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Predicting missing pairwise preferences from similarity features in group decision making

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ABSTRACT

In group decision-making (GDM), fuzzy preference relations (FPRs) refer to pairwise preferences in the form of a matrix. Within the field of GDM, the problem of estimating missing values is of utmost importance, since many experts provide incomplete preferences. In this paper, we propose a new method called the entropy-based method for estimating the missing values in the FPR. We compared the accuracy of our algorithm for predicting the missing values with the best candidate algorithm from state of the art achievements. In the proposed entropy-based method, we took advantage of pairwise preferences to achieve good results by storing extra information compared to single rating scores, for example, a pairwise comparison of alternatives vs. the alternative's score from one to five stars. The entropy-based method maps the prediction problem into a matrix factorization problem, and thus the solution for the matrix factorization can be expressed in the form of latent expert features and latent alternative features. Thus, the entropy-based method embeds alternatives and experts in the same latent feature space. By virtue of this embedding, another novelty of our approach is to use the similarity of experts, as well as the similarity between alternatives, to infer the missing values even when only minimal data are available for some alternatives from some experts. Note that current approaches may fail to provide any output in such cases. Apart from estimating missing values, another salient contribution of this paper is to use the proposed entropy-based method to rank the alternatives. It is worth mentioning that ranking alternatives have many possible applications in GDM, especially in group recommendation systems (GRS).

1. Introduction: predicting pairwise preferences based on individual similarities

In a group decision-making (GDM) problem, a group of individuals (experts) evaluate different options (alternatives) to find the best option in terms of some given criteria. Applications of GDM are vary, and can be as simple as the recommendation of music or a book to an online group of customers of a store or as crucial as vital decisions made by politicians. In real-life situations, GDM is based on information available in relation to an individual's preferences. Therefore, typically, one uses pairwise comparisons, for instance, to understand to what extent a customer prefers music A to music B. However, such problems suffer from a ubiquitous lack of available data. In most cases, especially when the number of alternatives is high, several pairwise comparisons may be missing.

This paper addresses the question of how to estimate missing pairwise comparisons for properly predicting customers' preferences for a specific alternative, proposing a matrix factorization framework that permits discovery of latent features of experts and alternatives. In this manner, similarity measures between different experts and between different alternatives can be better captured using our framework and used for predicting the pairwise preferences of experts.

We know that some alternatives, such as cuisines, can be similar. For example, in the case of Chinese and Japanese food, the most notable similarity between these two culinary cultures is the use of fresh ingredients. Whether we think of fresh seafood or fresh vegetables, the dishes almost always require fresh meat and other fresh products. Therefore, people who like the fresh Chinese dishes probably also like Japanese dishes. As a result, we can say that these types of food are similar, and both differ from

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some well-cooked cuisines, such as Turkish cuisine. The same concepts can be considered in regard to the similarity between experts who have similar tastes. Hence, in the above example, we say two experts are similar if they have the same taste and food preferences. For example, they both like pizza and Italian food, but they are less fond of Mexican dishes.

Motivated by the idea of similarity, in this paper, we tried to estimate the missing preferences of one expert by determining the preferences of a similar expert. The most important advantage of this method is that even in situations where there are too many missing values, we have an excellent opportunity to estimate the missing values while some other methods fail to work [1,2]. The problem of estimating missing values of pairwise preferences has a long tradition in GDM [1–8]. The better the missing value estimation is, the more accurate the group ranking of the alternatives is. While experts have their own preferences, the final group decision will be made as a result of the aggregation of all of them. Experts usually express their preferences in different ways, including (i) rankings, in which a list of alternatives is ordered from the most to the least preferred [9], (ii) the so-called utility vectors, in which the utility of every alternative is indicated by its corresponding element in the vector [10,11], (iii) preference relations, where the binary relation of the alternatives is considered as the preference [12], and (iv) so-called fuzzy preference relations (FPRs), in which the experts express their pairwise preferences on a set of alternatives that are usually stored in the form of a preference matrix.

In this paper, we have chosen to use pairwise preferences from among the different ways of expressing the preferences. This is because compared to single rating scores, pairwise preferences give more information about the alternatives. In the following, we explain this more clearly with an example from the single rating method. Typically, experts rate food A as a 5 (from 1 to 5) if they like it. However, if they also like food B, they will also rate it as a 5. Therefore, although the experts like both foods, they cannot say which one they like more using this rating system. Accordingly, ranking the foods they like would not be very accurate. On the other hand, instead of a single rating, if they compare the foods they like by giving pairwise scores, they would have a better chance of obtaining a more accurate ranking of their food preferences. The idea of comparing alternatives by using pairwise scores when recommending alternatives to a group of experts with different personalities was introduced by Abolghasemi et al. [13]. This approach resulted in very precise and fair recommendations.

In most works on the application of FPR in GDM such as [1], only the preferences of the expert in question are used to estimate the missing pairwise preferences. For instance, if the expert likes food A more than food B, B more than C, and C more than D, the classical methods will conclude that the expert likes food A more than food D. The main drawback of this family of methods is that if one pair in this chain is missing, we may not be able to estimate the pairwise preferences of another related pair. For instance, in the above example, if the pairwise preference of B over C is missing, then it will be impossible to calculate that of A and D. Therefore, in such situations, calculating some pair scores is impossible. To overcome this problem, in this paper, we not only take the experts' pairwise preferences into account but also use the similarity between the experts and the similarity between the alternatives.

In a GDM, experts express their preferences on the alternatives differently. In GDM-related problems, experts often provide insufficient information about the alternatives [1,14–18]. For instance, in an FPR, there are some missing values. Therefore, it is important to find an accurate method to estimate the missing values in the FPR and then rank the alternatives correctly. In this section, we will detail some research that has tried to estimate the missing values in an FPR.

In [19], Liang et al. proposed an interactive GDM approach to make rankings with incomplete additive preference relations. One aspect of this approach is that it incorporates the strength of social ties and social influence calculated by social network analysis methods. Moreover, to complete the missing preference values of the incomplete additive preference relations, a linear programming model is used. The main advantage of resorting to the linear programming model is maintaining consistency. In [1], a learning procedure for estimating the missing values in an incomplete FPR based on additive consistency is introduced. This method uses the known values to estimate the unknown values using the transitivity property. The main drawback of this method is that, at first, it only works well if data are consistent, and the second is that in some conditions, it does not succeed in the completion of the FPR. More clearly, if the expert did not provide a pairwise preference score for one alternative over the others, it would be impossible to estimate that expert's preferences. In this paper, we introduce a method that can predict the pair scores even when all the pairwise preferences of one alternative are missing. Since our model works based on similarity, it will determine the expert's taste in food by considering their preferences for the other alternatives and their similarities to the other experts. Some works have used collaborative filtering to deal with incomplete information in the field of group decision making [20–22]. Making recommendations for groups is a well-known GDM problem that benefits greatly from the idea of similarity in collaborative filtering [23]. For instance, in [24], using the idea of collaborative filtering in GDM, the authors constructed a social trust network for travel recommendations that resulted in a better online booking experience for travelers.

Matrix factorization (MF) [25] is a type of collaborative filtering method that finds the relationships between the experts and the alternatives by learning their latent features. MF aims to find the similarity between different experts and different alternatives. In [26–28] the idea of MF in GDM is implemented to find the aggregation between the experts used to reach a final decision. Although MF was successful in addressing the group decision-making problems, its input is a single rating. More precisely, the datasets are made up of scores indicating the experts' preferences on the alternatives. As we explained earlier in this section, pairwise preferences can better show the experts' preferences. Thus, they are more informative than the single rating scores. In this paper, we applied the idea of pairwise preferences in MF for the GDM problem, which allowed us to better predict missing values and rank the alternatives.

In GDM, reaching a consensus is a key concept. In [29], a distance measure for detecting the inconsistencies among experts is used to calculate the consensus level. Then, a feedback mechanism (benefit or cost) is developed to obtain the individual consensus evaluation matrix. The final group evaluation matrix is obtained by integrating all individual consensus evaluation matrices. In [30], a group consensus-based travel destination evaluation method is proposed, which uses online reviews, estimates missing preferences, and reaches a group consensus. This method contains steps: (1) represent the decision opinions through the sentiment matrix with the percentage distribution. (2) obtain the weight vector of attributes by the preference of attributes given by group experts and then estimate missing values. (3) reach consensus based on the minimum adjustment cost feedback mechanism and obtain ranking. In [31], a comprehensive star rating approach for cruise ships based on interactive group decision-making with personalized individual semantics is presented. This paper presents a novel weight calculation method for assigning the importance of the main cruise indicators. Moreover, the personalized individual semantics (PIS) model is adopted to effectively address uncertainty. In comparison, in our algorithm, after predicting the missing pairwise preferences, we created a group-based score for every alternative based on all individual scores. As a result, with a straightforward Borda count method, we could obtain the group opinion. It is worth mentioning that the main aim of our paper is predicting missing pairwise values and not designing a novel consensus algorithm; thus, more sophisticated approaches for aggregating preferences and achieving consensus can be used.

2. Theoretical background: From fuzzy preference relations (FPR) matrices to ranking vectors

The framework we will apply to derive predictions of pairwise preferences is based on a matrix of pairwise preferences, usually called the fuzzy preference relation (FPR) matrix. Additionally, we will also use a Bayesian optimization criterion called Bayesian personalized ranking (BPR) [32]. In this section, we describe the framework, including a background that defines and implements these tools and concepts.

To introduce both FPR matrices and the output ranking vectors, we assume that $E = \{e_1, \dots, e_m\}$ is a set of experts deciding between and comparing a set of alternatives $X = \{x_1, \dots, x_n\}$. For each expert, we can then define an FPR P on a set of alternatives X as a relation characterized by function μ_P on the product set $X \times X$. For each pair of alternatives in the product, μ_P retrieves a value in the interval [0, 1], which is considered to be the expert's comparison score on that pair of alternatives [33]. In other words, we will write each pairwise comparison weight as $p_{ij} = \mu_P(x_i, x_j)$ and interpret its value as the *preference degree* of alternative x_i over alternative x_j. In particular, if $p_{ij} > 0.5$, we say that x_i is preferred to x_j and if $p_{ij} = 0.5$, we say that we equally prefer alternatives x_i and x_j. Note that, for consistency we assume P to be additive reciprocal, i.e. $p_{ij} + p_{ji} = 1$, for every $i, j \in \{1, \dots, n\}$. Typically, relation P is represented as an $n \times n$ matrix (with $n = |X|$), which we will call the preference matrix, whose entries are p_{ij}.

Preference matrices help assess how and when consensus in the group of m experts is reached. In fact, when using a GDM approach to monitor collective opinions, all individual preference matrices would be aggregated, and the deviations of the individual matrices from the collective (average) would be observed. This is usually called an indirect approach to GDM [11]. A direct approach would predict the collective opinion by looking at the individual rankings of the alternatives according to experts' preferences. This can be done by transforming FPR preference matrices into vectors whose entries measure the ranking of the alternatives.

More formally, a weighted ranking can be defined as a function $X \mapsto \mathbb{R}$, which maps each alternative in X into its *single rating scores*. Ordering these single rating scores from the highest to the lowest yields the ranking vector, and aggregating the individual ranking vectors yields a direct GDM approach to assess consensus dynamics.

There are several ways to deduce a ranking vector based on a fuzzy preference relation matrix. Two relevant approaches are *the quantifier-guided dominance degree* (QGDD), where the rank of each alternative represents the dominance or importance of the alternative over the rest of the alternatives, and *the quantifier-guided nondominance degree* (QGNDD), where the rank of each alternative represents the degree to which the alternative is not dominated by the rest of the alternatives [34]. Another method is the *Netflow method* [35], which defines the rank of an alternative as the difference between the *inflow* and the *outflow* preference of the alternative. This method has also been defined in [36] as the broad Borda count.

Instead of the processes above, we have used the approach introduced recently in [11,37,38], where the vector of single rating scores is given by the stationary distribution π solving the equation

$$\pi = \pi S \, , \tag{1}$$

where S is an irreducible aperiodic stochastic matrix defined by the entries s_{ij} as

$$s_{ij} = \frac{1}{n-1} p_{ji} \quad \text{if } i \neq j \, , \tag{2a}$$

$$s_{ii} = 1 - \sum_{j=1, j\neq i}^{n} s_{ji} \, . \tag{2b}$$

Note that the stationary distribution π can be either computed iteratively via a random walk on the Markov chain defined by the stochastic matrix S or analytically via the computation of the eigenvector associated with the highest eigenvalue of the matrix S.

Finally, regardless of which approach one chooses, direct or indirect, or method one uses to derive ranking vectors, there are also different ways to aggregate the corresponding individual preference values (corresponding entries in FPR matrices or ranking vectors) into a collective preference value. This is typically carried out using an aggregation operator.

3. Methods and data

We assume that each of the m experts expresses their preferences independently and in the form of an FPR. Following the terminology from the previous section, we will denote $\mu^{(k)}$ the FPR of the kth expert and let $P^{(k)} = [p_{ij}^{(k)}]_{n\times n}$ be the corresponding preference matrix. Consequently, $p_{ij}^{(k)} = \mu_P^{(k)}(x_i, x_j)$ for all $i, j = 1, \dots, n$ and $k = 1, \dots, m$.

3.1. The cross-entropy approach to predict pairwise preferences

This section introduces our approach to predicting pairwise preferences based on the concept of cross-entropy. In general, the entropy of a given distribution $\rho(x)$ measures the level of uncertainty of that distribution, quantified by the aggregated value of $\log(\rho(x))$ weighted by the distribution itself, i.e., $\int \rho(x) \log(\rho(x))dx$. In case the stochastic variable is discrete, the integral is substituted by a sum over all possible values. Cross-entropy measures the relative entropy of a distribution or set of probabilities, p_i, compared to another one, $\hat{p}_i$, usually a model of the first ones. In this manuscript we will deal with cross-entropy loss functions of discrete stochastic variables, which compare the "true" probability values, p, with the ones estimated by some model, $\hat{p}$, namely, $L(\hat{p}_i) = \sum_i p_i \log(\hat{p}_i)$. In particular, since we will consider only pairwise preferences between two classes, we will use a binary cross-entropy loss function.

We start by considering the matrix factorization pair-score prediction (MFP), introduced in [39], as an aggregation method that derives the weighted ranking of alternatives, $x_1, \dots, x_n$, by aggregating the FPRs of experts, $e_1, \dots, e_m$.

We now introduce d factors whose values compose a vector that characterizes experts and alternatives. In other words, we assume that each expert e_k and each alternative x_i are characterized by d factors, which we represent as vectors $f^{(k)} = (f_1^{(k)}, \dots, f_d^{(k)})$ and $g^{(i)} = (g_1^{(i)}, \dots, g_d^{(i)})$ respectively.

The MFP method consists of estimating a matrix R as

$$R = FG^T \, , \tag{3}$$

where matrix F is an $m \times d$ matrix, usually called the expert-feature matrix, and G is an $n \times d$ matrix, usually called the

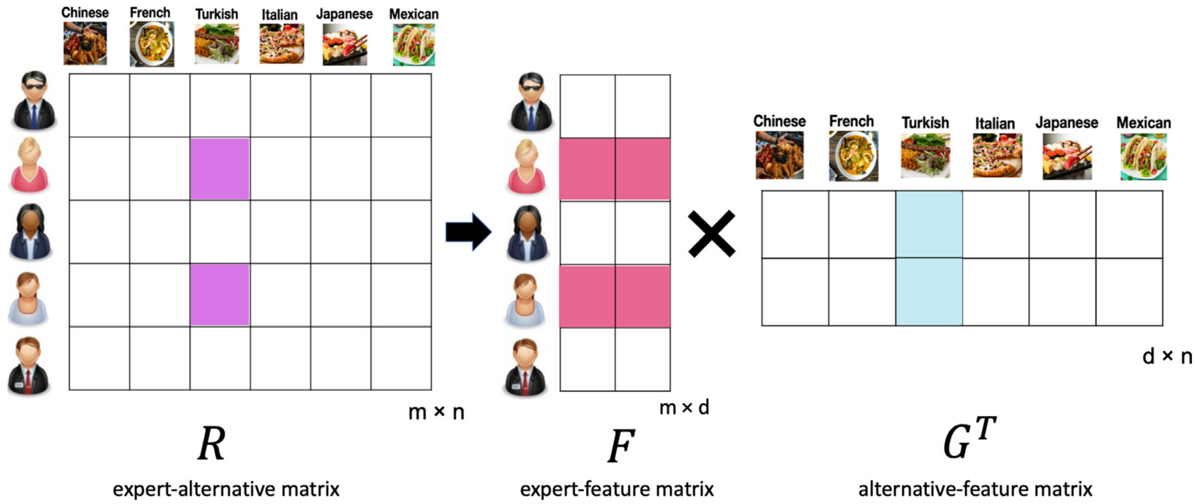

Fig. 1. Matrix R is factorized into two matrices:expert-feature matrix (F) and alternative-feature matrix (G).

alternative-feature matrix. Fig. 1 illustrates how matrix R is factorized into two matrices F and G. In this figure, $d = 2$ means every expert, and every alternative is considered as a vector of two features. Experts that have similar food preferences will be represented with vectors that are similar according to some similarity metric such as cosine similarity. Therefore, similar alternatives (restaurants such as Chinese and Japanese) will be clustered together in this two-dimensional space.

With the elements r_{ki}, $(k = 1, \dots, m$ and $i = 1, \dots, n)$ we can estimate the pairwise comparisons of expert k as

$$\hat{p}_{ij}^{(k)} = b_{ij} + r_{ki} - r_{kj} = b_{ij} + \sum_{\ell=1}^{d} f_\ell^{(k)} \left(g_\ell^{(i)} - g_\ell^{(j)} \right) \tag{4}$$

where $b_{ij} = \tilde{\beta}_i - \tilde{\beta}_j$ with β_i the bias of alternative x_i, i.e., the intrinsic value of the alternative, independent of the expert.

To find the optimal values of parameters β_i, $f_\ell^{(k)}$ and $g_\ell^{(j)}$ the following loss function is used:

$$L(\hat{p}_{ij}^{(k)}) = -p_{ij}^{(k)} \log \left(h(\hat{p}_{ij}^{(k)}) \right) - (1 - p_{ij}^{(k)}) \log \left(1 - h(\hat{p}_{ij}^{(k)}) \right) \tag{5}$$

where $h(x) = \frac{1}{1+e^{-x}}$ is the logistic function used to map the predicted data between 0 and 1. Function $L(\hat{p}_{ij}^{(k)})$ is the binary cross-entropy loss (log-loss), which is typically used to predict the error [40]. In other words, function $L(\hat{p}_{ij}^{(k)})$ can be interpreted as a binary classification of the pairwise alternatives, indicating whether those alternatives are correctly ordered or not. The objective function is defined as

$$\mathcal{L}(\Theta) = \sum_{i,j=1}^{n} L(\hat{p}_{ij}^{(k)}) + R(\Theta) \tag{6}$$

where $R(\Theta)$ is used for regularization and Θ is the set of model parameters, namely,

$$\Theta = (\beta_i, \beta_j, f_\ell^{(k)}, g_\ell^{(i)}, g_\ell^{(j)}), \tag{7}$$

for $i, j = 1, \dots, n$, $k = 1, \dots, m$ and $\ell = 1, \dots, d$. To optimize the model, we search for the minimum value of the objective function $\mathcal{L}(\Theta)$, which is found using a stochastic gradient descent (SGD) algorithm. The SGD algorithm updates each model's parameters according to

$$\Theta\text{new} \leftarrow \Theta\text{old} - \alpha \left(\left(h(\hat{p}_{ij}^{(k)}) - p_{ij}^{(k)} \right) \nabla_\Theta \hat{p}_{ij}^{(k)} + \lambda_\Theta \Theta \right), \tag{8}$$

where α is the learning rate, λ_Θ is the regularization coefficient for each parameter $\Theta = (\beta_i, f_\ell^{(k)}, g_\ell^{(j)})$, and ∇_Θ is the gradient of preference estimations in the parameter space.

Considering each parameter separately, Eq. (8) is decomposed into the following system of equations

$$\begin{aligned} \left(f_\ell^{(k)}\right)_{\text{new}} \leftarrow & \left(f_\ell^{(k)}\right)_{\text{old}} \\ & - \alpha \left(\left(h(\hat{p}_{ij}^{(k)}) - p_{ij}^{(k)} \right) \left(\left(g_\ell^{(i)}\right)_{\text{old}} - \left(g_\ell^{(j)}\right)_{\text{old}} \right) + \lambda_{f_\ell^{(k)}} \left(f_\ell^{(k)}\right)_{\text{old}} \right), \end{aligned} \tag{9a}$$

$$\begin{aligned} \left(g_\ell^{(i)}\right)_{\text{new}} \leftarrow & \left(g_\ell^{(i)}\right)_{\text{old}} \\ & - \alpha \left(\left(h(\hat{p}_{ij}^{(k)}) - p_{ij}^{(k)} \right) \left(f_\ell^{(k)}\right)_{\text{old}} + \lambda_{g_\ell^{(i)}} \left(g_\ell^{(i)}\right)_{\text{old}} \right), \end{aligned} \tag{9b}$$

$$\begin{aligned} \left(g_\ell^{(j)}\right)_{\text{new}} \leftarrow & \left(g_\ell^{(j)}\right)_{\text{old}} \\ & + \alpha \left(\left(h(\hat{p}_{ij}^{(k)}) - p_{ij}^{(k)} \right) \left(f_\ell^{(k)}\right)_{\text{old}} - \lambda_{g_\ell^{(j)}} \left(g_\ell^{(j)}\right)_{\text{old}} \right), \end{aligned} \tag{9c}$$

$$(\beta_i)_{\text{new}} \leftarrow (\beta_i)_{\text{old}} - \alpha \left(\left(h(\hat{p}_{ij}^{(k)}) - p_{ij}^{(k)} \right) + \lambda_{\beta_i} (\beta_i)_{\text{old}} \right), \tag{9d}$$

$$(\beta_j)_{\text{new}} \leftarrow (\beta_j)_{\text{old}} + \alpha \left(\left(h(\hat{p}_{ij}^{(k)}) - p_{ij}^{(k)} \right) - \lambda_{\beta_j} (\beta_j)_{\text{old}} \right). \tag{9e}$$

To see how we obtained Eqs. (9a)–(9e) from Eq. (6), we refer the reader to Appendix A.

Having estimated in this way matrices $F^{(k)}$, A and B, i.e., all parameters $\beta_i, f_\ell^{(k)}$ and $g_\ell^{(j)}$, for $i, j = 1, \dots, n$, $k = 1, \dots, m$ and $\ell = 1, \dots, d$, we can now predict the missing relative preference $\hat{p}_{ij}^{(k)}$ by using Eq. (4).

Finally, the rankings of alternatives can also be estimated as follows. We first aggregate the estimated relative preferences $\hat{p}_{ij}^{(k)}$ to compute what we call a personalized alternative score $s_i^{(k)}$, given as the average of relative preferences, namely,

$$s_i^{(k)} = \frac{1}{n} \sum_{j=1}^{n} \hat{p}_{ij}^{(k)}. \tag{10}$$

The score $s_i^{(k)}$ is the personalized score of expert k for alternative i and measures how much expert k prefers alternative i with respect to the universe of possible alternatives. From these personalized scores, we compute the so-called group alternative scores as

$$a_i = \frac{1}{m} \sum_{k=1}^{m} s_i^{(k)}. \tag{11}$$

The group ranking of alternatives will be achieved by sorting the a_i in descending order.

Fig. 2. Illustration of the front-end and back-end of the interface used to perform the online experiment (see text). **(Left)** Front-end: A expert is comparing and evaluating different pairs of dishes. This data is used as pairwise preference scores in the dataset. **(Right)** Back-end: The collected dataset contains a FPR matrix for each expert.

3.2. The datasets: a social experiment on consensus

The datasets used in this paper were collected in an online experiment called *Consens@OsloMet*, performed at Oslo Metropolitan University (Norway). The experiment aimed to study two different ways groups can reach consensus (general agreement) when presented with multiple choices regarding food preferences, namely, Chinese food, French food, Turkish food, Italian food, Japanese food, and Mexican food. The participants were grouped into groups of five. Then, each group was asked to update – or not – their food choices according to their knowledge about the average opinion within the group. The experiment has been registered in and approved by the Norwegian Centre for Research Data, the *Norsk Senter for Forskningsdata* (NSD) with reference number 631862.

The authors developed an online interface that allows experts to compare different food pairs and enter their pairwise scores. A view of the front-end interface is illustrated in Fig. 2 (left). In the example shown in the figure, the expert compares a Japanese dish with an Italian dish. A score of 0.4, indicated on the screen, means that the expert would choose Japanese with a 40% probability, compared to 60% for the Italian option. In terms of the back-end of the interface, the data concerning the choices – values between 0 and 1 – were collected as preference matrices for each expert in each group. An illustration of the preference matrix is given in Fig. 2 (right). The exact values of the preference matrices are given in Appendix B. From each matrix, we compute the corresponding vector of single rating scores, as defined in Eq. (1), according to the following order of choices: Chinese (entry 1 of the vector), French, Turkish, Italian, Japanese, and Mexican (last entry).

3.3. Data processing and evaluation metrics

To process the data, we then generated "incomplete" preference matrices by deleting some of the entries in the original preference matrices. Note that only the upper (or lower) triangular part of these matrices has independent values (see the bold numbers in Appendix B). Since we have 6×6 entries, we need to consider between 1 and $15 = 6 \times 5/2$ missing values in each matrix. Since we have a total of 10 matrices (experts), we will have a number N between 1 and 150 missing values in each trial.

For each value $N = 1, \dots, 150$, we sampled 100 trials. Each trial was conducted as follows:

- We randomly select N of the independent values in the upper triangle of all ten matrices.
- We remove those selected values and the corresponding ones in the lower triangle part of all ten matrices.
- We perform our algorithm, described in Section 3.1, to estimate each of the N missing values.
- We evaluate how accurate these estimates are compared to the original matrices (see below).

Additionally, we compared the average ranking computed from the original matrices with the ranking computed from the "estimated" matrices. This was done by averaging both the original and estimated matrices, computing the vector of single rating scores, as in Eq. (1), and finally ordering these single rating scores. Please see the logical diagram of the proposed method in Fig. 3. The codes of the proposed method will be published on GitHub after the paper is accepted.

To check the quality of the proposed method, we define two different prediction errors. The prediction error is $E_{\hat{P}}$ for the estimated pairwise comparisons in each preference matrix, and the ranking error is $E_{\hat{R}}$ of the estimated average ranking.

The former is given by

$$E_{\hat{P}} = \frac{1}{N} \sum_{k=1}^{m} \sum_{i,j=1}^{n} \left| \hat{p}_{ij}^{(k)} - p_{ij}^{(k)} \right| . \tag{12}$$

Note that for the "nonmissing" values, the terms in the sum in Eq. (12) are zero.

The latter prediction error is given by

$$E_{\hat{R}} = \frac{1}{n} \sum_{i=1}^{n} \left| \hat{r}_i - r_i \right| , \tag{13}$$

where r_i and $\hat{r}_i$ are the entries of the average ranking vector $\vec{r}$ and its estimate $\hat{\vec{r}}$.

We have illustrated the framework described in this section with one specific example. For this experiment, we used the pairwise preferences of $m = 5$ experts (matrices P1 to P5 in Appendix B), with $n = 6$ alternatives and $d = 2$ factors:

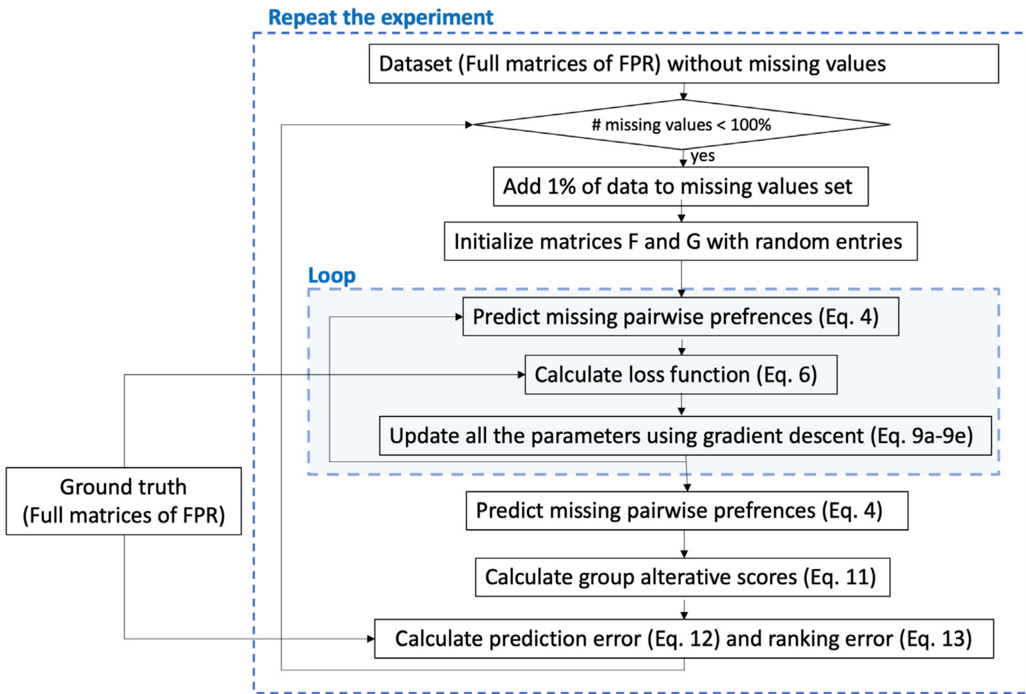

Fig. 3. Logical diagram of the process of the proposed method.

- Step 1: initialize matrices F and G in Eq. (3) with dimensions 5×2 and 6×2 with random entries in the range [0, 0.01]:

$$F = \begin{bmatrix} 0.008 & 0.001 \\ 0.006 & 0.002 \\ 0.004 & 0.0006 \\ 0.007 & 0.004 \\ 0.009 & 0.003 \end{bmatrix}, \qquad G = \begin{bmatrix} 0.003 & 0.003 \\ 0.004 & 0.008 \\ 0.004 & 0.005 \\ 0.005 & 0.008 \\ 0.002 & 0.006 \\ 0.001 & 0.006 \end{bmatrix}.$$

- Step 2: choose $N = 10$ out of 75 pairwise scores as the missing values and estimate them according to Eq. (4).
- Step 3: update the matrices F and G entries according to Eqs. (9a)–(9e).
- Step 4: predict all the missing values using Eq. (4).
- Step 5: repeat iteratively steps 3 and 4.
- Step 6: after 1000 iterations, the estimation is assumed to converge to an optimal value. In this example, we reach the final F and G matrices, which retrieve

$$R_{\text{final}} = F_{\text{final}} G^T_{\text{final}} = \begin{bmatrix} -0.656 & -0.900 & 0.597 & -0.316 & 0.718 & 0.110 \\ 0.123 & -0.181 & 0.913 & -0.567 & -0.886 & 0.260 \\ -0.178 & 0.683 & -0.255 & 0.222 & -0.160 & -0.318 \\ 0.911 & 0.279 & 0.200 & 0.117 & -0.625 & -0.882 \\ -0.530 & 0.793 & -0.396 & 0.249 & -0.257 & -0.122 \end{bmatrix}. \quad (14)$$

Finally, the predicted missing values for the pairwise preferences are used to calculate the group alternative scores, as defined in Eq. (11). In this example, these scores (0.47, 0.52, 0.52, 0.44, 0.53, 0.50) correspond to alternatives 1 to 5. To see how the proposed method works, we have compared our proposed entropy-based model with the one introduced in [1], which we call the ACHH model. The group alternative scores for the ACHH model are (0.47, 0.51, 0.50, 0.46, 0.51, 0.52).

All of these scores are compared with the "ground truth", i.e., the group scores calculated using Eq. (11) based on FPR matrices in the food dataset with entries of pairwise preferences with no missing values. For the above example, these scores are (0.47, 0.53, 0.51, 0.44, 0.53, 0.50). Hence, the prediction error – Eq. (12) – for our model is $E_{\hat{P}} = 0.16$, while for the ACHH model, it is $E_{\hat{P}} = 0.23$.

For the group ranking of the alternatives, our model yields (5, 2, 3, 6, 1, 4), whereas the ACHH model yields (6, 5, 2, 3, 1, 4). Knowing that the ground truth rankings are (5, 2, 3, 6, 1, 4), it is concluded that, unlike the ACHH model, our model predicts the correct ranking in all cases.

4. Experiment and discussion

In this section, we show the validity of the proposed method on two different datasets.

4.1. Experiment on the food dataset

The first experiment is performed on the "Food dataset" explained in Section 3.2, and the results are shown in Fig. 4. The prediction error average is shown for the ACHH model (red dashed lines) and the entropy-based model (solid green lines) together with the corresponding standard deviations (error bars). The average prediction and ranking error for the entropy-based model increases linearly, while the ACHH model shows a different trend. It shows very little error change in the beginning, while when the percentage of missing values exceed 50%, the errors increase drastically until it stops completely in the best case with 80% missing values. This is because in the ACHH method, as the number of missing values increases, less information is available to calculate the missing values. The increasing number of missing values, however, does not lose too much information in the proposed entropy-based method. This is because we use experts' preferences and because we use the similarity of the experts and similarity of the alternatives, which helps substantially in estimating the missing values. Therefore, even if the above-mentioned stop condition is fulfilled, we can still estimate the missing alternative score.

In the insets shown in Fig. 4, we indicate the fraction of trials that stopped during execution when the percentage of the

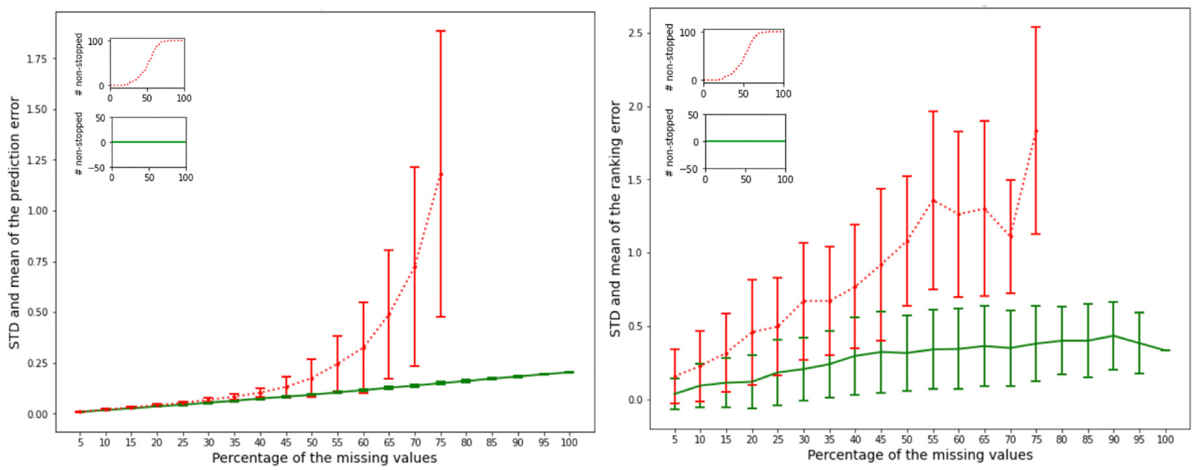

Fig. 4. (Left): Mean prediction error for the entropy-based model (green) and ACHH model (red) (Left): Mean ranking error for the entropy-based model (green) and ACHH model (red). In the insets we show the corresponding number of stopped experiments out of 100 trials. The plot is related to the food dataset.

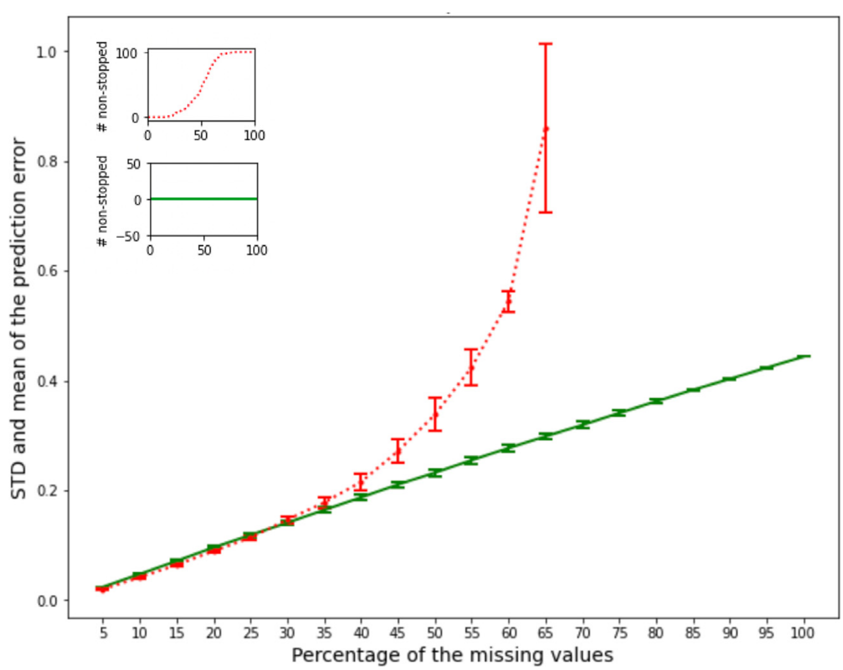

Fig. 5. Mean prediction error for the entropy-based model (green) and ACHH model (red) for the car dataset. In the insets, we show the corresponding number of stopped experiments out of 100 trials. The plot is related to the car dataset.

missing values increased. As seen in the figures, for more than 80% of the missing values, there is no mean or STD in the ACHH model. This is because this model stops working when there is a special condition.

Based on [1], if a row in the FPR matrix is missing, then all the pairwise scores of an expert on an alternative are missing, the ACHH model does not have enough information to estimate the expert's preferences on that alternative. Therefore, this model stops working. Since the missing values are chosen randomly in the experiment, the above-mentioned condition will eventually happen at different times. This stop completely depends on the position of the missing values. To see when this model stops on average, we repeated the experiment 100 times, and the average stop was when the number of missing values reached 75% of all the data. However, as the figures confirm, the entropy-based model never stops, which is a significant advantage of our model.

4.2. Experiment on the car dataset

A dataset of car preferences[1] provided by Abbasnejad et al. [41] in 2013 was used in this experiment. The data were collected from 60 different experts from the United States using Amazon's Mechanical Turk. From this dataset we used the pairwise comparison data of 50 experts on ten different cars (alternatives). Each expert provided answers for all 45 possible pairs of alternatives, giving 90 observations for each expert. In this dataset, if expert u prefers alternative i over alternative j, then pairwise preference $p_{ij}^{(k)}$ is considered as 1, otherwise as 0. If two alternatives are equally preferred, then $p_{ij}^{(k)} = 0.5$.

We tested our algorithm and ACHH model with this car dataset. The mean and standard deviation of the prediction error for 10 experiments are shown in Fig. 5. As in the case of the food

1 http://users.cecs.anu.edu.au/~u4940058/CarPreferences.html

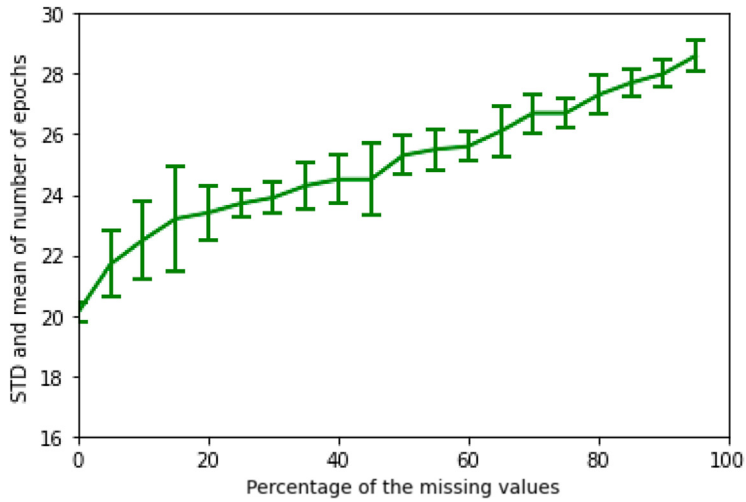

Fig. 6. Mean and standard deviation of the number of epochs in which the learning process of the proposed algorithm is completed. This experiment is executed 10 times on the food dataset.

dataset, we can see that our model has less prediction error than the ACHH model. Moreover, as we expected, the ACHH model stops at some points when the percentage of missing values increases.

To analyze the complexity of the proposed method, we recorded the number of epochs needed for the learning process to be completed. The stopping criterion is when the change in error after 20 consecutive epochs is less than 0.0001. In Fig. 6, the mean and standard deviation of the number of epochs for 10 different experiments on the food dataset are illustrated. Based on this figure, the average number of epochs increases as the percentage of missing values increases. However, this increase is linear.

5. Conclusion

In this paper, we focused on improving the estimation of missing values in fuzzy preference relations (FPRs) by using the idea of similarity and pairwise preferences. We proposed a new method called the entropy-based method to estimate the missing values in FPR based on the idea of similarities between alternatives and experts. Pairwise preferences contain more information than the single rating scores, and the similarity of experts and alternatives helps find the missing values even with minimal data.

We also used the entropy-based method to rank the alternatives, which has many applications in group decision-making (GDM), especially in group recommendation systems (GRS). We have compared the accuracy of our algorithm with the best candidate from the state of the art achievements. Entropy-based methods, and in particular the cross-entropy loss function used in our framework, are particularly useful for estimating probabilities, since they weigh probabilities associated with high certainty more than the lowest probabilities. However, similar to other methods, the overall model fitting, when minimizing the cross-entropy loss function, generates larger errors if the dimension of our problem – number of alternatives – increases.

A study on "the possibility of adding information related to the experts' personalities" or "having dynamic expert preferences in which experts can change their opinions during the voting phase" is suggested for future work.

CRediT authorship contribution statement

Roza Abolghasemi: Conceptualization , Software, Validation, Writing – original draft, Writing – review & editing, Visualization. **Rabindra Khadka:** Software, Validation, Visualization, Writing – review & editing. **Pedro G. Lind:** Conceptualization, Methodology, Formal analysis, Supervision, Writing – review & editing. **Paal Engelstad:** Conceptualization, Supervision, Writing – review & editing. **Enrique Herrera Viedma:** Conceptualization, Methodology, Writing – review & editing. **Anis Yazidi:** Conceptualization, Methodology, Formal analysis, Supervision, Writing – review & editing, Project administration.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Derivations and auxiliary calculus

In this section, we explain how we have obtained Eqs. (9a)–(9e) from Eq. (6). Since we are looking for the minimum value of the objective function $\mathcal{L}(\Theta)$, we intend to solve the vanishing of the gradient of $\mathcal{L}(\Theta)$ with respect to the following set of parameters Θ:

$$\nabla_{\Theta}\mathcal{L}(\Theta) = \left(\frac{\partial \mathcal{L}}{\partial \Theta_1}, \dots, \frac{\partial \mathcal{L}}{\partial \Theta_{2n(d+1)+dm}}\right) = 0\,, \tag{A.1}$$

where the number of parameters can be computed by recalling the vector of parameters in Eq. (7).

Substituting the definition of $\mathcal{L}(\Theta)$ in Eq. (6) into Eq. (A.1) yields for each parameter Θ_i ($i = 2n(d+1)+dm$), see (A.2) given in Box I.

$$
\begin{aligned}
\frac{\partial \mathcal{L}(\Theta)}{\partial \Theta_i} &= \frac{\partial}{\partial \Theta_i}\left[\sum_{i,j=1}^{n} L(\hat{p}_{ij}^{(k)}) + R(\Theta)\right] \\
&= \frac{\partial}{\partial \Theta_i}\sum_{i,j=1}^{n}\left[-p_{ij}^{(k)}\log\left(h(\hat{p}_{ij}^{(k)})\right) - (1-p_{ij}^{(k)})\log\left(1-h(\hat{p}_{ij}^{(k)})\right)\right] + \frac{\partial R(\Theta)}{\partial \Theta_i} \\
&= \sum_{i,j=1}^{n}\left[-p_{ij}^{(k)}\frac{\partial \log\left(h(\hat{p}_{ij}^{(k)})\right)}{\partial \Theta_i} - (1-p_{ij}^{(k)})\frac{\partial \log\left(1-h(\hat{p}_{ij}^{(k)})\right)}{\partial \Theta_i}\right] + \frac{\partial R(\Theta)}{\partial \Theta_i} \\
&= \sum_{i,j=1}^{n}\left[-p_{ij}^{(k)}\frac{1}{h(\hat{p}_{ij}^{(k)})}\frac{\partial h(\hat{p}_{ij}^{(k)})}{\partial \hat{p}_{ij}^{(k)}}\frac{\partial \hat{p}_{ij}^{(k)}}{\partial \Theta_i} - (1-p_{ij}^{(k)})\frac{1}{1-h(\hat{p}_{ij}^{(k)})}\frac{\partial\left(1-h(\hat{p}_{ij}^{(k)})\right)}{\partial \hat{p}_{ij}^{(k)}}\frac{\partial \hat{p}_{ij}^{(k)}}{\partial \Theta_i}\right] + \frac{\partial R(\Theta)}{\partial \Theta_i} \\
&= \sum_{i,j=1}^{n}\left[-p_{ij}^{(k)}\left(1-h(\hat{p}_{ij}^{(k)})\right)\frac{\partial \hat{p}_{ij}^{(k)}}{\partial \Theta_i} - (1-p_{ij}^{(k)})\frac{1}{1-h(\hat{p}_{ij}^{(k)})}\frac{-\hat{p}_{ij}^{(k)}\left(1-h(\hat{p}_{ij}^{(k)})\right)}{1}\frac{\partial \hat{p}_{ij}^{(k)}}{\partial \Theta_i}\right] + \frac{\partial R(\Theta)}{\partial \Theta_i} \\
&= \sum_{i,j=1}^{n}\left[\left(-p_{ij}^{(k)} + p_{ij}^{(k)}h(\hat{p}_{ij}^{(k)})\right)\frac{\partial \hat{p}_{ij}^{(k)}}{\partial \Theta_i} + \left(h(\hat{p}_{ij}^{(k)}) - p_{ij}^{(k)}h(\hat{p}_{ij}^{(k)})\right)\frac{\partial \hat{p}_{ij}^{(k)}}{\partial \Theta_i}\right] + \frac{\partial R(\Theta)}{\partial \Theta_i} \\
&= \sum_{i,j=1}^{n}\left(h(\hat{p}_{ij}^{(k)}) - p_{ij}^{(k)}\right)\frac{\partial \hat{p}_{ij}^{(k)}}{\partial \Theta_i} + \frac{\partial R(\Theta)}{\partial \Theta_i}\,.
\end{aligned}
\tag{A.2}
$$

Box I.

According to Eq. (4), the derivatives of $\hat{p}_{ij}^{(k)}$ with respect to the parameter Θ are:

$$
\frac{\partial}{\partial \Theta}\hat{p}_{ij}^{(k)} = \begin{cases} (g_\ell^{(i)} - g_\ell^{(j)}) & \text{if } \Theta = f_\ell^{(k)} \\ f_\ell^{(k)} & \text{if } \Theta = g_\ell^{(i)} \\ -f_\ell^{(k)} & \text{if } \Theta = g_\ell^{(j)} \\ 1 & \text{if } \Theta = \beta_i \\ -1 & \text{if } \Theta = \beta_j \end{cases}
\tag{A.3}
$$

By replacing the derivations of different parameters Θ from Eqs. (A.3) into Eq. (A.2), we then obtain Eqs. (9a)–(9e).

Appendix B. The datasets

In this appendix, we provide the datasets used in our simulations. The datasets are provided from the experiment described in Section 3.2. In this paper, we only consider a small amount of the preference matrices from the above-mentioned experiment, namely, ten experts ($m = 10$) and six alternatives ($n = 6$). The preference matrices are:

$$
P_1 = \begin{bmatrix} 0.50 & \mathbf{0.23} & \mathbf{0.26} & \mathbf{0.71} & \mathbf{0.19} & \mathbf{0.13} \\ 0.77 & 0.50 & \mathbf{0.19} & \mathbf{0.21} & \mathbf{0.15} & \mathbf{0.18} \\ 0.74 & 0.81 & 0.50 & \mathbf{0.83} & \mathbf{0.13} & \mathbf{0.20} \\ 0.29 & 0.79 & 0.17 & 0.50 & \mathbf{0.17} & \mathbf{0.21} \\ 0.81 & 0.85 & 0.87 & 0.83 & 0.50 & \mathbf{0.15} \\ 0.87 & 0.82 & 0.80 & 0.79 & 0.85 & 0.50 \end{bmatrix},
$$

$$
P_2 = \begin{bmatrix} 0.50 & \mathbf{0.64} & \mathbf{0.37} & \mathbf{0.64} & \mathbf{0.19} & \mathbf{0.69} \\ 0.36 & 0.50 & \mathbf{0.28} & \mathbf{0.65} & \mathbf{0.22} & \mathbf{0.66} \\ 0.63 & 0.72 & 0.50 & \mathbf{0.69} & \mathbf{0.06} & \mathbf{0.71} \\ 0.36 & 0.35 & 0.31 & 0.50 & \mathbf{0.27} & \mathbf{0.74} \\ 0.81 & 0.78 & 0.94 & 0.73 & 0.50 & \mathbf{0.18} \\ 0.31 & 0.34 & 0.29 & 0.26 & 0.82 & 0.50 \end{bmatrix},
$$

$$
P_3 = \begin{bmatrix} 0.50 & \mathbf{0.37} & \mathbf{0.59} & \mathbf{0.62} & \mathbf{0.59} & \mathbf{0.15} \\ 0.63 & 0.50 & \mathbf{0.31} & \mathbf{0.38} & \mathbf{0.71} & \mathbf{0.56} \\ 0.41 & 0.69 & 0.50 & \mathbf{0.57} & \mathbf{0.40} & \mathbf{0.37} \\ 0.38 & 0.62 & 0.43 & 0.50 & \mathbf{0.61} & \mathbf{0.61} \\ 0.41 & 0.29 & 0.60 & 0.39 & 0.50 & \mathbf{0.64} \\ 0.85 & 0.44 & 0.63 & 0.39 & 0.36 & 0.50 \end{bmatrix},
$$

$$
P_4 = \begin{bmatrix} 0.50 & \mathbf{0.75} & \mathbf{0.69} & \mathbf{0.79} & \mathbf{0.78} & \mathbf{0.75} \\ 0.25 & 0.50 & \mathbf{0.74} & \mathbf{0.75} & \mathbf{0.72} & \mathbf{0.74} \\ 0.31 & 0.26 & 0.50 & \mathbf{0.80} & \mathbf{0.90} & \mathbf{0.62} \\ 0.21 & 0.25 & 0.20 & 0.50 & \mathbf{0.75} & \mathbf{0.71} \\ 0.22 & 0.28 & 0.10 & 0.25 & 0.50 & \mathbf{0.76} \\ 0.25 & 0.26 & 0.38 & 0.29 & 0.24 & 0.50 \end{bmatrix},
$$

$$
P_5 = \begin{bmatrix} 0.50 & \mathbf{0.14} & \mathbf{0.40} & \mathbf{0.38} & \mathbf{0.34} & \mathbf{0.31} \\ 0.86 & 0.50 & \mathbf{0.79} & \mathbf{0.82} & \mathbf{0.77} & \mathbf{0.71} \\ 0.60 & 0.21 & 0.50 & \mathbf{0.42} & \mathbf{0.36} & \mathbf{0.38} \\ 0.62 & 0.18 & 0.58 & 0.50 & \mathbf{0.37} & \mathbf{0.69} \\ 0.66 & 0.23 & 0.64 & 0.63 & 0.50 & \mathbf{0.64} \\ 0.69 & 0.29 & 0.62 & 0.31 & 0.36 & 0.50 \end{bmatrix},
$$

$$
P_6 = \begin{bmatrix} 0.50 & \mathbf{0.28} & \mathbf{0.20} & \mathbf{0.19} & \mathbf{0.19} & \mathbf{0.19} \\ 0.72 & 0.50 & \mathbf{0.78} & \mathbf{0.24} & \mathbf{0.24} & \mathbf{0.23} \\ 0.80 & 0.22 & 0.50 & \mathbf{0.80} & \mathbf{0.15} & \mathbf{0.19} \\ 0.81 & 0.76 & 0.20 & 0.50 & \mathbf{0.19} & \mathbf{0.74} \\ 0.81 & 0.76 & 0.85 & 0.81 & 0.50 & \mathbf{0.18} \\ 0.81 & 0.77 & 0.81 & 0.26 & 0.82 & 0.50 \end{bmatrix},
$$

$$
P_7 = \begin{bmatrix} 0.50 & \mathbf{0.13} & \mathbf{0.12} & \mathbf{0.15} & \mathbf{0.16} & \mathbf{0.15} \\ 0.87 & 0.50 & \mathbf{0.78} & \mathbf{0.25} & \mathbf{0.07} & \mathbf{0.64} \\ 0.88 & 0.22 & 0.50 & \mathbf{0.26} & \mathbf{0.25} & \mathbf{0.64} \\ 0.85 & 0.75 & 0.74 & 0.50 & \mathbf{0.26} & \mathbf{0.20} \\ 0.84 & 0.93 & 0.75 & 0.74 & 0.50 & \mathbf{0.78} \\ 0.85 & 0.36 & 0.36 & 0.80 & 0.22 & 0.50 \end{bmatrix},
$$

$$P_8 = \begin{bmatrix} 0.50 & \mathbf{0.33} & \mathbf{0.41} & \mathbf{0.40} & \mathbf{0.35} & \mathbf{0.71} \\ 0.67 & 0.50 & \mathbf{0.35} & \mathbf{0.31} & \mathbf{0.64} & \mathbf{0.39} \\ 0.59 & 0.65 & 0.50 & \mathbf{0.37} & \mathbf{0.61} & \mathbf{0.60} \\ 0.60 & 0.69 & 0.63 & 0.50 & \mathbf{0.62} & \mathbf{0.80} \\ 0.65 & 0.36 & 0.39 & 0.38 & 0.50 & \mathbf{0.54} \\ 0.29 & 0.61 & 0.40 & 0.20 & 0.46 & 0.50 \end{bmatrix},$$

$$P_9 = \begin{bmatrix} 0.50 & \mathbf{0.50} & \mathbf{0.35} & \mathbf{0.23} & \mathbf{0.23} & \mathbf{0.27} \\ 0.50 & 0.50 & \mathbf{0.23} & \mathbf{0.22} & \mathbf{0.30} & \mathbf{0.22} \\ 0.65 & 0.77 & 0.50 & \mathbf{0.22} & \mathbf{0.32} & \mathbf{0.31} \\ 0.77 & 0.78 & 0.78 & 0.50 & \mathbf{0.28} & \mathbf{0.28} \\ 0.77 & 0.70 & 0.68 & 0.72 & 0.50 & \mathbf{0.31} \\ 0.73 & 0.78 & 0.69 & 0.72 & 0.69 & 0.50 \end{bmatrix},$$

$$P_{10} = \begin{bmatrix} 0.50 & \mathbf{0.27} & \mathbf{0.33} & \mathbf{0.15} & \mathbf{0.39} & \mathbf{0.28} \\ 0.73 & 0.50 & \mathbf{0.72} & \mathbf{0.36} & \mathbf{0.64} & \mathbf{0.65} \\ 0.67 & 0.28 & 0.50 & \mathbf{0.21} & \mathbf{0.34} & \mathbf{0.34} \\ 0.85 & 0.64 & 0.79 & 0.50 & \mathbf{0.87} & \mathbf{0.85} \\ 0.61 & 0.36 & 0.66 & 0.13 & 0.50 & \mathbf{0.77} \\ 0.72 & 0.35 & 0.66 & 0.15 & 0.23 & 0.50 \end{bmatrix}.$$

As explained in Section 2, since preference matrices obey the relation $p_{ij} = 1 - p_{ji}$ for all off-diagonal entries ($i \neq j$), matrices of dimension 6×6 contain 15 independent terms, as highlighted above. The diagonal terms are always equal to one half.

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