Inverse problems of the dynamics of observation interpretation systems

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Inverse problems of the dynamics of observation interpretation systems

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Abstract. Based on the analysis and systematization of the inverse problems of dynamics, the study of the properties and features of the types of dynamic models under consideration, an approach is proposed for the development of appropriate methods of mathematical modeling based on the use and implementation of integral models in the form of Volterra equations of the I and II kind, their functional capabilities are determined in the study of various classes of problems , and also formulated the features that affect the choice of methods for their numerical solution. Methods for obtaining integral models are proposed, which are the basis for constructing algorithms for solving inverse problems of dynamics for a fairly wide class of dynamic objects. Integral methods for the identification of dynamic objects have been developed, which make it possible to obtain stable non-optimization algorithms for calculating the parameters of mathematical models. Recurrent methods of parametric identification of transfer functions of dynamic objects with an arbitrary input action are proposed (the obtained parameters of the transfer functions are also coefficients of the corresponding differential equations, which makes it possible to obtain equivalent mathematical models in the form of integral equations). The study of algorithms that implement the proposed identification methods allows us to conclude about their efficiency in terms of the amount of computation and ease of implementation, as well as the high accuracy of calculating the model parameters.

1. Introduction

One of the essential indicators of the development of methods of on-line information processing is the increase in the share of inverse problems in the general list of problems to be solved by systems of this class [1]. This means that, in principle, all problems of mathematical modeling can be divided into two groups: direct problems, that is, analysis problems, when the causes of certain processes are known and it is necessary to find the consequences, as well as inverse problems, when the consequences are known and the reasons need to be found. The theoretical and practical interest in the creation, research and application of methods and means for solving inverse problems is determined by the need for the development of new methods of signal processing, as well as the increased complexity of inverse problems in relation to direct problems, since the latter are not correct from a mathematical point of view and have a number of features [2]. The difficulty, in particular, lies in the need to take measures to ensure the conditions for the existence and uniqueness of the solution, as well as to take into account the absence of a continuous (regular) dependence of the solution on the initial data. Since the input information in inverse problems is experimental data determined with a certain error, the resulting solution can differ greatly from the exact solution [3].

The inverse problems of the dynamics of systems include the following tasks: restoration of external influences on the object according to the given parameters of the system and reactions; restoration of equations or parameters for given external influences and reactions; selection of control signals according to a given structure and the law of change of output coordinates. In systems theory, they correspond to the tasks of identification, restoration of input signals, and control [4].

The solution of problems of dynamics is carried out, as a rule, within the framework of some mathematical model of the object or system under study, in the formation of which the traditional approach is currently the use of ordinary differential equations for objects with lumped parameters and partial differential equations for objects with distributed parameters [5]. In the latter case, in relation to the generally accepted method of structural modeling of systems, partial differential equations are replaced by approximate dynamic models in the form of ordinary differential equations. The identification problem in this case consists in determining the coefficients of the equations by the known right-hand sides (external influences on the object) and solutions (reactions of the object). The task of restoring the input actions is to determine the right-hand side by the given coefficients (object parameters) and solutions (object reactions) [6].

When studying applied problems, a typical situation is when the desired characteristics of an object are inaccessible or difficult to access for direct observation (for example, the problem of non-destructive quality control of products and structures, identifying defects inside an operating object, and many other tasks), and the experiment itself may be impossible. In this case, some indirect information about the investigated object is used, which is determined either by the nature of the investigated object or by experimental data [7].

2. Research methods

Methods for solving inverse problems of dynamics are based on the use of optimization algorithms, the implementation of which can be difficult due to the complexity of search procedures, in particular, due to the nonlinear dependence of the minimized functional on the parameters of the model. The complication of models and difficulties in processing experimental data lead to the need to consider methods for constructing alternative forms of representing dynamic models and to further develop methods for solving inverse problems of dynamics [8].

An alternative form of representing dynamic models of real technical objects or systems when solving inverse problems of dynamics are integral equations of the Volterra type [9]. The advantages of integral models are the smoothing properties of integral operators, simplicity and high stability of numerical integration operations. Integral dynamic models are formed on the basis of the given dynamic characteristics of an object, its links or elements. Dynamic characteristics are functional dependencies that can be obtained in the form of experimental data or in an analytical form [4].

On the basis of the concept of inverse problems and their inherent mathematical models, problems of diagnostics and control, design of technical objects and their control are solved. The main types of inverse problems include the problem of identifying dynamic objects and the problem of recovering input actions.

We will consider a class of objects with the property of continuous functional dependence of output signals on input signals. This dependence in a fairly general case can be represented mathematically in the form of the following operator dynamic model.

$$A[Y(x,t);F(x,t);Q(x,t)]=0, \tag{1}$$

where $t \in [0,T]$ – time, $x=(x_1,x_2,\dots,x_m)$ – vector of spatial coordinates of the dynamic object model (DO), A − an arbitrary generally unknown operator satisfying the condition for the existence of a continuous implicit function Y, which is a vector of output coordinates (signals) DO, F − vector of input coordinates (signals), Q - is a vector, generally speaking, of unknown parameters. For dynamic objects described by models of the form (1), there are the following two main groups of identification problems: 1) the problem of finding the structure of the operator A, such that there is an operator continuous in the first variable *Φ* and, moreover, that in (1) the conditions the existence of an implicit function $Y(x,t)=Y(x,t,F,0)$ defined by the equality

$$Y(x,t)=\Phi[(F(x,t);Q(x,t)]$$

(the problem of identifying DO parameters in a broad sense); 2) the problem of finding, according to equation (1), a vector $Q(x,t)$ with a known structure of the operator *A* (the problem of identifying DO parameters in the narrow sense) satisfying sufficient conditions for the existence in (1) of an implicit function $Q(x,t)$ of the form

$$Q(x,t)=\psi[(Y(x,t);F(x,t)],$$

where $\psi[\ ]$ - is some rather arbitrary operator.

Methods for solving the identification problem can differ according to the following features: by the way of representing the characteristics of DO (in the time or frequency domain) or by the type of its model; by the method of the experiment on the DO: active (these methods are usually inapplicable in normal operation), passive and mixed, in which special test signals of low intensity are supplied to the DO that do not disrupt its normal operation; according to the accepted criterion of proximity (similarity) of DO and model; by methods of searching for unknown parameters of DO.

Parameter search methods can be divided into two groups - probabilistic and deterministic. The paper considers the methods of the second group, which require the least amount of a priori information for the problem being solved. The general scheme of the identification process is shown in Figure 1 (*J* is the assessment of the quality of the result of solving the problem, $\hat{Y}$ is the reaction of the model). It can be seen that the diagram reflects the optimization process of searching (selecting) parameters by minimizing the quality indicator of the obtained solution.

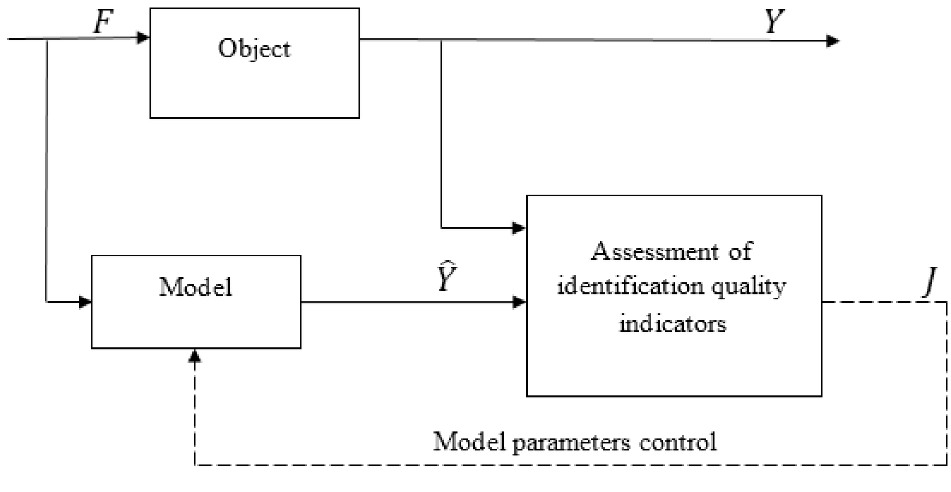

Figure 1. General diagram of the identification process.

The use of integral models makes it possible to study a wide variety of processes occurring in technical systems [5]. These models belong to the class of nonparametric. Therefore, the algorithms used to determine the parameters of differential models [6] cannot be used to simulate dynamic objects based on integral equations.

3. Results and discussion

Consider the problem of identifying a linear stationary object, the model of which is represented by the equation

$$u^{(m)}(t)+\sum_{j=1}^{m} p_j u^{(m-j)}(t)=f(t), \quad u^{(k)}(0)=c_k,\ k=\overline{0,m-1}, \tag{2}$$

Where, f is a given input signal; u is a given output signal; p - vector of unknown parameters of the model; c - vector of initial conditions; $t\in[0,T]$

Applying for (2) the method of successive integration, we obtain an equivalent Voltaire integral equation of the second kind:

$$\sum_{j=1}^{m} p_j\left[\int_0^t \frac{(t-\tau)^{j-1}}{(j-1)!}u(\tau)d\tau-\sum_{k=0}^{m-j-1} c_k \frac{t^{k+j}}{(k+j)!}\right]=\int_0^t \frac{(t-\tau)^{m-1}}{(m-1)!}f(\tau)d\tau+\sum_{j=0}^{m-1} c_j \frac{t^j}{j!}-u(t). \tag{3}$$

From (3) for the moments of measurement $t_i,\ \tau_j\ (i=\overline{1,N})$ of the form

$$0\le t_0<t_1<\ldots<t_N\le T;$$
$$0\le \tau_0<\tau_1<\ldots<\tau_N\le T$$

and, assuming that $m=n,$ we get the following system of linear equations with respect to $p_j,\ \ j=1,m$:

$$A\cdot p=b,$$

where $p=(p_1,\ldots,p_m)^T,\ b=(b_1,\ldots,b_m)^T,\ A=[A_{ij}]_{i,j=1}^m,$

$$A_{ij}=\int_0^{t_i}\frac{(t_i-\tau)^{j-1}}{(j-1)!}u(\tau)d\tau-\sum_{k=0}^{m-j-1} c_k\frac{t_i^{k+j}}{(k+j)!}, \tag{4}$$

$$b_j=\int_0^{t_i}\frac{(t_i-\tau)^{m-1}}{(m-1)!}f(\tau)d\tau-y(t_i)+\sum_{v=0}^{m-1} c_v\frac{t_i^v}{v!}. \tag{5}$$

To calculate integrals (4) and (5), we use quadrature formulas [7], which in the case of a stationary object will be written as follows

$$\int_0^{t_i}(t_i-\tau)^{j-1}u(\tau)d\tau=\sum_{k=0}^{N_i} W_{ij}(t_i-\tau_k)^{j-1}u(\tau_k)+r_{ij}[u], \tag{6}$$

$$\int_0^{t_i}(t_i-\tau)^{m-1}f(\tau)d\tau=\sum_{k=0}^{M_i} W_{ik}(t_i-\tau_k)^{m-1}f(\tau_k)+r_{im}[u], \tag{7}$$

where t_i, τ_k are the nodes of the partition ($t_0\le\tau_k\le t_i$); W_{ij} - weights of the quadrature formula; $1\le N_i,\ \ M_i\le N$, $r_{ij},\ \ r_{im}$ - remainder terms of the corresponding formulas.

Taking into account that the signal values were obtained experimentally with some errors, and discarding $r_{ij}[y]$ and $r_{im}[f]$, from (4) - (7) we obtain the following calculated expressions:

$$\tilde{A}_{ij}=\frac{1}{(j-1)!}\sum_{k=0}^{N_i} W_{ik}(t_i-\tau_k)^{j-1}\tilde{u}(\tau_k)-\sum_{l=0}^{m-j-1} c_l\frac{t_i^{l+j}}{(l+j)!},\ 1\le N_i\le N, \tag{8}$$

$$\tilde{b}_i=\frac{1}{(m-1)!}\sum_{k=0}^{M_i} W_{ik}(t_i-\tau_k)^{m-1}\tilde{f}(\tau_k)+\sum_{v=0}^{m-1} c_v\frac{t_i^v}{v!}-\tilde{u}(t_i),\ 1\le M_i\le N. \tag{9}$$

Thus, we arrive at the final system of equations for determining the approximate values of the vector $\tilde{p} = (\tilde{p}_1, \ldots, \tilde{p}_m)^T$:

$$\tilde{A} \cdot \tilde{p} = \tilde{b}, \tag{10}$$

where

$$\tilde{A} = \left[A_{ij}\right]_{i,j=1}^{m}, \quad \tilde{b} = \left[\tilde{b}_1, \ldots, \tilde{b}_m\right]^T.$$

The considered problem is a special case of a more general problem of identification of a nonstationary dynamic object with lumped parameters [8] described by the integral equation

$$\alpha_1(t)u(t) + \int\limits_{G_1(t)} K_1(t,\tau)u(\tau)d\tau + \lambda_1(t) = \alpha_2(t)f(t) + \int\limits_{G_2(t)} K_2(t,\tau)f(\tau)d\tau + \lambda_2(t), \tag{11}$$

where α_i, K_i, λ_i $(i = 1,\ 2)$ – unknown quantities; u and f - are defined in (2), $G_i(t)$ are the domains of integration ($G_1(t) = G_2(t) = [0, t], t \in [0, T]$).

Obviously, for a stationary object

$$\alpha_1(t) \equiv 1,\ \alpha_2(t) \equiv 0,\ \lambda_1(t) \equiv 0,\ K_1(t,\tau) = K(t-\tau) = \sum_{j=1}^{m} p_j \frac{(t-\tau)^{j-1}}{(j-1)!},\ m \in N,$$
$$K_2(t,\tau) = \frac{(t-\tau)^{m-1}}{(m-1)!},\ \lambda_2(t) = \sum_{j=1}^{m-1}\left(c_j \frac{t^j}{j!} + p_j \sum_{k=0}^{m-j-1} c_k \frac{t^{k+j}}{(k+j)!}\right). \tag{12}$$

To solve equation (11), we discretize the model at the nodes $t_i,\ i = \overline{0, N}$ and apply quadrature formulas, as a result of which we obtain an algebraic system with respect to unknown parameters containing the equation $N+1$:

$$\alpha_1(0)u(0) + \lambda_1(0) = \alpha_2(0)f(0) + \lambda_2(0),$$
$$\alpha_1(t_i)u(t_i) + \sum_{j=0}^{N_i} W_{ij} K_1(t_i, \tau_j)u(\tau_j) + \lambda_1(t_i) =$$
$$= \alpha_2(t_i)f(t_i) + \sum_{j=0}^{M_i} W_{ij} K_2(t_i, \tau_j)f(\tau_j) + \lambda_2(t_i),$$
$$M_i N_i = \overline{1, N},\ \ i = \overline{1, N}.$$

This system is underdetermined [6, 9], which can cause difficulties in solving it. To make sure that the method is effective, let us consider examples of solving test problems (the initial data of test problems provide the ability to obtain an exact solution and check the accuracy of the applied numerical algorithms, and also have the ability to reproduce the results of computational experiments).

Example 1: output signal: $u(t) = 1 - e^{-2t},\ t \in [0;2],\ h = 0.01;$ input signal: $f(t) = -14e^{-2t} - 0.2;$ $C_1 = 0,$ $C_2 = 2,$ $C_3 = -4,$ $C_4 = 8,$ $C_5 = -16$ – initial conditions for an equivalent differential equation

$$u^{(5)}(t) + p_1 u^{(4)}(t) + p_2 u^{(3)}(t) + p_3 u^{(2)}(t) + p_4 u^{(1)}(t) + p_5 u(t) = f(t),$$
$$u^{(i-1)}(0) = C_i,\ i = \overline{1{,}5}.$$

Need to determine the coefficients $\tilde{p}_i$ (exact values: $p_1 = 1.2$, $p_2 = -2$, $p_3 = 3.1$, $p_4 = 0.7$, $p_5 = -0.2$).

Using expressions (8) - (10) and the quadrature formula of trapezoids to approximate the integrals included in expressions (4), (5), we obtain a system with respect to unknown coefficients $\tilde{p}_i$. Applying the least squares method to solve it, we obtain the values of the sought coefficients:
$\tilde{p}_1 = 0{,}0113,\ \tilde{p}_2 = -3{,}3704,\ \tilde{p}_3 = 5{,}7472,\ \tilde{p}_4 = 1{,}9459,\ \tilde{p}_5 = 0{,}1995$,
wherein, the maximum error of the output signal $\max \Delta u$ is 0,000125.

Add to the input signal *f* a random noise distributed according to the normal law, taking into account the amplitude *f*. Table 1 shows the values of the coefficients $\tilde{p}_i$ obtained at various values of the interference.

Table 1. Values of the coefficients $\tilde{p}_i$, $i = \overline{1,5}$ with interference of the output signal described by the exponential law.

The amount of interference of the input signal in %	Maximum absolute error $\max \Delta u$	$\tilde{p}_1$	$\tilde{p}_2$	$\tilde{p}_3$	$\tilde{p}_4$	$\tilde{p}_5$
1	0.003053	0.0059	-1.6820	5.7503	-4.8095	-1.3727
5	0.014173	0.0059	-1.1268	5.5406	-11.4488	-0.7102
10	0.059389	0.0183	-5.6392	5.0458	9.8715	-3.2394

Example 2: output signal: $u(t) = e^{1-t} + 1;$ input signal: $f(t) = -0.5e^{1-t} - 1;$ initial conditions: $C_1 = 3.7183,\ C_2 = -2.7183$, $C_3 = 2.7183$; equivalent differential equation

$$u'''(t) + p_1u''(t) + p_2u'(t) + p_3u(t) = f(t),$$
$$u^{(i-1)}(0) = C_i,\ i = \overline{1,3};$$

exact solution: $p_1 = 2,\ p_2 = 0.5,\ p_3 = -1$. Need to determine the coefficients $\tilde{p}_i$ (Table 2).

In the process of solving the test case was obtained the value of the maximum absolute error of the output signal $\max \Delta u = 0{,}0024378$

Table 2 shows the values of the coefficients $\tilde{p}_i$ obtained at various values of the interference.

Table 2. The values of the coefficients $\tilde{p}_i$, $i = \overline{1,3}$ with the interference of the output signal described by the exponential law.

The amount of interference of the input signal in %	Maximum absolute error $\max \Delta u$	$\tilde{p}_1$	$\tilde{p}_2$	$\tilde{p}_3$
0	0.002438	0.0034	3.0077	-0.9974
1	0.037324	2.0267	0.5265	-0.9779
5	0.048346	1.9757	0.5471	-0.9859
10	0.017157	2.0083	0.5315	-1.0057

Example 3: output signal: $u(t) = t^3;$ input signal: $f(t) = -t^3 + 1.5t^2 + 12t + 6;$ initial conditions: $C_1 = 0$, $C_2 = 0$, $C_3 = 0$. Need to determine the coefficients $\tilde{p}_i$; equivalent differential equation $u'''(t) + p_1u''(t) + p_2u'(t) + p_3u(t) = f(t),\ u^{(i-1)}(0) = C_i,\ i = \overline{1,3}$.

Exact solution: $p_1 = 2,\ p_2 = 0.5,\ p_3 = -1$. The simulation results are presented in Table 3.

Table 3. The values of the coefficients $\tilde{p}_i$, $i=\overline{1,3}$, with interference of the output signal described by a polynomial of the 3rd degree

The amount of interference of the input signal in %	Maximum absolute error $\max \Delta u$	$\tilde{p}_1$	$\tilde{p}_2$	$\tilde{p}_3$
0	0.021668	2.0856	4.7155	-0.6312
1	0.024827	2.0828	0.2345	-0.7736
5	0.026551	2.2235	-0.5445	0.4487
10	0.028324	2.1337	-0.0666	-0.1927

4. Conclusions
The results of the computational experiments carried out indicate such properties of the integral identification method as high stability, efficiency in terms of computer time and computational volume, and ease of implementation. The considered method can be effectively used in solving problems of parametric identification of dynamic objects, represented by both differential equations and integral models in the case of their parametrization. Within the framework of this method, a particular case of a stationary object is considered, the use of which makes it possible to avoid the difficulties of solving an underdetermined system of algebraic equations, to which the algorithm of the proposed method is reduced in the general case.

References
[1] Samarskiy A A 2005 *Matematical models* **44** 320
[2] Verlan A F 2011 Methods of mathematical and computer modeling of measuring transducers and systems based on integral equations 336.
[3] Irmuhamedova R M and Sagatov M V 2015 *Journal of Multimedia Information System* **2**(4) 317-326.
[4] Brunner H 2004 Collocation Methods for Volterra Integral and Related Functional Equations 597.
[5] Sidorov D N 2013 Analysis methods for integral dynamic models 293.
[6] Verlan A F 1986 *Integral Equations* 544.
[7] Verzhbitskiy B M 2005 *Numerical Methods* **2** 400.
[8] Verlan A F 1988 Mathematical modeling of continuous dynamic systems 287.
[9] Denisenko A N 2010 *Computer processing of information* **7** 252.