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Reliability Test Framework for Electric Motors for Aerial Applications

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Reliability Test Framework for Electric Motors for Aerial Applications

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Reliability, Availability, Maintainability and Safety (RAMS)
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Preface

This report is completed as the master's thesis assignment of the TPK4950 - Reliability, Availability, Maintainability, and Safety (RAMS), Master's Thesis course. The content of this study is inspired by the 2-years RAMS international master's program course topics and further study about reliability tests, data analysis and statistics. The study is founded by the demands in rapidly evolving industry of electric motors for aerial applications and realized with the collaboration of Alva Industries AS and NTNU – Mechanical and Industrial Engineering Department/RAMS Program.

In the last year of my master's program, I have learned many great details of electric motors, reliability tests, data analysis, statistics, and reliability engineering. It has been very entertaining and enlightening to see real life applications of the topics in my interest and combining different research areas with the competence I gained during my master's program. I hope this study will help those who are studying with similar topics and inspire my colleagues. I would like to thank Alva Industries AS, NTNU, my supervisors Anna Olsen, Yiliu Liu and Alexey Matveev for providing me guidance, supervision, and valuable resources.

Executive Summary

This thesis presents relevant factors to consider when designing and analyzing an accelerated reliability test, such as reliability test types, stress factors, stress level calculations, resource allocation, data analysis and interpretation specialized for drone motors. The theory and system analysis are used to explain three case-studies for a concrete understanding of the subject which are composed based on the Alva X60 type motor data and design. These case studies are analyzed with reliability Python library and a reliability estimate is found accordingly.

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1 Introduction

Alva Industries AS (the name of the company will be referred as Alva or Alva Industries through the rest of the report) has developed various electric motors for aerial uses depending on their environment and it is certain that they will continue to produce different versions of the motors, add new designs and technologies to their catalog.

This quick change in the development and production requires adaptive reliability test plans for the unique designs. The slow design of experiments can lead to declined production or production of unreliable electric motors. At the best, poorly planned tests can cause waste of time and money when no conclusion or wrong conclusions are inferred.

1.1 Problem Description

Planning a correct reliability test in accordance with correct reliability goals can be challenging and time consuming for a newly established company because the resources are limited and prior information is missing for products that have state-of-the-art technology.

Moreover, expectations from the product in terms of reliability can be very high and these expectations can change rapidly for a company that has high adaptability to new technologies such as Alva Industries. Developing ways to convert these expectations into useful reliability goals and an adaptive technique to realize and measure these goals are the demands from this study.

1.2 Objective and Scope

The objective of this study is to develop a reliability test framework for the use of drone motors so that Alva Industries can use as a basis for designing and interpreting accelerated reliability tests for new motor designs and technologies.

The scope of the study involves presenting different types of reliability tests that can be useful for testing drone motors, the aims of each test; focusing on accelerated lifetime testing to enable the engineers in Alva Industries to produce their own experiments with providing steps to follow for designing the experiment and ways to analyze the collected data with pieces of Python codes.

1.3 Research Questions

- How can sample size be estimated without any prior knowledge about the data?
- What are the consequences of small sample size in analyzing the data and getting accurate results?

- How can the lifetime of the product during normal conditions be estimated with the results of the reliability tests?

1.4 Approach

First, the types of reliability tests are categorized in terms of their speed of testing and implementation to provide available information for further needs such as the parameters that are going to be measured in each test. Accelerated lifetime test (ALT) is highlighted because this type of test is the immediate need of the company at this stage of production.

Then, the available stress types are presented for the drone motors and most critical stresses are selected based on similar failure trends of complementary products.

After that, the application methods for the stress factors are developed in terms of the duration, level, and the pattern of exposure. The related analytical and graphical models are introduced to visualize and interpret the data. To simplify the analysis of the system (drone motor) for ALT, the entity is divided into subsections and parts. The types of life-stress relationships and data analysis methods are dedicated to such divisions.

After an understanding of the behavior of the product during the test, case studies are developed in such a way that Alva Industries can utilize during testing. Interpreting the data in terms of reliability along with how to present the product with setting the right reliability goals are demonstrated.

1.5 Methods

The methodology used in this study involves utilization of books on topics handling reliability tests, planning accelerated reliability tests, and statistical methods for data analysis of accelerated reliability tests.

Prior to the study, research is conducted to find an appropriate platform to conduct data analysis of completed tests with the criteria that Alva Industries can employ as well as having a large community inside data analysis. After this research, Python is decided to be used with an introduction to reliability library and its modules.

Another source type that are advanced through this report is journals with the topic of aerospace and aviation to find relevant information about the functions of the units, operation conditions, failure trends, mathematical models to describe the physics of operation.

1.6 Limitations

The first limitation comes from the lack of prior failure data of the Alva X60 drone motor. To find the correct sample size, which is an important step in reliability test planning, the standard deviation and the distribution of the data should be known. Since there is no failure data before this study, the distribution of the data and its parameters should be estimated with engineering knowledge, which can be erroneous. The solution proposed in this study is to estimate the test duration instead of the sample size considering the risks. However, this solution is also subjected to high deviations resulting from unknown former data.

The missing former data makes it difficult to explain the analytically found solutions because the engineers cannot visualize the meaning of the solutions without examples on the real system. Thus, case studies from similar problems are presented with random data.

The second limitation comes from the number of available samples for the reliability test. Since Alva is producing the motors not as mass production, there is a limited number of samples that can be allocated to the experiments. This makes estimating the distribution and its parameter with graphical and analytical methods harder because the variance of the results becomes quite high.

2 Background Research

Background research consists of information about the system and the theory that is needed to implement the objective of the study. To gather the essential information, data sheets from Alva Industries and the manufacturer of the components are utilized. Also, a literature review of fundamental information about reliability tests, statistical methods for data analysis, and Python libraries is conducted.

First, a general framework for accelerated reliability tests is demonstrated. Then, a review of the system is presented from the information retrieved from Alva Industries. After that, on the light of the system requirements, a review of the possible methods to implement the framework is demonstrated. This review is sectioned into four segments: possible reliability test methods, stress factors to use for the reliability test, determination of the stress levels of these factors and data analysis methods for a completed test.

2.1 Accelerated Reliability Test Framework

The need of this section comes from the necessity of a plan for reliability test and data analysis. The aim here is to combine the test planning with the data analysis plan.

The steps presented in this section are common phases of accelerated reliability test plans and data analysis for every system and gathered as a cumulative knowledge of other systems' accelerated reliability test analysis and the needs for design of experiments in this report.

Steps presented here are to be followed both before performing the accelerated reliability tests and after the tests, which provide the basis for the following chapters and taken as a framework.

Testing drone motors for reliability analysis is a comprehensive study that involves:

1. Defining what a failure means (complete failure, decrease in the performance, failure of a unit etc.)
2. Finding the relevant stresses that can affect the lifetime, performance etc.
3. Finding the parts of the product that is affected by that stress (Failure mode analysis)
4. Finding the level of the stress that will damage the product
5. Finding the level and duration of the stress that should be applied during the test
6. Determining the term to measure reliability (Failure rate, mean time to failure (MTTF), mean time between failures (MTBF) etc.)

7. Choosing an inspection method (Periodic inspection, continuous inspection and so on.)
8. Determining the sample size
9. Using a life-stress relationship
10. Determining the distribution type
11. Analyzing the data according to the distribution and life-stress relationship

A part of these items has been accomplished in the previous study (Mert, 2021) such as items 1,2,3,4,5. These items were applied only to a stress type (temperature) and their continuation in data analysis was missing as tests couldn't be realized due to the lack of material to test. In this study, a comprehensive approach to implement reliability tests will be shown by introducing steps 6-11 and mentioning the previous points to maintain the integrity of the process, so that the engineers in Alva Industries can apply practically with following the flow in this report.

The next section presents the system components and their physics of failures to determine the meaning of a failure for each component, which is the first step in the framework.

2.2 Information About the System

Alva Industries AS has an ambition to reflect the state-of-the-art technology of drone motors for multicopters to the field by planning the most affordable reliability test to determine the lifetime of "X60" motor model. The reason behind this reliability ambition is to use it for marketing and to determine the cost for repairs during the warranty period.

Alva X60 type motor is taken as a reference for the rest of the study. Its components are analyzed and statistical analysis is performed about its design and performance parameters. It consists of four basic elements of housing, bearing, stator and rotor. A Halbach rotor is used and the windings in the stator are fiberprinted, which is a unique technique that Alva Industries developed to produce the stators. Also, the windings are epoxy encapsulated to keep the windings away from damage, coiling and wobbling. The bearing is SKF 61900-2RS1, which is a ball bearing with NBR seals on both sides. Therefore, the system has high-end technology with high quality equipment to increase its lifetime.

The reliability block diagram, which can be demonstrated with Figure 1, is a simple series system where failure of a unit causes the failure of the system. The diagram can be enlarged by adding the components of the subsystems to the diagram in a series format. They are also presumed to be added to the series diagram because the functions of these are components are rather the main function of the subsystem or a protection function of the main unit. These units can be seen in Table 1.

The function of the winding is the same as function of the subsystem which is to provide magnetic field. Back Iron enhances that magnetic field and epoxy protects

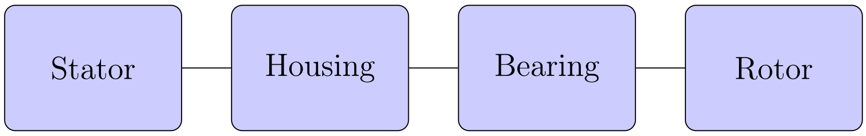

Figure 1: Reliability Block Diagram of Drone Motors

Table 1: Drone Motor Parts

Stator	Rotor	Bearing (Subsystem)
Winding	Magnet	Bearing
Back Iron	Lamination	Lubrication
Epoxy		Seal

the winding from environmental effects and dynamic forces. The function of the magnet is to create magnetic field which is the same as the function of the subsystem, and the lamination protects the magnets by keeping them in place. The function of the bearing is to support the rotor and transfer the load from shaft to the motor, where lubrication and seal protects the bearing from damage. Lubrication also has a function to increase the performance of the bearing and its gradual loss decreases the performance but a more critical effect is to cause the wear out of the bearing.

Overall, in this study, only the critical failures of the drone motor are studied which causes failure of the product during its objective mean lifetime. The next section shows the reliability objectives and defining an effective reliability goal.

2.3 Requirement Analysis

The reliability goal of Alva from the X60 motor is to have a mean life of 1500 hours. It is also important to observe the early failures to upgrade the product (assuming these failures aren't caused by burn-in period). The product life should be presented with 95% confidence level and 20 samples can be allocated for the reliability test to demonstrate a population of 2000 units.

Converting reliability the goals into requirements needs further information about the system and the testing equipment, such as the inspection method that is going to be used in the test setup. The inspection method is selected as a continuous method for the test arrangement of Alva so that the failures can be detected as soon as they occur. It is a crucial part of the test because realizing the failures after a time in an accelerated test can mean huge time variations in normal conditions.

It is also important to know what is meant by the mean life. Mean time to failure (MTTF) and mean time between failures (MTBF) are two commonly used terms to describe the expected lifetime. There are also other terms such as mean time between downing events (MTBDE), mean time between replacements and mean time between scheduled replacements. The main differences are the type of the system, categorized as repairable or non-repairable, and inclusion of the repair time (or down time) in the calculations. MTTF is used for non-repairable systems and the other terms are used for repairable systems. MTBF doesn't include the down time in the calculations, where MTBDE takes the down time into account. The mean time between replacements and mean time between scheduled replacements are for systems with scheduled replacement programs. For mean time between replacements, it is assumed that the system becomes as good as new after the replacement and for the mean time between scheduled replacements, it is not assumed as such (Hottinger Bruel Kjaer Inc, 2022c).

Alva Industries plans to provide a maintenance program for the customers that encompasses the replacement of the components that are damaged after failure. It is assumed that each subsystem is a non-repairable part of a repairable system. Scheduled replacements aren't considered by the company. Also, the down time isn't an important factor since failure of the system will not interrupt a continuous operation of a safety critical system or a system reliant on a continuous operation for economic reasons. Therefore, MTBF term is chosen to represent the reliability of the drone motor. The MTBF of a product is calculated as:

$$\mathrm{MTBF(t)} = \frac{t}{N(t)} \tag{1}$$

where, the product is observed during time t and $N(t)$ is the number of observed failures during t.

Thus, the reliability requirement of the Alva X60 model motor is transformed into: MTBF = 1500 hours under normal operation conditions where the normal operation conditions are inclusive of the extreme users with the conditions below:

Table 2: Normal Operation Conditions of Alva X60 Motor

Minimum Temperature	-15°C
Maximum Temperature	40°C
Temperature Cycling	-15°C - +5°C
Maximum Load	37N

Moreover, the product is recently designed, and the failure modes are unknown, observing the failure modes is essential to detect the problematic parts and the cause of early failures. If a test is designed that can cause competing failure modes, it will be hard to distinguish which failure mode occurred first and under which condition. Therefore, a lifetime test should be performed with triggering only a single failure mode. The next section reviews such tests.

2.4 Reliability Tests

The purpose of reliability tests can be divided into two categories:

1. Estimate the reliability parameters of the product (failure rate, MTTF, MTBF etc.).
2. Improve the design flows and eradicate imperfections.

For the first type of the test, the statistical theory and data analysis planning is the key to success, whereas for the second type of tests, the importance is to improve an already successful product to reach an ambition. Therefore, the first kind of reliability test type is referred as the statistics-based reliability testing (SRT) and the latter is mentioned as the engineering-based reliability testing (ERT) because for the second one engineering experience has a value.

Throughout the life cycle of the production, ERT and SRT are used together in different stages of the manufacturing process. As it can be seen on Figure 2, ERT is mostly used in the design and production phase, while SRT is used in the integration and the delivery phase. It is also important to know that for the SRT, the input from the user can be utilized to collect data, besides for the ERT, only manufacturer can create such a testing environment to collect data.

Because ERT has the objective to eliminate the design defects, it is more of an advance study and requires large samples to analyze the data. On the other hand, for SRT, the aim is to know the reliability properties of the product. Therefore, depending on the usage conditions, failure modes, failure mechanisms, test equipment and cost limits, SRT can be performed. Hence, asking the users about these factors and observing the failed products have value for SRT.

Depending on the testing objective, the reliability tests branches into the reliability growth test (RGT), reliability qualification test (RQT), reliability screening test (RST), reliability acceptance test (RAT), and reliability determination test (RDT) (Chen et al., 2018). RGT and RST are categorized as ERT methods. RQT, RAT, and RDT are SRT methods. RQT and RAT are categorized together as verification test because their statistical inference methods are the same, which is the hypothesis test. It means that a data is checked whether it fits to a known distribution, where the distribution and its parameters are known from the prior data. Besides, for the RDT, the statistical estimation is accomplished with parameter estimation, where the distribution is guessed but the parameters are unknown, which is suitable for products with unknown prior data (Chen et al., 2018).

These tests can be accelerated if the test time is too long under normal operation conditions. If ERTs are accelerated, the names of the tests become: accelerated RGT, highly accelerated life test (HALT), and highly accelerated stress screening test (HASS) (the last two are also called reliability enhancement test, RET). If SRTs are accelerated, the new tests comprise of accelerated life test (ALT) and the accelerated degradation test (ADT) depending on the stresses applied and their effects on the system (Chen et al., 2018).

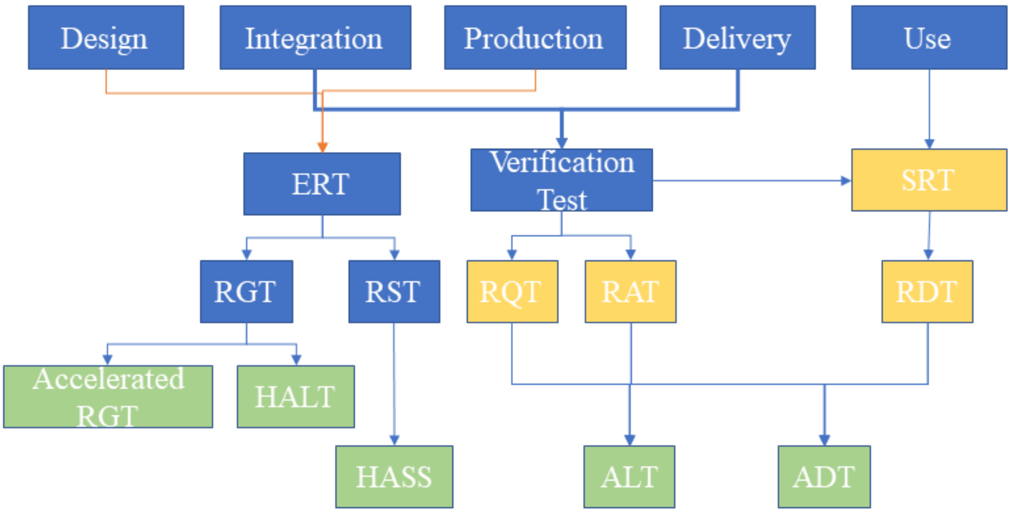

Figure 2: Reliability Test Types
(Chen et al., 2018)

Alva X60 model motor is considered to be in the use phase of the product development process, where the aim of the test is showing the reliability of the motor by emulating the conditions that the customers may create. Also, there is no prior data about the failure of the motor, which makes the test condition eligible for RDT. Furthermore, since the product is designed to withstand the possible usage conditions with a safety factor, which is assumed to endure at least 1500 hours on an average, the test is subjected to acceleration. Therefore, an accelerated RDT should be performed, which can be ALT or ADT. The type of test is to be determined by the type of stresses that are going to be applied during the testing process, which are shown in the next section.

2.5 Types of Stresses and Related Failure Mechanisms

To effectively use the stresses to estimate the reliability of a product, it is required to know the possible failures of the simpler units and their causes. For such a knowledge, the engineering experience in units' materials, functions, and the physics of stresses is important. To understand if a potential stress influences failure of the product, a hypothesis test can be conducted with two levels of each stress (low and high). An example of such a test can be ANOVA, where the null hypothesis is that there is no effect of the stress factor on the lifetime of the product. The suspected stress factors can be applied one by one at the low and high stress levels with m repetitions. Then, their significance on the lifetime of the product can be calculated by least squares method and be compared with the F-distribution value to understand their significance. With this method, the effect of each stress factor as well as the effect of interaction of factors can be seen on the lifetime. This is for a qualitative understanding of the data but it can also be applied after a quantitative test. Also, it is important to note that this method assumes a normal distribution for the failure of the samples.

It is essential to mention that the high level of the stress should not create such a failure mechanism that wouldn't happen for the normal operation conditions. If it is hard to gather such physics of failure, HALT can be conducted to verify that the high stress doesn't cause a failure than expected (Dohi and Nakagawa, 2013).

The stresses and the stress levels to be applied at the reliability tests are not the same for all the products of the company. For example, a standard design of experiment cannot be developed because different products have diverse goals, and they are designed accordingly to adapt to a new operation condition. The same experiment cannot be used even the product is going to be operated in the same operation conditions if a new version of the product is developed because it is apparent that a new design is adopted, where the component usage limits and the interaction between the components are not the same as before.

To ease the design of experiment process, types of stresses that are relevant for drone motors can be introduced. The stresses can be categorized into three sections:

1. Mechanical Stresses
2. Environmental Stresses
3. Electrical Stresses

It is essential to get informed about the types of stresses that may affect the system because for later stages, the stresses can be combined to emulate a similar model for normal operation conditions or to accelerate the reliability tests further.

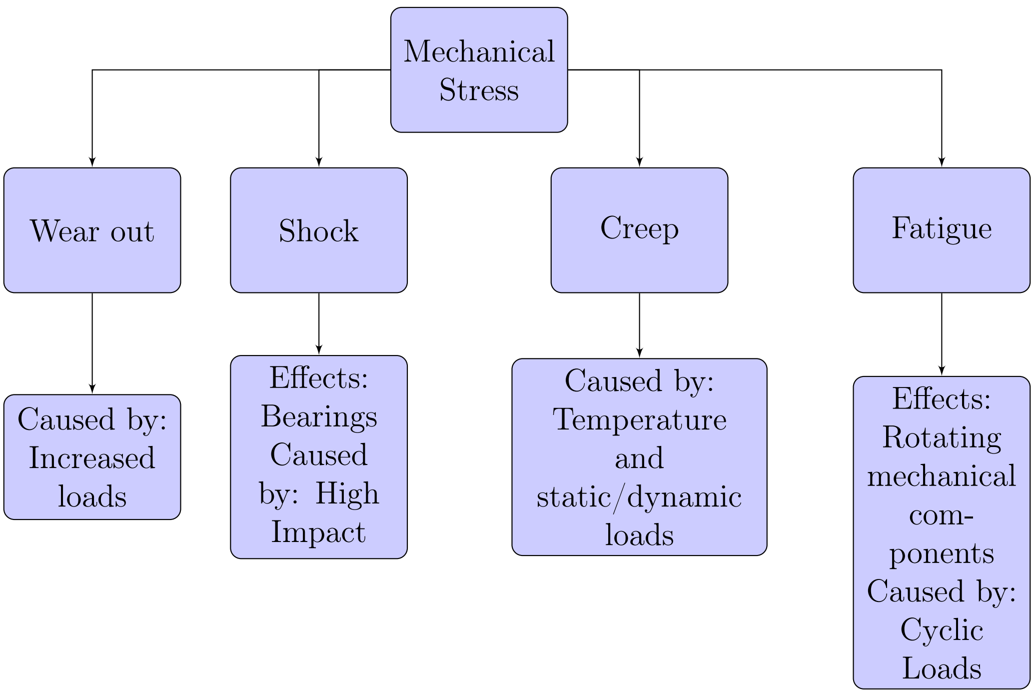

Figure 3: Mechanical Stresses, Their Effects and Causes

Mechanical stresses can be visualized as in Figure 3. These stresses can induce fatigue, creep, shock or/and wear out on the components of the product. Fatigue is very common for rotating equipment where the material loses its strength gradually. Thus, it can be measured and prevented by crack/stress propagation modelling in the design stage of the product development. Fatigue can be measured by assessing the ductility of the material over time because it is caused cumulative loading effects of mechanical stresses such as cyclic loads (Wasserman, 2002). Fatigue can also be induced with other types of stresses which are discussed further in this section.

Creep can occur when a product is subjected to elevated temperature levels and loads (static/dynamic). It is crucial to test for drone motors since drones has components where the temperature can increase due to high constant stress, such as the bearing. Therefore, when a failure mode has a failure mechanism of creep, the reliability test can be accelerated by increasing the time under high stress.

Wear out is a crucial failure mechanism for moving objects and the component that is subjected to this mechanism is the bearing. Wear out can be observed and accelerated by increasing the operational load on the motor. The operational load can be increased by additional wind force against the moving direction of the drone.

Shocks are temporary loads with high amplitude and short duration. They are transferred as waves and lose their damaging effect as the waves enter cross mechanical interfaces (Allen et al., 2018). A type of shock mechanism is impact shock which can be relevant for drone motors. An example for this failure mechanism can be the hard landing of the drone which has high amplitude and short duration which is significant for the bearings of the motors. Shocks cannot be simulated as sinusoidal waves but can be tested by rather emulating the same scenario for hard landing or can be accelerated by increasing the amplitude of the load or repeating the load after a random duration. A way to reduce the impact of shock failure mechanism is to use a material that can consume the kinetic energy produced by the impact by permanently deforming the material.

The failure mechanisms can be categorized by their immediate effect on the motors and their frequency of appearance during normal operation conditions. For instance, shocks can be categorized as critical by their immediate effect because its cumulative loading can damage the motors during their stated lifetime. On the other hand, it can be classified as user fault if the landing throttle level is specified on the product manual.

When it comes to creep, which can be effective in bearing seals, its immediate effect is not considered to alter the system critically. It means that if the bearing seal fails with creep failure mode, if there no other environmental stresses such as fine particle etc. its collapse won't influence the system.

As a result, wear out and fatigue failure mechanisms can have a significant effect on the drone motors during their stated lifetime.

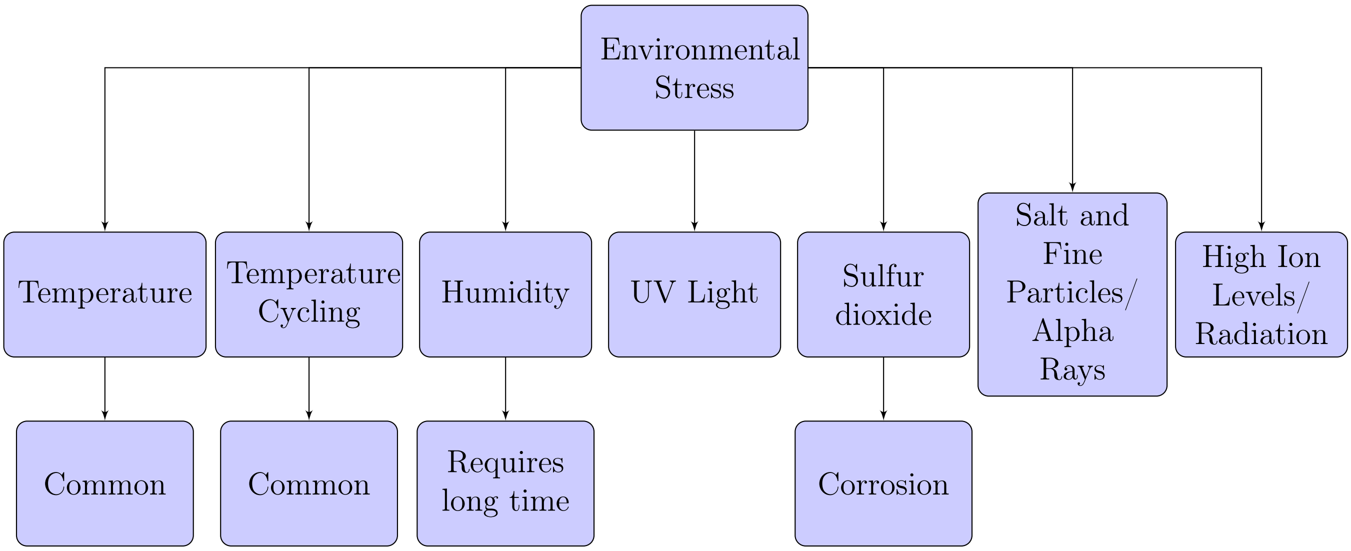

Figure 4: Environmental Stresses, Their Effects and Frequency of Application

Figure 4 shows the environmental stresses overall. The stresses other than the temperature, humidity and fine particles are irrelevant for this study because they are mostly applicable to space applications such as radiation. UV light is not significant because it is particularly effective to elastomers which are under the metal housing of drone motors. Sulfur dioxide, salt and fine particles/alpha rays are damaging to RAMs or similar units which are irrelevant for drone motors (Dohi and Nakagawa, 2013).

Therefore, the significant environmental stresses are temperature, temperature cycling, fine particles, and humidity, which is a less common application. Humidity is regarded as negligible because it takes long to damage units with the humidity of the air even if it is accelerated because drone operation is considered as discontinuous where humidity isn't applicable continuously to create cumulative damage. The continuous operation time of the drone motor is ≈ 30 minutes.

It is possible to mitigate the effects of humidity along with fine particles and sulfur dioxide by storing the product properly. Therefore, users should be clearly explained about how to store the product.

Temperature is a common application and relevant for drone motors because it causes two types of failure mechanisms and three failure modes:

1. Fatigue for stator lamination (epoxy) which causes propagated cracks,
2. Fatigue for the glue in between stator lamination and back iron; causes crack growth on the glue material or change of material properties such as ductility,
3. Creep for polymeric materials such as bearing seal etc.

Taking the most recorded failures as a reference, bearing and stator failures take the lead for AC motors with overheating failure mechanism as the major source (Holbert, Lin and Karady, 2006). Therefore, the reliability tests are designed for the stator and the bearing with temperature as the main stress factor in this study. The stator's stress levels are determined with the help of data sheets of the epoxy and the copper wire considering their temperature limits. The stress levels for the bearing are determined by inspecting the thrust forces of the motor for the mechanical stress and with the data sheets that describes the traffic light concept of the bearing greases for the temperature stress (Figure 5).

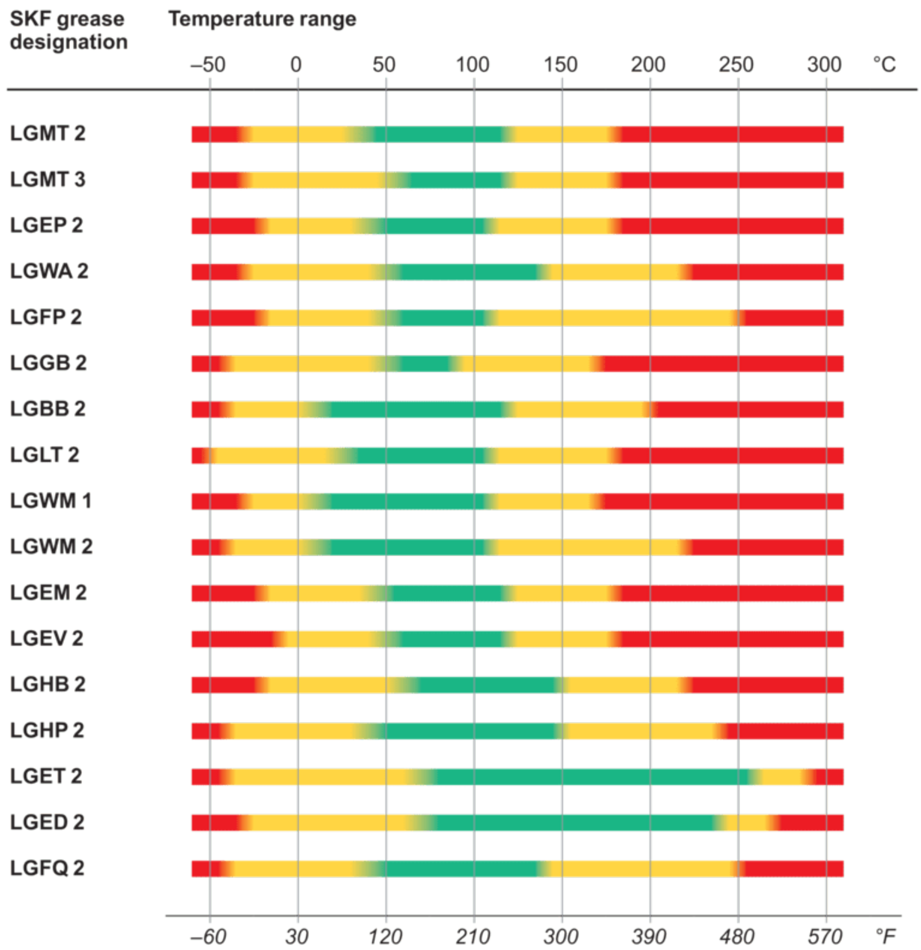

Figure 5: Traffic Light Concept for Standard Greases
(SKF, 2022)

Figure 5 shows the usage limits of the bearing greases with traffic light colors, where green is the safe zone for operation, yellow zone is a risky region that the usage duration should be limited and the red zones should be avoided. On the yellow zones, the performance of the bearing decreases but on the red zone, the functionality is affected because of the frictional torque (SKF, 2022). This concept is used to emulate a failure scenario of the motor when the motor is started with high payload on a cold day where the grease operation is represented with the red zone which is also still in the range of temperature operation limits of the motor. Another stress type is the electrical stress. It affects the electronic components of a drone but it is not under the scope of this study. For the future need a flowchart about it is attached in the appendix (Figure 17). For an overall view of stresses, affected components, and failure modes Table 9 in Appendix B can be referred.

Overall, the stress factors that are going to be applied to the accelerated reliability test are determined. Now, the type of accelerated reliability test can be decided as ALT because the stress factors are chosen to create a complete failure of the drone motor; they are not chosen to create partial failures which are related with performance degradation of the product.

After deciding on step 2 of the framework, next section discusses the approach on stress level determinations for ALT tests.

2.6 Stress Level Determination for ALT

After determining the stress types that are to be applied to the products, next stage is to discover the stress levels for the relevant failure modes. The levels are selected with the level categories demonstrated in Figure 6. The specification limits show the interval that the product should be used in normal conditions. This is the level that the manufacturer recommends being used. The operating limits are the levels that the product starts to lose its functionality which is reversible after the stress is removed. Such an example can be the elastic limit on a stress-strain curve. However, the destruct limits are the boundaries when the product is irreversibly damaged. The examples for this limit can be the plastic deformation of a material. When temperature is considered as the applied stress factor, it can be the phase change of a material; when vibration is considered, the destruct limit is when a unit breaks and so forth. (Wasserman, 2002).

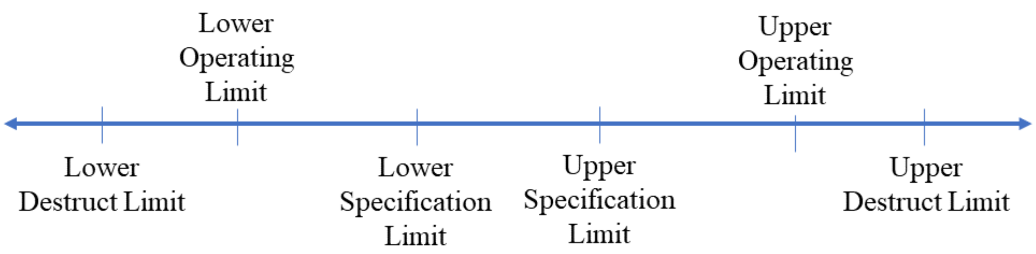

Figure 6: Stress Levels for Accelerated Reliability Tests
(Wasserman, 2002)

When performing the ALT, it is assumed that:

- No new failure modes are introduced during the test via high stresses or combination of stresses.
- The hazard rate of the failure distribution does not change.
- The shape parameter of Weibull distribution ,and the standard deviation of (log)normal distribution does not change. (Wasserman, 2002)

Therefore, when determining the ALT stress levels, the stresses shouldn't create an effect that is not expected during normal operation conditions. Thus, the stress levels should stay in the region of lower and upper operating limits. For example, such a test can be created with taking the low accelerated stress level as the upper specification limit and high accelerated stress level as the upper operating limit and this experiment can be executed several times. For stresses higher than the operating limit is suitable for HALT where it is expected to create failure modes different than normal operation conditions.

The simplification for the analysis of the data gathered from accelerated stress levels comes from the constant β and σ parameters for Weibull and (log)normal distribution respectively. Weibull and lognormal distributions are selected in this study for reasons explained in the next chapter in addition to the data analysis methods along with the constant parameter rule introduced above.

3 ALT Data Analysis

The previous chapters introduced the methods about design of experiment for ALT. This section handles the data analysis part of the framework, where it is assumed that ALT experiments are performed.

For the ALT to be understood, the data from the experiments should be interpreted properly. The data collected can be analyzed with parametric or non-parametric methods. Non-parametric methods include Quantile-Quantile plot (Q-Q plot), which matches the failure times of the normal and elevated stress levels on a line. The problem with the Q-Q method with small sizes is that it is insensitive to outlier points, it can include the outliers where it shouldn't have included.

Parametric methods are more useful if the collected data is small as in the case of ALT plan for drone motors. Parametric methods have two types, which are graphical and analytic methods. The steps for analyzing the data with parametric methods can be summarized as shown below:

1. Decide on the statistical distribution that the data best represents.
2. Make two empirical cumulative distribution (CDF) plots for both low stress and high stress data in the same graph.

   Using logarithmic scale for the life and probability scale for the cumulative distribution makes the plot more interpretable because it linearizes the plot and makes the iteration easier.
3. Use a stress-life relationship to iterate through normal operation condition's stress.
4. If the goal life is on the line or lies under the line, consider the goal has been met.
5. If the goal life is above the line, check if the distance (error) is within the confidence interval. If it is inside the interval, the error is not significant (the goal has been met); if it is outside the interval, the goal has not been met.

The first challenge with the data analysis is to find an appropriate probability distribution for the data. It can be guessed by experience or with the goodness of fit methods. A table presenting the common distribution types for different data categories is shown in Table 3.

From Table 3, it can be inferred that for the focus of the drone motor technology at Alva Industries, Lognormal and Weibull distributions can be used for data analysis of ALT because both distributions can be utilized to apply for fatigue related failure fitting. If it is uncertain which probability distribution should be used, goodness of fit tests can be performed.

Goodness of fit tests are primarily applied to analytic data analysis methods because they are hard to apply manually. For graphical methods, the correctness of the selected distribution is checked by observing the linearity and the parallelism of the CDF plots in the life-stress model. Hence, the correctness of the plots can only be revealed after fitting the model, which is not a very objective technique to assess because the evaluation of the linearity of the data depends on the observation of the analyst.

Both techniques have advantages and disadvantages. In the next two chapters the applicability of graphical and analytic methods with the theory behind them is discussed.

Table 3: Probability Distributions and Usage Areas

(Wasserman, 2002)

Distribution	**Properties**
Normal Distribution	Generally used for symmetric data with central tendency. It is used when the coefficient of variation $C = \sigma/\mu$ is lower then 0.1. Tolerances are a common application of normal distribution.
Lognormal Distribution	Mainly used for design for reliability applications. Common usage areas are data fitting of fatigue related applications and stress-strength visualization.
Exponential Distribution	Primarily used for data analysis of electronic components. It is used for demonstration of data with constant failure rate.
Weibull Distribution	It is a versatile distribution to explain time-to-failure of products.
Extreme-value Distribution	It is used for understanding the environmental data from extreme conditions, such as minimum rainfall, maximum load etc.

4 Theory Behind ALT Data Fitting

After deciding the distribution type, the data should be fit to the distribution to estimate the parameters, which are further used to estimate the lifetime at the normal conditions. In this chapter, the theory behind the ALT data fitting with compatible distributions and models for drone motors is demonstrated. Theory provides the basis for graphical and analytic methods for data analysis and the formulae are referred in the following chapters.

4.1 Life-Stress Relationships

The chapter begins with life-stress relationships because they are the linkages between the distribution at the accelerated stress levels and the normal operation stress levels. In the next sections, the association of life-stress relationships with the relevant distributions are presented.

Analysis methods benefit from life-stress relationships to gain information about the past and the future of the life of the product. Here, life can be represented with the mean, median or distribution percentiles on the graph. It is easy to extrapolate with life-stress relationships because they are transformed to logarithmic or other suitable scales to fit a straight line. They are applied to accelerated tests with certain failure mechanisms such as metal fatigue.

The life-stress relationship is not sufficient by itself to model the failure tendencies of the product; it should be combined with the statistical distribution of life. Each point on the life-stress relationship represents a percentage of failure of the product with related stress as well as the life of each unit. This explanation can be visualized with Figure 7, where the y-axis represents the life of the units (the time when a sample fails), x-axis represents the applied stresses and the data points on the graph represents the percent failure at that specified stress and time.

4.1.1 Arrhenius Relationship

Arrhenius relationship is a life-stress relationship used for products that have failure modes involving temperature. Hence, temperature stress type is applied to the product during ALT. It is widely used for insulation, plastic, grease, and lubrication applications. Therefore, it is relevant for drone motor applications such as copper winding insulation, bearing lubrication, and bearing seal. Arrhenius relationship explains the nominal time to failure (mean or median of the statistical distribution at that stress) with the Equation (2).

$$\tau = A \cdot e^{E/(k \cdot T)} \tag{2}$$

where;
A is a constant that is related with the product shape, size, manufacturing process,

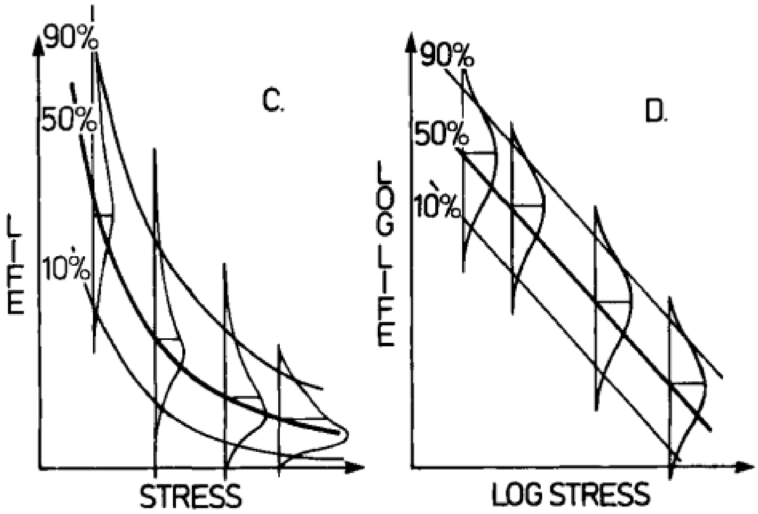

Figure 7: An Example Life-Stress Relationship Graph with Statistical Distribution Function

(Nelson, 2009)

test method etc.
E is the activation energy of the reaction. When more than one failure mode is observed during a test, each failure mode has different A and E values.
k is the Boltzmann's constant, which is equal to $8.6171 \cdot 10^{-5}$ electron-volts per $°C$.
t is the absolute temperature (in Kelvin).

As mentioned before, the life-stress relationships are useful because they can linearize the relationship and help extrapolating. Arrhenius equation can be linearized by taking the logarithm at base 10:

$$\log \tau = \gamma_0 + \frac{\gamma_1}{T} \tag{3}$$

where,

$$\gamma_1 = \log e \cdot \frac{E}{k} = 0.4343 \cdot \frac{E}{k}$$

Thus, $\log \tau$ is linearly related with $1/T$. Here, τ should represent the same life parameter for all the stress levels, such as mean (63.2^{th} percentile for Weibull), median (50^{th} percentile) or another specified percentile. Mean is preferred mostly but median can be preferable when the data has outliers or when it is skewed (Australian Bureau of Statistics, 2022). E is mostly in the range between 0.3 and 1.5 electron-volts (eV) but the exact number can differ even for the same material for different failure modes (Nelson, 2009) .

Acceleration factor is an important value to calculate when accelerating tests because it is used to estimate the parameters of the distribution in normal conditions.

The acceleration factor for the temperature relationship can be calculated with Arrhenius model by Equation (4).

$$K = \frac{\tau}{\tau'} = \exp\left(\frac{E}{k} \cdot \left(\frac{1}{T} - \frac{1}{T'}\right)\right) \tag{4}$$

where, τ is the life at normal conditions, τ' is the life at the accelerated condition. T is the temperature at normal conditions and T' is the temperature at the accelerated condition.

Arrhenius plot can be drawn as in Figure 8 where, life is fit with log-scale on the y-axis and linear inverse absolute temperature on the upper x-axis which has a nonlinear Centigrade temperature on the bottom x-axis. This part of the plot is the same for all probability distribution types. The linear percentiles are spaced accordingly with lognormal cumulative distribution in this example.

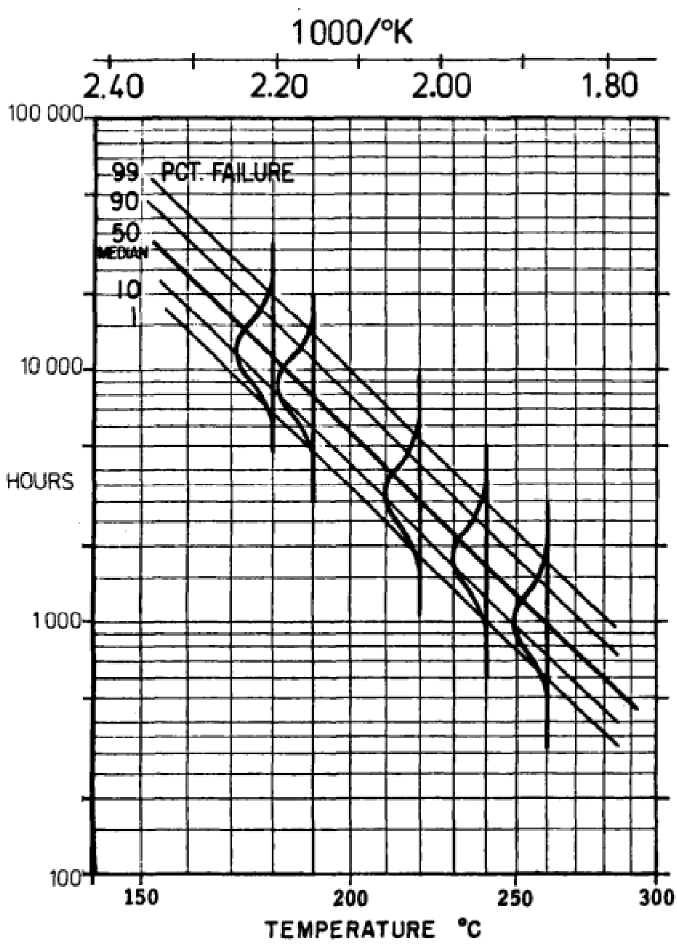

Figure 8: Arrhenius Plot with Lognormal Percentile Lines on Arrhenius Paper (Nelson, 2009)

Each percentile line determines the equation parameters: γ_0 ($\log A$) is the intercept of the line to the highest temperature on the plot and E is the slope of the line. It is important to note that the Arrhenius plot has a limit on the temperature axis, which can be shown with the operational stress limit of the unit.

Rather than plotting the relationship with temperature on the x-axis, failure percentages can be placed there. Then, the grids should be generated according to the probability distribution as on Figure 9.

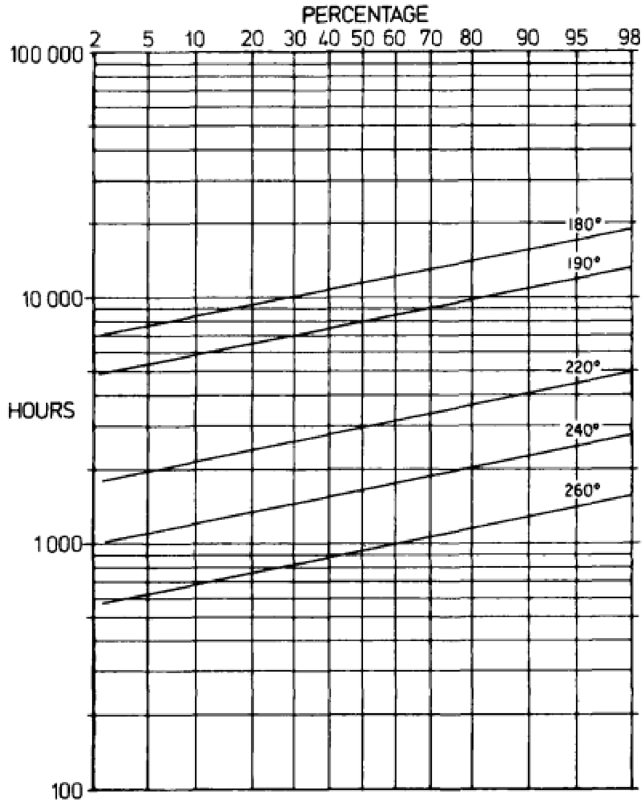

Figure 9: Arrhenius Plot with Lognormal Percentiles on the X-axis and Temperature Lines

(Nelson, 2009)

Arrhenius-Lognormal Model. Here, the slope of the lines depict the standard deviation σ, which is the same for all lines in the model.

Selecting the lognormal distribution with Arrhenius model depends on some assumptions such as:

- At the absolute temperature, the product life is described with lognormal distribution,
- The standard deviation is constant,
- The median and mean life can be depicted with linear equations:

$$Median : \log[\tau_{.50}(T)] = \gamma_0 + \frac{\gamma_1'}{T} \quad (5)$$

$$Mean : \mu(x) = \gamma_0 + \gamma_1 x \quad (6)$$

where, $\mu(x)$ is the mean of log lifes, $\gamma_1 = \gamma_1'/1000$ and $x = 1000/T$.

Fraction Failed at a Temperature. The fraction failed at a desired life for a temperature can be found with the cumulative distribution function:

$$F(t,T) = \Phi \left\{ \frac{[\log(t) - \mu(x)]}{\sigma} \right\} \tag{7}$$

where, Φ is the standard normal cumulative distribution function.

Life at Percentile. At a chosen temperature, the life at $100P^{th}$ percentile (P fractile) for failure is given as:

$$\tau_P(T) = \log^{-1}[\mu(x) + z_p\sigma] = \log^{-1}[\gamma_0 + \gamma_1(1000/T) + z_p\sigma] \tag{8}$$

where, z_p is the standard normal percentile which determines the vertical positions of the percentile lines in Figure 8. The following percentile for a goal life at a certain temperature is given as:

$$\eta_P(x) = \log\left[\tau_P(x)\right] = \mu(x) + z_P\sigma \tag{9}$$

The 50^{th} percentile has a special case for the equivalent log life, where:

$$\eta_{.50}(x) = \mu(x) = \gamma_0 + \gamma_1 x \tag{10}$$

Using the Arrhenius relationship on Figure 8, the design temperature can be found, the operation conditions can be renewed or more durable materials can be chosen for design temperature. Using the lognormal model, the temperature is found as follows :

$$T^* = \frac{1000\gamma_1}{\log(\tau_P*) - \gamma_0 - z_p\sigma} \tag{11}$$

With a known design temperature, the mean/median life can be calculated by the pursuit of τ_P. For the goal life, the required material temperature property can be found with the pursuit of T^* (Nelson, 2009).

Arrhenius-Weibull Model. Some failure data related to temperature failure mechanisms can be explained with Arrhenius-Weibull model. As it is with the lognormal model, Arrhenius-Weibull model also has some assumptions:

- At a certain temperature, the life data should have Weibull distribution.
- Weibull shape parameter (β) is constant for all stress levels.

- The natural logarithm of Weibull characteristic life (α) is linearly related with $(1/T)$ as in Equation (12).

$$\ln[\alpha(T)] = \gamma_0 + \frac{\gamma_1'}{T} \tag{12}$$

where, γ_0, γ_1' and β are characteristic properties of the product and the test (Nelson, 2009).

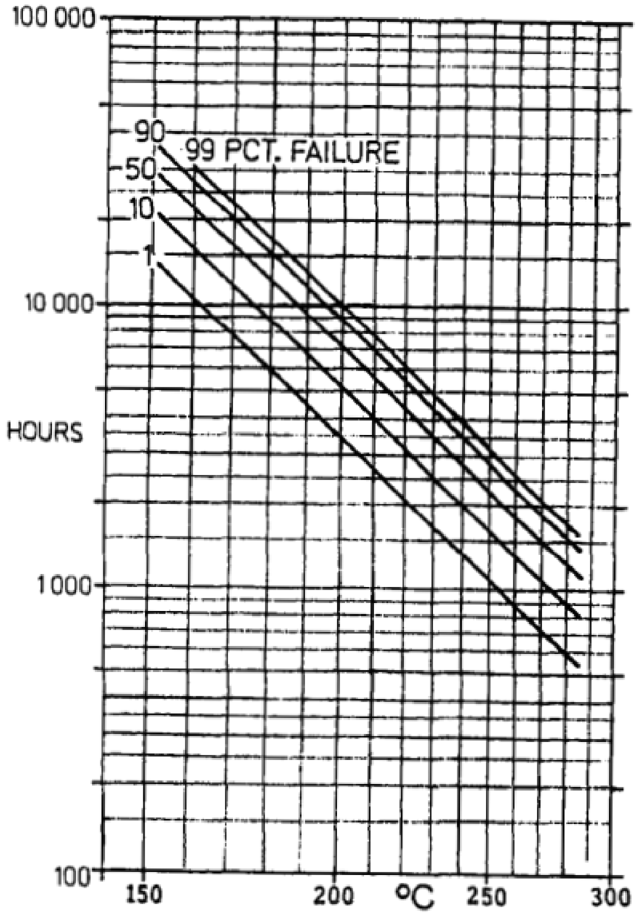

Figure 10: Arrhenius Plot with Weibull Percentiles
(Nelson, 2009)

Figure 10 is drawn with the same scale as Figure 8. The difference is the percentile lines which demonstrate the characteristic life ($\alpha(T)$), with the slope in Figure 10.

Fraction Failed. As for Arrhenius-lognormal model, the fraction failed at an absolute temperature can be found. Unlike lognormal model, here, the cumulative distribution function for the Weibull distribution is used:

$$F(t;T) = 1 - \exp\left[-\frac{t}{\alpha(T)}\right]^{\beta} = 1 - \exp\left\{-\left[t\exp\left[-\gamma_0 - (\gamma_1'/T)\right]\right]^{\beta}\right\} \tag{13}$$

Life at Percentile. At a certain temperature, the $100P^{th}$ percentile (P fractile) failed is estimated with Equation (14):

$$\tau_P(T) = \alpha(T)[-\ln{(1-P)}]^{1/\beta} = \exp{[\gamma_0 + \gamma_1(1000/T)]}[-\ln{(1-P)}]^{(1-\beta)} \quad (14)$$

To determine the life at the sought percentile, τ_P is pursued at the desired temperature. To find the percentile of failure at a given stress and temperature, the inverse exponential can be applied to Equation (16).

$$\eta_P(x) = \xi(x) + u_P\delta \quad (15)$$

where, $x = 1000/T$, and $u_P = \ln{[-\ln{(1-P)}]}$.

$$\tau_P(T) = \exp[\eta_P(x)] \quad (16)$$

For the 63.2^{th} percentile, $\tau_{.632}(T) = \alpha(T), \eta_{.632}(x) = \gamma_0 + \gamma_1 x$.

Design Temperature When there is a goal life for a percentile such as mean/median, the required design temperature can be found with Equation (17).

$$T^* = \frac{1000\gamma_1}{\ln{\{\tau_P^*/[-ln(1-P)]^{(1/\beta)}\}}} \quad (17)$$

4.1.2 Inverse Power Relationship

Inverse Power relationship is used for modelling the accelerated stress and life relationship of ball and roller bearings, which is useful for the drone motors. This is a model that explains the life data of products that are subjected to constant stress. The relationship between stress and life can be illustrated with Equation (18).

$$\tau(V) = A/V^{\gamma_1} \quad (18)$$

where, A and γ_1 are dependent on characteristic properties of the product, its geometry, manufacturing and test structure.

Coffin Manson Relationship. This relationship is used for describing the fatigue failure of metal components caused by temperature cycling. It can also be used for plastic encapsulants which is convenient for drone motors of Alva Industries where the epoxy lamination of the motor stator is exposed to thermal cycling. The relationship is explained with Equation (19).

$$N = A/(\Delta T)^B \tag{19}$$

where, N is the total number of cycles to failure, ΔT is the temperature range of the thermal cycle, A and B are the characteristic properties of the material. For metals, it is mostly employed with lognormal distribution and B is approximately 2. For plastic encapsulants, B is approximately 5.

Coffin-Manson relationship and standard inverse power relationship is the same but expressed with different stress factor.

The inverse power relationship should be linearized to ease the extrapolating for normal use stress level. The linearized version of Equation (18) is shown below:

$$\ln(\tau) = \gamma_0 + \gamma_1[-ln(V)] \tag{20}$$

The Coffin-Manson equation can be linearized with Equation 20 as well, where, V is changed with ΔT and γ_1 is changed with B.

Equation (20) can be presented as a straight line in Figure 11. This figure has time on the y-axis and stress levels on the x-axis. It can be shown vice-versa for metal fatigue and insulation applications. Also, the range of the life (time) axis may not be enough for some applications which are fit according to the relevant range in computer-based solutions.

Power Acceleration Factor The acceleration factor shows how much longer the life (mean, median or at a percentile) of the product be compared to the accelerated stress level. It is calculated for the power model as shown below:

$$K = \tau/\tau' = (V'/V)^{\gamma_1} \tag{21}$$

where, τ is the life at normal operation conditions, τ' is the life at the accelerated stress conditions, V is the stress at normal operation conditions and V' is the stress at accelerated conditions.

Power-Lognormal Model. The accelerated stress failure of some products can be demonstrated with lognormal distribution. These failures are mostly caused by metal fatigue. There are some examples that this model can be used for other materials' fatigue failure mechanism (Nelson, 2009). There are some rules that should be followed before using power-lognormal model:

- At each stress level, the failure times should be described with lognormal distribution.
- The standard deviation, σ, of log-life is constant and independent from the stress.

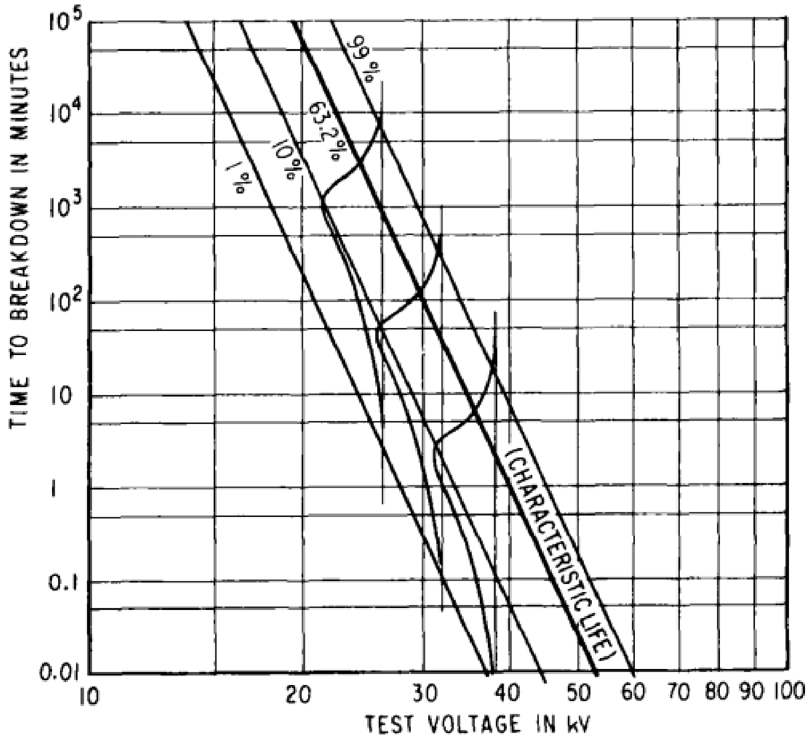

Figure 11: Inverse Power Relationship on a log-log paper with Weibull Percentile Lines

(Nelson, 2009)

- The median life $\tau_{.50}$ is the inverse power function of the stress:

$$\tau_{.50}(V) = 10^{\gamma_0}/V^{\gamma_1} \tag{22}$$

Similarly, the mean of log-life at base 10 is linearized as Equation (23), where $x = -\log V$.

$$\mu(x) = \gamma_0 + \gamma_1 x \tag{23}$$

Fraction Failed. The population fraction failed by age t can be found with the cumulative distribution function of lognormal distribution as in Equation (7). For this model, instead of T for temperature, V for stress level is used. Again, the formula represents the straight lines on a lognormal probability paper as in Figure 9.

Percentiles. The life at a certain fraction and stress level can be determined with the same function in Equation (8). When it is simplified, the equation looks as follows:

$$T_P(V) = (10^{\gamma_0}/V^{\gamma_1}) \cdot \log^{-1}(z_P \sigma) \tag{24}$$

Again, the equivalent percentile for goal log-life can be found with Equation (9). Here, the difference is how the x is defined:

$$\eta_P(x) = \log[\tau_P(V)] = \mu(x) + z_P \sigma \tag{25}$$

For the median log life, $z_{.50} = 0$.

When deciding the warranty inclusion of stress levels, it can be useful to know the stress at a desired percentile with desired life. Thus, it can be calculated accordingly with Equation (26).

$$V^* = (1/\gamma_1) \cdot \log^{-1}[\gamma_0 + z_P \sigma - \log(\tau_P^*)] \tag{26}$$

Since life-stress relationships are dependent on specimen geometry, characteristic material properties, fabrication and test method, there is an uncertainty with the life times. Thus, a safety factor should be defined to avoid this uncertainty. This safety factor is 3 for airplane engine and frame design, where these parts are replaced after $\tau_P/3$, where τ_P is the time of use (Nelson, 2009). Therefore, engineers at Alva Industries should consider implementing safety factors for proposing life time of the product for marketing purposes. The safety factors are in the range of 1.2 to 4 with an average of 2 (Wasserman, 2002). Since drone motors do not bear as much risk as an airplane comparing their safety critical level, safety factor in the range of 1.2 to 2 can be used.

4.1.3 Power-Weibull Model

The stress-life relationships of some products follow Power-Weibull model. Some of these products are:

- Ball and roller bearings when mechanical load is accelerated.
- Metal fatigue due to mechanical load; unlike lognormal model, where the acceleration stress is the thermal cycling.

The assumptions of this model is the same with the Arrhenius-Weibull model, except the last item. Here, the characteristic life is described with the power function:

$$\alpha(V) = e^{\gamma_0}/V^{\gamma_1} \tag{27}$$

The fraction failed at a certain time and stress is represented with the Weibull cumulative distribution function. When the parameters are put for the power model, the cumulative distribution function is as follows:

$$F(t;V) = 1 - \exp\left\{[-te^{-\gamma_0}V^{\gamma_1}]^\beta\right\} \tag{28}$$

As it is with the previous equations, $\gamma_0, \gamma_1, \beta$ are related to material, test, manufacturing and shape properties of the product. This function can tell if a unit is going to fail at a constant stress in a determined stress exposure duration. The cumulative distribution lines are straight lines on Weibull probability paper (Figure 12). When β is larger, the distribution lines are lined further.

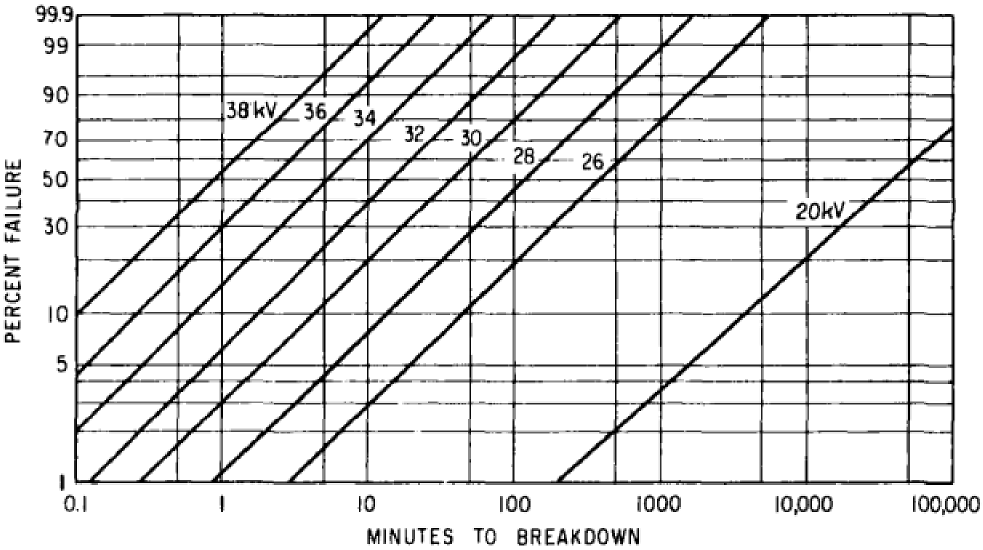

Figure 12: Cumulative Distribution Lines on Weibull Paper (Nelson, 2009)

At a certain stress level, the $100P^{th}$ percentile (P fractile) is calculated with the equation below:

$$\tau_P(V) = \alpha(V)[\ln(1-P)]^{1/\beta} = [e^{\gamma_0}/V^{\gamma_1}][-\ln(1-P)]^{1/\beta} \tag{29}$$

The percentile failed at an ln-life is given as:

$$\eta_P(x) = \ln[\tau_P(V)] = \xi(x) + u_P\delta \tag{30}$$

where, $u_P = \ln[-\ln(1-P)]$ and $\delta = 1/\beta$. The life at 63.2^{th} percentile is gathered as:

$$\tau_{.632}(V) = \alpha(V) = e^{\gamma_0}/V^{\gamma_1} \tag{31}$$

Accordingly,

$$\eta_{.632}(x) = \xi(x) = \gamma_0 + \gamma_1 x \tag{32}$$

The stress level that can be utilized on the design phase can be calculated with Equation (33).

$$V^* = \left\{e^{\gamma_0}[-\ln(1-P)]^{1/\beta}/\tau_P^*\right\}^{1/\gamma_1} \tag{33}$$

In the next chapter, the methods of analyzing the ALT data are discussed to choose a technique to interpret the data in the following chapters.

5 Graphical and Analytic ALT Data Analysis

To use the theory accurately, there are methods to accurately analyze the data. The compatibility of the techniques depends on the sample size, engineering knowledge and resources for analysis. Graphical and analytic methods are discussed briefly in this chapter depending on the factors introduced above.

Graphical and analytic ALT data analysis methods both use the plots in the previous chapter to interpret the data. They have advantages and disadvantages compared to each other.

The advantages of the graphical method relative to the analytic method are as follows:

- It is easy to "see" the effect of the stress.
- It can uncover unknown physical phenomena of the product such as competing failure modes.

The determination of competing failure modes is an important factor for not to fit the data to a wrong model. This is fulfilled by checking the linearity of the line on the life-stress relationship plot. If the line is not straight, and a peculiar data group is seen, it is understood that there is another failure mode during that stress level. If the line is straight, but the slope of the line is different from the other lines at different stress levels, it can be concluded that the stress level with a different slope has a different failure mode than others. Therefore, graphical method is sensitive detecting the different failure modes at a certain stress and between stresses. Besides, analytic methods are only sensitive to different failure modes between the stresses.

Also, with the graphical method, it is easy to distinguish the outliers, whereas the analytic method tends to include those data points which is decreasing the accuracy of the fit.

The disadvantages of the graphical method relative to the analytic method are as follows:

- The interpretation of the data is subjective because the analysis depends on the observation of the analyst.
- Graphical method has a more basic approach on finding the plotting positions of the data points, where they are fit by rank adjustments as it is the case for analytic method but then, the positions are optimized for the analytic method.

Moreover, the sensitivity of graphical method about the outliers and different failure modes among the same stress level is impractical when the sample size is small. For example, when there are three data points, which resembles an outlier, it is hard to find which point is the outlier. The analysis can continue with the assumption of common slope and the line can be fit accordingly.

Furthermore, the correctness of the model is tested with more basic methods. It is checked by observing the linearity and parallelism of the lines. It can be a very simple task with enough data and an experienced data analyst to detect the differences but it can be erroneous with an inexperienced analyst and a small sample size.

With all these information along with the possible resources, the advantages of the graphical method cannot be utilized for the ALT data testing of drone motors of Alva Industries because the sample size is small.

Analytic method can be utilized for the ALT data analysis further in this study because the data can be fit to a model with optimizers. The correctness of the fit can be checked simultaneously while fitting with goodness of fit methods, and the interpretation is not subjective. The downsides of this method are as follows:

- It is hard to follow up the method, when there is an error or mistake because analytic method uses more complex computational techniques.
- The fit can have a large error because of the small sample size and inclusion of outliers.

It is more coherent to use the analytic method for the data analysis of ALT plan in this study despite its disadvantages because it is more appropriate to use for small sample sizes and with the computational methods, it can be applicable to any ALT data. Therefore, in the following chapters, the application of analytic method is explained with the computational implementations.

6 Reliability Library for Python

Reliability library is a useful addition for Python-based data analysis. It is a tool that reliability engineers may need to use during test planning and data analysis after test. It allows to fit a data to a specific distribution, selecting best fit for the data, estimating distribution and model parameters, extrapolation to find the life at normal stress or at a higher stress, test duration planning, test sample size planning, choosing the best fit for ALT models and estimating parameters, and so forth. Functions that are useful for ALT test planning and data analysis is explained in this chapter with the theory behind them.

6.1 Sample Size and Test Duration

As it is shown in the reliability test framework, sample size determination is one of the milestones of ALT planning and data analysis, it should be calculated accurately because the rest of the analysis depends on the sample size. One of the provided functions of the reliability library is "reliability_test_duration" which allows the calculation of the test duration and sample size accordingly.

Sample size determination is an important part of reliability tests because large sample sizes can be uneconomical to the company, but small sample sizes can be inaccurate for quantitative methods. Therefore, sufficient sample size should be allocated to the reliability tests.

Determination of the sample size depends on the type of test termination method. If the test is planned to be terminated by the number of failed units, then the required sample size is calculated by designating a maximum number of failed units in a certain period of time. This type of tests is called success testing which can be categorized into two segments:

1. Success (Bogey) Test: For this type of test, no failure is allowed in a particular amount of time (bogey time). The reliability and confidence levels are predetermined before the test. It is based on binomial distribution and specifies if a single unit is able to pass the reliability requirement during the bogey time. This test determines how much test sample is required to reach that level of confidence.

2. Success - Failure Test: This type of test is a form of success test where a determined number of failures are allowed. This test is used for noncritical systems which has low probability of occurrence and severity.

However, a different test termination is chosen in this study which is called test-to-failure, where all units are waited until they fail and it is the recommended test type because failure modes and distribution parameters to estimate the life can be found (Wasserman, 2002). This type of test termination approach is chosen because if success tests are chosen and failed, there would be no evidence on the reliability and lifetime about the product.

The "reliability_test_duration" module is dedicated to find the duration of reliability tests that are time terminated. It uses the following equation to perform its function:

$$MTBF = \frac{2T}{\chi^2\left(\frac{1-CI}{n}, 2F + p\right)} \tag{34}$$

where,
$MTBF$ is mean time between failures and assumes that hazard rate is constant which is equal to MTTF,
T is the total test duration,
CI is the confidence interval, where only the lower bound is facilitated (one-sided confidence interval is assumed),
F is the number of failure expected in the test,
n is a constant which is either 1 for one-sided test, or 2 for two-sided test,
p is a constant which is either 2 for time terminated tests, or 0 for failure terminated tests.

The algorithm works in four steps:

1. One failure is assumed and plugged for F. $MTBF$ is the goal life and CI is $1 -$ Consumer's risk.
2. At this step, number of failures is kept the same. Designed MTBF (assumed life with the safety factor implemented) is used for $MTBF$. Using the test time from the previous step, CI is sought. This CI is the producer's risk and it is compared with the intended value. This value should be higher than the expected CI.
3. Steps 1 and 2 are repeated by increasing the number of failures and iterations are stopped when the producer's risk is lower than the sought value. Then, the value from the previous iteration is used for the test duration.

After the determination of the sample size with the algorithm demonstrated in this section, the reliability test should be performed with relevant stress factors and levels explained in the previous chapters. After that, the data analysis can be performed which is explained in the next section with the use of reliability module algorithm.

6.2 Fitting Model to Data

It is important to read the failure data as accurate as possible with least available samples. It is recommended to use at least 4 samples for tests to be accurate but in this study, with the determined failure mechanisms in the previous chapters, it is not possible to allocate 4 samples for every test. Therefore, the accuracy of the tests with less than 4 samples are going to be investigated.

The module "Fit_Everything_ALT" provides the functions for finding the appropriate life-stress model for the data with goodness of fit methods as well as finding the parameters of the model and the distribution it belongs to with maximum likelihood estimation.

The algorithm needs failure data in an array or list, the accelerated stresses in an array or list, if there is, the fraction of the right censored life data, the stresses belong to the censored life data, and the stress level at the normal usage conditions. In this study, the right censored data is not applicable, so the fraction of it is taken as zero, which can be seen in the code below (Reid, 2019):

```
from reliability.Other_functions import make_ALT_data
from reliability.ALT_fitters import Fit_Everything_ALT

ALT_data =
↪ make_ALT_data(distribution='Weibull',life_stress_model='Exponential',
a=7543.14,b=5.19e-8,beta=6,stress_1=[328.16,343.16],number_of_samples=3,
fraction_censored=0,seed=1)

Fit_Everything_ALT(failures=ALT_data.failures,
↪ failure_stress_1=ALT_data.failure_stresses,
↪ use_level_stress=313.16)
```

The piece of code that is used for fitting the randomly generated data for thermal exposure test is shown above. Here, the failure data is created by "make_ALT_data" module but Alva Industries is going to put failure times of the samples in both low accelerated stress level and high accelerated stress level in a single array. Also, for manual filling of the code, the stresses should be repeated to make the array have the same number of elements with the life data array (Reid, 2019). The example below can be followed by Alva Industries, where the numbers are random here:

```
from reliability.ALT_fitters import Fit_Everything_ALT
Fit_Everything_ALT(failures=[200, 231, 233, 500, 438, 568],
↪ failure_stress_1=[200, 200, 200, 120, 120, 120],
↪ use_level_stress=60)
```

The algorithm works as follows:

1. The data is fit to the module "Fit_Weibull_Exponential", which is presented as the Arrhenius-Weibull model in the previous chapters.
   (a) Perform least squares estimation which is accomplished as follows:
      i. Plotting positions are found for the life-stress model of concern, for example, Arrhenius.
      ii. The model is linearized by taking the logarithm, and model parameters are identified on the linearized equation.

iii. Linear regression is performed by taking the squares of errors of the positions with respect to y-axis, which is called RRY. Then, these squares are minimized with taking the derivative and equating to zero.

iv. Model parameters are retrieved again from the optimized fitted line. The gathered parameters are E/k and A.

(b) Divide the failure data into stress groups and fit to the distribution model of concern, such as Weibull, for each stress level. It is fitted by running another least squares estimation and using the information from that estimation by calculating the probability of each data point occurring. Then, the logarithms of the probabilities are summed, and a value arises.

(c) Try optimizers 'TNC', 'L-BFGS-B', 'nelder-mead' is order and stop when one of them returns a value. These optimizers perform this simulation by putting new values each time for the distribution parameter in the range of bounds which are predetermined. The gradient of each value is calculated and the parameter value which returns the minimum gradient is chosen (minimum slope), which means the position that returns zero for the derivative of the distribution function.

(d) Gather the distribution parameters for each fitted distribution and find a common parameter by taking the average. If it is Weibull distribution, β is found with this method. If it is lognormal distribution, σ is found.

(e) The parameters gathered from previous items are put as an input to the maximum likelihood estimation (MLE) to optimize the parameters. This function is performed as follows:

i. Log-likelihood function of the life-stress relationship is fed into the function "loglik_optimizer". An example is "Weibull-Arrhenius". This function performs the goals of the "ALT_MLE_optimization" module. It works as follows:

A. Log-likelihood function is of the life-stress model is taken (Weibull-Arrhenius). It is minimized for each parameter ($E/k, A, \beta$ by taking the derivative and equating the zero. For each minimization, the parameters from the least squares estimation are used.

B. The result of the previous calculation replaces the values in the log-likelihood function until the difference between the previous function and the current function is less than or equal to 0.001 or when the simulation runs for 5 iterations.

ii. This process is repeated for each distribution and life-stress relationship. When a life-stress relationship's log-likelihood gives the highest value, that distribution is selected with the parameters that return the highest log-likelihood.

iii. Using the parameters, the life at the use stress is calculated. This life is taken as the characteristic life parameter of the distribution (α for Weibull distribution and μ for lognormal distribution) where, the mean life of Weibull distribution can be different from 50^{th} percentile. The mean life is calculated by fitting the distribution with the characteristic life and the common parameter of the life-stress relationship.

The next chapters show the implementation of these modules in ALT tests planned for Alva Industries-X60 motor. Also, the interpretation of the results in terms of reliability requirements are discussed.

7 Case Studies

In this chapter, the possible failure modes of the Alva-X60 motor are planned to be stimulated with accelerated life tests that are realized with case studies. Each case study is planned to trigger one failure mode. Therefore, it is not expected to encounter competing failure modes. This chapter refers to steps 4,5,8 in the reliability test framework. The next section shows the required test duration for the sample size allocation of the case studies.

7.1 Test Duration

Determining the sample size is an important step before performing the ALT test because with a planned sample size, the risks are acknowledged. Without the knowledge about the risks, the whole ALT test can be in vain because after the test, it can reveal high risks for the customer, which makes the test results deceptive.

The test sample size is relevant for test planning of success-failure tests. In this study, the scope is on testing to failure. For this test, rather than the sample size, total time on test is important. The sample size can be decided by assigning test duration to samples. For example, longer durations can be designated to low stress units. If they fail before the determined duration, one more sample should be used. It is important to note that same sample size should be allocated to the high accelerated stress levels and low accelerated stress levels.

The test duration is calculated according to the consumer's risk and the producer's risk. This is called the risk control approach. Producer's risk is generated by rejecting a test when it meets the reliability requirements. Consumer's risk is originated from accepting a test while it fails the reliability requirements. Deciding on the consumer's and producer's risk is upon the company. Regarding the confidence level that Alva Industries prefer to demonstrate the reliability (lifetime) of their motors, the consumer's risk is specified as 0.05 (95% confidence level).

The test time and corresponding sample size can be reduced by increasing the producer's risk. There is a trade-off between increasing this risk (decreasing the test time) and not meeting the reliability requirement where it should have been met (false alarms). The producer's risk can push the producer to reject the test and end up with a new design of a product or a new test design, which is uneconomical. To overcome the dilemma, a risk level that is higher than consumer's risk in an acceptable limit is chosen in this study (0.1).

Another factor that should be taken into account is the safety factor that the design engineers use when designing the product. It is mentioned in the previous chapters that safety factor should be implemented while designing the components considering the uncertainties of the reliability tests. It is acknowledged that safety factor can be chosen between 1.2 to 2 for drone motors. Choosing the safety factor higher requires less test time because there is more confidence for the product that it will pass the test. To be on the safe side and to use less resources on testing, safety factor of 2 is assumed to be used for designing. The following code in the reliability

Python library is used to find the test duration with the factors explained above (Reid, 2019):

```
from reliability.Reliability_testing import reliability_test_duration
import matplotlib.pyplot as plt
reliability_test_duration(MTBF_required=1500, MTBF_design=3000,
consumer_risk=0.05, producer_risk=0.1)
plt.show()
```

The code returns the test time and a graph of consumer and producer's risks, which are shown below:

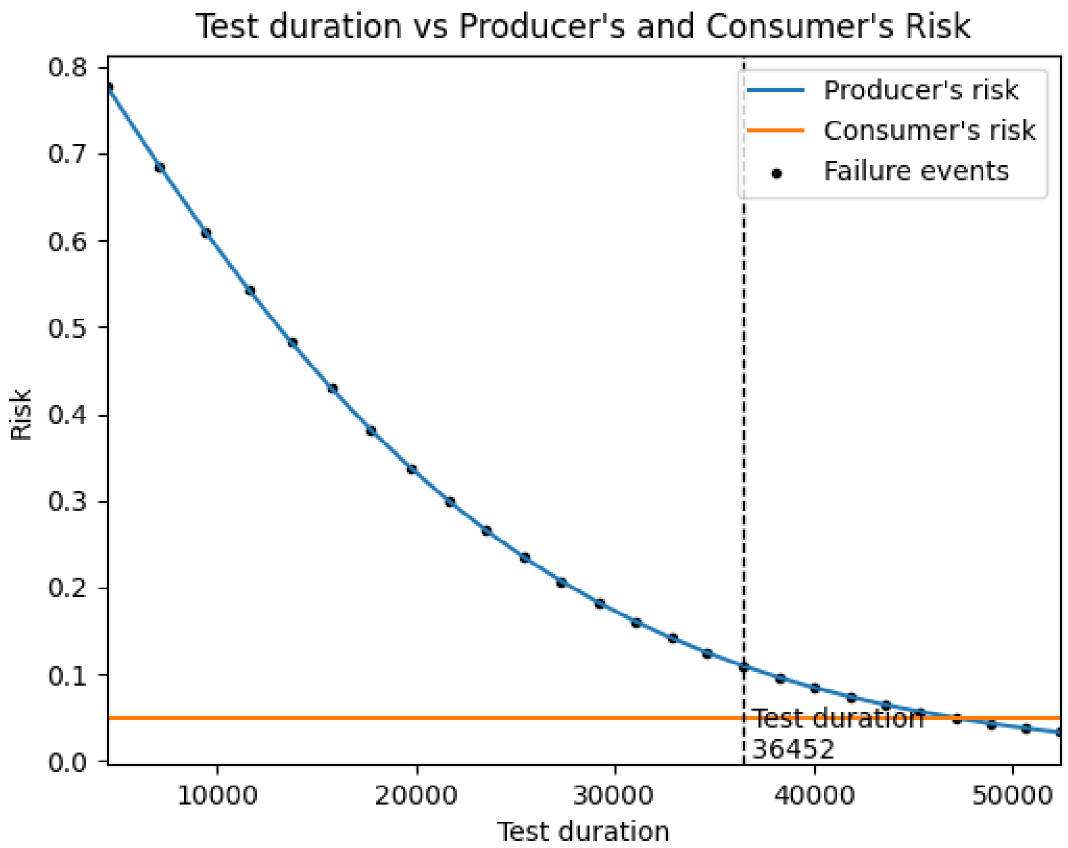

Figure 13: Test Duration, Consumer's Risk and Producer's Risk

The test time is 36452 hours when the goal mean time between failures (MTBF) is chosen as 1500 hours and the designed MTBF is chosen as 3000 hours. Thus, 25 units can be allocated in a non-accelerated test assuming they are going to fail at 1500^{th} hour. For accelerated tests, the effective time on test is $AF \cdot t$ which is the multiplication of acceleration factor with the test time (Wasserman, 2002). Thus, for accelerated reliability tests, the same risk levels can be achieved with less test duration because accelerated reliability tests are designed to emulate the operation conditions in a short amount of time with the use of higher stresses. Therefore, to estimate the required test time, acceleration factor should be estimated.

According to (Modarres, Amiri and Jackson, 2017) the acceleration factors for mechanical units are at least between 2-4 for low stress accelerations and at most 40-50 for high stress accelerations. However, the acceleration factors can only be estimated with experimental work. Therefore, to simulate the similar scenarios with a real ALT test setup, case studies are generated. Here, the acceleration factors can be estimated with assumed values and sample size (test duration) can be determined.

However, the generate the case studies, the sample size should be known. In the following examples, the maximum sample size requirement from Alva Industries is used, where 20 units can be allocated for the ALT tests in total. The 20 units are apportioned between the case studies in a way that the same number of samples are assigned for low and high accelerated stress levels.

7.2 Case Study - 1: Thermal Exposure Failure of the Winding

This example aims to illustrate the failure mode of the motor caused by high temperature exposure on the stator windings. This stress factor is chosen because the temperature upper specification limit of the motor is expected to increase the stator winding temperature to the upper specification limit of the manufacturer. Two stress levels are decided to be used to limit the samples used. Also, there are three repetitions of each level to be able to fit the data to a distribution.

The low and high stress levels are determined by choosing values that are under the upper specification limit of the wire manufacturer because the specification limit and the operating limit is very close ($192°C$ - $200°C$). The low level is selected as $55°C$ which will generate a temperature of $175°C$ on the windings. For high temperature, $70°C$ is chosen which generates $190°C$ temperature on the windings.

After stress levels are determined, the acceleration factors are estimated using Arrhenius model with an estimate of the activation energy of the winding insulation material which is polyurethane. The acceleration factor for low accelerated stress level becomes 3 and 8 for high stress level. This finding is in line with the estimate of Modarres, Amiri and Jackson, 2017.

Then, the acceleration factors are used to estimate the lifetimes of the units allocated for the test. For instance, for the samples that are accelerated with acceleration factor of 3 are assumed have mean failure times close to $1500/3 = 500$ and for the acceleration factor of 8, the mean failure times are assumed to be close to $1500/8 = 187.5$.

Using the "make_ALT_data" module, random failure times are generated assuming they follow Weibull distribution. The module needs the assumed distribution type, life-stress model, the accelerated stress levels in an array, the number of samples for each stress level, an assumed β value, a value which is E/k for Arrhenius model, and b value which corresponds to the constant A value for Arrhenius model. Therefore, these values are fit to the code with the help of acceleration factor, assumed accelerated lifetimes and assumed activation energy.

However, the failure times of real units can follow a different distribution; using "Fit_Everything_ALT" module reveals the distribution of the data and its parameters.

The accelerated stress levels and their generated failure times can be seen in Table 4. The presented stress levels are the values for the environment, so the outside temperature of the test units should be elevated to these levels.

Table 4: Failure of the Stator Winding with Temperature as the Stress Factor

	Temperature	Failure Time
Normal Conditions	$40°C$	Unknown
Accelerated Low Stress	$55\ °C$	309 488 526
Accelerated High Stress	$70\ °C$	117 163 155

The distribution fitting of the generated failure times to find the distribution parameters in the normal operation conditions is presented in the "Results and Discussion" chapter.

7.3 Case Study - 2: Thermal Cycling Failure of Epoxy Lamination

Epoxy around the stator windings can be subjected to rapid temperature changes due to operational stress, load on the drone, and different environmental conditions. Therefore, thermal cycling stress is chosen to be applied to trigger a fatigue failure mode, which can be encountered during operation conditions of the drone motor. It is important to know how much temperature cycling the motor can tolerate, which can be found by the physics behind the failure. The physics behind the failure can be explained with the following equation:

$$\sigma = \Delta\alpha \cdot E \cdot \Delta T \tag{35}$$

where, σ is the thermal stress, E is the elastic modulus, $\Delta\alpha$ is the difference between the thermal expansion coefficient of epoxy and steel (back iron) and ΔT is the temperature change. When this stress is higher than the yield strength of the epoxy, residual stress is left after the stress is removed (thermal cycling is stopped). When the residual stress accumulates to the fatigue strength point, it causes failure of the product. This phenomenon can be explained with the S-N curve (Equation (36)) and power relationship:

$$\sigma^n \cdot N = A \tag{36}$$

where, σ is the stress generated by thermal cycling, A is a constant, N is the total number of cycles to failure and n is the fatigue resistance of the component which is reliant on the frequency and amplitude of stress. The same equation can be expressed with Equation (19).

The normal temperature cycle is chosen from the static reliability test that Alva performed on X60 type motor. The outside temperature of the motor has increased from 20°C to 40°C during a cycle. This difference is taken as a reference to demonstrate the temperature level of the outside of the motor during the ALT test.

By using Equation (35), the thermal cycle level is determined. σ/E is taken as the strain of the epoxy at break (1.5% -2.5%) because there is no information about the elastic properties of the epoxy on its data sheet. At normal thermal cycling, the stress level doesn't reach to the strain values. The accelerated test is designed to fail the units by achieving to the strain levels. The thermal expansion coefficient of epoxy is $100{\cdot}10^{-6}/°C$ - $120{\cdot}10^{-6}/°C$. The thermal expansion coefficient difference between the epoxy and the steel is taken as $100{\cdot}10^{-6}/°C$ with the assumption of the thermal expansion coefficient of epoxy is $120{\cdot}10^{-6}/°C$, where the thermal expansion coefficient of steel is around $20{\cdot}10^{-6}/°C$. Equation (35) is used by putting 1.5% and 2.5% for σ/E. The temperature differences for the epoxy are gathered from this equation.

The outside temperature is found by a linear relationship of the temperature increase rate of the motor housing and the temperature increase rate of the epoxy, which means that the temperature increase of epoxy is 6 times of the motor housing when used in normal operation conditions. For example, when the drone is started at -15°C and used at normal mechanical load that are specified on the data sheet of the motor, the drone housing reaches 5°C whereas, the epoxy reaches 105°C.

To emulate the same condition of starting the drone at cold temperature, the outside temperatures of the motor should reach the levels as in Table 5. The values are found by using the strain of the epoxy and increasing the temperature using the temperature difference, where the outside temperature increases 6 times slower.

The presented stress levels are the values for the environment, therefore, the outside temperature of the test units should be elevated to these levels.

Table 5: Failure of the Epoxy Lamination by Thermal Cycling

	Temperature Range		
	Outside	Inside	Failure Cycle
Normal Conditions	-15°C - (+5°C)	-15°C - (+105°C)	Unknown
Accelerated Low Stress	-15°C - (+10°C)	-15°C - (+135°C)	609 961 1036
Accelerated High Stress	-15°C - (+20°C)	-15°C - (+190°C)	132 175 184

For generating the failure times, 3000 cycles are assumed for the normal life cycle because a normal flight duration is assumed 30 minutes by Alva Industries (2cycle/hr = 3000cycle/1500 hrs). The failure cycles are assumed to follow Weibull distribution with $\beta = 6$ from Weibull properties of common components database (Wasserman, 2002).

"make_ALT_data" module from reliability library is used to generate life data. Addition to β, a and n values are required to start the module. a is the A constant in Equation (19). n is represented with $1/B$ in the same equation. B is chosen as 5 which is the recommended value for polymers so, n is 0.2. a is estimated by putting the expected cycles (3000) in a normal temperature cycle (120 $°C$).

After providing these parameters into the module, it generates failure data on both low accelerated and high accelerated levels which are shown in Table 5.

The interpretation of the failure data in the accelerated conditions to find the distribution parameters of the life data in the normal conditions is discussed in the next chapter.

7.4 Case Study - 3: Wear-out Failure of the Bearing

Best performance of the bearing can be obtained with bearing lubrication. The performance of the lubrication changes with its viscosity and it is affected by the temperature. When high loads are combined with high viscosity of the bearing, failure can occur because of the friction. Friction can cause wear-out at the inner surface of the bearing, this can create vibration and it can speed up the failure process. This case study aims to emulate this scenario with increasing the payload during a cold day. The method is to generate higher thrust on the drone when the viscosity of the lubrication is higher, so that the inner surface of the bearing will be cumulatively damaged.

The lower accelerated stress level is chosen by the following formula:

$$\begin{aligned}\text{Payload Capacity} &= \text{Motor Thrust} \cdot \text{Number of Motors} \cdot \text{Hover Throttle\%} \\ &- \text{The weight of the system}\end{aligned} \tag{37}$$

Motor thrust is chosen to be the maximum thrust force that a single motor can generate, which is 93.9N. Number of motors is chosen as one because this test aims to calculate the payload limit on one motor. Hover throttle is 0.78 which is the ratio of the maximum throttle to the throttle value at hover thrust (37N). The weight of the system is the weight of a single drone motor, which is 3.7N.

With the procedure above, the highest payload that a drone motor can carry is 69N. This is the highest value that the drone can maneuver and hover as with no payload. This value is chosen to be the low accelerated stress. The high accelerated stress is chosen with the payload that the drone cannot maneuver but hover. This value is 90N and calculated with Equation (37) with a hover throttle ratio of 1.

Here, a flight cycle is assumed to be 30 minutes and the accelerated conditions will be generated during this cycle. The failure cycle values are randomly generated with "make_ALT_data" module. The needed parameters are the same with the previous case study. Here, module parameter n corresponds to $-\gamma_1$ of the power model (Equation (18)). It is guessed randomly as 2 and used for estimating the a value. 3.5 is used for β by using the Weibull properties database (Wasserman, 2002). The generated failure times can be seen in Table 6.

Eight samples are allocated to this test differently from the previous examples because 12 out of 20 of the samples are expected to fail during the previous two experiments. Also, four units can be fit to the drone frame to test on a dynamic rig.

Table 6: Failure of Bearing by High Load Cycle

	Thrust Range	Failure Cycle
Normal Conditions	0 N - 37 N	Unknown
Accelerated Low Cycle	0 N - 69 N	380 712 831 944
Accelerated High Stress	0 N - 90 N	237 382 418 779

The fitting of the accelerated stress failure data to a distribution to find the distribution parameters in the normal conditions are presented in the next chapter.

8 Results and Discussion

8.1 Results

The results refer to the steps 8-11 in the reliability test framework. Here, the generated data in the previous sections are analyzed with relevant life-stress relationships and distributions to be fitted in the normal stress conditions.

8.1.1 Test Duration

Starting with the test duration, it is calculated by using the effective accelerated test time, $AF \cdot t$. Acceleration factor is estimated with the estimated constants and accelerated stress levels by using Equations (4) and (21) for Arrhenius and power models. The acceleration factors for low and high stresses for the case studies can be seen in Table 7.

Table 7: Acceleration Factor of the Stresses

	Acceleration Factor	
	Low Stress	High Stress
Case Study 3	3.48	5.92
Case Study 2	3.05	14.55
Case Study 1	3	8.21

With the acceleration factors, the overall acceleration of the tests is found by taking the mean of the acceleration factors, which is 6.20. By using $t = AF/T$ as the test duration, where T is the test duration when the test is not accelerated, t is the accelerated test duration and AF is the acceleration factor, then the required test time is $36452/6 = 5878$ hours. However, the generated test times from the case studies are 5648 hours. This means that two more test samples should have been allocated, instead of one because the number of samples on one stress factor should be the same for low and high accelerated stress levels.

On the other hand, the reason of the difference between the required test duration and obtained one can be the uncertainty of the estimated β. The uncertainty is dependent on the sample size, where the coefficient of variation increases as the sample size decreases, which can be seen with the formula below:

$$\text{Relative Bias} = \frac{(\widehat{\beta} - \beta)}{\beta} \cdot 100 \tag{38}$$

where, β is the value that has put into the simulation and $\widehat{\beta}$ is the value that comes from the maximum likelihood estimation (MLE). The reason behind the relationship of relative bias and the sample size can be explained with MLE, which is more accurate as the sample size increases (Hottinger Bruel Kjaer Inc, 2022b). The reliance of relative bias on the sample size can be explained with the equation below:

$$\text{Average Relative Bias (ARB)} = 1.914n^{-1.032} \tag{39}$$

Since four units are used for each accelerated test, ARB is 46%, which means that the MLE can estimate the $\widehat{\beta}$ with 0.46 times different. The upper and lower β values are calculated as such:

$$\beta_U = \widehat{\beta} e^{\frac{K_a \sqrt{Var(\widehat{\beta})}}{\beta}} \tag{40}$$

$$\beta_L = \widehat{\beta} e^{-\frac{K_a \sqrt{Var(\widehat{\beta})}}{\beta}} \tag{41}$$

where, the average coefficient of variance is calculated as:

$$\text{Average} \frac{\sqrt{Var(\widehat{\beta})}}{\widehat{\beta}} = 1.4818n^{-0.654} \tag{42}$$

It means that the life at the percentiles (Equation (14)) can be guessed with a large variance, which can cause underestimation of the accelerated lifetimes. The variation in β can be seen by running the fitting modules. For example, if Weibull-Eyring model is shown as the best fit for the data, "Fit_Weibull_Eyring" module can be run.

8.1.2 Simulation Results from Case Studies

As discussed before, "Fit_Everything_ALT" module calculates the mean life at normal stress level. The results from the run case studies are as follows:

Table 8: Mean Life Data

Case Study 1	Case Study 2	Case Study 3
1612 hrs	4132·0.5 = 2066hrs	2711·0.5 = 1355.5hrs

The data means that the tests of case study 1 and case study 2 passes the requirement while the test with the payload fails because the mean life is expected to be lower than 1500 hours. However, it is important to note that this result comes from the samples generated, not real data. Therefore, the significance level of the result on the normal population size should be calculated. This can be realized by checking the statistical significance of the difference with Kolmogorov-Smirnov (KS) test.

To run the KS test a null hypothesis should be set. It can set as "the 2000 randomly generated data points with the parameters gathered from the "Fit_Everything_ALT" module comes from the same distribution with a characteristic life of 1500 hrs (3000 cycles)". Scale parameter (shape parameter for Weibull) of the accelerated stress level that is closest to the normal stress can be chosen for the data generated with a characteristic life of 1500 hours. The code below can be followed for case study 3 to understand the significance of the result (Reid, 2019):

```
from reliability.Distributions import Normal_Distribution
from reliability.Reliability_testing import KStest
import matplotlib.pyplot as plt

data = Normal_Distribution(mu=2711.186,
 ↪  sigma=205.41).random_samples(2000)
dist = Normal_Distribution(mu=3000,sigma=211.22)
KStest(distribution=dist, data=data)
plt.show()
```

The "KStest" calculates a critical value by using the significance level (0.05) and the number of data points to find a critical value. Also, by comparing the empirical cumulative distribution (CDF) and the fitted CDF, KS statistic is determined. If this value is lower than the critical value, it is confirmed that the null hypothesis is accepted, otherwise it is rejected.

For the example above, the null hypothesis is rejected with a statistic value of 0.51 and a critical value of 0.03. The difference between the two data plots can be seen in Figure 14.

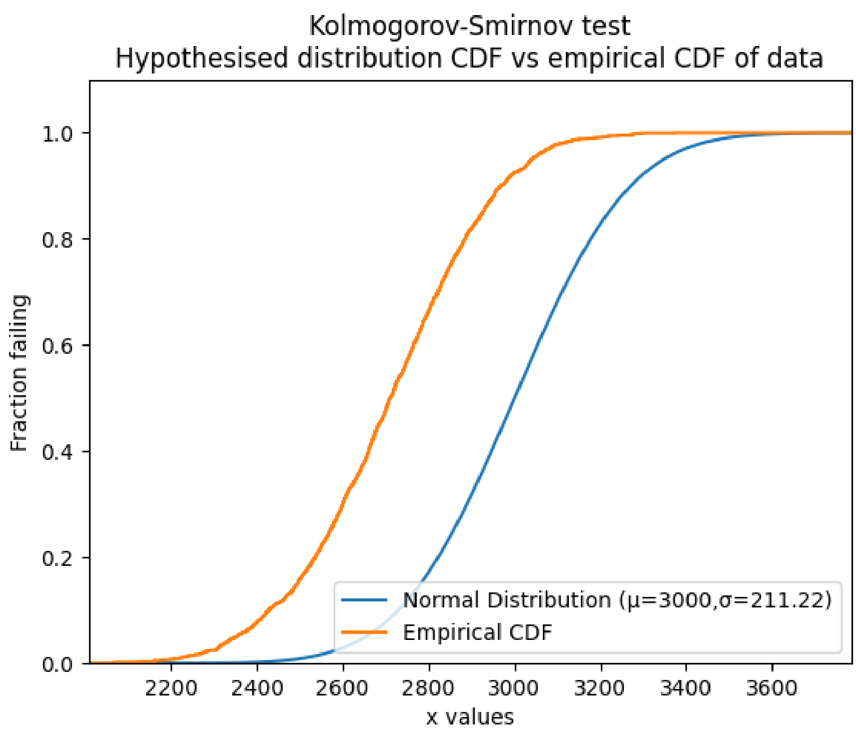

Figure 14: KS Test of Payload Stress Test

The figure can also be used for finding the fraction of 3000 cycles corresponds to on the empirical CDF. It can be seen that the value 3000 corresponds to 0.92^{th} quantile on the empirical CDF, which means that 92% of the units in the population (1840 units) are expected to fail by 1500 hours.

Considering the drone motor system as a series structure, where failure of a subsystem causes the system to fail, rejection of the null hypothesis of "mean life of the motor is 1500 hours" for a subsystem causes the rejection of the null hypothesis for the whole system. Therefore, it can be concluded that the mean life of the Alva X60 motor under normal operation conditions is 1355.5 hours, which is the mean life of the bearing subsystem.

On the other hand, the MTBF is different from the mean life. For instance, under normal conditions, while one product is observed until the second bearing failure, which is observing for 2711 hours, there will be three failures. First failure will be at 1355^{th} hour, after replacing the bearing, the second failure will be at 1612^{th} hour, after replacing the stator, the third will be at 2711^{th} hour, which is the second bearing replacement. Using Equation 1, the MTBF is found as $2711/3 = 904$hours.

Rather than explaining the lifetime of the product with MTBF, the reliability at the desired life can be found using the survival function because the mean life can imply the failure of 20%-90% percentile of the population depending on the type of distribution that the population failure represents. The reliability at a certain position on CDF is calculated as shown below (Reid, 2019):

For Case Study 3:

```python
from reliability.Distributions import Normal_Distribution
print(1-Normal_Distribution(mu=2711.186, sigma=205.41).CDF(3000))
```

For Case Study 2:

```python
from reliability.Distributions import Weibull_Distribution
print(1-Weibull_Distribution(alpha=4131.95, beta=7.644).CDF(3000))
```

For Case Study 1:

```python
from reliability.Distributions import Weibull_Distribution
print(1-Weibull_Distribution(alpha=1611.79, beta=7.643).CDF(1500))
```

The results are 0.08 for case study 3, 0.92 for case study 2, 0.56 for case study 1. Since the system is shown as a series structure with all the basic units by Figure 1 and Table 1, and the reliability tests in the case studies trigger the failure modes of different basic units, the reliability can be shown as:

$$R(1500\text{hrs}) = R_{TE}(1500) \cdot R_{TC}(3000) \cdot R_P(3000) = 0.04 = 4\% \tag{43}$$

So, the reliability at 1500^{th} hour is 4% according to the data of the case studies. It should be remembered that not all the components in Table 1 are tested in this study but the reliability is dependent on the weakest chain of the system, where high reliabilities of other components won't affect this result significantly. Therefore, the hypotheses of 1500 hours as the MTBF or demonstrating a reliability of at least 50% at 1500^{th} hour has failed.

8.2 Discussion

Following the steps in the reliability test framework, reliability requirements are defined, accelerated reliability tests are planned and data analysis of the tests are performed assuming the planned ALT tests (case studies) are conducted. There are expected and unexpected results originated from the data analysis part of the test framework. The expected results come from the estimated mean life of the data in normal conditions for case studies 1 and 2. The unexpected results are the distribution and life-stress estimations of the case studies, the mean lifetime of the data from case study 3 on normal conditions, and the type of failure mode that brings the lowest mean lifetime estimation, which is the wear-out of the bearing. This failure mode isn't expected to generate the lowest mean lifetime because more samples are allocated to that case study. The possible reasons for the unexpected results are discussed in this section.

It should be noted that the distribution fitting and the parameter estimation depends on the test design. It also means that it is as accurate as the correctly chosen stress levels and allocated samples. Taking the case studies into account, after generating random samples with a distribution and seeking the right distribution to explain the random samples, the fitter returned a different distribution and/or a different life-stress relationship than the default one. It was the case for the case study 3; the fitter suggested Normal-Eyring model, where it should have been explained with Weibull-Power model. For the other two case studies, only the life-stress relationships were incorrect, which are mistaken with Eyring model again instead of Exponential (Arrhenius).

The reason for these errors is the inadequate number of sample sizes. Graphically thinking, since there are only three samples for a stress level and they are tried to be fit to a straight line, it can be fit to any distribution easily. Analytically, the distribution and parameters are chosen by maximizing the log-likelihood function; the distribution and the parameters that maximizes this function is chosen as the best fit. Since there are not enough data points to fit, too many parameters for a distribution can be tried in the fitter because, small samples can be fit to almost every distribution. Therefore, the simulation only fits the data with changing the parameters limitlessly between the bounds. The stopping criteria is the number of iterations, which is 5, or the variance between the current and the previous log-likelihood function value. Since it is unknown which stopping criteria converged into the selected parameters, it could be still an immature representation of the data.

A factor that is affecting the precision of the results is the goodness of fit test used to rank the best match of distributions. Bayesian Information Matrix (BIC) is used for fitters in the reliability library. This test is useful in terms of fitting to a linear model because it tends to simplify the fit. It is also better in excluding the outliers but can be worse in noticing the competing failure modes.

Ability to reject the outlier may not be a good attribute in every situation. For some conditions, it can lead to oversimplifying of the model, which is the matter for case study 3. It fits the data to Normal-Eyring model where, it should have been fit to Weibull-Power because of the closeness of the data points when fit to the Normal-Eyring model. On the contrary, the data looks close even between the stress levels with that model, which isn't supposed to be that way. Weibull-Power model is rejected because there is a noticeable difference between the fitted line and empirical line. The difference between these lines can be because of the optimizer chosen, where the default optimizer is used instead of trying with every optimizer and selecting the best fitted one. The difference between these models can be seen on Figures 15 and 16.

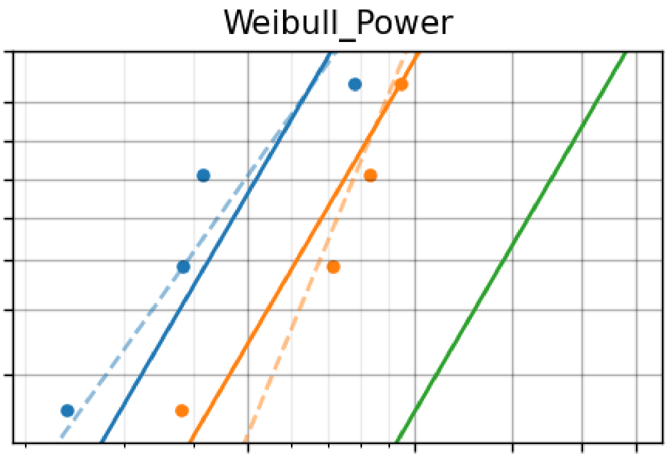

Figure 15: Weibull-Power Fit of the Case Study 3

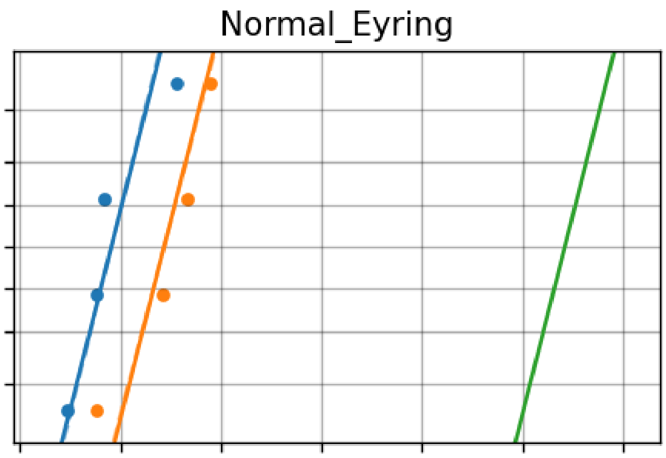

Figure 16: Normal-Eyring Fit of the Case Study 3

Another reason of nonparallel fit data can be because of a different failure mode at the high accelerated stress level. When the stress level is too high that doesn't follow any trend or that triggers the other components of the system to fail, fitted lines don't appear to be parallel. However, it is difficult to distinguish the competing failure modes for the case studies mentioned in this report because there are only two accelerated stress levels. If there were three or more accelerated stress levels, the incompatible stress level could be identified by checking if its scale parameter (shape parameter for Weibull distribution) is different from others.

To conclude, accelerated reliability test planning and data analysis with small sample sizes such as three and four for a stress level can give inaccurate results because of the variation of the common parameter of the distribution that the data belongs to, which is dependent on the sample size. Also, there is the contribution of the uncertainty of fitting the data to a distribution, where maximum likelihood estimation (MLE) is used. Since the accuracy of the MLE also depends on the sample size, the error of the results can be quite high. Therefore, a sample size that is more than the sample sizes allocated in this study should be used for design of experiment of a single stress.

9 Conclusion

In this study, the need of adaptive reliability tests for drone motors is handled. With the pursuit of this problem, the basics of designing experiments and performing data analysis of hypothetical case-studies with assumed failure modes are presented in a framework. This study has found solutions to estimating the sample size for reliability tests without prior knowledge and demonstrated the outcome of small sample sizes with data analysis of three case studies. The data analysis of the case studies revealed the lifetime and the distribution parameters of the Alva X60 motor at normal operation conditions. Analytic data analysis method is used for fitting the data to a distribution and gathering the parameters with a Python library called reliability. Graphical method is utilized while the discussion of the data and parameters to acquire a better observation of the data.

When the data analysis considered, it revealed significant variations between the lifetime of the case studies, which was a result of small sample sizes allocated for each stress levels. Also, the solution for the estimation of the sample size was estimating the test duration, where sample size could be found implicitly. This solution gave reasonable estimates of the test duration, but as a result it revealed that two more samples should be allocated for the ALT tests overall.

In conclusion, Alva Industries can utilize the case studies and theories in this study as a framework, realize the experiments with allocating more sample sizes for each test and continue with the further work.

9.1 Further Work

The design of experiments can continue with the acceleration of electrical stresses, impact stress and vibration stress (sinusoidal stress) after gaining more information about the vibration characteristics of the design during operation in normal environmental conditions. The data analysis methods in this study can be used for interpreting the results and further module in the reliability library of Python can be utilized. After the critical tests are finalized, performance tests can be conducted with accelerated degradation tests.

Bibliography

Allen, Christopher S et al. (2018). 'Spaceflight environment'. In: *Space Safety and Human Performance*. Elsevier, pp. 87–138.

Alva Industries AS (2022). *X60*. URL: https://www.alvaindustries.com/product/x60 (visited on 26/05/2022).

Australian Bureau of Statistics (2022). *Statistical Language - Measures of Central Tendency*. URL: https://www.abs.gov.au/websitedbs/D3310114.nsf/Home/Statistical+Language+-+measures+of+central+tendency (visited on 07/06/2022).

Chen, Wen-Hua et al. (2018). 'Design of accelerated life test plans—Overview and prospect'. In: *Chinese Journal of Mechanical Engineering* 31.1, pp. 1–15.

Dohi, Tadashi and Toshio Nakagawa (2013). *Stochastic reliability and maintenance modeling*. Springer.

Elektrisola (2022). *Litz Wire*. URL: https://www.elektrisola.com/en-us/Litz-Wire/Info#design-and-calculation (visited on 26/05/2022).

Guo, Huairui, Edward Pohl and Athanasios Gerokostopoulos (2013). 'Determining the Right Sample Size for Your Test: Theory and Application'. In: *2013 Annual RELIABILITY and MAINTAINABILITY Symposium*.

Holbert, Keith E, Kang Lin and George G Karady (2006). 'Enhancement of electric motor reliability through condition monitoring'. In: *IFAC Proceedings Volumes* 39.7, pp. 255–260.

Hottinger Bruel Kjaer Inc (2022a). *Characteristics of the Weibull Distribution*. URL: https://www.weibull.com/hotwire/issue14/relbasics14.htm (visited on 25/05/2022).

— (2022b). *Determining the Sample Size for a Life Test Based on the Shape Parameter of the Weibull Distribution*. URL: https://www.weibull.com/hotwire/issue126/hottopics126.htm (visited on 25/05/2022).

— (2022c). *MTTF, MTBF, Mean Time Between Replacements and MTBF with Scheduled Replacements*. URL: https://www.weibull.com/hotwire/issue94/relbasics94.htm (visited on 20/05/2022).

James, MN et al. (2007). 'Residual stresses and fatigue performance'. In: *Engineering Failure Analysis* 14.2, pp. 384–395.

Liu, Yiliu (2021). *TPK 5165 – RAMS Engineering and Management - Lecture 7*. University Lecture.

Mert, Cagla (2021). 'Design of Experiment for Accelerated Environmental Testing of Electric Motors for Aerial Applications'.

Modarres, Mohammad, Mehdi Amiri and Christopher Jackson (2017). *Probabilistic physics of failure approach to reliability: modeling, accelerated testing, prognosis and reliability assessment*. John Wiley & Sons.

Nelson, Wayne B (2009). *Accelerated testing: statistical models, test plans, and data analysis*. John Wiley & Sons.

Reid, Matthew (2019). *Reliability A Pyhton Library for Reliability Engineering*. URL: https://reliability.readthedocs.io/en/latest/index.html (visited on 15/05/2022).

Sarkar, Bijit Kumar (1995). 'Fatigue of brittle materials—a critical appraisal'. In: *Bulletin of Materials Science* 18.6, pp. 755–772.

SKF (2022). *Selecting a suitable grease.* URL: https://www.skf.com/group/products/rolling-bearings/principles-of-rolling-bearing-selection/bearing-selection-process/lubrication/selecting-a-suitable-grease (visited on 24/05/2022).

Stephens, Matthew P (2010). *Productivity and reliability-based maintenance management.* Purdue University Press.

Wang, Shengguo et al. (2002). 'Crack resistance test of epoxy resins under thermal shock'. In: *Polymer testing* 21.2, pp. 195–199.

Wasserman, Gary (2002). *Reliability verification, testing, and analysis in engineering design.* Marcel Dekker, Inc.

Yadav, Shubham, Monika Sharma and Abhishek Borad (2017). 'Thrust Efficiency of Drones (Quad Copter) With Different Propellers and Their Payload Capacity'. In: *International Journal of Aerospace and Mechanical Engineering* 4.2.

Yin, Shen (2021). *Diagnostic - Prognostic - Maintenance – Safety Analysis.* University Lecture.

Appendix

A Electrical Stress Factors for Future Use

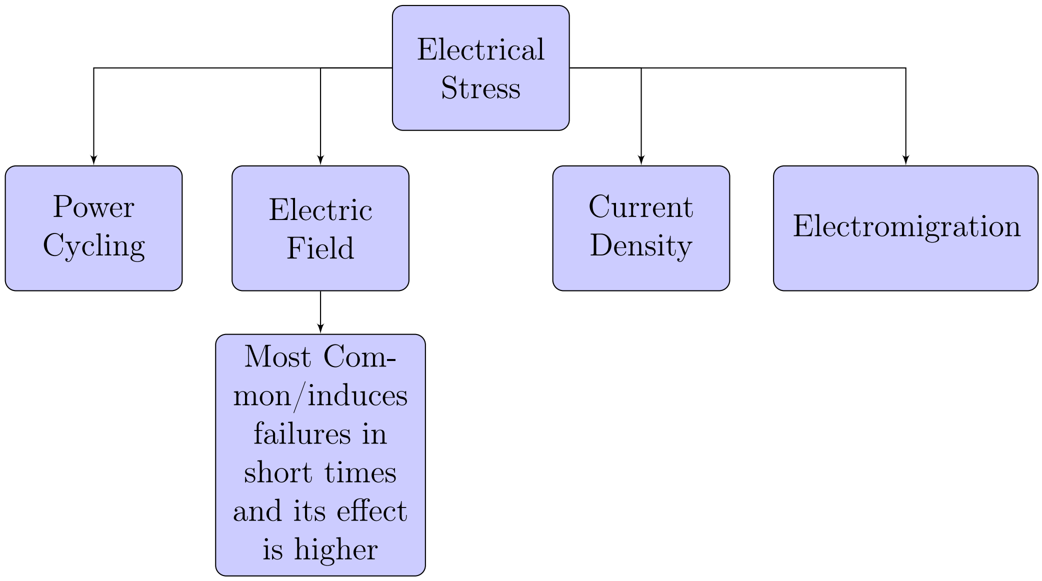

Figure 17: Electrical Stress Types and Frequency of Use in Industry (Dohi and Nakagawa, 2013)

B Failure Modes and Stresses for Future Use

Table 9: Failure Modes, Related Components and Stresses

(Wasserman, 2002)

Failure Mode	**Acceleration Stresses**	**Related Products**
Corrosion Oxidation Rusting	Temperature, Humidity, Voltage, Residual Stress	Oxidation of metal surfaces, electrical connections etc.
Creep and Creep Rupture	Mechanical stress, temperature	Plastics, welds, bonds, joints, lubricants etc.
Diffusion	Temperature, concentration gradient	plastics, lubricants etc.
Electromigration	Current density, temperature, temperature gradient	Electronic Circuits
Fatigue	Cyclic mechanical stress, cyclic temperature, frequency of usage	Metals, plastics, composite materials
Wear	Contact force, relative sliding velocity, temperature, lack of lubrication	Solid surfaces in contact (for plastics and metals), coatings

C Guidance for the Test Engineer at Alva Industries

1. Refer to section 2.1 for to find what is needed to start.
2. If the planned tests in this study is going to be performed, execute case studies one by one.
3. For case study 1, apply accelerated low stress level to the environment and put three units to test until they fail and repeat the same procedure for the accelerated high stress level.
4. For case study 2, apply the accelerated low stress cycle to the environment, which is referred as outside. 3 units should be tested at that level. Each cycle should be 30 minutes. Run the motors until they fail.
5. Repeat the previous step for accelerated high stress cycle.
6. For case study 3, put four motors into the test for accelerated low cycle. The thrust range means that the motor should reach to the final level as fast as possible and run for 30 minutes. The units should repeat this procedure until they fail.
7. The same procedure is repeated for the accelerated high stress level.
8. After getting the results of the case studies, the data should be fit to the "Fit_Everything_ALT" module as explained in section 6.2 for each case study.
9. The found mean lives for each case study should be put to Equation 1 as explained in section 8.1.2. The found solution is the mean time between failures.

However, it is recommended to read the report because the significance of the results are planning new experiment designs are explained.