Exploiting Wired-Pipe Technology in an Adaptive Observer for Drilling Incident Detection and Estimation

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Abstract

We deploy an adaptive observer recently developed for general hyperbolic partial differential equation (PDE) systems, to detect and diagnose various drilling faults. The well is modeled by a distributed PDE which, contrary to lumped models, preserves fundamental properties of well flow dynamics enabling faster and more accurate fault detection and estimation. Wired drill-pipe technology with pressure sensors are needed to locate and isolate faults. Four realistic simulation case studies demonstrating various properties of the observer are presented. Drilling incidents treated in the case studies include, pack-off in the annulus, formation inflows, loss of circulation, and various combinations of these. While simulation results show that the developed observers successfully estimate properties of the incidents that they are tailored for, they do not constitute a fault detection system for drilling. But they provide a part of the data on which an overall fault detection system can rely.

Introduction

A variety of *faults* can impose significant threats to both operational safety and efficiency in oil well drilling. Fast detection and identification of faults are therefore of uttermost importance to ensure that the appropriate counteractive steps are taken. It is almost equally important to avoid *false alarms* and false classification so that the operational efficiency is not reduced by unnecessary actions (**?**).

The well flow is accurately described by a hyperbolic partial differential equation (PDE) model. Unfortunately, up until recently, few results in the field of observer design for hyperbolic systems have been developed. In addition, a complicating factor in the field of fault detection and estimation is that a fault incident is by definition unknown, so that the corresponding model parameters can not be a-priori known. To handle such systems, the observer design must be adaptive to parameter uncertainties.

The use of lumped ODE models for fault detection and estimation, however, have been studied extensively. The model presented in **?** is commonly used to describe the lumped pressure in the annulus and drill-string and the drill-bit flow rate. Based on this model, various fault detection, estimation and localization schemes using an adaptive observer have been developed in **????**, which in addition to pump flow measurement utilized wired-pipe technology with distributed pressure sensor for fault localization. The lumped model is derived by simplifying a distributed PDE model. Distributed flow dynamics are ignored, with the effect that the finite time propagation property is lost. An adaptive observer for fault detection and estimation in drilling based on low order lumped models is also presented in **?**. In our recent work **???**, we derive adaptive observers for general hyperbolic systems utilizing distributed measurements. Our recently developed observer design for PDEs is an extension of the adaptive observer for ODEs developed in **??** which were used for flow and parameter estimation in systems described by the low order lumped well model in **?**.

Various logging tools, collectively referred to as *logging while drilling* (LWD), are embedded into the bottom hole assembly. While these measurements are fairly accurate, the transmission bandwidth is often insufficient. Traditionally, mud pulse telemetry has been used to transmit LWD data to the rig in real-time. The bandwidth of mud pulse telemetry is typically in the range of $10-40\,\mathrm{bit/second}$, but can drop to as low as $0.5\,\mathrm{bit/s}$ in long wells (see e.g. **?**). Either way, it is too low for kick & loss detection. As an alternative, *wired pipe* technology offers bandwidths up to $1\,\mathrm{Mbit/s}$. However, possibly due to high cost and complexity of deployment, this technology has so far seen limited use. Another interesting application of wired drill-pipe is along-string pressure sensing where pressure sensors are installed at a fixed interval inside the annulus. This possible application is discussed in **?**. In this paper, we apply the adaptive observer **???** for fault detection and estimation by utilizing wired-pipe technology with pressure measurements. The distributed flow model and theoretical results from **??** are briefly presented in the Appendix. We study three types of faults; pack-off, lost circulation and influx which are described in following section. Next, the main contribution of the paper is the construction of 4 realistic case studies which are designed to illustrate the broad range of possible applications for the observer. For each case a detailed description of the model parameters and simulation setup is discussed and the corresponding results from computer simulations are presented.

The list of possible applications mentioned above is by no means exhaustive. However, the drilling incidents studied in the case studies are among the most studied problems in drilling, which facilitates comparisons with previous model-based estimation schemes in drilling. In particular, all 3 drilling incidents studied in this paper are also studied in **????**. On the other hand, oil and gas drilling is associated with a wide range of possible incidents. We make no claim to cover them all. The possibility of applying the proposed method to detect and identify drilling incidents such as pipe sticking, hole deviation, pipe failures, bore hole instability, mud contamination, formation damage, hole cleaning, H2S bearing formations and shallow gas influx etc, is an area for further research. Our expectation, however, is that characterizing all such incidents will require a large set of different models. Alternatively, an attempt to incorporate all possible effects in any single model will yield a high order model too complicated to be useful in any real-time estimation or control scheme. Furthermore, measurements corrupted by noise and un-modeled dynamics make the model output uncertain. The problem of fault *detection* and *identification* is consequently a statistical problem, where competing explanatory models should be weighted against each other based on uncertain observed and estimated data.

Flow rate and parameter estimation

To illustrate the observer design, we will focus on single-phase flows in the annulus. However, the method can also be applied to interconnected flow systems with an arbitrary number of subsystems, for example a drill-string – annulus interconnection, or linearized multi-phase systems.

Model derivation. To model the flow in the annulus, we use the classical *water-hammer* equations (see e.g. **???**) which are derived under the assumption of unidirectional, axis-symmetric flow of a compressible fluid in a rigid pipe. The validity of the unidirectional flow assumption in pipe systems have been studied extensively in e.g. **??**. The local, area-averaged, water-hammer equation can be written in conservative form

$$\frac{\partial(\rho A)}{\partial t} + \frac{\partial(\rho q - \rho q_{in})}{\partial x} = 0 \tag{1}$$

for the mass balance, and

$$\frac{\partial(\rho q)}{\partial t} + \frac{1}{A}\frac{\partial(\eta \rho q^2)}{\partial x} + A\frac{\partial p}{\partial x} = -\frac{\partial G}{\partial x} - \frac{\partial F}{\partial x} \tag{2}$$

for the momentum balance, where ρ is fluid density, q is volumetric flow, p is pressure, η is a momentum correction coefficient, A is cross-sectional area, q_{in} is volumetric formation inflow, $G = \rho A dxg \sin\alpha$ accounts for gravitational forces and α is the inclination of the well relative to vertical, and F are other external momentum sources such as frictional forces or momentum in-flux from the surrounding formation. Using that the density ρ is dependent on the pressure p, the mass and momentum balance can be written in the non-conservative form

$$A\frac{1}{c^2}\frac{\partial p}{\partial t} + q\frac{1}{c^2}\frac{\partial p}{\partial q} + \rho\frac{\partial q}{\partial x} = q_{in}\frac{1}{c^2}\frac{\partial p}{\partial x} + \rho\frac{\partial q_{in}}{\partial x} \tag{3a}$$

$$q\frac{1}{c^2}\frac{\partial p}{\partial t} + \rho\frac{\partial q}{\partial t} + \frac{1}{A}\frac{\partial(\eta\rho q^2)}{\partial x} + A\frac{\partial P}{\partial x} = -\frac{\partial G}{\partial x} - \frac{\partial F}{\partial x} \tag{3b}$$

where $c = \sqrt{\frac{dp}{d\rho}}$ is the pressure wave velocity. Since the pressure wave velocity is much larger than the convective flow velocity $c \gg q$, i.e. a low Mach number, terms linear in q or q_{in} are neglected, yielding

$$A\frac{1}{c^2}\frac{\partial p}{\partial t} + \rho\frac{\partial q}{\partial x} = \rho\frac{\partial q_{in}}{\partial x} \tag{4a}$$

$$\rho\frac{\partial q}{\partial t} + A\frac{\partial p}{\partial x} = -\frac{\partial G}{\partial x} - \frac{\partial F}{\partial x}. \tag{4b}$$

In the following, we will also assume constant pressure wave velocity c and linearize around a constant ρ. The structure of the inflow-model q_{in} and the momentum sources F are application specific and will be specified in later sections. For now, let $\frac{\beta}{A}\frac{\partial q_{in}}{\partial x} = \phi_1(p,x)\theta_1$ and $-\frac{1}{\rho}\frac{\partial G}{\partial x} - \frac{1}{\rho}\frac{\partial F}{\partial x} = \phi_2(p,q,x)\theta_2$ where ϕ_1 and ϕ_2 are general nonlinear functions, of dimension $\mathbb{R}^{1\times n_1}$ and $\mathbb{R}^{1\times n_2}$ for some $n_1, n_2 > 0$ respectively, that can be used to model various mass and momentum sources and sinks and θ_1 and θ_2

are unknown parameters of dimension $\mathbb{R}^{n_1}$ and $\mathbb{R}^{n_2}$ respectively. To sum up, assuming unidirectional, axis-symmetric flow in a rigid pipe, and linearizing around $q = 0$ and $\rho = const$, the flow in the annulus is thus described by

$$\frac{\partial}{\partial t}p(x,t) + \frac{\beta}{A}\frac{\partial}{\partial x}q(x,t) = \phi_1(p(x,t),x)\theta_1 \tag{5a}$$

$$\frac{\partial}{\partial t}q(x,t) + \frac{A}{\rho}\frac{\partial}{\partial x}p(x,t) = \phi_2(p(x,t),q(x,t),x)\theta_2 \tag{5b}$$

where the pressure $p(x,t)$ and volumetric flow $q(x,t)$ are functions of the along-string distance $x \in [0,L]$ (where $x = 0$ is bottom-hole, $x = L$ is top-side at the rig, and L is the total well length), and time $t \in [0,\infty)$. The model in Eq. (5) is common model, often used as an alternative to the low order lumped model in **?**, in drilling applications where distributed phenomena, such as fast pressure oscillations (e.g. the heave problem in offshore drilling) or pack-off and loss localization, are significant (see e.g. **????**). The left hand side of Eq. (5) represent a sufficient description of the pressure waves in the well, while the right hand side represents mass and momentum sources and sinks which are associated with more uncertainty, such as friction and various fault situations. The purpose of the observer is to estimate the uncertain elements using a stream of real-time measurement from the drilling system, and as such compensate for any model deficiencies.

The boundary conditions are given in terms of bottom-hole bit flow and top-side choke pressure (or atmospheric pressure for conventional drilling),

$$q(0,t) = q_{bit}(t) \tag{6a}$$

$$p(L,t) = p_{choke}(t) \tag{6b}$$

where the boundary functions q_{bit} and p_{choke} are arbitrary. We assume that the initial conditions $p(x,0)$ and $q(x,0)$ are compatible with Eqs. (5) and (6). In the following, for readability, the argument in time will be dropped when unambiguous.

Note that only the fast propagating pressure waves are modeled in Eq. (5). Thermal conditions, lithology and rheology that affect the parameters of the model (β,ρ) vary extremely slowly compared to the time-scale at which faults are detected using Eq. (5).

Problem statement and theoretical results. We assume that wired-pipe technology with along-string pressure sensors are being used and make the simplifying assumption that continuous distributed pressure measurements are available (**?**). Considering the level of smoothness of the pressure profile, this is a reasonable assumption for a sufficiently high pressure sensor density. In addition, we assume that the bottom-hole bit flow is known. If the pump flow into the drill-string is measured, the bottom-hole bit flow can be estimated in finite time.

The objective is to estimate the distributed flow rate q and the unknown parameters θ_1, θ_2. The technical details of the adaptive observer are given in the Appendix where the two main results are stated in Theorem 1and Conjecture 1. Here, we just ascertain the significance of the main property established for the adaptive observer which is that we can obtain estimates of θ_1 and θ_2, which characterize faults, even though $q(x,t)$ is not measured. The needed level of variation in the measurement data, however, is a limiting factor. The expression *variation in the (measurement) data* or *excitation* is precisely mathematically defined in the appendix. In systems with no mass-influx (i.e $\theta_1 = 0$ which is considered in Theorem 1), the flow estimates still converge in the absence of sufficient excitation needed for parameter convergence. Nevertheless, for many of the faults modeled in the next section, parameter convergence is equally important. However, an important property of the parameter estimation scheme for θ_2 is that the needed excitation is achieved locally. Meaning that increasing the number of spatial discretization points does not demand a higher level of excitation. The estimates of θ_1 on the other hand require global excitation. Consequently, if θ_1 is used to model local phenomena, the level of excitation is usually insufficient and parameter convergence is not achieved. However, as stated in Conjecture 1, the estimates will still be bounded and as the simulations in later sections will show, the estimate will often be sufficiently accurate to be useful in many fault detection and localization applications.

Fault detection, estimation and localization

We will in this paper limit the study to 3 types of faults; *pack-off* in annulus, *lost circulation*, *influx* of formation fluids and various combinations of these. The method can also be used to estimate and locate incidents such as drill-string washout or bit nozzle plugging. For these incidents, a model of the flow inside the drill-string must be included, and the multi-dimensional observer design presented in the Appendix can be used to estimate the flow and unknown parameters in the drill-string – annulus interconnected system.

Pack-off is a local build-up of debris around the drill-string, consequently limiting the flow of drilling fluid. Pack-off is commonly caused by inadequate transportation of cuttings triggered by a low circulation rate, or a collapse of the well-bore wall (**?**). A pack-off will affect the local friction and can be modeled by a frictional force term F_f on the right hand side of Eq. (5b).

Lost circulation is loss of drilling fluids into the formation, which occur in high-permeability regions or where fractured formations are either naturally encountered or caused by a drilling-pressure exceeding the fracturing pressure (**?**). Loss of circulation will first and foremost affect the mass-balance where q_{in} is negative (a sink) in Eq. (5a), but will often also lead to a *loss* of momentum modeled by an equivalent force term F_r on the right hand side of Eq. (5b).

Influx of formation fluids or a *kick* occur when the formation pore pressure exceeds the well pressure. The volumetric inflow q_{in} will by positive and act as a mass source in Eq. (5a). In addition, an influx will often lead to a *gain* of momentum modeled by a force term F_r in Eq. (5b). A kick related incident can pose a serious threat to operational safety and has, as such, been studied extensively. See e.g. **????**.

Combinations of the three fault types can lead to many interesting detection and estimation problems. For example, a loss of circulation will result in lower pressure downstream due to a lower frictional head, which in turn might trigger a kick further downstream. Or, combinations of a local pack-off and an influx will both result in increased local friction. A sufficient level of variation in the data is needed to distinguish between the two types of faults.

Case studies

The objective of the case studies is to illustrate the fault size estimation and localization capabilities of the adaptive observer proposed in **???**. Fault detection and identification in the broader sense of correctly detecting and identifying a single fault among a set of possible faults is mainly a statistical problem, and is not considered.

All four cases are simulated in MATLAB using the `ode45` solver and *method of lines* with 200 spatial discretization points. For each case, both system states and the estimation error signals are shown in figures. Note that the true well flow and parameter estimation error signals are not available in a real-world implementation, but are computable in computer simulations.

Case 1: Annulus pack-off. In the first case, we examine the observers ability to locate and estimate the size of isolated regions with pack-off. We assume no mass influx so that the only momentum sink is frictional loss. In addition to the local friction loss due to pack-off, denoted $F_{local}(q)$, we assume a global uniform wall friction $F_{global}(q)$. Let

$$F_f(q) = F_{local}(q) + F_{global}(q). \tag{7}$$

We assume laminar flow in regions without pack-off and use the linear model

$$F_{global}(q) = f_w q dx. \tag{8}$$

The pack-off region is modeled as an orifice plate where the flow through the orifice is proportional to the square root of the pressure difference. We use the model

$$F_{local}(q) = f_p q|q|dx. \tag{9}$$

Considering the model Eq. (5), we obtain a model with source terms $\phi_1^T = \theta_1 = 0$ and

$$\phi_2(q(x),x) = \begin{bmatrix} -Ag\sin\alpha(x) \\ -q(x) \\ -q(x)|q(x)|\chi(x) \end{bmatrix}, \theta_2 = \begin{bmatrix} 1 \\ \frac{f_w}{\rho} \\ \frac{f_{p,0}}{\rho}. \end{bmatrix}^T \tag{10}$$

We assume that both frictional coefficients are unknown while the first element in θ_1 is obviously known. For the local frictional forces we discretize the well into $N = 10$ subintervals. The number of intervals is arbitrary, but the computational cost is approximately of order N. The indicator function $\chi(x)$ is of dimension N where each element has local support in a region centered at x and with width $\frac{L}{N}$. In the simulation, we use

$$f_p(x) = \chi(x) f_{p,0} \tag{11}$$

where

$$\begin{aligned} f_{p,0} = &\begin{bmatrix} 0 & 0.5 & 0 & 0 & 1 & 0 & 0 & 0 & 0.7 & 0 \end{bmatrix}^T \\ &\times 10^5 \mathrm{kg\,m^{-6}\,s^{-2}} \end{aligned} \tag{12}$$

The inclination $\alpha(x)$ is selected as

$$\alpha(x) = \frac{\pi}{2}\frac{x}{L}. \tag{13}$$

The top-side choke pressure is kept constant at $p(L,t) = 10\,\mathrm{bar}$ (1×10^6 Pa) and the bit flow rate is initially zero and then ramped up to $2\,\mathrm{m^3\,min^{-1}}$ as shown in **??**. This is meant to simulate a realistic pack-off incident, which is often caused by a too-low bit flow rate, e.g. during a connection, which is detectable only after the bit flow is ramped up again. Numeric values for the remaining model parameters are given in Table 1.

The system is initially in steady state for the given bit flow rate and choke pressure. The resulting flow and pressure profiles are shown in **????** respectively. The flow is estimated accurately throughout the well, as can be seen in **????**. **??** shows that the global friction factor estimate $\hat{f}_w$ does not converge to the true value f_w. Nevertheless, the pack-off friction factor estimates are sufficiently accurate and the pack-off regions are easily located as illustrated in **??**, which shows the estimated coefficient at three distinct times together with the true coefficient values. As can be seen, the size of the pack-off can also be estimated fairly accurately after $t = 25\,\mathrm{s}$.

Case 2: Open hole with non-local uncertain parameters. One of the most common causes of kicks is during a connection when the bit flow rate is ramped down. The frictional head provided by the circulation of drilling fluid keeps the well overbalanced. In the second case, we examine in/out-flow size estimation in a vertical well ($\alpha(x) = \pi/2$) with an open hole region during bit flow ramp down. We use the pressure-dependent inflow model

$$q_{in}(x) = J(x)(p_r(x) - p(x))dx \tag{14}$$

where J, called the *productivity index*, is uniform and equal to $J(X_1) = 2\times10^{-11}\,\mathrm{m^2/s/Pa}$ in the open hole region $X_1 \subset [L/2, L]$ and zero in the sealed off region $[0, L/2]$. The formation pressure $p_r(x)$ is assumed to follow the linear model

$$p_r(x) = \rho_r g x + p_{r,0} \tag{15}$$

where ρ_r and $p_{r,0}$ are known constants. The same global friction model Eq. (8) as for Case 1 is used. In addition, we model the momentum influx from the reservoir as a force equivalent

$$F_r(q_{in}(x)) = f_r q_{in}(x)dx. \tag{16}$$

The source terms in Eq. (5) are implemented as

$$\phi_1(p,x) = \frac{\beta}{A}\begin{bmatrix}\psi(x)gh(x)\\ \psi(x)\\ -\psi(x)p(x)\end{bmatrix},\ \theta_1 = \begin{bmatrix}J\rho_r\\ Jp_{r,0}\\ J\end{bmatrix} \tag{17a}$$

$$\phi_2(p,q,x) = \frac{1}{\rho}\begin{bmatrix}-Ag\sin\alpha(x)\\ -q\\ -\psi(x)gh(x)\\ -\psi(x)\\ \psi(x)p(x)\end{bmatrix},\ \theta_2 = \begin{bmatrix}1\\ f_r\\ f_wJ_0\rho_r\\ f_wJp_{r,0}\\ f_wJ\end{bmatrix} \tag{17b}$$

where the indicator function ψ is equal to one in the open-hole region and zero elsewhere. Numerical values for all model parameters are given in Table 2. Compared to case 1, we have changed the friction factor and drilling mud density so that the system is initially in steady state with no inflow. That is,

$$q_{in}(x,0) = 0 \Leftrightarrow p_r(x) = p(x,0)\forall x \in [0,1] \tag{18}$$

which is satisfied if

$$\rho = \rho_r - \frac{f_w}{Ag}q_{bit}(0), \quad p(0,0) = p_{r,0}(0) \tag{19}$$

The bit flow rate is initially at $2\,\mathrm{m^3\,min^{-1}}$ before being ramped down to $0\,\mathrm{m^3\,min^{-1}}$ and then back up to $2\,\mathrm{m^3\,min^{-1}}$ again as can be seen in **??**. As the bit flow rate is reduced, the formation pore pressure will exceed the well pressure which will result in a kick. The top-side choke pressure is kept constant at $10\,\mathrm{bar}$ (1×10^6 Pa). We assume that the formation pressure is known and the only unknown parameter is the productivity index J. This is a reasonable assumption as the well is initially in steady state and the formation properties can be computed from Eq. (19). The productivity index however is unobservable as long as the inflow is zero.

The flow and pressure profiles are shown in **????** and the inflow is shown in **??**. As predicted, the inflow increases as the bit flow is ramped down, and vanishes when the bit rate is ramped back up again. Interestingly, a small loss of circulation occurs in the transient ramp up phase between around $t = 30\,\mathrm{s}$ to $t = 50\,\mathrm{s}$. **????** show that the well flow rate estimation error converges to zero. In addition, **??** shows that the inflow estimation error (which is based on the parameter estimates) also converge to zero.

Case 3: Geological faults causing zero net gain. In the third case we introduce a geological fault that divides the formation into two zones with two distinct pore pressures. The case is constructed so that the well pressure is lower than the pore pressure in the fist zone and higher than the pore pressure in the second zone (see **??**). Consequently we will have a region where the drilling fluid is lost and another region with formation influx. The net inflow as observed top-side will be less than the total absolute in- and out-flow. This case will test the observers ability to identify local in- and out-flow phenomena.

The same inflow model Eq. (14) as in case 2 is used, but the reservoir pressure is now given as

$$p_r(x) = \begin{cases} 203\,\mathrm{bar}, & x \in [0, 0.75L] \\ 183\,\mathrm{bar}, & x \in [0.75L, L] \end{cases} = \begin{cases} 2.03 \times 10^7\,\mathrm{Pa}, & x \in [0, 0.75L] \\ 1.83 \times 10^7\,\mathrm{Pa}, & x \in [0.75L, L] \end{cases} \tag{20}$$

which with $N = 12$ sub-intervals is implemented as

$$p_r(x) = \chi(x) p_{r,0} \tag{21}$$

where

$$p_{r,0} = \begin{bmatrix} \mathbb{1}_3 183\,\mathrm{bar} & \mathbb{1}_9 203\,\mathrm{bar} \end{bmatrix} = \begin{bmatrix} \mathbb{1}_3 1.83 \times 10^7\,\mathrm{Pa} & \mathbb{1}_9 2.03 \times 10^7\,\mathrm{Pa} \end{bmatrix}. \tag{22}$$

The indicator function $\chi(x)$ is of dimension N where each element has local support in a region centered at x and with width $\frac{L}{N}$. In addition, we assume that the open hole region is $X_1 = [0.5L, L]$ with productivity index $J(X_1) = 2 \times 10^{-11}\,\mathrm{m^2/s/Pa}$ in the open hole region (and zero in the sealed region $[0, 0.5L]$). Additional simulation parameters are given Table 3. The well inclination is

$$\alpha(x) = \begin{cases} 0, & x \in [0, 0.7L] \\ \frac{\pi}{4}, & x \in [0.7L, L], \end{cases} \tag{23}$$

the top-side choke pressure is kept constant at $10\,\mathrm{bar}$ ($1 \times 10^6\,\mathrm{Pa}$), while the bit flow rate, similar to case 2, is initially at $2\,\mathrm{m^3\,min^{-1}}$ before being ramped down to $0\,\mathrm{m^3\,min^{-1}}$ and then back up to $2\,\mathrm{m^3\,min^{-1}}$ again as can be seen in **??**. This gives the initial well pressure and formation pore pressure profiles shown in **??**. The source terms in Eq. (5) have the form

$$\phi_1(p, x) = \frac{\beta}{A} \begin{bmatrix} \psi(x)\chi(x) \\ -\psi(x)p(x) \end{bmatrix}, \theta_1 = \begin{bmatrix} J p_{r,0} \\ J \end{bmatrix} \tag{24a}$$

$$\phi_2(p, q, x) = \frac{1}{\rho} \begin{bmatrix} -Ag\sin\alpha(x) \\ -q \\ -\psi(x)\chi(x) \\ \psi(x)\chi(x)p(x) \end{bmatrix}, \theta_2 = \begin{bmatrix} 1 \\ f \\ f_w J p_{r,0} \\ f_w J \end{bmatrix}. \tag{24b}$$

where the indicator function ψ is equal to one in the open-hole region and zero elsewhere.

The complete flow, inflow and pressure profiles can be seen in **????????** respectively. Comparing the bit flow rate in **??** with the choke flow rate in **??**, it can be observed that the net accumulated inflow is small, and thus not easily detected using top-side sensing only. The observer is able to estimate both the distributed flow, formation inflow, the formation pore pressure and the fault location fairly accurately. The flow estimation error is shown in **????** while the inflow estimation error and formation pressure estimation error are shown in **????** respectively.

Case 4: Mass influx and pack-off. In the final case study, we simulate both a region with pack-off and an open-hole section with formation in- and out-flow. Both pack-off and formation inflow will act as momentum sinks, but the frictional loss caused by a pack-off will be flow dependent, while the formation inflow is pressure dependent. Thus, for sufficient variation in the data, the observer should be able to distinguish between the two types of fault and successfully locate and estimate the size of the two faults.

The pack-off will be modeled by the same orifice equation Eq. (9) used in case 1, but with

$$f_p(x) = \chi(x) \begin{bmatrix} 0 & 0 & 0 & f_{p,0} \end{bmatrix}^T \tag{25}$$

where $f_{p,0} = 1 \times 10^5\,\mathrm{kg\,m^{-6}\,s^{-2}}$ and the indicator function $\chi(x)$ is of dimension N where each element has local support in a region centered at x and with width $\frac{L}{N}$. The same inclination Eq. (23) as in case 3 is used, the topside choke pressure is kept constant at $10\,\mathrm{bar}$ ($1 \times 10^6\,\mathrm{Pa}$) and the bit flow rate is ramped up and down between $1\,\mathrm{m^3\,min^{-1}}$ and $2\,\mathrm{m^3\,min^{-1}}$ as shown in **??**, which is needed in order to achieve the necessary variation in the data for parameter convergence. The inflow model Eq. (14)

from case 2 is used (which is also used in case 3), but the reservoir pressure is assumed to be known and constant at $p_r = 193\,\mathrm{bar}$ ($1.93\times10^7\,\mathrm{Pa}$) (the well is horizontal in the open-hole section). The productivity index is implemented as

$$J(x) = \chi(x)\begin{bmatrix}0 & 0 & 0 & J_0.\end{bmatrix} \tag{26}$$

where $J_0 = 2\times10^{-11}\,\mathrm{m}^2/\mathrm{s}/\mathrm{Pa}$. The remaining simulation parameters are given in Table 3. We discretized the well into $N = 4$ sub-intervals, so that 4 pack-off friction factor estimates and 4 productivity index estimates are generated. Again, we stress that the number of sub-intervals does not affect the level of variation in the data required, only the computational cost. The source terms in Eq. (5) take the form

$$\phi_1(p,x) = \frac{\beta}{A}\psi(x)\chi(x)(p_r - p(x)) \tag{27a}$$

$$\theta_1 = J_0 \tag{27b}$$

$$\phi_2(p,q,x) = \frac{1}{\rho}\begin{bmatrix}-Ag\sin\alpha(x)\\ -q\\ -q(x)\\ -q(x)|q(x)|\chi(x)\\ -\psi(x)\chi(x)(p_r - p(x))\end{bmatrix} \tag{27c}$$

$$\theta_2 = \begin{bmatrix}1 & f_w & f_p & f_wJ_0\end{bmatrix}^T. \tag{27d}$$

The resulting flow, pressure and inflow profiles are shown in **??????**. As can be seen in **??**, the pressure variations caused by the changing bit flow rate, makes the inflow alternate between being positive and negative. The pressure profile is also shown for $t = 5$ and $t = 20$ in **??** where the bit flow rate is $q_{bit}(t = 5) = 1\,\mathrm{m}^3\,\mathrm{s}^{-1}$ and $q_{bit}(t = 20) = 2\,\mathrm{m}^3\,\mathrm{s}^{-1}$ respectively. The pressure drops due to pack-off and formation inflow are clearly visible. As can be seen in **????**, the flow estimation error is small, but does not converge to zero. More interestingly, the unknown parameters are estimated fairly accurate. The pack-off friction factor estimates are shown in **??** while the productivity index estimates are shown in **??**. The pack-off region (**??**) and inflow region (**??**) are easily identified, and the size estimates are fairly accurate.

Concluding remarks

In this paper, a recently developed adaptive observer has been used to estimate the flow, and locate and estimate the size of a set of faults affecting either the mass balance or the momentum balance in the annulus. The observer is demonstrated though 4 simulation case studies involving 1) annulus pack-off, 2) formation inflow, 3) geological faults resulting in simultaneous losses and gains, and 4) a combination of formation inflow and annulus pack-off. The method can be extended to handle multi-phase flows, interconnected pipe systems with distinct flow regimes, or other related drilling incidents. The observer utilizes wired-pipe technology for pressure measurements, which makes it robust to local variations in the pressure profile and local momentum sources and sinks can easily be identified. To identify faults only affecting the momentum balance, such as a pack-off, the necessary level of variation in the data is easy to achieve. This was demonstrated in case 1. Here, the localization accuracy is only affected by the number of distributed pressure sensors. For faults affecting the mass balance, boundary flow measurements must be utilized, which makes the necessary level of variation harder to achieve. However, most faults resulting in mass inflow are also likely to affect the local momentum balance, and a redundant set of parameter estimates can be generated. Cases 2,3,4 demonstrate that sufficient estimation accuracy is possible to achieve even without the necessary level of variation in boundary data needed to achieve parameter estimation convergence for the parameters affecting the mass balance. The fault *detection* problem is a statistical problem which involves the identification of a single explanatory model from a large set of possible models (one for every conceivable drilling incident). The observers that we have demonstrated in this paper may be elements of an overall fault detection system.

Nomenclature

List of symbols

p = well pressure, Pa

q = well flow, $\mathrm{m}^3\,\mathrm{s}^{-1}$

β = bulk modulus, Pa

A = cross-sectional area, m^2

$\rho =$ density of drilling mud and formation fluids (assumed equal), $\mathrm{kg\,m^{-3}}$

$\phi_1\theta_1 =$ mass balance source term, Pa

$\phi_2\theta_2 =$ momentum balance source term, $\mathrm{m^3\,s^{-1}}$

$G =$ gravitational forces, $\mathrm{kg\,m\,s^{-2}}$

$F_f =$ flow dependent frictional forces, $\mathrm{kg\,m\,s^{-2}}$

$F_r =$ force equivalent formation momentum inflow term, $\mathrm{kg\,m\,s^{-2}}$

$q_{in} =$ well inflow from formation, $\mathrm{m^3\,s^{-1}}$

$q_{bit} =$ bit flow, $\mathrm{m^3\,s^{-1}}$

$p_{choke} =$ choke pressure, Pa

$F_{local} =$ local frictional forces, $\mathrm{kg\,m\,s^{-2}}$

$F_{global} =$ global frictional forces, $\mathrm{kg\,m\,s^{-2}}$

$f_r =$ formation momentum inflow factor, $\mathrm{kg\,m^{-3}\,s^{-1}}$

$\alpha =$ well inclination angle, dimensionless

$\chi =$ indicator function, dimensionless

$L =$ well length, m

$f_p =$ friction factor due to pack-off, $\mathrm{kg/m^6/s^2}$

$f_w =$ wall friction factor, $\mathrm{kg\,m^{-3}\,s^{-1}}$

$J =$ productivity index, $\mathrm{m^2/s/Pa}$

$p_r =$ formation pressure, Pa

$\rho_r =$ formation density, $\mathrm{kg\,m^{-3}}$

$g =$ gravitational acceleration, $9.81\,\mathrm{m\,s^{-2}}$

$X_1 =$ open-hole region, m

$\psi =$ indicator function, dimensionless

$t =$ time, s

$x =$ spatial position, m

Acronyms

LWD Logging While Drilling

ODE Ordinary Differential Equation

PDE Partial Differential Equation

PE Persistently Exciting (or Persistent Excitation)

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Appendix A: Overview of observer design

The equations needed to implemented the observer are presented in the following. First, the simple design from **??** for scalar 2×2 systems is given. This design was extended to multi-dimensional $n \times n$ systems in **?** which is presented next. We first state the following main technical results.

Theorem 1 (Modified from **?**). *Consider the state observer Eq.* (A.4) *and let* $\hat{q}(x) = \hat{\zeta}(x) - lp(x)$ *for some constant* $l > 0$. *If* $\theta_1 = 0$ *and estimates of* θ_2 *are generated with the adaptive law specified by Eqs.* (A.10)–(A.13). *Then, the state estimation error* $||\tilde{q}||$ *and parameter estimation error* $(\theta_2 - \hat{\theta}_2)$ *are bounded and*

$$||\tilde{q}|| \to 0. \tag{A.1}$$

Conjecture 1 (Modified from **?**). *Consider the state observer Eq.* (A.4) *with* $\hat{\theta}_1$ *generated by the adaptive law Eqs.* (A.5)–(A.8) *and* $\hat{\theta}_2$ *generated with the adaptive law specified by Eqs.* (A.10)–(A.13), *and let* $\hat{q}(x) = \hat{\zeta}(x) - lp(x)$ *for some constant* $l > 0$. *Then, the state estimation error* $||\tilde{q}||$ *and parameter estimation error* $(\theta_1 - \hat{\theta}_1)$, $(\theta_2 - \hat{\theta}_2)$ *are bounded. Moreover, if PE condition Eq.* (A.9) *is satisfied, then*

$$||\tilde{q}|| \to 0. \tag{A.2}$$

Notation For some matrices A, B, the notation $A \succ B$ is used to indicate that $A - B$ is positive definite, and equivalently $A \succeq B$ is used to indicate that $A - B$ is positive semi-definite. The notation $:=$ is used to indicate that the left hand side is a defined variable. The symbol $\cdot$ is used as a place-holder when a function is not strictly a function of some variable. E.g. for a function $u : [0,1] \times [0,t) \to \mathbb{R} : (x,t) \mapsto u(x,t)$, we write e.g. $||u(\cdot,t)||$ to denote the $L_2([0,1])$ norm $\sqrt{\int_0^1 u^2(x,t)dx}$ which is a function of $t \in [0,\infty)$ only.

Observer design for 2×2 hyperbolic systems. Consider the 2×2 hyperbolic system given in Eq. (5) and define $a = \frac{\beta}{A}, b = \frac{A}{\rho}$. Let $\zeta(x) = lp(x) + q(x)$ for some l such that $\lambda := la > 0$. We have

$$\begin{aligned} \zeta_t(x) + \lambda \zeta_x(x) =& (l^2 a - b) p_x(x) \\ &+ l\phi_1^T(p(x),x)\theta_1 + \phi_2^T(q(x),x)\theta_2 \end{aligned} \tag{A.3a}$$

$$\zeta(0) = lp(0) + q(0) \tag{A.3b}$$

To estimate the unknown state ζ, consider the observer

$$\begin{aligned} \hat{\zeta}_t(x) + \lambda \hat{\zeta}_x(x) =& (l^2 a - b) p_x(x) \\ &+ l\phi_1^T(p(x),x)\hat{\theta}_1 + \phi_2^T(\hat{q}(x),x)\hat{\theta}_2 \end{aligned} \tag{A.4a}$$

$$\hat{\zeta}(0) = \zeta(0) \tag{A.4b}$$

where $\hat{q}(x) = \hat{\zeta}(x) - lp(x)$ and $\hat{\theta}_1, \hat{\theta}_2$ are estimates of θ_1, θ_2.

For estimates of θ_1, let the operators Ψ, Ω and Δ be defined as

$$\Psi[p] := \int_0^1 p(x)dx \tag{A.5a}$$

$$\Omega[p] := \int_0^1 \phi_1(p(x),x)dx \tag{A.5b}$$

$$\Delta[q] := -a(q(1) - q(0)) \tag{A.5c}$$

and consider the filters

$$\dot{\nu} = -\varsigma \nu + \Omega[p] \tag{A.6a}$$

$$\dot{\rho} = -\varsigma(\rho - \Psi[p]) + \Delta[q] \tag{A.6b}$$

for some $\varsigma > 0$ and let

$$\bar{\Psi} := \rho + \nu^T \theta_1. \tag{A.7}$$

For some $\Gamma_1 = \Gamma_1^T \succ 0$, we select the adaptive law

$$\dot{\hat{\theta}}_1 = \Gamma_1 \varepsilon_1 \nu \tag{A.8}$$

where $\varepsilon_1 := \Psi[p] - \rho - \nu\hat{\theta}_1$. For parameter convergence, we say that $\Omega[y]$ is PE (persistence of excitation) if

$$\alpha_0 I \preceq \frac{1}{T}\int_t^{t+T} \Omega[y]\Omega[y]^T d\tau \preceq \alpha_1 I, \tag{A.9}$$

for some $\alpha_0, \alpha_1, T > 0$.

For estimates of θ_2, consider the signal $\hat{\sigma}$ defined by

$$\dot{\hat{\sigma}} = \int_0^1 \eta[\hat{q}](x)\left(-bp_x + \phi_2^T(\hat{q}(x),x)\hat{\theta}_2\right)dx \tag{A.10a}$$

$$\hat{\sigma}(0) = \hat{\theta}_2^*(0) \tag{A.10b}$$

where

$$\eta[\hat{q}] = \lambda^{-1}\Gamma_2\Phi[\hat{q}], \tag{A.11}$$

and let

$$\hat{\theta}_2 = \hat{\sigma} - \Xi[\hat{q}]. \tag{A.12}$$

where the operator Ξ is computed as

$$\begin{aligned}&\Xi[\hat{q}(\cdot,t_1)] = +\Xi[\hat{q}(\cdot,t_0)]\\ &+\int_0^1\int_0^1 \eta\left[\hat{q}(\cdot,t_0)+\gamma[\hat{q}(\cdot,t_1)-\hat{q}(\cdot,t_0)]\right](x)\\ &\times(\hat{q}(x,t_1)-\hat{q}(x,t_0))\,dxd\gamma.\end{aligned} \tag{A.13}$$

Observer design for $n\times n$ hyperbolic systems. The observer design for 2×2 hyperbolic systems can be generalized to $n\times n$. Consider the system

$$y_t + Ay_x + Bz_x = f(y,x) \tag{A.14}$$

$$z_t + Cy_x + Dz_x = g(y,x) + \phi(y,z)\theta \tag{A.15}$$

where y is a m-dimensional measured signal, z is a $(n-m)$-dimensional unknown signal, A,B,C,D are constant parameters describing the flux densities, and f,g are general non-linear functions. For any diagonal Λ_1 and Λ_2 with distinct entries, and any K_2, there exist matrices K_1, L and $P = \mathrm{diag}(P_1,P_2)$ such that $(\alpha,\beta) := P(y,z)$ maps system Eq. (A.14) into

$$\alpha_t + \Lambda_1\alpha_x + \bar{B}\beta_x = \bar{f}(\alpha,x) - \bar{K}_1\alpha_x \tag{A.16a}$$

$$\begin{aligned}\beta_t + \bar{C}\alpha_x + \Lambda_2\beta_x =& \bar{g}(\alpha,x)\\ &+\bar{\phi}((\alpha,\beta),x)\theta - \bar{K}_2\alpha_x\end{aligned} \tag{A.16b}$$

$$(\alpha(x,0),\beta(x,0)) = P(y_{ic}(x),z_{ic}(x)) \tag{A.16c}$$

where

$$\bar{K}_1 = P_1K_1P_1^{-1} \tag{A.17a}$$

$$\bar{K}_2 = P_2K_1P_1^{-1} \tag{A.17b}$$

$$\bar{B} = P_1BP_2^{-1} \tag{A.17c}$$

$$\bar{C} = P_2(C-(D+LB)L+LA-K_2)P_1 \tag{A.17d}$$

and

$$\begin{bmatrix}\bar{f}(\alpha,x)\\ \bar{g}(\alpha,x)\end{bmatrix} = P\begin{bmatrix}f(P_1^{-1}\alpha,x)\\ g(P_1^{-1}\alpha,x)\end{bmatrix} \tag{A.18}$$

$$\bar{\phi}((\alpha,\beta),x) = P_2\phi(P^{-1}(\alpha,\beta),x). \tag{A.19}$$

Based on Eq. (A.16) we propose the observer

$$
\begin{aligned}
\hat{\alpha}_t + \Lambda_1 \hat{\alpha}_x + \bar{B}\hat{\beta}_x &= \bar{f}(\alpha, x) - \bar{K}_1 \alpha_x && \text{(A.20)}\\
\hat{\beta}_t + \bar{C}\hat{\alpha}_x + \Lambda_2 \hat{\beta}_x &= \bar{g}(\alpha, x) + \bar{\phi}((\alpha, \hat{\beta}), x)\hat{\theta} \\
&\quad - \bar{K}_2 \alpha_x && \text{(A.21)}\\
(\hat{y}, \hat{z}) &= P^{-1}(\hat{\alpha}, \hat{\beta}) && \text{(A.22)}
\end{aligned}
$$

where each element $\hat{\theta}_i$, in $\hat{\theta}$ is generated as

$$
\begin{aligned}
\hat{\theta}_i &= \hat{\sigma}_i - \Xi_i[(\alpha, \hat{\beta})] && \text{(A.23)}\\
\dot{\hat{\sigma}}_i &= \left\langle \eta_i^{\alpha}[(\alpha, \hat{\beta})], -(\Lambda_1 + \bar{K}_1)\alpha_x + \bar{f}(\alpha, \cdot) \right\rangle \\
&\quad + \left\langle \eta_i^{\hat{\beta}}[(\alpha, \hat{\beta})], -P_2 D P_2^{-1} \hat{\beta}_x + \Sigma \right\rangle && \text{(A.24)}\\
\Sigma(x) &= \bar{g}(\alpha, x) + \bar{\phi}((\alpha, \hat{\beta}), x)\hat{\theta} - \bar{C}\hat{\alpha}_x - \bar{K}_2 \alpha_x && \text{(A.25)}\\
\hat{\sigma}_i(0) &= \hat{\theta}_0 + \Xi[(\alpha_{ic}, \hat{\beta}_{ic}] && \text{(A.26)}
\end{aligned}
$$

and where the operator Ξ is computed as

$$
\begin{aligned}
&\Xi_i[(\alpha(\cdot,t), \hat{\beta}(\cdot,t))] \\
&\quad = \int_0^1 \left\langle \eta_i^{\alpha} \left[\begin{pmatrix} \alpha(\cdot,0) + \gamma(\alpha(\cdot,t) - \alpha(\cdot,0)) \\ \hat{\beta}(\cdot,0) + \gamma(\hat{\beta}(\cdot,t) - \hat{\beta}(\cdot,0)) \end{pmatrix} \right], \right. \\
&\qquad\qquad \left. \alpha(\cdot,t) - \alpha(\cdot,0) \right\rangle d\gamma \\
&\qquad + \int_0^1 \left\langle \eta_i^{\hat{\beta}} \left[\begin{pmatrix} \alpha(\cdot,0) + \gamma(\alpha(\cdot,t) - \alpha(\cdot,0)) \\ \hat{\beta}(\cdot,0) + \gamma(\hat{\beta}(\cdot,t) - \hat{\beta}(\cdot,0)) \end{pmatrix} \right], \right. \\
&\qquad\qquad \left. \hat{\beta}(\cdot,t) - \hat{\beta}(\cdot,0) \right\rangle d\gamma && \text{(A.27)}
\end{aligned}
$$

where $(\eta_i^{\alpha}, \eta_i^{\hat{\beta}})$ solves

$$
\begin{aligned}
(P_1 B P_2^{-1})^T \eta_i^{\alpha}[(\alpha, \hat{\beta})] &= -\gamma_i \Phi_i[(\alpha, \hat{\beta})] && \text{(A.28)}\\
(P_2 L B P_2^{-1})^T \eta_i^{\hat{\beta}}[(\alpha, \hat{\beta})] &= \gamma_i \Phi_i[(\alpha, \hat{\beta})]. && \text{(A.29)}
\end{aligned}
$$

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Table 1—Simulation parameters case 1.

Parameter	Value	Description
L	$1650\,\mathrm{m}$	Well length
β	$1.78\times10^{9}\,\mathrm{Pa}$	Bulk modulus
ρ	$1210\,\mathrm{kg\,m^{-3}}$	Drilling fluid density
A	$0.0636\,\mathrm{m^{3}}$	Annulus cross sectional area
g	$9.81\,\mathrm{m\,s^{-2}}$	Gravitational acceleration
f_w	$50\,\mathrm{kg\,m^{-3}\,s^{-1}}$	Wall friction factor

Table 2—Simulation parameters case 2.

Parameter	Value	Description
L	$7000\,\mathrm{m}$	Well length
β	$1.78\times10^{9}\,\mathrm{Pa}$	Bulk modulus
ρ	$1196\,\mathrm{kg\,m^{-3}}$	Drilling fluid density
A	$0.024\,\mathrm{m^{3}}$	Annulus cross sectional area
g	$9.81\,\mathrm{m\,s^{-2}}$	Gravitational acceleration
f_w	$100\,\mathrm{kg\,m^{-3}\,s^{-1}}$	Wall friction factor
f_r	$100\,\mathrm{kg\,m^{-3}\,s^{-1}}$	Inflow-induced friction factor
ρ_r	$1210\,\mathrm{kg\,m^{-3}}$	Formation pressure parameter #1
$p_{r,0}$	$10\times10^{5}\,\mathrm{Pa}$	Formation pressure parameter #2

Table 3—Simulation parameters case 3 and 4.

Parameter	Value	Description
L	$7000\,\mathrm{m}$	Well length
β	$1.78\times10^{9}\,\mathrm{Pa}$	Bulk modulus
ρ	$1210\,\mathrm{kg\,m^{-3}}$	Drilling fluid density
A	$0.024\,\mathrm{m^{3}}$	Annulus cross sectional area
g	$9.81\,\mathrm{m\,s^{-2}}$	Gravitational acceleration
f_w	$100\,\mathrm{kg\,m^{-3}\,s^{-1}}$	Friction factor