’s look at the two parts of the term mathematical literacy. The first part, mathematical, connects the term mathematical literacy unambiguously to mathematics. Thus, it should be clear that mathematical literacy is something that differs from other subject literacies, whether it be in the humanities, social sciences or natural science subject areas. This does not imply that there may be no bonds to other disciplines, but the connection to mathematics must be of a kind that necessitates distinguishing it, and not only speak of literacy or scientific literacy. The other part of the term is the word literacy. If we use literacy as part of the concept, whether it is in the form of mathematical literacy or quantitative literacy, we create other connotations than if we had called it numeracy. By letting literacy be a part of the concept, one gives a hint that there are connections to other concepts that contain literacy as part of the concept, and in particular one has connected to

the literacy concept itself. In connecting to literacy, a connection is made to a dominant discourse in literacy research, viz. social practice theory, seeing literacy as a situated social practice (Brandt and Clinton 2002). Along these lines, Fisher (2019) argues that literacies are socially constructed and that to become literate involves a change in identity, joining a new social group, or adopting a new culture (10). Craig et al. (2019) argue similarly when they point to how the social world is being integrated with numbers leading to numeracy emerging as a partner to literacy.

## Mathematical Literacy, Numeracy, or Quantitative Literacy?

In this section I will discuss the relationship between the terms mathematical literacy, numeracy, and quantitative literacy, and look at some definitions.

In oft-cited parts of the research literature within mathematics education there is no distinction between the terms mathematical literacy, numeracy, or quantitative literacy. Among those belonging to the tradition of not distinguishing between terms are Merrilyn Goos and her colleagues in Australia and Lynn Arthur Steen and colleagues in USA. Which word that is used by these researchers depends on which country you are from. Goos et al. (2011) say that “*numeracy* is a term used in many English-speaking countries, such as the UK, Canada, South Africa, Australia and New Zealand, in the USA, and elsewhere, it is more common to speak of *quantitative literacy* or *mathematical literacy*” (131). Likewise, Steen (1999) considers quantitative literacy and numeracy as equivalent, claiming that numeracy is more common in British English while quantitative literacy is the term used by American writers. In a survey of the first ten years of the journal *Numeracy*, Vacher (2017) also found that quantitative literacy seems to be the term preferred by American writers whereas numeracy is more commonly used by non-Americans. Even so, the main point here is that for these writers the use of different terms does not point to differences in the concept itself.

So what do the terms mean? Merriam-Webster (n.d.) defines numeracy as “ability to understand and work with numbers: the quality or state of being numerate” and says that the first known use of the term was in 1959. About the word numerate, it is said that this means “having the ability to understand and work with numbers.” Thus, the definition in the dictionary connects directly to the work with numbers. It seems that most people regard the British Crowther report from 1959 as the main source for the word numeracy, at least in English speaking countries (McIntosh et al. 1992; Madison and Steen 2008; Geiger et al. 2015b; Karaali et al. 2016). Numeracy is here seen as mirroring, or being an analogy to, the literacy concept in reading and writing. Being numerate is then seen as an analogy to being literate. McIntosh et al. (1992) point to the fact that since

numeracy was associated with literacy, it led to a narrow interpretation, viz. “to mean only an ability to cope with the basic mathematical demands of everyday life” (2). Madison and Steen (2008) on the other hand, found that the original connection of numeracy to literacy paved the way for sophisticated interpretations involving for example problem solving, but that numeracy “gradually came to mean just basic arithmetic skills normally acquired in childhood” (2). Steen (1999) also considers the different terms used and warns against a too narrow definition:

Quantitative literacy—or numeracy, as it is known in British English—means different things to different people. Although quantitative literacy is often confused with its close relatives, such as basic skills, elementary statistics, logical reasoning, or advanced mathematics, none of these by itself offers a complete or balanced view of numeracy. (8)

Steen et al. (2007) mention quantitative literacy, functional mathematics, and quantitative reasoning as concepts that are in use and are considered synonymous to mathematical literacy. They point out:

In some contexts and some nations, these terms are used narrowly to mean just “basic skills”—arithmetic plus a bit more. This is a corruption of the terms, just as literacy means much more than spelling, grammar and syntax. These skills are necessary but far from sufficient. (294)

Steen and his colleagues add, “Mathematics plays a parallel role in mathematical literacy to that of language in literacy” (287), and that this was the original definition of numeracy in the Crowther report in 1959 but that this definition has been distorted. They thus claim that the numeracy concept, or mathematical literacy as it is named in their article, embraces more than numbers, manipulations, calculations, and algorithmic skills. Steen et al. (2007) tie mathematical literacy unambiguously to the mathematics needed for everyday life. They say that “Mathematical literacy is the capacity to make effective use of mathematical knowledge and understanding in meeting challenges in everyday life” (285). This functional perspective on mathematical literacy is analogous to how functional literacy has been defined within reading and writing. In 1956, Gray wrote that “a person is functionally literate when he has acquired the knowledge and skills in reading and writing which enable him to engage effectively in all those activities in which literacy is normally assumed in his culture or group” (Gray 1956, 24). Steen et al. (2007) regard mathematical literacy in contrast to school mathematics which they find too scholastic and too focused on tying the line to Euclid, Newton, and Euler, something that at best equips a minority of students with the mathematics they need in their future life and career. In everyday life there is seldom any use of advanced mathematics, they claim, but rather advanced use of elementary mathematics. “Whereas school mathematics stresses elementary uses of sophisticated mathematics, mathematical literacy focuses on sophisticated uses of (often) elementary mathematics” (Steen et al. 2007, 289). This view is shared by Gillman (2006) who refers to quantitative literacy as being able to “use fairly

elementary mathematical tools in sophisticated manners in a wide variety of contexts” (vii), and contrasts this with (pure) mathematics which use tools and techniques that have little value for the common citizen in everyday life. A step in the right direction, according to Steen et al. is the definition of mathematical literacy given by OECD in connection with the PISA (Programme for International Student Assessment) tests. In the 2003 version that Steen et al. referred to, mathematical literacy is defined as

an individual’s capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgements and to use and engage with mathematics in ways that meet the needs of that individual’s life as a constructive, concerned and reflective citizen. (OECD 2003, 24)

In the framework for PISA 2021, the definition has been slightly changed. It now reads

Mathematical literacy is an individual’s capacity to reason mathematically and to formulate, employ, and interpret mathematics to solve problems in a variety of real-world contexts. It includes concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to know the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective 21<sup>st</sup> century citizens. (OECD 2018, 7)

It is common to both definitions that mathematical literacy is defined as an individual’s capacity. In 2003 that individual was constructive, concerned, and reflective. In 2018 s/he is still constructive and reflective but is now engaged rather than concerned. A more important change concerns the capacity of the individual. In 2003 it was to identify and understand the role mathematics plays in the world, whereas in 2018 it is the capacity to reason mathematically. To identify and understand is a more passive role by the individual than to reason mathematically. This is not made up for by the 2003 formulations about using and engaging with mathematics versus the 2018 definition about formulating, employing, and interpreting.

The individual in a variety of contexts, including social life, is also in focus in the definition given by the Special Interest Group of the Mathematical Association of America on Quantitative Literacy (Mathematical Association of America 2004), who describe quantitative literacy as

the ability to adequately use elementary mathematical tools to interpret and manipulate quantitative data and ideas that arise in individuals’ private, civic, and work lives. Like reading and writing literacy, quantitative literacy is a habit of mind that is best formed by exposure in many contexts. (Article I Section 2)

It is worth noting that OECD choose to use different terms in PISA and PIAAC (Programme for the International Assessment of Adult Competencies). For PIAAC,

which surveys adults between 16 and 65 years of age, the term used is numeracy rather than mathematical literacy. In the PIAAC context, numeracy is defined as

the ability to access, use, interpret and communicate mathematical information and ideas, in order to engage in and manage the mathematical demands of a range of situations in adult life .... Numeracy is further defined in terms of the concept of “numerate behaviour” that involves managing a situation or solving a problem in a real context by responding to mathematical information and content represented in various ways. (OECD 2019, 24)

The choice of numeracy instead of mathematical literacy for PIAAC might be simply because PISA is related to school, where mathematics is an explicitly named subject, whereas PIAAC is related to adult life. Even if different terms are used in the PISA and PIAAC frameworks, the functional aspect of mathematics is dominant. In both cases it is about mathematics necessary for managing life in modern society. Oughton (2018) challenges the PIAAC definition of numeracy through a social practice lens. In particular, she questions the phrase “real contexts” and argues that survey test items are not genuinely real. On the contrary, solving problems related to numeracy in a real setting is fundamentally different from solving test items (Oughton 2018, 9), and the result of such tests thus underestimate adults’ genuine real-world skills.

While the OECD definitions stress the individual citizen, a social practice definition would put more emphasis on communities. Fisher (2019), arguing that literacies are socially constructed, suggests the following definition of quantitative literacy: “Quantitative literacy is the facility to participate in the intersecting quantitative practices of many different communities (each with their own patterns of discourse)” (10).

To participate in quantitative practices of different communities can be compared to OECD’s words about reasoning mathematically in a variety of real-world contexts. While real-world contexts seemingly exclude pure mathematics, a community with quantitative practices could also be the community of mathematics.

## **A Hierarchy of Terms?**

Having discussed some definitions where any distinction between mathematical literacy, numeracy, or quantitative literacy are absent or downplayed in the previous section, I am in this section going to consider whether the terms may rather be linked in a hierarchy.

In de Lange (2006), mathematical literacy is explored within the perspective of Realistic Mathematics Education (RME). De Lange has been central in the development of the PISA framework for mathematical literacy as leader of the PISA Mathematics Expert Group in 2000, 2003, 2006, and 2009. De Lange does not see numeracy, quantitative literacy, and mathematical literacy as synonyms. He sees mathematical literacy as an overarching concept involving knowledge of



mathematics in a broad sense which he finds is not captured by the terms quantitative literacy or numeracy. Instead, quantitative literacy, numeracy, and spatial literacy are regarded as sub-concepts of mathematical literacy. Spatial literacy concerns how we perceive space and orientate in the world we are part of. Numeracy in this model concerns the ability to handle numbers, measures, and data, and being able to assess statements related to problems and situations that invite thinking and estimation in real contexts. In other words, numeracy is strongly connected to handling of and understanding of numbers, but also to how to make use of this knowledge in society. Quantitative literacy includes several subcategories, like quantity, change, and relationships, and uncertainty. The four categories Space and Shape (subcategory of Spatial literacy), Quantity (subcategory of both numeracy and quantitative literacy), Change and Relationships (subcategory of quantitative literacy), and Uncertainty (subcategory of quantitative literacy) are exactly what OECD has chosen, because they “reflect both the mathematical phenomena that underlie broad classes of problems, the general structure of mathematics, and the major strands of typical school curricula” (OECD 2018, 22).

De Lange stresses that using the term literacy together with mathematics, i.e., making the term mathematical literacy, underlines that we are not talking about the school subject mathematics as it is defined in the curricula. “Instead, what we have in mind is mathematical knowledge put into functional use in a multitude of contexts in varied, reflective, and insight-based ways” (de Lange 2006, 16). In contrast to, for example, Goos and her colleagues (Goos et al. 2011; see next section) de Lange (2006) holds that attitudes and emotions, including self-efficacy and the urge to explore, are not components of mathematical literacy but important prerequisites for developing mathematical literacy, as he claims that “it is possible to possess mathematical literacy without possessing such attitudes and emotions” (16). De Lange’s model differs from several of the models discussed earlier in that the terms mathematical literacy, numeracy, and quantitative literacy are defined differently and are not seen as synonymous; rather, mathematical literacy is seen as the overarching concept. The models discussed so far agree in the sense that mathematical literacy concerns the use of mathematics in society and not mathematics *per se* (i.e., mathematics for mathematics’ sake without relating it to any practical use outside of mathematics itself).

An in-depth analysis of terms used is undertaken by Karaali et al. (2016). Starting with the terms numeracy, quantitative literacy, and quantitative reasoning, Karaali et al. (2016) use a four-dimensional framework to come to grips with the content and description of the terms. See Table 1.

**Table 1**  
**Four-dimensional Overview of Numeracy, Quantitative Literacy, and Quantitative Reasoning**  
**(Karaali et al. 2016)**

	Numeracy	Quantitative literacy	Quantitative reasoning
Quality of desired outcome	Ability to make use of some skills (basic)	Ability and habit of mind (intermediate to advanced)	Habit of mind (advanced)
Knowledge domain	Mathematics/mathematical, arithmetic/quantitative, and logical (arithmetic-mathematics-logic spectrum)	Data, mathematics, mathematical and arithmetic/quantitative, and logical (data-arithmetic-mathematics-logic spectrum)	Statistics, statistical, data, mathematics/mathematical, and arithmetic/quantitative and logical (statistics-data-arithmetic-mathematics-logic spectrum)
Display of expertise	Understand and appreciate and cope (passive, reactive)	Analyze, appreciate, decide, understand, use (active, reactive)	Analyze, appreciate, critique, decide, understand, use (active, reactive, proactive)
Context	Information and practical situations (daily life, possibly work)	Citizen, information, practical situations (daily life, work, civic life)	Citizen, information, practical situations (daily life, work, civic life)

From their analysis, Karaali et al. (2016) find that there is a hierarchy of terms, with numeracy as a basis, or prerequisite, for the other two, and quantitative reasoning represents a level of mastery and sophistication that goes beyond the other two terms (17). They conclude that a common thread relating numeracy, quantitative literacy, and quantitative reasoning concerns “representations of the real world, in contexts of daily life, work situations, and the civic life” (25). Karaali et al. (2016) also discuss other terms related to numeracy, like mathematical literacy and statistical literacy, noting that “mathematical and statistical literacy often incorporate some aesthetic expectations” (25), as for example geometry fits within mathematical literacy whereas numeracy and quantitative literacy/reasoning focus on numerical and logical aspects of mathematics.

Both de Lange (2006) and Karaali et al. (2016) operate with a hierarchy of terms but disagree on how these terms are related. Whereas de Lange sees mathematical literacy as the overarching term having numeracy and quantitative literacy (together with spatial literacy) as subterms, Karaali et al. see numeracy as the basis upon which the other terms are built, including mathematical literacy which builds on numeracy but has a wider perspective by including for instance aesthetics.

So far, we have seen that mathematical literacy, numeracy, and quantitative literacy are defined in different ways, but in all cases relating the use of something connected with mathematics in the world.

## Mathematical Literacy in a Critical-mathematical Perspective

In models with a social practice aspect, the use of mathematics/numeracy in society is addressed, and the word critique can be found in Table 1. In this section I address criticality explicitly and discuss how mathematical literacy, numeracy, or quantitative literacy can play a role in having a critical view on the world or can contribute to changing the world.

Even if the OECD/PISA definition relates to the use of mathematics in society, it does not explicitly address critical thinking within mathematics, nor does it address critical use of mathematics. OECD itself being an authority, it is probably not to be expected to propose the use of mathematics to challenge authority. A model where the critical aspect is on the frontline has been launched by Marilyn Goos (e.g., Goos et al. 2011). Goos and her colleagues use the term numeracy but consider numeracy as synonymous to mathematical literacy or quantitative literacy, saying “numeracy, sometimes known as quantitative literacy or mathematical literacy” (Goos et al. 2014, 81). In addition to attempting to give an updated view of what numeracy means in the 21<sup>st</sup> century, their model is meant to provide teachers (at primary and secondary school) with a tool for planning and reflection. Goos and her colleagues tie numeracy, and its numerous synonyms, tightly to what a person needs to function well in society. “Numeracy is the capacity to make effective use of mathematics in contexts related to personal life, the workplace, and in exercising civil responsibilities” (Geiger et al. 2015a, 611). They emphasize that numeracy is something more than basic computational skills, viz. how to apply mathematics in society: “how to connect the mathematics learnt in more formal situations, such as classrooms, to real world problems” (Geiger et al. 2015b, 531). According to their model, developing numeracy requires having an active relationship with real life contexts, managing to make use of mathematical knowledge, and using tools which can be digital, physical, or representations. To have a positive disposition for the use of mathematics is important for the will and necessary self-confidence to engage in activities where mathematics is demanded. Disposition is an integrated part of the model in contrast to the OECD/PISA definition. It should be noted that a critical orientation is an explicit premise in the model, where a critical orientation to numeracy has at least two dimensions: critical to the choice of models and methods, e.g., what quantitative data are being collected, what is being measured and how; and critical to the use and interpretation of mathematics in society, e.g., how quantitative data can be (mis)used to manipulate media and shape opinions about political and social issues. The critical orientation underlying the Goos model is consistent with the findings made by Craig (2018). In analyzing the numeracy discourse, he found that numeracy promises to reflect modern society and empower people, whereas innumeracy

promises to have social costs. Along the same lines, Craig and Guzmán (2018) argue that a social theory of numeracy takes into account and is explicit about the power relations inherent in society.

For interpretations that emphasize the political aspects of mathematical literacy, Marilyn Frankenstein and Ole Skovsmose are central. Marilyn Frankenstein (2009) uses the terms *criticalmathematical numeracy* and *criticalmathematical literacy* to underline that mathematics is to be used in a critical way in society in the struggle for social justice. The goal of criticalmathematical literacy is not to understand mathematics in itself, but to use mathematics to make the world a better place. Skovsmose introduces the concept *mathemacy* as a parallel to the way Paulo Freire has used literacy, namely by linking it to being able to function in a society structured by mathematics, and thus not just being able to do calculations in a narrow sense (Skovsmose 1998, 199). Thus, Skovsmose looks at mathematics and mathematics teaching from a completely different perspective, or with different glasses, than G. H. Hardy. In *A Mathematician's Apology* Hardy (1940/1993, 119) emphasized that *real* mathematics plays a useless and harmless role in society: “The ‘real’ mathematics of the ‘real’ mathematicians, the mathematics of Fermat and Euler and Gauss and Abel and Riemann, is almost wholly ‘useless.’” On the other hand, Hardy admitted that there are branches of what he labelled *trivial* mathematics that can be used and even exploited for war applications. Hardy mentioned ballistics and aerodynamics as examples but says of these that they cannot claim to be called real mathematics as they “are indeed repulsively ugly and intolerably dull” whereas “[r]eal mathematics has no effects on war” (140). Hardy mentioned number theory and relativity as examples of “real mathematics” with no war applications. Alas, World War II proved him wrong on both accounts.

The view of mathematics as a neutral subject outside the ravages of war machinery was also strongly opposed by Anthony Judge. He wrote in connection with the many ongoing conflicts, including in Kosovo, for which mathematicians cannot abdicate responsibility:

Mathematicians—having lent the full support of their discipline to the weapons industry supplying the missile delivery systems—would claim that their subtlest thinking is way beyond the comprehension of those seated around a negotiating table. They have however failed to tackle the challenge of the packing and unpacking of complexity to render it comprehensible without loss of relationships vital to more complex patterns. As with the protagonists in any conflict, they would deny all responsibility for such failures and the manner in which these have reinforced unsustainably simplistic solutions leading to further massacres. (Judge 2000)

Mathematics plays a role in war but as we have seen, relating numeracy to social practices makes it clear that mathematics plays a role throughout society and the power relations inherent therein. Skovsmose (2008) refers to two different views on the connection between mathematics and power. He calls one view the

thesis of indifference, which states that there is no connection between mathematics and power. It is reasonable to place Hardy within this view of mathematics. Against such a principle of indifference, Skovsmose sets the thesis of significance: “mathematics interacts with power, and this interaction has a political, technological and economic significance” (8). The importance of mathematics in the construction and building of society is well known and is perhaps even more visible in the age of technology. Skovsmose shows cryptography as an example and how important it is in the globalized economy, and cryptography is mathematics in use. Performing work tasks in society often involves dealing with operations where mathematics plays an important role, but at the same time is hidden from the operator. In this way man is alienated, in the Marxist sense, from mathematics. This applies just as much when we look at citizens as consumers. Skovsmose believes that much of the mathematics teaching in the school has been built up with the purpose of educating students to be consumers: “Consumers’ mathematics has been developed from a highly pragmatic perspective. This pragmatism has dominated many textbooks with elaborated examples of mathematics in daily-life situations” (14). Against this, we can ask to what extent mathematics teaching can educate students to become critical citizens who can challenge authorities.

D’Ambrosio (1999, 2003) questions the school system and its role in a society dominated by oppression, inequity, and environmental crises. As part of a program for a new curriculum he suggests three strands called literacy, *matheracy*, and *technoracy*. These three strands are supposed to “provide, in a critical way, the communicative, analytical, and technological instruments necessary for life in the twenty-first century” (D’Ambrosio 2003, 237). For D’Ambrosio, literacy not only includes all types of reading and writing, but more broadly the ability to process information, such as the use of spoken and written language, of signs and gestures, of codes and numbers. He further says that “Nowadays, reading also includes the competence of numeracy, the interpretation of graphs and tables, and other ways of informing the individual. Reading even includes understanding the condensed language of codes” (237). It is worth emphasizing that for D’Ambrosio, quantitative literacy (or numeracy) is included in the literacy concept. By expanding what reading means, he seems at the same time to narrow down what is meant by numeracy if compared to most definitions mentioned earlier. He claims that his interpretation of numeracy as a subconcept within literacy is in line with mathematics as we know it from the ancient Babylonians and Egyptians, where the surviving written sources show examples of concrete calculations and solution methods in or outside contexts. This differs from mathematics as it was developed and understood in ancient Greece, with emphasis on logical deductions of general relationships in an axiomatic deductive system. D’Ambrosio uses *matheracy* as a contrast to literacy, and thereby also in contrast to quantitative literacy, in that it is an analytical instrument in line with classical Greek mathematics. *Matheracy* is

therefore not about counting or measuring, but about thinking and philosophy, about the ability to deduce, propose hypotheses and draw conclusions from data.

Through a social practice lens Oughton (2018) aligns with Skvovmose, Frankenstein, and D'Ambrosio in being critical of numeracy models that employ a functional perspective and emphasize economic effectiveness, and global competition. In contrast, a social practice model embraces learners' personal goals, purposes, beliefs, and attitudes. This may include learning numeracy skills needed for the workplace or civic life, developing the necessary quantitative literacy to be a critical citizen, but also learning mathematics for the enjoyment of exploring mathematics for its own sake. To study mathematics for the pure enjoyment of mathematics itself is otherwise not prevalent in the models discussed. Rather, commonalities between the views of mathematical literacy considered so far are that 1) they hold that mathematical literacy concerns the use of mathematics in society, viz. what mathematics is necessary for the individual to master life, and what mathematics society needs its citizens to master; 2) they are not concerned with pure mathematics as research field, but are concerned with mathematics as applied in society and thus with school mathematics and adults' numeracy practices in society; 3) the majority of models don't distinguish between mathematical literacy, numeracy, or quantitative literacy. There are exceptions though, as exemplified by de Lange (2006) and the hierarchy found by Karaali et al. (2016), and the distinction between quantitative literacy (as part of a broader literacy concept) and matheracy made by D'Ambrosio (1999, 2003). One does, however, see distinctions between those who emphasize mathematical literacy, quantitative literacy, or numeracy as a tool for social change and emancipation, and those with a less normative view. With a social practice perspective, numeracy (mathematical literacy, quantitative literacy) is what we do, not something that we have or should have.

We have seen an abundance of terms used, of which the most prominent are mathematical literacy, numeracy, quantitative literacy, criticalmathematical literacy, and matheracy. Not only are there several terms, but these terms have different meanings to different people. Some people consider the terms to just be different names for the same concepts, some regard them as slightly different, and some regard them as being part of a hierarchy, each covering different aspects within a world rooted in something connected to mathematics. The situation is therefore problematic on several accounts. There is a communication problem as any use of any of the terms needs explanation whenever you use it, and this comes in addition to the problems with translations to non-English languages. To bring research within mathematical literacy further, it is probably also necessary to find a common ground, a common understanding of the concepts involved. Otherwise, the base upon which the research is built would be shaky, and still far from fulfilling

the guidelines Scheaffer (2008) deemed necessary for research in quantitative literacy to be scientifically based.

## **Mathematics in Content Area and Disciplinary Literacy**

As if the situation was not complicated enough, I will in this section bring in two more concepts related to literacy, which will make the picture even more complicated.

Looking at literacy research from a standpoint outside mathematics education research, there is a distinction between disciplinary literacy and content area literacy. This distinction is made clear by for example Shanahan and Shanahan (2008, 2012, 2018). I am going to look closer at these concepts and compare them to the understanding of mathematical literacy, numeracy and quantitative literacy discussed above.

Already 30 years ago, McKenna and Robinson (1990) writing about the definition of content literacy expressed frustration at how the term literacy had spread to encompass knowledge in specific disciplines, changing the literacy concept to “denote mere knowledgeability of a specific subject” (184). As examples they mention computer literacy and cultural literacy. In their opinion this makes it difficult to obtain consistency in the use of the term. For McKenna and Robinson content literacy is the ability to apply reading and writing to attain new knowledge in a given discipline. Mathematical literacy is thus “not to know mathematics per se, but to be able to read and write about the subject as effective means of knowing still more about it” (185). This understanding of mathematical literacy is different from, and narrower than, the models described above in that it does not concern the use of mathematics in society, only the learning of mathematics. We also note that it concerns in an unambiguous way the use of reading and writing as tools to learn mathematics. Thereby it can be considered as reading and writing literacy in mathematics rather than mathematical or quantitative literacy.

Writing about content-area literacy, Siebert and Draper (2008) investigated why generic reading and writing strategies do not appeal to mathematics teachers. Attempts at implementing teaching of reading and writing in the subject areas have therefore been unsuccessful. Siebert and Draper claim that ever since 1925, with Gray’s “A modern program of reading instruction for the grades and high school,” it has been repeated as a mantra from literacy proponents that every teacher is a reading teacher. In their view, mathematics is a good test to whether it is meaningful to talk about reading and writing across subjects. Suggestions from literacy-research about how to improve literacy do not appeal to mathematics teachers because a vast majority of such messages “neglect, deemphasize, or misrepresent the nature and content of the discipline of mathematics” (Siebert and Draper 2008,

231). This may be seen as a harsh statement, but the fact is that all content-area literacy documents investigated by Siebert and Draper contained at least one problematic statement, and many of the documents were problematic on three important categories: neglecting the influence of mathematics, deemphasizing, or minimizing the uniqueness of mathematics, or misrepresenting mathematics. An example of neglecting the influence of mathematics is to have a too narrow view of what is considered text, thereby missing out on, for example, formulas, tables, and graphs. Messages that deemphasize the uniqueness of mathematics would likewise ignore how mathematics is full of non-traditional text, like uniquely mathematical symbols, the use of letters in formulas, etc. It is quite common for mathematics textbooks to have short texts about historical or philosophical facts connected with the introduction of new themes. Such texts may contribute as background and motivation for a particular piece of mathematics (think of a short biography on Pythagoras as introduction to a chapter on Pythagoras' theorem). But as Siebert and Draper point out, such texts are not in themselves mathematics and don't contribute to the understanding of the mathematical content. For focusing on literacy to be meaningful for mathematics teachers, it is necessary to understand what constitutes text, reading, and writing in mathematics. Siebert and Draper (2008) say that one must include everything that people use to create, communicate, and negotiate meaning. This would include, but not be limited to, diagrams, pictures, calculator printouts, manipulatives, equations, group- and plenary-discussions, as well as explanations and justifications related to concepts. Siebert and Draper suggest that in this way mathematics teachers will understand that their classrooms are text-rich learning environments and so realize that literacy is an important and necessary part of learning mathematics.

As content area literacy educators, Johnson et al. (2011) point out that we use different tools for different jobs, and even if tools with the same name are used by workers in different crafts, they are used differently. One should think similarly when it comes to literacy, viz. that literacy has different meanings within different content areas. Thereby they disagree with textbook authors and colleagues that believe the same literacy tools can be used universally. Instead, for content-area literacy to be meaningful, one should focus on the distinguishing features of each discipline. According to Johnson et al., literacy teaching in a subject must have as its goal to build understanding of how knowledge is produced *in* the subject and not communication of knowledge *about* the subject. This includes an understanding of what the most important theoretical ideas are, which questions are important, and how one can work to try to answer the questions (107).

As we have seen, content-area literacy, with mathematics as content-area, concerns the use of (reading and writing) literacy in mathematics teaching. This is a different perspective from any of the models of mathematical literacy, numeracy, and quantitative literacy discussed earlier, where the focus was on the role of



mathematics in the world, including everyday life outside educational and academic settings.

Next, I am going to look at disciplinary literacy and how it compares with content area literacy. Shanahan and Shanahan (2008) argue against the myth that mastering of basic reading and writing skills automatically leads to an ability to attain advanced knowledge. Research they and others have done shows, on the contrary, that explicit instruction in how to master advanced subjects within the discipline are necessary. In Shanahan and Shanahan (2012), the authors highlight the distinction between disciplinary and content area literacy. They make it clear that this is not just a distinction in which words are used, but that the two terms have a qualitatively different content. By content area literacy, Shanahan and Shanahan mean the use of general literacy strategies in specific subject areas, as we have discussed above. Thus, they will include the use of generic reading and writing strategies used in mathematics teaching. Like Siebert and Draper (2008), Shanahan and Shanahan (2008) point out that content area literacy has not appealed to mathematics teachers. Shanahan and Shanahan maintain that content area literacy is about techniques that beginners in the subject can use to understand texts in the subject. Disciplinary literacy, on the other hand, is about the unique tools that the experts in the subject use, as Johnson et al. (2011) also emphasized. Johnson et al. relate being literate in mathematics to mathematics as a research field, saying that it involves being well-read in the field, being able to discuss major results, and being able to read research papers. In this interpretation, disciplinary literacy does not concern the use of mathematics in everyday life but rather concerns life within the research field of mathematics. Both Johnson et al. (2011) and Shanahan and Shanahan (2008, 2012) maintain that content area literacy is a look at the subject from the outside, with general tips on how to read texts. Disciplinary literacy, on the other hand, is about giving students a look inside the subject with a perspective on the unique features of the subject. The perspective from within is important because subject experts read and write differently both compared with beginners in the subject and compared with experts in other subjects. Thus, disciplinary literacy in mathematics would mean to be able to understand the main theoretical ideas in mathematics, to understand which questions are important and how answers are sought within mathematics. Again, this points toward mathematics as an academic field rather than as life-mastering tool.

## Discussion

We have seen that the concept of literacy is used in several ways in connection to mathematics. The first thing we learn is that there is no single term in use that all scholars agree upon. Mathematical literacy, quantitative literacy, and numeracy are the most commonly used terms. Why do we have competing terms? Some scholars

point to mere geography as a reason, which points to linguistic and cultural backgrounds within different communities. There does however seem to be more to it. As a word, numeracy makes linguistic connections with numbers, which, while being an important part of mathematics is definitely not the whole of mathematics, as geometry, shapes and space, and even algebra, seem to be excluded. Thereby there might be reasons for not using numeracy as the sole term. Quantitative literacy likewise makes linguistic connections with quantity; and while quantity is certainly an important part of mathematics, it could be argued that it seems to exclude shapes and space and logical reasoning. Concerning both numeracy and quantitative literacy, these seeming shortcomings have been amended by giving definitions that include more than numbers and quantity in the narrow sense. For example, in describing what quantitative literacy might mean, Gillman (2006) includes not only numeracy aspects like numbers, magnitudes, algorithmic skills, probability and statistics, but also geometry, algebra, problem solving ability, and the ability to capture and summarize information and make decisions (vii). This is a very wide definition of quantitative literacy, encompassing de Lange's notion of mathematical literacy.

All three terms mathematical literacy, quantitative literacy, and numeracy are primarily used to make it clear that we are talking about something else than the pure world of mathematics, rather connecting to the world of everyday life. The PISA (OECD 2003, 2018) definition of mathematical literacy implies that mathematical literacy is something a person has, "an individual's capacity." SIGMAA QL (Mathematical Association of America 2004) relates quantitative literacy to a person's ability, defining quantitative literacy as a habit of mind. Within a social practice theory literacy is seen as something people *do* and not something people have, emphasizing that the literacy practices take place in the world and not only in people's heads (Craig et al. 2019). If we agree that mathematical and/or quantitative literacy is something people do and not simply something they have, it may not only be difficult or inconvenient but simply wrong to have a fixed definition of quantitative literacy. As quantitative practices change, quantitative literacy must change, as Craig and Guzmán (2018) argue. It is also a fact that in the field of mathematics definitions change or evolve over time. Take for example the definition of a *point* in Euclid's Elements: A point is that which has no part. At the time of Euclid this was a perfectly acceptable definition which the community of mathematicians could agree upon. (But see also the discussion in Joyce (2013).) The definition was later challenged, for what does it mean to have no part? Saying that it means to have no length, breadth, or width does not make it more precise, for what does length, breadth, or width mean? Centuries later it was agreed that some notions must be left undefined, as for example the term *point* in Hilbert's *The Foundations of Geometry* (1902). A related non-mathematical example is the atom: in ancient Greece an atom was that which could not be cut

into smaller parts (*atomos* meaning “uncuttable”). In the 20<sup>th</sup> century it was first found that an atom consists of a nucleus and electrons, then that the nucleus consists of protons and neutrons, and then that protons and neutrons are composed of quarks. There are numerous other examples of how definitions of terms change over time both in mathematics and other subject areas, often after a time of naïve approaches with fuzzy or unclear definitions. As example one can look at the evolution of mathematical analysis, from infinitesimals to limits to set theory and topology, thereby going from intuitive approaches to making precise what we mean by differentials and integrals, continuity, power series, etc., at the same time also abandoning the fights between Newton and Leibniz about names and notations. Are we finding ourselves in a similar situation regarding mathematical literacy, numeracy, and quantitative literacy?

Forest Fisher (2019) writes about literacies as being socially constituted and that to become literate means to change your identity and join a new social group or adopt a new culture. For Fisher, quantitative literacy is about participation in “intersecting quantitative practices of many different communities” (10). One would assume that the community of mathematicians could be one of these groups, but definitely not the only one. This contrasts with an interpretation of mathematical literacy as disciplinary literacy, where the agenda is to facilitate becoming a mathematician, joining the group of (professional) mathematicians, for example as a master student, joining the community of mathematicians as a legitimate peripheral participant in the sense of Lave and Wenger (1991).

Another interpretation lies in the complement of abstract mathematics, as for instance Craig and Guzmán (2018) emphasize when they claim that numeracy is not the same as mathematics or abstract statistics since it cannot be abstracted from reality. This also points to a main division between mathematics on the one hand and numeracy or quantitative literacy on the other hand. In mathematics, it is precisely the abstractions that are sought, unless we are working within applied mathematics or statistics. Pure mathematics itself is abstract, whereas quantitative literacy (numeracy) is about participating in “the intersecting quantitative practices of many different communities” (Fisher 2019, 10).

It is possible to discern a difference between mathematical literacy on the one hand and numeracy and quantitative literacy on the other. Numeracy and quantitative literacy more clearly relate to a viewpoint concerning the use of mathematics in society (private or working life), whether this is with a critical orientation (Goos et al. 2011), with a social practice view (which can include a critical orientation) (Craig and Guzmán 2018; Oughton 2018; Fisher 2019), or with an explicit view toward social change (Skovsmose 1998, 2008; D’Ambrosio 2003; Frankenstein 2009). The term mathematical literacy on the other hand can also have synonymous meaning to the different contents associated with numeracy and quantitative literacy. Mathematical literacy has also been opened for a wider

definition, one being that of literacy in mathematics in the form of content-area literacy, the use of writing and reading in mathematics teaching; another being disciplinary literacy, which does/can connect directly to what it means to develop into a mathematician and at least what it means to work like a mathematician. In this sense, mathematical literacy is a more problematic concept than numeracy or quantitative literacy. Numeracy and quantitative literacy both unambiguously connect to everyday life use of (mostly elementary) mathematics (in possibly sophisticated ways) (Gillman 2006; Steen et al. 2007), whereas mathematical literacy may have two competing meanings: 1) as quantitative literacy or numeracy, so about learning/being able to make use of mathematics in everyday life; and 2) becoming (like a) mathematician, learning mathematics the way mathematicians work, with inquiry and experimentation as dominant ways of working rather than rote learning and following recipes. The last meaning strikes a chord with mathematics education researchers emphasizing that mathematics should be taught in ways so that students learn how to work like a mathematician, grasping the essence of mathematics, emphasized in particular with sociocultural and socio-constructivist mathematics education theories (e.g., Simon 1995; Liljedahl 2016).

## Conclusion

In mathematics it is common to have several terms for the same concept, for example that a mapping may be called one-to-one, 1-1, or injective. There are different names, but they all describe/define the same concept. However, it is not regarded as a good thing to have competing definitions of a concept, as with mathematical literacy and quantitative literacy, which are not only different names but also have so many different meanings. The several competing notions regarding mathematical literacy make it unclear what we are talking about. Lack of clarity affects 1) the academic discourse, with several communities talking across each other; and 2) communication with the world outside of academia: if mathematical literacy is to have an impact people need to know what we are talking about, including teachers and students who are supposed to engage with it. The communication argument is even stronger considering the lack of native terms in several languages.

There are lessons to be learned by examining the different approaches to mathematical literacy and its sister terms. Content-area literacy concerns reading and writing in the subject areas. By discussing content-area literacy in mathematics, we learn that mathematics also contains text. Texts in mathematics are both of a general nature and text that must be considered particular to mathematics. The shortcomings of general text-competence approaches are thereby revealed, calling for more subject specific approaches. Disciplinary literacy concerns the unique tools, techniques, and uses in each discipline. By discussing disciplinary literacy in

mathematics, we learn to appreciate mathematics as a subject with its own language, tools, techniques, and ways of thinking that emphasize inquiry and experimentation following the heritage from Euclid, Archimedes, Newton, and Euler. Within the community of mathematicians, the focus is on carrying on this legacy. From discussing mathematical literacy, numeracy, and quantitative literacy, we learn about the importance of mathematics in the world at large, including both how mathematics is necessary for everyday life but also how it can be used to change the world. The discourse in the communities of quantitative literacy, numeracy, and mathematical literacy teaches us about the importance of elementary mathematics and its uses in society. In this paper I have provided insights into different terms and approaches associated with mathematics and literacy and have shown what we can learn from analyzing each of them.

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