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Estimation in the Mathematics Curricula of Denmark, Norway and Sweden: Inadequate Conceptualisations of an Essential Competence

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ABSTRACT
Acknowledging evidence that the ability to estimate has major consequences for both later mathematics learning and real-world functionality, this paper examines the national mathematics curriculum for compulsory school for each of Denmark, Norway and Sweden for the estimation-related opportunities it offers children. Framed against four conceptually and procedurally different forms of estimation (computational, measurement, quantity and number line), each of which is implicated differently in the later learning of mathematics, analyses indicated that none of the four forms of estimation were addressed explicitly in the Norwegian curriculum. Expectations of computational and measurement estimation were present in both the Danish and the Swedish curricula, although neither referred to either quantity or number line estimation. Even when estimation-related learning outcomes were articulated, there was no evidence of the processes by which they might be realised. Finally, there were no acknowledgements that estimation may contribute to the learning of other mathematical topics.

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Introduction

Estimation "is a pervasive activity in the lives of both children and adults" (Booth & Siegler, 2006, p. 189) and one of the three most important goals of mathematics education (Sriraman & Knott, 2009). However, despite its importance, it is internationally problematic. Four decades ago, a study of US preservice elementary teachers' perspectives on estimation concluded that the "cursory treatment given to estimation in most mathematics programs is insufficient to build any appreciable estimation" (Bestgen et al., 1980, p. 124). Today, the problem persists. American preservice teachers have poor conceptions of estimation, whether at elementary (Son et al., 2019) or secondary levels (Subramaniam, 2014). Internationally, textbooks rarely address estimation adequately, as found in studies comparing textbooks used in Korea and the United States (Hong et al., 2018) and Finland, Singapore and Sweden (Sayers et al., 2019). That being said, an issue missing in the research literature is the role of estimation in curricula. In this paper, motivated by the importance of estimation in children's learning of mathematics (Schneider et al., 2009) and the paucity of estimation-related

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opportunities found in Swedish textbooks, we examine how estimation is presented in the mathematics curricula of the compulsory schools of Denmark, Norway and Sweden.

What is Estimation and Why is it Important?

Our reading of the literature indicates that the role of estimation in children's learning of mathematics is warranted in two ways. The first, essentially psychological, refers to the developmental role of estimation in children's learning of various mathematics topics, alongside its potential for identifying developmental problems. The second, essentially pragmatic, refers to the significance of various forms of estimation in different real-world contexts. In the following, we focus principally on the first, while not ignoring the second. Also, as highlighted in Sowder's (1992) earlier review, estimation has historically been assumed to take three forms; *computational estimation, measurement estimation* and *quantity (or numerosity) estimation*. Today, almost thirty years later, a fourth form, *number line estimation,* has come to dominate the research interests of cognitive psychologists, mathematics educators and special needs educators around the globe. In the following, we examine, in as much detail as the limits of this paper will allow, the international literature on these four forms of estimation, their characteristics and relevance to school mathematics.

Computational Estimation

The everyday conception of estimation is as an aid to computation. Indeed,Sowder's (1992) well-known review focused almost exclusively on the forms and functions of computational estimation. Defined as "the process of simplifying an arithmetic problem using some set of rules or procedures to produce an approximate but satisfactory answer through mental calculation" (Ainsworth et al., 2002, p. 28), computational estimation "is more important and practical than precise calculation for many everyday uses of mathematics" (Bestgen et al., 1980, p. 124), because "it takes less time and attentional resources than exact calculation, and thus can be used in circumstances where time or attention resources are limited" (Ganor-Stern, 2018, p. 2).

Computational estimation is an important aid to children's understanding of place value and standard algorithms (Sowder, 1992). It is a predictor of general mathematical competence (Seethaler & Fuchs, 2006; Star et al., 2009) and a positive correlate of mathematics self-concept (Gliner, 1991). It is a skill, drawing on a wide range of strategies (Dowker, 1992), dependent on both the maturity of the estimator and the complexity of the task (Dowker, 1997), not least because maturation brings with it a range of experiences supportive of a wider range of appropriate strategies (LeFevre et al., 1993). Children's computational estimations are largely unaffected by the size of the numbers (Liu, 2013), providing tasks remain within their arithmetical base-competence (Dowker, 1997). They make better estimates when tasks are presented contextually (Gliner, 1991) and visually rather than orally (Liu, 2009). However, partly due to instruction-derived beliefs that answers to problems should be precise, invitations to estimate may confuse children, who then resort to the mental application of standard algorithms (Liu, 2009; Yang, 2005). Finally, adults' computation estimation competence is influenced by mathematics anxiety (Si et al., 2016), educational background (Dowker et al., 1996) and cultural affiliation (Xu et al., 2014).

Measurement Estimation

Measurement estimation, or measuring without measurement tools, invokes various forms of mental referents to provide a measure of the object under scrutiny (Sowder, 1992), typically in "everyday situations in which precise calculation or measurement are contextually defined as either impossible or unnecessary" (Forrester & Pike, 1998, p. 334). However, while both children and adults tend to be poor estimators (Joram et al., 1998), measurement estimation is probably the most widespread in other disciplines, particularly those of science, in all its forms, engineering and technology. However, in respect of school mathematics, it is poorly taught, frequently due to a lack of teacher confidence (Joram et al., 2005). Finally, highlighting the importance of context, estimations of distance, particularly when addressing the question, "how far is it to … ?", are typically framed by time and not distance (Grootenboer & Sullivan, 2013).

Middle grade students are generally poor estimators of objects' lengths, although they tend to be more accurate when using non-standard units than standard (Desli & Giakoumi, 2017). Also, while there is limited evidence of estimation maturation over the same time period, children become more competent with respect to the dimensionality of the objects being estimated (Forrester & Shire, 1994). When estimating, secondary-aged students typically employ some form of individual frame of reference, which is one element of a complex interaction with the nature of the estimation activity and the physical context in which it occurs (Gooya et al., 2011). Indeed, others have highlighted the value of reference points - fixed values against which estimations may be calibrated – not only because context familiarity improves estimates (Jones et al., 2012) but also because children who employ reference points are more accurate than those who do not (Joram et al., 2005). Finally, measurement estimation has been positively implicated in mathematics achievement (Kramer et al., 2018) and is an everyday tool of professional users of mathematics (Jones & Taylor, 2009).

Number Line Estimation

Number line estimation is a competence that develops with age, although young children tend to construe smaller numbers as more widely spaced than larger numbers, creating, as with quantity estimation, a logarithmic pattern (Ashcraft & Moore, 2012). Integer number line estimation, which is improved by means of reference points (Sullivan & Barner, 2014a), underpins mathematical competence in general (Schneider et al., 2018) and, in particular, the acquisition of arithmetical competence (Schneider et al., 2009) and the learning of fractions (Bailey et al., 2014). It is a predictor of children's algebraic readiness and equation solving (Booth et al., 2014) and mathematical learning difficulties (Siegler & Opfer, 2003), particularly developmental dyscalculia (Huber et al., 2015). In similar vein, decimal number line estimation competence is a better predictor of algebraic competence than either integer or fraction number line estimation (DeWolf et al., 2015).

Number line estimation improves with intervention, especially from the perspective of replacing logarithmic patterns by linear (Siegler & Opfer, 2003). Number line estimation efficacy is more accurate on bounded – typically where an upper limit is placed on the number line to facilitate estimations – than unbounded tasks, with children using proportional judgements on the former and magnitude-based judgements on the latter (Jung et al., 2020). Adults' number line estimation performance is not only inconsistent but dependent on the nature of the task (Huber et al., 2017). With respect to large numbers, around half of all adults erroneously place one million halfway between one thousand and one billion, highlighting an underdeveloped sense of large numbers (Landy et al., 2013).

Quantity Estimation

Quantity estimation concerns the ability to discern or produce the number of objects in a set without recourse to counting. It is reciprocally dependent on the ability to count (Barth et al., 2009) and diminishes in accuracy as the number of objects grows (Smets et al., 2015). The development of quantity estimation is a complex interplay of set size and maturity that become more accurate with age (Ebersbach & Wilkening, 2007). As with number line estimation, reflecting a greater familiarity with small numbers, young children's estimates are thought to shift gradually from logarithmic models to linear (Siegler et al., 2009; Sullivan & Barner, 2013). However, others have argued that the logarithmic model for young children may be better construed as two linear forms, one reflecting children's relatively accurate representation of small numbers and the other their lack of knowledge of large numbers (Stapel et al., 2015), while others have found that the linear model applies to children's estimates in continuous conditions and logarithmic in discrete (Odic et al., 2013). That said, irrespective of any accuracy, young children estimate ordinally; in the correct direction relative to previous estimates (Sullivan & Barner, 2014b).

Quantity estimation is a precursor of later arithmetical competence (Bartelet et al., 2014), although some studies have shown that its influence is inextricably linked with the influence of number line estimation, which correlate with each other but independently predict arithmetical competence (Wong et al., 2016). Interestingly, most quantity estimation-related research has been undertaken with adults, whose performance varies according to context (Crollen et al., 2011), although regular dot patterns elicit better estimates than random (Ginsburg, 1976) and groups estimate quantities better than individuals (Bonner et al., 2007).

The Current Study and its Methods

Acknowledging the above, and the particular paucity of estimation-related opportunities in Swedish school textbooks, whether Swedish-authored or adapted Finnish and Singaporean imports (Sayers et al., 2019), it would seem reasonable to examine the extent to which curricula structure opportunities for children, of all ages, to develop not only the competences associated with each form of estimation but also, in the longer term, the various benefits derived from those competences. In the following, therefore, we examine the mathematics curricula of Denmark, Norway and Sweden to examine, through the mathematical lenses described above, how estimation is construed across Scandinavia, not least because all three countries have experienced both internal and external criticisms for allegedly low levels of mathematics achievement on international tests of achievement. However, as a consequence of the unique and culturally situated nature of school mathematics (Andrews, 2016), our objective is not to undertake a comparative examination of the three curricula but three parallel analyses, each independently undertaken, focused on determining how estimation is construed by each of the three curricular authorities in order to provide a snapshot of the situation across Scandinavia.

In addressing this ambition, we acknowledge that the curriculum, as an object of analysis, may be construed differently in different cultural contexts. For example, in countries like Cyprus, mandated learning outcomes are presented in government-produced textbooks that all teachers are obliged to follow (Xenofontos, 2019). In such contexts, curriculum analysts would typically focus on the content of these textbooks as they represent the "contract" between the state and the individual teacher. In other countries, like the United States, there is no centrally-produced curriculum with, historically, each school district identifying both curricular goals and the textbooks to address them (Reys, 2001). Here, depending on the extent to which such curricular goals are represented in some form of steering document, analyses might focus on the steering documents themselves or the textbooks chosen to reify them. In other systems, government-mandated goals are specified in steering documents, or national curricula, that explicate for schools, teachers and parents not only what is to be taught but when it is to be taught. In these systems, textbooks, produced by commercial publishers and subject to limited systemic regulation, fall outside the "contract" between the state and the teacher, not least because teachers are free to decide whether or not they use textbooks and, if they do, which. Consequently, analyses of curricula and analyses of textbooks are qualitatively different enterprises.

The three Scandinavian countries of Denmark, Norway and Sweden fall into this latter group, whereby government expectations are manifested in centrally-produced documents that outline, in varying degrees of detail, both the content and the broad timing of the mathematics to be taught. In this paper, we present qualitative analyses of the ways in which the curricula for compulsory school mathematics, including the final year of pre-school, of Denmark, Norway and Sweden structure opportunities for children to develop estimation-related competence. In so doing, as indicated above, we focus on the curriculum documents themselves as they, rather than any textbooks, form the "contract" between the state and individual teachers. Importantly, not least because our interest lies explicitly in the opportunities the different systems offer today's learners of mathematics, analyses of current curricula allow insights into how estimation is construed by the different curricular authorities as a form of "what is" rather than any historical "what was". In other words, by examining curricula through the mathematical lenses we have articulated above, we aim to examine the extent to which Scandinavian curricula explicitly facilitate students' acquisition of this important competence.

That being said, all three sets of analysed documents are recent revisions of earlier documents, each originally introduced to address perceived failings identified by early iterations of the OECD's Programme of International Student Assessment or PISA. Thus, we suggest, it would be reasonable to assume, acknowledging PISA's goal of assessing "the capacity of students to put mathematical knowledge into functional use in a multitude of different situations in varied, reflective and insight-based ways" (Schleicher, 2007, p. 351), that such curricula would privilege the teaching of functionally important competences like estimation. However, none of these earlier curricula, whether Swedish (Skolverket, 2011), Danish (Undervisningsministeriet, 2009), or Norwegian (Utdanningsdirektoratet, 2013), offer estimation-related opportunities different from those of the recent revisions discussed below. Moreover, while it would be of interest to engage in a critique of PISA's influence on curriculum development, limitations of space and a desire not to obscure the explicit focus on opportunities for children to acquire estimation-related competence suggest that such a narrative would be better left to another paper.

Across the three countries, current national curricula and, with one exception, supporting documents are available electronically as searchable pdf files. At this stage, it is important to note that while all documents have been produced by government departments or agencies, the three curricular authorities present both expected learning outcomes and exemplificatory material in different ways. Consequently, the number of documents analysed varied from one country to another.

The relevant documents for each country were analysed by two members of the authorial team. With respect to Denmark and Norway, one of these was a native speaker of Danish and Norwegian respectively, while the second, acknowledging the mutual intelligibility of the Scandinavian languages, was a speaker of a different Scandinavian language. In the case of Sweden, both analysts were native speakers of Swedish. All members of the authorial team were involved in at least one such analysis. The documents were subjected to a three-stage process, involving the same two people for each country. First, each of these two project colleagues undertook independent searches for any occurrences of the words, *estimation, approximation*, *check*, *round* and their variants in their allocated curricula. Searches were also undertaken for any occurrences of the four key terms of *computation*, *measurement*, *number line* and *quantity*, their variants, as well as alternatives like *calculation* or *numerosity*. In so doing, we were aware that in all three languages, synonyms and processes related to estimation abound, and our reading of the documents reflected this. For example, while the commonly used verbs for estimate are, in Denmark (Dk), Norway (No) and Sweden (Se), anslå, anslå and uppskatta respectively, we were conscious of synonyms like estimere (Dk), estimere (No) and estimera (Se), as well as nouns like overslagsregning (Dk), overslagsregning (No) and överslagsräkning (Se) and estimation-related processes like rounding, as manifested in the words afrunde (Dk), avrunde (No) and avrunda (Se). Each colleague independently copied and pasted the statements he or she had identified into a his or her own text document. Second, the two text documents for each country were compared and contrasted by the same two colleagues and a set of statements and their relationship to the four forms of estimation agreed. In practice, little disagreement arose as both the nature of the vocabulary and the context in which it was used typically determined colleagues' interpretations. Third, one of the two analysts synthesised, from the agreed and categorised statements, a summary narrative for each form of estimation across the various years of phases of the country's curricula. This summary was then read and edited by the second colleague to form the material on which the following is based.

Results

In the following, in accordance with our aim of offering independent analyses, we present the results of the analytical process described above for, alphabetically, Denmark, then Norway and finally Sweden. At this stage, the reader is informed that throughout the following we use the terms *statutory* and *non-statutory* to qualify the different curricular statements on which our narratives are based. In this context, a statutory statement is a legally binding reference to something that must be taught, while a non-statutory statement has no legal basis and is typically presented as advisory or illustrative.

Denmark

The Danish mathematics curriculum, published by the Ministry for Children and Education or Børne-og Undervisningsministeriet, comprises three parts, each of which is available as a downloadable pdf. These are the common goals or fælles mål, part of which, as is explained below, is statutory (Børne-og Undervisningsministeriet, 2019a), the statutory learning plan or matematik læseplan (Børne-og Undervisningsministeriet, 2019b) and the non-statutory mathematics teaching guidance or matematik undervisningsvejledning (Børne-og Undervisningsministeriet, 2019c). However, all three documents have been incorporated in their entirety into the mathematics handbook or matematik faghæfte (Børne-og Undervisningsministeriet, 2019d), with the consequence that our analysis is based solely on this document.

The common goals, of particular importance due to their essential framing of both the learning plan and the teacher guidance, comprises three sections. The first, the competence goals (kompetencemål), presents in tabular form statutory expectations for each of the three phases of education against three broad content areas of *number and algebra; geometry and measure; statistics and probability* alongside a vaguely specified *mathematical competence*. The second section, also presented in tabular form, highlights seven statutory points of special attention (opmærksomhedspunkter). These are spread across the three phases, with three focused on the first phase, two on the second and two on the third. Finally, the third section of the common goals comprises thirteen pages of tabulated and non-statutory learning outcomes for each phase of education. These are framed, essentially, in two ways. On the one hand they are framed by the content knowledge domains and, on the other hand, by the six process competences (kompetenceområde) of *problem solving, modelling, reasoning and mathematical thinking, representation and symbol processing, communication,* and *aids and tools* respectively. In short, the common goals comprise statutory competence goals, statutory points of special attention and non-statutory recommended learning outcomes. Finally, an overarching aim of critical thinking and evaluation is apparent throughout the curriculum.

Computational Estimation

Across the curriculum, the role of computational estimation seems transparently clear. For example, by the end of year three, it is recommended that children should have "knowledge of mental arithmetic, estimation (rough calculation) and calculation with written notation and digital tools", while the learning plan adds that they should be able to "round to the nearest ten or hundred" and, explicitly with reference to "mental arithmetic, estimation (rough calculation) and calculation with written notation and digital tools", "participate actively in developing methods" and "work with appropriate strategies for calculation". By the end of year six, a point of special attention asserts that children will be able to "carry out calculations within all four operations, including the use of estimation and calculators". Moreover, the learning plan adds that

> in many practical situations it will be sufficient for students to be able to perform an estimation of the calculation rather than perform an accurate calculation. In situations where an accurate calculation is needed an estimation can help in identifying incorrect keypresses on the calculator.

By the end of year nine, a point of special interest is that students will be “able to carry out simple percentage calculations by means of estimation and calculator”. This is supported by the learning plan’s assertion, effectively an extended repetition of that presented for the end of the first phase, that

> in many practical situations it will be sufficient for the students to be able to perform an estimation of the calculation rather than perform an accurate calculation. In situations where an accurate calculation is needed an estimation can aid in detecting possible calculator mistakes.

In sum, computational estimation finds warrants in the learning plan as a tool to be used in problematic practical situations. However, its manifestation is found in expectations, alongside invocations to round, focused on the checking of calculations, particularly with respect to calculator error.

Measurement Estimation

When read in context, the situation with respect to measurement estimation seems unambiguous. By the end of year three, a point of special attention asserts that children should be able to “estimate length, time and weight in simple everyday situations”. This is supported by the recommended outcome that the “student can estimate and measure length, time and weight”, alongside the statement in the learning plan that children’s “work with time includes both estimating and measuring time, reading and setting the time, as well as using dates and calendars”. With respect to the second phase of education, the learning plan asserts that

> students (should) build on the concept of measurement they developed in the first (phase), which includes estimating and measuring lengths, weights, time and angles, using relevant measuring units and measuring instruments and being able to evaluate a measurement result based on the accuracy of the measurement.

The same document, implying a connection between estimation and non-standard units, adds that

> Students are expected to be able to take measurements with non-standard units after the third grade. It is also expected that they have developed an initial sense of estimating goals in everyday contexts.

These goals are supported by several statements in the recommended learning outcomes, typically repeating that of the learning plan. For example, children should “estimate and determine circumference and area” and have “knowledge of different methods to estimate and determine circumference and area, including digital tools”. In addition, students should be able to “estimate and determine volume”, having knowledge of “methods to estimate and determine volume”. However, the learning plan also asserts that at “the beginning of this level the focus is on measuring and calculating circumference and area, including estimation and use of digital tools”, while at the end of the level the “focus is on making the students able to estimate, measure and calculate the volume of simple polyhedrons, including simple prisms”. There are no references to measurement estimation for the third phase of education.

In sum, there seems a clear developmental progression in the presentation of measurement estimation, with, from the perspective of mensuration, incremental shifts from one to two to three dimensions. Also, in expectations that children should “estimate and determine volume” is an implicit expectation that the former may be a computational estimation of the latter. Expectations of measurement also apply to time, weight and angle and the use of non-standard units.

Number Line Estimation

There is no reference to number line estimation in the recommended learning outcomes, although there are hints in the learning plan. For example, during the first three years, as part of children’s developing understanding of number, children should “place numbers on a number line”. In the second three years children should be encouraged to see "fractions as numbers on a number line" and in the last three years "place irrationals on the number line". In sum, we construe the number line placement of integers, fractions or irrationals as an implicit estimation.

Quantity Estimation

There are no references, in either the statutory or the non-statutory sections of the document relating to quantity estimation.

Norway

It is important for readers unfamiliar with Scandinavia to know that there are two parallel forms of Norwegian, Nynorsk and Bokmål, currently in use. The two languages are remarkably similar but political and cultural sensitivities necessitate the curriculum being presented in both. Thus, the Norwegian national curriculum for mathematics, published by the Directorate for Education in preparation for implementation in 2020 and representing statutory expectations, can be found as downloadable pdfs in both Nynorsk (Utdanningsdirektoratet, 2019a) and Bokmål (Utdanningsdirektoratet, 2019b). Importantly, the content of the two documents is identical, so only the former has been analysed below. Similarly, the Nynorsk version of the framework document for pre-school (Utdanningsdirektoratet, 2017) was consulted for references to estimation but yielded nothing of relevance.

The curriculum (læreplan i matematikk) presents mathematics within a set of six broad competences comprising exploration and problem solving; modelling and applications; reasoning and argumentation; representation and communication; abstraction and generalisation; and mathematical topics. All six are summarised in single paragraphs and are underpinned by expectations of critical thinking and evaluation. The curriculum is loosely specified, with all mathematical content knowledge, presented year on year, summarised in the form of 95 bullet points. Within these are no explicit references to the verb estimate, its variants or alternatives, although there are indications that it may be an embedded expectation. Additionally, there are online support materials[1] showing how each of the above statements matches one or other of the six broad competences and clarifying the authority's construal of the verbs embedded in them. However, this material offers no additional estimation-related insights, with the consequence that the following, as indicated earlier, draws solely on the Nynorsk curriculum document (Utdanningsdirektoratet, 2019a).

Computational Estimation

Within the curriculum there is a broad aim that children should be able to use mathematical representations, concepts and procedures to undertake calculations and evaluate the validity of their solutions. Our view is that it is difficult to evaluate the accuracy or validity of a solution without some form of computational estimation competence. Also, there are particular references that we construe as indicative of estimation. For example, in year three, children should "explore and explain the connections between addition and subtraction and use them in mental calculation and problem solving", while in year four, they should explore, use and describe various division strategies. In similar vein, year five finds the expectation that children should "develop and use different calculation strategies with positive integers and fractions and explain their thinking", year six expects them to "explore decimal calculation strategies and compare with integer calculation strategies", while year eight finds them being expected to "develop and communicate strategies for mental arithmetic in calculations".

In sum, our view is that such statements, when interpreted against expectations of critical thinking and evaluation, may implicate forms of computational estimation. That being said, we are conscious that such an inference may be an over-generous interpretation.

[1]https://www.udir.no/lk20/mat01-05/kompetansemaal-og-vurdering/kv21?lang=nob

Measurement Estimation

Although there are no explicit references to measurement estimation, there are several statements that we have construed as indicative of it. For example, in year two, children should "measure and compare lengths and areas using non-standard and standard units". By year three, they should "use different measurement units for length and mass in practical situations and justify the choice of measurement unit", while in year four they are expected to "use non-standard measurement units for area and volume in practical situations and justify the choice of measurement unit". Our view is that iterations of non-standard units for measuring different properties of geometrical objects, particularly against expectations of critical thinking and evaluation, could be suggestive of measurement estimation. Indeed, while such matters are not made explicit, it is not difficult to imagine a task whereby children are asked to calculate the area of their classroom's floor in terms of the area of their desktops. Such tasks, we suggest, would typically invoke estimation in some form.

Number Line Estimation

Number line estimation, although not presented explicitly, appears less implicit due to several statements concerning the placing of numbers on a number line. The placement of numbers on a number line may involve elements of estimation, whether it is the placement of integers in year two, fractions in year five or decimals in year six, although we concede this may be a hypothetical inference.

Quantity Estimation

Of all forms of estimation, quantity estimation seems the least visible in the new curriculum. Indeed, irrespective of the lack of any reference to the word estimation, there are only three references to "quantity", two of which can be loosely connected to estimation. The first asserts that children in year two should order "quantities … according to properties, compare them and reflect on whether they can be arranged in several ways". The second, for those in year three, is that children should "describe similarities and differences in … quantities … and use equal- and unequal signs". However, the extent to which such expectations can be construed as connecting to estimation is questionable.

Overall, although there are no explicit references to estimation anywhere in the curriculum, there are several implied, typically focused on computational and measurement estimation, although, as indicated above, we are aware that our interpretation may have been generous.

Sweden

The 2011 Swedish national curriculum for mathematics, which was subjected to its sixth revision in 2019, is structured in three phases, with outcomes specified for the end of years three, six and nine respectively. The statutory curriculum, or läroplan (Skolverket, 2019), incorporates three sections. The first presents the general aims for the learning of mathematics and includes five broad mathematical process goals. In brief, these are that pupils will formulate and solve problems; use and analyse mathematical concepts; choose and use appropriate mathematical methods; apply and follow mathematical reasoning; and use mathematical forms of expression to discuss and reason. The second covers the expected learning outcomes for each phase, framed by six broad content categories focused on, respectively, number sense and number use, algebra, geometry, probability and statistics, relationships and change, and problem solving. The third includes the benchmark outcomes for different achievement levels of the national tests for each phase. In addition, teachers' curriculum implementation is supported by means of a non-statutory commentary on the content of the formal document (Skolverket, 2017) and a third document (Skolverket, 2014), addressing assessment for learning in the context of mathematics, is presented at too general a level to be helpful in the context of this paper and is not referred to again. All Swedish documents are available as downloadable pdfs.

Computational Estimation

Expectations of computational estimation seem to be well-represented in the Swedish national curriculum throughout all stages in school. For example, while it is not explicitly focused on computational estimation, the broad goal that pupils should be able to "select and use appropriate mathematical methods for calculating and solving problems", implies a role for computational estimation, a role that is made more explicit in the details. The curriculum asserts that by the end of year three, pupils should know the "central methods for calculation with natural numbers, including mental and rough calculation" and be able to make "reasonable judgements concerning simple calculations and estimations". These statutory expectations are supported by the commentary, whereby pupils should be able to "develop methods for undertaking both mental and rough calculations with natural numbers" and, with respect to any calculation, "use mental calculations to judge if the results are reasonable".

By the end of year six, the statutory expectations have been augmented with the introduction of simple decimals and that reasonable judgements now include "estimations and calculations in everyday situations". Moreover, the commentary indicates that pupils should be able to "use several functional methods with natural numbers and simple decimals for mental calculations and rough calculations" and "reflect on and assess the reasonableness of rough calculations and mental calculations … and also round whole numbers in everyday situations". Similar statutory expectations are presented for the end of year nine, albeit with the inclusion of fractions and no explicit restrictions on decimals. In similar vein, the non-statutory document also includes references to negative integers and the expectation that pupils will "use the rules of rounding".

In sum, while details beyond broad statement of principle are scant, the Swedish curriculum includes a variety of expectations with respect to computational estimation, ultimately focused on expectation that children should be able to "reflect on and assess the reasonableness of rough calculations … in everyday situations". However, the processes by which children may make computational estimation, particularly with respect to rounding, are absent.

Measurement Estimation

With respect to measurement estimation, the statutory expectations for the end of year three appear, at least initially, a little vague. For example, the curriculum for years one through three specifies, with respect to geometry, that pupils should "compare and estimate mathematical quantities", before indicating that such a goal is related to the "measurement of length, mass, volume and time in usual current and older units". These goals are further clarified by the commentary, which suggests that students in years one through three should "gain experience of comparing and estimating different quantities, such as distance, area or volume, before moving on to measuring and using different units of measurement". By the end of year six, pupils should know "methods for how perimeters and areas of different two-dimensional geometric figures can be determined and estimated" and be able to "compare, estimate and measure length, area, volume, mass, time and angles with standard units of measure". In addition, the commentary adds that pupils should "make reasonable estimations" of such quantities "in different contexts". Finally, by the end of year nine, the commentary add that pupils should not only be able to "compare, estimate and measure" the same quantities as earlier but "use appropriate measurement instruments and measuring systems".

In sum, despite ambiguity over the use of the word quantity (the word *storhet* can also be interpreted as magnitude) the curriculum clearly expects pupils to be able to make reasonable estimations of a variety of properties of increasingly sophisticated geometrical objects. That said, expectations that children should "estimate and determine volume" imply that the former may be a computational estimation of the latter. Expectations of measurement also apply to time, weight and angle, although the expected use of historical units masks an omission, addressed in the assessment for learning document, that pupils should "compare and estimate quantities with the aid of non-standard units".

Number Line Estimation

There is no reference to number line estimation, implicit or explicit, in any context.

Quantity Estimation

There are no explicit references to quantity estimation, although within the geometrical expectations for the first phase can be found both statutory and non-statutory statements concerning the "comparison and estimation of quantities". That being said, there is a commentary expectation that by the end of year three, children will be able to "perceive objects (fewer than ten) without needing to count them one by one". On the face of it, such a goal may be construed as an encouragement to subitise. However, since most adults are able to subitise only up to four or five objects (Starkey & Cooper, 1995), our view is that a larger collection may require some form of estimation. Overall, though, there is no clear expectation of quantity estimation.

Discussion and Implications

In the above, we presented a summary of the literature concerning the form and function of different forms of estimation in order to frame independently conducted analyses of the national curricula of the three Scandinavian neighbours. In the following, we discuss these three curricula from the perspectives of their similarities and differences with respect to the opportunities they present for children's learning of these important skills.

First, the estimation-related emphases of the three countries' curricula varied considerably. On the one hand, the Norwegian curriculum seems to offer only the barest of allusions to the role of estimation, irrespective of its differing forms, in children's learning. On the other hand, while both the Danish and Swedish curricula clearly had estimation-related expectations, these focused almost exclusively on computational and measurement estimation. Thus, in none of the three curricula was there any evidence of systemic expectations of either quantity estimation or number line estimation.

Second, with respect to computational estimation, the Norwegian document has nothing to offer, other than its implicit role in its expectation of critical thinking and evaluation. Alternatively, both the Danish and the Swedish curricula warrant the inclusion of computational estimation by reference to "problematic practical situations" and "everyday situations" respectively. Both also, albeit in different ways, justify computational estimation as a means of checking calculations performed in other ways. However, while the Swedish curriculum encourages children to "reflect on and assess the reasonableness of rough calculations … in everyday situations", in neither the Danish nor the Swedish curricula was there anything but an implicit sense that computational estimations may take "less time and attentional resources than exact calculation, and thus can be used in circumstances where time or attention resources are limited" (Ganor-Stern, 2018, p. 2). That is, the importance of computational estimation as an essential life skill was left implicit in both documents, which we argue is an issue that could be addressed in any revision of the curriculum support documents. Also, with the exception of the Danish invocation to round, there was no evidence in any curricula of expectations concerning the processes by which computational estimations may be undertaken. Finally, none of the curricula materials, particularly the various non-statutory guidance materials, offered any explicit indication that the skills of computational estimation are implicated in the later learning of mathematics, whether in respect of particular topic areas (Ganor-Stern, 2018; Sowder, 1992), mathematics in general (Sekeris et al., 2019) or problem solving (Star & Rittle-Johnson, 2009). If there were any such expectations, they have been left implicit.

Third, measurement estimation was a common thread across both Danish and Swedish curricula, typically focused on time in different contexts and the physical properties of objects presented in sequences of expectation indicative of a developmental progression. However, while it may be unreasonable for curricular specifications to alert teachers to the implications of measurement estimation competence, it would not be unreasonable for curricular support materials to do so. With respect to the former, it could be construed, therefore, as an omission that neither the Danish nor the Swedish support materials alert teachers to research showing that children with mathematical learning difficulties overestimate time (Hurks & van Loosbroek, 2014) and that time estimation accuracy is a predictor of general mathematical competence (Kramer et al., 2018). In other words, at least from the perspective of measurement estimation, curriculum support materials might benefit from the inclusion of research-based warrants and indications of the importance of such estimation-related competence.

Finally, with respect to measurement estimation, the only common thread across the three curricula was the use of non-standard units, which research has shown to be a powerful underpinning of later conceptual and procedural competence (Chang et al., 2011). In this respect, the Danish curriculum clearly connected non-standard units to estimation, the Norwegian emphasised the use of non-standard units in general but offered nothing in respect of estimation, while the Swedish had nothing related to non-standard units, with the exception of a single reference in the non-statutory guide to assessment for learning. In other words, the extent to which the different systems expect non-standard units to be visible in children's learning of estimation is not only variable but unclear.

Fourth, both quantity estimation and number line estimation are effectively absent in all three curricula, although, with its link to subitising, the former is tangentially present in the Swedish. These seem disappointing omissions, not least because the former is reciprocally tied to the ability to count (Barth et al., 2009) and a predictor of later arithmetical competence (Bartelet et al., 2014). With respect to the latter, number line estimation is effectively absent in all three countries' curricula. Such an omission seems unfortunate, as number line estimation is a strong predictor of both later mathematical learning difficulties (Wong et al., 2017) and mathematical learning (Schneider et al., 2018).

In conclusion, the curricula of all three Scandinavian countries, especially the Norwegian, offer few, if any, explicit opportunities for children to acquire any form of estimational competence. Moreover, both Danish and Swedish expectations resonate with Smart's (1982) historical view of estimation as a utilitarian tool to support calculation and measurement. That is, both reflect a mathematical history rather than a mathematical future and, in so doing, fail to acknowledge the two forms of estimation with the greatest developmental implications, number line estimation and quantity estimation, with, we posit, profound implications for the later learning of other areas of mathematics (Wong et al., 2016). Consequently, we suggest that all three countries would be well-served by either introducing or refining curriculum support documents to explain and exemplify the different forms of estimation in ways that would help teachers understand why they matter as both curriculum goals and supports for later learning and real-world functioning. Finally, in comparison with countries with strongly framed mathematics curricula, whereby teachers work within tightly defined objectives for each grade and have little personal autonomy (Andrews, 2016), the three Scandinavian curricula can be construed as weakly framed (Bernstein, 1981), leaving teachers with the responsibility for deciding what should be taught, when it should be taught and how it should be taught (Skott, 2004). Thus, from the perspective of future research, it would be timely to examine how teachers in the three countries construe and reify estimation in their fulfilment of the "contract" between them and the state. In similar vein, it would seem pertinent to examine curriculum developers' perspectives on estimation, not least because all three curricula, notwithstanding their weak framing, offer remarkably limited conceptions of this essential competence.

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