

Xiaolun Xu

Collision avoidance for multiple autonomous surface vehicles utilizing information exchange

A centralized optimization based approach

Master's thesis in Marine Technology

Supervisor: Dong Trong Nguyen

Co-supervisor: Melih Akdag, Tom Arne Pedersen

June 2022

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Norwegian University of Science and Technology
Faculty of Engineering
Department of Marine Technology



MSC THESIS DESCRIPTION SHEET

Name of the candidate: Xiaolun Xu

Field of study: Marine control engineering

Thesis title (English): Collision avoidance for multiple autonomous surface vehicles utilizing information exchange - A centralized optimization-based approach

Background

Ship collision is the main and most hazardous accident which can cause huge losses. In congested waterways, narrow channels or seaports, multiple ships encountering situations are highly frequent, which increases the collision risk. Vessel traffic services (VTS) are responsible for the pilotage, sending hazard warnings and giving advice to vessels in the regulated areas. To assist the vessels' anti-collision actions, a collision-avoidance optimization algorithm can be developed for VTS considering both other vessels' intention and navigation rules from a centralized perspective.

Work description

1. Perform a background and literature review to provide information and relevant references on:

- Ship motion prediction.
- Collision risk indices.
- Path planning.
- Swarm intelligence optimization.
- Vessel Traffic Services.

Write a list with abbreviations and definitions of terms, explaining relevant concepts related to the literature study and project assignment.

2. Build guidance and control systems for multiple vessels in Simulink.
3. Find and implement proper methods for motion prediction and collision risk indices assessment.
4. Model the optimization problem based on rolling horizon optimization approach and path planning considering other vessels' intention.
5. Solve the optimization problem based on swarm intelligence optimization algorithm.
6. Perform verification by simulations (several scenarios regarding head on, crossing and overtaking).
7. Tentative: Consider static obstacles and non-cooperative vessels.

Specifications

The scope of work may prove to be larger than initially anticipated. By the approval from the supervisor, described topics may be deleted or reduced in extent without consequences with regard to grading.

The candidate shall present personal contribution to the resolution of problems within the scope of work. Theories and conclusions should be based on mathematical derivations and logic reasoning identifying the various steps in the deduction.

The report shall be organized in a logical structure to give a clear exposition of background, results, assessments, and conclusions. The text should be brief and to the point, with a clear language. Rigorous mathematical deductions and illustrating figures are preferred over lengthy textual descriptions. The report shall have font size 11 pts., and it is not expected to be longer than 60-80 A4 pages, from introduction to conclusion, unless otherwise agreed upon. It shall be written in English (preferably US) and contain the following elements: Title page, abstract, acknowledgements, thesis specification, list of symbols and acronyms, table of contents, introduction with objective, background, and scope and delimitations, main body with problem formulations, derivations/developments and results, conclusions with recommendations for further work, references, and optional appendices. All figures, tables, and equations shall be numerated. The

original contribution of the candidate and material taken from other sources shall be clearly identified. Work from other sources shall be properly acknowledged using quotations and a Harvard citation style (e.g. *natbib* Latex package). The work is expected to be conducted in an honest and ethical manner, without any sort of plagiarism and misconduct. Such practice is taken very seriously by the university and will have consequences. NTNU can use the results freely in research and teaching by proper referencing, unless otherwise agreed upon.

The thesis shall be submitted with a printed and electronic copy to the main supervisor, with the printed copy signed by the candidate. The final revised version of this thesis description must be included. The report must be submitted according to NTNU procedures. Computer code, pictures, videos, data series, and a PDF version of the report shall be included electronically with all submitted versions.

Start date: 1 January 2022 **Due date:** As specified by the administration.

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DEPARTMENT OF MARINE TECHNOLOGY

TMR4930 - MARINE TECHNOLOGY
MASTER'S THESIS

**Collision avoidance for multiple
autonomous surface vehicles
utilizing information exchange**

A centralized optimization based approach

Author:
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Co-supervisors: Melih Akdag (NTNU), Tom Arne Pedersen (DNV)

June, 2022

Abstract

Centralized collision avoidance algorithms for multiple autonomous surface vehicles are frequently proposed, but most of them assume ideal condition. However, vessels' non-collaboration and non-compliance motion in Convention on the International Regulations for Preventing Collisions at Sea (COLREGS) may happen in real maritime world. This thesis aims to find the optimal maneuvering in both ideal and non-ideal conditions. A centralized optimization system utilizing information exchange is developed and implemented for multiple vessels encountering. Particle swarm optimization method is used for solving the optimization problem. Simulation tests are performed to validate the effectiveness and robustness of the proposed optimization system.

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Abbreviations

AIS Automatic Identification System

ASV Autonomous Surface Vehicles

BFGS Broyden-Fletcher-Goldfarb-Shanno

COLREGs Convention on the International Regulations for Preventing Collisions
at sea

CRI Collision Risk Index

DCPA Distance at Closest Point of Approach

DLSA Distributed Local Search Algorithm

DOF Degrees of Freedom

DSSA Distributed Stochastic Search Algorithm

DTSA Distributed Tabu Search Algorithm

HMI Human-Machine-Interface

IALA International Association of Lighthouse Authorities

IMO International Maritime Organization

KKT Karush-Kuhn-Tucker

LOS Line-of-Sight

MPC Model Predictive Controller

NED North-East-Down

PSO Particle Swarm Optimization

PSV Platform Supply Vessel

SOLAS International Convention for the Safety of Life at Sea

STM Sea Traffic Management

TCPA Time at Closest Point of Approach

VDES VHF Data Exchange System

VHF Very High Frequency

VTS Vessel Traffic Services

1 Introduction

1.1 Background

The ship industry is developing continuously and the traffic is becoming congested in maritime waterways, including international commercial waters, ports, channels and inland waterways. The growing of the numbers of ships increases the risk of maritime accidents, especially collision between ships. Ship collision can cause huge losses on human lives and economy, thus it is an imperative task to handle for navigation safety. Many maritime accident investigations show that 75-96% of marine accidents are caused by or related to human errors (Zhang et al., 2013). Therefore, developing Autonomous Surface Vehicles (ASV) is an effective way to decrease the possibility of collision caused by human. ASV also could release the burden of vessel operators and reduce greenhouse gas emission. An example is that *YARA Birkeland* has been in commercial operation with only five crews in Norway, which aims to be the world's first fully electric and autonomous ship and could "reduce noise and dust emissions, improve the safety of local roads, and reduce NO_X and CO_2 emission", said Svein Tore Holsether, CEO of Yara (Skredderberget, 2018).

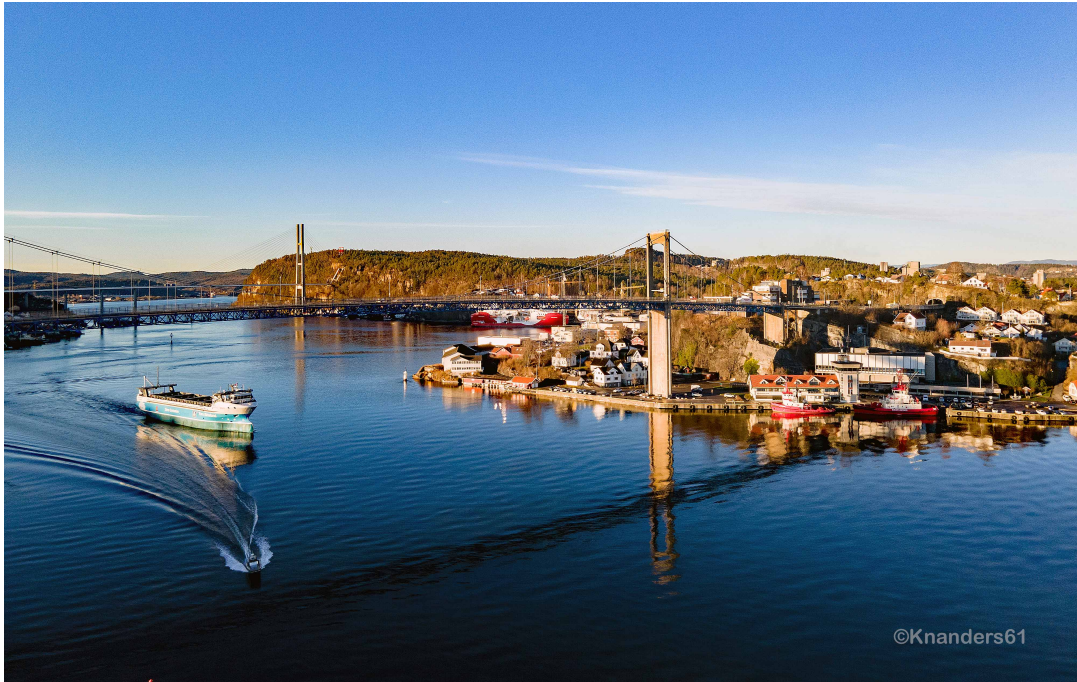


Figure 1: *Yara Birkeland*, the world's first electric autonomous ship navigating in Oslo fjord (YARA, 2022).

Relling et al. (2019) pointed out a future with autonomous vessels will change the maritime industry not only in itself, but also the way it interacts with others in maritime traffic systems such as Vessel Traffic Services (VTS) centre. International

Maritime Organization (IMO) describes VTS as any service implemented by a competent authority, designed to improve safety and efficiency of maritime traffic (IMO, 1985). A VTS centre is responsible for the pilotage, sending hazard warnings and giving advice to vessels in the regulated areas. For Norwegian waters, the Norwegian Coastal Administration executes the responsibility for VTS, and there are five VTS centres, four responsible for territorial waters and one for international waters. The future VTS will face the challenges of the interaction between the autonomous vessels and VTS centre as well as safety regarding the co-existence of autonomous and conventional vessels (Relling et al., 2019). In high density VTS-dominate area, multiple vessels encounter happen frequently, and hence collision avoidance problem should attract more attention.

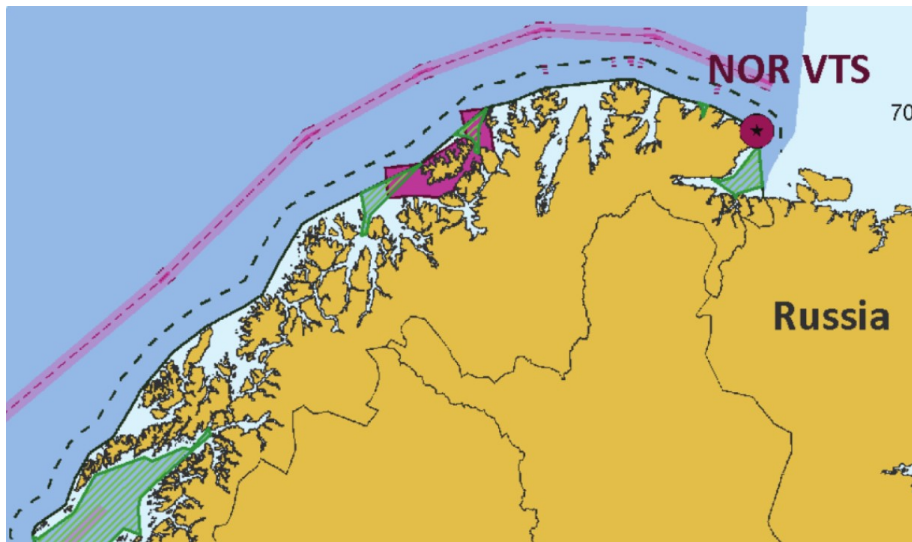


Figure 2: Northern Norwegian coastline and the Vardø VTS (shown with its call signal NOR VTS). (Bye and Schaathun, 2015)

1.2 Information Exchange Channels in Maritime Domain

Following the IMO International Convention for the Safety of Life at Sea (SOLAS) Regulation, it requires that Automatic Identification System (AIS) class A transceivers to be carried on-board for all vessels of 300 gross tonnage and upwards engaged on international voyages and all passenger ships irrespective of size (IMO, 1974). Non-SOLAS recreational small vessels carry the AIS class B transceivers, which provides limited functionality and usually silences itself to not interfere class A receivers in congested waters. Systems based on AIS technology can support vessel-to-vessel and vessel-to-shore communication and are becoming more and more important in ensuring navigation safety and preventing vessels collision (Ho et al., 2018).

The main purpose of AIS is to compile and transmit vessel's static, dynamic and voyage related information to the other vessels and VTS centre by using Very High Frequency (VHF) band. Some critical information like vessel identity, position,

course and speed are included and used for assisting collision avoidance. Shore side VTS centre can use such information to monitor vessels motion in its authority region, and can also use AIS channel to send located weather information. However, false data and information errors may be contained due to lack of integrity checks of AIS messages (Akdağ et al., 2022).

Nowadays the demand for information communication is increasing and the VHF channel is overloaded in crowded areas. As an improvement of AIS, VHF Data Exchange System (VDES) has been developed, which can support two way data transfer between vessels and has high confidence of data reception by additional integrity check. Other new techniques like high-speed broadband terrestrial networks are also investigated to transfer more data over long ranges and apply in marine domain such as remote control and monitoring of ASV (Akdağ et al., 2022).

To integrate existing and new navigational tools together, the International Association of Lighthouse Authorities (IALA) and IMO have adopted e-navigation strategy to enhance navigation safety and commercial efficiency (Patraiko, 2007). Through e-navigation projects, route exchange is explored that each vessel could create and share waypoints as their intention of navigation. Two types of route exchange are strategic and tactical route exchange. Strategic route exchange means that vessels share all of their waypoints in the navigation plan with the shore authority center, while tactical route exchange shares only a few of waypoints in vessel-to-vessel communication for safety purpose (Porathe et al., 2015).

1.3 Literature Review on Multi-vessel Collision Avoidance

Many studies have been investigated on automatic collision avoidance, but most of them are only available to a single vessel-to-vessel encounter. The existing methods for multi-vessel's collision avoidance can be divided in two groups, decentralized and centralized. Decentralized or distributed methods always label the self-ship as own-ship and the encounter-ship as target-ship and consider vessel-to-vessel communication. While centralized methods consider all the vessels from a global perspective, and every individual vessel will communicate with a central control station.

1.3.1 Decentralized Methods

Decentralized methods assume that encountering vessels can communicate and share intention with each other and reach a consensus for their collision-free actions by negotiation (Szlapczynska, 2015). The collision avoidance problem can be solved by individual vessel locally without any help from outer party.

Hu et al. (2006) proposed a two-vessel negotiation framework which enable vessels to negotiate with each other and optimize the cost of collision avoidance in the high-

cost situations at open sea. Later, Liu et al. (2008) developed a negotiation-based multi-vessel planning algorithm, where the negotiation aims to find a joint decision and reach each vessel's individual goal. The negotiation process considers an initiator to form the strategy and the rest respond on the proposal in each cycle. However, no simulation tests are presented to validate the performance in the research.

Kim et al. (2014) and Kim et al. (2015) used Distributed Local Search Algorithm (DLSA) and Distributed Tabu Search Algorithm (DTSA) to calculate the distributed solution, where each vessel's course alteration is ranked by its proposed course to reducing the collision risk. As an extension, Kim et al. (2017) introduced a Distributed Stochastic Search Algorithm (DSSA) approach, which allows a simultaneously stochastic intention exchange for eliminating quasi-local minimum. Li and Liu (2019) proposed a distributed coordination mechanism for multi-vessels encounter, where the collision avoidance plan with optimal rudder angles and the rudder steering time are found by distributed constraint optimization. The communication and computation costs are evaluated through simulations. Huang et al. (2020) investigated a Human-Machine-Interface (HMI) oriented collision avoidance system, which can visualize the solution space of collision avoidance to human operators. Human operators can also take over the autonomous vessels and modify the rule-violation behaviour.

1.3.2 Centralized Methods

IMO (1985) suggested that a VTS centre is particularly suitable in high traffic density areas and narrow channels and has the authority to organize the vessels movements. The VTS-oriented centralized methods assign a master unit to make decision and assist all the collaborating vessels finding their optimal collision-free action (Akdağ et al., 2022).

Tam and Bucknall (2013) proposed a cooperative deterministic path planning algorithm to obtain a centralized solution for the collision avoidance. To solve the collision-free trajectory problem, all the vessels are ranked based on their priority in Convention on the International Regulations for Preventing Collisions at sea (COLREGs) as well as maneuverability. The priority in COLREGs means that a stand-on vessel can stay on its original path, while a give-way vessel has to change its navigation plan. When considering the limits of COLREGs rules in multi-vessel encounter, vessel's maneuverability is also an important aspect where larger vessels or vessels with lower capability in steering will be given more priority in keeping their courses. Their proposed collision avoidance algorithm finds the collision-free trajectories by the iteration process which deals with the vessel with the highest priority first, and then proceeds to the vessels with lower priority. Several simulations are performed to validate the algorithm whereas the ideal assumption that the vessels are collaborative has been made. Figure 3 illustrates the process of proposed algorithm.

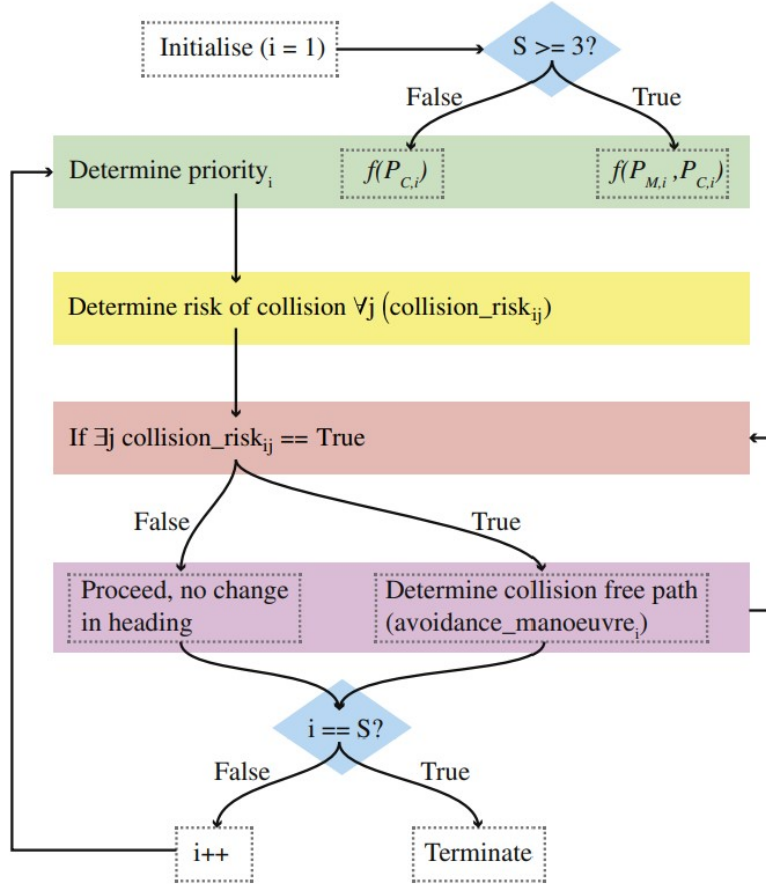


Figure 3: Flow chart of a deterministic cooperative trajectory planning algorithm (Tam and Bucknall, 2013)

Szlapczynska (2015) proposed a semi-distributed maneuver auto-negotiation system based on the centralized collaborative methods. To be independent of the central control station, the collision avoidance problem in the open sea encounter can be solved by assigning a leader ship and considering the others as participants. The assigned leader ship is responsible for data collection, determination and optimization and finally distributing the collision-free trajectories to the others through communication systems. However, no relevant simulation results are performed in the article.

A rolling horizon optimization approach is proposed by Li et al. (2019), which aims at speeding up the optimization process through diving continuous time into a set of discrete time slots and thus reducing the whole computational complexity. This is a meaningful method since the centralized optimization will suffer the increasing of variables' dimension when considering complex multi-vessels encounter. Figure 4 presents the time horizon regarding the optimization procedure. The overall collision avoidance can be splitting into a series of smaller optimization problems which will be carried at every time slot. In each time slot, the future states of vessels can be predicted with a prediction horizon by using vessels current states and vessel maneuverability models. A novel objective of this rolling horizon optimization

problem is to minimize the heading angle deviations and time costs for collision avoidance maneuvering instead of maximizing the safety. Several simulation tests show good performance of proposed rolling horizon approach in complex dynamic scenarios, whereas they also make ideal assumption that all vessels can follow the solution.

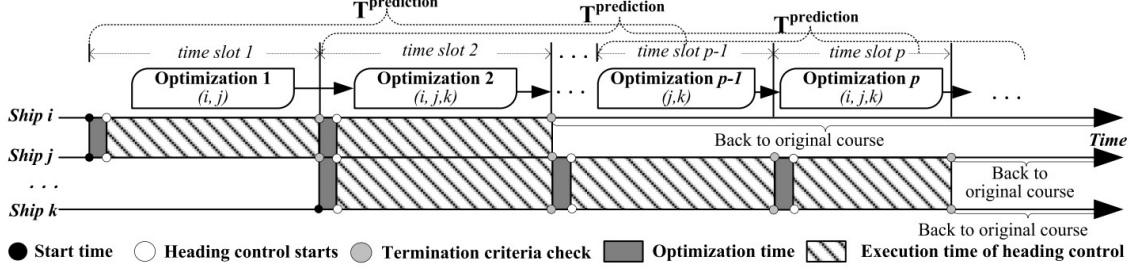


Figure 4: Time horizon regarding the optimization procedure (Li et al., 2019)

Papadimitrakis et al. (2021) introduced a centralized multi-ship Model Predictive Controller (MPC) to compute optimal evasive control actions. The motion prediction of the controlled vessels is based on vessels' course model, while the obstacle trajectory is generated from the offline AIS data-driven model. The objective of the multi-ship MPC controller is to generate risk-free and efficient trajectories under model uncertainties, and an emergency constraint as well as COLREGs non-compliance penalties are considered. By solving the constrained optimization problem, the optimal trajectories are calculated for multiple steps ahead and execute in every controller timestep. Figure 5 shows the framework of the proposed MPC in the online process combined with an offline trajectory prediction. A case study is performed at the Miami port area and better performance is achieved by comparing with the normal MPC with straight-line obstacle prediction.

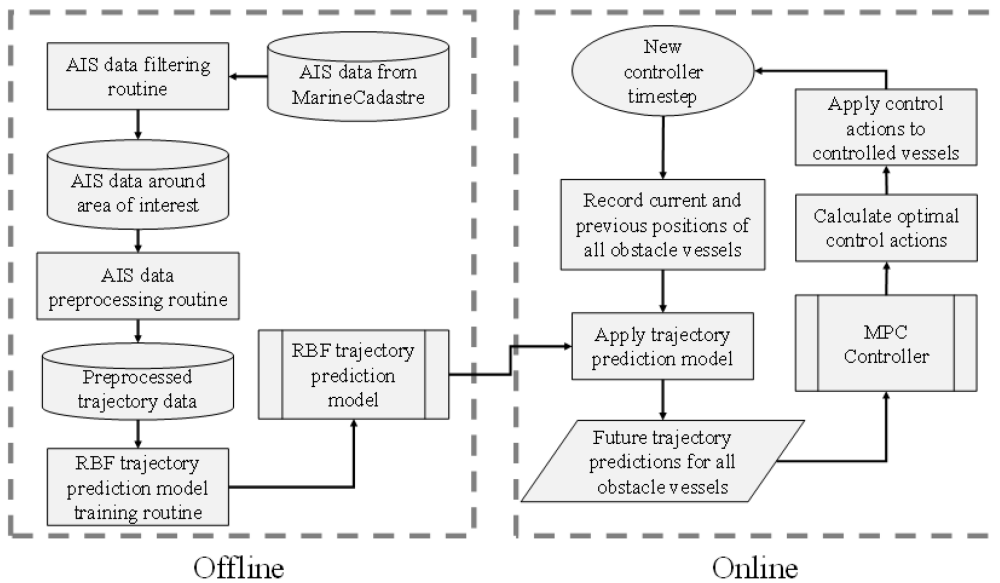


Figure 5: Control framework of multi-ship MPC (Papadimitrakis et al., 2021)

1.4 Motivation and Contributions

Nowadays, there are more and more discussions like Sea Traffic Management (STM) route exchange projects about giving larger authority to VTS centre, which could better organize the traffic (Akdağ et al., 2022). However, most of the existing centralized methods assume ideal condition to achieve global optimality but rarely consider non-ideal situations.

Some vessels are not able to receive the instruction from the centralized solution, for example when they suffer communication failure. These vessels are called non-cooperative vessels in this thesis and are not in control of centralized control station. Hence dealing with these non-cooperative vessels in centralized method is important. Further more, IMO (1985) stated that VTS operations should not encroach upon each vessel and remain the actual authority of navigation and maneuvering to the seafarer. It is unlikely for VTS centre to consider all the information, for instance some engaged-in vessels are restricted in their ability to manoeuvre and may not collaborate with centralized proposal. So it is necessary to consider the communication between the cooperative vessels and VTS center, and cooperative vessels should be allowed to make its own actions based on its mission and environment.

The research question to be answered in this thesis are: *In the ideal condition, can the optimal maneuvering be found for multiple vessels?* and *Can a collision avoidance system be robust to non-ideal conditions?*

To answer these questions, the contribution of this master thesis is threefold:

1. A dynamic centralized COLREGs-complaint optimization system for multi-vessels' collision avoidance is developed and implemented.
2. A scheme of co-existence of cooperative and non-cooperative vessels in centralized optimization system is proposed.
3. Information exchange between cooperative vessels and shore is implemented to enhance the robustness of the optimization system.

1.5 Outline

The remainder of this thesis is organized as follows: Section 2 describes the basic theory of the vessel's collision avoidance systems, and Section 3 elaborates the design of optimization-based collision avoidance system. Section 4 contains the simulation results to validate the proposed optimization system, and relevant discussion are also given. Finally, Section 5 draws a conclusion of this study and gives direction for future work.

2 Basic Theory

2.1 Nonlinear 3-DOF maneuvering model of Marine Vessel

In this thesis, a nonlinear 3 Degrees of Freedom (DOF) maneuvering model of marine vessel will be used for developing the simulator. 3-DOF represents surge, sway and yaw, which means only the horizontal plane motion will be considered. Three components of nonlinear 3-DOF maneuvering model are the kinematic equation, rigid-body and hydrodynamic kinetics (Fossen, 2011). No environmental forces are assumed in this thesis as suggested by Fossen (2011) for optimal path following and control in horizontal plane, thus the relative velocity between vessel and current $\nu_r = \nu$. Then we have the following equations:

$$\dot{\eta} = R(\psi)\nu \quad (1)$$

$$M\dot{\nu} + C(\nu)\nu + D(\nu)\nu = \tau_{controller} \quad (2)$$

Equation 1 represents the kinematics of the marine vessel, where ψ is the vessel heading angle in North-East-Down (NED) frame, η is the position vector in NED frame, R is the rotation matrix, $\dot{\eta}$ is the velocity vector in NED frame and ν is the velocity vector in body-fixed frame. Equation 2 is based on a spring-mass-damper system suffering external forces, where M , $C(\nu)$ and $D(\nu)$ are the mass matrix, Coriolis and centripetal matrix and damping matrix respectively, and $\tau_{controller}$ is the command force from controller.

2.1.1 Kinematics

Kinematics describes geometrical aspects of motion without considering mass and forces: reference frames, variables and transformations. As a reference frame for the vessel motion, the NED, and body-fixed frame are used.

The NED-frame is a tangent plane to the earth's surface and is denoted as (Fossen, 2011):

$$\{n\} = [x_n, y_n, z_n]^T \quad (3)$$

where x_n follows true north, y_n follows east and z_n is positive pointing downwards.

The body-fixed frame is fixed at vessel's body centre and corresponds to vessel's different directions, which is denoted as (Fossen, 2011):

$$\{b\} = [x_b, y_b, z_b]^T \quad (4)$$

where x_b corresponds to the vessel's longitudinal direction, y_b corresponds to transverse direction and z_b points in a direction normal to x_b and y_b (Fossen, 2011).

Transformation

Because the velocity and the total forces of the vessel are defined in the body-fixed frame while the measurements from sensors are used in NED frame, the conversion between these two frames are important and necessary before the signal flows into the controller. The horizontal rotation matrix from body-fixed frame to NED-frame can be acquired by heading angles (Fossen, 2011):

$$R(\psi) = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (5)$$

Course, heading and sideslip angles

Relationship between the angular variables course, heading, and sideslip is important for maneuvering a vessel in the horizontal plane. The terms course and heading are used interchangeably in much of literature of marine vessels and can lead to confusion easily.

Course χ is defined as the angle from the x-axis (north) of the NED frame to the velocity vector U of the vehicle, heading (yaw) ψ is the angle from NED x-axis to the body x-axis, and sideslip (drift) β is the angle from body x-axis to the velocity vector of the vessel (Fossen, 2011). The sideslip angle of a vessel is caused by nonzero sway velocity due to the environmental forces, which makes the heading and course different. All three angles satisfy the positive rotation about z-axis frame by the right-hand screw convention (Fossen, 2011). The relations between these three angles are depicted in Figure 6 and can be expressed as follows:

$$\chi = \psi + \beta \quad (6)$$

$$\beta = \arcsin\left(\frac{v}{U}\right) \quad (7)$$

2.1.2 Kinetics

Mass matrix in Equation 2 contains rigid-body and added-mass term M_{RB} and M_A . The expression is given as follows (Fossen, 2011):

$$M = M_{RB} + M_A \quad (8)$$

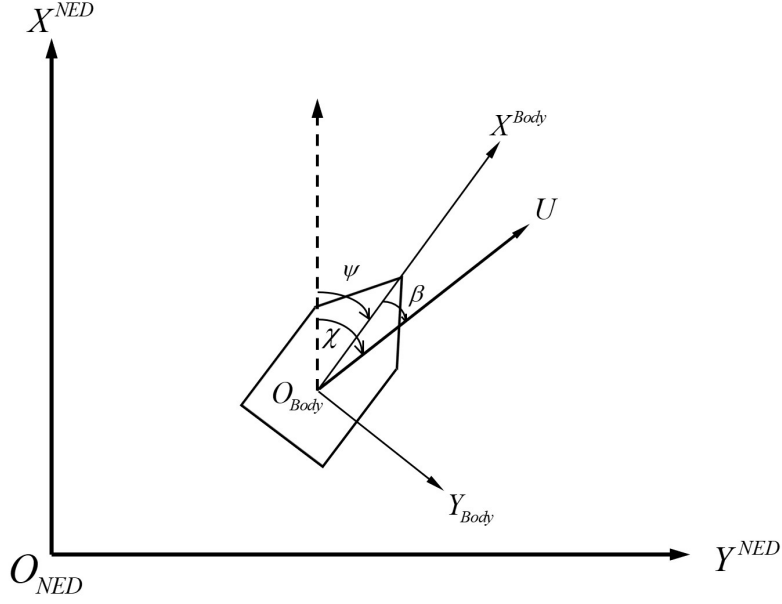


Figure 6: Illustration of vessel's course, heading and sideslip angles in body-fixed and NED frames.

where

$$M_A = \begin{bmatrix} -X_{\dot{u}} & 0 & 0 \\ 0 & -Y_{\dot{v}} & -Y_{\dot{r}} \\ 0 & -Y_{\dot{r}} & -N_{\dot{r}} \end{bmatrix}, \quad M_{RB} = \begin{bmatrix} m & 0 & 0 \\ 0 & m & mx_g \\ 0 & mx_g & I_z \end{bmatrix}$$

The added mass matrix M_A captures the effect of hydrodynamics when a vessel moves in the water (Fossen, 2011). The hydrodynamic derivatives in added mass matrix are assumed as constant values. Symmetric hull are assumed and we have the rigid-body matrix, where m is the mass of the vessel, x_g is the distance along the x-axis between the center of gravity and the body-fixed origin and I_z is the moment of inertia about the z-axis (Fossen, 2011).

The Coriolis and centripetal matrix is similar and has both the rigid-body and hydrodynamic component. The expression is given by Fossen (2011):

$$C(\nu) = C_{RB}(\nu) + C_A(\nu) \quad (9)$$

where

$$C_{RB}(\nu) = \begin{bmatrix} 0 & 0 & -m(x_g r + v) \\ 0 & 0 & mu \\ m(x_g r + v) & -mu & 0 \end{bmatrix}$$

$$C_A(\nu) = \begin{bmatrix} 0 & 0 & Y_{\dot{v}}v + Y_{\dot{r}}r \\ 0 & 0 & -X_{\dot{u}}u \\ -Y_{\dot{v}}v - Y_{\dot{r}}r & X_{\dot{u}}u & 0 \end{bmatrix}$$

Linear damper D matrix is assumed for low-speed maneuvering and also guarantees that the velocity converges exponentially to zero (Fossen, 2011).

$$D = \begin{bmatrix} -X_u & 0 & 0 \\ 0 & -Y_v & -Y_r \\ 0 & -N_v & -N_r \end{bmatrix} \quad (10)$$

2.2 Guidance, Navigation and Control Systems

The motion control systems for a marine vessel consist of guidance, navigation and control systems and can perform motion control through data communication and signal transmission between each component (Fossen, 2011).

2.2.1 Guidance

The guidance systems can generate the desired trajectories according to the control objective, which contain information about own vessels' position, heading and velocity. Three main components of the guidance systems of a marine vessel are waypoint generator, waypoint management system and the reference computing algorithms (Fossen, 2011).

The waypoint generator can establish the desired waypoints according to mission requirements, operator decision, weather information and so on. For example, collision-free waypoints can be generated by optimization methods for collision avoidance. When the current position of the vessel is close to the target waypoint, the waypoint management system can then update the active waypoint, thus the vessel can move to the next target waypoint. In addition, the reference computing algorithms or guidance law can generate a smooth feasible trajectory between two waypoints.

Path Following with LOS Guidance Laws

Path following is the task of following a predefined, desired path independent of time (Fossen, 2011). Line-of-Sight (LOS) guidance is frequently used for the path following problems, where a LOS vector from the vessel to a point on the path between two waypoints can be used for both course and heading control. Generally there are two different guidance principles can be used to steer along the LOS vector, enclosure-based and lookahead-based (Fossen, 2011).

The lookahead-based LOS steering law calculates the desired course angle χ_d , which is necessary to make the vessel follow a straight line path between two waypoints p_k

and p_{k+1} . The course angle assignment is separated into two parts:

$$\chi_d = \chi_p + \chi_r \quad (11)$$

where χ_p is the path-tangential angle, and χ_r is a velocity-path relative angle which contains the information of LOS vector.

$$\chi_p = \text{atan2}(y_{k+1} - y_k, x_{k+1} - x_k) \quad (12)$$

$$\chi_r = \arctan\left(\frac{-e}{\Delta}\right) \quad (13)$$

$$e(t) = -(x(t) - x_k)\sin(\chi_p) + (y(t) - y_k)\cos(\chi_p) \quad (14)$$

x_k, y_k and x_{k+1}, y_{k+1} are the position information of waypoint p_k and p_{k+1} in NED reference frame, while $x(t), y(t)$ represent the current position of the ship. e is the cross-track error, which is normal to the path. Δ is called the lookahead distance, a large value for Δ will make the vessel converges slowly towards the path, while a small value can make the vessel oscillate around the path (Fossen, 2011). The LOS guidance law is depicted in Figure 7.

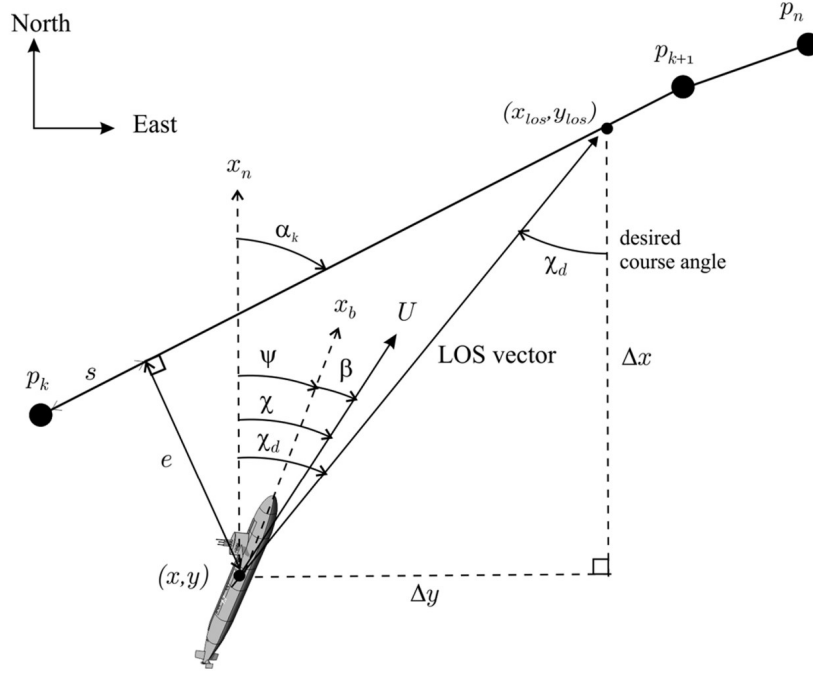


Figure 7: LOS guidance where the desired course angle χ_d is chosen to point toward the LOS intersection point. (Fossen, 2011)

The heading (yaw) command ψ_d can be transformed from the course angle command χ_d by using the information of sideslip (drift) angle calculated from the body velocity (Fossen, 2011).

$$\psi_d = \chi_d - \beta \quad (15)$$

$$\beta = \arcsin\left(\frac{v}{U}\right) \quad (16)$$

Reference Models

To prevent abrupt change from the output from LOS guidance, reference model is used here to synthesize the desired position, velocity and acceleration, which results in a smoother transition between waypoints.

Inspired by Minne (2017), two different reference models are used in this thesis, one to generate the desired velocity, and one to generate yaw angle, both are implemented in NED frame.

The velocity reference model is suggested to at least be order two to obtain smooth reference signals for the desired velocity (Fossen, 2011). The mathematical model of the second-order low-pass filter in NED frame is given as follows:

$$\ddot{\nu}_d^n + 2\Delta\Omega\dot{\nu}_d^n + \Omega^2\nu_d^n = \Omega^2\nu_{ref}^n \quad (17)$$

$$\lim_{t \rightarrow \infty} \nu_d(t) = \nu_{ref}^n \quad (18)$$

Here, ν_d is used to denote a desired velocity for the ship model, then the controller is aim to make the ship to follow this desired value. ν_{ref} is computed by the guidance steering law for path following. The steady state velocity will reach the reference signal ν_{ref} .

The attitude reference model for desired heading angle ψ_d is typically chosen to be of third order for filtering the steps in ψ_{ref} (Fossen, 2011), which contains a first order low pass filter cascaded with a mass-damper-spring system.

$$\ddot{\psi}_d^n + (2\Delta + I)\Omega\dot{\psi}_d^n + (2\Delta + I)\Omega^2\psi_d^n + \Omega^3\psi_d^n = \Omega^3\psi_r^n \quad (19)$$

$$\lim_{t \rightarrow \infty} \psi_d(t) = \psi_{ref}^n \quad (20)$$

Here, Δ and Ω are diagonal design matrices of relative damping ratios and natural frequencies, where $\zeta_{1,2}$, and $\omega_{1,2}$ are used for velocity reference model, ζ_3 and ω_3 are used for heading reference model.

$$\Delta = \text{diag} \{ \zeta_1, \zeta_2, \zeta_3 \} \quad (21)$$

$$\Omega = \text{diag} \{ \omega_{n_1}, \omega_{n_2}, \omega_{n_3} \} \quad (22)$$

2.2.2 Feedback Linearization Controller

Control is the action of generating appropriate force commands (Fossen, 2011). Here, we want to achieve the control objective for trajectory tracking, which requires the position and velocity of the marine craft track the desired time-varying

reference from the guidance. Inspired by Minne (2017), two different control systems: a speed controller and a heading controller are used in this thesis. They are responsible for making the own-vessel follow the desired heading angle ψ_d and the desired speed ν_d computed by the two reference models. The controller command forces are implemented in vessel body-fixed frame.

Speed controller

The idea of the feedback linearizing speed controller for marine vessels is to transform the nonlinear system dynamics into a linear system (Fossen, 2011). The nonlinear term in 3-DOF maneuverability model is $n(\nu) = C(\nu)\nu + D(\nu)\nu$, however the time-variant Coriolis matrices is difficult to cancel in real operation. Hence, we use a simple proportional controller to compensate only the damper matrix in this project.

$$\tau_{controller}^b = D\nu^b - MK_p(\nu^b - \nu_d^b) \quad (23)$$

Here, $\nu^b = [u, v, r]^\top$ is the speed vector of the vessel, $\nu_d^b = [u_d, v_d, r_d]^\top$ is the desired speed vector in body frame, and $K_p \in R^3$ is the tuning controller gain.

Heading controller

The heading controller is implemented as a proportional controller, which calculates the desired yaw rate r_d^n to be used in the feedback linearizing speed controller.

$$r_d^n = -k_{p,\psi}(\psi^n - \psi_d^n) \quad (24)$$

ψ_d^n is the desired value calculated from the heading reference model, and $k_{p,\psi}^n$ is the tuning proportional controller gain.

2.3 COLREGs

When two vessels encountering each other, they are supposed to follow the navigation rule, which is the Convention on the International Regulations for Preventing Collisions at sea (COLREGs) published by the IMO in 1972. The COLREGs describe potential collision scenarios between encountering vessels and provide a set of guidelines for safe maneuvering at sea. Hence it is important to develop the collision avoidance systems of ASV with COLREGs compliance.

2.3.1 COLREGs Rules

The COLREGs rules and regulations are divided into 5 parts (General, Steering and Sailing, Lights and Shapes, Sound and Light signals and Exemptions) and four annexes containing technical requirements. For a power driven marine vessel, the rules in part B are of primary interest. From rule 13 to 17 in Part B, COLREGs rules state a set of regulations of compliant actions considering a two vessels encounter. Figure 8 illustrates the evasive actions for the vessels who are supposed to take according to the regulations for three scenarios: overtaking, head-on and crossing.

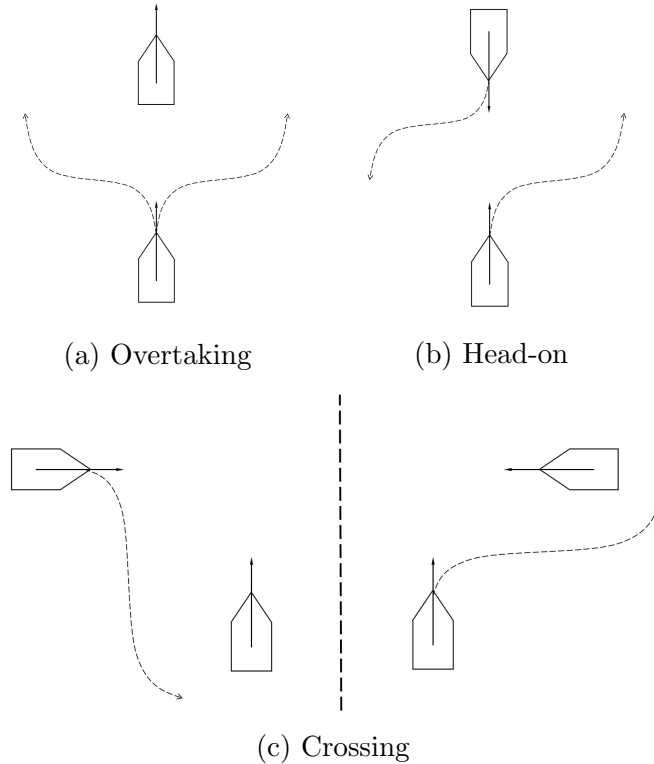


Figure 8: Evasive action which should be taken in encounter scenarios mentioned in COLREGs rules

By rule 13 of COLREGs, *a vessel shall be deemed to be overtaking when coming up with another vessel from a direction more than 22.5 degrees abaft her beam* (Guard et al., 2012). The overtaking ship is located within the visibility range of the stern light of the front ship. As depicted in Figure 8a, if the overtaking ship is overtaking the target ship, the overtaking ship must stay out of the path of the target ship.

By rule 14 of COLREGs, *When two power-driven vessels are meeting on reciprocal or nearly reciprocal courses so as to involve risk of collision each shall alter her course to starboard so that each shall pass on the port side of the other* (Guard et al., 2012). If two ships are in a head-on encounter, each shall alter its course to starboard so that each should pass on the port side of the other as shown in Figure 8b.

By rule 15 of COLREGs, a crossing encounter between two ships can be deemed when the paths of two ships are crossing and they are at risk of collision. If two ships are in a crossing encounter, the ship which has the other on her starboard side must do steering action and avoid crossing ahead if the circumstances admits. In addition, two actions stand-on and give-way can be done by the own ship in crossing scenario according to the rule 16 and 17 of COLREGs as illustrated in Figure 8c. Stand-on action means a vessel should keep its speed and course, while give-way action means a vessel should change its speed and course in order to clear the collision risk. Thus the stand-on vessel has higher priority than the give-way one.

2.3.2 COLREGs-based Encounter Type Classification

Although COLREGs rules have restricted vessels' action during collision avoidance, they are not explicit enough to provide clear role classification. Furthermore, only fundamental guidelines with no detailed quantitative criteria are provided for the sake of mariners to make their own decision based on experience, whereas it could make troubles for the automatic collision avoidance (Cho et al., 2020). Hence specific values are desirable to be assigned to classify the encounter type.

A relative bearing angle and a relative course angle are extensively used in the existing studies (Tam and Bucknall, 2010). The relative bearing angle, β_{ij}^R , between two vessels i and j indicates the position direction of the target ship j with respect to the body-fixed coordinates of the own ship i . While the relative course angle, χ_{ij}^R , indicates the velocity direction of the target ship j with respect to the body-fixed coordinates of the own vessel i (Cho et al., 2020).

$$\beta_{ij}^R(t) = \arctan \left(\frac{y_j(t) - y_i(t)}{x_j(t) - x_i(t)} \right) \quad (25)$$

$$\chi_{ij}^R(t) = \arctan \left(\frac{U_{y,j}(t) - U_{y,i}(t)}{U_{x,j}(t) - U_{x,i}(t)} \right) \quad (26)$$

Within a certain distance, the encounter type can be classified into six types based on COLREGs rules, head-on, give-way, stand-on, overtaking, overtaken, and safe. All the classified encounter type represents the action of the own vessel i should take when it encounter the target vessel j , while vessel j should also take corresponding action as shown in Table 1. As shown in Figure 9, the relative bearing angles β_{ij}^R are divided into four regions in the body-fixed coordinate of own vessel with the angle of $\{\frac{\pi}{8}, \frac{5\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}\}$. Each region can then be divided based on the relative course angle χ_{ij}^R and COLREGs rules.

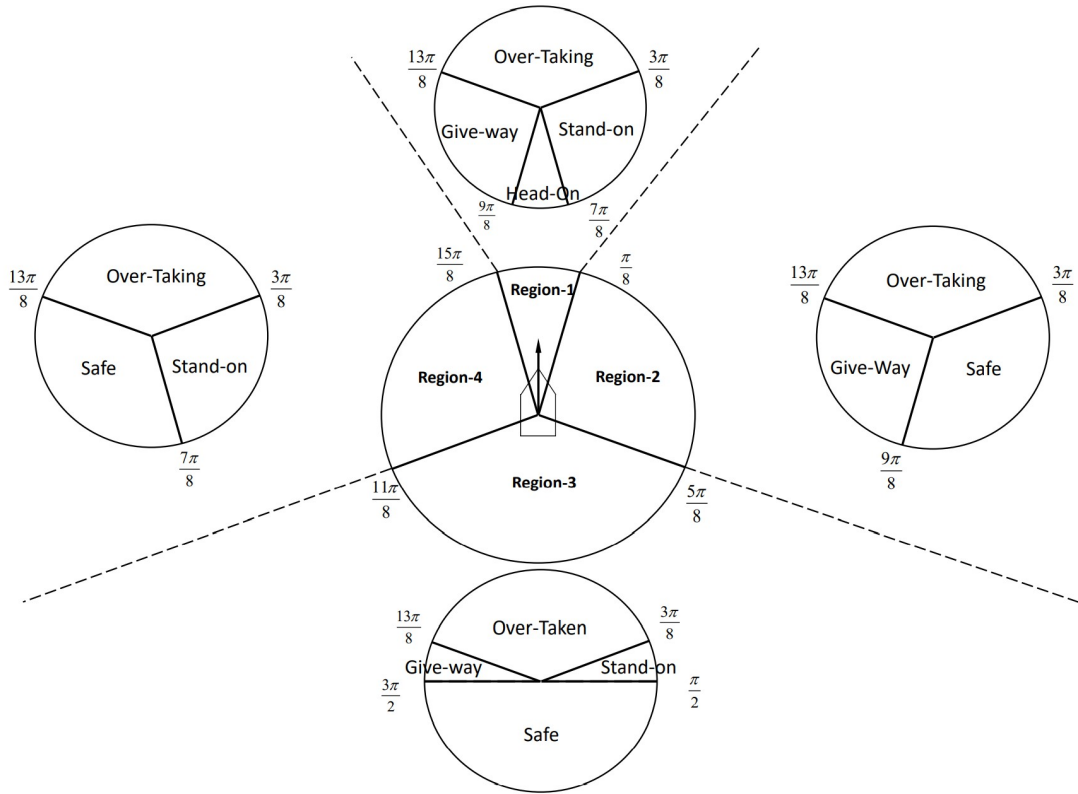


Figure 9: COLREGs-based encounter type classification

Table 1: Encounter type of two encountering vessels

| Own Vessel | | Target Vessel | |
|----------------|---------------------|----------------|---------------------|
| Encounter Type | Action | Encounter Type | Action |
| Head-on | Starboard turn | Head-on | Starboard turn |
| Give-way | Starboard turn | Stand-on | Keep course |
| Stand-on | Keep course | Give-way | Starboard turn |
| Overtaking | Starboard/Port turn | Overtaken | Keep course |
| Overtaken | Keep course | Overtaking | Starboard/Port turn |
| Safe | Keep course | Safe | Keep course |

2.4 Optimization

Martins and Ning (2021) interpreted optimization mathematically as a process of finding the best solution by changing the variables that can be controlled. An optimization problem can be any problem where a decision needs to be made. And to mathematically formulate the problem relevant knowledge is required to consider as much as possible. The statement of a general optimization problem is (Martins

and Ning, 2021):

$$\begin{aligned} \min_{x \in R^n} \quad & f(x) \\ \text{s.t.} \quad & \hat{c}_e(x) = 0, \quad e = 1, 2, \dots, \hat{m} \\ & c_i(x) \geq 0, \quad i = 1, 2, \dots, m \end{aligned} \tag{27}$$

where $f(x)$ is the cost function which is a quantifiable criterion to determine the optimality, x is the design variables which are independent and can describe the system, $\hat{c}(x)$ and $c(x)$ are equality and inequality constraints respectively, and they can show the feasible region of design variables (Martins and Ning, 2021).

For problems with no constraints, line search is an iterative gradient-based method, where we firstly choose a suitable descent direction from the current point, and then determine the step length which shows how far to move in descent direction. The descent direction can be obtained by using steepest descent, conjugated method, and Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm for quasi-Newton method (Martins and Ning, 2021). The step length can be determined by satisfying sufficient decrease and curvature condition.

To use the gradient information, the derivatives of the cost function must be calculated. If the cost function is explicit to design variables, symbolic calculation can be performed to get the exact values. If we only have access to function values, we can use finite differences which is subject to subtractive errors. If we have all the information of implicit cost function, then complex step, algorithmic differentiation and analytic methods can be chosen based on the design goal (Martins and Ning, 2021).

For constrained optimization gradient-based problem, we can add a penalty function according to constraints. Another method is the sequential quadratic programming, which is the combination of linear search method and Karush-Kuhn-Tucker (KKT) condition (Martins and Ning, 2021).

However gradient-based methods are sensitive to noisy and discontinuous functions and limited to continuous design variables (Martins and Ning, 2021). Gradient-free optimization is then used to handle the problems that is hard for the gradient-based methods. Particle Swarm Optimization (PSO) is a stochastic, population-based gradient-free algorithm and solves the optimization problem by applying the concept of swarm intelligence. Basic idea of PSO is that each particle represents a design point and moves in n-dimensional space looking for the best solution, it can also adjust its movement according to the effects of cognitivism (self experience) and sociocognition (social interaction). Furthermore, PSO has the advantage of high speed of convergence for solving the single-objective optimization (Coello et al., 2004).

3 Development of Optimization Systems

3.1 Overall Scheme of Collision Avoidance Systems

In centralized approach, a master unit is assigned to assess the collision risk and solve the collision avoidance problem from a global perspective. The master unit can communicate with the cooperative vessels and acquire vessels' mission and current states. For instance, it could be a VTS centre monitoring vessels in its governed region, or a shore-control center for a group of ASVs. Szlapczynska (2015) also suggested that a smart agent is able to be the master unit in the open water which can communicate and advice other vessels in the adjacent.

In VTS centre governed region, vessels who equip with the communication tools are cooperative vessels, and they share their short-term intention to the master unit and also receive the suggestions of collision avoidance from the master unit. Non-cooperative vessels could be the ones which do not have the AIS onboard or suffer the communication failure during the navigation. In the case of shore-control center, the controlled ASVs are the cooperative vessels and non-controlled obstacle vessels are the non-cooperative vessels.

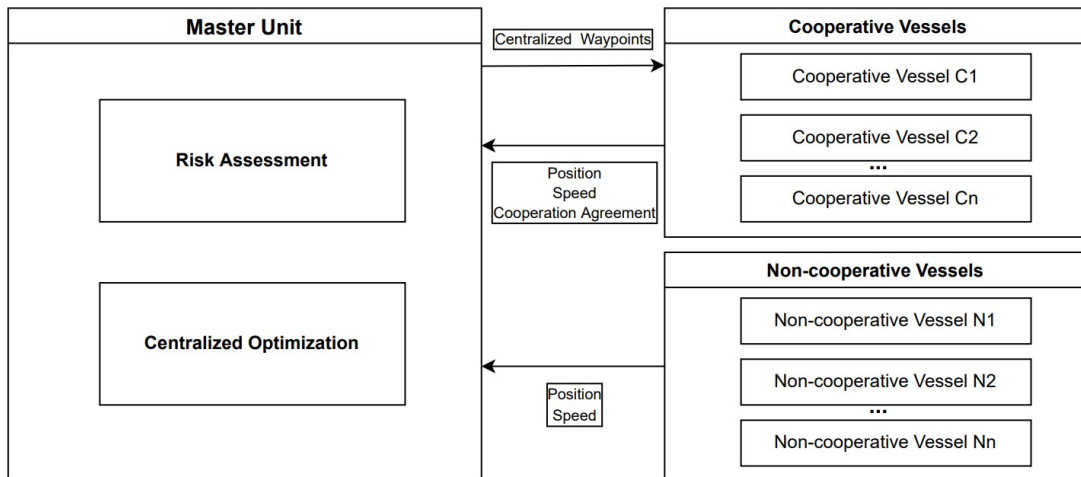


Figure 10: Overall scheme of the proposed centralized collision systems

The existing studies focus on achieving good performance in the ideal conditions with assumption that vessels can received and follow the centralized solutions. However, non-ideal conditions like cooperative vessels' non-collaboration and non-cooperative's COLREGs non-compliance motion are also important to consider in real operation. Hence a centralized scheme is proposed to enhance the robustness in non-ideal conditions utilizing information exchange.

Figure 10 shows the overall scheme of the information exchange between the master unit and both cooperative vessels and non-cooperative vessels. The master unit can

monitor all vessels' motion through AIS and radar and then acquire their position and velocity. Non-cooperative vessels cannot receive any suggestions from the master unit. To include such non-cooperative vessels, the master unit can predict their motion according to the historical data (Papadimitrakis et al., 2021) or just by assuming it will keep its course and speed constant. Through motion prediction, the master unit can then assess the collision risk and formulate the optimization problem. When the master unit detects the risk of collision, it will solve the centralized optimization problem with all the vessels in the scenario and assign the centralized collision-free waypoints to the cooperative vessels. Cooperative vessels can agree to collaborate with the master unit and achieve collision avoidance. Whereas it is unlikely for the master unit to consider all the information. For example, engaged-in towing vessels are restricted in their ability to maneuver, sailing boats have to keep constant angle of attack and some vessels are constrained by their draught. In these scenarios, cooperative vessels are unable to collaborate with the master unit. To handle this issue, cooperative vessels can send their planned waypoints or course and speed over a period of time. When the master unit receives such information, adjustment can be made for the optimization problem.

3.2 Implementation of Guidance and Control Systems

The implementation of guidance and control systems of cooperative ASV in simulink are shown in Figure 11. The blue ports on the left-hand side are three inputs to the vessel, where the first one and third one represent for the optimal solutions from the master unit. Task waypoints are predefined vessel's navigation plan and input from the second port. The guidance systems can generate reference heading and velocity between the assigned waypoints. Then two reference models can generate smooth references. The smooth references and the state feedback can be used by the controller to generate command force to the vessel and complete the waypoint tracking and collision avoidance tasks. The outputs are vessels position, heading, velocity as well as the task waypoints, which will be transmitted to the master unit.

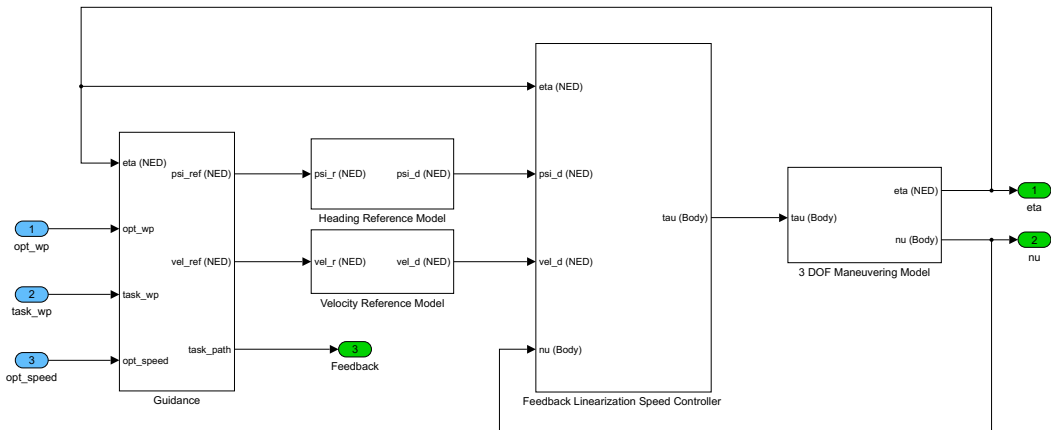


Figure 11: Implementation of guidance and control systems of cooperative vessels

Vessel's 3-DOF maneuvering model

The vessel model used in this thesis is a 116-meter long Platform Supply Vessel (PSV). All vessel's parameters used are shown in Table 2. These parameters are provided by DNV.

Table 2: Model parameters for the ship

| Parameter | Value | Unit |
|---------------|------------------------|-----------------------|
| Mass | 15524000 | <i>kg</i> |
| Length | 116 | <i>m</i> |
| Width | 25 | <i>m</i> |
| I_z | $1.0437 \cdot 10^{10}$ | <i>kg \cdot m^2</i> |
| x_g | -3.7 | <i>m</i> |
| $X_{\dot{u}}$ | -979290 | <i>kg</i> |
| $Y_{\dot{v}}$ | -10727527 | <i>kg</i> |
| $Y_{\dot{r}}$ | -11357800 | <i>kg \cdot m</i> |
| $N_{\dot{r}}$ | $-6.2422 \cdot 10^9$ | <i>kg/m^2</i> |
| X_u | -1650 | <i>kg/s^2</i> |
| Y_v | -1050060 | <i>kg/m^2</i> |
| Y_r | 0 | <i>kg \cdot m/s</i> |
| N_v | 0 | <i>kg \cdot m/s</i> |
| N_r | -2452793600 | <i>kg \cdot m^2/s</i> |
| T_{surge} | 10000 | <i>s</i> |
| T_{sway} | 2500 | <i>s</i> |
| T_{yaw} | 2550 | <i>s</i> |

With the vessel's parameters, the nonlinear 3-DOF maneuvering model can be implemented in Simulink as shown in Figure 12. It contains vessel's rigid-body kinetics and kinematics. The input is the command force from the controller in body-fixed frame. The first output includes vessel's position and heading in NED frame, while second output is the velocity in body-fixed frame.

Guidance system

Figure 13 illustrates the implementation of vessel's guidance system which includes the waypoint management and LOS guidance law.

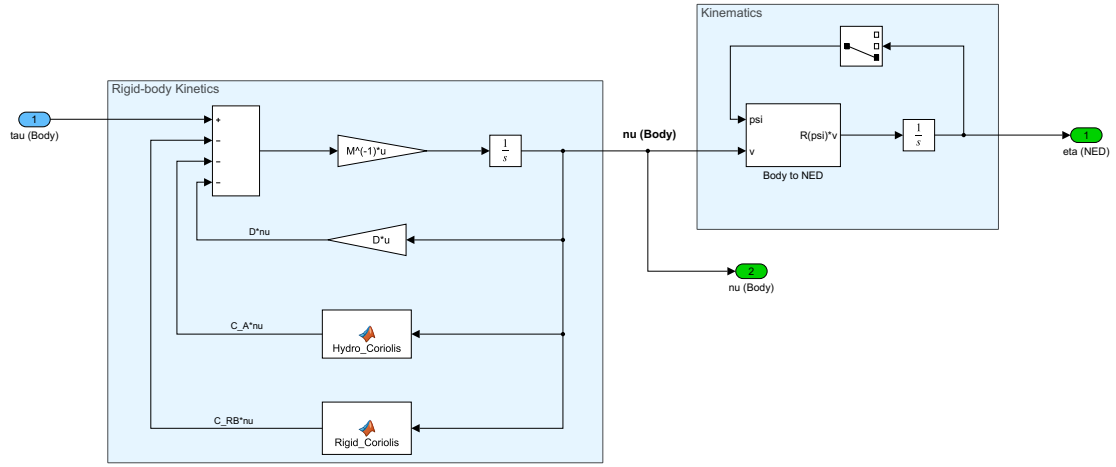


Figure 12: Implementation of nonlinear 3-DOF maneuvering model

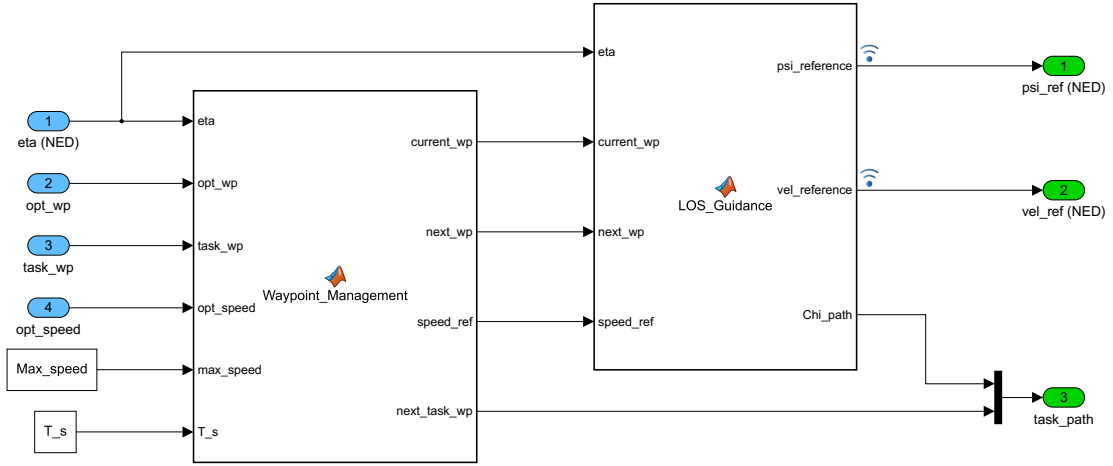


Figure 13: Implementation of vessel's guidance system

To achieve collision avoidance using centralized optimization, the guidance systems have two assignments to complete. The first one is in global sense and to guide the vessel to its task waypoints or destination, and the second one is in local sense and to track the centralized waypoints assigned by the master unit. Thus the waypoint management should have the ability to switch the navigation goal temporarily during the collision avoidance. Moreover, since the centralized waypoint is received at one time instant, the guidance system should be able to store this waypoint for a time period until vessel arrives.

Algorithm 1 presents the details of the waypoint management. The outputs are two coordinates of waypoints and reference speed used for lookahead-based LOS guidance law. In the beginning in safe situation, two output waypoints are the task waypoints. When there is a collision risk, the next waypoint for LOS guidance is then changed by the assigned collision-free waypoint. To store centralized waypoints, a persistent variable regarding time is used, which is presented by N_{time} in the algorithm. After finishing the collision avoidance, the vessel continues to go to its destination.

Algorithm 1 Waypoints Management Algorithm for Guidance and Collision Avoidance

Input: Task Waypoints List WP_{task} , Optimal Centralized Waypoint WP_{opt} , Maximum speed V_{max} , Optimal speed V_{opt} , Vessel's current position Pos

Output: Current waypoint for LOS guidance $WP_{current}$, Next waypoint for LOS guidance WP_{next} , Reference Speed for LOS guidance V_{ref}

- 1: The list of task waypoints WP_{task} contains number n of waypoints
 - 2: Initialize N_{task} as 1, which is used for the counting the number of current waypoint in the task list.
 - 3: Initialize N_{opt} as 0, which is used for counting the number of optimal waypoints.

 - 4: Initialize N_{time} as 0, which is used for keeping the optimal waypoints.
 - 5: Initialize R_{switch} as 250, which is used for switching the task waypoints.
 - 6: Initialize R_{stop} as 100, which is used for stop at the final destination.
 - 7: Initialize $WP_{store,1}$ as $(0,0)$, which is used for storing the previous optimal waypoints.
 - 8: Initialize $WP_{store,2}$ as $(0,0)$, which is used for storing the current optimal waypoints.
 - 9: Initialize V_{store} as 0, which is used for storing the optimal speed.
 - 10: Initialize T_{slot} as 120, which is the length of a time slot.
 - 11: Initialize T_{sample} as 0.1, which is the fixed sample time size in Simulink.
 - 12: Compute the distance between vessel current position and the current target waypoint $D = norm(WP_{task}(N_{task} + 1) - Pos)$
 - 13: **if** no WP_{opt} received and $N_{time} \leq 0$ **then**
 - 14: **if** $N_{opt} = 0$ **then**
 - 15: $WP_{current} = WP_{task}(N_{task}), WP_{next} = WP_{task}(N_{task} + 1), V_{ref} = V_{max}$
 - 16: **else**
 - 17: $WP_{current} = WP_s, WP_{next} = WP_{task}(N_{task} + 1), V_{ref} = V_{max}$
 - 18: **end if**
 - 19: **else if** WP_{opt} received and $N_{time} \leq 0$ **then**
 - 20: **if** $N_{opt} = 0$ **then**
 - 21: $WP_{store,1} = Pos, WP_{store,2} = WP_{opt}, V_{store} = V_{opt}$
 - 22: **else**
 - 23: $WP_{store,1} = WP_{store,2}, WP_{store,2} = WP_{opt}, V_{store} = V_{opt}$
 - 24: **end if**
 - 25: $WP_{current} = WP_{store,1}, WP_{next} = WP_{store,2}, V_{ref} = V_{store}, N_{time} = T_s, N_{opt} = N_{opt} + 1$
 - 26: **else**
 - 27: $WP_{current} = WP_{store,1}, WP_{next} = WP_{store,2}, V_{ref} = V_{store}$
 - 28: **end if**
 - 29: $N_{time} = N_{time} - T_{sample}$
 - 30: **if** $N_{task} < n - 1$ and $D \leq R_{switch}$ **then**
 - 31: $N_{task} = N_{task} + 1, N_{opt} = 0$
 - 32: **end if**
 - 33: **if** $N_{task} < n - 1$ and $D \leq R_{stop}$ **then**
 - 34: $V_{ref} = 0$
 - 35: **end if**
 - 36: **return** $WP_{current}, WP_{next}, V_{ref}$
-

Reference models

The second order velocity reference model and the third order heading reference model are implemented to generate smooth velocity and heading reference in NED frame for controller. The theory behind has been presented in details in Section 2. The implementation of two reference models are illustrated in Figure 14 and Figure 15 respectively.

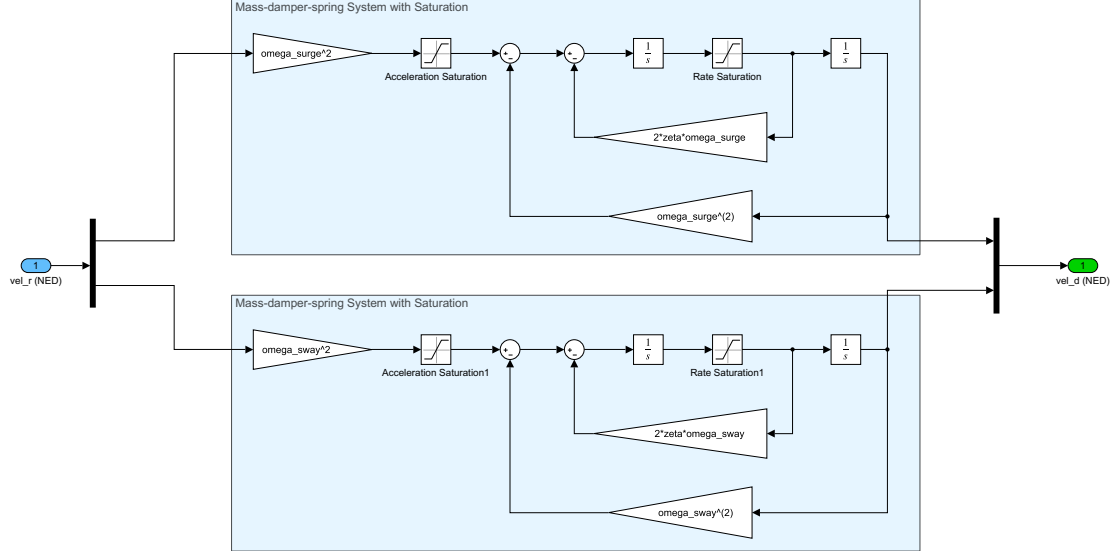


Figure 14: Implementation of velocity reference model

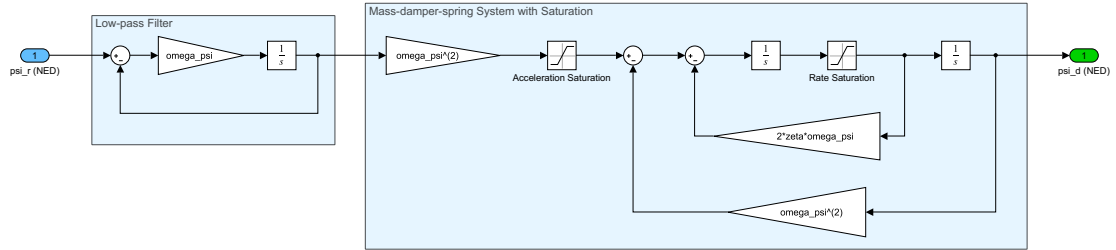


Figure 15: Implementation of heading reference model

The chosen parameters for relative damping ratios and natural frequencies can be seen in Table 3, where relative damping ratios is critical and the natural period of vessel model is assumed to be 75 seconds.

Control system

One proportional heading controller is implemented to generate desired heading rate in NED frame with the heading reference. Then through rotation from NED frame to body-fixed frame, the desired velocity can be used in the speed controller. The

Table 3: Parameters of the reference model and controller

| Parameter | Value |
|----------------|---|
| ζ_1 | 1 |
| ζ_2 | 1 |
| ζ_3 | 1 |
| ω_{n_1} | 0.0838 |
| ω_{n_2} | 0.0838 |
| ω_{n_3} | 0.0838 |
| K_P | $\begin{bmatrix} 0.5 & & \\ & 0.5 & \\ & & 0.5 \end{bmatrix}$ |
| $k_{p,\psi}$ | 0.5 |

implementation of the control system is shown in Figure 16. After the parameter tuning process, the final parameters of the controllers are shown in Table 3.

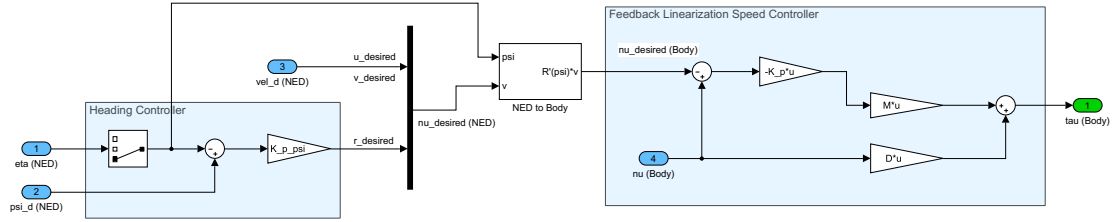


Figure 16: Implementation of heading controller and feedback linearization speed controller

3.3 Centralized Optimization Problem

The master unit can acquire the position and speed information of all the cooperative and non-cooperative vessels within its authority region. Then the whole multi-vessels' collision avoidance problem can be modeled as a centralized optimization problem from a global perspective. With the optimization problem solved, a sets of waypoints can be sent to cooperative vessels as a suggestion for collision avoidance.

3.3.1 Rolling Horizon Approach

To solve a dynamic optimization problem of multi-vessel collision avoidance, a rolling horizon optimization method is adopted. By dividing the total prediction time horizon into a sets of small time slots, the overall collision avoidance optimization

problem can be split into several smaller optimization problem, thus it can reduce the computational complexity and accelerate the optimization process (Li et al., 2019). Another benefit of implementing optimization in a series of time slots is that the VTS centre usually broadcast information at a fixed intervals (IMO, 1985). The rolling time horizon is depicted in Figure 17.

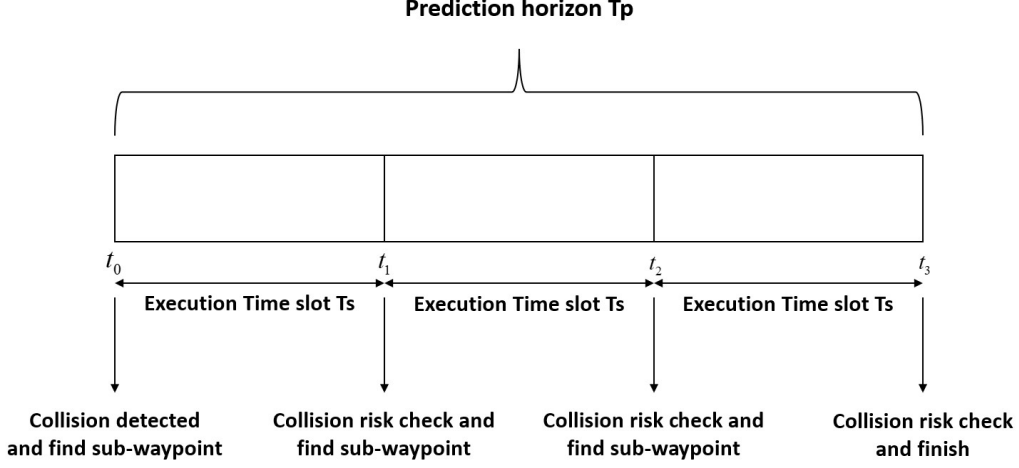


Figure 17: Illustration of rolling time horizon approach

At t_0 , the master unit predicts the vessels' motion in a prediction horizon t_p based on current states, assesses the collision risk and classifies the encounter type for all the vessels. When a collision risk is detected, the master unit finds the optimal anti-collision waypoints for vessels to navigate in a time slot T_s . After the execution, the motion prediction and collision risk check will be done once again at t_1 . If the vessels are still in danger, another centralized optimization problem will be solved. The whole process ends when all the vessels complete the collision avoidance. The prediction horizon t_p is chosen as 240 seconds, while a time slot is 120 seconds by tuning in this thesis.

3.3.2 Motion Prediction

Motion prediction is a process to predict the trajectories of the vessels (Huang et al., 2020). When vessels encounter with potential dangers, the predicted trajectories are used to determine the collision risk for conflict detection. The straight line prediction is chosen in the centralized optimization problem, because it is the simplest way to predict vessel's motion with assumption that the vessel will keep its course and speed and the environmental disturbance are neglected. Another reason is that vessel's complex mass and damping matrix are hard to acquire for the master unit. The equations of straight line prediction to predict vessel i 's future position can be

expressed as follows:

$$\hat{x}_i(t_0 + t_p) = x_i(t_0) + U_{x,i}(t_0) \cdot t_p \quad (28)$$

$$\hat{y}_i(t_0 + t_p) = y_i(t_0) + U_{y,i}(t_0) \cdot t_p \quad (29)$$

The time when the prediction starts is denoted by t_0 , and the prediction horizon is t_p . Estimated position \hat{x}_i and \hat{y}_i represent the north and east position of the vessel i after the prediction horizon. Speed $U_{x,i}$ and $U_{y,i}$ are the vessel's velocities in current time instant expressed in NED frame. The straight line prediction is used for both collision risk assessment and finding the optimal waypoints for the optimization problem.

3.3.3 Design Variables

Assume there are total number n of vessels detected by master unit, which include n_{co} cooperative vessels and n_{non} non-cooperative vessels. To find the collision-free waypoints, the course angle and speed of each vessel can be chosen as the design variables.

$$n = n_{co} + n_{non} \quad (30)$$

$$x \triangleq [\chi'_{1_{co}} \quad V'_{1_{co}} \quad \dots \quad \chi'_{n_{co}} \quad V'_{n_{co}} \quad \chi'_{1_{non}} \quad V'_{1_{non}} \quad \dots \quad \chi'_{n_{non}} \quad V'_{n_{non}}] \in \mathbb{R}^{2n} \quad (31)$$

With the optimal course and speed, the centralized collision-free waypoints for each cooperative vessel can then be calculated in one time slot ahead based on their current position.

$$WP_{i_{co}}^{T_s, x} = Pos_{i_{co}}^x + T_s * V_{i_{co}}^* * \cos(\chi_{i_{co}}^*) \quad i_{co} \in [1_{co}, n_{co}] \quad (32)$$

$$WP_{i_{co}}^{T_s, y} = Pos_{i_{co}}^y + T_s * V_{i_{co}}^* * \sin(\chi_{i_{co}}^*) \quad i_{co} \in [1_{co}, n_{co}] \quad (33)$$

where $WP_{i_{co}}^{T_s, x}$ and $WP_{i_{co}}^{T_s, y}$ are the coordinates of the collision-free waypoints in the NED frame, $Pos_{i_{co}}^x$ and $Pos_{i_{co}}^y$ are the position of each cooperative vessel at current time instant, $V_{i_{co}}^*$ and $\chi_{i_{co}}^*$ are the optimal course and speed of each cooperative vessel.

3.3.4 Objective Functions

To meet the requirement of collision avoidance and find the optimal waypoints for cooperative vessels, there are four objective functions and one penalty function could be set for the centralized optimization problem

Safety objective

The first and prime objective is to ensure safety for the vessels, which aims to reduce the risk among all the vessels during the collision avoidance process.

To quantify the collision risk at sea, two most popular indices to measure the risk are Distance at Closet Point of Approach (DCPA) and Time at Closest Point of Approach (TCPA), which depict the urgency of the collision danger. Based on the vessel's position and speeds after one time slot T_s , $DCPA_{ij}$ and $TCPA_{ij}$ between any two vessels i and j can be calculated. For example, Figure 18 shows the relative position and DCPA of two encountering vessels in the NED frame.

With known information of predicted position and velocity, we can calculate their relative distance D_{ij}^R and relative speed U_{ij}^R , then with relative bearing angle β_{ij}^R , and relative course angle χ_{ij}^R , the $DCPA_{ij}$ and $TCPA_{ij}$ can be calculated (Xie et al., 2020):

$$D_{ij}^R = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2} \quad (34)$$

$$U_{ij}^R = \sqrt{(U_{x,j} - U_{x,i})^2 + (U_{y,j} - U_{y,i})^2} \quad (35)$$

$$\alpha_{ij} = \chi_{ij}^R - \beta_{ij}^R - \pi \quad (36)$$

$$DCPA_{ij} = D_{ij}^R \sin(\alpha_{ij}) \quad (37)$$

$$TCPA_{ij} = D_{ij}^R \cos(\alpha_{ij}) \cdot \frac{1}{|U_{ij}^R|} \quad (38)$$

The Collision Risk Index (CRI) can be chosen as an exponential expression of DCPA

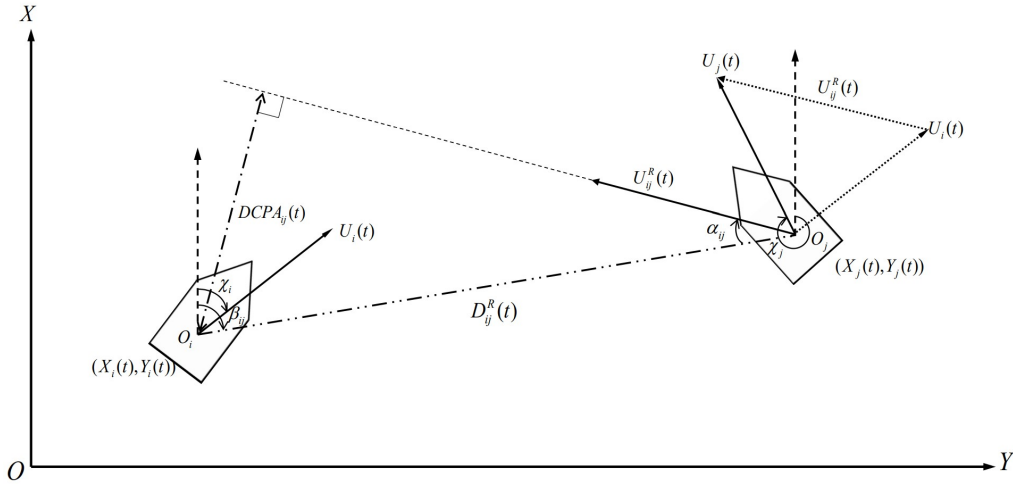


Figure 18: An example of a two-vessel encounter situation to present DCPA.

and TCPA with the safety thresholds d_{safe} and t_{safe} (Papadimitrakis et al., 2021):

$$CRI_{ij} = \begin{cases} \exp(a_1(d_{safe} - DCPA_{ij} + t_{safe} - TCPA_{ij})) - 1, \\ \quad \text{if } DCPA_{ij} \leq d_{min}, TCPA_{ij} \leq t_{min} \\ 0, \text{ if otherwise} \end{cases} \quad (39)$$

where a_1 is a scaling parameter equal to 10^{-3} and d_{safe} and t_{safe} are chosen based on tuning. The value of the CRI_{ij} will increase if DCPA and TCPA have a smaller value, which means that vessels i and j encounter in a more dangerous situation.

To get a set of collision-free waypoints for one time slot ahead, we can minimize both the average and maximum collision risk at the waypoints. Every possible and the worst collision risk are considered and represented by CRI in function $f_1^{T_s}(x)$.

$$f_1^{T_s}(x) = \max\{CRI_{ij}^{T_s}(x)\} + \frac{2}{n * (n - 1)} \sum_{i,j}^n CRI_{ij}^{T_s}(x) \quad \forall i, j \in \mathbb{R}^n, i < j \quad (40)$$

Only consider collision-free waypoints may not guarantee the safety, hence another two positions after half time slot and after one and a half time slot could also be included to reduce the collision risk.

$$f_1^{\frac{1}{2}T_s}(x) = \max\{CRI_{ij}^{\frac{1}{2}T_s}(x)\} + \frac{2}{n * (n - 1)} \sum_{i,j}^n CRI_{ij}^{\frac{1}{2}T_s}(x) \quad \forall i, j \in \mathbb{R}^n, i < j \quad (41)$$

$$f_1^{\frac{3}{2}T_s}(x) = \max\{CRI_{ij}^{\frac{3}{2}T_s}(x)\} + \frac{2}{n * (n - 1)} \sum_{i,j}^n CRI_{ij}^{\frac{3}{2}T_s}(x) \quad \forall i, j \in \mathbb{R}^n, i < j \quad (42)$$

Then the first objective function $f_1(x)$ can be a linear combination of the above functions:

$$f_1(x) = f_1^{T_s}(x) + a_2 f_1^{\frac{1}{2}T_s}(x) + a_3 f_1^{\frac{3}{2}T_s}(x) \quad (43)$$

where a_2 and a_3 are the weighted parameters regarding three future positions and chosen as $\frac{1}{2}$ and $\frac{1}{2}$ in the thesis. A smaller function value of the safety objective $f_1(x)$ indicates a safer solutions of collision-free waypoints.

Obstacles avoidance objective

Obstacles avoidance is also an important aspect to consider due to the environmental complexity in maritime world. Obstacles could be small floatages or shallow water region, then it can be interpreted as a forbidden region where vessels are not allowed to go inside. Since the CRI is a function of vessels' position and velocity, we can also use it to assess the collision risk between cooperative vessels and floating or static obstacles. Assume that there are numbers n_{obs} of obstacles detected by the master unit, the second objective function can be the sum of all the CRI between

cooperative vessels and the obstacles.

$$f_2^{T_s}(x) = \frac{1}{n_{co} * n_{obs}} \sum CRI_{ip}^{T_s}(x) \quad \forall i \in \mathbb{R}^{n_{co}}, p \in \mathbb{R}^{n_{obs}} \quad (44)$$

Maneuvering objective

The third objective is based on the good seafaring practice (Hu et al., 2019) and COLREGs rule 8 that large course change maneuvering are encouraged and preferred over speed ones. This is mainly because of energy conservation, faster response and better visibility of the vessel's maneuvering intention to outside observers.

$$f_3(x) = \sum (|V'_{i_{co}} - V_{i_{co}}| + \exp(-|\chi_{p,i_{co}} - \chi'_{i_{co}}|)) \quad \forall i_{co} \in \mathbb{R}^{n_{co}} \quad (45)$$

where $V_{i_{co}}$ and $\chi_{p,i_{co}}$ are cruising speed and path-tangential angle of each cooperative vessel. Small course angle change and large speed change will result in a big function value of $f_2(x)$, which is not favourable.

Efficiency objective

Due to the maneuvering objective function, the collision-free path could be high-cost. Thus the fourth objective is trying to find a more efficient path, which means to decrease the deviation from vessels' task path. The deviation can be reflected by calculating the sum of distances from each vessel's current position to the destination via the new generated waypoint as shown in Figure 19.

$$f_4(x) = a_4 \sum (Distance(Pos_{i_{co}}, WP_{n_{co}}^{T_s}) + Distance(WP_{i_{co}}^{T_s}, Des_{i_{co}})) \quad \forall i_{co} \in \mathbb{R}^{n_{co}} \quad (46)$$

where a_4 is equal to 10^{-3} , $Pos_{i_{co}}$ and $Des_{i_{co}}$ represent the current position and the destination of each cooperative vessel.

Constraint

In collision avoidance optimization problem, a hard constraint is required regarding the relative distance. To handle this constraint, penalty method (Martins and Ning, 2021) is adopted, which gives large penalty when solutions break the constraint. The distance between any two centralized waypoints and the distance between any centralized waypoint and obstacles should be greater than a minimum distance.

$$f_5 = \begin{cases} Inf, & \text{if } Distance_{wps} \leq Dis_{min} \text{ or } Distance_{wp-obs} \leq Dis_{min} \\ 0, & \text{if otherwise} \end{cases} \quad (47)$$

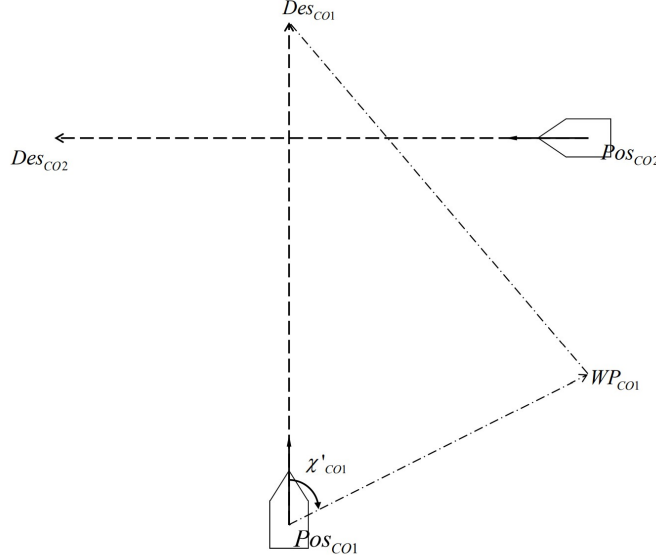


Figure 19: Illustration of the total distance when the vessel navigates towards the centralized waypoint assigned by the master unit.

where Dis_{min} is a threshold chosen as 150 meters. Function $f_5(x)$ will be infinity if any distance is lower than the threshold, and it equals to zero when the constraint is satisfied. Thus the worst situation of collision could be avoided.

3.3.5 Bounds for Decision Variables

Bounds of cooperative vessels

The bounds of design variables define the feasible searching region of the optimization problem, thus it is important to choose the upper and lower bounds appropriately. The bounds of the course and speed for cooperative vessels can be set according to their priority based on COLREGs rules, obstacle avoidance and cooperative agreement.

$$\begin{bmatrix} \Delta\chi_{i_{co},min} + \chi_{p,i_{co}} \\ V_{i_{co},min} \end{bmatrix} \leq \begin{bmatrix} \chi'_{i_{co}} \\ V'_{i_{co}} \end{bmatrix} \leq \begin{bmatrix} \Delta\chi_{i_{co},max} + \chi_{p,i_{co}} \\ V_{i_{co},max} \end{bmatrix}, \quad \forall i_{co} \in \mathbb{R}^{n_{co}} \quad (48)$$

As for one-to-one vessel encounter scenario, a stand-on vessel has a higher priority than the give-way vessel. The issue may arise when considering multi-vessel encounter, since a vessel could be both stand-on and give-way at the same time. To handle the COLREGs-compliance, the priority of cooperative vessels can be set according to the number of give-way and stand-on actions with the others based on the encounter type classification. When a cooperative vessel is safe or there is no course alteration, it has the priority to keep constant course and speed. Hence the

upper and lower bounds could be set as equality constraint as follows.

$$\begin{bmatrix} \Delta\chi_{i_{co},min} \\ V_{i_{co},min} \\ \Delta\chi_{i_{co},max} \\ V_{i_{co},max} \end{bmatrix} = \begin{bmatrix} 0 \\ V_{i_{co}} \\ 0 \\ V_{i_{co}} \end{bmatrix}, \text{ if cooperative vessel } i_{co} \text{ is safe or a stand-on vessel} \quad (49)$$

Based on the encounter type classification, we can decide the number of maneuvering actions to take for each vessel to avoid collision with the others in the encounter scenario. The bounds of design variables could be set as a feasible region for starboard turning with speed reduction. Woerner (2016) pointed out an apparent course maneuvering should be larger than 35° , hence the maximum course change is set as $\frac{\pi}{3}$ or $\frac{\pi}{2}$. When there is only one maneuvering action to take, the course change is set as $\frac{\pi}{3}$, while $\frac{\pi}{2}$ is set when there are more risks exists with the others in multi-vessel encounter situation.

$$\begin{bmatrix} \Delta\chi_{i_{co},min} \\ V_{i_{co},min} \\ \Delta\chi_{i_{co},max} \\ V_{i_{co},max} \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{V_{i_{co}}}{2} \\ \frac{\pi/3/\pi/2} \\ V_{i_{co}} \end{bmatrix}, \text{ if a cooperative vessel is supposed to do maneuver} \quad (50)$$

When an obstacle is detected around a cooperative vessel, both port-side and starboard-side turning is allowed. Thus the lower bound of the course can be set as $-\frac{\pi}{3}$ plus the original path-tangential course. When both obstacle and the other vessel is around, the searching region will be loosened to seek more possible collision-free solutions.

$$\begin{bmatrix} \Delta\chi_{i_{co},min} \\ V_{i_{co},min} \\ \Delta\chi_{i_{co},max} \\ V_{i_{co},max} \end{bmatrix} = \begin{bmatrix} -\frac{\pi/3/\pi/2} \\ \frac{V_{i_{co}}}{2} \\ \frac{\pi/3/\pi/2} \\ V_{i_{co}} \end{bmatrix}, \text{ if obstacles is detected around a cooperative vessel} \quad (51)$$

Last but not least, IMO (1985) stated that the authority of maneuvering still remains to the individual vessel. If a cooperative vessel wants to make its own decision on navigating, it could send disagreement feedback for non-collaboration with its planned course and speed for the next time slot to the master unit. The master unit can then consider these information in the optimization problem, and give such non-collaborating vessel the highest priority. The bounds of non-collaborating vessel will be set as equality constraints based on its proposed values. An important change is that anti-collision maneuvering should be done for those who was supposed to keep its way.

$$\begin{bmatrix} \chi'_{i_{co}} \\ V'_{i_{co}} \end{bmatrix} = \begin{bmatrix} \chi_{i_{co},proposed} \\ V_{i_{co},proposed} \end{bmatrix}, \text{ if cooperative vessel } i_{co} \text{ want to make its own decision} \quad (52)$$

Considering the coexistence with non-cooperative vessels, the bounds of cooperative vessels are set in case of non-cooperative vessels restricted in their ability to maneuver.

$$\begin{bmatrix} \Delta\chi_{i_{co},min} \\ V_{i_{co},min} \\ \Delta\chi_{i_{co},max} \\ V_{i_{co},max} \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{V_{i_{co}}}{2} \\ \frac{\pi}{2} \\ V_{i_{co}} \end{bmatrix}, \text{ if non-cooperative vessels are around} \quad (53)$$

Bounds of non-cooperative vessels

Non-cooperative vessels don't have the ability of information communication, thus the master unit can simply assume that these vessels will keep their course and speed. The bounds of design variables of non-cooperative vessels can then be set as equality constraints to their current values.

$$\begin{bmatrix} \chi'_{i_{non}} \\ V'_{i_{non}} \end{bmatrix} = \begin{bmatrix} \chi_{i_{non}} \\ V_{i_{non}} \end{bmatrix}, \quad \forall i_{non} \in \mathbb{R}^{n_{non}} \quad (54)$$

3.3.6 Particle Swarm Optimization

To consider all the objectives, we can use a composite weighted function to handle this multi-objective problem:

$$F(x) = \omega_1 f_1(x) + \omega_2 f_2(x) + \omega_3 f_3(x) + \omega_4 f_4(x) + \omega_5 f_5(x) \quad (55)$$

where $\omega_1, \omega_2, \omega_3, \omega_4$ and ω_5 are weighted parameters for each objective and chosen based on tuning.

The centralized problem can thus be formulated as

$$\begin{aligned} \min_{x \in \mathbb{R}^{2n}} \quad & F(x) \\ \text{s.t.} \quad & (48) - (54) \end{aligned} \quad (56)$$

In this thesis, PSO is adopted and implemented in Matlab function as the optimization method to solve the centralized problem.

First a set of particles are initialized with their positions and velocities randomly distributed throughout the 2n-dimension searching space limited by design variables bounds. By comparing the function values, we can get the local best particle position and global best particle position. At each iteration, the position and velocity of each

particle are updated by using previous information, the local and global best.

$$v_{k+1}^i = \omega v_k^i + c_1 r_1 (p_k^i - x_k^i) + c_2 r_2 (p_k^g - x_k^i) \quad (57)$$

$$x_{k+1}^i = x_k^i + v_{k+1}^i \quad (58)$$

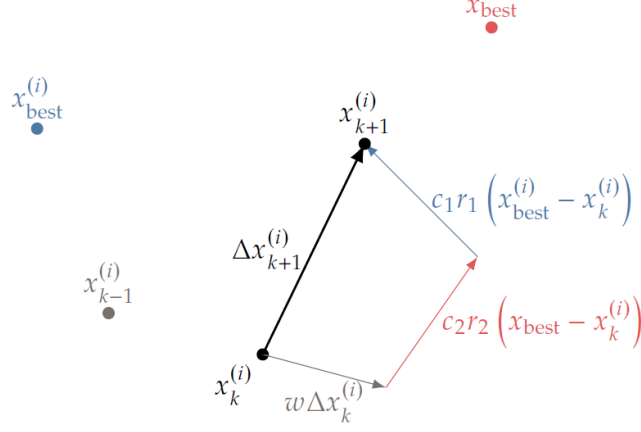


Figure 20: Components used for the PSO update in a two-dimensional case (Martins and Ning (2021))

where r_1 and r_2 are random numbers in the interval $[0, 1]$. The inertial parameter ω are in the interval $[0.8, 1.2]$ (Martins and Ning, 2021). The iteration with a lower value of ω will have a faster convergence, while with a higher value of ω more space will be explored. The cognitive parameter c_1 and social parameter c_2 are usually chosen in the interval $[0, 2]$ (Martins and Ning, 2021). The stopping criteria is set as the total iteration number reaches the maximum $iter_{max}$ or the average velocity of the swarm falls below a tolerance V_{tol} . Table 4 shows the value of PSO parameters chosen in this thesis.

Table 4: PSO parameters

| Parameter | Name | Value |
|--------------|---------------------|--------|
| ω | Inertial Parameter | 0.8 |
| c_1 | Cognitive Parameter | 0.8 |
| c_2 | Social Parameter | 1.5 |
| $iter_{max}$ | Maximum Iterations | 50,000 |
| V_{tol} | Velocity Tolerance | 0.001 |
| m | Number of Particles | 40 |

3.3.7 Implementation

The implementation of the whole optimization system is shown in Figure 21. , where the master unit can monitor both cooperative vessels and non-cooperative vessels. The position, heading as well as velocity information can be acquired by the master unit. The cooperative vessels can communicate with the master unit and share their task waypoints as well as agreement in collaboration. The master unit is responsible for the risk assessment as well as solving the centralized optimization for collision avoidance. Then optimal collision-free waypoints and speed will be assigned to each cooperative vessel.

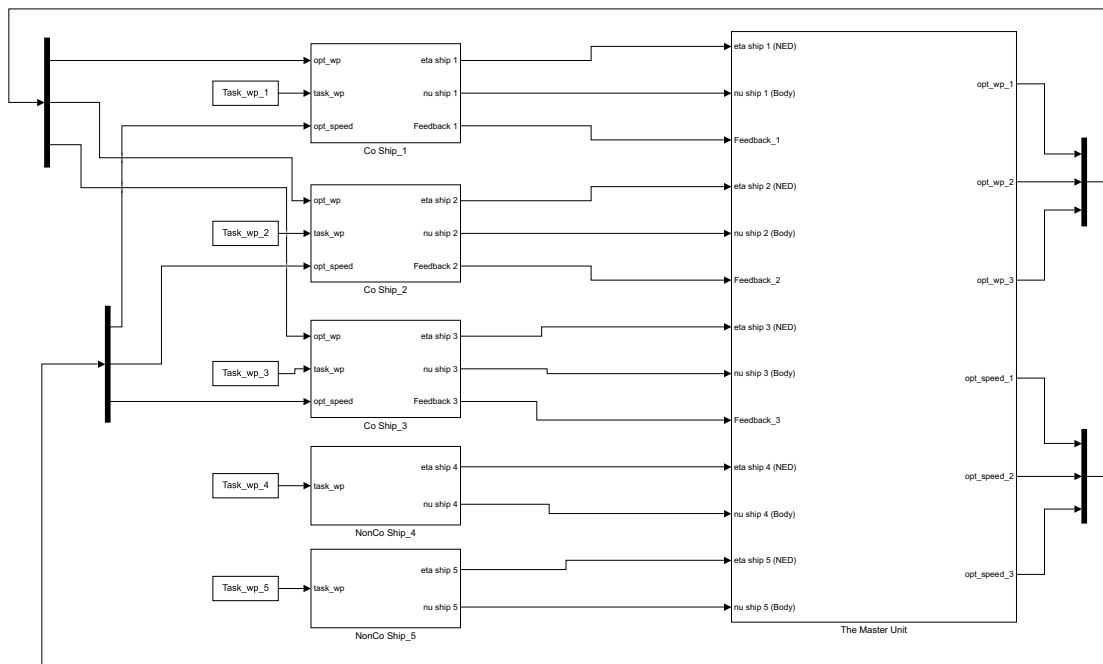


Figure 21: The implementation of the whole optimization system

4 Simulations and Discussions

In this section, several simulation tests have been manually designed to cover a wide range of encountering situations, which aims to validate the centralized optimization systems. The first simulation contains the encountering situations for two cooperative vessels defined in COLREGs rules such as overtaking, head-on and crossing. Then more complex five cooperative vessels encountering scenario are performed twice. In the first one, all the vessels will follow the suggestions from the master unit. Then as a comparison, one vessel is designed to do non-collaborate action and broadcast its intention to the master unit. The optimization systems show the robustness to handle this non-ideal situation. In the final simulation tests, three cooperative vessels and two non-cooperative vessels encountering are performed with static obstacles.

After the simulation analysis, the general performance of the proposed centralized method of collision avoidance is evaluated. Some suggestions regarding the future centralized method are also given for further works.

4.1 Simulation 1 - Collision Avoidance for 2 Vessels

Scenario 1-1 - Overtaking

The overtaking encountering scenario begins with vessel 1 navigating from position $(0, 0)$ towards destination $(5000, 0)$ in NED frame in the speed of 5 [m/s], while vessel 2 travels from $(1500, 0)$ towards $(4000, -1000)$ with 3 [m/s]. According to COLREGs rule 13, vessel 1 is an overtaking vessel and should give way to vessel 2.

The whole process of the collision avoidance is presented in Figure 22. There are four time instants chosen to illustrate the process. Each sub-figure shows the vessels' motion in the region from -3000 meters to 3000 meters in east and from -500 meters to 5500 meters in north. The solid line, solid dot and pentagram represent the trajectory, position and destination of vessels respectively. Color red is used for vessel 1 while color blue is used for vessel 2. The red circle is the centralized waypoint assigned to vessel 1 for collision avoidance.

The master unit detects the collision risk at 49.80 seconds and solves the optimization problem for collision avoidance. The coordinates of the collision-free waypoint $(519.62, 425.78)$ can be acquired by the optimal course $\frac{\pi}{3}$ and speed 5 [m/s] with one time slot 120 seconds ahead using straight-line prediction. With new centralized collision-free waypoint assigned, cooperative vessel 1 maneuvers through LOS guidance law. After one time slot for 120 seconds, vessel 1 reaches the vicinity of the assigned waypoint and now there is no collision risk detected. Two vessels then navigate to their destinations.

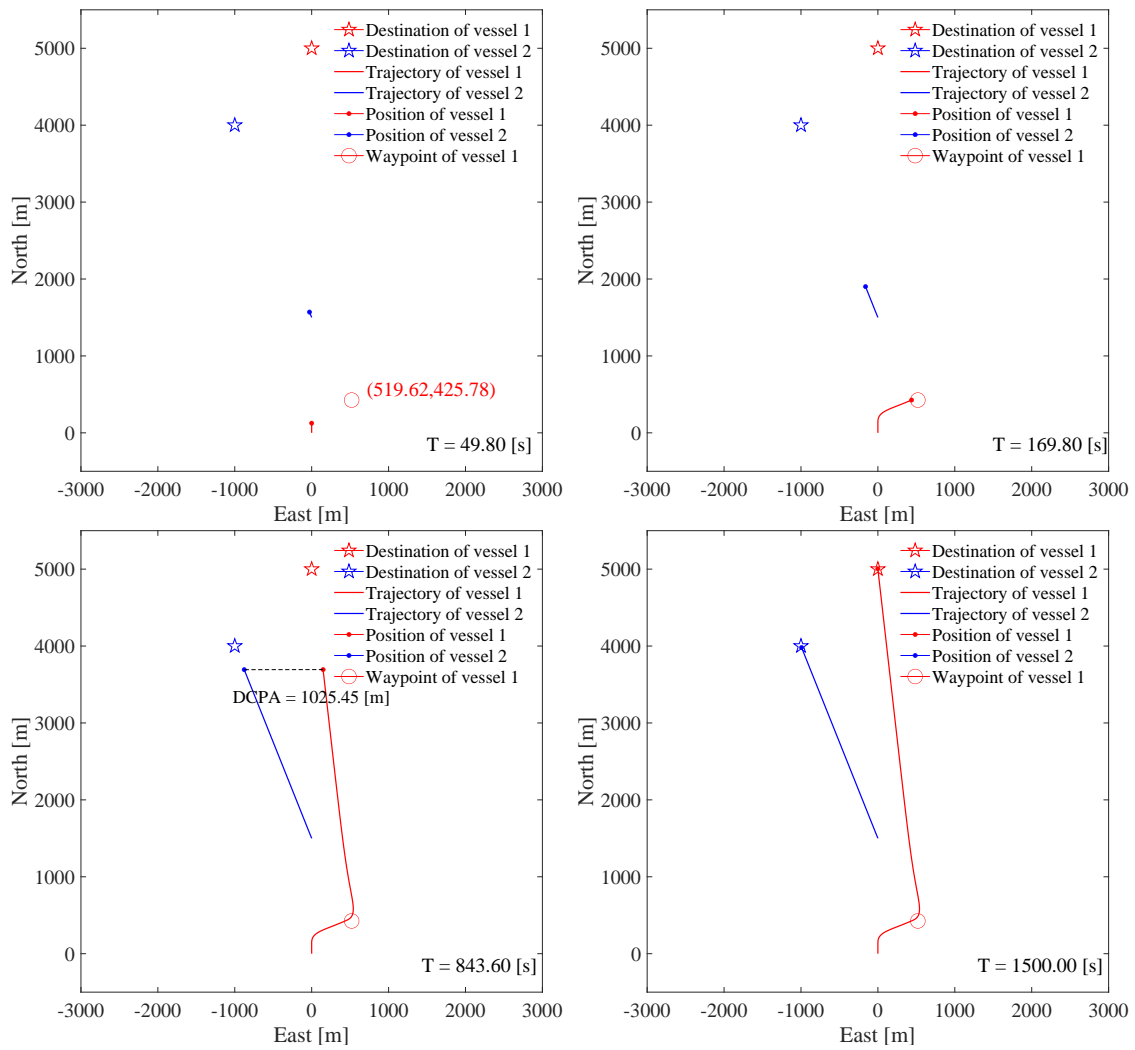


Figure 22: The process of collision avoidance for two cooperative vessels in overtaking situation.

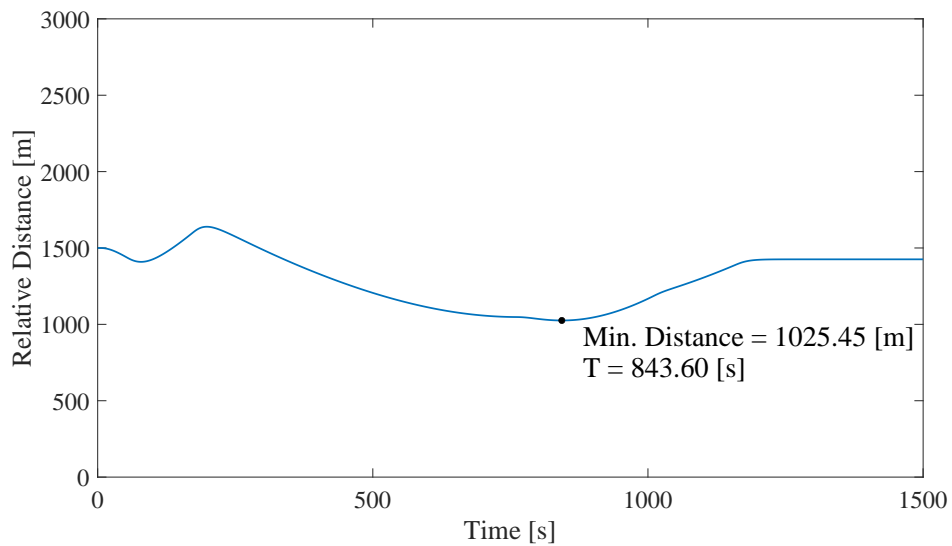


Figure 23: The relative and minimum distance between two vessels in overtaking situation.

Figure 23 shows how the relative distance between two vessels changes during the whole process of collision avoidance. Two vessels reach their closest point of approach at time 843.60 seconds, and the distance at closest point of approach is 1025.45 meters.

Scenario 1-2 - Head-on

To simulate head-on situation, vessel 1 is initially placed at (0,0) and navigates towards (5000,0), while vessel 2 starts from (5000,0) with the destination of (0,0). The setting maximum speed is 5 [m/s] for both vessels. At time 185.80 seconds, two collision-free waypoints are assigned to the corresponding vessel when master unit detects the collision risk as presented in Figure 24. Then give-way actions are made by both vessels, which is in accord with COLREGs rule 14.

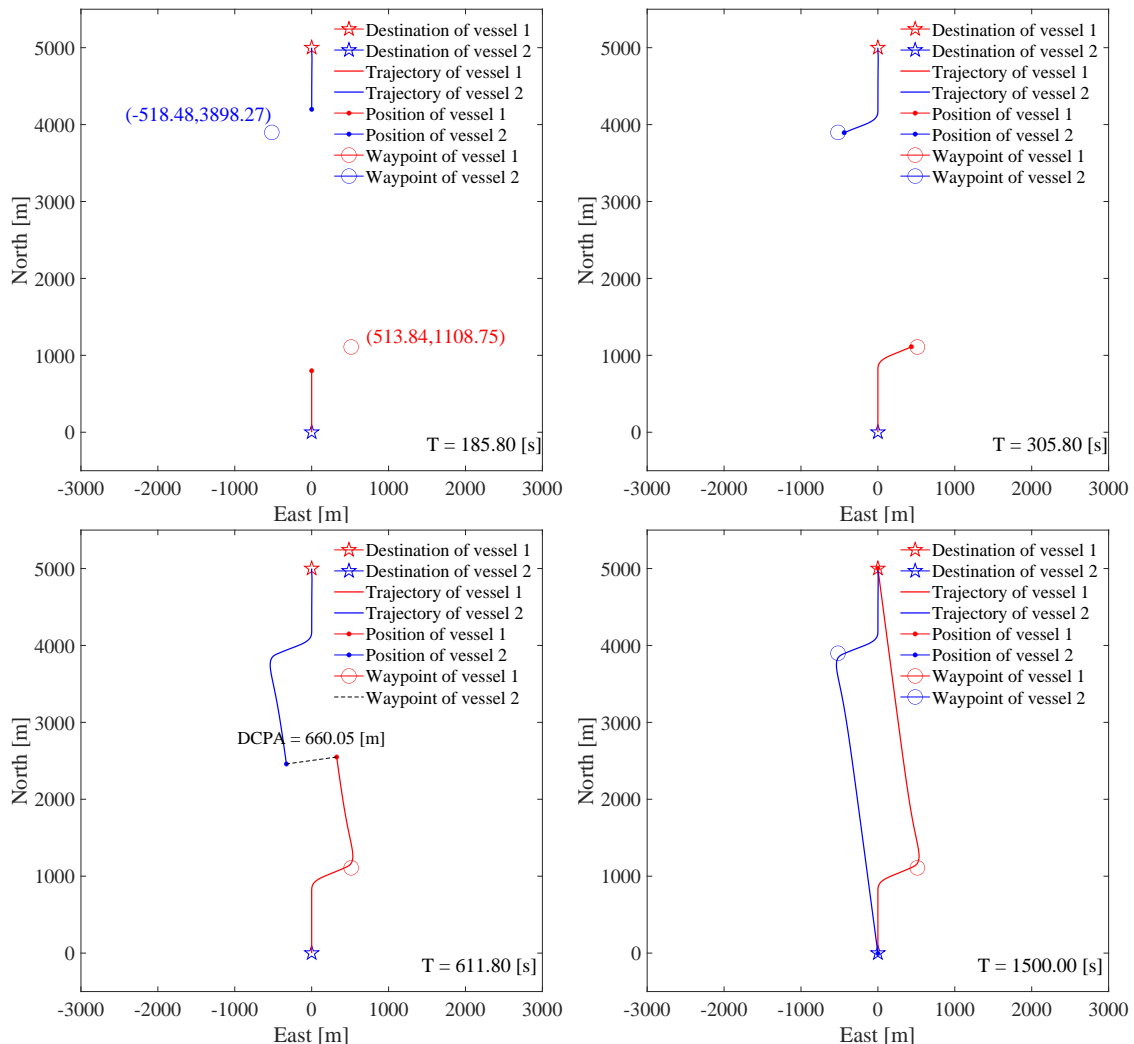


Figure 24: The process of collision avoidance for two cooperative vessels in head-on situation.

After a time slot at 305.80 seconds, no collision risk is detected and then vessels maneuver to their destinations through the guidance systems. Vessels accomplish the

navigation tasks with collision-free trajectories and the minimum relative distance is 660.05 meters at time 611.80 seconds.

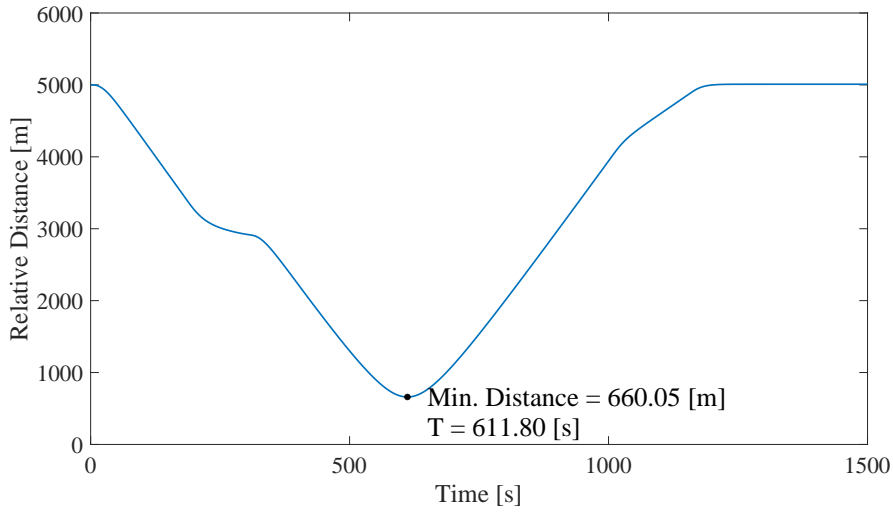


Figure 25: The relative and minimum distance between two vessels in head-on situation.

Scenario 1-3 - Crossing

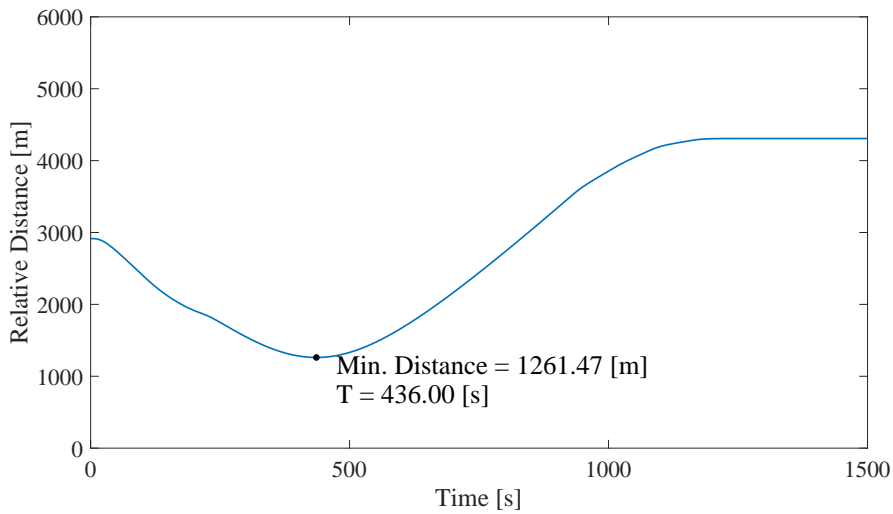


Figure 26: The relative and minimum distance between two vessels in crossing situation.

In scenario 1-3, crossing situation has been studied for two vessels encountering. Vessel 1 begins from (0,0) and travels to north, while vessel 2 navigates from (1500, -2500) in NED frame to east. Both vessels have the maximum speed of 5 [m/s]. Vessel 1 starts to change its course to the assigned collision-free waypoint (-1891.75,980.52) at time 86.80 seconds. After 120 seconds, it reaches the vicinity of the waypoint. Then vessel 1 continues its navigation towards the destination and no more collision risk is detected with vessel 2. The whole process is presented in Figure 27. The

relative distance between vessel 1 and vessel 2 reaches its minimum 1261.47 meters at time 436.00 seconds.

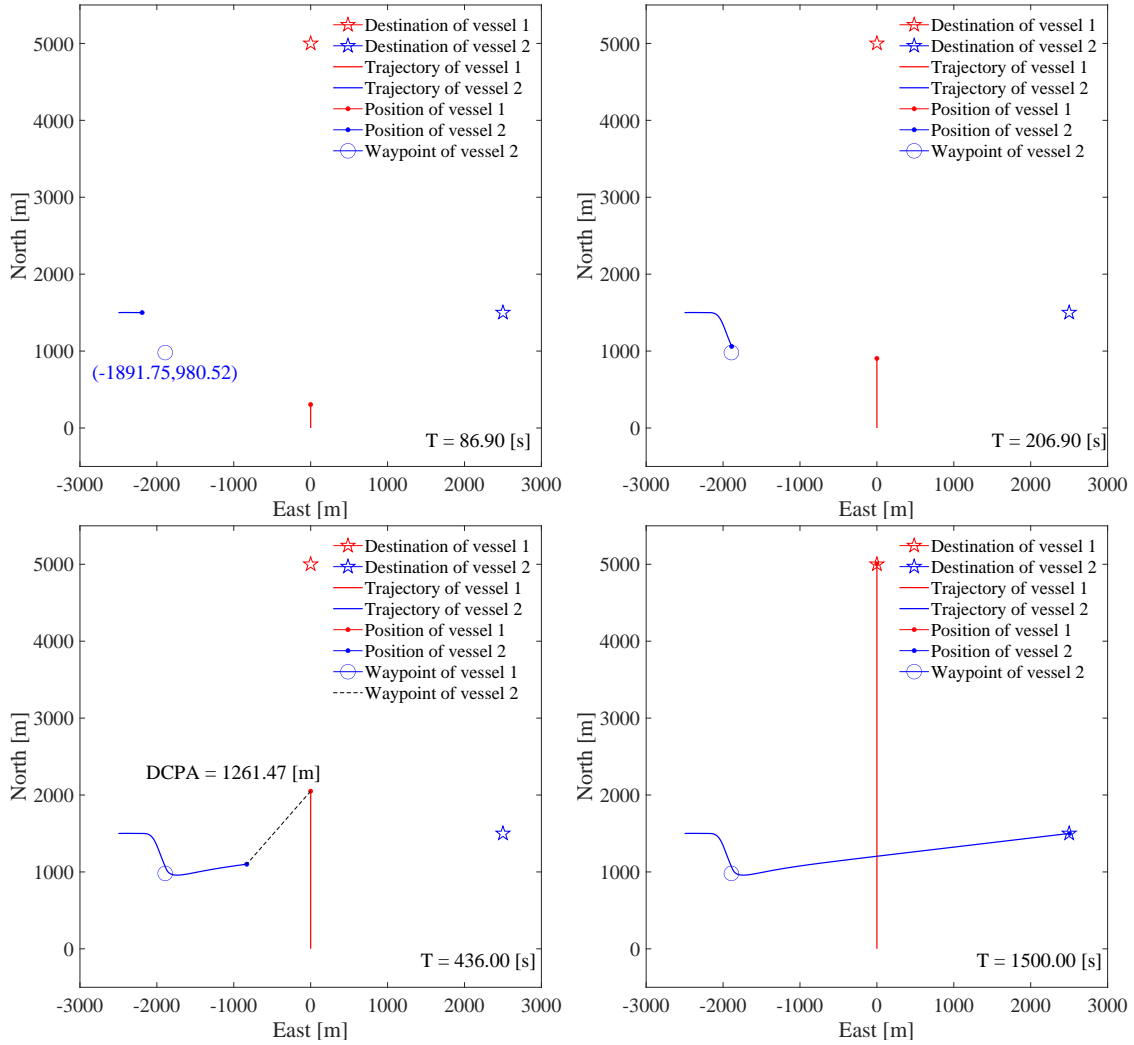


Figure 27: The process of collision avoidance for two cooperative vessels in crossing situation.

4.2 Simulation 2 - Collision Avoidance for 5 Cooperative Vessels

In simulation 2, five cooperative vessels encountering is manually designed not aiming for simulating real multiple vessels' encountering, but for showing the performance of the proposed optimization systems. Two scenarios are performed with the first one showing an ideal situation, while the second scenario is non-ideal due to one vessel's non-collaborating.

Scenario 2-1

In scenario 2-1, all five cooperative vessels will agree to collaborate with the master unit, which means that they follow the centralized suggestions and do collision-free maneuvering.

The initial position and final destination of five vessels are presented in Figure 28 and placed according to the COLREGs rule 15 and 17, where a vessel is supposed to give its way to the vessels on its starboard side in a crossing situation. For vessel 3 with green color, since there is no other vessels on its starboard side, it has the priority to keep its course and speed during the process. For the other vessels, there is one or more than one vessels on their starboard side, and thus they are supposed to do maneuvering to avoid collision. The maximum speed of each vessel are also set by manually. For vessel 1, vessel 3 and vessel 5, they have the maximum speed of 5 [m/s], while vessel 2 and vessel 4 have a slower speed of 4 [m/s].

The master unit assesses the collision risk with motion prediction at every time instant, and solves the centralized optimization when the risk is detected. Instead of solving the whole collision avoidance problem at once, the master unit only consider minimizing the collision risk for the next time slot which is equal to 120 seconds. And the whole rolling time horizon process ends when the process of collision avoidance finishes. Figure 29 shows how the course and speed of each vessel change during the collision avoidance as well as the optimal solutions of each sub-optimization problem. The hollow points means that optimization problem handles the bounds of the corresponding design variables with equality constraints. For example, vessel 1 is at safe when the first optimization problem is formulated, then the upper and lower bounds of its course and speed are equal to its current states, and no waypoint is assigned from the master unit. The solid points represent for the active solutions, where the upper and lower bounds are set based on COLREGs-compliant maneuvering. And the active waypoints as well as speed setpoints are then assigned to corresponding vessels for collision avoidance.

At time 519.70 seconds, optimal solutions with both course angle change and speed reduction are found for vessel 4 (black) to do maneuvering. Vessel 2 (blue) is assigned with a waypoint to give its way to vessel 3 (green). The waypoints of Vessel 1 (red) and vessel 5 (pink) are trade-off between apparent course changes and efficient solutions. At time 639.80 seconds, only vessel 1 is assigned with a waypoint to keep its maneuvering and prevent collision with vessel 3. No turning command is assigned with vessel 2, vessel 4 and vessel 5, and then they change their course to go to their destinations. However, at time 759.90 seconds, new collision risks are detected for vessel 4 and vessel 5 because they are facing vessel 3 from their starboard side, and vessel 2 is overtaking vessel 3 from behind in the mean time. Then another set of waypoints are assigned, as well as a speed reduction for vessel 5. For the last round of rolling horizon optimization at time of 1120.20 seconds, vessel 4 is keeping its maneuvering, vessel 1 has a waypoint assigned to overtake vessel 2 and no

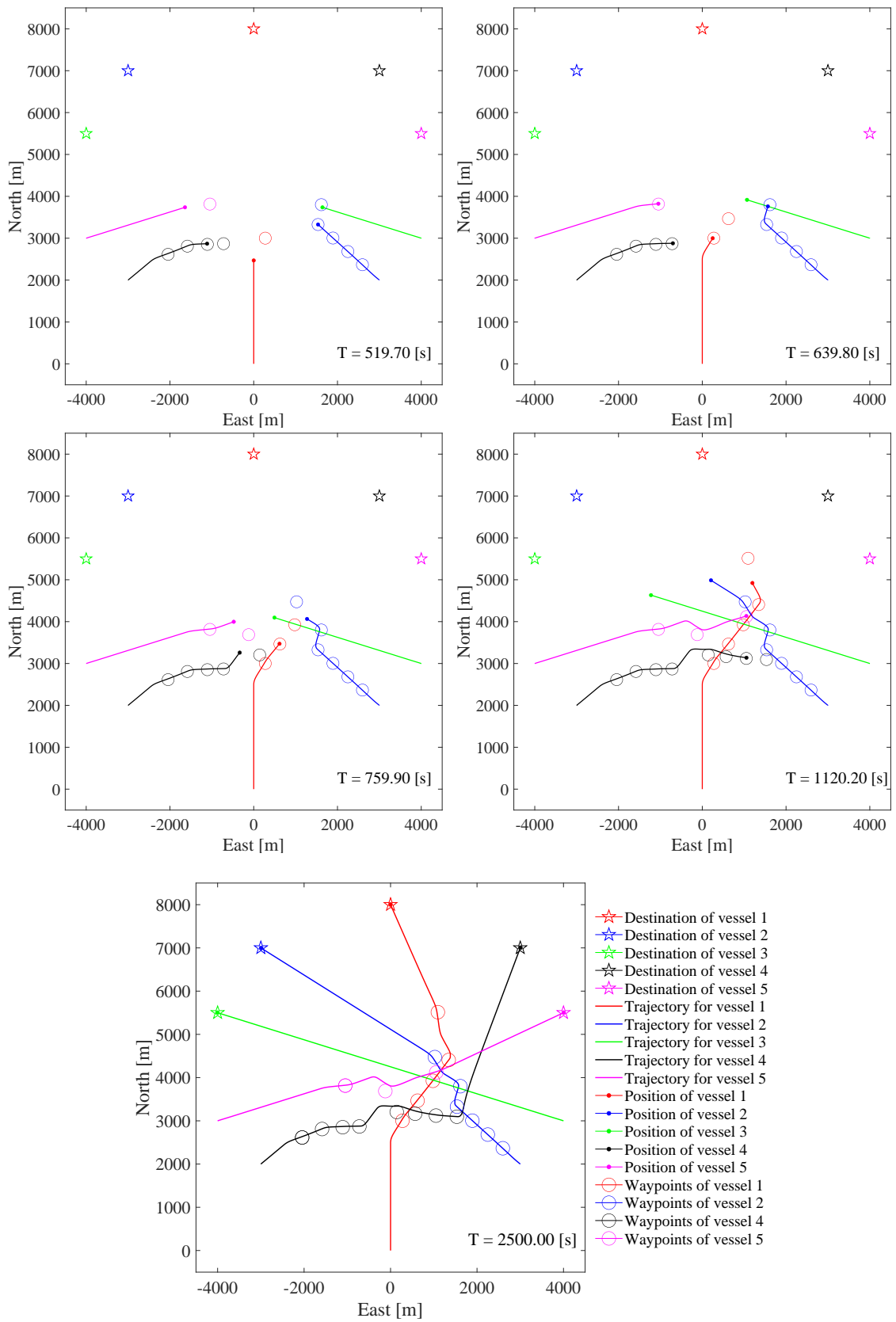


Figure 28: The process of collision avoidance for five cooperative vessels encountering.

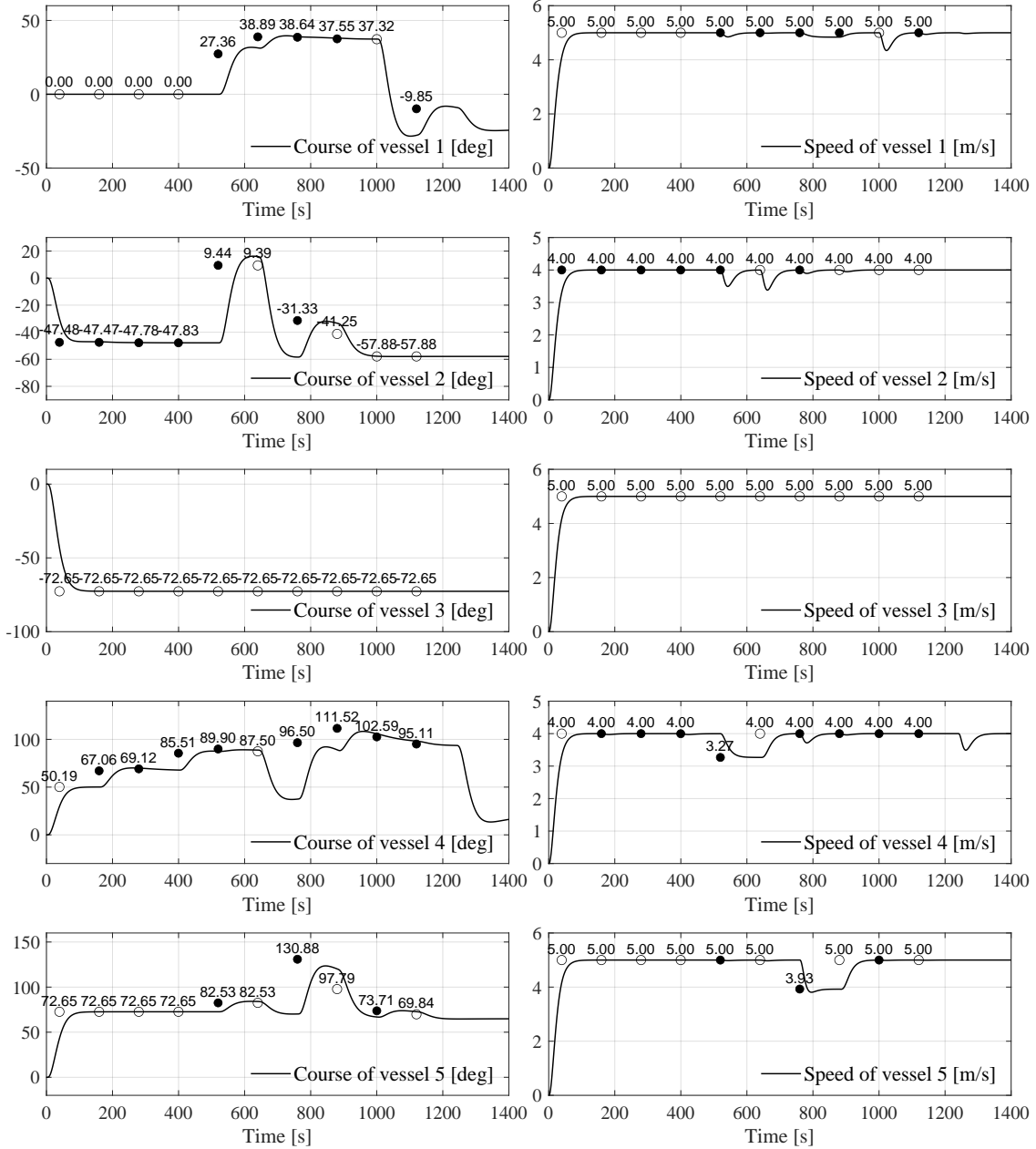


Figure 29: Course and speed of each vessel as well as the optimal solutions in scenario 2-1.

waypoints are assigned to vessel 2, vessel 3 and vessel 5. Figure 30 shows the relative distance during the whole process, and the minimum distance 328.14 meters among all the vessels is reached at time 575.80 seconds when vessel 2 is overtaking vessel 3.

In two vessels encountering, large and apparent course angle changes are encouraged to take according to COLREGs rule 8. However, in complex multi-vessels' encountering situation, a large course change may not guarantee the safety with all the other vessels. As shown in Figure 29, the optimization gives the optimal course change with more than 30 degrees in most cases, while sometimes small changes in heading angle can also guarantee the safety. For example, vessel 4 maneuvers with small course changes in the beginning since vessel 1 is on its starboard side. Same

action is taken by vessel 5 in its last maneuvering. Two speed reductions are also calculated by solving the formulated optimization system. In addition, small speed reductions are caused by vessel dynamics during the heading change. The last sub-figure in Figure 28 presents the whole trajectories and all the centralized waypoints, and all the vessels successfully avoid collision and reach their final destination.

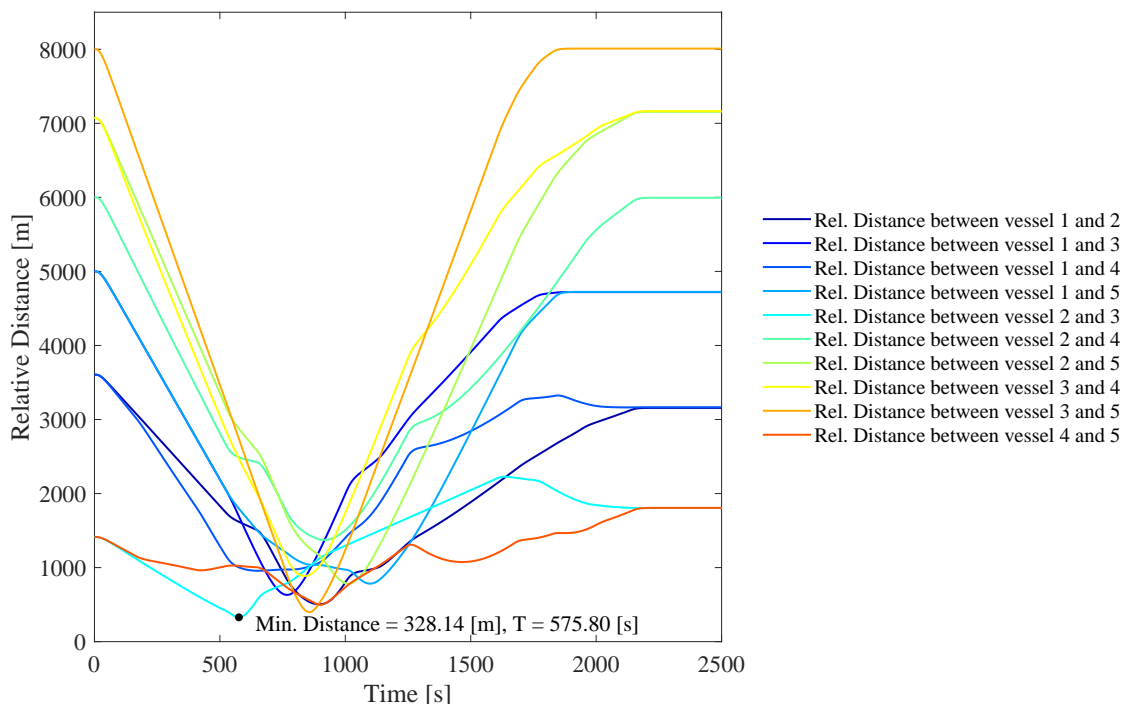


Figure 30: Relative distance between each pair as well as the minimum distance among all the vessels in scenario 2-1.

Scenario 2-2

Scenario 2-2 is aimed for showing the robustness of the system when one vessel doesn't want to collaborate with the master unit. This situation may happen in maritime world, for example an engaged-in vessel is restricted in its ability to maneuver. Such vessel can broadcast its intention to the master unit, and master unit can then utilize proposed course and speed in the centralized optimization problem.

In scenario 2-2, the initial conditions and final destinations of all the vessels stay the same. Vessel 5 is set to keep its course and speed during the whole process. As shown in Figure 31, at time 519.70 seconds, a waypoint is assigned to vessel 5 to do maneuvering, however vessel 5 doesn't want to follow this command and then sends feedback with its proposed the course and speed.

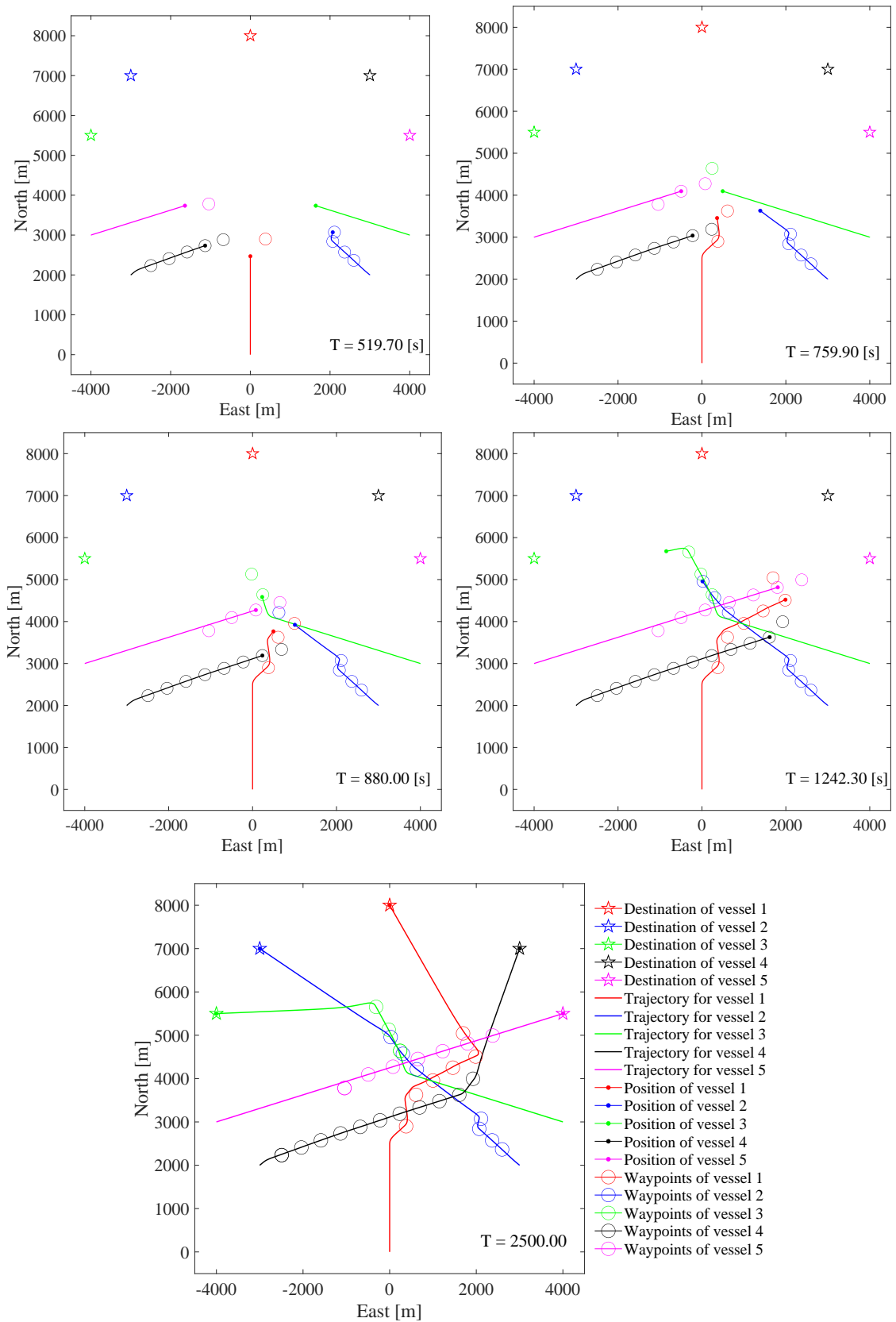


Figure 31: The process of collision avoidance for five cooperative vessels encountering with one vessel want to keep its course and speed.

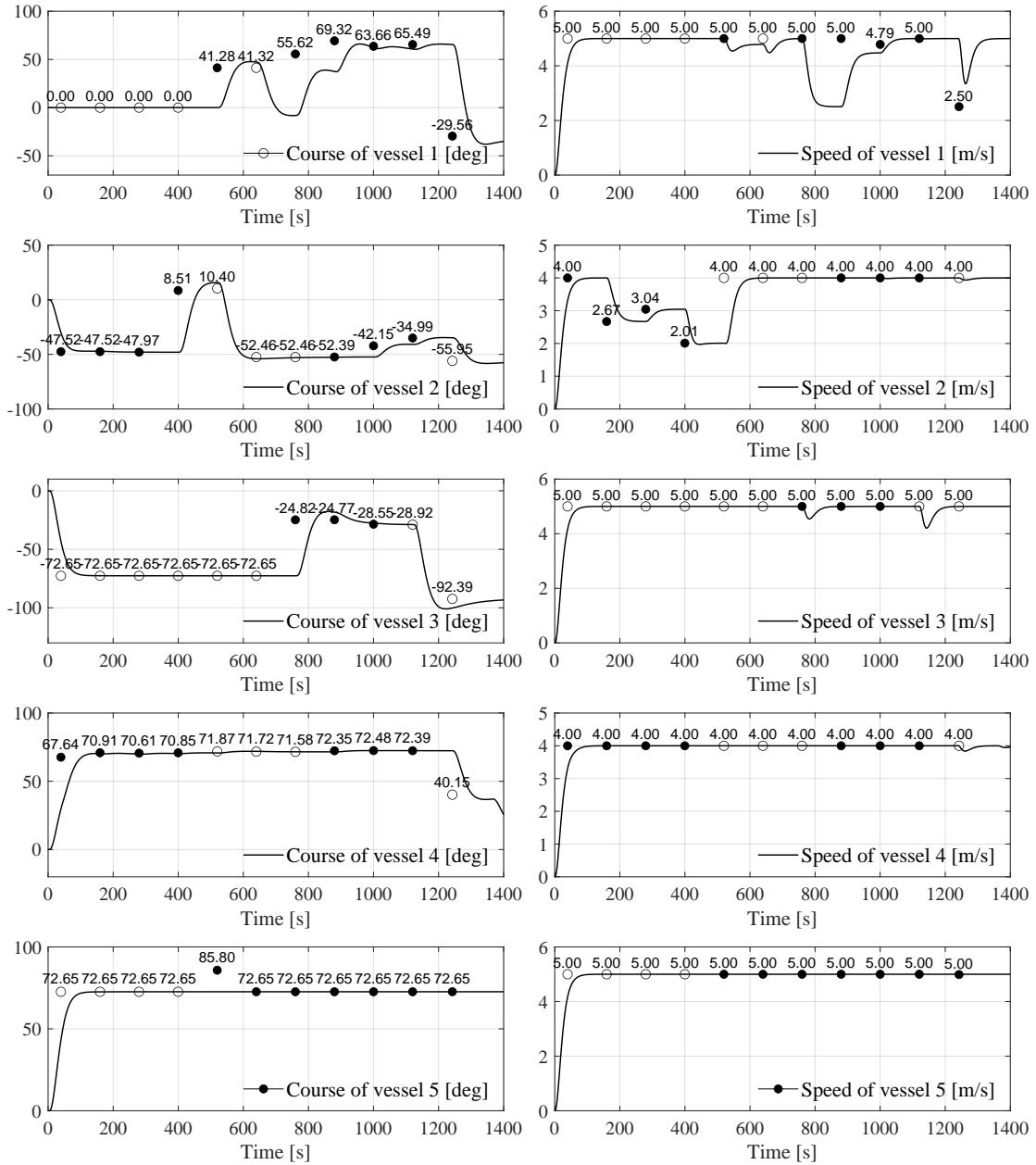


Figure 32: Course and speed of each vessel as well as the optimal solutions in scenario 2-2.

The master unit then consider such information in the centralized optimization for the next time slot. Then at time 759.90 seconds, a collision-free waypoint is assigned to vessel 3 who was supposed to stand on its way. The potential risk of collision with vessel 5 also forces vessel 1 to do another maneuvering with course angle change for 55 degrees. At time 880.00 seconds, all five vessels are assigned with collision-free waypoints from the master unit, where vessel 2 does maneuvering in a small course change to cross behind vessel 5. At last round of optimization, vessel 1 and vessel 4 are both assigned waypoints which are close to their final destination, and vessel 1 also suffers a speed reduction at the same time.

As presented in Figure 33, most of the relative distances of each vessel pair reach

their minimum during time period from 750 seconds to 1300 seconds when they are in a close encountering situation. The minimum relative distance among all the vessels is equal to 207.83 meters and appears at time 1295.90 seconds. All the vessels reach their destination with collision-free waypoints, and thus the optimization system shows good performance in handling this non-ideal situation.

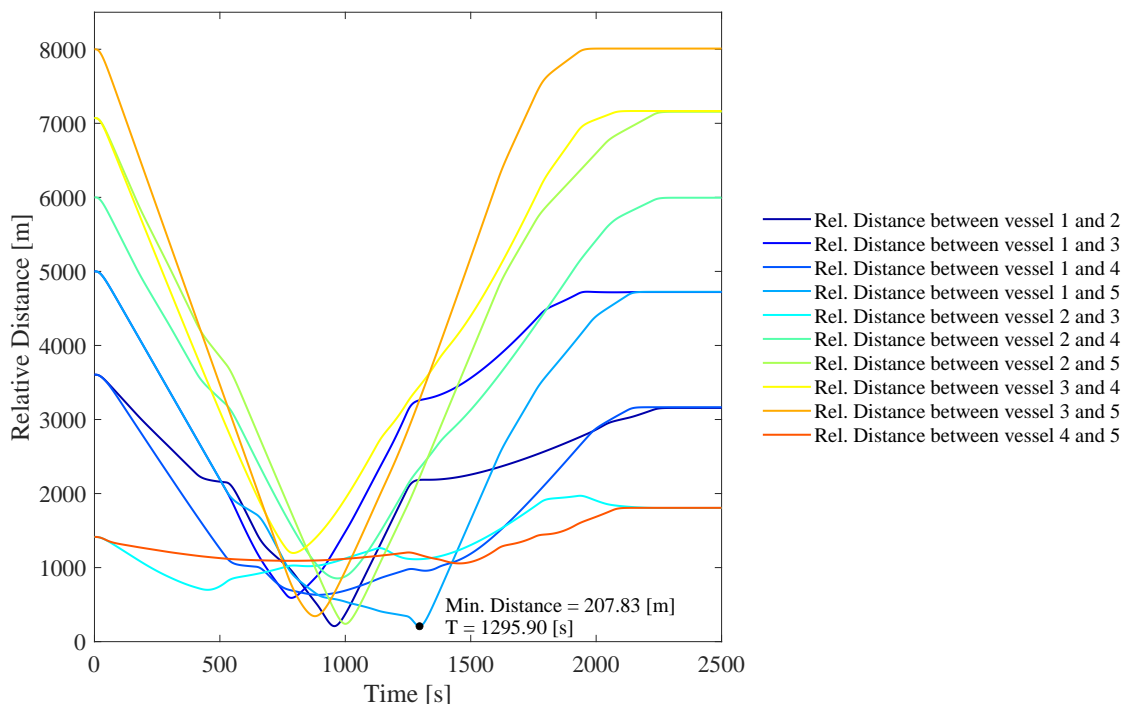


Figure 33: Relative distance between each pair as well as the minimum distance among all the vessels in scenario 2-2.

4.3 Simulation 3 - Collision Avoidance for 3 Cooperative Vessels, 2 Non-cooperative Vessels with Static Obstacles

To show the ability of proposed optimization systems working in complex environment, non-cooperative vessels as well as several static obstacles are manually designed in simulation 3.

A non-cooperative vessel doesn't have the ability to receive any information from the master unit, which could be a vessel suffering the communication failure or a vessel doesn't equip with AIS systems. The master unit cannot get any destination or intention from such vessels, and non-cooperative vessels also could not receive any collision-free waypoint. In this simulation test, two non-cooperative vessels are designed in the encountering situation, and they navigate in a predefined path and may do COLREGs non-compliant actions. The master unit can only acquire position and velocity information of these non-cooperative vessels and do straight line prediction for collision avoidance. Three cooperative vessels are also designed

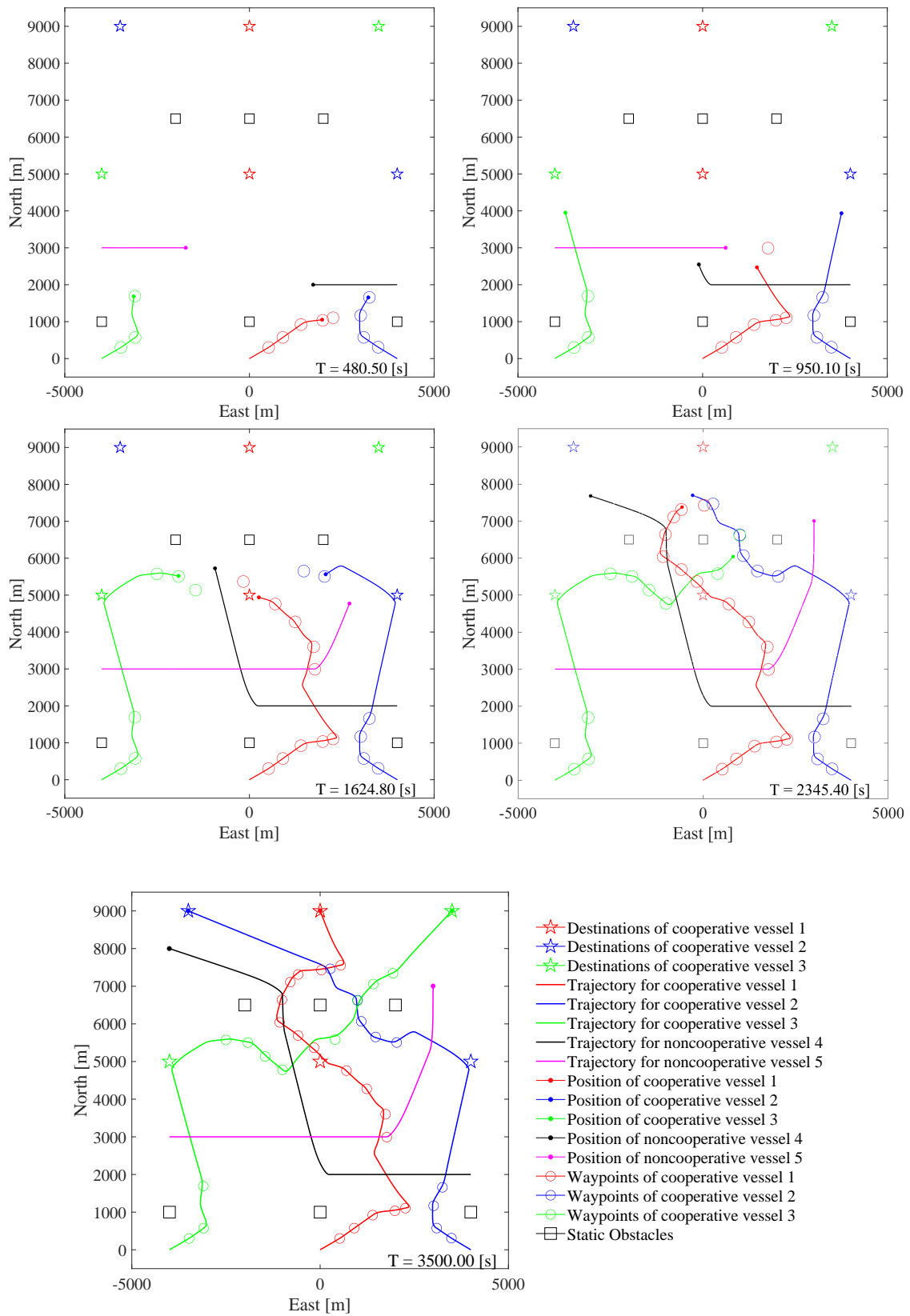


Figure 34: The process of collision avoidance for three cooperative vessels and two noncooperative vessels encountering with static obstacles.

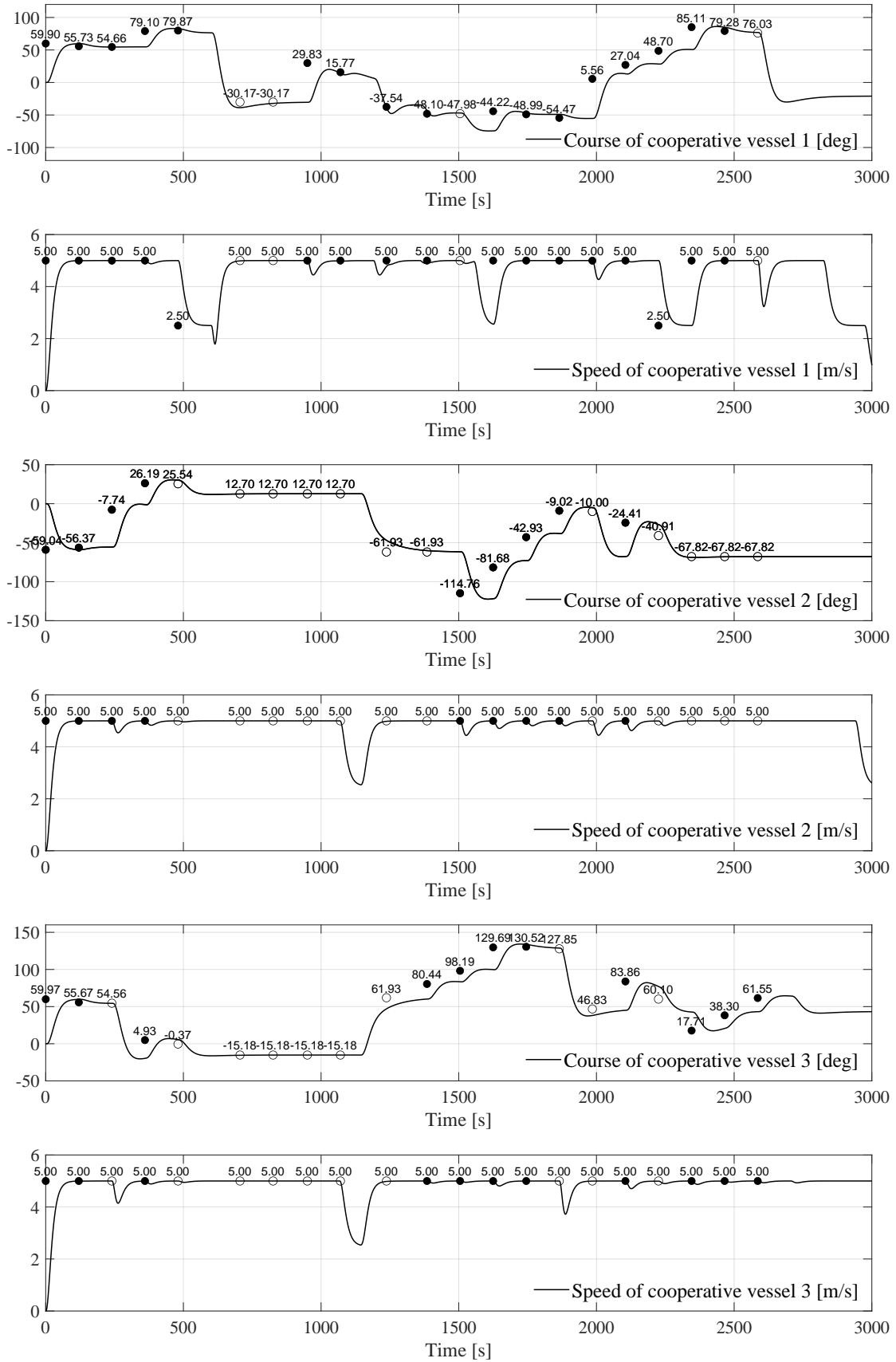


Figure 35: Course and speed of each cooperative vessel as well as the optimal solutions in simulation 3.

with following the centralized collision-free waypoints. In addition, each cooperative vessel has two task destinations to reach. Six static obstacles are manually placed in the environment and their coordinates in NED frame are known.

As presented in Figure 34, all three cooperative vessels have a static obstacle right ahead in the beginning. Vessel 1 and vessel 3 do a starboard maneuvering to avoid collision, while vessel 2 turns its heading in port-side. At time 480.50 seconds, vessel 1 receives a command of reducing its speed and waiting for vessel 2's passing by as shown in second subfigure of Figure 35. Then at time 950.10 seconds, cooperative vessel 1 is facing non-cooperative vessel 5 from its port side. Since the master unit predicts vessel 5's by assuming it will keep its course and speed for the next time slot, a collision-free waypoint is assigned to vessel 1 for COLREGs non-compliant maneuvering. The third subfigure shows the time instant at 1624.80 seconds when vessel 3 does starboard maneuvering to give its way to vessel 4 according to COLREGs rule 15 in crossing situation. In the meaning time, vessel 1 and vessel 3 are in a head-on situation, and both vessels take actions to avoid collision based on COLREGs rule 14. At time 2345.40 seconds, vessel 1 gives its way to vessel 2 in a crossing situation after a speed reduction. At the same time, vessel 3 is leading by the collision-free waypoints and passing through two static obstacles.

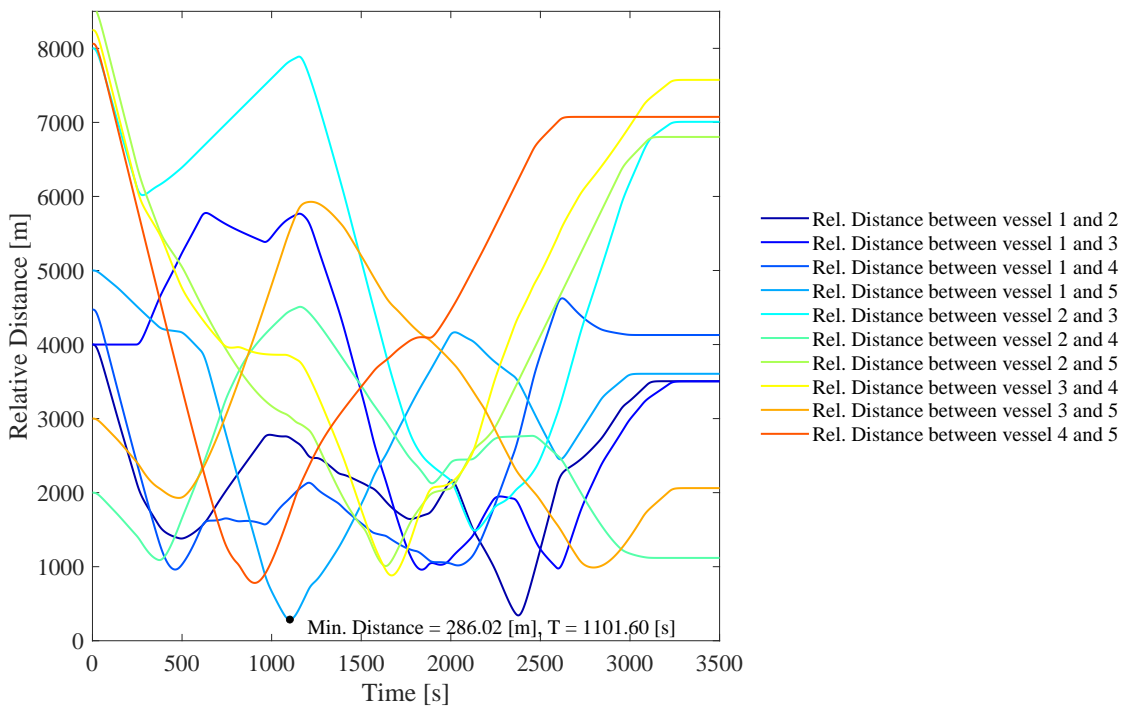


Figure 36: Relative distance between each pair as well as the minimum distance among all the vessels in simulation 3.

The minimum relative distance among all the vessels is 286.02 meters at time 1101.60 seconds when vessel 1 do maneuvering to avoid collision with vessel 5, which is marked in Figure 36. All the cooperative vessels then complete their mission to

reach the destinations and avoid any collision with non-cooperative vessels and static obstacles. Thus the optimization system shows good performance in handling the coexistence of both cooperative vessels and non-cooperative vessels in a complex environment with static obstacles.

4.4 Discussion

General Evaluation

From the simulation tests, the proposed centralized optimization system firstly shows good performance in two vessels' encountering situation for overtaking, head-on and crossing, where both vessels complete COLREGs-compliant collision avoidance. The existing path planning methods can also achieve good results in these scenarios (Hu et al., 2019). In simulation 2 with five cooperative vessels, the first scenario shows optimization system's ability of finding collision-free and COLREGs-compliant waypoints. Similar tests are also performed in the previous study in multi-vessels collision avoidance (Li et al., 2019). Rather than focusing on tackling the ideal encountering, this thesis emphasises more on non-ideal situation. The second scenario of simulation 2 considers vessel's non-collaborating behaviour, and the safety result validates the robustness of the optimization system. Further more, the proposed optimization system can also guide cooperative vessels with collision-free waypoints in a complex environmental condition in simulation 3 where non-cooperative vessels do actions in COLREGs non-compliance.

Solving the problem and giving a path or a set of waypoints at once may lose robustness to non-predicted actions in real operations such as vessels' non-collaborating and non-compliance motion. Rolling time horizon approach is thus adopted to separate the total collision avoidance process in several small sub-optimization problems, which improves system's ability and robustness in handling dynamic multiple vessels' encountering. In addition, it can also reduce the complexity of the problem. A short period of time slot can achieve safer results during the collision avoidance, while it may also lead to more frequent change in control input as well as increase the computation cost. This thesis chooses a fixed time slot as 120 seconds by tuning, which is a trade-off between safety and cost.

Instead of computing round collision-free trajectories, this thesis optimizes vessels' heading and speed to get coordinates of waypoints. Vessels could use their guidance system to track the assigned waypoints and complete the collision avoidance. In realistic maritime operations, such waypoints are more straightforward for human operators and requires less information exchange compared to round trajectories.

As shown in the simulation tests, the minimum distance don't break the setting threshold 150 meters in the penalty function. Furthermore, optimization system gives the solutions with apparent course changing in most cases, which is in accord

with COLREGs rule 8, *alteration of course alone may be the most effective action to avoid a close quarters situation*. With safety guaranteed, a few optimal solutions suggests small course alteration or speed reduction.

Future Works

The simulation results from this thesis indicate how to deal with non-ideal situations in multi-vessels' collision avoidance. However, several assumptions have been made to simplify the problem, which may cause problems in real operation. Moreover, there are some limitations of the centralized methods exist. Hence, more works could be done in the future to realize a better optimization systems for collision avoidance.

The centralized collision-free waypoints are calculated based on a straight line prediction of vessels' current position with optimal course and speed in a time slot ahead. Even though the simulation results show good performance in safety with straight line prediction, vessels are hard to reach the accurate position of waypoints especially when they maneuver with large course angle change. The existing physics-based motion prediction methods like extend Kalman Filter or using vessel's maneuvering model all require parameters of vessels' mass and damping matrices as well as control inputs (Huang et al., 2020), which could be inaccessible for a centralized station. Hence, a model-based motion prediction can be developed in the future if a vessel can share its mass and damping to the master unit.

No environmental disturbance is assumed in this thesis, however current, wave and wind are significant external forces for marine vessels, which could affect vessel's motion (Fossen, 2011). In real operations, VTS centre can obtain the hydrological and meteorological environmental data in the fairway (IMO, 1985). Thus objective functions regarding the effects of current and winds could be developed in the centralized optimization problem.

In this thesis, there are four objective functions with one penalty function considered to formulate the centralized optimization problem. Thus a set of weighting parameters should be chosen and tuned by manually, which could be a burden. Multi-objective optimization methods could be implemented and get solutions with Pareto optimal (Coello et al., 2004).

Other non-ideal situation for centralized methods in real maritime world could be the existence of undetected obstacles like small floatages or small recreational vessel. The centralized solutions may lead to collision due the lack of obstacles' position. However, such obstacles can be easily handled by the local collision avoidance for individual vessel. As a future work, a hybrid collision avoidance scheme could contain both local and global perspective. By developing a decision-making system, a ASV can evaluate the centralized and individual solutions and achieve more robustness in collision avoidance.

5 Conclusion

This thesis aims to answer the research question about whether or not a collision avoidance system can find the optimal maneuvering in both ideal and non-ideal condition in multiple vessels encounter. A centralized optimization scheme is proposed to handle the coexistence of cooperative vessels and non-cooperative vessels in collision avoidance. The information exchange is also utilized in solving cooperative vessels' non-collaboration. A centralized optimization problem is thus formulated with the objective regarding finding safe, efficient and COLREGs-compliant maneuver. Then a rolling time horizon approach is adopted to separate dynamic collision avoidance problem into a series of sub-optimization problem, and then particle swarm optimization is used for finding global minimal.

The whole systems is implemented in Matlab/Simulink, which includes vessel dynamics and centralized optimization. Several simulations tests in a wide range of scenarios have been performed to validate the proposed optimization system. All the tests achieves promising result in safety, and the optimization system shows robustness in designed non-ideal conditions of non-collaborating and non-compliance.

For further works, suggested model-based motion prediction methods can provide more effective collision-free waypoints. Moreover, objective functions regarding environmental disturbance can also be formulated in the centralized optimization problem. To release the burden of parameters tuning, multi-objective optimization methods can be adopted. In addition, a decision-making system can be developed for individual vessel to achieve robustness when considering undetected obstacles for the master unit.

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