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Global Asymptotic Position and Heading Tracking for Multirotors using Tuning Function-based Adaptive Hybrid Feedback

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***Abstract*—This letter considers the problem of global asymptotic position and heading tracking for multirotors. We propose a hybrid adaptive feedback control law that globally asymptotically tracks a position and heading reference in the presence of unknown constant disturbances in both the translational and rotational dynamics. By employing a tuning function-based backstepping approach, the number of parameter estimates are minimized. Moreover, we propose a novel control law for the translational subsystem, which leads to a simpler virtual control law when backstepping. Global asymptotic heading tracking is achieved through a novel construction of the desired rotation matrix. The theory is verified through experiments on a quadrotor.**

***Index Terms*—Flight control, adaptive control**

I. Introduction

MULTIROTOR unmanned aerial vehicles (UAVs) have become increasingly popular in recent years. Their low-cost, vertical take-off and landing, and hovering abilities make them well suited to perform a wide variety of tasks, such as inspection [1], parcel delivery [2], surveillance, mapping and even autonomous recovery of fixed-wing UAVs [3].

Multirotors are typically designed with co-planar propellers. Although such systems have full torque actuation, forces can only be produced along a single vehicle-fixed axis, known as the thrust axis. Since the propulsion system cannot produce an arbitrary three-dimensional force vector, these systems are underactuated mechanical systems. Due to the underactuation of the system, position and orientation tasks cannot be fully decoupled. Instead, control algorithms for quadrotors often employ a cascaded structure consisting of an inner- and outer-loop control law for orientation and position control, respectively [4]. For such schemes, the outer position control loop often computes a desired three-dimensional force. The norm of this vector then serves as the thrust input, while the vehicle orientation is controlled such that the thrust direction of the vehicle is aligned with the desired force direction in the inertial frame. This is the approach we will take in this letter.

There is an extensive amount of literature on the subject of trajectory tracking for multirotor UAVs, and the reader is referred to the surveys [5], [6] and the references therein. The following review is limited to earlier works on geometric control of multirotors, that is, the development of control laws based on quaternion or rotation matrix feedback. The control law proposed in [7] guarantees local exponential tracking for multirotors that can produce both negative and positive thrust along the thrust axis. An adaptive position tracking control scheme for underactuated multirotors is proposed in [8]. However, the control law does not enable a desired heading to be tracked. Moreover, the adaptive control law is overparametrized.

The aforementioned approaches rely on continuous state-feedback. However, the non-contractibility of the configuration space of a rigid body implies that these control laws are at most almost globally stabilizing [9]. This is referred to as a topological obstruction to global asymptotic stability, and can be overcome by employing hybrid feedback with a properly defined switching logic [10]. The hybrid feedback approach in [11] achieves global asymptotic position tracking using the thrust and angular velocity as inputs. However, by using a reduced orientation control approach, the rotation angle around the thrust axis is left uncontrolled. A saturated tracking control law for a quadrotor in the presence of unknown constant disturbances is developed in [12]. The control law ensures that the position error is contained in an arbitrarily small neighborhood of the origin, but leaves the heading uncontrolled and does not ensure convergence of the position and linear velocity errors to zero. In [13], the hybrid quaternion feedback strategy from [14] is employed together with the results on backstepping of hybrid feedback laws from [15] to synthesize a hybrid feedback control law that achieves global asymptotic tracking of a smooth position reference trajectory while minimizing the rotation angle to a given orientation configuration. Moreover, the controller includes an integral/adaptive term and is shown to work in the presence of additive disturbances in the translational dynamics. However, stability of the translational subsystem is shown using a Lyapunov function with a cross term which results in a complicated expression for the gradient, and hence, the virtual backstepping control law. Moreover, due to the construction

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of the desired rotation matrix, a desired heading (specified by a basic rotation matrix around the z-axis) can only be tracked provided that the roll and pitch angles are zero. Furthermore, the control law is overparametrized, as the number of parameter estimates is three times larger than the number of unknown parameters. Consequently, if the control law in [13] were to be extended to the case of a constant disturbance in the rotational dynamics, parameter convergence would be impossible. Another hybrid feedback approach is introduced in [16]. This approach achieves robust global trajectory tracking for multirotors. However, due to a lack of integral action, the tracking errors do not converge to zero in the presence of disturbances.

The goal of this letter is to achieve uniform global asymptotic tracking of both the position and heading of a multirotor in the presence of unknown constant disturbances in both the translational and rotational dynamics. To this end, we build on the work in [13], which we extend as follows. First, we propose a novel bounded adaptive control law for the translational subsystem, which leads to a simpler virtual control law when backstepping. Second, we propose a novel construction for the desired rotation matrix, which avoids the use of intermediary Euler angles, and is crucial in ensuring global asymptotic tracking of the desired heading reference. Third, we augment the rotational dynamics with a constant disturbance, and by employing tuning functions [17], the number of parameter estimates becomes equal to the number of unknown parameters. As a consequence, we can show that the disturbance estimates in both the translational and rotational dynamics converge to their true values.

This letter is organized as follows. In Section II, we introduce the equations of motion and give the problem statement. Section III introduces a bounded adaptive control law for the translational subsystem, before extending this control law using a backstepping approach to account for the rotational dynamics. Finally, Section IV presents experimental results verifying the theoretical developments, before Section V concludes the letter.

A. Preliminaries

The Euclidean inner product in $\mathbb{R}^n$ is written $\langle x, y\rangle$, and the Euclidean norm is denoted $|x| = \langle x, x\rangle^{1/2}$. The unit n-sphere embedded in $\mathbb{R}^{n+1}$ is given by $\mathbb{S}^n = \{x \in \mathbb{R}^{n+1} : |x| = 1\}$, and the closed ball of radius r in $\mathbb{R}^n$ is the set $r\mathbb{B}^n = \{x \in \mathbb{R}^n : |x| \leq r\}$. The standard basis vectors in $\mathbb{R}^n$ are denoted $e_1, e_2, \ldots, e_n$ and the entry of a matrix $a \in \mathbb{R}^{n\times n}$ corresponding to the ith row and jth column is denoted a_{ij}. The special orthogonal group of dimension three is denoted $\mathrm{SO}(3)$ and defined by $\mathrm{SO}(3) := \{R \in \mathbb{R}^{3\times 3} : \det R = 1, RR^{\mathsf{T}} = I\}$, where I is the identity matrix. The Lie algebra of the matrix Lie group $\mathrm{SO}(3)$ is denoted $\mathfrak{so}(3)$ and can be identified with the space of skew-symmetric matrices in $\mathbb{R}^{3\times 3}$. Define the mapping $(\cdot)_\times : \mathbb{R}^3 \to \mathfrak{so}(3)$ by $\alpha_\times\beta = \alpha \times \beta$ with $\alpha, \beta \in \mathbb{R}^3$. A set-valued mapping is denoted by $F : X \rightrightarrows U$, where $X \subset \mathbb{R}^n$ is the domain of the mapping and $U \subset \mathbb{R}^m$ is its codomain. The range of a mapping $f : \mathbb{R}^m \to \mathbb{R}^n$ is defined as $\operatorname{rge} f = \{y \in \mathbb{R}^n : \exists x \in \mathbb{R}^m$ such that $y = f(x)\}$. A unit quaternion is given by $z = (\eta, \epsilon) \in \mathbb{S}^3$, where $\eta \in \mathbb{R}$ and $\epsilon \in \mathbb{R}^3$, describe its real and imaginary components, respectively. Any unit quaternion is mapped to a rotation matrix through the surjective mapping $\mathcal{R} : \mathbb{S}^3 \to \mathrm{SO}(3)$ defined by $\mathcal{R}(z) := I_3 + 2\eta\epsilon_\times + 2(\epsilon_\times)^2$. Finally, we define the constant matrix $S := \left(\begin{smallmatrix} 0 & -1 \\ 1 & 0 \end{smallmatrix}\right)$.

II. Modeling and Problem Statement

Let $p \in \mathbb{R}^3$ denote the position of the vehicle in the inertial frame and let $R \in \mathrm{SO}(3)$ denote the orientation of the vehicle-fixed frame with respect to the inertial frame. Additionally, we define the heading relative to the inertial frame as $\nu := \frac{1}{|(R_{11}, R_{21})|}(R_{11}, R_{21}) \in \mathbb{S}$ for $|(R_{11}, R_{21})| \neq 0$. Furthermore, let $v \in \mathbb{R}^3$ and $\omega \in \mathbb{R}^3$ denote the linear and angular velocites of the vehicle in the inertial and vehicle-fixed frames, respectively. The equations of motion for a multirotor are given by [4]

$$\dot{p} = v \tag{1a}$$
$$\dot{R} = R\omega_\times \tag{1b}$$
$$m\dot{v} = -Re_3 f + mge_3 + b \tag{1c}$$
$$\mathcal{I}\dot{\omega} = -\omega_\times\mathcal{I}\omega + \mu + \theta, \tag{1d}$$

where $b, \theta \in \mathbb{R}^3$ are constant disturbances, $m > 0$ is the mass of the vehicle, $g > 0$ is the gravitational acceleration, $\mathcal{I} \in \mathbb{R}^{3\times 3}$ is the vehicle inertia matrix, $f \in \mathbb{R}$ is the total thrust generated by the rotors and $\mu \in \mathbb{R}^3$ is the total torque generated by the rotors in the vehicle-fixed frame.

Assumption 1. *The disturbances b, θ are upper and lower bounded by known constants $\overline{b}, \overline{\theta} \in \mathbb{R}^3$ and $\underline{b}, \underline{\theta} \in \mathbb{R}^3$, respectively.*

Assumption 1 implies that the disturbances are contained in the convex sets

$$\mathcal{P} := [\underline{b}_1, \overline{b}_1] \times [\underline{b}_2, \overline{b}_2] \times [\underline{b}_3, \overline{b}_3], \tag{2}$$
$$\Theta := [\underline{\theta}_1, \overline{\theta}_1] \times [\underline{\theta}_2, \overline{\theta}_2] \times [\underline{\theta}_3, \overline{\theta}_3]. \tag{3}$$

Let $\overline{\mathrm{Proj}} : \mathbb{R}^3 \times \mathcal{S} \rightrightarrows \mathbb{R}^3$ be the outer semicontinuous, convex-valued and locally bounded set-valued mapping defined by $\overline{\mathrm{Proj}}(\sigma, s) := (\overline{\mathrm{proj}}(\sigma_1, s_1), \overline{\mathrm{proj}}(\sigma_2, s_2), \overline{\mathrm{proj}}(\sigma_3, s_3))$, where $\underline{s}, \overline{s} \in \mathbb{R}^3$ and $\mathcal{S} := [\underline{s}_1, \overline{s}_1] \times [\underline{s}_2, \overline{s}_2] \times [\underline{s}_3, \overline{s}_3] \subset \mathbb{R}^3$ and

$$\overline{\mathrm{proj}}(\sigma_i, s_i) := \begin{cases} \sigma_i, & \text{if } \sigma_i \in \mathrm{T}_{[\underline{s}_i, \overline{s}_i]}(s_i) \\ [0, 1]\sigma_i, & \text{if } \sigma_i \notin \mathrm{T}_{[\underline{s}_i, \overline{s}_i]}(s_i) \end{cases} \tag{4}$$

where the tangent cone $\mathrm{T}_{[\underline{a}, \overline{a}]} : [\underline{a}, \overline{a}] \rightrightarrows \mathbb{R}$ is defined by

$$\mathrm{T}_{[\underline{a}, \overline{a}]}(\varphi) := \begin{cases} [0, \infty), & \text{if } \varphi = \underline{a} \\ (-\infty, \infty), & \text{if } \varphi \in (\underline{a}, \overline{a}) \\ (-\infty, 0], & \text{if } \varphi = \overline{a} \end{cases} \tag{5}$$

for $\underline{a}, \overline{a} \in \mathbb{R}$. Observe that the solutions to the constrained differential inclusion

$$\dot{s} \in \overline{\mathrm{Proj}}(\sigma, s), \quad s \in \mathcal{S}, \tag{6}$$

where σ is a hybrid input [18], include solutions arrived at if the discontinuous projection

$$\mathrm{proj}(\sigma_i, s_i) := \begin{cases} \sigma_i, & \text{if } \sigma_i \in \mathrm{T}_{[\underline{s}_i, \overline{s}_i]}(s_i) \\ 0, & \text{if } \sigma_i \notin \mathrm{T}_{[\underline{s}_i, \overline{s}_i]}(s_i) \end{cases} \tag{7}$$

would have been used instead. As a result, there always exists a flow direction contained in $\overline{\mathrm{Proj}}(\sigma, s)$ that steers s within

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$\mathcal{S}$, i.e. $\overline{\mathrm{Proj}}(\sigma, s) \cap \mathrm{T}_{\mathcal{S}}(s) \neq \emptyset$ for all $s \in \mathcal{S}$, where $\mathrm{T}_{\mathcal{S}}(s) = \mathrm{T}_{[\underline{s}_1,\overline{s}_1]}(s_1) \times \mathrm{T}_{[\underline{s}_2,\overline{s}_2]}(s_2) \times \mathrm{T}_{[\underline{s}_3,\overline{s}_3]}(s_3)$. Therefore, since $\mathcal{S}$ is compact, every maximal solution to (6) is complete [19, Proposition 6.10].

Lemma 1. *Let $s, \hat{s} \in \mathcal{S}$, $\tilde{s} = s - \hat{s}$ denote the estimation error and $\Gamma \in \mathbb{R}^{3\times 3}$ be a positive definite and diagonal matrix. Then*

$$-\langle \tilde{s}, \Gamma^{-1}\overline{\mathrm{Proj}}(\sigma, \hat{s})\rangle \leq -\langle \tilde{s}, \Gamma^{-1}\sigma\rangle. \tag{8}$$

Proof. If $\underline{s} < \hat{s} < \overline{s}$, or if $\hat{s} \in \mathcal{S}$ and $\sigma \in \mathrm{T}_{\mathcal{S}}(s)$, it follows that $\overline{\mathrm{Proj}}(\sigma, \hat{s}) = \sigma$ and (8) is satisfied with equality. Since Γ is diagonal with positive entries, we only have to verify (8) componentwise for the case $\hat{s}_i \in \{\underline{s}_i, \overline{s}_i\}$ and $\sigma_i \notin \mathrm{T}_{[\underline{s}_i,\overline{s}_i]}(s_i)$. Observe that $\hat{s}_i = \overline{s}_i$ and $\sigma_i \notin \mathrm{T}_{[\underline{s}_i,\overline{s}_i]}(s_i)$ implies that $\sigma_i > 0$ and $\tilde{s} \leq 0$. Similarly, $\hat{s}_i = \underline{s}_i$ and $\sigma_i \notin \mathrm{T}_{[\underline{s}_i,\overline{s}_i]}(s_i)$ implies that $\sigma_i < 0$ and $\tilde{s}_i \geq 0$. In both cases it follows that

$$\begin{aligned} -\langle \tilde{s}_i, \Gamma_i^{-1}\sigma_i\rangle &\geq -\langle \tilde{s}_i, \Gamma_i^{-1}[0,1]\sigma_i\rangle \\ &= -\langle \tilde{s}_i, \Gamma_i^{-1}\overline{\mathrm{proj}}(\sigma_i, s_i)\rangle \geq 0. \quad \square \end{aligned} \tag{9}$$

A desired trajectory for the multirotor consists of a desired position $p_d : \mathbb{R}_{\geq 0} \to \mathbb{R}^3$ of the multirotor relative to the inertial frame, and a desired heading $\nu_d : \mathbb{R}_{\geq 0} \to \mathbb{S}$ of the multirotor relative to the inertial frame. Given a continuously differentiable desired heading ν_d, the quantity $\dot{\nu}_d$ can always be expressed in terms of the scalar desired heading rate $\varpi_d(t) := \langle S\nu_d(t), \dot{\nu}_d(t)\rangle$. Then, $\dot{\nu}_d(t) = S\nu_d(t)\varpi_d(t)$.

Assumption 2. *The desired position p_d and its derivatives up to the fourth order are bounded and continuous. The desired heading ν_d and its derivatives up to the second order are bounded and continuous. The bias b is lower and upper bounded by the constants $\underline{b}$ and $\overline{b}$, respectively. Finally, it holds that*

$$m(g - \sup_{t\geq 0} \ddot{p}_{d,3}(t)) + \underline{b}_3 := c > 0. \tag{10}$$

Let $\chi = (p_d, \dot{p}_d, \ddot{p}_d, p_d^{(3)}, \nu_d, \varpi_d)$. For every desired trajectory satisfying Assumption 2, there exist scalars $c_1 > 0$, $c_2 > 0$ and a compact set $\Omega \subset \mathbb{R}^3 \times \mathbb{R}^3 \times \mathbb{R}^3 \times \mathbb{R}^3 \times \mathbb{S} \times \mathbb{R}$ such that the desired trajectory is a solution to the differential inclusion

$$\dot{\chi} \in F(\chi) := (\dot{p}_d, \ddot{p}_d, p_d^{(3)}, c_1\mathbb{B}^3, S\nu_d\varpi_d, c_2\mathbb{B}^1), \chi \in \Omega. \tag{11}$$

Assumption 2 is relatively mild, seeing as the supremum of the desired acceleration in the z-direction is often small compared to the gravitational acceleration. When the z-component of the desired acceleration is zero and $\underline{b}_3$ is negative, we require that the absolute value of the lower bound of the z-component of the disturbance force is smaller than the gravitational force.

Let $R_d \in \mathrm{SO}(3)$ denote the desired orientation, which will be defined in Section III. The desired angular velocity is given by $(\omega_d)_\times := R_d^{-1}\dot{R}_d$, and by introducing the error coordinates $\tilde{p} := p - p_d$, $\tilde{v} := v - \dot{p}_d$, $\tilde{R} := RR_d^\mathsf{T}$ and $\tilde{\omega} := \omega - \omega_d$, we obtain the error system

$$\left.\begin{aligned} \dot{\tilde{p}} &= \tilde{v} \\ \dot{\tilde{R}} &= \tilde{R}(R_d\tilde{\omega})_\times \\ m\dot{\tilde{v}} &= mge_3 - Re_3 f - m\ddot{p}_d + b \\ \mathcal{I}\dot{\tilde{\omega}} &= \mu - \omega_\times \mathcal{I}\omega + \theta - \mathcal{I}\dot{\omega}_d \\ \dot{\chi} &\in F(\chi) \end{aligned}\right\} \quad \chi \in \Omega \tag{12}$$

Problem statement: Design a hybrid feedback control law with output $(f, \mu) \in \mathbb{R} \times \mathbb{R}^3$ such that the compact set

$$\mathcal{A}_0 = \left\{(\tilde{p}, \tilde{R}, \tilde{v}, \tilde{\omega}, \chi) : \tilde{p} = 0, \tilde{R} = I, \tilde{v} = 0, \tilde{\omega} = 0\right\}, \tag{13}$$

is uniformly globally asymptotically stable for the system (12).

III. Control Design

As introduced in [20] for fully actuated marine vehicles, we define a modified velocity error $\xi := \tilde{v} - \zeta$, where ζ is generated by the dynamical system

$$\Lambda\dot{\zeta} = -k_1\vartheta(\tilde{p}) - \Xi\vartheta(\zeta), \tag{14}$$

where $\Xi, \Lambda \in \mathbb{R}^{3\times 3}$ are positive definite and diagonal, $k_1 > 0$ and $\vartheta : \mathbb{R}^n \to \mathbb{R}^n$ denotes the following saturation mapping

$$\vartheta(x) := \frac{\tanh|x|}{|x|}x. \tag{15}$$

It is clear that $\zeta = 0$ implies $\xi = \tilde{v}$, which entails that the velocity tracking control objective $\tilde{v} = 0$ can be restated as $(\xi, \zeta) = 0$. We propose the following adaptive control law for the translational subsystem

$$\left.\begin{aligned} &\Lambda\dot{\zeta} = -k_1\vartheta(\tilde{p}) - \Xi\vartheta(\zeta) \\ &\dot{\hat{b}} \in \overline{\mathrm{Proj}}(\Gamma\xi, \hat{b}) \\ &u = \hat{b} + mge_3 - m(\dot{\zeta} + \ddot{p}_d) + k_1\vartheta(\tilde{p}) + K\vartheta(\xi) \\ &f = |u| \end{aligned}\right\} \quad \hat{b} \in \mathcal{P}$$

where $\hat{b}$ denotes the estimate of the disturbance b and $K \in \mathbb{R}^{3\times 3}$ is positive definite and diagonal.

Given a desired heading ν_d and a desired thrust direction $\rho := \frac{u}{|u|} \in \mathbb{S}^2$, we define the desired vehicle orientation by

$$R_d := \begin{pmatrix} r & \rho_\times r & \rho \end{pmatrix} \tag{16}$$

$$r := \frac{\operatorname{sgn}\rho_3}{\sqrt{\rho_3^2 + (\rho_1\nu_{d,1} + \rho_2\nu_{d,2})^2}} \begin{pmatrix} \rho_3\nu_d \\ -\rho_1\nu_{d,1} - \rho_2\nu_{d,2} \end{pmatrix}. \tag{17}$$

Moreover, by defining $w := (\rho, r)$, the desired angular velocity can be computed according to

$$\omega_d = \begin{pmatrix} r^\mathsf{T}\rho_\times\dot{\rho} \\ r^\mathsf{T}\dot{\rho} \\ -r^\mathsf{T}\rho_\times\dot{r} \end{pmatrix} = \begin{pmatrix} r^\mathsf{T}\rho_\times & 0 \\ r^\mathsf{T} & 0 \\ 0 & -r^\mathsf{T}\rho_\times \end{pmatrix}\begin{pmatrix} \dot{\rho} \\ \dot{r} \end{pmatrix} =: A(w)\dot{w}. \tag{18}$$

Observe that R_d is well-defined for any $\nu_d \in \mathbb{S}$ and $\rho \in \mathbb{S}^2$ provided $\rho_3 \neq 0$. The desired orientation aligns the thrust axis of the multirotor with the desired thrust direction ρ. The vector $r \in \mathbb{S}^2$ should be interpreted as the desired configuration of the vehicle-fixed x-axis of the multirotor expressed in the inertial frame. It is chosen such that its projection onto the horizontal plane is aligned with the desired heading. Note that R_d as defined in (16) can also be constructed using the approach in [21]. However, [21] employs an intermediary step in which the desired roll and pitch angles are computed as a function of ρ and the desired yaw angle. Although a direct computation of the desired roll-pitch-yaw angles are required for any control algorithm based on a three-parameter representation of $\mathrm{SO}(3)$, it is unnecessary for any control scheme based on rotation matrix or quaternion feedback. Also, note that the approach in [7] does not yield the same R_d as (16) and only guarantees

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that the vehicle-fixed x-axis converges to the projection of the desired body-fixed x-axis onto the plane orthogonal to ρ.

Let $z = (\eta, \epsilon) \in \mathbb{S}^3$ be a unit quaternion satisfying $\mathcal{R}(z) = \tilde{R}$. Since z is not measured, we employ the path-lifting mechanism proposed in [22] in order to lift the solution $t \mapsto \tilde{R}(t)$ of $\dot{\tilde{R}} = \tilde{R}(R_d\tilde{\omega})_\times$ to a continuous path $t \mapsto z(t)$ that satisfies the kinematic equation

$$\dot{z} = \frac{1}{2}\begin{pmatrix}\epsilon^\mathsf{T} \\ \eta I_3 + \epsilon_\times\end{pmatrix} R_d\tilde{\omega} =: T(z)R_d\tilde{\omega}. \tag{19}$$

If ω_d were entirely known, it would be straightforward to design a backstepping control law using the angular velocity error as the virtual control input. However, due to the presence of the constant disturbance b in the translational dynamics, the following part of ω_d is problematic

$$A(w)\Big(\frac{\partial w}{\partial u}\dot{\hat{b}} + \frac{1}{m}\frac{\partial w}{\partial u}\frac{\partial u}{\partial \xi}\tilde{b}\Big). \tag{20}$$

Although the first term is known, it cannot be canceled without increasing the dynamic order of the system. In other words, we would need two additional estimators for the same bias due to its appearance in ω_d and $\dot{\omega}_d$. Note that this is the approach taken in [13]. To circumvent this, we will follow a tuning function based design procedure. For $i \in \{2, 3\}$, we define β_i as the known part of ω_d with $\dot{\hat{b}}$ replaced by $\Gamma\tau_i$

$$\beta_i := \omega_d - A(w)\left(\frac{\partial w}{\partial u}\dot{\hat{b}} + \frac{1}{m}\frac{\partial w}{\partial u}\frac{\partial u}{\partial \xi}\tilde{b}\right) + A(w)\frac{\partial w}{\partial u}\Gamma\tau_i,$$

which can be rewritten as

$$\begin{aligned}\beta_i = {} & A(w)\frac{\partial w}{\partial u}\Big(\Gamma\tau_i - m(\ddot{\zeta} + p_d^{(3)}) + k_1 J(\tilde{p})\tilde{v} \\ & + KJ(\xi)\big(ge_3 - \ddot{p}_d - \dot{\zeta} + \frac{1}{m}(\hat{b} - \mathcal{R}(z)u)\big)\Big) + A(w)\frac{\partial w}{\partial \nu_d}\dot{\nu}_d\end{aligned} \tag{21}$$

where τ_i is the ith tuning function and $J(\varepsilon) := \frac{\partial \vartheta(\varepsilon)}{\partial \varepsilon}$. Let $q \in \mathcal{Q} := \{-1, 1\}$ be a logic variable, let $k_2, k_z > 0$ and define the virtual control input α by

$$\alpha_q := \beta_2 + qR_d^\mathsf{T} h, \tag{22}$$

$$h = -k_2k_z\epsilon + \frac{2}{k_z}((\eta I + \epsilon_\times)u)_\times\xi. \tag{23}$$

Furthermore, following the tuning function-based backstepping approach in [23], we define the tuning functions

$$\tau_1 := \xi \tag{24}$$

$$\tau_2 := \tau_1 - \frac{k_z q}{m}\left(\frac{\partial u}{\partial \xi}\right)^\mathsf{T}\left(\frac{\partial w}{\partial u}\right)^\mathsf{T} A(w)^\mathsf{T} R_d^\mathsf{T}\epsilon \tag{25}$$

$$\tau_3 := \tau_2 - W(x)\mathcal{I}(\omega - \alpha_q), \tag{26}$$

$$\begin{aligned}W := {} & \frac{1}{m}\Bigg(\left(\frac{\partial u}{\partial \xi}\right)^\mathsf{T}\left(\frac{\partial w}{\partial u}\right)^\mathsf{T}\left(\frac{\partial \beta_2}{\partial w}\right)^\mathsf{T} + \left(\frac{\partial u}{\partial \xi}\right)^\mathsf{T}\left(\frac{\partial \beta_2}{\partial u}\right)^\mathsf{T} + \left(\frac{\partial \beta_2}{\partial \tilde{v}}\right)^\mathsf{T} \\ & + \left(\frac{\partial \beta_2}{\partial \xi}\right)^\mathsf{T} - \left(\frac{\partial u}{\partial \xi}\right)^\mathsf{T}\left(\frac{\partial w}{\partial u}\right)^\mathsf{T} A^\mathsf{T} R_d^\mathsf{T} T(z)^\mathsf{T}\left(\frac{\partial \beta_2}{\partial z}\right)^\mathsf{T} \\ & + \frac{k_z q}{m}\left(\frac{\partial u}{\partial \xi}\right)^\mathsf{T}\left(\frac{\partial w}{\partial u}\right)^\mathsf{T} A^\mathsf{T}(R_d^\mathsf{T}\epsilon)_\times A\frac{\partial w}{\partial u}\frac{\partial u}{\partial \xi}\Gamma\left(\frac{\partial w}{\partial u}\right)^\mathsf{T} A^\mathsf{T} \\ & + q\left(\frac{\partial u}{\partial \xi}\right)^\mathsf{T}\left(\frac{\partial h}{\partial u}\right)^\mathsf{T} R_d - q\left(\frac{\partial u}{\partial \xi}\right)^\mathsf{T}\left(\frac{\partial w}{\partial u}\right)^\mathsf{T} A^\mathsf{T}(R_d^\mathsf{T} h)_\times \\ & + q\left(\frac{\partial h}{\partial \xi}\right)^\mathsf{T} R_d - q\left(\frac{\partial u}{\partial \xi}\right)^\mathsf{T}\left(\frac{\partial w}{\partial u}\right)^\mathsf{T} A^\mathsf{T} R_d^\mathsf{T} T(z)^\mathsf{T}\left(\frac{\partial h}{\partial z}\right)^\mathsf{T} R_d\Bigg).\end{aligned} \tag{27}$$

Define the state space $\mathcal{X} := \mathbb{R}^3 \times \mathbb{S}^3 \times \mathbb{R}^3 \times \mathbb{R}^3 \times \mathbb{R}^3 \times \mathcal{P} \times \Theta \times \Omega$, the state vector $x := (\tilde{p}, z, \xi, \zeta, \tilde{\omega}, \hat{b}, \hat{\theta}, \chi)$, where $\hat{\theta}$ denotes the estimate of θ, and the flow and jump sets

$$\mathcal{C} := \{(x, q) \in \mathcal{X} \times \mathcal{Q} : q\Psi(x) \geq -\delta\}, \tag{28}$$

$$\mathcal{D} := \{(x, q) \in \mathcal{X} \times \mathcal{Q} : q\Psi(x) \leq -\delta\}, \tag{29}$$

where $\delta \in (0, 1)$ is the hysteresis half-width, and

$$\Psi(x) := \eta + \frac{1}{2k_z}(\omega - \beta_2 + A\frac{\partial w}{\partial u}\Gamma(\tau_2 - \tau_1))^\mathsf{T}\mathcal{I}o(x) \tag{30}$$

$$o(x) := R_d^\mathsf{T} h - \frac{k_z}{m}A\frac{\partial w}{\partial u}\Gamma\left(\frac{\partial u}{\partial \xi}\right)^\mathsf{T}\left(\frac{\partial w}{\partial u}\right)^\mathsf{T} A^\mathsf{T} R_d^\mathsf{T}\epsilon. \tag{31}$$

Consider the following hybrid control law

$$\begin{cases}\left.\begin{aligned}&\dot{\zeta} = -\Lambda^{-1}\left(k_1\vartheta(\tilde{p}) + \Xi\vartheta(\zeta)\right) \\ &\dot{\hat{b}} \in \overline{\operatorname{Proj}}(\Gamma\tau_3, \hat{b}) \\ &\dot{\hat{\theta}} \in \overline{\operatorname{Proj}}(\Gamma_2(\omega - \alpha_q), \hat{\theta})\end{aligned}\right\} & (x, q) \in \mathcal{C} \\ q^+ = -q & (x, q) \in \mathcal{D} \\ u = \hat{b} + mge_3 - m(\dot{\zeta} + \ddot{p}_d) + k_1\vartheta(\tilde{p}) + K\vartheta(\xi) & \\ f = |u| & \\ \mu = -\hat{\theta} + \omega_\times\mathcal{I}\omega - K_2(\omega - \alpha_q) - qk_zR_d^\mathsf{T}\epsilon & \\ \quad + \mathcal{I}\gamma_q - qk_z\mathcal{I}W(x)^\mathsf{T}\Gamma\left(\frac{\partial w}{\partial u}\right)^\mathsf{T} A(w)^\mathsf{T} R_d^\mathsf{T}\epsilon & \end{cases} \tag{32}$$

where $K_2 = K_2^\mathsf{T} \in \mathbb{R}^{3\times 3}$ is positive definite and

$$\begin{aligned}\gamma_q := {} & \left(\frac{\partial \beta_2}{\partial w}\frac{\partial w}{\partial u} + \frac{\partial \beta_2}{\partial u}\right)(\dot{u} - \frac{1}{m}KJ(\xi)\tilde{b}) + \frac{\partial \beta_2}{\partial z}T(z)R_d(\omega - \beta_3) \\ & + \frac{\partial \beta_2}{\partial \tilde{v}}(\dot{\tilde{v}} - \frac{1}{m}\tilde{b}) + \frac{\partial \beta_2}{\partial \xi}(\dot{\xi} - \frac{1}{m}\tilde{b}) + \frac{\partial \beta_2}{\partial y}\dot{y} + \frac{\partial \beta_2}{\partial \hat{b}}\dot{\hat{b}} \\ & - \frac{k_z q}{m}A\frac{\partial w}{\partial u}\Gamma\left(\frac{\partial u}{\partial \xi}\right)^\mathsf{T}\left(\frac{\partial w}{\partial u}\right)^\mathsf{T} A^\mathsf{T}(R_d^\mathsf{T}\epsilon)_\times\beta_3 + q(R_d^\mathsf{T} h)_\times\beta_3 \\ & + qR_d^\mathsf{T}\frac{\partial h}{\partial z}T(z)R_d(\omega - \beta_3) + \frac{1}{m}A\frac{\partial w}{\partial u}KJ(\xi)\mathcal{R}(z)u_\times(\omega - \beta_3),\end{aligned} \tag{33}$$

where $y := (\tilde{p}, \zeta, p_d^{(3)}, \nu_d, \varpi_d)$. The hybrid control law (32) leads to the following hybrid closed-loop system

$$\mathcal{H}: \begin{cases}\left.\begin{aligned}&\dot{\tilde{p}} = \mathcal{R}(z)\tilde{v} \\ &\dot{z} = T(z)\tilde{\omega} \\ &\dot{\xi} = ge_3 - \ddot{p}_d - \dot{\zeta} + \frac{1}{m}\Big(b - \mathcal{R}(z)u\Big) \\ &\dot{\zeta} = -\Lambda^{-1}(k_1\vartheta(\tilde{p}) + \Xi\vartheta(\zeta)) \\ &\dot{\tilde{\omega}} = \kappa(x) \\ &\dot{\hat{b}} \in \overline{\operatorname{Proj}}(\Gamma\tau_3, \hat{b}) \\ &\dot{\hat{\theta}} \in \overline{\operatorname{Proj}}(\Gamma_2(\omega - \alpha_q), \hat{\theta}) \\ &\dot{\chi} \in F(\chi)\end{aligned}\right\} & (z, q) \in \mathcal{C} \\ q^+ = -q & (z, q) \in \mathcal{D}\end{cases} \tag{34}$$

where

$$\begin{aligned}\kappa(x) = {} & \tilde{\theta} + \gamma_q - \dot{\omega}_d - \mathcal{I}^{-1}K_2(\omega - \alpha_q) \\ & - qk_z\mathcal{I}^{-1}\Big(I + \mathcal{I}W^\mathsf{T}\Gamma\left(\frac{\partial w}{\partial u}\right)^\mathsf{T} A^\mathsf{T}\Big)R_d^\mathsf{T}\epsilon.\end{aligned} \tag{35}$$

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Theorem 1. *If Assumptions 1 and 2 hold and the gains satisfy*

$$k_1 + K_{33} + mk_1\frac{1}{\Lambda_{33}} + m\frac{\Xi_{33}}{\Lambda_{33}} < c, \tag{36}$$

with c defined as in Assumption 2, then there exists $\varsigma > 0$ such that $\rho_3 \geq \varsigma$ for all solutions to $\mathcal{H}$, and the compact set

$$\begin{aligned}\mathcal{A} = \{(x, q) \in \mathcal{X} \times \mathcal{Q} : \tilde{p} = 0, z = qe_1, \\ \xi = 0, \zeta = 0, \hat{b} = b, \tilde{\omega} = 0, \hat{\theta} = \theta\},\end{aligned} \tag{37}$$

is uniformly globally asymptotically stable for $\mathcal{H}$.

Proof. It follows from Assumption 2, (36) and (32) that $u_3(t, j) \geq \inf_{(t,j)} u_3(t, j) > 0$. Therefore, there exists $\varsigma' > 0$ such that $u_3(t, j) \geq \varsigma'$ for all (t, j) in the hybrid time domain of the solution. Since u is bounded, it follows that there exists $\varsigma > 0$ such that $\rho_3(t, j) = \frac{u_3(t,j)}{|u(t,j)|} \geq \varsigma$. Let $\mathcal{U}$ be an open set containing $\mathcal{X}$ and consider the continuously differentiable function $V : \mathcal{U} \times \mathcal{Q} \to \mathbb{R}$ defined by

$$\begin{aligned}V(x, q) := k_1 \ln\cosh|\tilde{p}| + \frac{m}{2}\xi^\mathsf{T}\xi + \frac{1}{2}\zeta^\mathsf{T}\Lambda\zeta + \frac{1}{2}\tilde{b}^\mathsf{T}\Gamma^{-1}\tilde{b} \\ + \frac{1}{2}\tilde{\theta}^\mathsf{T}\Gamma_2^{-1}\tilde{\theta} + 2k_z(1 - q\eta) + \frac{1}{2}(\omega - \alpha_q)^\mathsf{T}\mathcal{I}(\omega - \alpha_q).\end{aligned}$$

For all $(x, q) \in \mathcal{C}$, the change in V along the solutions of $\mathcal{H}$ can be shown to be

$$\begin{aligned}\dot{V} \leq -\xi^\mathsf{T}K\vartheta(\xi) - \zeta^\mathsf{T}\Xi\vartheta(\zeta) - k_z k_2\epsilon^\mathsf{T}\epsilon \\ - (\omega - \alpha_q)^\mathsf{T}K_2(\omega - \alpha_q) \leq 0.\end{aligned}$$

Consequently, the growth of V along the flow solutions of $\mathcal{H}$ is bounded by

$$u_c(x) = \begin{cases} -\xi^\mathsf{T}K\vartheta(\xi) - \zeta^\mathsf{T}\Xi\vartheta(\zeta) - k_z k_2\epsilon^\mathsf{T}\epsilon & \\ \quad - (\omega - \alpha_q)^\mathsf{T}K_2(\omega - \alpha_q) & x \in \mathcal{C} \\ -\infty, & \text{otherwise}\end{cases}$$

For all $(x, q) \in \mathcal{D}$, the change in V across jumps is

$$V(x, -q) - V(x, q) = 2k_z q\Psi(x) < 0,$$

where the last inequality follows from the definition of the jump set $\mathcal{D}$. Since the continuously differentiable function V has compact sublevel sets and is positive definite with respect to the compact set $\mathcal{A}$, it follows that $\mathcal{A}$ is uniformly globally stable. Furthermore, $\mathcal{H}$ satisfies the hybrid basic conditions, and the time between jumps is lower bounded by a positive constant. Thus, it follows from [19, Corollary 8.7 (b)] that each solution to $\mathcal{H}$ converges to the largest weakly invariant subset Φ contained in $V^{-1}(\bar{r}) \cap \overline{u_c^{-1}(0)}$, for some $\bar{r} \in \mathbb{R}$, where

$$\overline{u_c^{-1}(0)} = \{(x, q) \in \mathcal{C} : \xi = 0, \zeta = 0, \epsilon = 0, \omega = \alpha_q\}.$$

Note that for every $z \in \mathbb{S}^2$, $\epsilon = 0$, implies $\eta = \pm 1$. The closed-loop system $\mathcal{H}$ is such that $\zeta \equiv 0$ implies $\tilde{p} \equiv 0$. Thus, $\xi \equiv 0$ implies that $\tilde{b} \equiv 0$, since $\epsilon = 0$ implies $\mathcal{R}(z) = I$. Finally, $\epsilon \equiv 0$ and $\xi \equiv 0$ imply that $\alpha_q = \beta_2$ and that $\tau_2 \equiv 0$, from which we can conclude that $\omega = \alpha_q = \omega_d$. It follows that $\dot{\tilde{\omega}} \equiv 0$ and hence that $\hat{\theta} = \theta$. Consequently, no solution ϕ makes $V(\phi(t, j))$ equal to a non-zero constant, and it follows from [19, Theorem 8.8] that $\mathcal{A}$ is uniformly globally asymptotically stable. □

Fig. 1. ModalAI Qualcomm Flight RB5

We remark that Theorem 1 implies that the problem statement is solved. Indeed, by employing the path-lifiting mechanism from [22], it follows from [22, Thm. 9] and Theorem 1 that $\mathcal{A}_0$ is uniformly globally asymptotically stable for the interconnection between (12) and (32).

IV. Experimental Results

The experimental platform is the ModalAI Qualcomm Flight RB5 depicted in Fig. 1. The mass of the quadrotor is $m = 1.4\,\mathrm{kg}$, the inertia matrix is $\mathcal{I} = \mathrm{diag}(0.029\,\mathrm{kgm}^2, 0.029\,\mathrm{kgm}^2, 0.052\,\mathrm{kgm}^2)$, the lower and upper bounds for the parameters are given by $\underline{\theta} = (-0.3, -0.12, -0.3)$, $\overline{\theta} = (0.2, 0.2, 0.2)$, $\underline{b} = (-1, -1, -0.3)$, and $\overline{b} = (1, 1, 3)$. Moreover, the reference trajectory is given by

$$p_d(t) = \begin{cases} (0, 0, -0.75) & t < 30 \\ \begin{pmatrix} \sin(0.5(t - 30)) \\ 1 - \cos(0.5(t - 30)) \\ -0.8 + 0.5(\cos(0.5(t - 30)) - 1)\end{pmatrix} & t \geq 30 \end{cases} \tag{38}$$

$$\nu_d(t) = (\cos\psi_d(t), \sin\psi_d(t)), \tag{39}$$

$$\psi_d(t) = \begin{cases} \frac{\pi}{2} & t < 30 \\ \frac{\pi}{2} - \frac{25\pi}{180}\sin(0.3(t - 30)) & t \geq 30 \end{cases} \tag{40}$$

and the control parameters are chosen as $k_1 = 2.2$, $K = \mathrm{diag}(2.9, 2.9, 3.3)$, $\Xi = K$, $K_2 = \mathrm{diag}(0.08, 0.08, 0.05)$, $k_z = 3.1$, $k_2 = 1$, $\Lambda = I$, $\Gamma = \mathrm{diag}(0.25, 0.25, 0.52)$ and $\Gamma_2 = 0.05I$. It is straightforward to verify that (36) is satisfied. In the implementation, we apply the approximation $\vartheta(x) \approx x$ when $|x| \leq 10^{-6}$. Moreover, we implement the maximal solution to (6), which employs the discontinuous projection (7), i.e. a componentwise saturation.

The experimental results are presented in Figs. 2 to 5. From Figs. 2 and 3 we observe that the quadrotor successfully tracks the position and velocity references. Fig. 4 shows that the estimate $\hat{b}_3$ approaches a value of approximately 1.7 N. This suggests that there are minor modelling errors in the mass of the quadrotor and the thrust produced by the propellers.

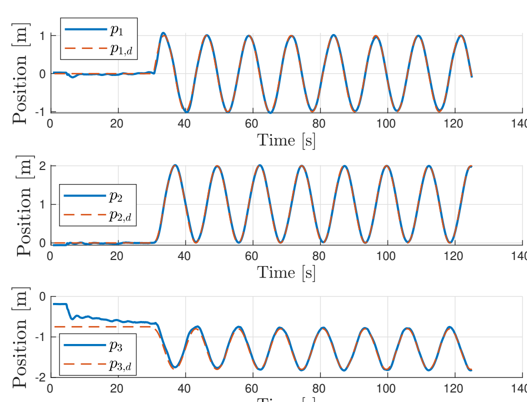

Fig. 2. The position $\boldsymbol{p} \in \mathbb{R}^3$ and the desired position $\boldsymbol{p_d} \in \mathbb{R}^3$.

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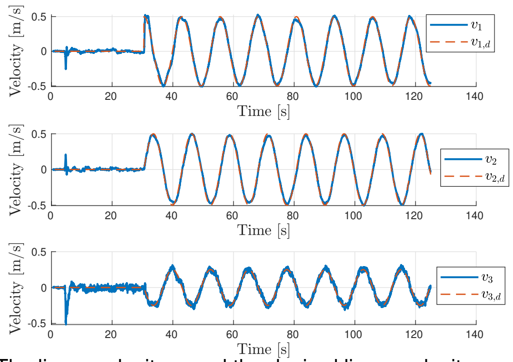

Fig. 3. The linear velocity $\boldsymbol{v}$ and the desired linear velocity $\boldsymbol{v_d}$.

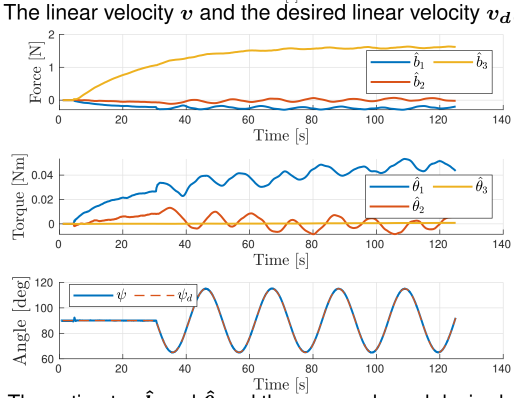

Fig. 4. The estimates $\hat{\boldsymbol{b}}$ and $\hat{\boldsymbol{\theta}}$ and the yaw angle and desired yaw angle $\boldsymbol{\psi}$ and $\boldsymbol{\psi_d}$.

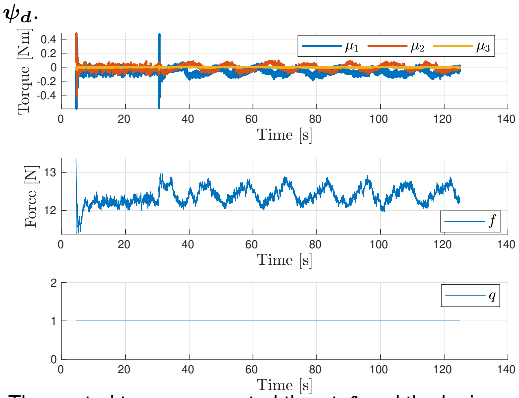

Fig. 5. The control torque $\boldsymbol{\mu}$, control thrust $\boldsymbol{f}$ and the logic variable $\boldsymbol{q}$.

Indeed, as $\hat{b}_3$ approaches its steady-state value, we see that p_3 approaches $p_{3,d}$. The estimates $\hat{b}_1$ and $\hat{b}_2$ remain very close to zero. The orientation disturbance $\hat{\theta}_1$ increases to a value of approximately $0.05\,\mathrm{Nm}$. This suggests that there are minor modeling errors in the roll-loop of the quadrotor, most likely in the position of the center of mass and the moments produced by the propellers. The two other orientation disturbances remain small. Moreover, we observe that the yaw angle of the quadrotor $\psi := \operatorname{atan2}(\nu_2, \nu_1)$, successfully tracks the desired yaw angle ψ_d. The desired propeller torque, total propeller thrust, and logic variable q, are plotted in Fig. 5. The two spikes in the desired torque correspond to lift off and the abrupt change in the desired trajectory at $t = 30\,\mathrm{s}$, respectively. The logic variable q does not change sign during the motion.

V. Conclusions

This letter has addressed the global position and heading tracking control problem for multirotor aerial vehicles. The proposed control law achieves uniform global asymptotic position and heading tracking. Global asymptotic heading tracking is achieved by utilizing a novel construction of the desired rotation matrix. Moreover, the use of tuning functions ensures that the number of parameter estimates is equal to the number of unknown parameters. In turn, this guarantees that the parameter estimates in both the translational and rotational dynamics converge to their actual values. The effectiveness of the control law has been demonstrated through experiments.

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