Term Premia in Norwegian Government Bond Yields

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Abstract

The typically observed upward sloping nominal yield curve implies that investors demand positive risk premia - or term premia - to hold long-term nominal bonds. Fundamentally, the term premium is compensation to investors for bearing interest rate risk and a component in the term structure of yields. There is substantial evidence of sizeable and time-varying term premia. As opposed to yields, term premia are not directly observable. In this paper we estimate term premia in Norwegian government bond yields from a set of dynamic term structure models (DTSM), covering the period from 2003/01 until 2021/04. In line with international studies, we find evidence of declining term premia over the sample period.

Keywords— Yield curve modelling, dynamic term structure models, term premia

JEL classification— C58, E4, G15, G120

1 Introduction

Government bonds issued by sovereigns in their domestic currency are considered default-free.[1] This is in contrast to bonds issued by financial institutions or corporates, for which yields contain an additional credit risk component to compensate the investor for the risk of issuer default. A common approach is to decompose the default-free yield into expectations of future short-rates and term premia.[2] The yield $y_t^{(n)}$ on an n-maturity bond at time t can be decomposed into to expected future short-rates and term premia;

$$y_t^{(n)} = \frac{1}{n}\mathbb{E}^{\mathbb{Q}}\left(y_t^{(1)} + y_{t+1}^{(1)} + \cdots + y_{t+n-1}^{(1)}\right) + TPy_t^{(n)}, \tag{1}$$

where $\mathbb{E}^{\mathbb{Q}}$ denotes the risk-neutral expectation and $TPy_t^{(n)}$ is the term premium.[3] As such, the term premium is a determinant of the term structure of interest rates. Hence, long-term risk-free yields are not solely impacted through the monetary policy channel, but also by changes in term premia.

Disentangling the informational content of the yield curve is important for central banks, regulatory bodies, financial institutions, corporates and investors. Policy makers need to judge the impact of monetary policy actions, such as policy rate changes or quantitative easing (QE) programs. In the context of macroeconomic implications it is crucial to understand whether changes in long-term yields are caused by changes in term premia or altered short-rate expectations, respectively. For investors, term premia determine bond excess returns and are important variables in asset allocation and portfolio con-

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[1] This is not always the case in emerging markets, where the creditworthiness of sovereigns might induce a risk premium for default risk.

[2] Strictly speaking, yields contain an additional convexity term - which relates to Jensen's inequality, see Rebonato (2018). However - unless yield volatility is very high or for very long maturities - the convexity effect is typically small. In line with the literature we ignore it for the purpose of this paper.

[3] Risk-neutral in this context refers to yields that would exist under the pure expectation hypothesis. That is yields with a market price of interest rate risk equal to zero and thus no term premium.

struction. For fixed rate debt issuers - both financial, corporate and sovereign - term premia represent excess funding cost over the short-term rate. Hence, fixed rate payers interpret the term premium as the cost of hedging short-term interest rate risk. Banks and other financial institutions use term premium estimates for risk management purposes. Models frequently applied for market- and credit risk quantification, take risk-premium adjusted market prices as parameters. Furthermore, accounting bodies in some instances require application of real-world expectations for financial reporting purposes.[4] To comply with this requirement, reporting entities distinguish term premia from short-rate expectations. Hence, numerical estimates of term premia are relevant for a diverse set of stakeholders.

The term premium can be interpreted as a risk premium required by risk averse investors to carry risk related to the future path of short-term interest rates. Researchers have documented that term premia are driven by uncertainty of macroeconomic variables, such as real growth and inflation (Rudebusch et al. 2006, Wright 2011, d'Amico et al. 2018). Complementary to this risk-based approach D'Amico et al. (2014) and Moessner (2018) document the impact on term premia from asset purchase programs employed by central banks. Hui et al. (2017) propose a convenience yield component in interest rates as an explanation for variation in term premia. Hence, it is plausible that yield term premia fundamentally reflect both risk premia and other factors unrelated to risk aversion.

While yields are observable in the bond market, risk premia and expectations of future short-rates are latent variables. The decomposition of yields into short-rate expectations and term premia requires application of econometric models. Early studies by Fama & Bliss (1987) and Campbell & Shiller (1988) relate the slope of the current yield curve to term premia through predictive linear regressions of excess bond returns onto measures of yield curve slope. Since the seminal contributions of Nelson & Siegel (1987),

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[4] See for instance IFRS9 Financial Instruments and IFRS17 Insurance Contracts, published by the International Accounting Standards Board.

Duffie & Kan (1996), Dai & Singleton (2000) and Duffee (2002) parsimonious dynamic term structure models have been the preferred econometric framework to estimate term premia.

In this paper we provide numerical estimates of term premia in Norwegian government bond yields from a representative set of dynamic term structure models. Our sample covers the period from 2003/01 through 2021/04, during which Norges Bank (the central bank of Norway) has executed monetary policy under a mandated fixed inflation target. Particular attention is given to the effect of the lower bound on term premium estimates. Furthermore, we discuss results in light of monetary policy actions, business cycles and financial market events. To our knowledge, this is the first paper to analyze Norwegian bond yields over this time period and using this set of models. The Norwegian Government Bond (NGB) results presented in this paper are interesting in their own right. Norway is one of only a few European countries having its own currency and thus have the ability run autonomous monetary policy, which potentially results in different term premium levels and dynamics. Similarly, limited sovereign funding needs combined with lower liquidity due to a limited number of market participants, might impact NGB term premia.

This paper contributes to the literature by providing updated estimates of term premia in Norwegian government bond yields. Sekkel (2011) and Wright (2011) use Norwegian data as part of their international comparative studies, but they cover shorter sample periods than we do. NGB term premia are not explicitly estimated by Sekkel (2011), but can be inferred from his analysis of excess return predictability. Wright's focus is to compare estimates from one single term structure model and survey-based estimates. Beyond using a longer time-series, our paper has a broader scope than Wright (2011), as we combine both Gaussian and shadow-rate term structure models.

The rest of this paper is organized as follows. In section 2 we give an overview of the empirical literature on term premia. In section 3 we describe the dataset. In section 4 we describe the econometric framework and estimation strategy applied in the study. In section 5 we present term premia estimates and discuss the results. Section 6 concludes and proposes ideas for further research.

2 Literature review

The vast majority of empirical research on term premia has been conducted using data from the U.S. Treasury market, see Fama & Bliss (1987), Campbell & Shiller (1988), Cochrane & Piazzesi (2005), Ang & Piazzesi (2003), Diebold & Li (2006), Joslin et al. (2011), Wu & Xia (2016), Bauer & Rudebusch (2016) and many others. The consolidated findings in these studies indicate that term premia in long-term U.S. Treasuries went through a minor cycle from 1961 to the early 1980s, peaking at about 2% in 1970. In the early 1980s, the term premia rose quickly to levels of up to 4% and then settled into a range around 2% from the mid-1980s to the early 2000s. Since then, term premia has trended lower and entered sub-zero levels in 2020. However, there have been several marked cycles against the trend, most notably peaks in 2007 and 2018.

A few papers analyze data from the euro area. McCoy et al. (2019) report a downward trend in term premia from 2002 to 2018 period and find that the term premium in 10y German bonds peaked at 2% in 2009 and has since 2014 been negative. Moessner (2018) finds a comparable trend from 2008 to 2015 using data for ten different European sovereigns. These results are supported by Cohen et al. (2018), who estimate ten-year euro area government bond term premia from 2000 to 2018 using three different term structure models. de los Rios & Shamloo (2017) estimate term premia for six different countries from 1999 to 2015. Their results for Germany resemble those of the studies mentioned above. For the U.K. de los Rios & Shamloo (2017) find that term premia were fairly stabile within the 0-1% range prior to 2008, when term premia peaked to 2%, and has since then trended downwards. Malik & Meldrum (2016) find similar results for the U.K., albeit using a slightly different sample. Jennison (2017), Callaghan et al. (2019) and Aydin et al. (2019) estimate term premia from affine term structure models using data from Australia, New Zealand and Turkey, respectively. The single country term premia estimates in these studies indicate that term premia are highly correlated across countries. This is indeed confirmed in international studies (Wright 2011, Dahlquist & Hasseltoft 2013, Jotikasthira et al. 2015).

NGB term premia has previously been estimated by Wright (2011), using a sample covering January 1998 to May 2009. Wright (2011) reports declining NGB term premia over the sample period, and explains this by the introduction of inflation targeting and increased independence of Norges Bank.

3 Data

3.1 Norges Bank dataset

Norges Bank publishes daily par rates based on observed yields of outstanding treasury bills and bonds.[5] We base our empirical analysis on this dataset.[6] Our sample period covers January 2003 through April 2021.

[FIGURE 1 (NGB yield-to-maturity) ABOUT HERE]

On 29 March 2001, the Government assigned a new operational mandate for the implementation of monetary policy to Norges Bank. From this date, the central bank sets the policy rate with an objective of maintaining low and stable inflation - defined as an explicit inflation target. To avoid possible structural breaks related to this formal change in monetary policy, we start our sample in January 2003.

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[5] The *par rate* is the market interest rate for a specific maturity, often referred to as yield-to-maturity. Thus, the par rate corresponds to the coupon rate of a bond that trades at face value.

[6] The dataset is available at `https://www.norges-bank.no/tema/Statistikk/Rentestatistikk/`

As is evident from Figure 1, NGB yields have generally declined since the beginning of the century. This is in line with the development of global yields. Bernanke (2013) assigns this long-run decline in yields to cyclical factors, including the slow pace of economic recovery, modest inflation rates, and *accommodative monetary policy.* Bauer & Rudebusch (2020) point to falling expectations of long-run inflation, real yields and related uncertainty. Although yields of different maturities tend to move in the same direction, the slope of the NGB yield curve has varied over time. The yield curve is typically upward sloping, but the slope has been negative at some occasions, such as during the global financial turbulence in 2008. These dynamics are consistent with an interpretation of the yield curve slope as a recession indicator (Harvey 1986).

We note that since august 2007 the Federal Reserve (FED) System's balance sheet has grown from around USD 1 trillion to USD 7,4 trillion by the end of March 2021. That is almost seven times the value of the Norwegian oil fund, just to put it in perspective. Over the last year the FED's balance has been relatively stable. Other central banks' balance sheets have also grown at unprecedented rates over this period, and there can be little doubt that this stimulus has severely impacted bond prices. Contrary to FED and ECB (European Central Bank), Norges Bank has not engaged in quantitative easing (QE) and consequently not influenced long-term NGB yields directly. Apart from a relatively short period following the 2007 global financial crisis (GFC) - where Norwegian Banks were allowed to borrow Norwegian Government Bonds through depositing Covered Bonds in Norges Bank - and during periods of stressed interbank money markets (such as the recent Covid 19 outbreak) - where Norges Bank has provided increased short-term lending facilities - the policy rate has been Norges Bank's most important monetary policy instrument.[7]

To promote liquidity in the government securities market, Norges Bank has entered into primary dealer agreements with four banks.[8] Under these agreements, primary dealers

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[7] Furthermore, Norges Bank employs *forward guidance* as a means to impact market expectations of future short-rates.

[8] Currently DnB, Nordea, Danske Bank and SEB

are obliged to quote bid and offer prices for treasuries at the Oslo Stock Exchange and are able to enter into repurchase agreements with Norges Bank. The primary dealers have committed to participate with bids in auctions of new issues, possibly on behalf of clients.

3.2 Zero coupon yields

Norges Bank does not report zero coupon yields for our full sample period - which are needed for the empirical purpose of this paper.[9] Norges Bank does, however, provide detailed disclosures of their application of the Nelson-Siegel-Svensson (NSS) model (Svensson 1994) to estimate zero-coupon yields from market prices of outstanding government bonds.[10] We take a similar approach and use the NSS framework correspondingly, adapted to our sample period. Appendix A.1 contains details on our NSS estimations.

To construct the short end of the zero coupon curve we follow Bauer & Rudebusch (2016) and use reported yields for maturities less than one year directly. Maturities beyond one year are estimated in our adapted NSS framework. Table 1 displays descriptive statistics for the zero coupon yields applied in this study.

[TABLE 1 (Descriptive statistics of zero coupon yields) ABOUT HERE]

Table 1 reveals some commonly observed stylized facts about yields. Short-term rates are on average lower than long-term rates. Furthermore, short-term yields are more volatile and less persistent than long-term yields.

Figure 2 contains a principal component decomposition (PCA) of basis point changes in yields. The results conform to the original findings of Litterman & Scheinkman (1991)

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[9] Norges Bank reports zero curves from 2019 and onward.

[10] The methodology and estimation procedure is thoroughly described in `https://www.norges-bank.no/contentassets/6bf78dd3f9934e89818acdd23f95b1e2/nullkupongrenter-forklaring.pdf`

which subsequently have been confirmed. The first three PCs explain most of the cross-sectional variance of yields (left panel). Furthermore, the factors have economic interpretation as level, slope and curvature factors respectively (right panel).

[FIGURE 2 (Principal Component Analysis) ABOUT HERE]

To further validate our NSS implementation, we compute par-rates from our estimated zero-coupon rates and compare these to the original par-rates reported by Norges Bank and displayed in figure 1. The deviation between actual and estimated par-rates, as measured by the Mean Absolute Error (MAE), is 3.2, 2.3 and 1.2 basis points for the 3, 5 and 10y maturities respectively. Thus, our NSS application reproduces the original Norges Bank dataset very accurately.

4 Methodology

Government bonds have risk premia if their returns covary with priced risk factors. To estimate these risk premia, the term structure literature has generally abstracted away from traditional asset pricing models and moved in the direction of vector autoregressive models with factors based on individual interest rates, linear combinations of these or latent factors filtered from the yield curve.

Cohen et al. (2018) illustrate the model-dependency of term premium estimates. Bauer & Rudebusch (2016) advocates against relying on a single model. We concur to this, and apply a set of four different dynamic term structure models (DTSMs) to estimate term premia in NGB yields. More specifically, we use models proposed by Joslin et al. (2011), Bauer et al. (2012), Adrian et al. (2013) and Bauer & Rudebusch (2016), henceforth referred to as JSZ, BRW, ACM and BR. ACM and JSZ have gained popularity, partly because they develop estimation procedures that resolve the maximum likelihood related problems associated with traditional affine models. BRW and BR address some of the theoretical shortcomings of JSZ, and serve as appropriate complement models for the purpose of this paper.

In this study, we use DTSMs as the tool to compute term premia estimates. Section section 4.1 gives a brief introduction to affine term structure models (ATSM). In section 4.2 we introduce definitions and common concepts of the models in our model set. In section 4.3 we describe our estimation of the different models and, more specifically, the corresponding term premia estimates.

4.1 Affine term structure models

Let $P_t^{(n)}$ denote the price at time t of an n-period zero coupon bond; let $y_t^{(n)} = log(P_t^{(n)})/n$ denote its yield; let M_{t+1} be the nominal pricing kernel; and let Π be the vector of cash flows. The price of the bond must be $P_t^{(n)} = \mathbb{E}_t(\Pi_{j=1}^{n} M_{t+j})$. ATSMs produces solutions for $P_t^{(n)}$ and and can be used to decompose yields into short rate expectations and term premia.

Gaussian affine term structure models (GATSM) have been widely favored by academics, central bankers and practitioners due to their analytical tractability. The first discrete-time GATSM appears in a seminal contribution by Ang & Piazzesi (2003). One theoretical shortcoming of the GATSMs is their ability to produce substantial probabilities of negative future short rates, and consequently produce inaccurate forecasts of short-rates and term premia when the short-rate is bounded from below. Prior to the global financial crisis this was not considered a significant issue, given the historical level of interest rates. Subsequently, short-rates across the globe have declined towards zero and in some currencies also below zero.

Shadow rate term structure models (SRTSMs) address the lack of an appropriate non-negativity restriction in GATSMs and account for the substantial asymmetry in the distribution of future short rates during periods of near-zero policy rates. SRTSMs are based on the shadow-rate concept introduced in Black (1995). Black interpreted observed interest rates as call options having zero strike on shadow rates, which can take any real number as value. Shadow-rate models are closely similar to their Gaussian counterparts, except that the affine short-rate equation is replaced by a shadow-rate specification; $r_t = max(s_t, r_{min})$. Hence, the distribution of the short-rate r_t is truncated from the shadow rate distribution s_t and bounded by r_{min}. Black (1995) argued that investors always have the option to hold physical currency and assumed a zero lower bound (ZLB) on the short-rate. In reality, there are storage costs and constraints related to holding physical currency, and sub-zero interest rates are observed in several currencies, not only for short-term interest rates but also for longer maturities.[11] The lower bound restriction can be relaxed and need not be set to zero. In addition to accounting for the zero lower bound, a key advantage of shadow-rate models is that away from the ZLB, they behave exactly as the corresponding Gaussian models. Furthermore, the probability of a zero future short rate is nonzero in shadow-rate models. This enables this class of models to represent the stickiness property of ZLB interest rates, which results from the highly asymmetric distribution around the expected short-rate path at the ZLB.

4.2 Econometric framework

The term structure models belonging to the model set in this study have some common features, which we will outline in the following.

The risk factors X_t are stationary and follow mean-reverting processes.[12] The vector of K risk factors follows a first-order Gaussian vector autoregression (VAR):

$$X_{t+1} = \mu + \phi X_t + \Sigma \epsilon_{t+1}, \tag{2}$$

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[11] For instance; in Germany short-term bond yields have been negative since mid 2014. Similarly, long-term yields were below zero from early 2019 until the end of 2021.

[12] The literature tends to use *risk factors, pricing factors* and *state variables* interchangeably.

where $\epsilon_{t+1} \sim (0, I_N)$ and Σ is lower triangular. The risk factors X_t are a set of K demeaned and normalised principal components of yields. The short-term interest rate r_t, is a function of the risk factors. For the Gaussian models, to which ACM, JSZ and BRW belong, the short-rate equation is

$$r_t = \delta_0 + \delta_1^{'} X_t \tag{3}$$

In the BR model the short-rate equation (3) is replaced by a shadow-rate specification:

$$r_t = max(s_t, r_{min}), \;\; s_t = \delta_0 + \delta_1^{'} X_t. \tag{4}$$

The shadow short rate in the BR model, s_t, is modeled as affine Gaussian, as the short rate in ACM, JSZ and BRW. Equation (4) ensures that the short rate and all other model-implied interest rates cannot go below r_{min}.

The models assume that the stochastic discount factor M_t that prices all assets under the absence of arbitrage exists, and is of the essentially affine form Duffee (2002):

$$-\log(M_{t+1}) = r_t + \frac{1}{2}\lambda_t^{\prime}\lambda_t + \lambda_t^{\prime}\epsilon_{t+1}, \tag{5}$$

where the K-dimensional vector of risk prices is affine in the risk factors

$$\lambda_t = \lambda_0 + \lambda_1 X_t. \tag{6}$$

As a consequence of these assumptions, the price $P_t^{(n)}$ at time t of an n-period zero coupon bond is an exponentially affine function of the risk factors:

$$P_t^n = \exp\left(\mathcal{A}_n, +\mathcal{B}_n^{\prime} X_t\right), \tag{7}$$

with loadings $\mathcal{A}_n = \mathcal{A}_n\left(\mu, \phi, \delta_0, \delta_1, \Sigma\ , \lambda_0\right)$ and $\mathcal{B}_n = \mathcal{B}_n\left(\phi, \delta_1\ , \lambda_1\right)$ that follow the recursions

$$\mathcal{A}_{n+1} = -\delta_0 + \mathcal{A}_n + (\mu - \Sigma\lambda_0)'\mathcal{B}_n + \frac{1}{2}\mathcal{B}_n'\Sigma\Sigma'\mathcal{B}_n \tag{8}$$

$$\mathcal{B}_{n+1} = (\phi - \Sigma\lambda_1)'\mathcal{B}_n - \delta_1, \tag{9}$$

with starting values $\mathcal{A}_0 = 0$ and $\mathcal{B}_0 = 0$. The recursions in (8) and (9) are the standard linear difference equations for affine term structure models with homoskedastic shocks (Dai & Singleton 2003).[13]

The bond prices in (7)-(9) are the same as though agents were risk-neutral ($\lambda_0 = \lambda_1 = 0$), but the risk factors followed an alternative law of motion:

$$X_{t+1} = \mu^{\mathbb{Q}} + \phi^{\mathbb{Q}} X_t + \Sigma\epsilon_{t+1}^{\mathbb{Q}}, \tag{10}$$

where

$$\mu^{\mathbb{Q}} = \mu - \Sigma\lambda_0, \qquad \phi^{\mathbb{Q}} = \phi - \Sigma\lambda_1. \tag{11}$$

Equations (3) and (10) are known as the physical and risk-neutral representations of the law of motion for the risk factors, respectively.

As common in the literature, we compute the the term premium as the difference between the physical and risk-neutral expectation of the short-rate.[14] From eq. (11), it is evident that the vector of risk prices in eq. (6) is a key determinant of term premium estimates, along with risk factor dynamics.

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[13] The difference equations for ACM contain one additional minor component compared to JSZ, BRW and BR. This is due to the ACM assumption of serially uncorrelated excess return pricing errors, as opposed to serially uncorrelated yield pricing errors (see section 2.4 of Adrian et al. (2013)).

[14] See for instance Wright (2011) pp. 1520-1521.

4.3 Estimation strategy

Our NGB term premia estimates are based on monthly zero coupon yields (see section 3.2 for further details).

Common to the four models in our model set is that the risk factors X_t are K demeaned and normalised principal components of yields. ACM uses $K = 5$, whereas JSZ, BRW and BR use $K = 3$. Thus, we begin the estimation procedure by performing a principal component analysis (PCA) of zero coupon yields. We then proceed by estimating each model as described in the following sections. Finally, term premia are computed as the difference between the physical and risk-neutral short-rate expectations for the different models.

4.3.1 ACM

ACM derive an expression for continuously compounded arbitrage free excess holding period returns $rx_{t+1}^{(n-1)}$. The log excess holding return of a bond maturing in n periods is defined as:

$$rx_{t+1}^{(n-1)} = \ln P_{t+1}^{(n-1)} - \ln P_t^{(n)} - r_t. \tag{12}$$

To estimate our ACM model, we follow the three-step series of linear regressions proposed by Adrian et al. (2013). The first step is to estimate eq. (2) via OLS, which decomposes X_t into a predictable component and an estimate of the innovation. The second step is a time series regression, where excess returns are regressed on a constant, lagged pricing factors and the estimated shock components collected from the first step. In the third step the price of risk parameters λ_0 and λ_1 are estimated via cross-sectional regression.

4.3.2 JSZ

Joslin et al. (2011) develop a novel canonical no-arbritrage Gaussian DTSM in which the risk factors can be any observable linear combinations of yields.[15] We use the first three principal components as risk factors in out JSZ specification. Joslin et al. (2011) propose a two-step procedure, in which physical and risk-neutral parameters can be estimated separately. Since the physical parameters in (2) follow an unconstrained vector autoregressive form, they can be consistently estimated by ordinary least squares in step 1. Joslin et al. (2011) propose an algorithm to estimate the risk-neutral parameters in (11) by maximum likelihood in step 2.

4.3.3 BRW

Small-sample bias is a well known challenge when estimating vector autoregressive systems (Hamilton 1994). In the context of DTSMs this bias is due to combined effects of the persistent nature of yields and limited sample sizes.[16] Since the bias causes the speed of mean reversion to be overestimated, model-implied interest rate forecasts will tend to be too close to their unconditional mean, especially at long horizons. Therefore, risk-neutral rates will be too stable, and too large a portion of the movements in nominal interest rates will be attributed to movements in term premia. To account for this, Bauer et al. (2012) suggest that the ordinary least squares parameters within affine DTSMs should be bias-corrected and propose a new estimation approach to achieve such bias-correction. We implement this procedure in our BRW model. The procedure involves adapting the two-step estimation approach of JSZ by replacing the OLS estimates of the autoregressive system in the first step by bias-corrected estimates de-

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[15] A canonical Gaussian DTSM is one that is maximally flexible in its parameterization of both the physical and risk-neutral distributions of X_t, subject only to normalizations that ensure econometric identification.

[16] Yields of different maturities tend to be highly correlated and the speed of mean reversion is relatively slow. Furthermore, yields display cyclical behaviour, and the number of cycles in a sample is typically low.

rived from a stochastic approximation bootstrap algorithm. The second step of the estimation procedure can then be carried out to recover the parameters determining the cross-sectional fit of the model, as in JSZ.

4.3.4 BR

The shadow-rate DTSM proposed by Bauer & Rudebusch (2016) is closely related to Joslin et al. (2011) except that the affine short-rate equation (2) is replaced by the shadow-rate specification in eq. (4). We set $r_{min} = 0$ as originally proposed by Black (1995). Furthermore, although the policy rate was set equal to zero by Norges Bank for the first time in 2020, the rate is yet to be set below zero.

Equation (4) introduces non-linearity into the BR model. A shadow-rate model does not lead to closed-form solutions for bond prices so that the need arises for approximate solution methods. When estimating our BR model we adapt the fast and accurate method proposed by Priebsch (2013) to discrete time, in combination with the extended Kalman filter.

5 Results

Our estimation of the ACM, JSZ, BRW and BR models provides a tight fit to actual NGB yields. Root Mean Squared Errors (RMSE) across the maturity spectrum is 10 basis points or lower for the four models, which is in line with the empirical literature.[17]

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[17] See for instance Diebold & Li (2006), Adrian et al. (2013), Bauer & Rudebusch (2016), Malik & Meldrum (2016) and Jennison (2017), who report comparable fitting errors.

5.1 Term premium estimates

The following discussion will focus on the 10y term premium. We consider this point of the yield curve to be of significant importance for practitioners and it is also among the most liquid maturities. Term premia estimates for shorter maturities are available from the authors upon request.

[FIGURE 3 (10y term premium estimates) ABOUT HERE]

NGB yields have declined over the sample period which is similar to what has been experienced by most other developed countries. Figure 3 shows that ACM, BR and JSZ agree on a substantial decline in the 10y term premium towards zero, whereas BRW depicts a more modest decline and significantly more stable term premia.

Trends and rapid changes in term premia can be related to important economic events, both global and domestic. Dahlquist & Hasseltoft (2013) note the importance of US bond risk and global business cycles in driving global yields, with increasing correlations between international bond risk premia over time. From the beginning of the century and up until the GFC, the Norwegian economy experienced strong growth and rising asset prices - which is consistent with a decline in term premia. GFC, commencing late 2007, induced uncertainty about future growth prospects and higher term premia. In 2010 a number of major central banks undertook unconventional monetary policy initiatives such as large scale asset purchase programs. Through this channel, the term premium is likely to fall for bonds purchased within these programs and second order effects might include falling term premium for other securities, such as NGBs. NGB term premia estimates surged following the euro area sovereign debt crisis in 2012 and the 2013 *taper tantrum*,[18] which is consistent with declining global risk appetite as the outlook for US monetary policy became less certain. In 2014 yields started to fall,

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[18] Comments from Fed officials led the market to expect an imminent removal of monetary accommodation.

largely due to the market's anticipation of the ECB's asset-buying Public Sector Purchase Program (PSPP) and its subsequent implementation (from early 2015). This is a plausible explanation for the sharp decline in estimated NGB term premia during 2014. The sharp increase in November 2016 coincides with a broad-based sell off in fixed income assets globally following the US election. As approximately two thirds of outstanding NGBs are held by foreign investors, such changes in global market sentiments are likely to impact NGB prices.[19] From 2018 to 2020 market volatility has been low across most asset classes, consistent with a downward trend in NGB term premia. Towards the end of the sample period term premium estimates rise sharply, most likely due to a combination of Covid19-related uncertainty about future economic growth and increased likelihood of higher inflation. ACM show negative NGB term premium estimates that might appear puzzling if viewed from the perspective of a risk averse investor. However, with consideration also given to other effects influencing the term premium, such as large scale asset purchases and global spillover effects, a negative term premium is entirely plausible.

Term premium estimates are computed as the residual of fitted yields and risk-neutral yields, implying that any deviation from the pure expectations hypothesis can move the term premium. As such, the models do not provide explanations as to what moves term premia. However, declining NGB term premia is consistent with the literature. According to Wright (2011) term premia have declined internationally since the 1990s, especially in countries that reduced inflation uncertainty by making substantial changes in their monetary policy frameworks. This might well be a plausible explanation for a decline in NGB term premia, since inflation has generally been low and stable following Norges Bank adaption of the inflation targeting regime in the early 1990s. Similar to Dahlquist & Hasseltoft (2013), Jotikasthira et al. (2015) find that world inflation and the US yield level together explain over two-thirds of the covariance of global yields,

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[19] See the Norges Bank annual report on government debt management: `https://www.norges-bank.no/contentassets/6ba8d7e5fb0546a58af07ea53a2bd8fd/arsrapport-statsgjeld-2021.pdf?v=03/31/2022144634&ft=.pdf/`

and that these effects operate largely through the term premium channel. In light of these findings, and the fact that Norway is a small, open economy likely to be impacted by international risk factors, a decline in term premia appears to be reasonable. On a similar note, Drought et al. (2018) argues that influencing the term premium in long term interest rates has become an important channel in the implementation of unconventional monetary policy. Although Norges Bank is yet to implement such monetary policy actions to influence NGB term premia directly, term premium spill-over effects from major economies are likely. Furthermore, central bank asset repurchases are not equilibrium outcomes in competitive markets, and are thus likely to cause prices changes that do not reflect investors' risk aversion. The diversion in monetary policy, with extensive QE programs in the U.S. and lack of comparable measures in the NGB market, is a plausible explanation for higher estimated NGB term premia compared to U.S Treasuries towards the end of sample.

[FIGURE 4 (ACM term premia vs. MOVE) ABOUT HERE]

Figure 4 compares our ACM estimates of the 10y NGB term premium to comparable estimates of the US Treasuries 10y term premium, as reported by the Federal Reserve Bank of New York. The series display similar trends and variability. The linear correlation is 0.82, indicating presence of common underlying risk factors. Estimates deviate significantly in the period preceding GFC. The correlation between US term premia and the Bank of America Merrill Lynch MOVE Index, a measure of bond market implied volatility frequently interpreted as a proxy for macro uncertainty, is well known. The NGB term premium displays as similar pattern. The correlation of MOVE vs NGB and US estimates are 0.61 and 0.70 respectively. These results of correlated term premia are in line with findings specific for term premia in yields (Wright 2011, Jotikasthira et al. 2015) and for asset risk premia in general (Bollerslev et al. 2018).

5.2 Impact of the zero lower bound

Gaussian DTSM are ill-suited to represent the dynamics of recent near-zero interest rates since they do not recognize that in the real world, with currency available as an alternative asset, interest rates are bounded at some level not too far from zero. Still, figure 3 indicates that JSZ and BR, the latter being the ZLB version of the former, produce close to identical term premium estimates.

[FIGURE 5 (JSZ-implied probabilities of negative future short rates) ABOUT HERE]

To assess the relevance of BR - which respects the ZLB - over the Gaussian JSZ model, we investigate to which extent JSZ violates the lower bound. One way to do this is to compute the model-implied probabilities of negative future short-rates. Figure 5 displays these probabilities at 6 months, 1 year and 2 years horizons. Another approach is to compare fitted yields to estimated shadow yields from the BR model. Figure 6 displays fitted yields and shadow yields and figure 7 displays the difference between the two - the ZLB wedge. Until 2015 the ZLB does not seem to have been binding in the NGB market; with probabilities of negative future rates close to zero and shadow yields not far from fitted rates. The only sub-period in the sample where BR produces different yield-curve dynamics compared to JSZ, is a relatively short period subsequent to the Covid19-outbreak in March 2020. Norges Bank lowered the policy rate to zero, leading to a spike in the ZLB wedge. Towards the end of the sample, long-term yields increase - predominantly due to risk of higher inflation - and the ZLB wedge reverts back to pre Covid19 levels. In the interim 2015-2020 period, the lower bound seems to be effective to some extent. However, the ZLB wedge varied in the range of 20 to 40 basispoints, which most likely is not sufficient to significantly impact term premia.[20]

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[20] In DTSMs, term premia estimates are equal to the difference in expected risk-neutral and empirical short-rates. Hence, differences in term premia between Gaussian and shadow-rate models are due to different modelling of the short-rate process which again arises from the truncated distribution of the short-rate in (4). The ZLB wedge represents this difference in drift of short-rates. The size of the ZLB wedge in this study indicates modest impact on term premia estimates.

[FIGURE 6 (Fitted yields vs shadow yields (BR model)) ABOUT HERE]

[FIGURE 7 (ZLB wedge) ABOUT HERE]

Insofar, we have fixed the lower bound of the short-rate at zero. The family of ZLB-models - to which BR belongs - are highly sensitive to model specification (Krippner 2015). To investigate the robustness of our term premium estimates, we estimate the BR model varying the lower bound from zero to 25 basis points. Figure 8 shows that shadow rate estimates are virtually unchanged, apart from the post Covid19 period. Even during this period of relatively short duration the differences in shadow rates are of limited magnitude.

[FIGURE 8 (Sensitivity of shadow rates to lower bound) ABOUT HERE]

We conclude that the ZLB is of minor importance for the purpose of this study, which is to estimate term premia - as opposed to yields. This is in line with earlier results found by Malik & Meldrum (2016) and Guimarães et al. (2014).

5.3 Impact of small sample bias

[FIGURE 9 (Impact of small sample bias) ABOUT HERE]

BRW is a bias-corrected version of JSZ. Figure 9 displays the decomposition of yields into term premia and risk-neutral expectations from the two models. Consistent with the literature, the bias-corrected BRW (right panel) results in more variability in risk-neutral rates and consequently, more stable term premium estimates. It is interesting that 10y term premium from BRW appears to be close to stationary around 1%. Implicitly, BRW attributes most of the decline in NGB long-term yields from 6% towards zero over the sample period to lowered short-rate expectations and less to changes in term premia. Furthermore, the variability of the term premium is relatively modest. This is somewhat surprising, especially in light of the global macroeconomic events spanned by the sample period. GFC, the Euro-area sovereign and banking crises, as well as the Covid19 pandemic, have created significant macroeconomic and financial uncertainty. Fundamentally, the term premium can be interpreted as a risk premium related to future economic conditions, and the BR estimates show little sensitivity to such factors.

Wright (2014) argues that term premium point estimates with and without a bias-correction lie within OLS confidence intervals and furthermore claims that bias-correcting is liable to overcorrect with respect to the true data generating process and thus lead to less plausible term premium and risk-neutral yield point estimates compared to making no correction. Malik & Meldrum (2016) and Guimarães et al. (2014) support this view.

The sample size of our study is comparable to that of Joslin et al. (2011) and Adrian et al. (2013). In light of the downward trend in yields, we find it likely that small-sample bias is present and might impact our NGB term premium estimates. However, our overall impression is that bias-correction using BRW does not produce plausible results.[21]

6 Conclusions

In this paper we present updated estimates of term premia in Norwegian government bond yields. We obtain estimates using well established dynamic term structure models covering the period from 2003/01 to 2021/04. Particular attention is given to impacts on term premia from a lower bound on yields and to small-sample bias. We find evidence of declining term premia over the sample period, which resembles international findings. Furthermore, the trend and variation in term premium estimates coincide with global

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[21] Unreported results, using the analytical bias correction proposed by Pope (1990) - which is accurate up to first order - produces similar term premium estimates.

macroeconomic events.

The results of this study can be extended in several directions. Including macro variables to test the spanning hypothesis could shed additional light on determinants of term premia and bond excess returns in the NGB market. Furthermore, spill-over analysis from global yields might reveal determinants specific to NGB term premia. From a technical perspective, relaxing the risk factor stationarity assumptions embedded in all term structure models we are aware of - including the models applied in this study - has potential to increase our understanding of yield curve dynamics. Machine learning methods, which have capability to deal with complex non-linear dynamics, appear particularly appealing in this context. The same applies to Bayesian VARs in state-space form. We leave this for future research.

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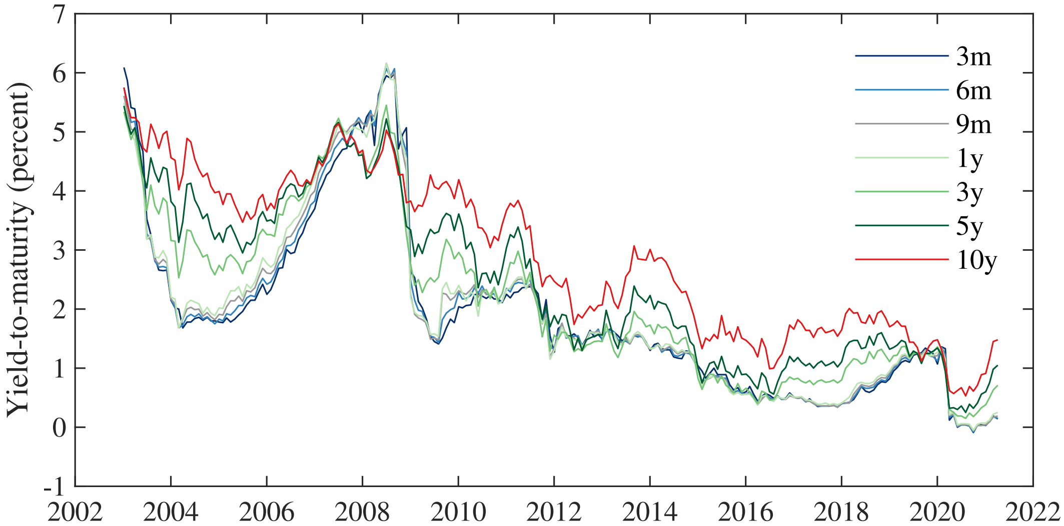
7 Tables and figures

Figure 1: **NGB yield-to-maturity**



Constant maturity yields on outstanding Treasury bills and bonds, as reported by Norges Bank. Sample period: 2003/01 to 2021/04.

Table 1: **Descriptive statistics of zero coupon yields**

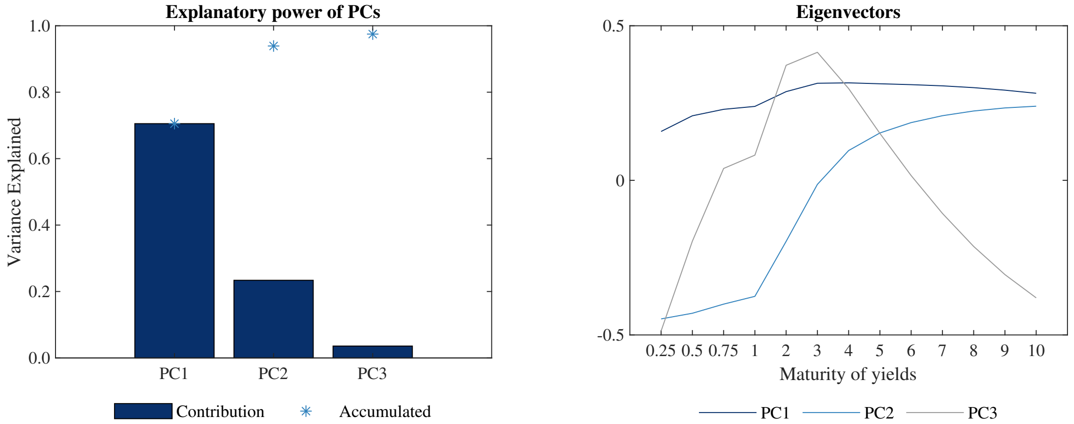
	Mean	Std.dev.	Min	Max	ρ_{1m}[1]	ρ_{1y}[2]
3m	1.92	1.47	(0.09)	6.09	0.96	0.53
6m	1.94	1.48	(0.08)	6.07	0.97	0.56
9m	1.96	1.49	(0.06)	6.16	0.97	0.58
1y	1.99	1.49	(0.05)	6.16	0.97	0.60
3y	2.20	1.41	0.15	5.45	0.97	0.71
5y	2.46	1.40	0.25	5.44	0.98	0.75
10y	2.92	1.34	0.53	5.75	0.98	0.75

Zero coupon yields as estimated by the authors (see section 3.) Negative numbers in parenthesis. Sample period: 2003/01 to 2021/04.

[1] ρ_{1m}: autocorrelations at 1 month lags.

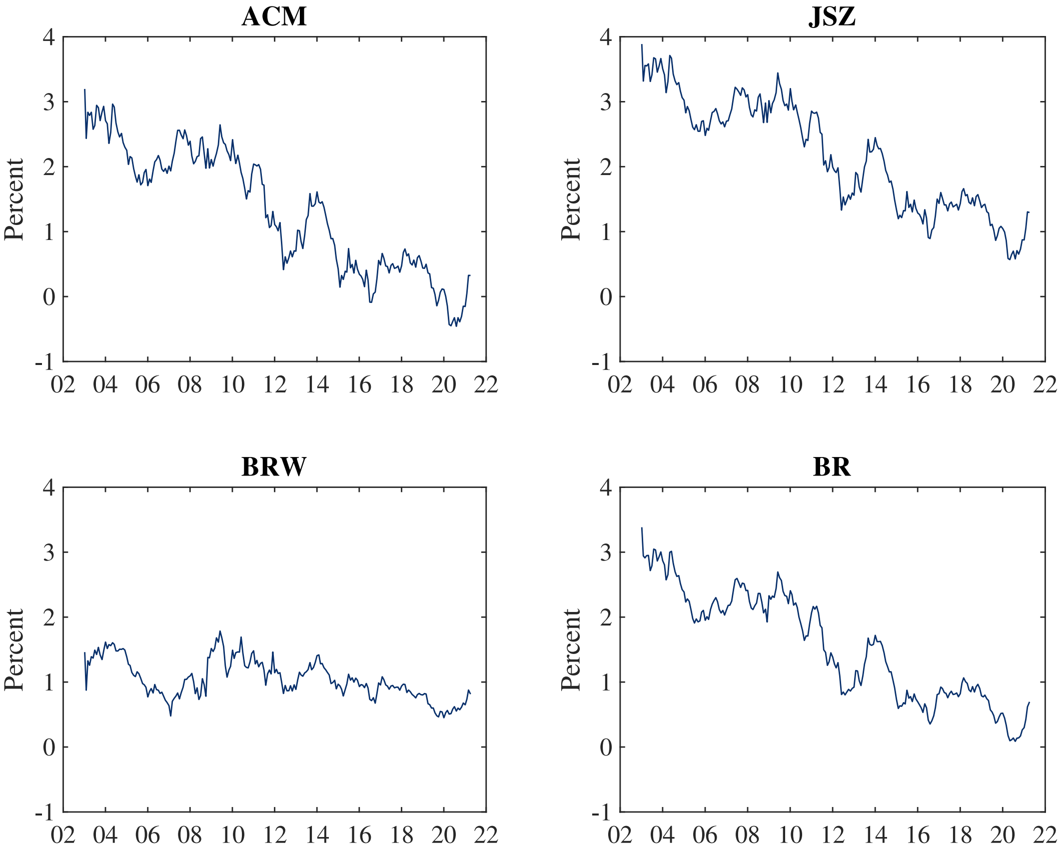
[2] ρ_{1y}: autocorrelations at 1 year lags.

Figure 2: **Principal Component Analysis**



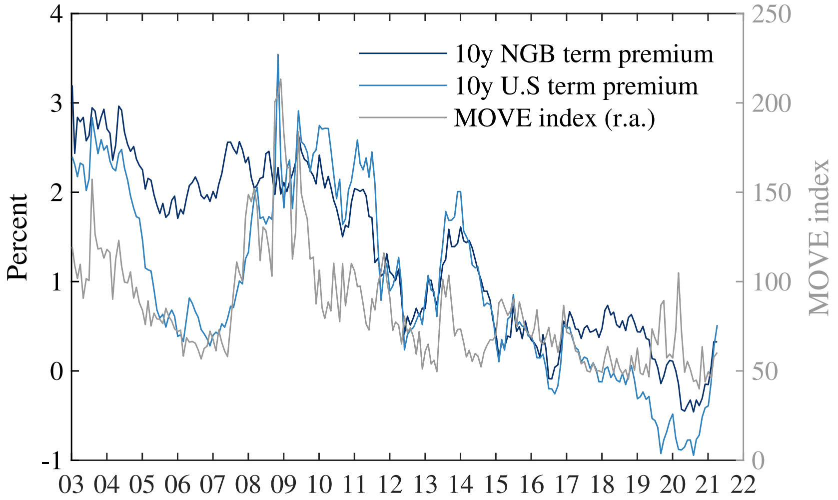
PCA of estimated zero-coupon yields as estimated by by the authors. Left panel: Contribution from each PC. Right panel: Shape of eigenvectors. Sample period: 2003/01 to 2021/04.

Figure 3: **10y term premium estimates**



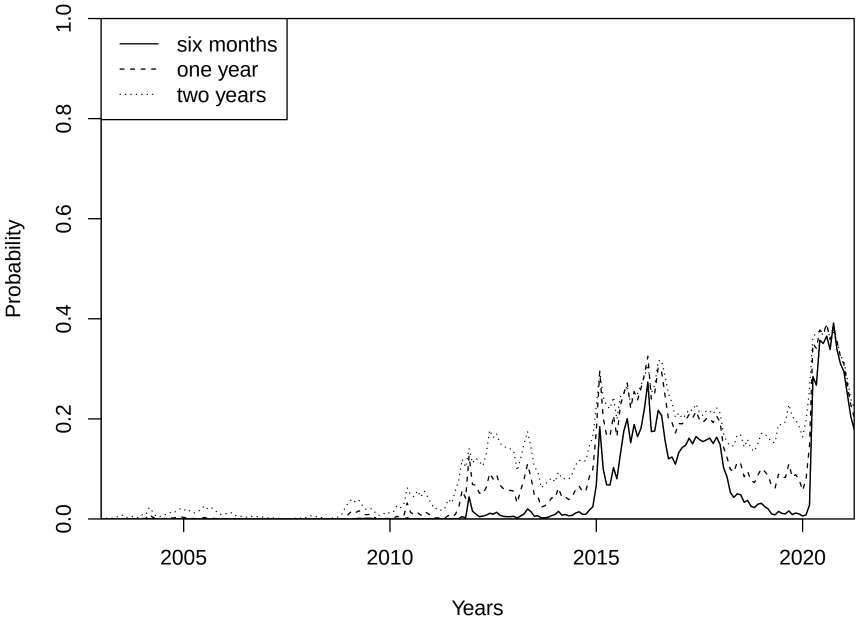
10y term premia estimates from ACM (Adrian et al. 2013), JSZ (Joslin et al. 2011), BRW (Bauer et al. 2012) and BR (Bauer & Rudebusch 2016). Sample period: 2003/01 to 2021/04.

Figure 4: **ACM term premia vs. MOVE**



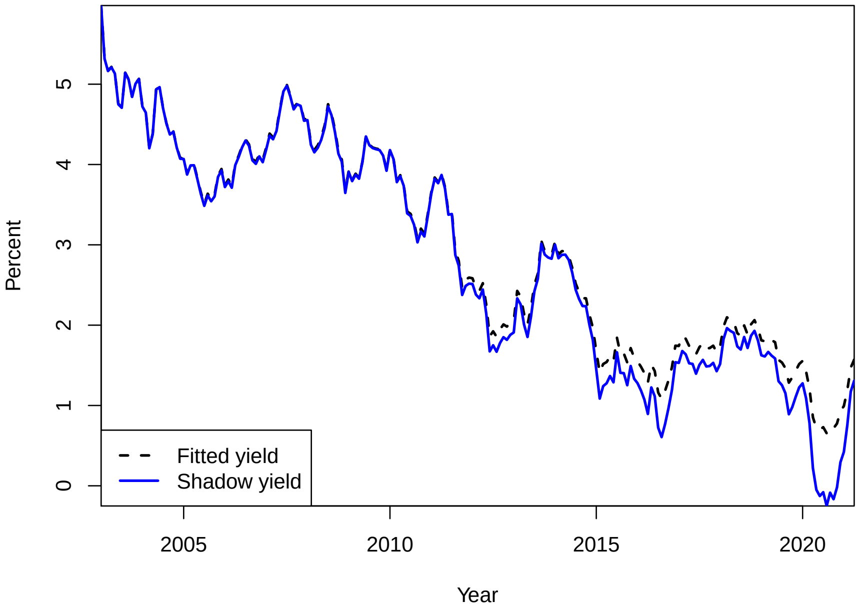
ACM 10y term premia (left axis) vs MOVE (right axis). NGB estimates by the authors, U.S estimates as reported by the Federal Reserve Bank of New York, MOVE Index downloaded from Bloomberg. Sample period: 2003/01 to 2021/04.

Figure 5: **JSZ-implied probabilities of negative future short rates**



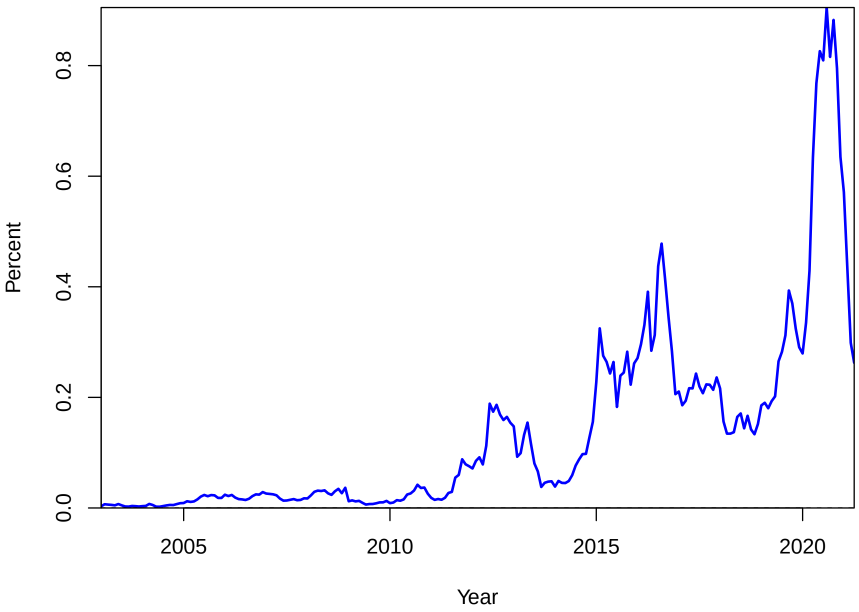
JSZ-implied real-world $\mathbb{P}$ probabilities of negative future short-term interest rates at horizons of 6 months, 1 year, and 2 years. Sample period: 2003/01 to 2021/04.

Figure 6: **Fitted yields vs shadow yields (BR model)**



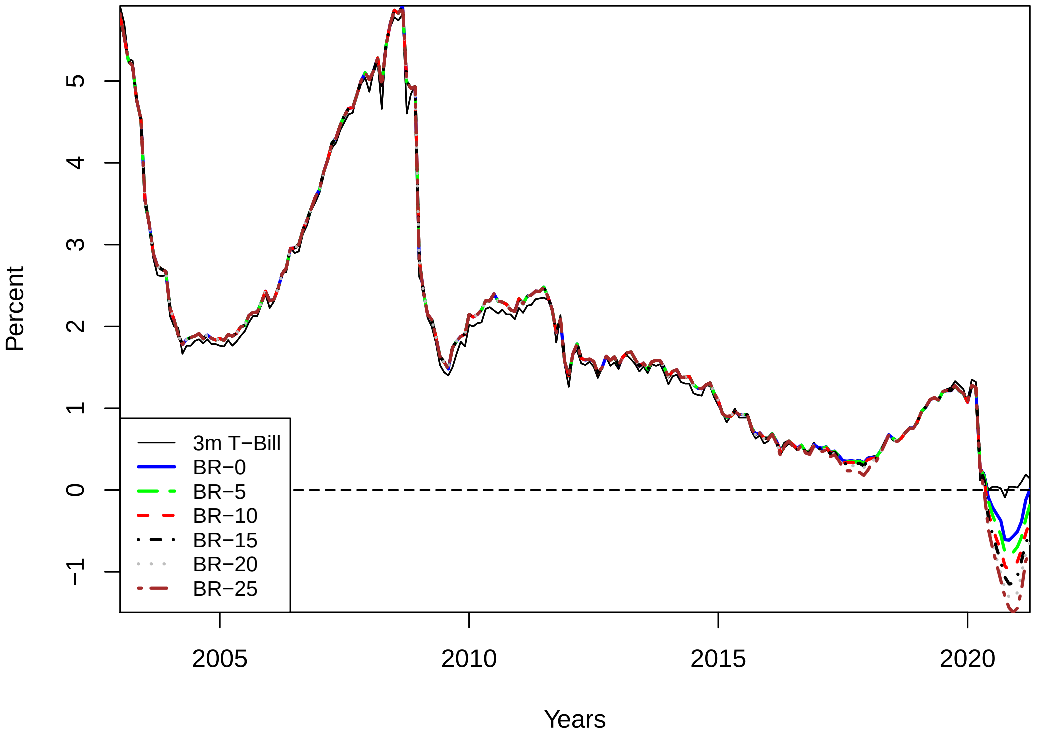
The fitted 10-year BR yield and the corresponding shadow yield. Sample period: 2003/01 to 2021/04.

Figure 7: **ZLB wedge**



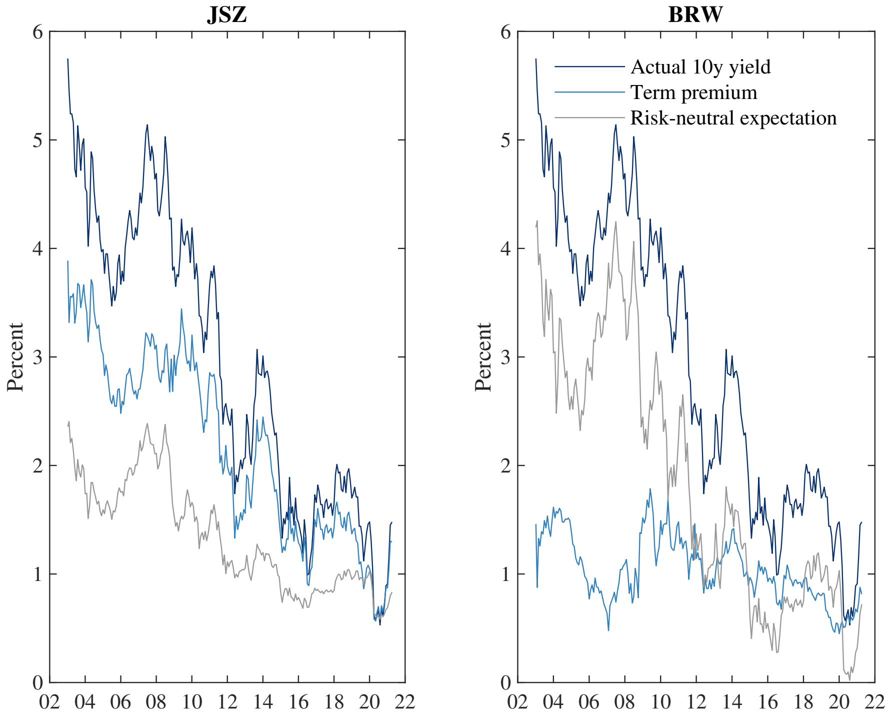
The difference between the fitted 10-year BR yield and the corresponding shadow yield. Sample period: 2003/01 to 2021/04.

Figure 8: **Sensitivity of shadow rates to lower bound**



Shadow short rates implied by the BR model and the three-month T-bill rate. Each panel shows estimated shadow short rates for the lower bound r_{min} varying from 0 to 25 basis points. Sample period: 2003/01 to 2021/04.

Figure 9: **Impact of small sample bias**



The effect of bias-correction (BRW - right panel) on the decomposition of yields into term premia and risk-neutral expectations, compared to OLS estimation (JSZ - left panel). Sample period: 2003/01 to 2021/04.

Appendices

A Appendices

A.1 Application of Nelson-Siegel-Svensson to NGB yields

The Nelson-Siegel-Svensson (NSS) method, proposed by Nelson & Siegel (1987) and developed by Svensson (1994), assigns a parametric functional form to forward rates;

$$f(m) = \beta_{0t} + \beta_{1t} e^{\left(\frac{-m}{\tau_1}\right)} + \beta_{2t} \frac{m}{\tau_1} e^{\left(\frac{-m}{\tau_1}\right)} + \beta_{3t} \frac{m}{\tau_2} e^{\left(\frac{-m}{\tau_2}\right)}, \tag{13}$$

where $f(m)$ is the instantaneous forward rate for maturity m and $[\beta_0, \beta_1, \beta_2, \beta_3, \tau_1, \tau_2]$ are parameters to be estimated at each t.

Given estimates for the parameter vector, zero coupon rates can be inferred by integrating (14) over time to maturity:

$$y(m) = \beta_{0t} + \beta_{1t} \frac{1 - e^{\left(\frac{-m}{\tau_1}\right)}}{\frac{m}{\tau_1}} + \beta_{2t} \left(\frac{1 - e^{\left(\frac{-m}{\tau_1}\right)}}{\frac{m}{\tau_1}} - e^{\left(\frac{-m}{\tau_1}\right)} \right) + \beta_{3t} \left(\frac{1 - e^{\left(\frac{-m}{\tau_2}\right)}}{\frac{m}{\tau_2}} - e^{\left(\frac{-m}{\tau_2}\right)} \right) \tag{14}$$

where $y(m)$ is the continuously compounded yield for maturity m.

Wahlstrøm et al. (2022) provide a discussion of optimal estimation of NSS models. They point out that the NSS factors have economic interpretations as level, slope and curvature factors respectively, and thus argue that their parameter estimates (β_0, β_1 and $\beta_{2:3}$) should be reasonably stable across time. Furthermore, they highlight that the two curvature factors often display high negative correlation and suggest that one is superfluous. More specifically, this high and negative correlation might lead to joint spurious changes in parameter estimates. To balance model flexibility and spurious fitting to

A.1: **Parameter restrictions in non-linear optimization**

	β_0	β_1	β_2	β_3	τ_1	τ_2
Upper bound	B1+[1]	0.1	1	2	3	1
Lower bound	B1-[2]	-1	-1	-1	0.0001	0.0001
Starting value	B1*[3]	S0-[4]	0	0.5	1	0.5

Yields as estimated by by the authors, 2003/01-2021/04

[1] B1+ : Yield-to-maturity of 10y NGB plus 200 basispoints

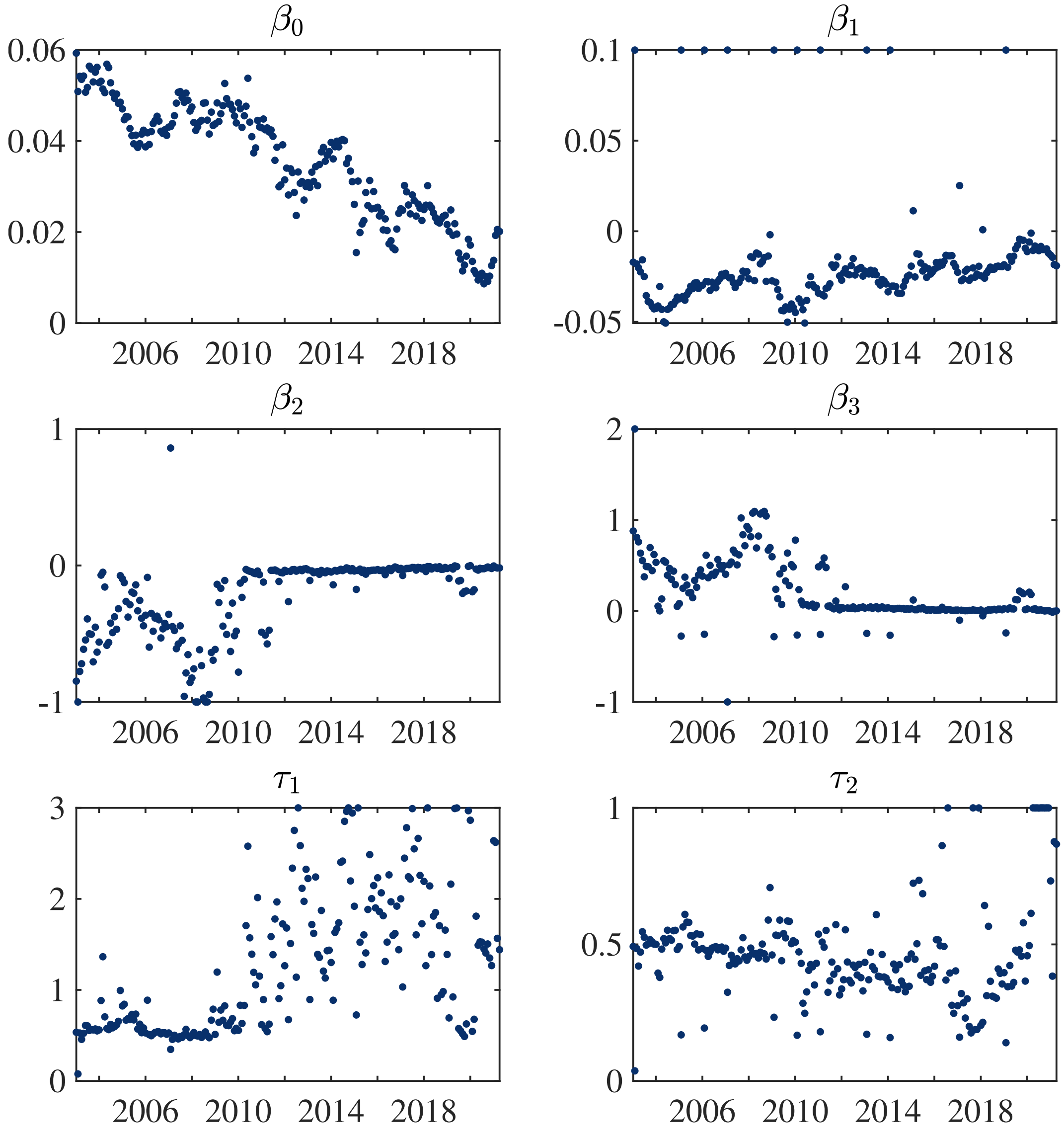
[2] B1- : The lower of 1 basispoint and the yield-to-maturity of 10y NGB minus 200 basispoints

[3] B1* : Average of 3/5/10y NGB yield-to-maturities

[4] S0- : Yield-to-maturity of 10y NGB minus B1*

noise, we run a trial and error procedure. The resulting starting values and bounds for parameter estimates are reported in Table 1. Parameter estimates are displayed in Figure 10.

A.10: **NSS parameter estimates**



Nelson-Siegel-Svensson parameter estimates. We estimate the NSS model by non-linear optimization in MATLAB.