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Innovative Applications of O.R.

Transmission investment under uncertainty: Reconciling private and public incentives[☆]

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ABSTRACT

Private companies (PCs) in restructured electricity industries determine facility investment timing and sizing. Such decisions maximize the PC's expected profit (rather than social welfare) under uncertainty. By anticipating the PC's incentives, a welfare-maximizing transmission system operator (TSO) shapes the network to align public and private objectives. Via an option-based approach, we first quantify welfare losses from the PC's and TSO's conflicting objectives. We show that by anticipating the optimal timing and capacity decisions of the profit-maximizing PC, the TSO is able to reduce, though not eliminate, welfare loss. Next, we exploit the dependence of the PC's capacity on the TSO's infrastructure design to devise a proactive transmission-investment strategy. Hence, we mitigate welfare losses arising from misaligned incentives even in relatively uncertain markets.

1. Introduction

Concern about climate change has prompted many OECD countries to set ambitious targets for adopting renewable energy (RE) technologies, such as solar and wind power.[1] Attaining these targets necessitates expansion of the transmission network because promising wind sites, for example, tend to be located remotely from population centers. However, the deregulated nature of the electricity industry means that decisions about transmission investment and generation adoption are made by separate entities with distinct and possibly conflicting objectives. In particular, generation capacity in RE technologies is installed by private, profit-maximizing power companies, whereas transmission investment is chiefly undertaken by state-regulated transmission system operators (TSOs), who are concerned about maximizing social welfare. Thus, although TSOs cannot directly intervene in electricity markets, they can, nevertheless, *orient* generation investment and operations by anticipating industry's incentives when building transmission capacity (Maurovich-Horvat, Boomsma, & Siddiqui, 2015; Sauma & Oren, 2007; Siddiqui, Tanaka, & Chen, 2019). Another complicating factor is that these are investment decisions under uncertainty about future market conditions. Consequently, decision makers have the discretion to defer adoption and to scale the capacity (Dangl, 1999).

In this article, we aim to analyze the potential of anticipative transmission planning when undertaking large infrastructure investments. More specifically, we focus on the investment decisions in a dynamic game between the private power generation company (PC) and the regulated transmission infrastructure provider (TSO) in a market characterized by uncertain demand.[2] In this game, the TSO moves first by providing transmission infrastructure,

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[1] For example, the EU has 2030 targets to reduce CO_2 emissions by at least 40% and to reach a share of at least 32% RE, compared to 1990 levels (European Union, 2018). The Nordic countries in particular want to lead by example and to commit to carbon neutrality (Regjeringen, 2019). The individual countries have ambitious climate policies such as the Swedish target of net-zero CO_2 emissions by 2045 (Regeringskansliet, 2017) and the Danish commitment to 100% RE use by 2050 (Energistyrelsen, 2015).

[2] Other examples of infrastructure investments with similar coordination problems are ports, telecommunication networks, and other utilities such as water or sewage.

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anticipating subsequent investment in generation capacity undertaken by the PC. We seek to uncover how conflicting objectives between a profit-maximizing PC and a welfare-maximizing TSO affect social welfare through their decisions regarding investment timing and size. The stylized nature of our model allows us to obtain closed-form solutions and, at the same time, capture the relevant tradeoffs involved in the investment decisions. This enables us to trace the nexus through which decisions taken by the private PC are affected by the TSO's transmission-capacity and timing decisions. We analyze the resulting welfare losses and investigate how conflicting objectives may be reconciled through the use of the infrastructure capacity and investment timing.

There exist several real-world investment cases that emphasize the pivotal role of transmission planning in the energy transition. One example is the prospect of building Europe's largest onshore wind park at Fosen in the county of Trøndelag in the central part of Norway based on a concession received in 2013. The realization of this wind park project was dependent on an addition to Norway's transmission grid amounting to an estimated cost of approximately NOK 6 billion by Norway's TSO, Statnett.[3] Statnett had promised in advance to extend the transmission grid so that new wind capacity could be connected. In order to achieve this outcome, Statnett was effectively depending on investment decisions for new generation capacity that are at the discretion of private companies.[4]

Many regions worldwide currently face similar challenges, where significant transmission investments are required to connect new renewable capacity to the main grid. For example, in Germany, where significant investments in new transmission lines are required to connect new wind farms in the north of the country to the more densely populated regions in the south where industry is concentrated (Kunz, 2013). In Northeast Asia, there have been several proposals to invest in new power grid interconnections between demand centers in the region and the wind and solar resources in Mongolia, as well as hydropower in Eastern Russia (Otsuki, Isa, & Samuelson, 2016).

Texas provides a cautionary tale about coordination of transmission investment with wind-farm expansion, where non-anticipative transmission planning has led to severe consequences for the electricity industry. Although Texas enjoys some of the highest on-shore wind speeds in the U.S. and has more installed wind capacity than any other state, most of the promising wind sites are in the sparsely populated northwest of the state. Due primarily to transmission constraints, these sites were unable to deliver electricity to the state's population centers and had to curtail 17% of electricity generated from wind in 2009. However, after construction of the necessary transmission capacity along with other market reforms, the curtailment rate declined to 0.5% by 2014 even as wind capacity continued to increase (Wiser et al., 2015). Hence, effective mechanisms to coordinate transmission and wind-power investments in a deregulated electricity industry are crucial to meeting environmental targets in a cost-effective manner.

Given this background, we address the following research questions. First, we investigate if market volatility prevents the TSO from enforcing the socially optimal transmission and generation investments. We find that although the TSO can naturally enforce the socially optimal outcome under certain market conditions, it cannot steer the PC toward the social optimum in a more uncertain environment. This is particularly relevant in the context of energy transition, as an increasing share of intermittent energy sources contributes to more volatile prices in electricity markets. Given that the investment valuation tools applied in practice are to a large extent static in nature, the absence of models that properly account for uncertainty in transmission planning may lead to sub-optimal investment decisions.

Second, we seek to quantify the welfare loss vis-à-vis a socially optimal integrated benchmark setting, as well as a setting in which the TSO does not anticipate the optimal decisions of the PC. We show that the former case in which the TSO maximizes social welfare and anticipates the PC's decisions in a deregulated industry still results in substantial social losses vis-à-vis an integrated one, especially under high market volatility. Nevertheless, anticipating the optimal decisions of the PC can spare the TSO from overinvestment and, therefore, prevent welfare losses relative to a situation in which the TSO behaves in a non-anticipative manner. Thus, in industries characterized by volatile demand, such as the electricity sector, anticipating the behavior of PCs is imperative for mitigating welfare losses from misaligned incentives between distinct decision makers. This is particularly important as in practice, infrastructure planning is often done without accounting for the optimal reaction of the industry actors. This non-anticipative case reflects, for example, the reactive planner of Sauma & Oren (2007), i.e., the TSO makes transmission investments without accounting for industry's generation expansion. By ignoring the possibility to shape more efficient generation investment, such a non-anticipative planner induces a welfare loss. These insights allow us to uncover the value of anticipative transmission planning by identifying the cost of coordination failures and bring to light the extent of existing inefficiencies given specific market conditions.

Finally, we assess if particular mechanisms are able to mitigate the welfare loss in the decentralized setting. In particular, we consider a lump-sum investment cost subsidy and the minimum capacity requirement that was actually employed in the case of Fosen's transmission planning. Although a TSO's measures to nudge private decisions to the social optimum are limited, we find that both mechanisms allow the TSO to enforce the social optimum for a wider range of market uncertainty. However, the investment cost subsidy has a limited effect in very uncertain environments in that it does not substantially reduce the welfare loss that is present even in the anticipated deregulated setting. A minimum capacity restriction imposed on the PC, however, reduces the welfare loss substantially also for high volatility. In fact, we derive a condition depending on volatility and other market parameters under which it is possible for the TSO to incentivize the PC to invest in a welfare-enhancing manner.

The rest of the article is organized as follows. Section 2 presents the review of the relevant literature. In Section 3, we introduce the model setup. Section 4 formulates the decision-making problems and provides analytical solutions in the decentralized case, where the TSO is responsible for the transmission investment only, whereas the PC decides upon the generation-capacity investment. Subsequently, Section 5 focuses on the results and derives their welfare implications. Mechanisms available to the TSO that mitigate the welfare loss of the decentralized setting are explored in Section 6, whereas Section 7 concludes. Proofs of analytical results are in Appendix Appendix A.

2. Literature review

In general, large infrastructure projects are often characterized by high uncertainty, irreversibility, and the dependence on different actors with distinct objectives. In order to account for these features and the dynamic nature of the problem in the decision-making process, we use real options analysis (Dixit & Pindyck, 1994). Traditionally, the theory of real options determines the optimal time to invest in a given capacity and concludes that un-

[3] https://www.tu.no/artikler/kostnadssmell-for-fosen-linjen/346906

[4] https://www.statnett.no/vare-prosjekter/region-midt/afjord-surna/nyheter/forbereder-utbygging-av-namsos-storheia/

certainty generates a value of waiting. Subsequent contributions to the literature allow for optimal capacity sizing of the investment as well (Bar-Ilan & Strange, 1999; Dangl, 1999; Kouvelis & Tian, 2014). A general result is that for higher levels of uncertainty, the firm invests later but in a larger quantity. Due to the strategic aspects arising in the considered problem, game-theoretic articles are also relevant for our work (Besanko, Doraszelski, Lu, & Satterthwaite, 2010; Chevalier-Roignant, Flath, & Trigeorgis, 2019; Huisman & Kort, 2015). Our model is most closely related to Huisman & Kort (2015) who consider both an investment's timing and capacity choice under uncertainty in a duopoly framework. They find that the capacity level of a welfare-maximizing social planner is twice the level of a profit-maximizing monopolist and that both agents invest in a larger capacity when uncertainty increases. As in Huisman & Kort (2015), we consider two different agents. However, compared to Huisman & Kort (2015), the two agents are not competing for market share. Rather, they are dependent on each other's investment decision with regards to both timing and sizing. Still, one can argue that competition arises in the sense that they have conflicting objectives.

Our study is related to a vast body of economic literature that investigates the effect of regulation on the infrastructure sector, cf. Braeutigam & Panzar (1993) and Gilbert & Newbery (1994). Recent articles contributing to this field acknowledge the importance of uncertainty when analyzing capital investment under regulation (Broer & Zwart, 2013; Dobbs, 2004; Evans & Guthrie, 2012; Guthrie, 2006; Teisberg, 1993; Willems & Zwart, 2018). Most of these studies specifically focus on welfare effects in a game between the social planner and regulated monopoly by taking a real options approach. However, they typically neglect the interactions between regulated entities and deregulated private firms by assuming that the regulated monopolist not only provides infrastructure but also makes production decisions. In practice, this assumption does not hold given the deregulated nature of most infrastructure industries in OECD countries. In the example of the electricity industry, a government agency (the energy ministry) is responsible for policies, the TSO (the regulated monopoly) provides the necessary grid infrastructure, whereas the production decisions are made by private PCs. These decisions together shape the evolution of the power sector. Taking them into account is, therefore, vital for achieving socially desirable outcomes. In our model, we disregard the interaction between the energy ministry and regulated monopoly by assuming that the their objectives are perfectly aligned, i.e., the TSO strives to maximize social welfare. We do so in order to focus on another aspect of the infrastructure investment problem, viz., the interaction between regulated entities and private firms, and study the mechanisms available to align the decisions of private companies with socially desirable outcomes.

This assumption is also typical for the operations research literature that focuses on transmission investment in a setting where the TSO maximizes social welfare in a game with profit-maximizing power companies (Maurovich-Horvat et al., 2015; Siddiqui et al., 2019). These studies, however, disregard discretion over timing or even market uncertainty. Traditionally, transmission planning in the power sector was based on the assumption of a cost-minimizing central planner that could rely upon a single mixed-integer linear program (MILP) to make transmission- and/or generation-capacity investments (Garver, 1970). Such a framework was adequate for the regulated paradigm in which decisions were made largely on the basis of engineering considerations rather than the profit motive. Indeed, even electricity prices were set administratively, which meant that decision makers could pass on the risk of cost over-runs to ratepayers (Hyman, 2010). Yet, as Hobbs (1995) points out, single-agent, deterministic models to support decisions may not be adequate in a deregulated electricity industry. In the context of transmission planning, subsequent work has deployed equilibrium and bi-level approaches to handle game-theoretic aspects, e.g., between firms competing over a congested transmission line (Borenstein, Bushnell, & Stoft, 2000) or between a cost-minimizing TSO and price-taking industry (Garcés, Conejo, Garcia-Bertrand, & Romero, 2009).

Such optimization models adapted to deregulated industries have assessed the interaction between transmission and generation investment in a now-or-never setting. Typically, they use a Stackelberg leader-follower framework in which transmission investment occurs at an upper level with generation investment at a lower level (Sauma & Oren, 2006; 2007) in order to explore how market power in generation would affect socially optimal transmission plans. Baringo & Conejo (2012) focus on transmission and wind power investment at an upper level with uncertain market operations at the lower level in order to explain how a wind subsidy drives investment decisions. Seeking to quantify welfare losses from alternative market designs, Maurovich-Horvat et al. (2015) show that a transmission planner at the upper level induces more wind power investment by lower-level power companies when they behave à la Cournot because it anticipates their strategic withholding and invests in countervailing transmission lines. They further find that a renewable portfolio standard (RPS) would increase wind investment regardless of whether industry were perfectly competitive or a Cournot oligopoly. Nevertheless, although such work incorporates conflicting objectives and uncertainty in wind output, decisions are modeled for a target test year, i.e., there is no deferral option.

Siddiqui et al. (2019) take a more stylized approach to the same problem as in Maurovich-Horvat et al. (2015) in order to formalize how proactive transmission investment may mitigate both market power by producers at the lower level and environmental externalities from carbon emissions. They prove that even a perfectly competitive industry without a carbon charge will not deliver the same generation mix as a centrally planned one because the cost of damage from emissions will not be internalized. Instead, the TSO at the upper level will reduce the size of the line in order to curb overconsumption. A regulatory mechanism for full alignment of incentives under perfect competition is a 100% carbon charge on emissions by generators, which sends the correct signal to consumers to reduce quantity demanded. However, such a regulation is not successful when power companies at the lower level behave à la Cournot because they already invest in less capacity in order to increase electricity prices. Thus, imposing a full carbon charge on industry will only exacerbate the loss in social welfare as it will further enable their ability to raise prices.

In spite of its contributions, the fact that this strand of the literature ignores discretion over investment timing means that it overlooks an important component of the regulatory toolkit to incentivize welfare-enhancing investments. Recent work by Henao, Sauma, Reyes, & Gonzalez (2017) and Willems & Zwart (2018) addresses the timing issue but ignores the interaction between regulated and deregulated stakeholders. In contrast to such works and Siddiqui et al. (2019), our focus here is on the role of market uncertainty and the value of the regulator's deferral option in driving capacity adoption and timing when transmission and generation are undertaken by distinct entities with conflicting objectives.

We also contribute to the literature on multi-objective infrastructure investments in infrastructure industries in general, e.g., airports and seaports (Jiang, Wan, & Zhang, 2017; Tsamboulas & Ballis, 2014; Xiao, Ng, Yang, & Fu, 2012; Zhang & Zhang, 2003). Despite taking a dynamic approach, these articles disregard uncertainty in the future profitability of such investments. In our model, however, uncertainty plays a crucial role in the TSO's ability to coordinate public and private investment. Balliauw, Kort, Meersman, de Voorde, & Vanelslander (2019) present an exception in accounting for uncertainty in the demand for cargo handling when ana-

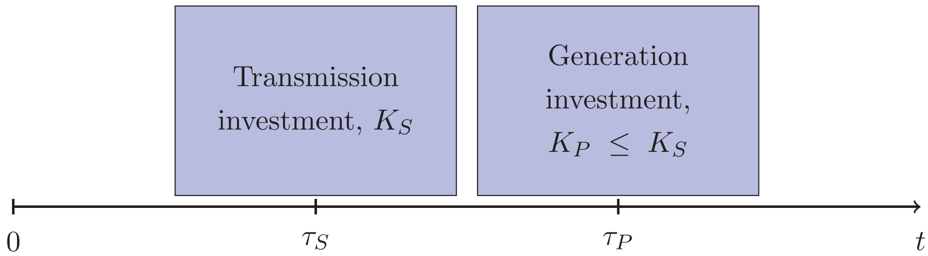


Fig. 1. Decision-making timeline.

lyzing optimal investment decisions in port infrastructure. Despite acknowledging that profit maximization is not the only objective of the infrastructure provider, they do not analyze the consequences of the conflicting objectives on social welfare. Regarding other industries, Sinha, Malo, Frantsev, & Deb (2013) consider a related problem including a regulator and a mining company. Their objectives are conflicting as the regulator strives to maximize social welfare through higher taxes and pollution reduction, whereas the mining company is profit maximizing. Our problem differs from that of Sinha et al. (2013) as the PC is assumed to operate in a deregulated industry, i.e., the TSO does not have a direct channel to influence the PC's profit. Instead, it can affect the investment decisions of the PC only indirectly by constraining the power company's timing and capacity choice. In addition to Sinha et al. (2013), we incorporate uncertainty into the model and derive analytical solutions for the optimal investment strategies that formalize policy insights.

3. Model setup

We consider two decision makers, a regulated entity (TSO) and a private power company (PC), that serve one market characterized by uncertain demand. he role of the TSO is to provide the necessary grid capacity so that the PC is able to transmit electricity. The TSO holds the option to invest in capacity at a time, τ_S, and a capacity, K_S (in MW), of its choosing. This can be, for example, an investment in a new infrastructure line to connect a power park to the main grid. Any subsequent investment by the PC is constrained by the availability of the infrastructure capacity. For example, if there is no transmission capacity available, then the PC is not able to transmit the generated electricity and, therefore, receive profits. We consider the decision of the PC to invest in new generation capacity of K_P (in MW) that is dependent on infrastructure provided by the TSO, i.e., $K_P \leq K_S$. Like the TSO, the PC can also choose both the timing, τ_P, and sizing, K_P, of its own possible investment. Both investments are characterized by substantial sunk investment outlay, and, hence, are considered to be irreversible. In addition, we focus on projects that are significant enough to motivate new transmission investment, as opposed to small incremental capacity expansions that are not likely to justify large infrastructure projects. The decision-making timeline of the model is depicted in Fig. 1.

In our model, the two agents have different objectives. The TSO maximizes social welfare, whereas the PC maximizes its profit. In what follows, we refer to this situation as decentralized planning. We assume perfect information implying that the TSO can anticipate the investment decision of the PC. This adds strategic aspects to the problem, which will influence the TSO's investment strategy as we assume that the TSO makes its investment decision before the PC. Therefore, the problem is similar to a Stackelberg game with the TSO as the leader and the PC as the follower. However, as transmission capacity complements generation capacity rather than substitutes for it, the considered problem does not have the same competitive aspects as the traditional Stackelberg model where companies compete on market share. Instead, each agent's value is dependent on the other agent's investment decisions. The PC is dependent on the decision of the TSO to invest in transmission capacity, whereas the TSO's objective is dependent on the amount of electricity produced by the PC. The TSO in this setting strives to align the decision of the PC with the social optimum by using the PC's is dependence on the capacity provided.

We assume that generation-capacity investment is a one-off lumpy decision as in most concession-based contracts.[5] Once a concession has been granted, the holder, i.e., the PC, has a perpetual option to build the generation capacity, viz., a wind farm. In effect, it can act as a *de facto* monopolist and influence the electricity price (hence, its revenues) via its timing and sizing decisions. Given that we consider large infrastructure investments, we assume that the power company is sufficiently large and exerts some market power. The European electricity industry, for example, shows a high degree of concentration on national and regional scales, suggesting market power.[6] Several studies have found evidence that producers in electricity markets exert some level of market power.[7]

The PC operates in a market characterized by uncertain demand. The inverse-demand function, $P(\theta_t, K_P)$, is measured in \$/MW and is given by:[8]

$$P(\theta_t, K_P) = \theta_t(1 - \eta K_P), \tag{1}$$

where θ_t (in \$/MW) is a stochastic demand shock (shift) parameter and η (in 1/MW) is a positive constant that is inversely proportional to the bound on the market size. We consider a continuous-time framework where the stochastic demand shift parameter is assumed to undergo geometric Brownian motion shocks, i.e., $\{\theta_t, t \geq 0\}$ follows a stochastic process of the form:

$$d\theta_t = \alpha\theta_t dt + \sigma\theta_t dW_t, \tag{2}$$

where $\alpha \in \Re_+$ is the trend parameter or drift, $\sigma \in \Re_+$ is the volatility parameter, and dW_t is an increment of a Wiener process. The current value of the demand parameter is known to the agents, but future values are uncertain and assumed to be log-normally distributed. A geometric Brownian motion (GBM) with positive drift is a reasonable representation of electricity demand shocks

[5] In Norway, for example, the Norwegian Water Resources and Energy Directorate (NVE) grants licenses for major power lines and other energy installations according to the Energy and/or the Water Course Act (NVE, 2018).

[6] Karthikeyan, Raglend, & Kothari (2013) provide a thorough review of electricity-sector market power worldwide.

[7] The European Commission (2011), for example, reports that the Italian energy market is highly concentrated, suggesting market power. Bosco, Parisio, & Pelagatti (2010), Bigerna, Bollino, & Polinori (2016), and Sapio & Spagnolo (2016) find empirical evidence of market power in the Italian energy market. Fleten & Lie (2013) conclude that Norway's largest hydropower producer has an incentive to reduce thermal production in order to increase the price, which both are signs of market power. Focusing on Texas, Woerman (2019) estimates that a 10% increase in demand causes markups to more than double, thereby showing that producers do have market power.

[8] We verify that the results of our model are robust when allowing for isoelastic demand function, which is used, for example, by Cohen, Lobel, & Perakis (2016) and non-linear investment costs. The derivations are available from authors upon request.

in view of expected future increase in demand for renewable energy due to an increase in consumers' willingness to pay stemming from the rise in their environmental awareness, increase in fossil fuel prices, and reduction in marginal costs for green energy producers (Bigerna & Polinori, 2014; Bigerna, Wen, Hagspiel, & Kort, 2019). In addition, mean reversion aspects in commodity prices play little role for investments in long term assets and, therefore, a GBM can be considered a reasonable assumption for such problems (Schwartz, 1998).

We assume that ρ is the exogenous discount rate, which is greater than α; otherwise, it would never be optimal to invest as both agents would prefer to wait forever. Furthermore, we assume that the PC's production is fixed and normalized to the capacity size, K_P.[9]

Without loss of generality, production costs are implicitly included in the sunk investment costs. Also, we assume that the TSO does not charge the PC for using the provided infrastructure. Thus, the continuous profit flow of the PC is equal to:

$$\pi(\theta_t, K_P) = P(\theta_t, K_P)K_P. \tag{3}$$

Similar to Sauma & Oren (2007), Huisman & Kort (2015), and Boonman, Hagspiel, & Kort (2015), we assume investment costs for each agent to be linear in capacity. Specifically, the total investment cost, including operating costs, for the TSO is γK_S, whereas the PC faces an investment cost of δK_P. Both γ and δ are in \$/MW and are assumed to include the effect of fixed payments from the PC to the TSO for the grid rental.[10]

4. Decentralized decision making

In this section, we formulate and solve the industry's transmission- and generation-capacity planning problem. As industry comprises diverse actors with conflicting objectives, we consider production and infrastructure decisions as being taken separately by two different agents, the PC and the TSO. Because the PC has no incentive to undertake an investment before the TSO provides the infrastructure, the TSO is the first mover in the model. We solve the problem via backward induction, starting with the decision of the second mover, i.e., here, the investment strategy of the PC given that the TSO has already installed transmission capacity. We then solve the problem of the TSO given the optimal strategy of the PC.

4.1. Decision of the PC

Given that the TSO has already undertaken an investment of K_S, the PC solves the following optimal stopping problem:

$$\sup_{\tau_P \geq \tau_S, K_P \leq K_S} \mathbb{E}\left[\int_{\tau_P}^{\infty} \theta_t(1-\eta K_P)K_P e^{-\rho t}dt - \delta K_P e^{-\rho\tau_P} \middle| \theta_0 = \theta\right]. \tag{4}$$

Due to the Markovian nature of θ_t, the state space of this stochastic process can be split into a continuation and a stopping region, separated by the optimal investment threshold, $\theta_P^*(K_S)$. If $\theta \geq \theta_P^*(K_S)$, then it is optimal for the PC to invest immediately. Otherwise, it is optimal for the PC to wait. The optimal investment time is defined as $\tau_P^*(K_S) \equiv \min\{t \geq \tau_S : \theta_t \geq \theta_P^*(K_S)\}$, and the corresponding optimal capacity is $K_P^*(\theta_P^*(K_S), K_S)$.

First, we derive the now-or-never optimal capacity investment for the PC in (4), denoted by K_P^*, for a given level of θ, i.e., the capacity that maximizes the value of the PC in the stopping region given that it is constrained by K_S from above:

$$\sup_{K_P \leq K_S} V(\theta, K_P) := \left[\frac{\theta(1-\eta K_P)K_P}{\rho-\alpha} - \delta K_P\right]. \tag{5}$$

Solving this constrained maximization problem yields the following optimal capacity of the PC:

$$K_P^*(\theta, K_S) = \min\left[K_{UP}^*(\theta), K_S\right], \tag{6}$$

where $K_{UP}^*(\theta) = \frac{1}{2\eta}\left(1 - \frac{\delta(\rho-\alpha)}{\theta}\right)^+$, which is the optimal unrestricted capacity of the PC.[11]

Depending on the TSO's capacity level and the current value of θ, two situations can occur:

- If $K_S > K_{UP}^*(\theta)$, then the PC's capacity optimization leads to an interior solution, $K_P^*(\theta, K_S) = K_{UP}^*(\theta)$.
- If $K_S \leq K_{UP}^*(\theta)$, then the PC's capacity optimization leads to a corner solution, $K_P^*(\theta, K_S) = K_S$.

Next, we consider these two cases separately in order to determine the optimal investment timing of the PC. Proposition 1 presents the optimal investment strategy in the case of the unconstrained PC, i.e., the PC's optimal capacity is an interior solution, $K_P^*(\theta, K_S) = K_{UP}^*(\theta)$.

Proposition 1. *If $K_S > K_{UP}^*(\theta)$, then the PC's capacity choice is not constrained by the TSO. The optimal investment threshold of the unconstrained PC is equal to*

$$\theta_{UP}^* = \frac{(\beta+1)}{(\beta-1)}\delta(\rho-\alpha), \tag{7}$$

and its optimal capacity choice at the investment threshold is given by

$$K_{UP}^*(\theta_{UP}^*) = \frac{1}{\eta(\beta+1)}, \tag{8}$$

where

$$\beta = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(-\frac{1}{2} + \frac{\alpha}{\sigma^2}\right)^2 + \frac{2\rho}{\sigma^2}} > 1. \tag{9}$$

If the current level of the stochastic process is such that $\theta \geq \theta_{UP}^*$, then the PC invests immediately and installs capacity equal to

$$K_{UP}^*(\theta) = \frac{1}{2\eta}\left(1 - \frac{\delta(\rho-\alpha)}{\theta}\right)^+. \tag{10}$$

If $\theta < \theta_{UP}^*$, then the PC will wait to invest.

Consider now the case of the constrained PC, i.e., the PC's optimal capacity is a corner solution supporting the constraint on the PC's capacity, K_S. Here, the optimal investment strategy is described by Proposition 2 as follows:

Proposition 2. *If $K_S \leq K_{UP}^*(\theta)$, then the PC's capacity choice is constrained by the TSO. The optimal investment threshold of the constrained PC is given by*

$$\theta_{CP}^*(K_S) = \frac{\beta}{\beta-1}\frac{\delta(\rho-\alpha)}{(1-\eta K_S)}, \tag{11}$$

[9] Renewable generation is generally largely dependent on weather conditions, making production highly variable both in the short and medium terms. However, production is more predictable and less variable in the long term, i.e., over yearly time scales. In the context of long-term investment decisions, we, therefore, do not consider variability in renewable energy generation for our analysis (Bigerna et al., 2019; Boomsma, Meade, & Fleten, 2012; Dalby, Gillerhaugen, Hagspiel, Leth-Olsen, & Thijssen, 2018).

[10] The marginal investment cost of the TSO depends on several factors like the voltage, thickness, and length of the power lines, whereas the marginal investment cost of the PC, among other things, depends on the type of power plant. Therefore, the two marginal investment costs will vary from project to project.

[11] In what follows, the subscript *UP* stands for *unconstrained private company* and reflects the situation in which the optimum is an interior solution of the PC's capacity optimization problem. The subscript *CP*, in turn, stands for *constrained private company* and reflects the situation in which the optimum is a corner solution supporting the constraint on the PC's capacity set by the TSO.

Table 1
Optimal strategy of the PC.

Timing/Capacity	Constrained	Unconstrained
Invest now	$K_P^*(\theta, K_S) = K_S$	$K_P^*(\theta, K_S) = \frac{1}{2\eta}\left(1 - \frac{\delta(\rho-\alpha)}{\theta}\right)^+$
Wait	Invest at θ $K_P^*(\theta, K_S) = K_S$ $\theta_P^*(K_S) = \frac{\beta}{\beta-1}\frac{\delta(\rho-\alpha)}{(1-\eta K_S)}$	Invest at θ $K_P^*(\theta, K_S) = \frac{1}{\eta(\beta+1)}$ $\theta_P^*(K_S) = \frac{\beta+1}{\beta-1}\delta(\rho-\alpha)$

and its optimal capacity choice at the investment threshold is given by

$$K_P^*(\theta, K_S) = K_S. \tag{12}$$

If the current level of the stochastic process is such that $\theta \geq \theta_{CP}^*$, then the PC invests immediately and installs capacity K_S. If $\theta < \theta_{CP}^*$, then the PC will wait to invest.

In general, the optimal investment threshold of the PC, $\theta_P^*(K_S)$, can be written as

$$\theta_P^*(K_S) = \mathbb{1}_{K_S < K_{UP}^*(\theta_{UP}^*)}\theta_{CP}^*(K_S) + \mathbb{1}_{K_S \geq K_{UP}^*(\theta_{UP}^*)}\theta_{UP}^*. \tag{13}$$

Table 1 summarizes the results of Propositions 1 and 2.

Note that for the case when $K_S > K_{UP}^*(\theta)$, the TSO can directly influence neither the optimal investment threshold nor the optimal capacity level of the PC. If, however, $K_S \leq K_{UP}^*(\theta)$, then the capacity installed by the TSO directly affects both the investment timing of the PC and its capacity choice. Depending on the level of K_S, there are two strategies available for the PC for a given value of θ: either invest immediately at the current level of θ or wait until θ increases to the level of $\theta_{CP}^*(K_S)$. Intuitively, immediate investment is possible only for low values of K_S, because in this case, being restricted by smaller available infrastructure, the PC does not have the incentive to wait for higher values of θ in order to install a larger capacity. If, however, K_S is large enough, then the PC would wait for more profitable opportunities to install a larger generation capacity. The capacity level of the TSO such that the constrained PC is indifferent between investing now and postponing is denoted by $\widehat{K}_S(\theta)$. This level can be found by solving $\theta = \theta_{CP}^*(K_S)$ for $K_S \leq K_{UP}^*(\theta)$, which insures that the PC is constrained by the TSO. This yields

$$\widehat{K}_S(\theta) = \frac{1}{\eta}\left(1 - \frac{\beta}{\beta-1}\frac{\delta(\rho-\alpha)}{\theta}\right)^+. \tag{14}$$

We distinguish among the following strategies of the PC in terms of investment timing and capacity:

I *Constrained-Simultaneous* The PC invests at the same time as the TSO and installs the same capacity.
II *Constrained-Sequential* The PC invests later than the TSO and installs the same capacity.
III *Unconstrained-Simultaneous* The PC invests at the same time as the TSO, and installs a smaller capacity.
IV *Unconstrained-Sequential* The PC invests later than the TSO and installs a smaller capacity.

Fig. 2 depicts the state-space diagram, illustrating for which levels of the demand shock, θ, and the infrastructure capacity size, K_S, the different strategies in Table 1 can occur.

4.2. Decision of the TSO

As the TSO's objective is to maximize social welfare, we define total surplus as the sum of the consumer and producer surplus net of investment costs for both agents (Huisman & Kort, 2015; Sauma & Oren, 2006). Note that the instantaneous consumer surplus depends on the generation capacity, K_P, and is given by:

$$CS(\theta_t, K_P) = \int_{\theta_t(1-\eta K_P)}^{\theta_t} \frac{1}{\eta}\left(1 - \frac{P}{\theta_t}\right)dP = \frac{1}{2}\theta_t K_P^2 \eta. \tag{15}$$

The instantaneous producer surplus, on the other hand, is equal to the profit flow of the PC given in (3). Therefore, the instantaneous part of total surplus is equal to:

$$TS(\theta_t, K_P) = \frac{1}{2}\theta_t K_P^2 \eta + \theta_t(1 - \eta K_P)K_P = \theta_t\left(1 - \frac{1}{2}\eta K_P\right)K_P. \tag{16}$$

The TSO solves the following optimal stopping problem, anticipating the PC's decision:

$$\sup_{\tau_S, K_S} \mathbb{E}\left[\int_{\tau_P^*(K_S)}^{\infty} \theta_t\left(1 - \frac{1}{2}\eta K_P^*(\theta_t, K_S)\right)K_P^*(\theta_t, K_S)e^{-\rho t}dt - \gamma K_S e^{-\rho\tau_S} - \delta K_P^*(\theta_{\tau_P^*(K_S)}, K_S)e^{-\rho\tau_P^*(K_S)}\Big|\theta_0 = \theta\right]. \tag{17}$$

Similar to the problem of the PC (4), the solution to (17) is a threshold-type strategy, i.e., for θ_t levels greater than the optimal investment threshold of the TSO, denoted by θ^*, we are in the stopping region where it is optimal for the TSO to invest immediately. For $\theta_t < \theta^*$, demand is too low to undertake the investment, and it is optimal for the TSO to wait. The optimal investment time is given by $\tau_S^* \equiv \min\{t \geq 0 : \theta_t \geq \theta^*\}$. The corresponding optimal capacity is denoted by $K_S^*(\theta^*)$.

Note that the solution to (17) depends on the optimal strategy of the PC, as depicted in Fig. 2. The next proposition states that it is always optimal for the TSO to choose the capacity level K_S low enough to ensure that the PC invests in the same capacity as the TSO.

Proposition 3. *The optimal capacity choice of the TSO is always such that $K_S \leq \min\left\{\widehat{K}_S(\theta), K_{UP}^*(\theta)\right\}$, implying the PC invests at the same time and in the same capacity as the TSO.*

The result in Proposition 3 implies that the solution to the TSO's optimization problem is always such that we end up in Scenario I (Constrained-Simultaneous) in Fig. 2. Hence, (17) becomes

$$\sup_{\tau_S, K_S \leq \min\left\{\widehat{K}_S(\theta), K_{UP}^*(\theta)\right\}} \mathbb{E}\left[\int_{\tau_S}^{\infty} \left(\theta_t(1 - \frac{1}{2}\eta K_S)K_S\right)e^{-\rho t}dt - (\delta + \gamma)K_S e^{-\rho\tau_S}\Big|\theta_0 = \theta\right]. \tag{18}$$

Note that this problem is similar to the one that would arise in an integrated planning setting where the TSO can make both production and generations decisions by itself. The only difference is that the TSO is facing a capacity constraint.

The intuition behind this result is as follows. If the capacity of the PC is not restricted by the TSO (i.e., the optimal capacity of the PC is smaller than that of the TSO), then the TSO cannot exert any influence on the PC's investment threshold. At the same time, some transmission capacity will stay idle resulting in a larger sunk cost for the TSO. Thus, it is never optimal for the TSO to provide more transmission capacity than the PC is going to use. If, however, the TSO always chooses the capacity level such that the PC is constrained (i.e., capacity of the PC is always equal to that of the TSO), then it is never optimal for the TSO to invest earlier than the PC. This is because earlier investment by the TSO affects neither the timing nor the capacity of the PC; however, the TSO does incur the sunk cost earlier. This option is always dominated by one in which investment is delayed until the optimal investment threshold of the PC is reached.

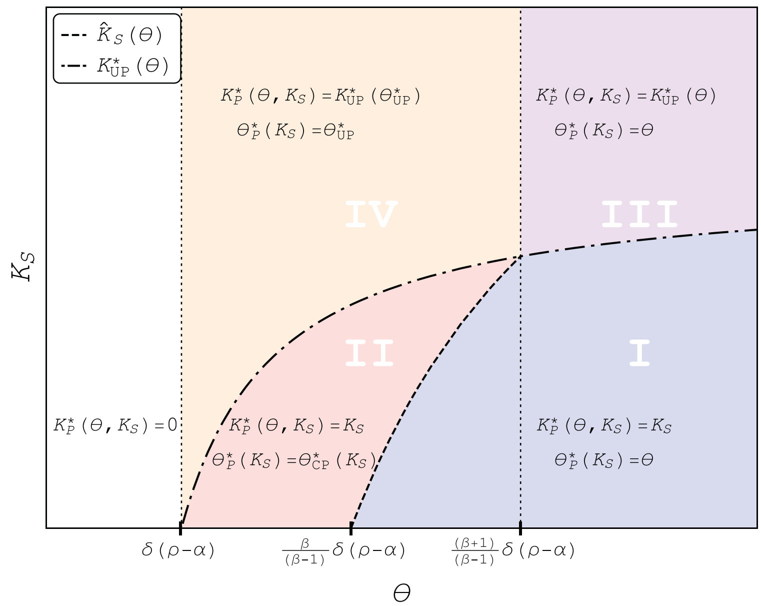


Fig. 2. Illustration of different strategies of the PC.

The optimal capacity in the constrained problem (18) is given in the following proposition.

Proposition 4. *The optimal capacity of the TSO in the simultaneous investment problem is given by*

$$K_S^*(\theta) = \begin{cases} \widehat{K}_S(\theta) & \text{if } \theta < \frac{(\beta+1)}{(\beta-1)}\delta(\rho-\alpha) \text{ and } \beta \leq \frac{\delta}{\gamma}+1, \\ K_{SO}^*(\theta) & \text{if } \theta < (\delta+2\gamma)(\rho-\alpha) \text{ and } \beta > \frac{\delta}{\gamma}+1, \\ K_{UP}^*(\theta) & \text{if } \theta \geq \max\left\{(\delta+2\gamma)(\rho-\alpha), \frac{(\beta+1)}{(\beta-1)}\delta(\rho-\alpha)\right\}, \end{cases} \tag{19}$$

where $K_{SO}^*(\theta)$ denotes the socially optimal capacity level and is given by[12]

$$K_{SO}^*(\theta) = \frac{1}{\eta}\left(1 - \frac{(\delta+\gamma)(\rho-\alpha)}{\theta}\right)^+. \tag{20}$$

Fig. 3 illustrates the result in Proposition 4, where the solid curve represents the socially optimal capacity, the dot-dashed curve is the interior optimum of the PC, and the dashed curve is the maximum capacity that ensures simultaneous investment if the PC is constrained. The gray shaded area in Fig. 3 refers to the Scenario I (Constrained-Simultaneous).

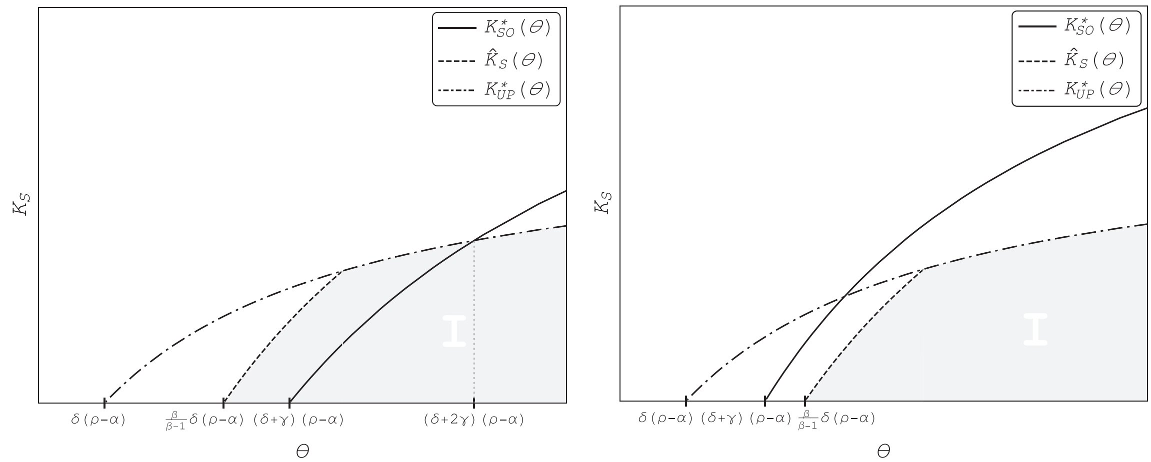
Comparing the expressions for $K_{SO}^*(\theta)$ and $K_{UP}^*(\theta)$, one can easily conclude that the solid curve in Fig. 3 representing the socially optimal capacity is always steeper than the dot-dashed curve representing the PC's optimal unconstrained capacity choice, cf. (20) and (10). This is because the TSO also takes into account the consumer surplus in addition to the producer surplus in its optimization problem. Moreover, the solid curve always requires a larger value of θ than the dot-dashed one to be positive. This is because the TSO takes into account both infrastructure and generation capacity investment costs.

[12] In the socially optimal case where the TSO decides on both production and transmission investments, it is never optimal to install transmission capacity that is not used for production, i.e., $K_P = K_S$ and $\tau_P = \tau_S$. Therefore, the TSO solves the following optimal stopping problem: $\sup_{\tau_S,K_S} \mathbb{E}\left[\int_{\tau_S}^{\infty} \theta_t\left(1-\frac{1}{2}\eta K_S\right)K_S e^{-\rho t}dt - (\delta+\gamma)K_S e^{-\rho\tau_S}\middle|\theta_0=\theta\right]$. The solution of this problem is known from Huisman & Kort (2015).

Depending on the parameters, two cases arise. Consider, for example, a change in the infrastructure investment cost, γ, which affects the social optimum but not the decision of the PC. If infrastructure investment becomes more expensive, then the socially optimal capacity decreases for a given demand level. Fig. 3a illustrates the case when γ is sufficiently large. Here, for a sufficiently low demand, the socially optimal capacity is small enough for the TSO to be able to restrict the PC in both capacity and in timing, i.e., it is optimal for the TSO is to install $K_S^*(\theta) = K_{SO}^*(\theta)$. If, however, $\theta > (\delta+2\gamma)(\rho-\alpha)$, then the TSO overinvests compared to the capacity choice of the PC. As the PC is not restricted from above in such a case, it chooses an optimal capacity that is smaller than the socially optimal capacity. Some of the infrastructure capacity would then be left idle, which is always suboptimal for the TSO because it increases its costs but does not increase consumer surplus. Thus, in this case, the TSO cannot achieve the social optimum and invests in a smaller capacity, i.e., $K_S^*(\theta) = K_{UP}^*(\theta)$, which is the maximum capacity that ensures that the PC invests in an equal amount at the same time.

Fig. 3 b depicts the situation in which γ is relatively small. In this case, the socially optimal capacity level is so large that the social optimum cannot be enforced for two reasons. First, similar to Fig. 3a, for high levels of demand, the TSO is not able to constrain the PC in capacity by choosing the socially optimal level. The optimal choice of the TSO is then $K_S^*(\theta) = K_{UP}^*(\theta)$. Second, unlike in Fig. 3a, for the low demand levels the social optimum cannot be reached either. This is because a large enough infrastructure capacity incentivizes the PC to invest at a higher threshold. This way, the PC is able to capitalize on uncertainty anticipating that it can install larger capacity in the future. The TSO should, therefore, invest at the level below social optimum and install the maximal capacity that still ensures simultaneous investment, i.e., $K_S^*(\theta) = \hat{K}_S(\theta)$. Proposition 3 states that sequential investment is not optimal from the TSO's point of view because it wants to avoid leaving capacity idle. If the TSO installs relatively little capacity, then the PC does not have any incentive to wait for investment, which will lead to simultaneous investment.

The next step is to determine the optimal investment threshold given the optimal capacity choice in different regions of θ. We summarize the results in the following proposition.



(a) $\beta > \frac{\delta}{\gamma} + 1$. (b) $\beta \leq \frac{\delta}{\gamma} + 1$.

Fig. 3. Optimal investment threshold and optimal capacity level of the TSO as functions of θ. $\gamma = 100$; $\eta = 0.05$.].

Proposition 5. *If it holds that $\beta < \frac{2\delta}{\gamma} + 3$, then the optimal investment threshold is given by*

$$\theta^* \equiv \theta^*_S = \frac{(\rho-\alpha)\left(\beta(2\gamma+3\delta) + \sqrt{4\gamma^2\beta^2 + 3\delta(4\gamma+3\delta)}\right)}{3(\beta-1)} \tag{21}$$

and the optimal capacity level is equal to $K^*_S(\theta) = K^*_{UP}(\theta)$. If $\beta \geq \frac{2\delta}{\gamma} + 3$, then the optimal investment threshold is equal to the socially optimal threshold level given by

$$\theta^* \equiv \theta^*_{SO} = \frac{(\beta+1)}{(\beta-1)}(\delta+\gamma)(\rho-\alpha), \tag{22}$$

and the optimal capacity level is equal to $K^*_S(\theta) = K^*_{SO}(\theta)$. Note that it holds that $\theta^*_S = \theta^*_{SO}$ when $\beta = \frac{2\delta}{\gamma} + 3$.

Proposition 5 shows that two cases can arise depending on the investment cost and demand shock parameters. In the first case, the TSO cannot force the PC to invest at the social optimum and, therefore, adjusts its investment choice such that the PC invests exactly in the same capacity as the TSO. In the second case, it is optimal for the PC to invest at the socially optimal threshold and in the socially optimal capacity level. In the following section, we will provide intuition for the condition that separates these two cases.

5. Results

In this section and the next (Section 6), we formalize the research contributions alluded to in Section 1, viz.,

1. Determine the impact of uncertainty on the TSO's ability to enforce the socially optimal transmission and generation investments.
2. Examine the effects of market structure and uncertainty on social welfare.
3. Investigate the effectiveness of a capacity restriction imposed on the PC by the TSO in enforcing the social optimum.

5.1. Impact of key parameters on enforceability of the social optimum

The crucial result of the previous section is that the constraints in terms of timing and capacity are sufficient in ensuring that the TSO is able to enforce the social optimum under certain conditions. The next proposition states that there exists a unique threshold for the volatility, $\hat{\sigma}$, below which the TSO can achieve the social optimum.

Proposition 6. *The TSO can enforce the social optimum if and only if the following condition holds:*

$$\sigma < \hat{\sigma} \equiv \sqrt{\frac{\left[\rho - \alpha\left(\frac{2\delta}{\gamma}+3\right)\right]^+}{\left(\frac{\delta}{\gamma}+1\right)\left(\frac{2\delta}{\gamma}+3\right)}}, \tag{23}$$

where $\frac{\partial\hat{\sigma}}{\partial\alpha} < 0$, $\frac{\partial\hat{\sigma}}{\partial\rho} > 0$, and $\frac{\partial\hat{\sigma}}{\partial\left(\frac{\delta}{\gamma}\right)} < 0$.

Intuitively, Proposition 6 implies that the TSO is unable to attain the socially optimal level of simultaneous investment in a relatively uncertain environment. In addition, it is also not possible to reach the social optimum when δ is relatively large compared to γ, i.e., the marginal investment cost of the PC is relatively large in comparison to the marginal cost faced by the TSO. This is typical for the electricity industry, where the marginal investment cost of transmission lines is comparatively much lower than the marginal investment cost of, for example, wind power plants (Baringo & Conejo, 2012).

Proposition 6 also states that $\hat{\sigma}$ decreases in the demand shock drift parameter, α. This means that the range for which the TSO can enforce the social optimum decreases as the drift rate increases. Likewise, as generation investment becomes more costly, the TSO's opportunity to attain the social optimum vanishes. For example, in an extreme case, i.e., when $\frac{\delta}{\gamma} > \frac{\rho-3\alpha}{2\alpha}$, it becomes impossible to enforce the social optimum as $\hat{\sigma} < 0$. However, the opposite holds for the effect of the discount rate, ρ, because its increase renders the future less important and facilitates achievement of the social optimum. These results are driven by the fact that it is always optimal for the TSO to align both the timing and size of infrastructure and production investment. Whether it is able to do so, depends on the TSO's possibility to constrain the PC. If the TSO can constrain the PC, then the TSO can enforce the social optimum. Note that $\hat{\sigma}$ does not depend on the demand parameter, η, that represents the bound on the market size. This is because the optimal capacities of the TSO and the PC are inversely proportional to η, and, thus, their resulting order does not depend on η. In addition, the optimal investment thresholds do not depend on η either. This implies that this parameter does not impact the ability of the TSO to align the size and timing of the production and transmission investment.

Table 2
Timing and size for both production and transmission investment under different planning strategies for the $\sigma > \hat{\sigma}$.

Setting	Transmission investment	Production investment
Integrated planning	$(\theta^*_{SO}, K^*_{SO}(\theta^*_{SO}))$	$(\theta^*_{SO}, K^*_{SO}(\theta^*_{SO}))$
Decentralized planning	$(\theta^*_S, K^*_{UP}(\theta^*_S))$	$(\theta^*_S, K^*_{UP}(\theta^*_S))$
Private planning	$(\theta^*_{SO}, \frac{1}{2}K^*_{SO}(\theta^*_{SO}))$	$(\theta^*_{SO}, \frac{1}{2}K^*_{SO}(\theta^*_{SO}))$
Non-anticipative planning	$(\theta^*_{SO}, K^*_{SO}(\theta^*_{SO}))$	$(\theta^*_{SO}, K^*_{UP}(\theta^*_{SO}))$

5.2. Impact of market structure on investment timing and capacity sizing

In order to provide context for these findings, we compare our decentralized results from solving (17) to the three benchmarks mentioned in Section 3:

- Integrated planning when the TSO decides on both production and transmission investments.
- Private planning, which is a hypothetical scenario in which the PC undertakes all of the decisions itself.
- Non-anticipative planning, in which the TSO does not take into account the private decisions about production capacity when investing in transmission.

Table 2 summarizes the resulting investment timing and capacity pairs, $\left(\theta^*, K^*_S(\theta^*)\right)$, for different planning configurations for the case when the social optimum cannot be enforced in the decentralized setting, i.e., for $\sigma > \hat{\sigma}$.

The relationship between investment timing and capacity sizing for the different settings is summarized in Proposition 7.

Proposition 7. *For $\sigma > \hat{\sigma}$, it holds that $\theta^*_{SO} > \theta^*_S$ and $K^*_{SO}(\theta^*_{SO}) > K^*_{UP}(\theta^*_{SO}) > K^*_{UP}(\theta^*_S) > \frac{1}{2}K^*_{SO}(\theta^*_{SO})$.*

Comparing the integrated and decentralized settings, one can see that in the decentralized case, the TSO and PC still invest at the same time, θ^*_S, and in the same capacity, $K^*_{UP}(\theta^*_S)$, albeit at lower demand and capacity levels than socially optimal. In other words, in the decentralized case, the TSO has to adjust by investing earlier and in a smaller capacity than is socially optimal if condition (23) does not hold. Failing to do so and, thus, disregarding the discretion of the PC over timing and capacity choice would result in a welfare loss. More specifically, under such non-anticipative planning, the TSO will invest in infrastructure at the socially optimal time and capacity level, i.e. at θ^*_{SO} and $K^*_{SO}(\theta^*_{SO})$, respectively. The PC will enter the market at the same time as the TSO, but it will invest in less capacity, $K^*_{UP}(\theta^*_{SO})$. Interestingly, in this case, both infrastructure and production capacity are larger than those in the decentralized setting and, thus, closer to the social optimum. The investment timing is also higher. Nevertheless, this scenario leads to a lower welfare in comparison to the decentralized case, as the capacity decisions of the PC and the TSO are misaligned. In particular, the installed production capacity is smaller than the infrastructure capacity, i.e. $K^*_{UP}(\theta^*_{SO}) < K^*_{SO}(\theta^*_{SO})$, implying that the latter will partially remain idle. Thus, by correctly accounting for the optimal decisions of the PC and adjusting *both* the investment timing and size, TSO is able to spare the costs associated with unused capacity. These results are different from the literature that only focuses on the interaction between social planner and regulated monopolies and implicitly assumes that the deregulated firms do not impact the decisions of higher level entities (Broer & Zwart, 2013; Evans & Guthrie, 2012; Evans, Quigley, & Guthrie, 2012; Willems & Zwart, 2018). In an application to transmission planning, Saphores, Gravel, & Bernard (2004) take a real options approach to tackle the interaction between a transmission planner and a regulator under uncertainty. In their example (loosely based on the case of Hydro-Québec's application for an interconnection with Ontario), however, both decisions (when to start the regulatory process and when to begin construction of the line) are conducted by the transmission planner. Thus, although they account for the deferral option, they do not reflect conflicting objectives between distinct entities.

Comparing the integrated and the private-planning settings, we find that a private investor halves the installed capacity that a TSO would have adopted, i.e. $\frac{1}{2}K^*_{SO}(\theta^*_{SO})$, whereas investment timing would be unchanged, cf. Huisman & Kort (2015). Despite the fact that in this case the timing of investment is socially optimal, the capacity installed is the lowest among different settings. In the decentralized setting, the TSO is able to increase welfare by installing larger capacity at the expense of an earlier investment. These results emphasize the importance of the flexibility both in the choice of the timing and size in coordination of infrastructure investment decisions. In practice, the boost in wind capacity in Texas beginning in 2008 without the concomitant increase in transmission capacity is indicative of the problems that could be created by such lack of coordination (Wiser et al., 2015). Next, we analyze how costly the coordination failures under the abovementioned settings are in terms of social welfare.

5.3. Impact of volatility on optimal decisions and social welfare

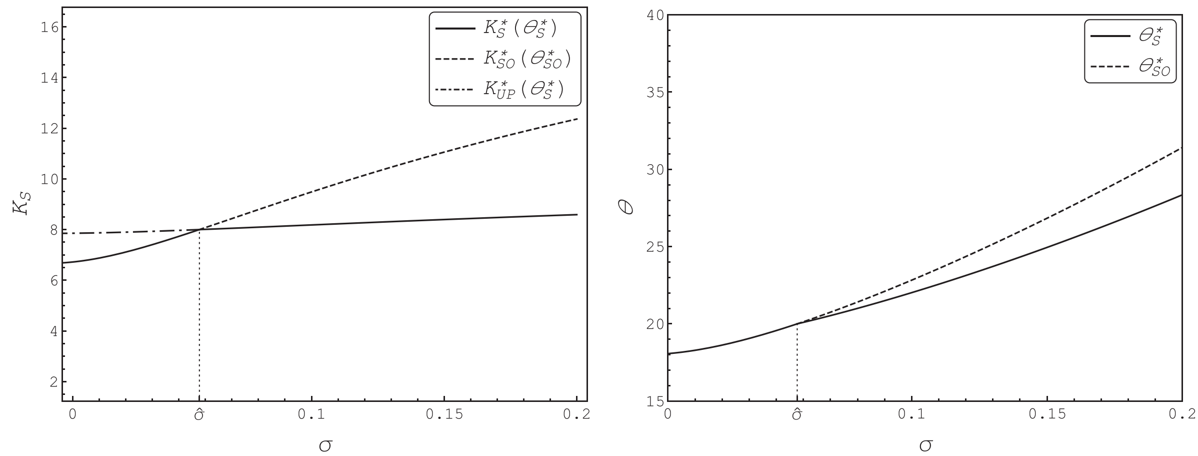
Given the importance of volatility in enabling the TSO to enforce the social optimum, cf. (23), we here analyze how volatility can affect both decisions and social welfare. First, we examine the effect of volatility on both the optimal investment timing and capacity sizing for the decentralized setting compared to the integrated one. Next, we study the welfare loss in the decentralized setting vis-à-vis the integrated one. Finally, we quantify the welfare loss if the TSO does not anticipate the PC's investment-timing and capacity-sizing decisions.

Proposition 8 summarizes the impact of volatility on the optimal decisions:

Proposition 8. *In both the integrated and the decentralized settings, the investment thresholds of the TSO, as well as its optimal capacities evaluated at these thresholds, are increasing with volatility σ, i.e., $\frac{\partial \theta^*_{SO}}{\partial \sigma} > 0$, $\frac{\partial \theta^*_S}{\partial \sigma} > 0$, $\frac{\partial K^*_{SO}(\theta^*_{SO})}{\partial \sigma} > 0$, and $\frac{\partial K^*_{UP}(\theta^*_S)}{\partial \sigma} > 0$. For $\sigma > \hat{\sigma}$, it holds that $\frac{\partial(\theta^*_{SO} - \theta^*_S)}{\partial \sigma} > 0$ and $\frac{\partial(K^*_{SO}(\theta^*_{SO}) - K^*_S(\theta^*_S))}{\partial \sigma} > 0$.*

Proposition 8 states that as volatility increases, the TSO invests at a higher threshold and in a larger capacity for both the decentralized and the integrated settings. This result is consistent with the real options literature that studies optimal capacity choice, where Dangl (1999) Bar-Ilan & Strange (1999) and Kouvelis & Tian (2014) among others, find that in more uncertain environments firms invest later and in larger capacity. In addition, the difference between the integrated and the decentralized settings in terms of both investment thresholds and capacity sizes widens as volatility increases. Fig. 4 illustrates these effects.[13]

[13] For our baseline parameter values, we choose $\delta = 50$ and $\gamma = 100$, which reflects the fact that marginal generation investment costs are generally lower



(a) Optimal capacity level. (b) Optimal investment threshold.

Fig. 4. Optimal investment threshold and optimal capacity level of the TSO as functions of σ. [Parameter values: $\rho = 0.1$; $\alpha = 0.02$; $\sigma = 0.1$; $\delta = 50$; $\gamma = 100$; $\eta = 0.05$.].

The investment timing of the TSO is driven by two factors: the ability of the TSO to restrict the capacity of the PC and the fact that it is always optimal to invest at the same time as the PC. If volatility is low, then it is optimal to invest early where it is possible to constrain the PC such that it invests at the socially optimal level of θ. If volatility is relatively high, then the TSO is no longer able to constrain the PC in either its capacity or its timing choice. The reason for that is that the PC installs less capacity than the social optimum. Therefore, the PC can justify investment for lower values of θ, i.e., earlier. In order to ensure that timing and size are the same for both infrastructure and production investment, the TSO adapts to the PC's investment timing choice. These findings contradict the result of Maurovich-Horvat et al. (2015) that the presence of a transmission planner leads to more wind power investments. The key difference is that their model does not account for the possibility to defer investments, which becomes particularly valuable in more uncertain environments emphasizing the importance of considering discretion over agents' investment timing.

We now analyze how social welfare is affected by a decentralized setting compared to an integrated one. Furthermore, we study the potential welfare loss when the TSO does not anticipate the fact that the PC chooses timing and capacity sizing with the objective to maximize profit.

The relative welfare loss for a specific choice of investment threshold, transmission capacity, and generation capacity, (θ, K_S, K_P), evaluated at θ_0 (such that $\theta_0 < \theta$ and $\theta_0 < \theta_{SO}^*$) is defined as

$$L(\theta, K_S, K_P) = 1 - \frac{W(\theta, K_S, K_P)}{W\left(\theta_{SO}^*, K_{SO}^*(\theta_{SO}^*), K_{SO}^*(\theta_{SO}^*)\right)}, \tag{24}$$

where $W(\theta, K_S, K_P) = \left(\frac{\theta_0}{\theta}\right)^{\beta}\left(\frac{\theta K_P\left(1-\frac{1}{2}\eta K_P\right)}{\rho-\alpha} - \delta K_P - \gamma K_S\right)$ is the discounted welfare value for the specific choice of (θ, K_S, K_P).

In what follows, we describe some analytical properties of the welfare loss functions. First, by definition, the welfare loss under integrated planning is equal to zero. Second, comparing the rest of the alternatives, it is obvious that under decentralized planning, the TSO is able to achieve the highest social welfare value and, thus, the lowest relative loss. This is because the triplet $(\theta, K_S, K_P) = (\theta_S^*, K_{UP}^*(\theta_S^*), K_{UP}^*(\theta_S^*))$ gives the optimal solution to the infrastructure investment problem when the TSO cannot directly influence the PC. Furthermore, it follows from

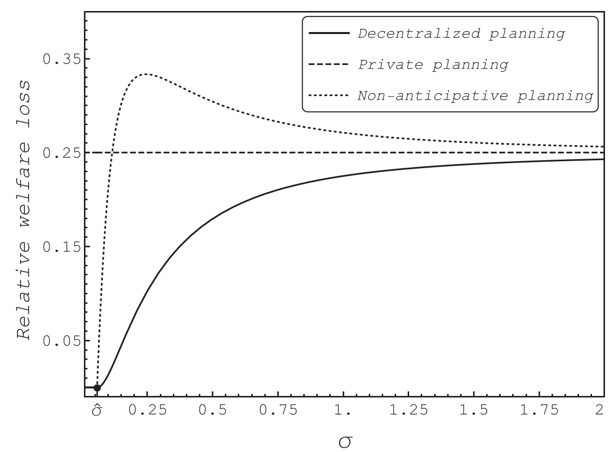


Fig. 5. Relative welfare loss. [Parameter values: $\rho = 0.1$; $\alpha = 0.02$; $\sigma = 0.1$; $\delta = 50$; $\gamma = 100$; $\eta = 0.05$.].

Proposition 8 that the relative welfare loss under decentralized planning increases with σ, as the difference from the social optimum both in terms of timing and capacity becomes larger as σ increases. Under private planning, the relative welfare loss is constant and equal to 25%. This result directly follows from Huisman & Kort (2015), as the social planner invests at the same time as the private planner, but in twice as much capacity. In the case of non-anticipative planning, it is possible to show that the relative welfare loss may even exceed that under the private planning. This happens for relatively large values of volatility.[14] Lastly, it is easy to verify that as $\sigma \to \infty$ and, thus, $\beta \to 1$, the relative welfare loss converges to the value under the private planning because all the investment thresholds tend to infinity, whereas the capacity levels $K_{UP}^*(\theta_S^*), K_{UP}^*(\theta_{SO}^*)$ and $\frac{1}{2}K_{SO}^*(\theta_{SO}^*)$ converge to $\frac{1}{2\eta}$, as follows from (20) and (10). This is because in our problem, there exists an exogenous bound on the market size.

Fig. 5 illustrates the relative welfare loss from the integrated benchmark as a function of volatility for the decentralized setting and two alternative settings. The dashed line illustrates the relative welfare loss when both infrastructure and generation invest-

than marginal transmission investment costs (DeSantis, James, Houchins, Saur, & Lyubovsky, 2021; Energy Information Administration, 2020).

[14] The difference between the discounted welfare value under non-anticipative planning and private planning is given by $W(\theta_{SO}^*, K_{SO}^*(\theta_{SO}^*), K_{UP}^*(\theta_{SO}^*)) - W(\theta_{SO}^*, \frac{1}{2}K_{SO}^*(\theta_{SO}^*), \frac{1}{2}K_{SO}^*(\theta_{SO}^*)) = \left(\frac{\theta_0}{\theta_{SO}^*}\right)^{\beta} \frac{\gamma\left(3\beta - \frac{4\delta}{\gamma} - 7\right)}{8(\beta+1)\eta\left(\frac{\delta}{\gamma}+1\right)}$, which is smaller than 0 for $\beta < \frac{7}{3} + \frac{4\delta}{2\gamma}$ (i.e., for large σ) and is larger than 0 otherwise.

ment are made by a private investor with the objective to maximize producer surplus disregarding consumer surplus. The dotted line illustrates the relative welfare loss when the TSO does not anticipate the PC's profit-maximizing behavior. Finally, the solid line illustrates the relative welfare loss when the TSO anticipates the PC's optimal investment behavior in a decentralized market.

As can be seen in Fig. 5, if the PC decides upon both infrastructure and generation investments, then the social optimum can be never reached with a welfare loss equal to 25%. It is also worth noting that the welfare loss is non-monotonic in σ in the case of non-anticipative planning. Given that the investment timing is the same in the social optimum and the non-anticipative planning case, the differences in the welfare are primarily driven by capacity choice of the PC. When $\sigma = \hat{\sigma}$, the capacities in the socially optimal and the non-anticipative cases are equal and the welfare loss is zero. As σ starts increasing, these capacities diverge, where the socially optimal capacity, K^*_{SO}, increases faster because the social planner is taking into account both producer's and consumer's surplus in its objective. The difference in capacities negatively affects the welfare, thereby implying that the welfare loss increases sharply with σ. At some point, however, the difference in the PC's capacities stops increasing as rapidly as a result of an increase in σ and becomes constant in the limit as σ goes to infinity. This, together with the fact that the welfare function is quadratic in the PC's capacity, leads to the result that the relative welfare loss starts declining. Thus, if the TSO invests in a decentralized setting without anticipating the PC's optimal choices, then the welfare loss can rise significantly above 25% for intermediate values of σ. In other words, when volatility is large enough, non-anticipative planning becomes particularly costly from the welfare perspective as the regulated entity does even worse than the profit-maximizing monopolist that undertakes both infrastructure and production decisions. When σ is relatively high, the welfare loss approaches 25% from above as in the private-planning setting.

If the TSO decides upon infrastructure capacity anticipating the PC's investment decision, then it is able to hold the welfare loss to below 25% as illustrated by the solid curve even though the welfare loss asymptotically approaches the same level as in the private-planning setting. For a low volatility, i.e., $\sigma \leq \hat{\sigma}$, it can even obtain the socially optimal outcome.

The policy implications here are that in less volatile markets, the TSO is able to achieve the social optimum even if it chooses a myopic strategy and does not anticipate the PC's profit-maximizing behavior. As volatility increases, however, such a myopic strategy results in substantial welfare losses. In moderately uncertain environments, the ability of the TSO to orient the decisions of the PC by choosing investment timing and capacity size of the infrastructure investment in itself helps to reduce welfare losses substantially in comparison to non-anticipative planning. In more volatile markets, however, additional mechanisms are necessary in order to achieve a significant reduction of the welfare loss, as the ability to restrict the PC in timing and capacity alone does not bring a significant advantage over the myopic strategy of ignoring the optimal decisions of the PC. These insights are particularly relevant for future energy systems as increasing penetration of highly volatile and inflexible renewable energy technologies contributes to a substantial increase in the electricity price volatility (Rintamäki, Siddiqui, & Salo, 2017). The welfare losses in our model are also in line with real world examples, such as Texas, where the extent of the curtailment to wind production as a result of poor coordination between generation and transmission expansion is an indicator of welfare loss. Likewise, the *Energiewende* in Germany also presents a cautionary tale about the consequences of fostering renewable-energy technologies without considering their impact on a transmission system that was configured for a starkly different pattern of consumption and production. Indeed, Kunz (2013) argues that insufficient transmission capacity will lead to substantially higher congestion management costs absent correct price signals.

6. Welfare-enhancement mechanisms

In practice, the possibilities for the TSO to steer private decisions towards the social optimum are limited. Typical measures that would affect investment decisions of PCs in deregulated industries, such as subsidies and other incentive schemes, are at the discretion of higher-level entities. In this paper, we analyze the effects of two welfare-enhancement mechanisms. The first one that we consider is motivated by the fact the TSO can demand that certain conditions are met before it commits to providing infrastructure. Such a dispensation was deployed in Norway, where Statnett set a minimum restriction on generation capacity in the case of the Fosen wind park (Lie, 2015). In particular, Statnett conditioned its approval of the required transmission capacity on a commitment by the wind-park investor to install a minimum of 1000 MW of generation capacity. The PC in the Fosen case eventually met this condition by adopting six onshore wind farms with a combined capacity of 1000 MW.

Another mechanism that we consider is a lump-sum investment cost subsidy (see, e.g. Boomsma et al. (2012) and Nagy, Hagspiel, & Kort (2021) that study investment timing and capacity choice of profit-maximizing energy producers), where the TSO subsidizes a fraction of the investment cost paid by the PC. Such a subsidy could serve as an effective incentive-alignment mechanism as it reduces the PC's investment cost, thereby stimulating investment at a lower threshold and leading to an increase of the volatility region where the social optimum is attained.

6.1. Minimum capacity requirement

To account for the minimum capacity requirement, we incorporate an additional decision variable for the TSO in our model, which is the minimum required generation capacity denoted by $K_{\min}$. This restriction can either be set such that $K_{\min} < K_S$ or $K_{\min} = K_S$ as the generation capacity is naturally limited from above by the size of the infrastructure provided. For the TSO in a decentralized industry, it is, in fact, always optimal to set the capacity requirement such that $K_{\min} = K_S$. This minimum capacity requirement essentially implies that the TSO eliminated the PC's choice over capacity. This leads to the following optimal stopping problem of the TSO in anticipation of the PC's decision:

$$\sup_{\tau_S, K_S} \mathbb{E}\Bigg[-\gamma K_S e^{-\rho\tau_S} + \int_{\tau^*_{CP}(K_S)}^{\infty} \theta_t \Big(1 - \frac{1}{2}\eta K_S\Big) K_S e^{-\rho t} dt \\ - \delta K_S e^{-\rho \tau^*_{CP}(K_S)} \Big| \theta_0 = \theta\Bigg], \tag{25}$$

where $\tau^*_{CP}(K_S) = \min\{t : \theta_t \geq \theta^*_{CP}(K_S)\}$.

In other words, for any choice of infrastructure capacity size, K_S, the TSO will impose exactly the same minimum restriction on generation capacity, $K_{\min} = K_S$, such that if the PC decides to invest, it will always install exactly K_S. Here, we assume that the PC is not able to back out of the deal after the TSO undertakes an investment. In reality for such agreements, the regulators often have levers to secure the PC's capacity commitment.[15] Given the level of planned infrastructure, the PC, however, still has some flexibility with respect to investment timing and, in principle, can delay its investment. In the following, we show that if the TSO anticipates the decisions of the PC, it can always ensure that it is never optimal for the PC to delay investment.

[15] For example, in renewable energy auctions, it is typical to employ various measures to enforce the contracts, such as penalties or pre-qualification requirements (Matthäus, Schwenen, & Wozabal, 2020).

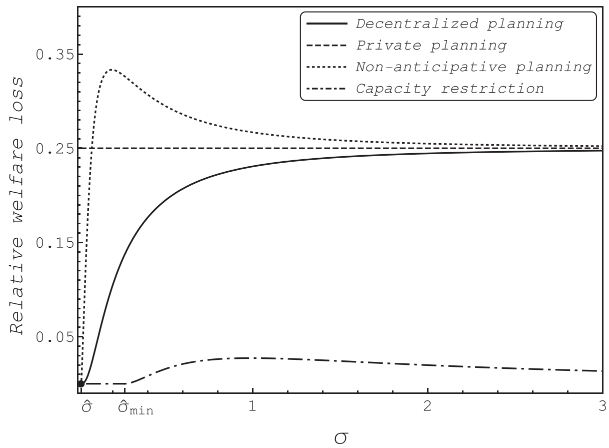


Fig. 6. Relative welfare loss. [Parameter values: $\rho = 0.1$; $\alpha = 0.02$; $\delta = 50$; $\gamma = 50$; $\eta = 0.05$.].

The solution to the investment problem in the presence of such a capacity restriction (25) is presented in the following proposition.

Proposition 9. *In the presence of the minimum generation capacity restriction, the TSO and the PC always invest at the same time and in the same capacity level.*

- *If $\beta < \frac{\delta}{\gamma} + 1$, then the optimal investment threshold is given by*

$$\theta^*_{min} = \frac{(\rho-\alpha)}{(\beta-1)}\left(\beta(\gamma+\delta)+\frac{\sqrt{(\beta-1)^2(\gamma+\delta)^2+\left(\beta^2-1\right)(\delta-(\beta-1)\gamma)^2}}{(\beta-1)}\right), \tag{26}$$

  and the optimal capacity level is equal to $\widehat{K}_S(\theta)$.
- *If $\beta \geq \frac{\delta}{\gamma} + 1$, then the optimal investment threshold is equal to the socially optimal threshold given by*

$$\theta^*_{SO} = \frac{(\beta+1)}{(\beta-1)}(\delta+\gamma)(\rho-\alpha), \tag{27}$$

  and the optimal capacity level is equal to $K^*_{SO}(\theta)$.

In the presence of the minimum capacity restriction, the TSO effectively forces the PC to install generation capacity of the same size as the infrastructure capacity. This implies that the TSO can essentially always constrain the capacity of the PC. However, the TSO is not able to affect the PC's investment timing directly. Proposition 9 states that, in this case, sequential investment is never optimal. Thus, the TSO has to make sure that its capacity is small enough to convince the PC to invest immediately rather than to defer investment.

Although the social optimum cannot always be enforced with a minimum capacity restriction mechanism, it, nevertheless, allows the TSO to enforce the social optimum for a wider range of parameters as illustrated in Fig. 6. Specifically, it manages to do so if $\beta \geq \frac{\delta}{\gamma} + 1$ (or, equivalently $\sigma \leq \hat{\sigma}_{\min}$), i.e., if volatility is relatively low and/or the marginal infrastructure investment cost is relatively high compared to the marginal generation investment cost.[16] Note that this condition is less restrictive than in the original model without minimum capacity requirement in which the TSO needs to make sure that the PC invests both in the same capacity and at the same time as the TSO. This is because now the TSO is less constrained in choosing its investment strategy as it effectively controls the PC's choice over capacity and only needs to ensure that both production and infrastructure investments occur at the same time. Furthermore, the social optimum is achieved in the limit as $\sigma \to \infty$.[17] This leads to a non-monotonicity in the relative welfare loss with respect to σ, which in this case is driven by the difference in the investment timing. While this wedge is large for the intermediate values of σ, for higher values of volatility, the investments in the case of the minimum capacity restriction and in the social optimum are delayed substantially and, thus, this difference is diminishing due to discounting. This indicates that such a mechanism allows the TSO to reduce the welfare losses substantially even in very uncertain environments. This result is qualitatively different from the cases considered previously where the TSO was able to achieve a substantial reduction of the welfare losses for only low levels of volatility.

Compared to the case without the minimum capacity mechanism, the TSO is able to install a larger capacity size that is closer to the social optimum albeit at the cost of delayed investment. In fact, the TSO installs the maximum capacity that allows simultaneous investment. Note that the mechanism only allows the TSO to fix the capacity size for the PC but not the timing choice. If the TSO were to adopt an even larger infrastructure capacity, then the PC would be forced to invest later in order to justify such large generation capacity given it is forced to install capacity size equal to the size of infrastructure investment in this case.

Proposition 10 formalizes features of the corresponding optimal threshold and capacity.

Proposition 10. *In the setting with a minimum capacity restriction, the optimal investment threshold of the TSO as well as its optimal capacity are increasing with volatility, $\frac{\partial \theta^*_{\min}}{\partial \sigma} > 0$ and $\frac{\partial \widehat{K}_S(\theta^*_{\min})}{\partial \sigma} > 0$. In addition, for $\sigma > \hat{\sigma}_{\min}$ it also holds that $\theta^*_{\min} > \theta^*_{SO}$ and $K^*_{UP}(\theta^*_S) < \widehat{K}_S(\theta^*_{\min}) < K^*_{SO}(\theta^*_{SO})$.*

Fig. 7 illustrates the findings of Proposition 10 concerning the optimal capacity (Fig. 7a) and optimal investment timing (Fig. 7b) as functions of volatility for three settings: the socially optimal one under integrated planning (dashed line), the decentralized one with capacity restriction (dot-dashed line), and the decentralized one without capacity restriction (solid line). Evidently, the optimal investment threshold in the setting with the minimum capacity restriction is larger, whereas the optimal capacity level is small than in the social optimum. However, as volatility increases, the optimal capacity choice approaches the socially optimal level.

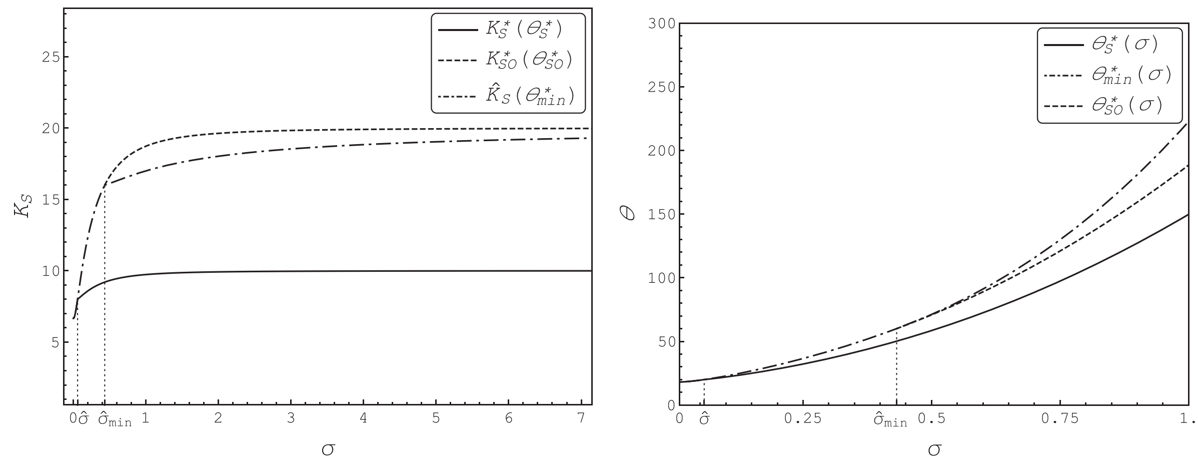
Although the minimum production capacity restriction reduces welfare loss in a decentralized setting, there may be some disadvantages to using this instrument. In particular, the reduction of welfare loss comes at a cost of delayed investment. This can be crucial in the case of renewable energy, where the policymakers are especially concerned about meeting timing targets. In such instances, there is a need to balance ambitious renewable targets with broader welfare objectives.

6.2. Investment cost subsidy

Another mechanism available to the social planner that may help to steer the PC towards the social optimum is an investment cost subsidy. In our model, such a subsidy results in a redistribution of the investment cost between the TSO and the PC. Let us denote the investment cost subsidy per unit of capacity installed

[16] Here, $\hat{\sigma}_{\min}$ is the unique solution to $\beta = \frac{\delta}{\gamma} + 1$. Note that because β is a monotonic function in σ, this allows us to conclude that $\hat{\sigma}_{\min} > \hat{\sigma}$.

[17] The relative welfare loss can be expressed as $L(\theta^*_{\min}, \widehat{K}(\theta^*_{\min}), \widehat{K}(\theta^*_{\min})) = 1 - \frac{\left(\frac{1}{\theta^*_{\min}}\right)^{\beta}\left(\theta^*_{\min}-(\rho-\alpha)(\gamma+\delta)\right)}{\left(\frac{1}{\theta^*_{SO}}\right)^{\beta}\left(\frac{2\theta^*_{SO}}{\beta+1}\right)}$ in this case. Taking the limits as $\sigma \to \infty$, both the numerator and the denominator of the fraction tend to 1 because $\beta \to 1$ and both $\theta^*_{SO} \to \infty$ and $\theta^*_{\min} \to \infty$, thereby leading to zero relative welfare loss.



(a) Optimal capacity level.

(b) Optimal investment threshold.

Fig. 7. Optimal investment threshold and optimal capacity level with the restriction on minimum generation capacity as a function of σ. [Parameter values: $\rho = 0.1$; $\alpha = 0.02$; $\sigma = 0.1$; $\delta = 50$; $\gamma = 100$; $\eta = 0.05$.].

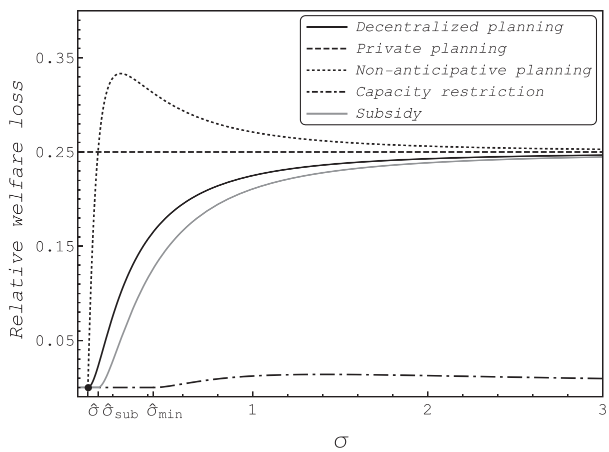


Fig. 8. Relative welfare loss. [Parameter values: $\rho = 0.1$; $\alpha = 0.02$; $\delta = 50$; $\gamma = 100$; $\epsilon = 50$; $\eta = 0.05$.].

by the PC by $\epsilon \in (0, \delta]$. In the decentralized setting, the PC solves the following problem

$$\sup_{\tau_P \geq \tau_S, K_P \leq K_S} \mathbb{E}\left[\int_{\tau_P}^{\infty} \theta_t (1-\eta K_P) K_P \mathrm{e}^{-\rho t} dt - (\delta - \epsilon) K_P \mathrm{e}^{-\rho \tau_P} \Big| \theta_0 = \theta\right], \tag{28}$$

which is the same problem as in (4) but with a lower marginal investment cost. Thus, its solution directly follows from Propositions 1 and 2. Let us denote the optimal capacity of the PC in this case by $K_{P,sub}^*$ and its optimal investment time by $\tau_{P,sub}(K_S)$. Then, the TSO's investment problem becomes:

$$\begin{aligned}\sup_{\tau_S, K_S} \mathbb{E}\Bigg[&\int_{\tau_{P,sub}^*(K_S)}^{\infty} \theta_t \left(1 - \frac{1}{2}\eta K_{P,sub}^*(\theta_t, K_S)\right) K_{P,sub}^*(\theta_t, K_S) \mathrm{e}^{-\rho t} dt \\ &- (\gamma + \epsilon) K_S \mathrm{e}^{-\rho \tau_S} \\ &- (\delta - \epsilon) K_{P,sub}^*(\theta_{\tau_{P,sub}^*(K_S)}, K_S) \mathrm{e}^{-\rho \tau_{P,sub}^*(K_S)} \Big| \theta_0 = \theta\Bigg].\end{aligned} \tag{29}$$

Similar to the case of the PC, this optimal problem is solved analogously to (17). Thus, the result of Proposition 3 sill applies, viz., that the TSO always chooses its optimal capacity such that the PC invests in the same capacity and at the same time as the TSO also in the case of investment cost subsidy. This leads to the following proposition.

Proposition 11. *For a marginal investment cost subsidy, $\epsilon \in (0, \delta]$, if it holds that $\beta < \frac{2(\delta-\epsilon)}{(\gamma+\epsilon)} + 3$, then the optimal investment threshold is given by*

$$\theta_{sub}^* = \frac{(\rho - \alpha)\left(\beta(2\gamma + 3\delta - \epsilon) + \sqrt{4(\gamma + \epsilon)^2 \beta^2 + 3(\delta - \epsilon)(4\gamma + \epsilon + 3\delta)}\right)}{3(\beta - 1)}, \tag{30}$$

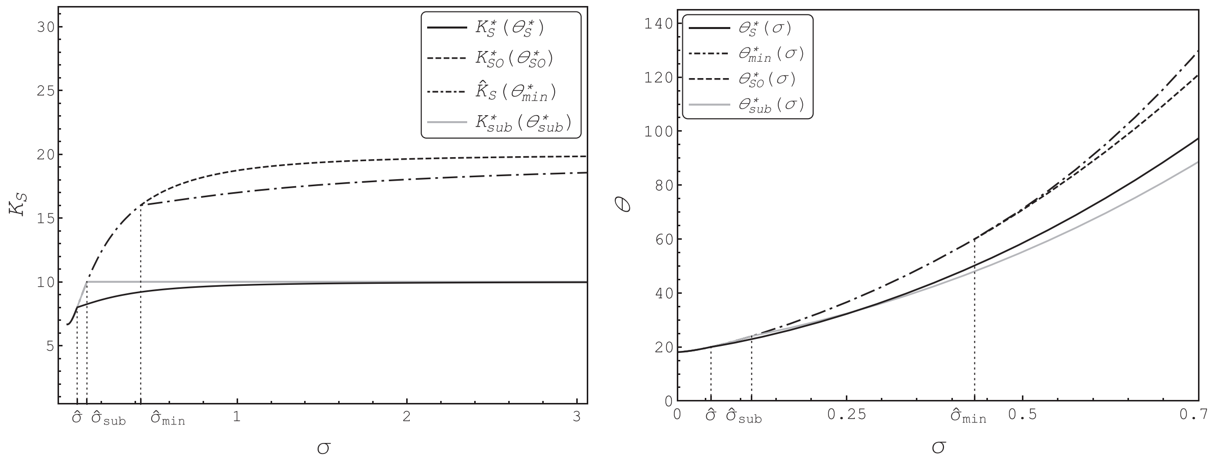
and the optimal capacity level is equal to $K_{sub}^*(\theta) = \frac{1}{2\eta}\left(1 - \frac{(\delta-\epsilon)(\rho-\alpha)}{\theta}\right)$. If $\beta \geq \frac{2(\delta-\epsilon)}{\gamma+\epsilon} + 3$, then the optimal investment threshold is equal to the socially optimal threshold level given by $\theta_{sub}^* = \theta_{SO}^*$ and the optimal capacity level is equal to $K_{sub}^*(\theta) = K_{SO}^*(\theta)$.

Also, in the presence of a subsidy, the TSO can only enforce the social optimum for low values of volatility such that

$$\sigma < \hat{\sigma}_{sub} \equiv \sqrt{\frac{\left[\rho - \alpha\left(\frac{2(\delta-\epsilon)}{\gamma+\epsilon} + 3\right)\right]^+}{\left(\frac{\delta-\epsilon}{\gamma+\epsilon} + 1\right)\left(\frac{2(\delta-\epsilon)}{\gamma+\epsilon} + 3\right)}}. \tag{31}$$

Note that the region where the social optimum can be attained increases in the presence of an investment cost subsidy, i.e. $\hat{\sigma} < \hat{\sigma}_{sub}$. This is because $\frac{\delta-\epsilon}{\gamma+\epsilon} < \frac{\delta}{\gamma}$ and $\frac{\partial \hat{\sigma}}{\partial\left(\frac{\delta}{\gamma}\right)} < 0$ from Proposition 6. Nevertheless, the positive effect of the subsidy on the social welfare is restricted due to the natural upper boundary of $\epsilon = \delta$, which corresponds to the case in which the TSO fully covers the PC's investment cost. Furthermore note that for $\delta \leq \gamma$, which is typically the case for the electricity industry (DeSantis et al., 2021; Energy Information Administration, 2020), it is easy to show that $\hat{\sigma}_{sub} < \hat{\sigma}_{\min}$.[18] This implies that the minimum capacity restriction still leads to the largest region where the social optimum can be attained. If, however, $\delta > \gamma$, a sufficiently large subsidy $\epsilon > \gamma$ is sufficient to ensure that the social optimum is attained for larger values of σ than under the minimum capacity restriction when the marginal investment cost of the PC is not too large, i.e. $\delta < \frac{2\gamma^2}{\epsilon-\gamma}$. However, even in this case, the TSO is not able to reduce the welfare loss in very uncertain environments as opposed to the situation when it imposes the minimum capacity requirement. This is because as $\sigma \to \infty$, the relative welfare converges to the value under the private planning similarly to the case without the subsidy. Fig. 8 illustrates the relative welfare loss as a function of volatility for all the

[18] For $\delta \leq \gamma$, $\frac{\delta}{\gamma} + 1 \leq 2 < 3 + \frac{2(\delta-\epsilon)}{\gamma+\epsilon}$, from which it directly follows that $\hat{\sigma}_{sub} < \hat{\sigma}_{\min}$.



(a) Optimal capacity level.

(b) Optimal investment threshold.

Fig. 9. Optimal investment threshold and optimal capacity level with the restriction on minimum generation capacity and investment cost subsidy as a function of σ. [Parameter values: $\rho = 0.1$; $\alpha = 0.02$; $\sigma = 0.1$; $\delta = 50$; $\gamma = 100$; $\epsilon = 50$; $\eta = 0.05$.].

settings considered in our model for our base case when $\delta < \gamma$ and the TSO chooses to provide the maximum possible subsidy level, i.e. $\epsilon = \delta$.

Proposition 12 formalizes features of the corresponding optimal threshold and capacity.

Proposition 12. *In presence of an investment cost subsidy,* $0 < \epsilon \leq \delta$, *the optimal investment threshold of the TSO as well as its optimal capacity are increasing with volatility,* $\frac{\partial \theta^*_{sub}}{\partial \sigma} > 0$ *and* $\frac{\partial K^*_{sub}(\theta^*_{sub})}{\partial \sigma} \geq 0$. *In addition, for* $\sigma > \hat{\sigma}_{sub}$, *it also holds that* $\theta^*_{sub} < \theta^*_{SO}$ *and* $K^*_{UP}(\theta^*_S) < K^*_{sub}(\theta^*_{sub}) < K^*_{SO}(\theta^*_{SO})$.

As can be seen, in the case of subsidy, the TSO invests in a larger capacity than in the case of decentralized planning (thus, closer to the socially optimal level). However, for high values of volatility, it does so at a lower level of θ than the one in the decentralized setting. This is because the capacity choice of the PC becomes less sensitive with respect to the investment timing in the presence of a large subsidy of the investment cost. Thus, investment at a higher threshold by the TSO does not lead to any substantial increase in the capacity of the PC for larger values of volatility. Hence, the TSO invests at a lower threshold than in the decentralized setting, which leads to the net effect that the welfare loss is reduced due to discounting (Fig. 9).

7. Conclusions

Deregulation of most infrastructure industries has decoupled network and production decisions, which were once undertaken by a central authority. This issue of coordinating complementary investments by distinct entities with conflicting objectives is particularly evident in the energy sector, which faces legally binding targets for integrating more renewable energy sources into the power system. Indeed, substantial investment in new transmission capacity may be required given the remoteness of promising renewable sites.

In this article, we analyze the optimal timing and optimal sizing of such transmission investments taking into account the misaligned objectives of the involved decision makers. In our model, the TSO aims to maximize social welfare, whereas the PC maximizes its profit. Their decisions are, however, connected in the sense that the PC cannot operate the generation capacity unless the infrastructure is installed. Therefore, by undertaking an investment in the infrastructure of a certain size, the TSO can naturally restrict the PC by installing a maximum capacity and a lower bound for investment timing.

Vis-à-vis the extent literature, we make three distinct contributions:

1. Determine market conditions under which the TSO may enforce the socially optimal transmission and production investments.
2. Relative to a socially optimal integrated benchmark setting, calculate the welfare loss resulting from a lack of coordination.
3. Provide insights about the viability of the minimum capacity requirement and the investment cost subsidy in the decentralized setting.

In particular, first, we find that the TSO can naturally enforce the social optimal outcome when market uncertainty is relatively low and/or the marginal investment cost of the TSO is relatively high compared to the marginal generation investment cost. Intuitively, in a more uncertain environment, the TSO cannot align the decision of the PC with the social optimum. In this scenario, however, anticipating the optimal decisions of the PC spares the TSO from overinvestment and, therefore, avoidable welfare loss.

Second, the relative welfare under private planning by the PC is a constant with respect to the volatility. This is related to the result from Huisman & Kort (2015) in which a monopolist invests at the same time as a social planner but installs half of the socially optimal capacity. By contrast, non-anticipative planning by the TSO can exacerbate welfare losses even vis-à-vis private planning by ignoring the PC's decisions for moderate levels of volatility. However, the relative welfare loss decays for higher volatility as more waiting is optimal regardless of the setting. Finally, decentralized planning in which the TSO anticipates the PC's optimal decisions has relatively low welfare losses and can even attain the social optimum provided that the volatility is below the critical threshold, $\hat{\sigma}$.

Third, we find that although the social optimum cannot always be reached with a minimum capacity restriction mechanism or an investment cost subsidy, it, nevertheless, allows the TSO to enforce the social optimum for a wider range of market conditions, especially in the case of the minimum capacity restriction. Specifically, we prove rigorously that enforceability of the mechanism depends on volatility and its relationship to the marginal infrastructure and generation investment costs. Hence, this finding may be used by policymakers to enable TSOs to incentivize socially optimal behavior by PCs in a decentralized setting.

This study provides several promising directions for further research. First, one could disentangle the objectives of the regulated monopolist and the regulator by assuming a profit-maximizing merchant transmission investor (Maurovich-Horvat et al., 2015). Second, it would be also interesting to investigate the effect of different renewable energy support schemes similar to Boomsma et al. (2012) that studies investment timing and capacity choice of profit-maximizing energy producers. Therefore, a valuable extension of this analysis would be to compare the effects of existing support mechanisms such as feed-in tariffs, feed-in premiums, or renewable energy certificate trading on the investment timing and size. Third, as part of our analysis, we did not include transmission congestion or competition at the level of the power companies (Baringo & Conejo, 2012; Murphy & Smeers, 2005; Sauma & Oren, 2007). Hence, a pertinent extension for future work could be to explore oligopolistic competition at the lower level and separate nodes with congested transmission lines.

Appendix A. Proofs of propositions

Proof of Proposition 1.. First, we derive the now-or-never optimal capacity investment, $K^*_{UP}(\theta)$, for a given level of θ, i.e., the capacity that maximizes the value of the PC in the stopping region:

$$\sup_{K_P} V_{UP}(\theta, K_P) = \sup_{K_P}\left[\frac{\theta(1-\eta K_P)K_P}{\rho-\alpha} - \delta K_P\right]. \tag{32}$$

Optimizing with respect to capacity yields

$$K^*_{UP}(\theta) = \frac{1}{2\eta}\left(1 - \frac{\delta(\rho-\alpha)}{\theta}\right)^+. \tag{33}$$

Second, we derive the optimal timing of the investment. The PC solves the following optimal stopping problem

$$\sup_{\tau} V_{UP}(\theta), \tag{34}$$

where $V_{UP}(\theta) = V_{UP}(\theta, K^*_{UP}(\theta))$.

The solution to the optimal stopping problem is defined by a threshold that separates a continuation, $\mathcal{C}$ and a stopping region $\mathcal{S}$. Let the value in the continuation region be denoted by F. Then the optimal value function is

$$V^*_{UP}(\theta) = \begin{cases} F(\theta) \text{ on } \mathcal{C}, \\ V_{UP}(\theta) \text{ on } \Re\setminus\mathcal{C}, \end{cases} \tag{35}$$

where the continuation and stopping regions are defined as

$$\mathcal{C} = \{\theta \in \Re | V^*_{UP}(\theta) > V\theta)\}, \tag{36}$$

$$\Re\setminus\mathcal{C} = \{\theta \in \Re | V^*_{UP}(\theta) = V_{UP}(\theta)\}. \tag{37}$$

The optimal stopping time can be written as

$$\tau = \inf\{t > 0; \theta_t \notin \mathcal{C}\}. \tag{38}$$

To solve the optimal stopping problem in (34), we need to find the function V^*_{UP}, which is the smallest superharmonic function dominating the gain function V (Peškir & Shiryaev, 2006), i.e., the following conditions must be satisfied:

$$\begin{cases} \mathcal{L}V^*_{UP} - \rho V^*_{UP} \le 0,\ (F^* \text{ minimal}), \\ V^*_{UP} \ge V,\ (V^*_{UP} > V \text{ on } \mathcal{C}\ \&\ V^*_{UP} = V \text{ on } \Re\setminus\mathcal{C}), \end{cases} \tag{39}$$

where $\mathcal{L}$ is infinitesimal generator of θ.[19]

As the value function $V_{UP}(\theta)$ is increasing, the continuation region can be written as $\mathcal{C} = (0, \theta^*_{UP})$, and the solution can be expressed in terms of the optimal investment threshold, θ^*_{UP}. This threshold can be found by applying the following value matching and smooth pasting conditions (Dixit & Pindyck, 1994):

$$F(\theta^*_{UP}) = V_{UP}(\theta^*_{UP}), \tag{40}$$

$$\frac{\partial F(\theta)}{\partial\theta}\bigg|_{\theta=\theta^*_{UP}} = \frac{\partial V_{UP}(\theta)}{\partial\theta}\bigg|_{\theta=\theta^*_{UP}}, \tag{41}$$

where F is the solution of

$$\frac{1}{2}\sigma^2\theta^2\frac{\partial^2 F(\theta)}{\partial\theta^2} + \alpha\theta\frac{\partial F(\theta)}{\partial\theta} - \rho F(\theta) = 0. \tag{42}$$

It is well known from the literature (see, e.g. Huisman & Kort (2015)) that in this case $F = A\theta^\beta$, where A is a constant and β is given by

$$\beta = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(-\frac{1}{2} + \frac{\alpha}{\sigma^2}\right)^2 + \frac{2\rho}{\sigma^2}}. \tag{43}$$

Note that as $\frac{\partial V_{UP}(\theta,K_P)}{\partial K_P}\big|_{K_P=K^*_{UP}(\theta)} = 0$, it holds that $\frac{\partial V_{UP}(\theta,K^*_{UP}(\theta))}{\partial\theta} = \frac{\partial V_{UP}(\theta,K_P)}{\partial\theta}\big|_{K_P=K^*_{UP}(\theta)}$. Thus, (40) is equivalent to solving the following system of equations for θ first and then taking into account the expression for $K^*_{UP}(\theta)$:

$$A\theta^\beta = \frac{\theta(1-\eta K_P)K_P}{\rho-\alpha} - \delta K_P, \tag{44}$$

$$\beta A\theta^{\beta-1} = \frac{(1-\eta K_P)K_P}{\rho-\alpha}. \tag{45}$$

The resulting investment threshold is

$$\theta^*_{UP}(K_P) = \frac{\beta}{\beta-1}\frac{\delta(\rho-\alpha)}{(1-\eta K_P)}. \tag{46}$$

In the unconstrained case, i.e., when $K_S > K^*_{UP}(\theta)$, the firm installs capacity $K^*_P(\theta, K_S) = K^*_{UP}(\theta)$. Therefore, the resulting investment threshold and capacity level at the threshold satisfy the following system of equations

$$\begin{cases} \theta &= \frac{\beta}{\beta-1}\frac{\delta(\rho-\alpha)}{(1-\eta K_P)}, \\ K_P &= \frac{1}{2\eta}\left(1 - \frac{\delta(\rho-\alpha)}{\theta}\right)^+. \end{cases} \tag{47}$$

The resulting $K^*_{UP}(\theta^*_{UP})$ and θ^*_{UP}

$$\theta^*_{UP} = \frac{(\beta+1)}{(\beta-1)}\delta(\rho-\alpha), \tag{48}$$

$$K^*_{UP}(\theta^*_{UP}) = \frac{1}{\eta(\beta+1)}. \tag{49}$$

If the current value of θ is such that $\theta \ge \theta^*_{UP}$, then it is optimal for the firm to invest immediately and install $K^*_{UP}(\theta) = \frac{1}{2\eta}\left(1 - \frac{\delta(\rho-\alpha)}{\theta}\right)$. Note that for $\beta > 1$ and any $\theta \ge \theta^*_{UP}$ it holds that the optimal $K^*_{UP}(\theta) < \frac{1}{\eta}$. If $\theta < \theta^*_{UP}$, then the firm will wait with investment until θ reaches θ and install $K^*_{UP}(\theta^*_{UP}) = \frac{1}{\eta(\beta+1)} < \frac{1}{\eta}$. This yields the result of the proposition. □

Proof of Proposition 2. The PC is constrained by TSO and will choose the investment level $K^*_P(\theta, K_S) = K_S$. Then, the value in the stopping region is equal to

$$V_{CP}(\theta) = \frac{\theta(1-\eta K_S)K_S}{\rho-\alpha} - \delta K_S. \tag{50}$$

[19] $\mathcal{L}f(\theta) = \lim_{t\to 0}\frac{\mathbf{E}_\theta[f(\theta)] - f(\theta)}{t}$.

Value matching and smooth pasting yield

$$A\theta^{\beta} = \frac{\theta(1-\eta K_S)K_S}{\rho-\alpha} - \delta K_S, \tag{51}$$

$$\beta A\theta^{\beta-1} = \frac{(1-\eta K_S)K_S}{\rho-\alpha}. \tag{52}$$

The resulting investment threshold is

$$\theta^*_{CP}(K_S) = \frac{\beta}{\beta-1}\frac{\delta(\rho-\alpha)}{(1-\eta K_S)}. \tag{53}$$

If the current value of θ is such that $\theta \geq \theta^*_{CP}(K_S)$, then it is optimal for the firm to invest immediately, if $\theta < \theta^*_{CP}(K_S)$, the firm will wait with investment. The capacity level such that the firm is indifferent between investing now or postponing is denoted by $\widehat{K}_S(\theta)$ and is determined as follows

$$\theta = \frac{\beta}{\beta-1}\frac{\delta(\rho-\alpha)}{(1-\eta K_S)}, \tag{54}$$

$$\widehat{K}_S(\theta) = \frac{1}{\eta}\left(1-\frac{\beta}{\beta-1}\frac{\delta(\rho-\alpha)}{\theta}\right)^{+}. \tag{55}$$

- If $K_S \leq \frac{1}{\eta}\left(1-\frac{\beta}{\beta-1}\frac{\delta(\rho-\alpha)}{\theta}\right)^{+} \equiv \widehat{K}_S(\theta)$, then the PC invests immediately at θ and installs capacity K_S. Combining with the condition for constrained case, we get
$$K_S \leq \min\left\{\widehat{K}_S(\theta), K^*_{UP}(\theta)\right\}. \tag{56}$$
- If $K_S > \frac{1}{\eta}\left(1-\frac{\beta}{\beta-1}\frac{\delta(\rho-\alpha)}{\theta}\right)^{+} \equiv \widehat{K}_S(\theta)$, then the PC postpones investment until $\theta^*_{CP}(K_S)$ and installs capacity K_S. Combining with the condition for constrained case, we get that $\widehat{K}_S(\theta) < K_S \leq K^*_{UP}(\theta)$. So, it must hold that $\widehat{K}_S(\theta) \leq K^*_{UP}(\theta)$. This condition can be transformed as follows
$$\begin{aligned}\frac{1}{\eta}\left(1-\frac{\beta}{\beta-1}\frac{\delta(\rho-\alpha)}{\theta}\right) &\leq \frac{1}{2\eta}\left(1-\frac{\delta(\rho-\alpha)}{\theta}\right),\\ \frac{1}{2} &\leq \frac{\delta(\rho-\alpha)}{\theta}\left(\frac{\beta}{\beta-1}-\frac{1}{2}\right),\\ \theta &\leq \frac{(\beta+1)}{(\beta-1)}\delta(\rho-\alpha).\end{aligned} \tag{57}$$

□

Proof of Proposition 3. First, consider the case where the PC's optimal capacity is an interior solution, i.e., the PC is not constrained by the TSO, $K_S > K^*_{UP}(\theta, K_S)$. This corresponds to Scenarios III (Unconstrained-Simultaneous) and IV (Unconstrained-Sequential) in Fig. 2. In Scenario III, the TSO's optimal stopping problem becomes

$$\sup_{\substack{\tau_S \geq \tau^*_{UP},\\ K_S > K^*_{UP}(\theta)}} \mathbb{E}\left[-\gamma K_S e^{-\rho\tau_S} + \int_{\tau_S}^{\infty}\theta_t\left(1-\frac{1}{2}\eta K^*_{UP}(\theta_t)\right)K^*_{UP}(\theta_t)e^{-\rho t}dt \right. \\ \left. -\delta K^*_{UP}(\theta_{\tau_S})e^{-\rho\tau_S}\middle|\theta_0=\theta\right], \tag{58}$$

where $\tau^*_{UP} = \min\{t \geq \tau_S : \theta_t \geq \theta^*_{UP}\}$.

In the stopping region, the TSO optimizes its value with respect to capacity. The value in the stopping region is then given by:

$$\begin{aligned}&\sup_{K_S > K^*_{UP}(\theta)}\left(\mathbb{E}\left[\int_0^{\infty}\theta_t\left(1-\frac{1}{2}\eta K^*_{UP}(\theta_t)\right)K^*_{UP}(\theta_t)e^{-\rho t}dt \right.\right.\\ &\qquad \left.\left. -\delta K^*_{UP}(\theta)\middle|\theta_0=\theta\right]-\gamma K_S\right)\\ &=\mathbb{E}\left[\int_0^{\infty}\theta_t\left(1-\frac{1}{2}\eta K^*_{UP}(\theta_t)\right)K^*_{UP}(\theta_t)e^{-\rho t}dt - \delta K^*_{UP}(\theta)\middle|\theta_0=\theta\right]\\ &\quad + \sup_{K_S > K^*_{UP}(\theta)}(-\gamma K_S).\end{aligned} \tag{59}$$

Thus, the optimum is a corner solution, leading to the case when $K_S = K^*_{UP}(\theta)$, which is incorporated in Scenario I in Fig. 2.

In Scenario IV, the TSO's optimal stopping problem becomes

$$\sup_{\substack{\tau_S < \tau^*_{UP},\\ K_S > K^*_{UP}(\theta)}} \mathbb{E}\left[-\gamma K_S e^{-\rho\tau_S} + \int_{\tau^*_{UP}}^{\infty}\theta_t\left(1-\frac{1}{2}\eta K^*_{UP}(\theta_t)\right)K^*_{UP}(\theta_t)e^{-\rho t}dt \right. \\ \left. -\delta K^*_{UP}(\theta_{\tau^*_{UP}})e^{-\rho\tau^*_{UP}}\middle|\theta_0=\theta\right]. \tag{60}$$

Similar to the above case, the value in the stopping region is then given by:

$$\begin{aligned}&\sup_{K_S > K^*_{UP}(\theta)}\left(\mathbb{E}\left[\int_{\tau^*_{UP}}^{\infty}\theta_t\left(1-\frac{1}{2}\eta K^*_{UP}(\theta_t)\right)K^*_{UP}(\theta_t)e^{-\rho t}dt \right.\right.\\ &\qquad \left.\left. -\delta K^*_{UP}(\theta_{\tau^*_{UP}})e^{-\rho\tau^*_{UP}}\middle|\theta_0=\theta\right]-\gamma K_S\right)\\ &=\mathbb{E}\left[\int_{\tau^*_{UP}}^{\infty}\theta_t\left(1-\frac{1}{2}\eta K^*_{UP}(\theta_t)\right)K^*_{UP}(\theta_t)e^{-\rho t}dt \right.\\ &\qquad \left. -\delta K^*_{UP}(\theta_{\tau^*_{UP}})e^{-\rho\tau^*_{UP}}\middle|\theta_0=\theta\right] + \sup_{K_S > K^*_{UP}(\theta)}(-\gamma K_S).\end{aligned} \tag{61}$$

Again, the supremum is reached at the corner, i.e., $K_S = K^*_{UP}(\theta)$, which is already incorporated in Scenario I in Fig. 2.

Lastly, we show that even if the PC is constrained in capacity, it is not possible for the TSO to allow for the sequential investment of the PC, i.e., it is not possible to end up in Scenario II (Constrained-Sequential) in Fig. 2.

In the Scenario II, the TSO's optimal stopping problem becomes

$$\sup_{\substack{\tau_S < \tau^*_{CP}(K_S),\\ \widehat{K}_S(\theta) < K_S < K^*_{UP}(\theta)}} \mathbb{E}\left[-\gamma K_S e^{-\rho\tau_S} + \int_{\tau^*_{CP}(K_S)}^{\infty}\theta_t\left(1-\frac{1}{2}\eta K_S\right)K_S e^{-\rho t}dt \right. \\ \left. -\delta K_S e^{-\rho\tau^*_{CP}(K_S)}\middle|\theta_0=\theta\right], \tag{62}$$

where $\tau^*_{CP}(K_S) = \min\{t \geq \tau_S : \theta_t \geq \theta^*_{CP}(K_S)\}$.

The value in the stopping region of the TSO is

$$\begin{aligned}V(\theta, K_S) &= \left(\frac{\theta}{\theta^*_{CP}(K_S)}\right)^{\beta}\left(\frac{\theta^*_{CP}(K_S)(1-\frac{1}{2}\eta K_S)K_S}{\rho-\alpha} - \delta K_S\right) - \gamma K_S\\ &= \left(\frac{\theta(\beta-1)(1-\eta K_S)}{\beta\delta(\rho-\alpha)}\right)^{\beta}\left(\frac{\beta(1-\frac{1}{2}\eta K_S)}{(\beta-1)(1-\eta K_S)} - 1\right)\delta K_S - \gamma K_S.\end{aligned} \tag{63}$$

Suppose the TSO's value function in (63) is maximized at $\tilde{K}_S(\theta)$. Then, to determine the investment threshold the following system must be solved:

$$A\theta^{\beta} = V(\theta, \tilde{K}_S(\theta)), \tag{64}$$

$$\beta A\theta^{\beta-1} = \frac{\partial V(\theta, \tilde{K}_S(\theta))}{\partial\theta} + \frac{\partial V(\theta, K_S)}{\partial K_S}\bigg|_{K_S=\tilde{K}_S(\theta)}\frac{\partial \tilde{K}_S(\theta)}{\partial\theta}. \tag{65}$$

Plugging the expression for $V(\theta, K_S)$ yields

$$A\theta^{\beta} = \left(\frac{\theta}{\theta^*_{CP}(\tilde{K}_S(\theta))}\right)^{\beta}\left(\frac{\beta(1-\frac{1}{2}\eta\tilde{K}_S(\theta))}{(\beta-1)(1-\eta\tilde{K}_S(\theta))} - 1\right) \\ \delta\tilde{K}_S(\theta) - \gamma\tilde{K}_S(\theta), \tag{66}$$

$$A\theta^{\beta} = \left(\frac{\theta}{\theta^*_{CP}(\tilde{K}_S(\theta))}\right)^{\beta}\left(\frac{\beta(1-\frac{1}{2}\eta\tilde{K}_S(\theta))}{(\beta-1)(1-\eta\tilde{K}_S(\theta))} - 1\right)\delta\tilde{K}_S(\theta) \\ + \frac{\theta}{\beta}\frac{\partial\tilde{K}_S(\theta)}{\partial\theta}\frac{\partial V(\theta, K_S)}{\partial K_S}\bigg|_{K_S=\tilde{K}_S(\theta)}. \tag{67}$$

Table A1
Optimal capacity of the TSO given the constraint $K_S \le \min\left\{\widehat{K}_S(\theta), K^*_{UP}(\theta)\right\}$.

$\beta \le \frac{\delta}{\gamma}+1$		
$\theta < (\delta+2\gamma)(\rho-\alpha)$	$(\delta+2\gamma)(\rho-\alpha) \le \theta < \frac{(\beta+1)}{(\beta-1)}\delta(\rho-\alpha)$	$\theta \ge \frac{(\beta+1)}{(\beta-1)}\delta(\rho-\alpha)$
$\widehat{K}_S(\theta) \le K^*_{SO}(\theta) < K^*_{UP}(\theta)$ Optimum at $\widehat{K}_S(\theta)$	$\widehat{K}_S(\theta) < K^*_{UP}(\theta) \le K^*_{SO}(\theta)$ Optimum at $\widehat{K}_S(\theta)$	$K^*_{UP}(\theta) \le \widehat{K}_S(\theta) \le K^*_{SO}(\theta)$ Optimum at $K^*_{UP}(\theta)$
	$\beta > \frac{\delta}{\gamma}+1$	
$\theta < \frac{(\beta+1)}{(\beta-1)}\delta(\rho-\alpha)$	$\frac{(\beta+1)}{(\beta-1)}\delta(\rho-\alpha) \le \theta < (\delta+2\gamma)(\rho-\alpha)$	$\theta \ge (\delta+2\gamma)(\rho-\alpha)$
$K^*_{SO}(\theta) < \widehat{K}_S(\theta) < K^*_{UP}(\theta)$ Optimum at K^*_{SO}	$K^*_{SO}(\theta) < K^*_{UP}(\theta) \le \widehat{K}_S(\theta)$ Optimum at K^*_{SO}	$K^*_{UP}(\theta) \le K^*_{SO}(\theta) < \widehat{K}_S(\theta)$ Optimum at $K^*_{UP}(\theta)$

However, for $\tilde{K}_S(\theta) > 0$, combining (66) and (67) yields

$$A\theta^\beta - A\theta^\beta = -\gamma \tilde{K}_S(\theta) - \frac{\theta}{\beta}\frac{\partial \tilde{K}_S(\theta)}{\partial \theta}\frac{\partial V(\theta, K_S)}{\partial K_S}\bigg|_{K_S=\tilde{K}_S(\theta)} > 0. \quad (68)$$

To see why the last inequality holds observe the following. First, suppose $\tilde{K}_S(\theta)$ to be an interior optimum. Then, the last term in (68) disappears as the first-order condition applies. Second, suppose now that the interior optimum does not exist. Then, for a given value of θ, this would lead to a corner solution at either i) the capacity level that insures simultaneous investment, $\tilde{K}_S(\theta) = \widehat{K}_S(\theta)$, or ii) the capacity level that ensures investment of equal size, i.e., $\tilde{K}_S(\theta) = K^*_{UP}(\theta)$.[20] The first case is a part of Scenario I where the PC is constrained both in timing and in capacity. In the second case, note that $\frac{\partial V(\theta,K_S)}{\partial K_S}\Big|_{K_S=K^*_{UP}(\theta)} < 0$, otherwise $V(\theta, K_S)$ is not maximized at $K^*_{UP}(\theta)$, because we can find a point in the interior, which leads to a larger value. Together with the fact that $\frac{\partial K^*_{UP}(\theta)}{\partial \theta} > 0$, this implies that the left hand side of (68) is always positive. Hence, the optimal investment threshold does not exist and it is always optimal for the TSO to wait. □

Proof of Proposition 4. In Scenario I (Constrained-Simultaneous), the TSO solves the following capacity optimization problem:

$$\sup_{K_S \le \min\left\{\widehat{K}_S(\theta), K^*_{UP}(\theta)\right\}} V(\theta, K_S) = \sup_{K_S}\left[\frac{\theta(1-\frac{1}{2}\eta K_S)K_S}{\rho-\alpha} - (\delta+\gamma)K_S\right]. \quad (69)$$

Note that the unconstrained optimum of this problem is equal to the social optimum. This is because if the TSO decides on both production and infrastructure investments, then it is never optimal to install infrastructure capacity that is not used for production, i.e., $K_P = K_S$. Moreover, once the infrastructure is installed, there is no incentive to leave it unused. Therefore, both investments occur at the same time, and the TSO solves the following optimal stopping problem:

$$\sup_{\tau_S, K_S} \mathbb{E}\left[\int_{\tau_S}^{\infty} \theta_t\left(1 - \frac{1}{2}\eta K_S\right)K_S e^{-\rho t}dt - (\delta+\gamma)K_S e^{-\rho\tau_S}\Big|\theta_0 = \theta\right]. \quad (70)$$

Let θ^*_{SO} denote the optimal investment threshold, θ^*_{SO}. The optimal investment time, τ^*, is equal to the first time the stochastic process hits the optimal level, θ^*_{SO}, i.e., $\tau^* \equiv \min\{t \ge 0 : \theta_t \ge \theta^*_{SO}\}$. The corresponding optimal capacity is denoted by $K^*_{SO}(\theta^*_{SO})$. Analogously to the proof of Proposition 1 and similar to Huisman & Kort (2015), it can be shown that that the solution of this problem is given as follows. The socially optimal investment threshold is

$$\theta^*_{SO} = \frac{(\beta+1)}{(\beta-1)}(\delta+\gamma)(\rho-\alpha), \quad (71)$$

and the socially optimal capacity choice is equal to

$$K^*_{SO}(\theta^*_{SO}) = \frac{2}{\eta(\beta+1)}, \quad (72)$$

where $\beta = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(-\frac{1}{2} + \frac{\alpha}{\sigma^2}\right)^2 + \frac{2\rho}{\sigma^2}}$.

If the current level of the stochastic process is such that $\theta > \theta^*_{SO}$, then the TSO invests immediately and installs capacity equal to

$$K^*_{SO}(\theta) = \frac{1}{\eta}\left(1 - \frac{(\delta+\gamma)(\rho-\alpha)}{\theta}\right)^+. \quad (73)$$

Therefore, the solution of (69) will depend on the relationship between $\widehat{K}_S(\theta)$, $K^*_{SO}(\theta)$ and $K^*_{UP}(\theta)$, which we formally state below.

- If $\theta \ge \frac{(\beta+1)}{(\beta-1)}\delta(\rho-\alpha)$, then $\widehat{K}_S(\theta) \ge K^*_{UP}(\theta)$, and otherwise $\widehat{K}_S(\theta) < K^*_{UP}(\theta)$.It can be shown that $\theta \ge \frac{(\beta+1)}{(\beta-1)}\delta(\rho-\alpha)$ implies $\widehat{K}_S(\theta) \ge K^*_{UP}(\theta)$ and vice versa as follows
$$\theta \ge \frac{(\beta+1)}{(\beta-1)}\delta(\rho-\alpha), \frac{1}{2} \ge \left(\frac{\beta}{\beta-1} - \frac{1}{2}\right)\frac{\delta(\rho-\alpha)}{\theta},$$
$$\frac{1}{\eta}\left(1 - \frac{\beta}{\beta-1}\frac{\delta(\rho-\alpha)}{\theta}\right) \ge \frac{1}{2\eta}\left(1 - \frac{\delta(\rho-\alpha)}{\theta}\right),$$
$$\widehat{K}_S(\theta) \ge K^*_{UP}(\theta).$$
- If $\theta \ge (\delta+2\gamma)(\rho-\alpha)$, then $K^*_{SO}(\theta) \ge K^*_{UP}(\theta)$, and otherwise $K^*_{SO}(\theta) < K^*_{UP}(\theta)$.It can be shown that $\theta \ge (\delta+2\gamma)(\rho-\alpha)$ implies $K^*_{SO}(\theta) \ge K^*_{UP}(\theta)$ and vice versa as follows
$$\theta \ge (\delta+2\gamma)(\rho-\alpha),$$
$$\frac{1}{2} \ge \left(\delta - \frac{\delta}{2} + \gamma\right)\frac{(\rho-\alpha)}{\theta},$$
$$\frac{1}{\eta}\left(1 - \frac{(\delta+\gamma)(\rho-\alpha)}{\theta}\right) \ge \frac{1}{2\eta}\left(1 - \frac{\delta(\rho-\alpha)}{\theta}\right),$$
$$K^*_{SO}(\theta) \ge K^*_{UP}(\theta).$$
- If $\beta \le \frac{\delta}{\gamma}+1$, then $K^*_{SO}(\theta) \ge \widehat{K}_S(\theta)$ and $(\delta+2\gamma)(\rho-\alpha) \le \frac{(\beta+1)}{(\beta-1)}\delta(\rho-\alpha)$, and otherwise $K^*_{SO}(\theta) < \widehat{K}_S(\theta)$ and $(\delta+2\gamma)(\rho-\alpha) > \frac{(\beta+1)}{(\beta-1)}\delta(\rho-\alpha)$.

It can be seen that $K^*_{SO}(\theta) \ge \widehat{K}_S(\theta)$ if and only if $\delta+\gamma \le \delta\frac{\beta}{\beta-1}$, implying that $\beta \le \frac{\delta}{\gamma}+1$.

Finally, if $\beta \le \frac{\delta}{\gamma}+1$, it holds that

$$\gamma \le \frac{\delta}{\beta-1}, 2\gamma \le \delta\left(\frac{(\beta+1)}{(\beta-1)} - 1\right),$$
$$(\delta+2\gamma)(\rho-\alpha) \le \frac{(\beta+1)}{(\beta-1)}\delta(\rho-\alpha).$$

[20] The case when the corner solution is at $K_S = 0$ is trivial because it would result into zero value of investment for TSO, and therefore will be equivalent to waiting.

Table A.3 summarizes these findings and states the optimal capacity choice in each of the cases.

□

Proof of Proposition 5. In what follows, we determine the optimal threshold for the optimal capacity choice in different cases: when $\beta > \frac{\delta}{\gamma}+1$ and when $\beta \leq \frac{\delta}{\gamma}+1$.

- $\beta > \frac{\delta}{\gamma}+1$

From Table A.3, the optimal capacity choice in this case is

$$\begin{cases} K^*_{SO}(\theta), & \text{if } \theta < (\delta+2\gamma)(\rho-\alpha), \\ K^*_{UP}(\theta), & \text{if } \theta \geq (\delta+2\gamma)(\rho-\alpha). \end{cases} \tag{74}$$

Note that the value for the TSO in the stopping region in this case is an increasing smooth function. In particular, the stopping value is equal to

$$V(\theta) = \begin{cases} V(\theta, K^*_{SO}(\theta)) = \frac{(\theta-(\delta+\gamma)(\rho-\alpha))^2}{2\eta\theta(\rho-\alpha)}, & \text{if } \theta < (\delta+2\gamma)(\rho-\alpha), \\ V(\theta, K^*_{UP}(\theta)) = \frac{(\theta-\delta(\rho-\alpha))(3\theta-(4\gamma+3\delta)(\rho-\alpha))}{8\eta\theta(\rho-\alpha)}, & \text{if } \theta \geq (\delta+2\gamma)(\rho-\alpha). \end{cases} \tag{75}$$

Now note that $\frac{\partial V(\theta,K^*_{SO}(\theta))}{\partial\theta} = \frac{\theta^2-(\delta+\gamma)^2(\rho-\alpha)^2}{2\eta\theta^2(\rho-\alpha)}$ and $\frac{\partial V(\theta,K^*_{UP}(\theta))}{\partial\theta} = \frac{3\theta^2-\delta(4\gamma+3\delta)(\rho-\alpha)^2}{8\eta\theta^2(\rho-\alpha)}$ are positive for $\theta > (\delta+\gamma)(\rho+\alpha)$ and $\theta \geq (\delta+2\gamma)(\rho-\alpha)$, respectively. Evaluating these derivatives at $\theta = (\delta+2\gamma)(\rho-\alpha)$, we get $\frac{\partial V(\theta,K^*_{SO}(\theta))}{\partial\theta}|_{\theta=(\delta+2\gamma)(\rho-\alpha)} = \frac{\partial V(\theta,K^*_{UP}(\theta))}{\partial\theta}|_{\theta=(\delta+2\gamma)(\rho-\alpha)} = \frac{\gamma(3\gamma+2\delta)}{2\eta(\delta+2\gamma)^2(\rho-\alpha)}$. Thus, the stopping value is monotonically increasing and smooth.

Using this result, we can derive the optimal thresholds in the regions $\theta < (\delta+2\gamma)(\rho-\alpha)$ and $\theta \geq (\delta+2\gamma)(\rho-\alpha)$ separately, and then verify if the constraint is satisfied.

We start with the region $\theta < (\delta+2\gamma)(\rho-\alpha)$, where the optimal capacity choice for the TSO is $K^*_{SO}(\theta)$. Recall that in the integrated case, the TSO's optimal capacity is equal to $K^*_{SO}(\theta)$, and the corresponding optimal investment threshold is equal to

$$\theta^*_{SO} = \frac{(\beta+1)}{(\beta-1)}(\delta+\gamma)(\rho-\alpha). \tag{76}$$

This expression is also the solution for the optimal stopping problem in the decentralized case if the constraint $\theta^*_{SO} < (\delta+2\gamma)(\rho-\alpha)$ is satisfied, i.e., when $\frac{(\beta+1)}{(\beta-1)}(\delta+\gamma)(\rho-\alpha) \leq (\delta+2\gamma)(\rho-\alpha)$. It is easy to verify that this is true if and only if $\beta > \frac{2\delta}{\gamma}+3$.

In the complementary case, when $\theta \geq (\delta+2\gamma)(\rho-\alpha)$, the optimal θ solves the following equation:

$$V(\theta, K^*_{UP}(\theta)) - \frac{\theta}{\beta}\left(\frac{\partial V(\theta, K^*_{UP}(\theta))}{\partial\theta} + \frac{\partial V(\theta, K_S)}{\partial K_S}\bigg|_{K_S=K^*_{UP}(\theta)} \frac{\partial K^*_{UP}(\theta)}{\partial\theta}\right) = 0. \tag{77}$$

Plugging the expression for $K^*_{UP}(\theta)$ into (77) yields

$$\frac{3(\beta-1)\theta^2 - 2\beta\theta(2\gamma+3\delta)(\rho-\alpha) + (\beta+1)\delta(4\gamma+3\delta)(\rho-\alpha)^2}{8\beta\eta\theta(\rho-\alpha)} = 0. \tag{78}$$

The function of θ in the numerator is a convex parabola, such that its largest root is the solution for the optimal threshold.[21] To verify whether the constraint $\theta \geq (\delta+2\gamma)(\rho-\alpha)$ holds we evaluate this expression at $\theta = (\delta+2\gamma)(\rho-\alpha)$, and get $4\gamma(\rho-\alpha)^2((\beta-3)\gamma-2\delta)$. This expression is negative for $\beta \leq \frac{2\delta}{\gamma}+3$, implying that the constraint is satisfied and the optimal solution is the positive root of the parabola given by

$$\theta^*_S = \frac{(\rho-\alpha)\left(\beta(2\gamma+3\delta)+\sqrt{4\gamma^2\beta^2+3\delta(4\gamma+3\delta)}\right)}{3(\beta-1)}. \tag{79}$$

If $\beta > \frac{2\delta}{\gamma}+3$, note that $\theta = (\delta+2\gamma)(\rho-\alpha)$ is larger than the value of θ for which the parabola reaches its minimum, i.e., $\theta = \frac{\beta}{\beta-1}\frac{(3\delta+2\gamma)(\rho-\alpha)}{3}$, if $\gamma > \frac{3\delta}{2(2\beta-3)}$. Thus, for $\gamma > \frac{2\delta}{\beta-3} > \frac{3\delta}{2(2\beta-3)}$, the candidate threshold value is lower than $(\delta+2\gamma)(\rho-\alpha)$ and the constraint is not satisfied.

To summarize, the optimal threshold for the parameter values such that $\beta > \frac{\delta}{\gamma}+1$ the optimal threshold is equal to

$$\begin{cases} \theta^*_{SO}, & \text{if } \beta > \frac{2\delta}{\gamma}+3, \\ \theta^*_S, & \text{if } \beta \leq \frac{2\delta}{\gamma}+3. \end{cases} \tag{80}$$

- $\beta \leq \frac{\delta}{\gamma}+1$.

From Table A.3, the optimal capacity choice in this case is

$$\begin{cases} \widehat{K}_S(\theta), & \text{if } \theta < \frac{(\beta+1)}{(\beta-1)}\delta(\rho-\alpha), \\ K^*_{UP}(\theta), & \text{if } \theta \geq \frac{(\beta+1)}{(\beta-1)}\delta(\rho-\alpha). \end{cases} \tag{81}$$

Consider first the situation when $\theta < \frac{(\beta+1)}{(\beta-1)}\delta(\rho-\alpha)$ so that the TSO chooses its optimal capacity at the level $\widehat{K}_S(\theta)$. In this case, the optimal θ solves the following equation:

$$V\left(\theta, \widehat{K}_S(\theta)\right) - \frac{\theta}{\beta}\left(\frac{\partial V(\theta, \widehat{K}_S(\theta))}{\partial\theta} + \frac{\partial V(\theta, K_S)}{\partial K_S}\bigg|_{K_S=\widehat{K}_S(\theta)} \frac{\partial \widehat{K}_S(\theta)}{\partial\theta}\right) = 0. \tag{82}$$

Plugging the expression for $\widehat{K}_S(\theta)$ into (82) yields

$$\frac{(\beta-1)^2\theta^2 - 2(\beta-1)\beta\theta(\delta+\gamma)(\rho-\alpha) + \beta(\beta+1)\delta(\rho-\alpha)^2\left(\frac{(\beta-2)\delta}{\beta-1}+2\gamma\right)}{2\eta(\rho-\alpha)\beta(\beta-1)\theta} = 0. \tag{83}$$

The function of θ in the numerator is a convex parabola, such that its positive root is the solution for the optimal threshold[22] Evaluating this function at $\theta = \frac{(\beta+1)}{(\beta-1)}\delta(\rho-\alpha)$ yields $-\frac{(\beta+1)\delta^2(\rho-\alpha)^2}{\beta-1} < 0$. This implies that for $\theta < \frac{(\beta+1)}{(\beta-1)}\delta(\rho-\alpha)$ the investment threshold does not exist, and it is optimal to wait.

Thus, the TSO will optimally wait until it is optional to install $K^*_{UP}(\theta)$, i.e., $\theta \geq \frac{(\beta+1)}{(\beta-1)}\delta(\rho-\alpha)$. The threshold in this case is defined by (79). To verify that $\theta^*_S \geq \frac{(\beta+1)}{(\beta-1)}\delta(\rho-\alpha)$, we evaluate the parabola in (78) at $\theta = \frac{(\beta+1)}{(\beta-1)}\delta(\rho-\alpha)$. This yields $-\frac{4(\beta+1)\gamma\delta(\rho-\alpha)^2}{\beta-1} < 0$, implying that $\theta^*_S > \frac{(\beta+1)}{(\beta-1)}\delta(\rho-\alpha)$. Thus, the value function of the TSO in the waiting region is given by $\left(\frac{\theta}{\theta^*_S}\right)^\beta V(\theta, K^*_{UP}(\theta^*_S))$.

The value function in the stopping region is increasing, however, it is no longer smooth as in the previous case. In addition Now note that both $\widehat{K}_S(\theta)$ and $K^*_{UP}(\theta)$ are smaller than $K^*_{SO}(\theta)$ where the maximum of the stopping value is reached (see Table A.3). Hence, we can conclude that $V(\theta, \widehat{K}_S(\theta)) < V(\theta, K^*_{UP})$ for $\theta < \frac{(\beta+1)}{(\beta-1)}\delta(\rho-\alpha)$ and $V(\theta, \widehat{K}_S(\theta)) \geq V(\theta, K^*_{UP})$ otherwise. Thus, it is easy to verify that for $\theta < \theta^*_S$, $V(\theta, \widehat{K}_S(\theta)) < V(\theta, K^*_{UP}) < \left(\frac{\theta}{\theta^*_S}\right)^\beta < V(\theta, K^*_{UP}(\theta^*_S))$. This sufficient to ensure that the optimal value function dominates the gain function.

[21] This is because $V(\theta, K^*_{UP}(\theta)) - \frac{\theta}{\beta}\left(\frac{\partial V(\theta,K^*_{UP}(\theta))}{\partial\theta} + \frac{\partial V(\theta,K_S)}{\partial K_S}\Big|_{K_S=K^*_{UP}(\theta)} \frac{\partial K^*_{UP}(\theta)}{\partial\theta}\right)$ must be smaller than zero for the values below the optimal threshold, otherwise stopping would be optimal in the waiting region.

[22] This is because $V(\theta, \widehat{K}_S(\theta)) - \frac{\theta}{\beta}\left(\frac{\partial V(\theta,\widehat{K}_S(\theta))}{\partial\theta} + \frac{\partial V(\theta,K_S)}{\partial K_S}\Big|_{K_S=\widehat{K}_S(\theta)} \frac{\partial \widehat{K}_S(\theta)}{\partial\theta}\right)$ must be smaller than zero for the values below the optimal threshold, otherwise stopping would be optimal in the waiting region.

To summarize, for $\beta \le \frac{\delta}{\gamma}+1$, the optimal capacity choice and the optimal investment threshold are $K^*_{UP}(\theta)$ and θ^*_S, respectively. Combining this result with the one for $\beta > \frac{\delta}{\gamma}+1$, we arrive at Proposition 6. □

Proof of Proposition 6. Here derive $\hat{\sigma}$, which is the value of σ such that $\beta = \frac{2\delta}{\gamma}+3$. As β solves $1/2\sigma^2\beta^2 + (\alpha - 1/2\sigma^2)\beta - \rho = 0$, it is straightforward to derive that $\hat{\sigma}$ is unique and given by

$$\hat{\sigma} = \sqrt{\frac{\left[\rho - \alpha\left(\frac{2\delta}{\gamma}+3\right)\right]^+}{\left(\frac{\delta}{\gamma}+1\right)\left(\frac{2\delta}{\gamma}+3\right)}}. \tag{84}$$

In addition, the comparative statics for $\hat{\sigma} > 0$ with respect to α, ρ, and $\frac{\delta}{\gamma}$ are given below

$$\frac{\partial \hat{\sigma}}{\partial \alpha} = -\frac{\gamma}{2(\delta+\gamma)\sqrt{\frac{\rho-\alpha\left(\frac{2\delta}{\gamma}+3\right)}{\left(\frac{\delta}{\gamma}+1\right)\left(\frac{2\delta}{\gamma}+3\right)}}} < 0, \tag{85}$$

$$\frac{\partial \hat{\sigma}}{\partial \rho} = \frac{\gamma^2}{2(\delta+\gamma)(3\gamma+2\delta)\sqrt{\frac{\rho-\alpha\left(\frac{2\delta}{\gamma}+3\right)}{\left(\frac{\delta}{\gamma}+1\right)\left(\frac{2\delta}{\gamma}+3\right)}}} > 0, \tag{86}$$

$$\frac{\partial \hat{\sigma}}{\partial \left(\frac{\delta}{\gamma}\right)} = -\frac{\rho\left(\frac{4\delta}{\gamma}+5\right) - \alpha\left(\frac{2\delta}{\gamma}+3\right)^2}{2\left(\frac{\delta}{\gamma}+1\right)^2\left(\frac{2\delta}{\gamma}+3\right)^2\sqrt{\frac{\rho-\alpha\left(\frac{2\delta}{\gamma}+3\right)}{\left(\frac{\delta}{\gamma}+1\right)\left(\frac{2\delta}{\gamma}+3\right)}}} < 0. \tag{87}$$

To derive the last inequality, consider the numerator of (87). As $\rho > \alpha\left(\frac{2\delta}{\gamma}+3\right)$ for $\hat{\sigma} > 0$, the following holds

$$\begin{aligned}\rho\left(\frac{4\delta}{\gamma}+5\right) - \alpha\left(\frac{2\delta}{\gamma}+3\right)^2 &> \alpha\left(\frac{2\delta}{\gamma}+3\right)\left(\frac{4\delta}{\gamma}+5\right) \\ &-\alpha\left(\frac{2\delta}{\gamma}+3\right)^2 \\ &= \alpha\left(\frac{4\delta}{\gamma}+5\right) - \alpha\left(\frac{2\delta}{\gamma}+3\right) = 2\alpha\left(\frac{\delta}{\gamma}+1\right) > 0.\end{aligned} \tag{88}$$

□

Proof of Proposition 7. First, we show that for $\sigma > \hat{\sigma}$, it holds that $\theta^*_{SO} > \theta^*_S$. Consider the following difference:

$$\begin{aligned}\theta^*_{SO} - \theta^*_S &= \frac{(\beta+1)}{(\beta-1)}(\delta+\gamma)(\rho-\alpha) - \frac{(\rho-\alpha)\left(\beta(2\gamma+3\delta) + \sqrt{4\gamma^2\beta^2 + 3\delta(4\gamma+3\delta)}\right)}{3(\beta-1)} \\ &= \frac{(\rho-\alpha)\left(3\delta + (\beta+3)\gamma - \sqrt{4\gamma^2\beta^2 + 3\delta(4\gamma+3\delta)}\right)}{3(\beta-1)} \\ &= \frac{(\rho-\alpha)\left(3\delta + (\beta+3)\gamma - \sqrt{(3\delta+(\beta+3)\gamma)^2 - 3(\beta+1)\gamma^2\left(\frac{2\delta}{\gamma}+3-\beta\right)}\right)}{3(\beta-1)} \\ &> 0.\end{aligned} \tag{89}$$

The last inequality holds, because $\beta < \frac{2\delta}{\gamma}+3$ for $\sigma > \hat{\sigma}$. Thus, $\theta^*_{SO} > \theta^*_S$.

Second, we show that $K^*_{SO}(\theta^*_{SO}) > K^*_{UP}(\theta^*_{SO}) > K^*_{UP}(\theta^*_S) > \frac{1}{2}K^*_{SO}(\theta^*_{SO})$. From Table A.3, it holds that $K^*_{SO}(\theta) > K^*_{UP}(\theta)$ for $\theta > (\delta+2\gamma)(\rho-\alpha)$. Now note that

$$\theta^*_{SO} - (\delta+2\gamma)(\rho-\alpha) = \gamma\left(\frac{2\delta}{\gamma}+3-\beta\right)\frac{(\rho-\alpha)}{\beta-1} > 0. \tag{90}$$

Thus, $K^*_{SO}(\theta^*_{SO}) > K^*_{UP}(\theta^*_{SO})$. In addition, as $K^*_{UP}(\theta)$ is an increasing function and from (89) $\theta^*_{SO} > \theta^*_S$, we can establish that $K^*_{UP}(\theta^*_{SO}) > K^*_{UP}(\theta^*_S)$. Lastly, consider the following difference:

$$\begin{aligned}K^*_{UP}(\theta) - \frac{1}{2}K^*_{SO}(\theta^*_{SO}) &= \frac{1}{2\eta}\left(1 - \frac{\delta(\rho-\alpha)}{\theta}\right) - \frac{1}{\eta(\beta+1)} \\ &= \frac{1}{2\eta}\left(\frac{\beta-1}{\beta+1} - \frac{\delta(\rho-\alpha)}{\theta}\right).\end{aligned} \tag{91}$$

This implies that $K^*_{UP}(\theta) > \frac{1}{2}K^*_{SO}(\theta^*_{SO})$ for $\theta > \frac{\beta+1}{\beta-1}\delta(\rho-\alpha)$. Thus, to conclude that $K^*_{UP}(\theta^*_S) > \frac{1}{2}K^*_{SO}(\theta^*_{SO})$ it is enough to show that $\theta^*_S > \frac{\beta+1}{\beta-1}\delta(\rho-\alpha)$. We show that this holds below.

$$\begin{aligned}\theta^*_S - \frac{\beta+1}{\beta-1}\delta(\rho-\alpha) &= \frac{(\rho-\alpha)\left(2\beta\gamma - 3\delta + \sqrt{4\gamma^2\beta^2 + 3\delta(4\gamma+3\delta)}\right)}{3(\beta-1)} \\ &= \frac{(\rho-\alpha)\left(2\beta\gamma - 3\delta + \sqrt{(2\beta\gamma-3\delta)^2 + 12(\beta+1)\gamma\delta}\right)}{3(\beta-1)} > 0.\end{aligned} \tag{92}$$

Thus, we conclude that $K^*_{UP}(\theta^*_S) > \frac{1}{2}K^*_{SO}(\theta^*_{SO})$. □

Proof of Proposition 8. First, consider the sensitivity of the investment thresholds with respect to volatility. In what follows we will use the fact that β decreases with σ and, thus, $\frac{\partial\beta}{\partial\sigma} < 0$. In the integrated case we have

$$\frac{\partial \theta^*_{SO}}{\partial \sigma} = \frac{2(\rho-\alpha)(\gamma+\delta)\left(-\frac{\partial\beta}{\partial\sigma}\right)}{(\beta-1)^2} > 0. \tag{93}$$

In the decentralized setting, we have

$$\frac{\partial \theta^*_S}{\partial \sigma} = \frac{(\rho-\alpha)\left(-\frac{\partial\beta}{\partial\sigma}\right)\left((2\gamma+3\delta)\sqrt{4\gamma^2\beta^2+3\delta(4\gamma+3\delta)} + 4\gamma^2\beta + 3\delta(4\gamma+3\delta)\right)}{3(\beta-1)^2\sqrt{4\gamma^2\beta^2+3\delta(4\gamma+3\delta)}} > 0. \tag{94}$$

Consider now the difference between these two thresholds, $\theta^*_{SO} - \theta^*_S$ for $\sigma > \hat{\sigma}$ (or equivalently, $\beta < \frac{2\delta}{\gamma}+3$).

$$\begin{aligned}\frac{\partial\left(\theta^*_{SO} - \theta^*_S\right)}{\partial\sigma} &= \frac{(\rho-\alpha)\left(-\frac{\partial\beta}{\partial\sigma}\right)\left(\left(\sqrt{4\gamma^2\beta^2+3\delta(4\gamma+3\delta)} - 3\delta\right)(4\gamma+3\delta) - 4\gamma^2\beta\right)}{3(\beta-1)^2\sqrt{4\gamma^2\beta^2+3\delta(4\gamma+3\delta)}} \\ &= \frac{(\rho-\alpha)\left(-\frac{\partial\beta}{\partial\sigma}\right)(4\gamma^2\beta + 3\delta(4\gamma+3\delta))\left(\sqrt{\frac{(4\gamma+3\delta)^2(4\gamma^2\beta^2+3\delta(4\gamma+3\delta))}{\left((4\gamma+3\delta)^2 - 4\gamma^2\left(-\beta+\frac{3\delta}{\gamma}+4\right)\right)^2}} - 1\right)}{3(\beta-1)^2\sqrt{4\gamma^2\beta^2+3\delta(4\gamma+3\delta)}}.\end{aligned} \tag{95}$$

Consider the last expression in the numerator of (95):

$$\begin{aligned}&\sqrt{\frac{(4\gamma+3\delta)^2(4\gamma^2\beta^2+3\delta(4\gamma+3\delta))}{\left((4\gamma+3\delta)^2 - 4\gamma^2\left(-\beta+\frac{3\delta}{\gamma}+4\right)\right)^2}} - 1 \\ &= \sqrt{\frac{(4\gamma+3\delta)^2}{(4\gamma+3\delta)^2 - 4\gamma^2\left(-\beta+\frac{3\delta}{\gamma}+4\right)} + \frac{(4\gamma+3\delta)^2(4\gamma^2\beta(\beta-1))}{\left((4\gamma+3\delta)^2 - 4\gamma^2\left(-\beta+\frac{3\delta}{\gamma}+4\right)\right)^2}} - 1 \\ &> \sqrt{\frac{(4\gamma+3\delta)^2}{(4\gamma+3\delta)^2 - 4\gamma^2\left(-\beta+\frac{3\delta}{\gamma}+4\right)}} - 1 > 0.\end{aligned} \tag{96}$$

Hence, we conclude that $\frac{\partial(\theta^*_{SO}-\theta^*_S)}{\partial\sigma} > 0$.

Now consider the optimal capacities evaluated at the optimal investment thresholds. Both $K^*_{SO}(\theta)$ and $K^*_{UP}(\theta)$ are increasing in θ. In addition, from (93) and (94), $\frac{\partial\theta^*_{SO}}{\partial\sigma} > 0$ and $\frac{\partial\theta^*_S}{\partial\sigma} > 0$. Thus, it immediately follows that $\frac{\partial K^*_{SO}(\theta^*_{SO})}{\partial\sigma} > 0$, and $\frac{\partial K^*_{UP}(\theta^*_S)}{\partial\sigma} > 0$.

Consider now the difference between these capacity levels for $\sigma > \hat{\sigma}$. As $K^*_{SO}(\theta)$ and $K^*_{UP}(\theta)$ do not depend directly on σ we can write

$$\frac{\partial\left(K^*_{SO}(\theta^*_{SO}) - K^*_{UP}(\theta^*_S)\right)}{\partial\sigma} = \frac{\partial K^*_{SO}(\theta)}{\partial\theta}\frac{\partial\theta^*_{SO}}{\partial\sigma} - \frac{\partial K^*_{UP}(\theta)}{\partial\theta}\frac{\partial\theta^*_{UP}}{\partial\sigma}. \tag{97}$$

Consider the difference $\frac{\partial K^*_{SO}(\theta)}{\partial\theta} - \frac{\partial K^*_{UP}(\theta)}{\partial\theta}$:

$$\frac{\partial K^*_{SO}(\theta)}{\partial\theta} - \frac{\partial K^*_{UP}(\theta)}{\partial\theta} = \frac{(\rho-\alpha)(2\gamma+\delta)}{2\eta\theta^2} > 0. \tag{98}$$

Thus, $\frac{\partial K^*_{SO}(\theta)}{\partial\theta} > \frac{\partial K^*_{UP}(\theta)}{\partial\theta}$. In addition, from (96) it follows that $\frac{\partial\theta^*_{SO}}{\partial\sigma} > \frac{\partial\theta^*_{UP}}{\partial\sigma}$ for $\sigma > \hat{\sigma}$. Together these observations imply that $\frac{\partial K^*_{SO}(\theta)}{\partial\theta}\frac{\partial\theta^*_{SO}}{\partial\sigma} > \frac{\partial K^*_{UP}(\theta)}{\partial\theta}\frac{\partial\theta^*_{UP}}{\partial\sigma}$ and, thus, from (98) it follows that $\frac{\partial(K^*_{SO}(\theta^*_{SO}) - K^*_{S}(\theta^*_{S}))}{\partial\sigma} > 0$ for $\sigma > \hat{\sigma}$. □

Proof of Proposition 9. Analogous to the derivations in Section 4.1, in the stopping region the PC solves the following problem:

$$\sup_{K_{min}\le K_P\le K_S} V(\theta, K_P) = \sup_{K_{min}\le K_P\le K_S}\left[\frac{\theta(1-\eta K_P)K_P}{\rho-\alpha} - \delta K_P\right]. \tag{99}$$

The difference is now that K_P is also restricted by $K_{\min}$ from below. The resulting optimal capacity level of the PC is given by

$$K^*_P(\theta, K_S, K_{min}) = \max\left[K_{min}, \min\left[K^*_{UP}(\theta), K_S\right]\right]. \tag{100}$$

This leads to the following two situations:

1. If $K_{\min} \ge K^*_{UP}(\theta)$, then the PC will choose the investment level $K^*_P(\theta, K_S) = K_{\min}$;
2. If $K^*_{UP}(\theta) > K_{\min}$, then the minimum capacity restriction is not binding.

Note that Situation 2 leads to exactly the same optimal strategies of the TSO as in the case without the minimum constraint on generation capacity.

Situation 1 however, leads to the following optimal stopping problem of the TSO

$$\sup_{\tau_S, K_S > K_{min}} \mathbb{E}\left[-\gamma K_S e^{-\rho\tau_S} + \int_{\tau^*_{CP}(K_{min})}^{\infty} \theta_t\left(1 - \frac{1}{2}\eta K_{min}\right)K_{min}e^{-\rho t}dt - \delta K_{min}e^{-\rho\tau^*_{CP}(K_{min})}\Big|\theta_0 = \theta\right], \tag{101}$$

where $\tau^*_{CP}(K_{\min}) = \min\{t \ge \tau_S : \theta_t \ge \theta^*_{CP}(K_{\min})\}$.

In this case, too, the K_S only enters as a cost, and, therefore, it is optimal for the TSO always to set the minimum capacity possible. Thus, in order to find the optimal strategies of the TSO and PC in case when the minimum constraint is in place, it is sufficient to consider the problem when $K_S = K_{\min}$. That way, the PC is always constrained to install exactly $K^*_P(\theta, K_S) = K_S$. This leads to the following optimal stopping problem of the TSO anticipating the PC's decision:

$$\sup_{\tau_S, K_S} \mathbb{E}\left[-\gamma K_S e^{-\rho\tau_S} + \int_{\tau^*_{CP}(K_S)}^{\infty} \theta_t\left(1 - \frac{1}{2}\eta K_S\right)K_S e^{-\rho t}dt - \delta K_S e^{-\rho\tau^*_{CP}(K_S)}\Big|\theta_0 = \theta\right]. \tag{102}$$

This problem is essentially the same as in Section 4.2, with the difference that the TSO no longer has to take into account that the PC might install the capacity that is smaller than K_S. Similar to the problem without capacity restriction, the sequential investment in this case is never optimal. The proof is analogous to that of Proposition 3. In the case of simultaneous investment, the TSO solves the following problem

$$\sup_{\tau_S, K_S \le \widehat{K}_S(\theta)} \mathbb{E}\left[\int_{\tau_S}^{\infty}\left(\theta_t\left(1 - \frac{1}{2}\eta K_S\right)K_S\right)e^{-\rho t}dt - (\delta+\gamma)K_S e^{-\rho\tau_S}\Big|\theta_0 = \theta\right], \tag{103}$$

which is essentially the same problem as (70), but now with a capacity constraint. Recall that the optimal capacity choice in an unconstrained problem is $K^*_{SO}(\theta)$ from (73). Then, the optimal capacity in the constrained problem will depend on the relationship between $\widehat{K}_S(\theta)$ and $K^*_{SO}(\theta)$ as follows:

$$K^*_S(\theta) = \begin{cases}\widehat{K}_S(\theta) & \text{if } \beta \le \frac{\delta}{\gamma} + 1,\\ K^*_{SO}(\theta) & \text{if } \beta > \frac{\delta}{\gamma} + 1.\end{cases} \tag{104}$$

In the following we determine the optimal threshold for the optimal capacity choice in different regions of θ.

- $K^*_{SO}(\theta)$ if $\beta > \frac{\delta}{\gamma} + 1$.

In this case, the solution for the optimal threshold is exactly the same as that in the case of integrated planning, θ^*_{SO}.

- $\widehat{K}_S(\theta)$ if $\beta \le \frac{\delta}{\gamma} + 1$.

In this case, the optimal θ solves the following equation:

$$V\left(\theta, \widehat{K}_S(\theta)\right) - \frac{\theta}{\beta}\left(\frac{\partial V(\theta, \widehat{K}_S(\theta))}{\partial\theta} + \frac{\partial V(\theta, K_S)}{\partial K_S}\Big|_{K_S = \widehat{K}_S(\theta)}\frac{\partial\widehat{K}_S(\theta)}{\partial\theta}\right) = 0. \tag{105}$$

Plugging the expression for $\widehat{K}_S(\theta)$ into (105) yields

$$\frac{(\beta-1)^2\theta^2 - 2(\beta-1)\beta\theta(\delta+\gamma)(\rho-\alpha) + \beta(\beta+1)\delta(\rho-\alpha)^2\left(\frac{(\beta-2)\delta}{\beta-1} + 2\gamma\right)}{2\eta(\rho-\alpha)\beta(\beta-1)\theta} = 0. \tag{106}$$

The function of θ in the brackets is a convex parabola, such that its positive root is the solution for the optimal threshold:[23]

$$\theta^*_{min} = \frac{(\rho-\alpha)}{(\beta-1)}\left(\beta(\gamma+\delta) + \frac{\sqrt{(\beta-1)^2(\gamma+\delta)^2 + \left(\beta^2-1\right)(\delta-(\beta-1)\gamma)^2}}{(\beta-1)}\right). \tag{107}$$

□

Proof of Proposition 10. First, we show that for $\sigma > \hat{\sigma}$, it holds that $\theta^*_{\min} > \theta^*_{SO}$.

$$\theta^*_{SO} - \theta^*_{min} = \frac{(\beta+1)}{(\beta-1)}(\delta+\gamma)(\rho-\alpha) - \frac{(\rho-\alpha)}{(\beta-1)}\left(\beta(\gamma+\delta) + \frac{\sqrt{(\beta-1)^2(\gamma+\delta)^2 + \left(\beta^2-1\right)(\delta-(\beta-1)\gamma)^2}}{(\beta-1)}\right)$$
$$= \frac{(\rho-\alpha)\left((\beta-1)(\gamma+\delta) - \sqrt{(\beta-1)^2(\gamma+\delta)^2 + \left(\beta^2-1\right)(\delta-(\beta-1)\gamma)^2}\right)}{(\beta-1)^2} < 0. \tag{108}$$

Thus, $\theta^*_{SO} < \theta^*_{\min}$.

Next, we prove that $K^*_{UP}(\theta^*_S) < \widehat{K}_S(\theta^*_{\min}) < K^*_{SO}(\theta^*_{SO})$ for $\beta < \frac{\delta}{\gamma} + 1$.

First, we show that $K^*_{UP}(\theta^*_S) < \widehat{K}_S(\theta^*_{\min})$. From Table A.3, for $\theta > \frac{(\beta+1)}{(\beta-1)}\delta(\rho-\alpha)$ it holds that $\widehat{K}_S(\theta) > K^*_{UP}(\theta)$. In addition, combining (89) and (108) we get that $\theta^*_S < \theta^*_{SO} < \theta^*_{\min}$. This implies that $\theta^*_{\min} > \theta^*_{SO} = \frac{(\beta+1)}{(\beta-1)}(\delta+\gamma)(\rho-\alpha) > \frac{(\beta+1)}{(\beta-1)}\delta(\rho-\alpha)$. Thus, $\widehat{K}_S(\theta^*_{\min}) > K^*_{UP}(\theta^*_{\min}) > K^*_{UP}(\theta^*_S)$.

Second, we show that $\widehat{K}_S(\theta^*_{\min}) < K^*_{SO}(\theta^*_{SO})$.

$$\widehat{K}_S(\theta) - K^*_{SO}(\theta^*_{SO}) = \frac{1}{\eta}\left(1 - \frac{\beta}{\beta-1}\frac{\delta(\rho-\alpha)}{\theta}\right) - \frac{2}{\eta(\beta+1)}$$

[23] This is because $V(\theta, \widehat{K}_S(\theta)) - \frac{\theta}{\beta}\left(\frac{\partial V(\theta, \widehat{K}_S(\theta))}{\partial\theta} + \frac{\partial V(\theta, K_S)}{\partial K_S}\Big|_{K_S=\widehat{K}_S(\theta)}\frac{\partial\widehat{K}_S(\theta)}{\partial\theta}\right)$ must be smaller than zero for the values below the optimal threshold, otherwise stopping would be optimal in the waiting region.

$$= \frac{(\beta-1)^2\theta - \beta(\beta+1)\delta(\rho-\alpha)}{\eta(\beta-1)(\beta+1)\theta}. \tag{109}$$

Hence, if $\theta < \frac{\beta(\beta+1)\delta(\rho-\alpha)}{(\beta-1)^2}$, then $\widehat{K}_S(\theta) < K^*_{SO}(\theta^*_{SO})$. Now consider the following difference

$$\begin{aligned}\theta^*_{SO} - \theta^*_{sub} &= \frac{(\beta+1)}{(\beta-1)}(\delta+\gamma)(\rho-\alpha) \\ &- \frac{(\rho-\alpha)\Big(\beta(2\gamma+3\delta-\epsilon)+\sqrt{4(\gamma+\epsilon)^2\beta^2+3\delta(4\gamma+3\delta+\epsilon)}\Big)}{3(\beta-1)} \\ &= \frac{(\rho-\alpha)\Big(3\delta+(\beta+3)\gamma+\beta\epsilon-\sqrt{4(\gamma+\epsilon)^2\beta^2+3\delta(4\gamma+3\delta+\epsilon)}\Big)}{3(\beta-1)} \\ &= \frac{(\rho-\alpha)\Big((\beta+3)\gamma+\beta\epsilon+3\delta-\sqrt{(3\delta+(\beta+3)\gamma+\beta\epsilon)^2-3(\beta+1)(\gamma+\epsilon)^2\big(\frac{2(\delta-\epsilon)}{\gamma+\epsilon}+3-\beta\big)}\Big)}{3(\beta-1)} \\ &> 0.\end{aligned} \tag{112}$$

$$\begin{aligned}\theta^*_{min} - \frac{\beta(\beta+1)\delta(\rho-\alpha)}{(\beta-1)^2} &= \\ &= \frac{(\rho-\alpha)\Big(\sqrt{(\beta^2-1)(\delta-(\beta-1)\gamma)^2+(\beta-1)^2(\gamma+\delta)^2}-\beta\gamma\big(\frac{2\delta}{\gamma}+1-\beta\big)\Big)}{(\beta-1)^2} \\ &= \frac{(\rho-\alpha)\Big(\sqrt{\beta^2\gamma^2\Big((\beta-1)^2+\frac{2(\beta-1)\delta}{\beta\gamma}\big(\frac{\delta}{\gamma}+1-\beta\big)\Big)}-\beta\gamma\sqrt{\big(\frac{2\delta}{\gamma}+1-\beta\big)^2}\Big)}{(\beta-1)^2} \\ &= \frac{\beta\gamma(\rho-\alpha)\Big(\sqrt{(\beta-1)^2+\frac{2(\beta-1)\delta}{\beta\gamma}\big(\frac{\delta}{\gamma}+1-\beta\big)}-\sqrt{(\beta-1)^2+\frac{4\delta}{\gamma}\big(\frac{\delta}{\gamma}+1-\beta\big)}\Big)}{(\beta-1)^2} \\ &< 0.\end{aligned} \tag{110}$$

The inequality holds because for $1 < \beta < \frac{\delta}{\gamma}+1$ and $\frac{2(\beta-1)\delta}{\beta\gamma} < \frac{4\delta}{\gamma}$. Hence, we conclude that $\theta^*_{\min} < \frac{\beta(\beta+1)\delta(\rho-\alpha)}{(\beta-1)^2}$ and, as a result, $\widehat{K}_S(\theta^*_{\min}) < K^*_{SO}(\theta^*_{SO})$.

Next, consider the derivative of $\theta^*_{\min}$ with respect to σ for $\sigma > \hat{\sigma}_{\min}$ (i.e., $\beta < \frac{\delta}{\gamma}+1$):

$$\frac{\partial\theta^*_{min}}{\partial\sigma} = \frac{(\rho-\alpha)(\gamma+\delta)\big(-\frac{\partial\beta}{\partial\sigma}\big)}{(\beta-1)^2} \left(\frac{\gamma^2(\beta-1)\beta+\gamma\delta\big(-\beta^2+\frac{2\delta}{\gamma}\beta+\frac{\delta}{\gamma}+1\big)}{(\gamma+\delta)\sqrt{(\beta-1)\beta\big(\gamma(\beta-1)(\gamma\beta-2\delta)+2\delta^2\big)}}+1\right) > 0. \tag{111}$$

This is because $\frac{\partial\beta}{\partial\sigma} < 0$ and for $1 < \beta < \frac{\delta}{\gamma}+1$, it holds that $\big(-\beta^2+\frac{2\delta}{\gamma}\beta+\frac{\delta}{\gamma}+1\big) > 0$. Here, we used the fact that this expression is a concave parabola and $\big(-\beta^2+\frac{2\delta}{\gamma}\beta+\frac{\delta}{\gamma}+1\big)\Big|_{\beta=1} = \frac{3\delta}{\gamma} > 0$ and $\big(-\beta^2+\frac{2\delta}{\gamma}\beta+\frac{\delta}{\gamma}+1\big)\Big|_{\beta=\frac{\delta}{\gamma}+1} = \frac{\delta^2}{\gamma^2}+\frac{\delta}{\gamma} > 0$.

Lastly, consider the optimal capacity evaluated at the optimal investment threshold. From Proposition 9, for $\beta < \frac{\delta}{\gamma}+1$ the TSO optimally chooses $\widehat{K}_S(\theta)$, which is increasing in θ. As $\frac{\partial\theta^*_{\min}}{\partial\sigma} > 0$ from (111), we conclude that $\frac{\partial\widehat{K}_S(\theta^*_{\min})}{\partial\sigma} > 0$. □

Proof of Proposition 11. The proof follows directly from the proof of Proposition 5 by substituting the marginal investment costs of the PC and the TSO by $(\delta-\epsilon)$ and $(\gamma+\epsilon)$, respectively. □

Proof of Proposition 12. The proof for $\frac{\partial\theta^*_{sub}}{\partial\sigma} > 0$ and $\frac{\partial K^*_{sub}(\theta^*_{sub})}{\partial\sigma} > 0$ follows directly from the proof of Proposition 8 by substituting δ by $(\delta-\epsilon)$ and γ by $(\gamma+\epsilon)$ in the expressions for θ^*_S and $K^*_{UP}(\theta^*_S)$, respectively.

We now show that $\theta^*_{SO} > \theta^*_{sub}$ for $\sigma > \hat{\sigma}_{sub}$. For $\beta < \frac{2(\delta-\epsilon)}{\gamma+\epsilon}+3$, it holds that

Next, we show that $K^*_{SO}(\theta^*_{SO}) > K^*_{sub}(\theta^*_{sub}) > K^*_{UP}(\theta^*_S)$. It holds that $K^*_{SO}(\theta) > K^*_{UP}(\theta)$ for $\theta > (\delta+2\gamma+\epsilon)(\rho-\alpha)$. Now note that

$$\begin{aligned}&\theta^*_{SO} - (\delta+2\gamma+\epsilon)(\rho-\alpha) \\ &\quad= (\gamma+\epsilon)\left(\frac{2(\delta-\epsilon)}{\gamma+\epsilon}+3-\beta\right)\frac{(\rho-\alpha)}{\beta-1} > 0.\end{aligned} \tag{113}$$

Thus, $K^*_{SO}(\theta^*_{SO}) > K^*_{sub}(\theta^*_{SO})$. In addition, as $K^*_{sub}(\theta)$ is an increasing function and from (112) $\theta^*_{SO} > \theta^*_{sub}$, we can establish that $K^*_{UP}(\theta^*_{SO}) > K^*_{sub}(\theta^*_{sub})$. Lastly, we show that $K^*_{sub}(\theta^*_{sub}) > K^*_{UP}(\theta^*_S)$. Note that $K^*_{UP}(\theta^*_S)$ can be written as

$$K^*_{UP}(\theta^*_S) = \frac{1}{2\eta}\left(1 - \frac{\frac{\delta}{\gamma}(r-\alpha)}{\frac{r-\alpha}{3(\beta-1)}\Big(\beta(\frac{3\delta}{\gamma}+2)+\sqrt{4\big(\beta^2-1\big)+(\frac{3\delta}{\gamma}+2)^2}\Big)}\right) \tag{114}$$

Then,

$$\frac{\partial K^*_{UP}(\theta^*_S)}{\partial\big(\frac{\delta}{\gamma}\big)} = \frac{3\Big(4+\frac{3\delta}{\gamma}-2\beta^2-\beta\sqrt{4\big(\beta^2-1\big)+(\frac{3\delta}{\gamma}+2)^2}\Big)}{(\beta+1)\eta(4+\frac{3\delta}{\gamma})^2\sqrt{4\big(\beta^2-1\big)+(\frac{3\delta}{\gamma}+2)^2}} < 0. \tag{115}$$

As $K^*_{sub}(\theta^*_{sub})$ is obtained directly from $K^*_{UP}(\theta^*_S)$ by substituting $\frac{\delta-\epsilon}{\gamma+\epsilon} < \frac{\delta}{\gamma}$ instead of $\frac{\delta}{\gamma}$, we conclude that $K^*_{sub}(\theta^*_{sub}) > K^*_{UP}(\theta^*_S)$. □

Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.ejor.2022.04.038.

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