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Whiteboards as a problem-solving tool

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Previous research shows there is more discussion, participation and persistence when pupils work on vertical whiteboards. In this study, we investigate how neighbouring whiteboards support two pupils solving problems in mathematics. We analyse classroom observations using characteristics of "thinking classrooms" to explore the pupils' work on whiteboards, examining what they notice, what they use from other pupils' whiteboards and how they process this in their work. The findings indicate that whiteboards give pupils opportunities to a) discuss different elements of the task, b) support each other during the problem-solving process and c) work independently to find knowledge in interaction with classmates and classmates' work.

Keywords: Problem solving, vertical whiteboards, communication, thinking classroom

Introduction

Problem solving has long been considered an important element in both teaching and learning of mathematics. This has had an impact on mathematics curricula around the world where the aim is that pupils learn to solve problems and learn through solving problems (Liljedahl, Santos-Trigo, Malaspina, & Bruder, 2016). Problem solving is becoming a key part of teaching in Norwegian schools as it is one of the core elements in the new mathematics curriculum (Kunnskapsdepartementet, 2018).

The new mathematics curriculum will place demands on developing a classroom culture where communication will have an important focus. Pupils will interact verbally, and think creatively and productively together (Littleton & Mercer, 2013). Important elements of mathematical conversations include explaining, arguing and defending mathematical ideas (Walshaw & Anthony, 2008). These components may be closely linked to new core elements in mathematics, such as communication, reasoning, argumentation, abstraction and generalisation.

In the 1990s Wells started to use whiteboards in physics classes with the aim of strengthening and developing discussion among the pupils (Wells, Hestenes, & Swackhamer, 1995). Whiteboards, which had already been used in some mathematics courses at universities in Australia in the mid-1970s (Forrester, Sandison, & Denny, 2017), can support the development of understanding concepts, the use of multi-representations and the development of academic discussions in the classroom (Wenning, 2005). Reports on the use of whiteboards have so far referred to using them lying on the pupils' desks.

It appears that whiteboards had their renaissance in 2016. Liljedahl (2016) used vertical whiteboards to change a classroom culture from what he calls a non-thinking classroom into a thinking classroom. He finds that pupils get down to work quicker, have better persistence, are more eager and collaborate better when working with vertical whiteboards. Megowan-Romanowicz (2016) shows that whiteboards make it possible to change the focus from finding the correct answer to creating meaning. Our study refers to pupils using vertical whiteboards.

Forrester et al. (2017) explore what pupils think about working on whiteboards and present research on how whiteboards are used. The pupils report that they have more motivation and a higher activity level in the subject, even if a few of them state that they are uncomfortable when their work is visible to all the others. When Liljedahl (2016) uses the vertical whiteboards to change a classroom into a thinking classroom, he rearranges the room. The pupils` working space shifts from sitting at their desks to standing and working on vertical whiteboards. By making this change, Liljedahl discovers that students work more perseveringly, they start working more quickly, they participate more actively in mathematical activities and there is greater mobility of knowledge in the classroom. The focus of this study is on how the pupils use the information from other whiteboards. We will look into what kind of information they use and how they use this information in their work. This leads to our research question: *How can (neighbouring) whiteboards contribute in pupils` mathematical problem-solving processes?*

This study is part of a larger research project where the aim has been to explore how pupils and teachers benefit from working on vertical whiteboards. The focus of this study is to examine how two pupils, in collaboration, use input from other pupils' whiteboards in their work. To answer our research question, we will analyse and discuss episodes from pupils' work on whiteboards, using characteristics of thinking classrooms. Pupil engagement is decisive in a thinking classroom, where proxies for engagement include: Time before pupils get started on their tasks, and time before the initial mathematical notation is made, pupil eagerness to get started, discussion, participation, persistence, non-linearity of pupil work and knowledge mobility between the groups (Liljedahl, 2016).

We have chosen to use four of the proxies in our analysis to help us understand the work process of pupils as a whole. The proxies we have excluded are related to minor parts of the work process. We use knowledge mobility, discussion, participation and persistence. Knowledge mobility means that pupils construe knowledge together, and this knowledge is dispersed amongst them in the classroom. It also refers to the interactions of pupils across groups. Participation and discussion refer to the degree to which the group members take part while working on the tasks and in the discussions, and how they discuss the task in their group and between groups. Persistence refers to how long pupils work with a focus on the task, how they work with challenges without giving up and how they try out new approaches if they get stuck (Liljedahl, 2016). Pupils show resilience and rarely quit out of boredom and frustration (Liljedahl, 2018). We use descriptions of the proxies for engagement to describe the students' problem-solving process.

Methodology

The term whiteboard is used about different types of non-permanent writing surfaces in the research literature. In our study, whiteboards are large (approximately A2 size) static electric surfaces hanging on the wall with similar functions as other whiteboards. It is easy to write, erase and write over again. Due to their static-electricity property, they can be attached to walls and windows, and they can easily be moved.

Data collection

The data material for this study comprises video and audio recordings of one teaching session, lasting around 90 minutes. Pupils in Year 7 (12-13 years old), worked with the task *Talltårn* (Number towers) on whiteboards. We were responsible for the data collection. As we both are mathematics teachers, Kristin, the teacher, wanted us to participate in the teaching while collecting the data. We have in part been participants-as-observers in work with the task (Gold, 1958). The pupils were informed that we might ask them questions and have brief conversations with them. The video material was transcribed and together with the audio material recorded into a chart with columns for the time, what was happening, what was said and what was written.

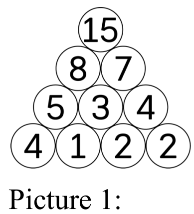
The given task was selected in collaboration with the mathematics teacher, Kristin (all names are fictitious), and us. Kristin planned and carried out the teaching with input from us. The teacher and her class have thus been selected through purposeful sampling (Creswell & Poth, 2018). We wanted to examine pupils who were accustomed to cooperating on mathematical problem solving, and to collaborate with a teacher who wanted to develop her teaching practice. We also wanted to focus on pupils who were able to cooperate and verbalise how they were thinking, and who would not be too shy in front of the camera. As Kristin knew the pupils best, she chose two *focus pupils*, Mari and Johan, before the teaching session. According to Kristin, Mari and Johan had varying mathematical competence. Kristin picked randomly collaboration partners for all the pupils. Mari and Hanne formed one *focus pair* and Johan and Colin another.

Kristin presented the task orally, formed random collaboration pairs and the pupils started working with the task. After a while, Kristin stopped and asked them to go to their neighbouring group. The pupils were to look at how this group had worked, comparing solution strategies and solutions with their own work. They wrote similarities on a green slip and differences on a pink slip. After some minutes at the neighbouring whiteboard, the pupils returned to their whiteboard and continued with the task. This step in the work process differs from Liljedahl`s (2016) work. We included this step to encourage the pupils to use the neighbouring whiteboards. We monitored each our focus pair with a video camera and audio recorder throughout the lesson.

This paper explores one teaching session, the second in a series of five. The pupils have thus worked on whiteboards earlier, are familiar with the tool and know that while working they should compare what they are doing with what their neighbours are doing. The pupils are working with *Number towers*, a task with relatively many solutions and two main strategies.

The task

The task *Number towers* has been derived from mattelist.no (Norwegian Centre for Mathematics Education – Matematikksenteret, 2019). The task is introduced by starting with four whole positive numbers on the ground floor (zero cannot be used). Two and two neighbouring numbers are added, and the sum is placed immediately above the two numbers on the next floor. The same procedure is repeated until there is one number at the top (see Picture 1).



Picture 1: Number tower example

The task is to choose four start-numbers which will give the number 15 at the top. The questions the pupils must answer along the way are: "How many solutions can you find?" and "Can you find systems that can help you in your work?". There are two main strategies for solving the task. The pupils may either choose four start-numbers to arrive at 15 or start with 15 at the top and work down. If pupils start with 15 at the top, they may make discoveries that can help them find additional solutions. They may find, for example, that they cannot have all possible subdivisions of 15 on the third floor, and that 1 can only be placed on the ground floor. Another discovery is that the number in the middle of the first floor is used twice to form the numbers on the second floor. For that reason, the highest number must be placed on the outermost side of the floor. Pupils who start from the top may also find that four start-numbers in different orders will yield different *number towers*.

Pupils starting on the ground floor will be more dependent on a trial and error strategy and must adjust their start numbers when they find numbers that do not yield 15 on top. They may make the same discoveries as those who start at the top, but these findings are not as obvious. Different premises for the task lead to several different solutions, such as that 4-2-1-2 and 2-2-1-4 may be considered two different sets of start numbers, or they may be viewed as the same four start-numbers and give one solution.

In this paper, we have chosen to study how pupils work with number towers because the task can be solved using two main strategies, the task has several solutions and the pupils have the opportunity to make different discoveries while working.

The analysis

To identify episodes in the data material where the pupils used their neighbouring whiteboards as tools we used open coding with the constant comparative analysis method (Postholm, 2019). In this section of the analysis we found two main categories relating to pupils` use of neighbouring whiteboards, either encouraged by the teacher or based on their own need. After completing this step of the analysis, we decided to focus on Colin and Johan's work on the task. The main reason for this choice was that the boys tried both strategies, one in the initial phase of their work, and the other after having studied their neighbours' whiteboard. The episodes during which the boys used the neighbouring whiteboard in some way, were analysed in more detail to interpret and describe how whiteboards supported the pupils' work on solving mathematical problems. We studied what the pupils talked about when looking at other whiteboards, what they saw and how they applied this in their subsequent work.

To increase the credibility of the analysis, we have watched the video material, transcribed, read, noted and commented on the material separately. Where we have disagreed, we have discussed together to arrive at a shared understanding of what takes place in the different episodes. We have examined the data material over and over, reflected and remained sceptical to our first impressions during the analysis process (Stake, 1995).

Findings and discussion

We will now present excerpts from the dialogue between Colin and Johan, where we find the characteristics of thinking classrooms. We present our findings according to the three chosen proxies for engagement: participation and discussion, persistence and knowledge mobility (Liljedahl, 2016).

Participation and discussion

In the following sequence, the pupils study the whiteboard of the neighbouring group, and they are to note down what is similar between their work and the work of the neighbouring group on a green slip, and differences are to be written on a pink slip.

At the neighbouring whiteboard, the following takes place:

The boys write "They work from the top" on the pink slip and "they have found the same solutions as we have" on the green slip. Here is the subsequent conversation between the pupils and Ingunn, the teacher/researcher:

1 Ingunn: What do you think about what this group has done?

2 Johan: It looks right.

3 Colin: It's a bit special that they have started from the top.

4 Ingunn: Do you think that you could have used this strategy?

5 Johan: Yes, because then they'll certainly find what could become 15, what can become eight and what can become 7. It's pretty smart.

6 Colin: They take, like, they have like 15, which is the answer. They don't make any mistakes.

7 Ingunn: Could you look at the first and second floors, whether there is something they have found out?

8 Johan: They have used 7 and 8 quite a lot.

9 Colin: They have used 10, 0 was not allowed.

10 Johan: But that was on the ground floor. We have also used 10.

11 Ingunn: Do you think that they have found all the solutions with 7 and 8?

12 Johan: No. We have just as many correct ones as they have.

According to Liljedahl (2016), this excerpt from the work session shows that both boys take an equal part in the discussion and that they find support in the problem-solving process by looking at the whiteboard of their neighbouring group. Ingunn supports them in this process of investigating the neighbouring whiteboard (1), (4), (7), (11), inviting the boys to consider the work of the neighbouring group. The boys assess the strategy of starting from the top, and it appears that they consider this strategy with curiosity and scepticism. They also see that the strategy has a strong side to it; they cannot find any number tower which will not result in 15 on the top floor. Using this strategy, they will end up with four start numbers that will guarantee 15 at the top (5).

Knowledge mobility

We identify two different types of knowledge mobility in Colin and Johan's work (Liljedahl, 2016). One type comes about when the pupils are instructed by the teacher to study the whiteboard of their neighbouring group and then attempt to test the strategy this group has used:

1 Johan: Should we start with this, then (pointing to the pink slip)? Then we have to write 15. Should we try something we don't have (on the second floor), 3 and 12? No, 3 doesn't

work because 1 can’t be there (points to the first floor). What is max on the second floor is 4 and 11. 1 can only be at the bottom, because 0 isn’t allowed, and then you can’t divide 1 (if it is on the first floor).

Johan replaces 3 and 12 with 4 and 11 on the second floor. On the floor below he enters 2-2-9.

2 Colin: But we already have this (points to a solution on the whiteboard).

In this excerpt, Colin and Johan test the strategy of starting at the top of the number tower. They make some discoveries. They see that 3 and 12 cannot be on the second floor because that gives 1 on the first floor. Here it appears that the pupils are using information from the neighbouring whiteboard that they did not comment on while studying the neighbouring group’s work. The neighbouring whiteboard said: “Cannot have 1 on the first floor”.

The pupils use what they have seen but not discussed. Johan states (1) that 1 cannot be on the first floor. The boys do not dwell on this at all, and it may thus appear that they have noticed the information given on the neighbouring whiteboard, using it now when they see that it provides meaning. This information is important if one starts from the top of the number tower and works downward, but since the boys always have started from the bottom, they have had no need for this information. They have never had 1 on the first floor.

We see another form of knowledge mobility further into the work session. The pupils are struggling to make progress in their work. On their own initiative, they turn away from their whiteboard and look at other whiteboards in the classroom:

1 Johan: I’m not sure whether there are more (solutions).

Colin turns to a whiteboard on the other side of the classroom.

2 Colin: OMG, they have a lot of solutions, really, take a look. They have a lot!

3 Johan: But the blue ones are wrong!

4 Colin: Is it allowed just to switch the placement of the bottom numbers (addressing Ingunn)?

5 Ingunn: Look how the number towers will look like then. Try it!

Colin tries an example.

In the excerpt above the boys are looking for confirmation that they have found all the solutions. Turning around, they look at the whiteboards of classmates and find that one of the groups has found many more solutions than they have. Discovering this, they start comparing the different solutions, seeing that they are not working according to the same premises as the other group. The other group has interpreted 4-2-1-2 and 2-2-1-4 as two different sets of start numbers, while Colin and Johan have deemed them to be the same start numbers (4).

We see that knowledge mobility helps the pupils to be more independent in the problem-solving process as they find knowledge in interaction with classmates and in the work of their classmates. If the boys had been working in their workbook, they might have asked the teacher whether they had finished instead of finding out for themselves. By studying, interpreting and discussing the work of other pupils, they acquire the opportunity to construe knowledge together (Liljedahl, 2016). They look for tips, ideas, strategies and procedures that may be effective, and which may contribute to their problem-solving process.

Persistence

Throughout their work, Colin and Johan are on the brink of giving up or running out of ideas several times, but they keep finding new numbers they can try. When one of them states that he is giving up, the other one offers new ideas:

1 Colin: Nothing, absolutely nothing. It doesn't work so well when we've come this far.

2 Johan: What if we try 3 and 6 there now, (pointing to the ground floor of a number tower on the whiteboard). I'm not saying that it'll work. 3+1 is 4, 2-7, 6-9, 15. Like that!

Later in the session:

3 Johan: I don't know. I'm really out of ideas.

4 Colin: What if we start at the top, and then do it with seven?

5 Johan: Let's try it. This is the easiest one.

Towards the end of the work session:
The pupils carry on working for some time using their initial strategy. After around ten minutes' work they are becoming uncertain as to whether there are more solutions. Colin turns towards a whiteboard on the opposite side of the classroom.

6 Colin: OMG, they have a lot of solutions, really, take a look. They have a lot!

According to Liljedahl (2018), Colin and Johan do not quit even though they are frustrated. They help each other to remain in the problem-solving process by offering several challenges (2), (4), and they use their neighbouring whiteboards as inspiration and guides in terms of whether they can claim to have finished the work (6). These findings correspond to Liljedahl's (2016) description of persistence. The pupils work on the task for 45 minutes.

Concluding remarks

The analysis of the work performed by Colin and Johan indicates that whiteboards contribute to their problem-solving process in different ways. They use the neighbouring whiteboards as inspiration; they find ideas and compare the number of solutions with their findings. In line with Liljedahl (2016), we find that whiteboards support the problem-solving process by opening for knowledge mobility.

The boys are highly active and participate in their problem-solving process. Ingunn supports them in using the neighbouring whiteboard by asking questions to help them specify what they are looking for. The boys discuss the strengths and weaknesses of various strategies and premises for the task, show persistence throughout the work session, do not give up and try again if they get stuck. According to Liljedahl (2016) and Megowan-Romanowicz (2016), Colin and Johan used their classmates' work as a guide for their problem-solving process. These findings indicate that whiteboards may support a teacher's work with problem solving and communication in mathematics classrooms as also found by Wells et al. (1995).

References

Creswell, J. W., & Poth, C. N. (2018). *Qualitative inquiry and research design: choosing among five traditions*. Thousand Oaks: Sage Publications.

Forrester, T., Sandison, C. E., & Denny, S. (2017). Vertical whiteboarding: Riding the wave of student activity in a mathematics classroom. *The Australian Mathematics Teacher, 73*(4), 3–9.

Gold, R. L. (1958). Roles in sociological field observations. *Social Forces, 36*(3), 217–223.

Kunnskapsdepartementet (2018). Fornyer innholdet i skolen. Retrieved from https://www.regjeringen.no/no/aktuelt/fornyer-innholdet-i-skolen/id2606028/?expand=factbox2606064

Liljedahl, P. (2016). Building thinking classrooms: Conditions for problem-solving. In P. Felmer, J. Kilpatrick, & E. Pekhonen (Eds.), *Posing and Solving Mathematical Problems: Advances and New Perspectives* (pp. 361–386). New York: Springer.

Liljedahl, P. (2018). On the edges of flow: Student problem-solving behavior. In S. Carreira, N. Amado, & K. Jones (Eds.), *Broadening the scope of research on mathematical problem solving: A focus on technology, creativity and affect* (pp. 505–524). New York: Springer.

Liljedahl, P., Santos-Trigo, M., Malaspina, U., & Bruder, R. (2016). Problem solving in mathematics education. In G. Kaiser (Ed.), *Problem Solving in Mathematics Education. ICME-13 Topical Surveys* (pp. 1–39). Hamburg: Springer Open.

Littleton, K., & Mercer, N. (2013). *Interthinking: Putting talk to work*. Abingdon: Routledge.

Megowan-Romanowicz, C. (2016). Whiteboarding: A tool for moving classroom discourse from answer-making to sense-making. *The Physics Teacher, 54*(2), 83–86.

Postholm, M. B. (2019). *Research and development in school: Grounded in cultural historical activity theory*. Leiden: Brill | Sense.

Stake, R. E. (1995). *The art of case study research*. Thousand Oaks: Sage Publications.

Walshaw, M., & Anthony, G. (2008). The teacher's role in classroom discourse: A review of recent research into mathematics classrooms. *Review of Educational Research, 78*(3), 516–551.

Wells, M., Hestenes, D., & Swackhamer, G. (1995). A modeling method for high school physics instruction. *American Journal of Physics, 63*(7), 606–619.

Wenning, C. J. (2005). Whiteboarding and Socratic dialogues: Questions and answers. *Journal of Physics Teacher Education Online, 3*(1), 3–10.