Master's thesis

NTNU
Norwegian University of Science and Technology
Faculty of Economics and Management
Dept. of Industrial Economics and Technology Management

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Evaluation of Tieback Developments for Marginal Oil Fields With Timing Flexibility

Master's thesis in Industrial Economics and Technology Management
Supervisor: Hagspiel-Janssen, Verena
Co-supervisor: Fedorov, Semyon and Haseldonckx, Sophie (NPD)
June 2022

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NORWEGIAN PETROLEUM
DIRECTORATE

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Preface

This thesis is submitted as the concluding part of our Master of Science degrees in Industrial Economics and Technology Management, specializing in Financial Engineering, at the Norwegian University of Science and Technology (NTNU). The thesis is written as an academic paper and is planned to be submitted to peer-reviewed international journal in the area of petroleum and energy economics. The paper is conducted in collaboration with the Norwegian Petroleum Directorate (NPD) and is part of the BRU21–NTNU Research and Innovation Program on Digital and Automation Solutions for the Oil and Gas Industry (`www.ntnu.edu/bru21`).

The authors of this thesis, Anders Rønning, Johannes H. Haugsgjerd, and Richard W. H. Rogstad, combined their versatile backgrounds in finance, optimization, and computer science on a real case given by NPD. The result is a financial model used to evaluate tiebacks for marginal oil fields on the Norwegian Continental Shelf.

We sincerely thank Professor Verena Hagspiel and Doctor Semyon Fedorov for their helpful guidance and much-appreciated feedback. Their inputs have been vital for the outcome of this thesis as they allowed us to explore and better grasp the fields of real options and E&P investments. Last but not least, we would like to thank Sophie Haseldonckx at NPD for sharing her expertise and willingness to provide valuable insight into the industry and data in a specific case.

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Abstract

The average size of new oil discoveries on the Norwegian Continental Shelf (NCS) is steadily decreasing, which increases the risks associated with investment decisions in the Exploration and Production (E&P) of petroleum fields. Therefore, marginal fields risk remaining unexploited, thereby losing value for the Norwegian society. As standalone developments are often not economically viable for such fields, tiebacks to existing production facilities are usually considered. At the same time, many production facilities in mature production areas have spare capacity due to depleted reservoirs. Furthermore, E&P projects are subject to significant market and technical uncertainties in terms of volatile market prices and uncertain estimations of the field potential. Additionally, marginal oil fields are relatively more uncertain than large ones because there is generally less data gathered before development. In this context, novel solutions must be developed to commercialize small discoveries under prominent uncertainties.

We establish a model that: (1) evaluates tieback development concepts; (2) determines the optimal choice of host facility for the field operator; and (3) optimizes the timing of development. Firstly, we develop a mathematical optimization model that maximizes petroleum production given the field potential, well capacity, and spare host capacity. Next, we model future oil and gas prices using two-factor stochastic models, where the gas price is correlated with the oil price. CAPEX is modeled as a GBM and also correlated with the oil price. By following a real options approach (ROA), we allow for managerial flexibility in terms of waiting-to-invest, which we solve by applying a Least-Squares Monte Carlo (LSM) algorithm. Finally, the proposed model is applied to a real case on the NCS.

The results suggest that marginal field developments carry large upside potential, which can be exploited by our methodology. Secondly, we found that no factors alone were able to change the optimal choice of host in our case because one host was evidently much more attractive. Anyway, the analyses we performed suggest that reducing the field operator's CAPEX or altering the parameters of the tariff scheme in combination with extending the lifetime of the host are the most efficient measures for the host to become the preferred choice. Thirdly, we identified that the value of timing flexibility increases as the profitability of the project decreases or the uncertainty of the investment increases. As marginal oil field developments often are characterized by relatively low profitability and prominent uncertainties, managerial flexibility is usually of high importance. Hence, ROV proves itself as a better valuation method as it allows us to capture the value of flexibility, while NPV tends to underestimate such investments. The combination of ROA and mathematical optimization in our methodology constitutes a novel contribution to the literature.

***Keywords* -** Petroleum economics, marginal fields, optimal tieback selection, real options

Sammendrag

Gjennomsnittlig størrelse på nye oljefunn på norsk kontinentalsokkel synker stadig, noe som øker risikoen knyttet til investeringsbeslutninger i leting og produksjon av petroleumsfelt. Derfor setter det marginale felt i fare for å forbli uutnyttede og dermed miste verdier for det norske samfunnet. Siden frittstående feltutbygginger ofte ikke er lønnsomme for slike felt, vurderes vanligvis tilknytning til eksisterende produksjonsanlegg istedenfor. Samtidig har mange produksjonsanlegg i modne produksjonsområder ledig kapasitet på grunn av uttømte reservoarer. Videre er leting- og produksjonsprosjekter gjenstand for betydelig markedsmessig og teknisk usikkerhet når det gjelder volatile markedspriser og usikre estimeringer av feltpotensialet. I tillegg er marginale oljefelt relativt mer usikre enn store oljefelt fordi det generelt innhentes mindre data før utvikling. I denne sammenhengen må det utvikles nye løsninger for å kommersialisere små funn under fremtredende usikkerheter.

Vi etablerer en modell som: (1) verdsetter utviklingskonsepter for tilbakekoblinger, (2) bestemmer det optimale valget av vertsanlegg for feltoperatøren, og (3) optimaliserer investeringstidspunktet. For det første utvikler vi en matematisk optimaliseringsmodell som maksimerer petroleumsproduksjonen gitt volum av olje og gass i feltet, feltpotensialet og ledig kapasitet på produksjonsanlegget. Vi modellerer fremtidige olje- og gasspriser ved å bruke tofaktors stokastiske modeller, der gassprisene er korrelert med oljeprisen. CAPEX er modellert som en GBM, og er også korrelert med oljeprisen. Ved å følge en realopsjonstilnærming, tillater vi fleksibilitet i form av å vente med å investere, som vi løser ved å bruke en Least-Sqaures Monte Carlo (LSM) algoritme. Den foreslåtte modellen anvendes på en reell case på norsk kontinentalsokkel.

Resultatene tyder på at marginale feltutbygginger har et stort oppsidepotensial, som kan identifiseres av vår metodikk. For det andre fant vi ut at ingen faktorer alene var i stand til å endre det optimale valget av vert i vårt tilfelle fordi én vert var vesentlig mer attraktiv. Analysene vi utførte tyder på at endring av parametrene til tariffordningen i kombinasjon med å forlenge levetiden til verten er de mest effektive tiltakene for at verten skal bli det foretrukne valget. For det tredje identifiserte vi at verdien av tidsfleksibilitet øker når lønnsomheten til prosjektet reduseres eller usikkerheten til investeringen øker. Siden marginale oljefeltutbygginger ofte er preget av relativt lav lønnsomhet og fremtredende usikkerhet, er fleksibilitet vanligvis av stor betydning. Derfor viser ROV seg som en bedre verdsettelsesmetode da den lar oss fange opp verdien av fleksibilitet, mens NPV har en tendens til å undervurdere slike investeringer. Kombinasjonen av ROA og matematisk optimalisering i vår metodikk utgjør et nytt bidrag til litteraturen.

***Nøkkelord* -** Petroleumsøkonomi, marginale felt, optimalt valg av tilbakekobling, realopsjoner

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Acronyms

ABEX Abandonment Expenditures

BBL Barrels of petroleum liquids

BTU British Thermal Units

CAPEX Capital Expenditures

CAPM Capital Asset Pricing Model

DA Decision Analysis

DCF Discounted Cash Flow

E&P Exploration & Production

GBM Geometric Brownian Motion

LSM Least Squares Monte Carlo

MS Markov Switching

NPV Net Present Value

NCS Norwegian Continental Shelf

NGL Natural Gas Liquids

NOK Norwegian Krone

NPD Norwegian Petroleum Directorate

NTNU Norwegian University of Science and Technology

OPEX Operational Expenditures

PDE Partial Differential Equations

R&D Research and Development

RM Reservoir Management

ROA Real Options Analysis

ROV Real Options Valuation

Sm^3 o.e. Standard Cubic Meter Oil Equivalents

USD United States Dollar

1 Introduction

The petroleum industry has been central to today's welfare society in Norway. It has been the most critical industry for the Norwegian economy regarding revenue to the Treasury, investments, and share of total value creation for decades (Norwegian Petroleum Directorate n.d.*b*). However, the Norwegian petroleum industry has recently encountered many critical challenges. Firstly, there is an increasing focus on transitioning to a more sustainable and climate-friendly society due to growing concern about global warming. Naturally, the role of the petroleum industry in society is being questioned, and many consumers and executives are calling for the end of O&G production. However, petroleum will be necessary for many years as no genuine alternative currently exists (Rystad Energy 2021). Secondly, the average size of discoveries has been steadily decreasing over the last decades. The average size of discovered reserves on the NCS in the '80s was between 80 and 100 million Sm^3 o.e. In contrast, the corresponding figures in the last 20 years have been below 10 million Sm^3 o.e. (see Figure 1). Since exploration and development of smaller oil fields require expensive technology and advanced engineering solutions to access them (Lund 1999), it is usually considered less attractive by E&P companies. The economic value of so-called *marginal oil fields* is small compared to significant discoveries and usually does not warrant standalone production facilities. Lower market prices can make the project not economically viable, where as previously most discoveries were highly profitable. Hence, there is a growing need for thorough economic analyzes of marginal oil fields before investments are made.

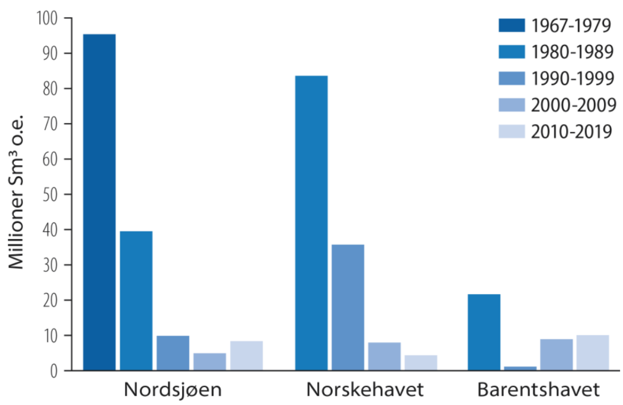


Figure 1: The average discovery size during the past decades, distributed on sea areas (Norwegian Petroleum Directorate 2020).

At the same time, many existing production facilities are approaching the end of their lifetimes as the production volumes are declining. At the start of 2021, 22% of the petroleum fields on the NCS had already reached maturity, and several of the largest reservoirs contained less than 10% of their original potential (Norwegian Petroleum Directorate 2020). Since the production is declining, it enables new petroleum sources to be connected to the existing infrastructure. These existing production facilities are becoming relevant for tiebacks of small, neighboring discoveries. The NPD estimates that less than 50% of the recoverable petroleum on the NCS is extracted and that value creation from further exploration is between NOK 1,200 billion and NOK 1,700 billion (Norwegian Petroleum Directorate 2021). Hence, smaller reservoirs may still provide substantial value as long as the decision-making process adequately addresses the field development risks and upside potentials.

Novel solutions must be established to commercialize marginal oil fields. In the present paper, we consider this problem from the perspectives of a field operator, a host facility owner, and the Norwegian society. Specifically, we study an undeveloped marginal oil field and two existing production facilities nearby, to which a tieback is technically feasible. The decision problem for the field operator consists of assessing if any or both of the tiebacks are economically feasible. If more than one tieback development concept is profitable, the decision is to identify the *most* profitable one. Moreover, if there is at least one profitable tieback, the decision problem also includes determining the optimal timing of investment. This is because waiting for favorable market conditions can enhance the project value.

The decision problems mentioned above relate to the field operator and the Norwegian society as both parties seek to maximize the value of the marginal oil field. However, in an extended section (see Section 6), we discuss how their objectives may conflict with each other in an optimization problem that consists of several potential tieback developments. The decision problem of the host facility owner consists of maximizing its profits by charging as much tariffs as possible while being the preferred choice among alternative host facilities for the field operator. Considerable costs are associated with abandoning the host facility, so any additional production due to tiebacks are of great value for the host owner.

We propose a novel methodology to evaluate tieback development concepts of marginal oil fields. Firstly, we develop a mathematical optimization model, which estimates production profiles based on: (1) the spare capacity at the host facility; (2) the initial O&G in place in the field, and; (3) the field potential, i.e. the maximum rate at which extracting the petroleum is possible. Secondly, we construct future O&G prices by using a two-factor stochastic prices process, which is calibrated using the Kalman filter on historical market data. As preliminary literature suggests, we let CAPEX and gas prices be correlated to oil prices by following the procedure of Thomas & Bratvold (2015) and Fedorov et al. (2022). This framework allows us to replicate the characteristics of the petroleum market in the real world. Thirdly, we estimate the development and operational expenses throughout the lifetime of the field. Through the combined simulations of production profiles, revenues, and costs, we construct several sets of the expected net present value (NPV) associated with the different timing of developments. To account for the managerial flexibility, we follow a real option approach (ROA), which is solved by using the least-squares Monte Carlo (LSM) framework proposed by Longstaff & Schwartz (2001). This allows us to identify the optimal timing of investment for each simulated case. We then calculate the value of the development concepts and determine the optimal tieback for the undeveloped field.

Our key contributions can be summarized as follows: (1) we establish an optimization model that maximizes O&G production rates; (2) we evaluate tiebacks for marginal oil fields under market, and; (3) we account for managerial flexibility in terms of postponing investment pending more favorable conditions. Our results lead to insights that can facilitate and enhance the allocation of tiebacks for marginal oil fields. Furthermore, the results help us to determine key drivers of optimal host selection. Thus, the proposed methodology is expected to give both academic and industry value.

1.1 Literature Review

We seek to contribute to two main strands of literature. The first strand pertains to option valuation problems in the petroleum industry, where we initially give a historical context of the matter. Tourinho. (1979) was arguably the first to evaluate oil reserves by using option-pricing techniques[1]. Later, Siegel et al. (1987), Paddock et al. (1988), and Chorn & Carr (1997) developed a methodology for the valuation of claims on an offshore petroleum release. They conclude that their methodology has several advantages over the static NPV as it requires less data, has lower computation costs, and provides a guide for optimal timing of development. Bjerksund & Ekern (1990) demonstrated that if an option exists to delay the investment, managerial flexibility of temporarily shutting down production or permanently abandoning the project will only have minor additional effects. This finding is relevant to our work as we only consider the flexibility of postponing the oil field development investment. In the work of Galli et al. (1999), three different methods to evaluate E&P projects are discussed; Monte Carlo simulation, decision-tree, and option pricing. They address several issues that are relevant to our methodology. Firstly, although there exists evidence of a correlation between variables (e.g., oil and gas prices), these are commonly treated as independent. Secondly, the time value of money is handled differently for ROV methods (use risk-free discount rate) than for traditional methods (use risk-adjusted discount rate). We discuss these issues in further detail in Section 3.

A growing body of literature has proposed the use of ROA in the petroleum industry during the last 20 years. In the following, we will present a selection of papers closest related to our work. Jafarizadeh & Bratvold (2009*a*) and Soares & Baltazar (2010) compare different real option approaches using relevant O&G investment cases. Cortazar & Schwartz (1998) developed and implemented a ROV model to value an undeveloped oil field and determine the optimal timing of investment. They assume the oil price follows a two-factor model, proposed initially by Gibson & Schwartz (1990), and solves the problem using the LSM suggested by Barraquand & Martineau (1995). They found that postponing the production would increase the field's value by approximately 10%. This paper is highly relevant to the present paper as the same problem is examined. However, our model is more sophisticated as we address more complexities in modeling market uncertainty by including gas prices and the correlation between CAPEX and oil and gas prices.

Furthermore, we incorporate an optimization model for production profiles. Jafarizadeh & Bratvold (2009*b*) also developed a ROV model for an undeveloped oil field, which is solved using the LSM framework of Longstaff & Schwartz (2001). They implement the two-factor price model by Schwartz & Smith (2000) and conclude that the choice of commodity price model significantly affects the real option value. Many advantages of the LSM framework are highlighted, and the results support that of Cortazar & Schwartz (1998); allowing for managerial flexibility adds value to the field. Jafarizadeh & Bratvold (2012) discuss termination flexibility in an E&P project by applying ROV and demonstrate how it can serve as a decision tool for petroleum managers. Other notable studies that have been conducted on ROV in E&P development include Laughton (1998), Chorn & Croft (2000), Lazo et al. (2003), and Costa Lima & Suslick (2006).

The previously mentioned papers all have in common that they only account for market uncertainty. Nonetheless, technical uncertainty constitutes another crucial risk concerning E&P investments. We review literature that incorporates both market and technical uncertainty. Dias (1997) analyzed the timing of investment in E&P of oil fields while accounting for both uncertainties. The conclusion suggests that economic uncertainty (e.g., oil prices and costs) is dominant in the development decisions.

In contrast, technical uncertainty (e.g., existence, quality, and volume of reserves) is dominant in exploratory investment decisions. This finding is relevant to the present paper as a development investment is considered, which is why we focus on market uncertainty. Dias (2002) investigates the effect of timing and drilling games with strategic interaction in E&P projects by

[1] The first study on the economic feasibility of exploring an oil field can be traced back to the works of Allais (1956), who applied probability theory to model sequential stages of exploration. Other relevant research during this period include Drew (1967), Harbaugh (1984), and Harris (1990), among many others.

explicitly modeling the value of learning through a dynamic real options model. Dias (2004) presents a set of selected real options models to evaluate E&P investments in petroleum under market and technical uncertainties. Chorn & Shokhor (2006) combined dynamic programming with a ROV algorithm to value investment opportunities related to petroleum exploration. Willigers & Bratvold (2008) demonstrated how LSM simulation could be used to value O&G options, where they account for gas price, OPEX, and rate of production decline. Parra-Sanchez (2010) combined deterministic studies with stochastic modeling and risk analysis to assess decision-making under uncertainty for a reservoir.

The sensitivity analysis suggests that the original oil in place and the oil price are the most influential variables in the optimization of a reservoir. Thomas & Bratvold (2015) considered uncertainties in oil and gas reserves and production as well as in transition costs when evaluating the time to switch from oil to gas production in a production facility, which is solved by the LSM framework. Castro & Singh (2019) studies the investment problem of an undeveloped oil field under market and technical uncertainty. They discuss different types of managerial flexibilities and different approaches to model the behavior of future oil prices. Their findings suggest that there is considerable value in accounting for managerial flexibility (similar to Majd & Pindyck (1987), Cortazar & Schwartz (1998)). In addition, there is a significant value in obtaining more information about the reservoir size, and this value increases with low oil prices.

The second strand of literature pertains to the economic assessment of marginal oil fields. Laine (1997) presented a binomial option valuation model to calculate values of deferral, expansion, and abandonment options of two real fields: Brage and Asgard. The results showed that option valuation technique could add substantial value to marginal oil fields. Lund (2000) developed a stochastic dynamic programming model for evaluating offshore petroleum projects under both market and reservoir uncertainty. To illustrate the qualities of the developed framework, the model was implemented for a small oil field, Midgard, where the results reveal a significant value of flexibility. Galli et al. (2001) used a real option framework on a satellite gas field close to a large gas and condensate field. This paper presents a new way of sequentially updating the technical parameters (e.g., size of reserves, the productivity of wells) in response to new well information. Armstrong et al. (2004) incorporated technical uncertainty in the ROV of small oil projects by combining it with Bayesian updating based on Archimedean copulas. Stoisits et al. (2010) developed a model to assess development concepts for two offshore satellite fields near two existing offshore production facilities. They concluded that it would be most profitable to tie the satellite fields to existing facilities rather than develop a dedicated standalone facility. Fleten et al. (2011) valued two real options related to offshore petroleum production. They consider expanding an offshore oil field by tying in a satellite field and the option of early decommissioning. Even if the satellite field is not profitable to develop at current oil prices, the option to tie in such satellites can have a significant value if the oil price increases. Two sources of uncertainty are considered: oil price risk and production uncertainty, and the option valuation is based on LSM. Lin et al. (2013) presented a methodology that evaluates three kinds of flexibility as a means to mitigate uncertainty in subsea tiebacks: the ability to tie back new fields, the ability to expand the capacity of a central processing facility, and the dynamic allocation of processing capacity to the connected fields. The case study results were that the expected NPV of an offshore oil field developed could be raised by as much as 76% through subsea tiebacks and more. Elmerskog (2016) described the added value of co-producing the adjacent oil fields Johan Castberg and Alta-Gohta, concerning capital expenditures, while Lei et al. (2021) presented a mathematical programming approach to evaluate tiebacks for the same fields. Fedorov et al. (2020) and Fedorov et al. (2021) combined real options approach and decision analysis to identify the value created by a sequential drilling strategy for marginal oil field development in the face of a great market and technical uncertainty. The current paper's authors have previously developed a similar methodology in Rønning et al. (2021), which they applied to a synthetic case. However, the methodology in the present paper is more sophisticated as it is built on multiple levels, and its underlying models account for a more realistic market environment.

The remainder of the paper is structured as follows. Section 2explains the importance of developing a novel methodology for economically assessing marginal discoveries. Section 3thoroughly presents the problem description we aim to solve and the methodology we have developed and implemented. In Section 4, we apply the proposed methodology to a real case provided by the NPD. The following results and sensitivity analysis are presented and discussed in Section 5. In Section 6, which is an extension section, we discuss how our methodology represents an essential building block for a global oil field optimization model. Finally, we summarize our main findings and identify future work in Section 7.

2 Background

In this section, we describe the motivation for developing a flexible valuation tool for marginal oil fields. The role of subsea tiebacks in the economic recovery of O&G is growing as it is proving itself to be a valuable option for the exploitation of marginal oil fields (Heng et al. 2000). However, there are several issues associated with tieback development concepts. Husy (2011) gave an overview of the main technical challenges for tieback of marginal fields. He addresses flow assurance, wax/hydrate/sand management, and appropriate artificial lift to boost production as the most important challenges. In addition, preservation, start-up, and sometimes pigging are also key issues. Ball (2006) and Lin et al. (2013) highlight another issue: the demand for new technology to cope with long offset satellite field developments[2]. In the past decades, significant technological advances have enabled long-distance tiebacks for development projects. Some recent technological advancements include Davalath & Wiles (2017), Gassert et al. (2019), Mikalsen & Loper (2019), Wiles et al. (2019), Rajaratnam et al. (2020), and (Bacati et al. 2020), but will not be further elaborated on here. Karimaie et al. (2018) provided a technical evaluation of the feasible development concepts for selected oil discoveries in the Barents Sea; Goliat, Johan Castberg, Alta, Gohta, and Wisting.

Investment decisions in the O&G industry are subject to extensive riskiness in terms of market and technical uncertainty. The revenue from such investments comes from the sales of oil and gas, which depends on their respective prices and production volume. It is widely acknowledged that O&G prices are highly volatile, in particular compared to other commodities (Asche et al. 2013). The volatility of O&G prices is mainly due to the low responsiveness of supply and demand to price changes in the short run, as it takes years to develop new supply sources, and it is hard for consumers to switch to other fuels (Askari & Krichene 2010, U.S. Energy Information Administration n.d.). Moreover, the volatility is also driven by events that can disrupt the flow of O&G to the markets, such as temporary supply shocks, weather-related aspects, and geopolitical developments. This has been particularly highlighted in the ongoing conflict between Ukraine and Russia, as it saw crude oil prices soar to $140 a barrel on March 7th, 2022.

Besides the market uncertainty, there is a large degree of subsurface uncertainty that affects the production volume. The uncertainty is related to gross rock volume and oil/water contact (OWC), reservoir architecture, faults and fractures and reservoir compartmentalization, degree of dolomitization, permeability and level of heterogeneity, fluid properties, amongst several more factors (O'Dell & Lamers 2005). Usually, appraisal wells are drilled before investments to reduce subsurface uncertainty, but fewer appraisal wells are typically drilled for small oil fields compared to large oil fields as the costs are considered to be inadequately high compared to the potential of information revelation (Dias 2004, Fedorov et al. 2021). Consequently, the development of marginal oil fields is relatively more uncertain than most other E&P projects.

Traditionally, E&P companies evaluate investment opportunities by applying the classical DCF approach, which calculates the net present value (NPV) by discounting the cash flows at the risk-adjusted discount rate given by:

$$NPV = \sum_{t=t_0}^{T} \frac{\mathbb{E}\left[CF_t\right]}{\left(1+R_a\right)^t}, \tag{1}$$

where $\mathbb{E}\left[CF_t\right]$ is the expected cash flow of period t, T is the number of periods, and R_a is the risk-adjusted discount rate. The discount rate reflects both the time value of money and the risk associated with holding an asset (Brealey et al. 2012). The NPV rule states that a project should only be accepted if and only if its NPV is positive and exceeds the NPV of all mutually exclusive alternative projects.

The DCF approach provides sufficiently accurate valuation in cases with stable and predictable cash flows. Furthermore, it has gained popularity due to its simplicity and intuitive approach. However, it is static as decisions are only based on available information at the time of investment, while additional information in the future is ignored. Cukierman (1980) and Bernanke (1983) found that uncertainty regarding future returns of irreversible investments creates an incentive to wait for more information before investing. This finding is supported by Cortazar & Schwartz (1998), who conclude that a significant part of the value of petroleum investments arises due to the option to wait.

Furthermore, Arrow & Fisher (1974) and Henry (1974) found that investment decisions are mainly influenced by three factors: (1) uncertain future cash flows; (2) investment expenditure cannot be fully recovered, and; (3) there is some degree of flexibility in the timing of investment. These factors are all relevant to oil field development projects. As a result, the DCF approach is not an appropriate valuation tool as it does not accurately capture the value of a decision-maker's ability to change course during the project[3]. Hence, the real options approach (ROA) has proved itself a better method as it accounts for managerial flexibility and the value of new information (Paddock et al. 1988, Chorn & Carr 1997). The first theoretical foundation for the real options[4] were published by Brennan & Schwartz (1985) and McDonald & Siegel (1985), who showed that the value of postponing an irreversible investment increases as the uncertainty of future profits increases[5].

[2]Currently, the longest subsea tieback is 220km long and corresponds to the gas field of Zohr near the coast of Egypt (Vielliard et al. 2019).

[3]Jafarizadeh & Bratvold (2009*a*) and Soares & Baltazar (2010) classify typical flexibilities in oil field development projects as; Wait-to-Invest Flexibility, Termination Flexibility, Temporary Start/Stop Flexibility, and Operational Flexibility.

[4]The term 'real options' was coined by Myers (1977), who considered the 'growth opportunity' of a company as a call option on a real asset. Myers introduced this concept in seminars already in 1975, shortly after financial option theory was developed by Black & Scholes (1973) and Merton (1973).

[5]In the wake of their papers, the literature on real options rapidly expanded, and several studies were conducted for valuing real option projects, many of which are summarized in textbooks such as Dixit & Pindyck (1994), Trigeorgis (1996) and Amram & Kulatilaka (1998).

Despite the early enthusiasm in academia, real option valuation (ROV) has not lived up to the expectations in the industry. According to a survey done by Horn et al. (2015), only 6% of CFOs from Scandinavia's 1500 largest companies report using real options for capital budgeting decisions. Corresponding surveys have been done for other regions (see e.g., Sandahl & Sjögren (2003), Block (2007), and Baker et al. (2010)), all of which report similar results. As written by Horn et al. (2015), the most important reason for non-use is a lack of familiarity with real options. Nonetheless, there exists strong empirical evidence of real option effects (see e.g.,, Quigg (1993), Moel & Tufano (2002), and the literature presented in Section 1.1). That being the case, this paper not only seeks to contribute to academia, but also to provide insight and a better understanding of real options for decision-makers in the O&G industry.

Although there are more technical challenges associated with subsea tiebacks than for standalone facilities, they are important to drive the profitability of small oil fields. Abbott et al. (1995) conclude that if subsea tiebacks to an existing platform are feasible, the cost will be difficult to beat. However, marginal oil field developments are relatively more uncertain than large oil fields. Low market prices and lower field size than initially estimated may jeopardize the profitability of the development, which is why marginal oil fields, in general, are considered less attractive by industry majors[6]. However, as marginal oil fields can still provide substantial value for E&P companies and society, it is crucial to develop novel methodologies to assess the economic viability of such fields. Due to the adverse price environment in the petroleum industry, there is considerable value in waiting for more information before investment decisions are made. Hence, the value of flexibility must be added to the methodology, which is why we apply ROA.

3 Methodology

This section presents our proposed methodology to evaluate the tiebacks for marginal oil fields. In Section 3.1, we describe the problem we solve. In Section 3.2, we provide a conceptual overview of the model and explain the flow of information throughout the calculations. Section 3.3 describes the optimization model we have incorporated to estimate production profiles for the tieback development. In Section 3.4, we present the commodity price model we use to simulate future O&G prices. Finally, Section 3.5 discusses solution approaches for ROV problems and describe the implementation of the LSM algorithm.

3.1 Problem Description

In this study, we seek to maximize the economic value of marginal field development projects on the NCS. We will look at the problem from the perspectives of the field operator, host owner, and the Norwegian society. We also refer to the latter as NPD[7]. Its primary objective is to ensure that 'the greatest possible value is achieved from oil and gas activities in Norway for the Norwegian society through efficient and responsible resource management' (Norwegian Petroleum Directorate n.d.*a*). Specifically, we consider a field operator with a production license[8] for an undeveloped O&G reservoir. We assume that the initial O&G in place is not big enough to warrant a standalone development. However, we assume there are N existing host facilities nearby, all of which are technically feasible for a tieback. Each tieback is associated with specific capital, operational, and abandonment costs along with a tariff that the field operator pays to the facility owner. In addition, the host facilities have different spare capacities, which will affect the production rate of the undeveloped reservoir. Finally, the revenue generated by the investment comes from the sale of O&G and is therefore conditioned on their respective market prices, p^{oil} and p^{gas}, and the production volume q^{oil} and q^{gas}. The field operator company must consider all the aspects mentioned above to assess the economical viability of a tieback investment in the presence of considerable market and technical uncertainty. The problem of the field operator can be summarized in a decision flowchart, as shown in Figure 2. The net present value (NPV) of the tieback investment is evaluated by discounting expected cash flows, given by:

$$
\begin{aligned}
NPV_{field} = \sum_{t=0}^{T} (p_t^{oil} \cdot q_t^{oil} + p_t^{gas} \cdot q_t^{gas} \\
- \text{CAPEX}_t - \text{OPEX}_t \\
- \text{Tariff}_t - \text{ABEX}_t) \cdot e^{-\gamma \cdot t},
\end{aligned}
\tag{2}
$$

where t is the year, T is the lifetime of the field, $CAPEX_t$ is the yearly capital expenditures, $OPEX_t$ is the yearly operational costs of the field, $Tariff_t$ is the yearly fees paid to the host owner, $ABEX_t$ is the yearly abandonment costs, and γ is the opportunity cost of capital. In the face of uncertainty, the field operator has an incentive to delay the investment, e.g., to

[6]For instance, ExxonMobil sold its Norwegian assets to Vår Energi in 2019. Neil Chapman (senior vice president of ExxonMobil) said that they are achieving their objective of having the most robust portfolio in the industry by (among other things) divesting assets that have lower long-term strategic value (see `https://www.offshore-energy.biz/exxonmobil-sells-norway-offshore-fields-to-var-energy-for-4-5-billion/`).

[7]The Norwegian Petroleum Directorate (NPD) is a governmental specialist directorate and administrative body established in 1972. It acts as an adviser and reports to the Ministry of Petroleum and Energy (Norwegian Petroleum Directorate n.d.*a*). NPD holds important data from NCS, and together with analyses they constitute a crucial factual basis on which O&G activities are founded.

[8]A production license is a concession that grants exclusive rights to conduct exploration drilling and production of oil and gas within a delimited area on the Norwegian Continental Shelf (Norwegian Petroleum Directorate n.d.*c*).

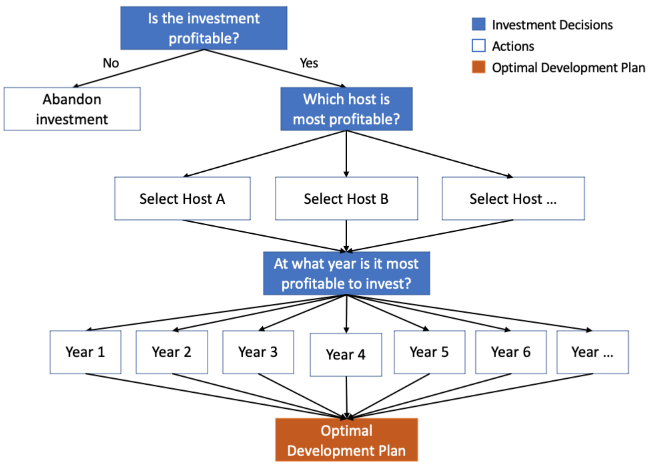


Figure 2: Decision flowchart from the perspective of a field operator.

wait for more favorable market conditions and potentially increase the value of the investment. However, waiting to invest also reduces the present value of the investment due to the time value of money.

The host owner seeks to maximize profits by charging as much tariff as possible, but is constrained by the guidelines of Forskrift om andres bruk av innretninger (2005). However, the host risks being opted out in favor of alternative hosts if the tariffs are priced too high. Another critical driver for the facility owner is to postpone abandonment. Since abandoning the host facility is costly, any additional production that moves this cost out in time will increase the profitability of the host. We assume that the host charges a tariff that corresponds to all its associated operational costs and a profit margin s (expressed as a percentage)[9]. Furthermore, we assume the host modifications and decommissioning costs due to the tieback are fully paid by the field operator. Thus, no CAPEX or ABEX is associated with the tieback for the facility owner. The NPV for the host is then given by

$$NPV_{host} = \sum_{t=0}^{T} (\text{Tariff}_t \cdot s) \cdot e^{-\gamma \cdot t}. \tag{3}$$

NPD seeks to maximize total value on the NCS by optimizing the allocation of oil fields and hosts. The NPV for the NPD is given by

$$NPV_{NPD} = NPV_{field} + NPV_{host}. \tag{4}$$

This problem is more interesting, particularly when we consider a portfolio of undeveloped oil fields. The problem calls for a novel optimization model that determines the optimal allocation of potential tieback developments, which we will discuss in more detail in Section 6.

3.2 Model Overview

Based on the problem description above, we propose a model to evaluate the optimal tieback development for a field operator. First, we develop a production optimization model. Then, we employ ROA in order to account for the managerial flexibility of waiting to invest. Figure 3 shows the main building blocks of the proposed model and describes the information flow of a tieback development project. The first key component is the optimization model we have developed in Section 3.3. It is incorporated into the valuation model and estimates the *production profile*. Based on *host spare capacity* and *field potential*, it calculates the yearly production rate of the undeveloped field during its whole lifetime. As previously mentioned, the host facilities have different spare capacities, depending on how much they are currently producing from other reservoirs. Moreover, the host capacities will vary in time as hosts enter or continue their decline phase.

The revenues of the field operator is the product of O&G produced and their respective uncertain future market prices. In order to make a *revenue prediction*, we employ a commodity pricing model that can simulate future O&G prices. This is the second key component of the model, and is presented in further detail in Section 3.4. Since it has been demonstrated by Villar & Joutz (2006) and Brown & Yucel (2008) that oil and gas prices have been historically related, we assume that the gas price is correlated to the oil price.

OPEX and *tariff* represent recurring negative cash flows during the lifetime of the field. OPEX mainly consists of the costs associated with facility maintenance, staffing, fuel, and storage vessel leasing, and will normally increase as the production rate increases. The tariff is an economic compensation paid by the field operator to the host owner, for the use of the host's facilities. Tariff schemes are bilaterally negotiated contracts between the field operator and the host and could be

[9]Instead of including OPEX of the field in Equation 3, we assume instead that the yearly profit of the host is the product of the tariff and the profit margin

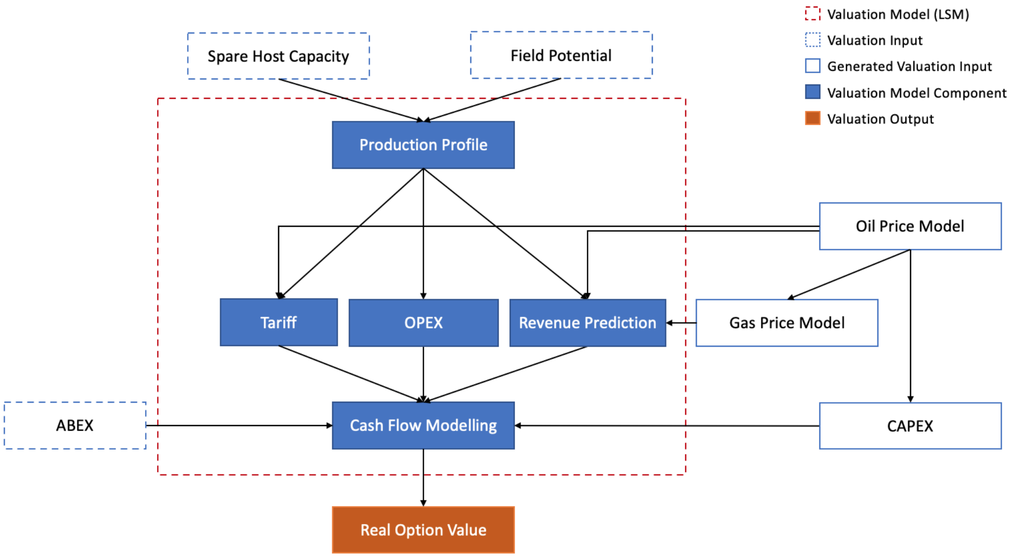


Figure 3: An overview of the model with its components and the information flow of a tieback development.

designed in numerous different ways. We want to analyze a tariff scheme model that is considered to be close to those frequently used in practice, given by:

$$\text{Tariff}_t = \alpha + \beta_0 q_t^{oil} + \beta_1 p_t^{oil} + \beta_2 q_t^{gas}, \qquad (5)$$

where α is a fixed minimum amount and β_0, β_1 and β_2 are the coefficients for the oil volume, oil price, and gas volume, respectively.

CAPEX includes expenditures for host modification, subsea production system, drilling of production wells, SURF (Subsea Umbilicals, Risers, and Flowlines), and project management for all these events. CAPEX is modeled as a Geometric Brownian Motion (GBM), given by:

$$d\theta_t = \mu_\theta \theta_t dt + \sigma_\theta \theta_t dz_\theta, \qquad (6)$$

where θ_t is CAPEX for year t, μ_θ is the drift rate, σ_θ is the volatility, and dz_θ is the Brownian increment. We apply the discretized version of the GBM, given by

$$\theta_{t+1} = \theta_t \cdot e^{\left[\left(\mu_\theta - 0.5\sigma_\theta^2\right)\Delta t + \sigma_\theta \varepsilon_\theta \sqrt{\Delta t}\right]}. \qquad (7)$$

In general, we observe in the O&G industry that CAPEX follows the movements of oil prices. Hence we model this by making CAPEX correlated to the oil price. This will be studied more closely in Section 3.4.

ABEX are the one-off decommissioning costs for the field operator at the end of the project, including plugging of wells, subsea facility removal, and other necessary costs associated with the disconnection from the host. These costs are assumed to occur the first year after the field's operative period. To the best of our knowledge, there exists no empirical evidence of a correlation between the field operator's ABEX and O&G prices. Thus they are assumed uncorrelated as in Jafarizadeh & Bratvold (2012).

Lastly, we estimate the future cash flows of the project in *cash flow modeling*, by taking into account CAPEX, ABEX, OPEX, tariff, and revenue prediction. These data will serve as an input to the ROV model, solved using the LSM framework, which allows for optimizing the field value by postponing the investment decision. This is another key component in the model and will be described thoroughly in Section 3.5. Since we want to compare the outcomes of ROV with traditional NPV, we perform a symmetric analysis of the two valuation methods based on equal O&G prices, production assumptions, and discount rate.

3.3 Production Profiles

In order to estimate the revenues of the investment, we construct the production profiles for the tieback developments. We do this by establishing an optimization model that maximizes the yearly production rate of O&G, using estimations of the field's contents, and yearly production. If the investment decision is made, it is economically optimal to produce as much petroleum as possible, as quickly as possible, due to the time value of money. However, the main factors that limit the production are: (1) initial O&G in place; (2) spare host capacity, and; (3) the field potential. In our case, hosts are considered to be existing oil production platforms with available spare capacity. The spare capacity may appear either due to the production decline in the field(s) connected to these facilities or due to built-in extra capacity. The field production must be adjusted in accordance with the existing host spare capacity, which in some cases means that field production start must be delayed. Table 1 presents the parameters of the established optimization model.

The objective function is defined in Equation 8, where q_t^{oil} and q_t^{gas} are the yearly produced volumes of O&G from the field to a specific host, t is time in years, γ is the discount rate, and T is the lifetime of the project. Once the investment decision is made, the field operator seeks to maximize the project NPV by producing as much O&G as possible as quickly as possible, given certain constraints. First of many, the yearly production volume of O&G cannot exceed the yearly spare host capacity of oil, c_t^{oil}, and gas, c_t^{gas}, as defined in Equation 9 and Equation 10. In addition, the yearly production volume of oil and water cannot exceed the yearly host capacity of liquid: c_t^{liquid}, as defined in Equation 11. Furthermore, the total production volume of oil cannot exceed the initial oil in place S^{oil}, and correspondingly for the total production of gas and the initial gas in place S^{gas}, as defined in Equation 12 and Equation 13.

The aforementioned equations handle the main constraints (1) and (2), and are straightforward to calculate as the yearly spare host capacities and the initial O&G in place are all direct inputs into the model. It is more complicated to calculate the field potential, because it is dependent on more factors. The field potential states the yearly maximum volume that is technically possible to extract from the O&G reservoir. It is easier to extract petroleum in the first years of production than in the later, due to high pressure. However, as more petroleum is extracted and the pressure decreases, the field potential declines. Thus, it becomes harder to extract the remaining petroleum in the reservoir. Before presenting the field potential constraints, we first describe some of its necessary components. The accumulated produced oil: U_t^{oil}, and gas, U_t^{gas}, are defined to be zero in $t = 0$ in Equation 16 and Equation 17. Later they are defined by Equation 14 and Equation 15. They state that the accumulated O&G for year t equals the accumulated O&G from the previous year and the average of the current and produced oil from the previous year. The reason for this is to get the average amount of accumulated O&G within the current year, transitioning into more accurate yearly values for Equation 20 and Equation 21.

The recovery factor for oil, r_t^{oil}, and gas, r_t^{gas}, are given by Equation 20 and Equation 21, respectively. The recovery factors for O&G are respectively the proportion of the current accumulated produced O&G and the initial O&G in place. They state how much of the initial petroleum is produced in relative terms. We define the field potential f_t^{oil} and f_t^{gas} in Equation 20 and Equation 21. The field potential is firstly dependent on the maximum well capacity W, which is a product of the maximal extraction rate of the well and the amount of drilled wells in the field. The recovery factor cannot exceed the maximum recovery factor R as it would result in negative field potential values. The component: $(1\text{-}r_t^{oil}/R)$, reflects the remaining field pressure and will steadily decline as more petroleum is extracted. This means that the field potential will also steadily decline unless W is increased by drilling more wells. Lastly, the ratio of oil production and field potential for oil cannot exceed that of water and field potential for water, as defined in Equation 22. This is in reality a simplification of a host's water constraint.

$$\max \sum_{t=0}^{T} (q_t^{oil} + q_t^{gas}) \cdot e^{-\gamma \cdot t} \tag{8}$$

$$q_t^{oil} \leq c_t^{oil}, \quad \forall t \in \mathcal{T} \tag{9}$$

$$q_t^{gas} \leq c_t^{gas}, \quad \forall t \in \mathcal{T} \tag{10}$$

$$q_t^{oil} + q_t^{water} \leq c_t^{liquid}, \quad \forall t \in \mathcal{T} \tag{11}$$

$$\sum_{t=0}^{T} q_t^{oil} \leq S^{oil}, \quad \forall t \in \mathcal{T} \tag{12}$$

$$\sum_{t=0}^{T} q_t^{gas} \leq S^{gas}, \quad \forall t \in \mathcal{T} \tag{13}$$

$$U_t^{oil} = U_{t-1}^{oil} + \frac{1}{2} \cdot \left(q_{t-1}^{oil} + q_t^{oil}\right), \quad \forall t \in \mathcal{T}/0 \tag{14}$$

$$U_t^{gas} = U_{t-1}^{gas} + \frac{1}{2} \cdot \left(q_{t-1}^{gas} + q_t^{gas}\right), \quad \forall t \in \mathcal{T}/0 \tag{15}$$

$$U_0^{oil} = 0 \tag{16}$$

$$U_0^{gas} = 0 \tag{17}$$

$$r_t^{oil} = \frac{U_t^{oil}}{S^{oil}}, \quad \forall t \in \mathcal{T} \tag{18}$$

$$r_t^{gas} = \frac{U_t^{gas}}{S^{gas}}, \quad \forall t \in \mathcal{T} \tag{19}$$

$$f_t^{oil} = W \cdot \left(1 - \frac{r_t^{oil}}{R}\right), \quad \forall t \in \mathcal{T} \tag{20}$$

$$f_t^{gas} = W \cdot \left(1 - \frac{r_t^{gas}}{R}\right), \quad \forall t \in \mathcal{T} \tag{21}$$

$$\frac{q_t^{oil}}{f_t^{oil}} \leq \frac{q_t^{water}}{f_t^{water}}, \quad \forall t \in \mathcal{T} \tag{22}$$

Table 1: Parameters of the optimization model.

Parameter	Description
q_t^{oil}	Produced oil
q_t^{gas}	Produced gas
q_t^{water}	Produced water
W	Maximum well capacity
c_t^{oil}	Host spare oil capacity
c_t^{gas}	Host spare gas capacity
c_t^{liquid}	Host spare liquid capacity
f_t^{oil}	Field potential oil
f_t^{gas}	Field potential gas
S^{oil}	Initial oil in place
S^{gas}	Initial gas in place
r_t^{oil}	Oil recovery factor
r_t^{gas}	Gas recovery factor
R	Maximum recovery factor
U_t^{oil}	Accumulated produced oil
U_t^{gas}	Accumulated produced gas

Figure 4 illustrates all the different constraints and their impact on the oil production. This oil production profile is not based on a real case, but highlights the functions of the constraints. Oil can be produced at the field production potential rate up until Year 6. In Year 7, the oil production has to be below the field's production potential due to limited spare capacity at the host. In Year 9, the liquid production (the combined volume of oil and water production) reaches the host's spare liquid capacity, and the oil production has to be reduced and remains below the field's oil production potential. From Year 10 and onward the production follows the field potential constraint.

3.4 Oil and Gas Price Modelling

Since we follow ROA, we use a stochastic price model[10] for oil and gas that resembles the real market uncertainty. There has been performed considerable research on commodity price models to enhance the quality of investment valuation under price uncertainty. In early applications, it was originally assumed that commodity prices follow simple one-factor stochastic processes similar to that of stock prices. The most common one-factor process is the Geometric Brownian Motion (GBM), which among others has been suggested for commodity prices by Brennan & Schwartz (1985), Cox et al. (1985), and Smith & McCardle (1999). Some researchers criticized the simplistic assumption that commodity prices and stock prices follow the same stochastic process, arguing that commodity prices have different properties than financial stock prices (Goodwin 2013). As opposed to financial assets, commodities are physically produced, making their prices dependent on the cost of production and future scarcity. Additionally, storage of commodities might impose several costs and risks, as well as commodities can be subject to seasonal fluctuations. Particularly, researchers noted a mean-reverting characteristic inherent to commodity prices as a result of producers' ability to respond to market conditions (Bessembinder et al. 1995, Lund 1999). Various models have been proposed, so-called

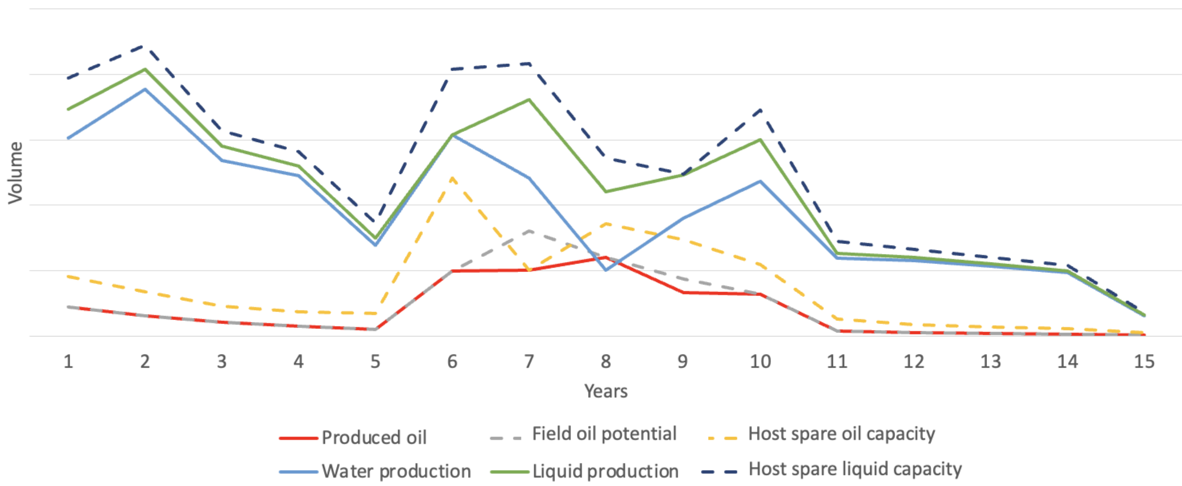


Figure 4: A given oil production profile with capacity constraints (only for illustration purposes).

[10]Although stochastic price models have been the main approach for commodity pricing (Cortazar et al. 2013), asset pricing models have also been investigated in commodity pricing literature. The starting point of this line of research can be found in Dusak (1973), Bodie & Rosansky (1980) and Carter et al. (1983).

N-factor models[11], with different specifications dependent of the number of state variables and the interpretation of these. The optimal number of factors depends on the stochastic behavior of the term structure of the commodity that is being modeled (Cortazar & Schwartz 1994) and on the complexity that the modeler is willing to accept (Cortazar & Naranjo 2006).

In this study, we assume the future O&G prices follow the two-factor stochastic price model proposed by Schwartz & Smith (2000). The model is simple enough to be communicated to decision-makers, who are generally not experts in financial modeling or option theory, while it is being sufficiently realistic (Thomas & Bratvold 2015). As its name suggests, the price model is decomposed into two factors; a long-term equilibrium price (ξ_t) and a short-term deviation from this equilibrium price (χ_t). In contrast to pure mean-reversion models, the model's long-term equilibrium is uncertain, allowing for the possibility that changes in the spot price are of a long-term nature (Goodwin 2013). At the same time, the short-term deviation from the equilibrium prices reflects events in the market that affect the price in the short-term, but are not expected to persist in the long term due to market participants' ability to respond to different market conditions (Fedorov et al. 2021). We denote P_t as the commodity price at time t, given by

$$\ln(P_t) = \xi_t + \chi_t. \tag{23}$$

The long-term equilibrium price is assumed to follow a Geometric Brownian Motion (GBM) process with drift μ_ξ and volatility σ_ξ, given by

$$d\xi_t = \mu_\xi dt + \sigma_\xi dz_\xi. \tag{24}$$

The short-term deviation is assumed to follow an Ornstein-Uhlenbeck (OU) process that reverts toward zero[12], given by:

$$d\chi_t = -\kappa \chi_t dt + \sigma_\chi dz_\chi, \tag{25}$$

where κ is the mean-reversion coefficient (it determines the rate at which the short-term deviation reverts towards zero), σ_χ is the short-term volatility, and dz_χ and dz_ξ are the correlated increments of a standard Brownian motion process with $dz_\chi dz_\xi = \rho_{\chi\xi} dt$.

We adopt a risk-neutral pricing approach, which is considered appropriate when the investment opportunity is exposed to various uncertainties (Cox et al. 1985, Smith & Nau 1995, Smith & McCardle 1999). This applies in our case as the risk natures of the market and technical uncertainty are different. By taking such an approach, we risk-adjust each uncertainty individually in the model, instead of risk-adjusting the entire cash flow[13]. The two factors can then be described as:

$$d\xi_t = (\mu_\xi - \lambda_\xi)\, dt + \sigma_\xi dz^*_\xi, \tag{26}$$

$$d\chi_t = (-\kappa\chi_t - \lambda_\chi)\, dt + \sigma_\chi dz^*_\chi, \tag{27}$$

where dz^*_χ and dz^*_ξ are the correlated increments of a standard Brownian motion process with $dz^*_\chi dz^*_\xi = \rho_{\chi\xi} dt$, and λ_χ and λ_ξ represent the risk premiums that constitute constant reductions in the drift rates of the two factors. Hence, the risk-neutral short-term factor reverts towards $-\lambda_\chi/\kappa$, and the risk-neutral long-term factor's drift corresponds to $\mu^*_\xi = \mu_\xi - \lambda_\xi$

Since we generate O&G cash flows by using Monte Carlo simulations, we must discretize the price processes. The discretized version of the long-term component is given by:

$$\xi^*_t = \xi^*_{t-1} + \mu^*_\xi \Delta t + \sigma_\xi \epsilon_\xi \sqrt{\Delta t}, \tag{28}$$

where μ^*_ξ is the drift rate of the Brownian motion, while σ_ξ is the long-term volatility, and ϵ_ξ is the long-term standard normal random variable. The discretized short-term risk-neutral component is given by:

$$\begin{aligned} {\chi^*}_t =& {\chi^*}_{t-1} e^{-k\Delta t} - (1 - e^{-k\Delta t})\frac{\lambda_\chi}{k} \\ &+ \sigma_\chi \epsilon_\chi \sqrt{\frac{1 - e^{-2k\Delta t}}{2k}}, \end{aligned} \tag{29}$$

where ϵ_χ and ϵ_ξ are standard normal random variables that are correlated in each time period with correlation $\rho_{\xi\chi}$. As proposed by Wiersema (2008), Cárdenas (2017), and Fedorov et al. (2022), the correlation coefficient for the two error terms is given by

$$\epsilon_\xi = \rho_{\xi\chi} \cdot \epsilon_\chi + \sqrt{1 - \rho^2_{\xi\chi}} \cdot \epsilon. \tag{30}$$

We employ the commodity price model above for oil and gas, but in order to make gas prices correlated to oil prices, we apply Equation 30 to their respective short-term errors, such that

$$\epsilon_{\chi^{gas}} = \rho_{\chi^{gas}\chi^{oil}} \cdot \epsilon_{\chi^{oil}} + \sqrt{1 - \rho^2_{\chi^{gas}\chi^{oil}}} \cdot \epsilon. \tag{31}$$

The relation between CAPEX (θ) is oil prices are handled by correlating the oil price's long-term errors with

[11] Some notable N-factor models include Laughton & Jacoby (1993), Ross (1997), Schwartz (1997), Gibson & Schwartz (1990), Longstaff & Schwartz (1992), Cortazar & Schwartz (2003), Cortazar & Naranjo (2006) and Trolle & Schwartz (2006), among many others.

[12] It reverts towards zero because we set the long-term mean (θ) equal to zero in the general definition of an OU-process: $d\chi_t = \kappa\,(\theta - \chi_t)\, dt + \sigma_\chi dz_\chi$

[13] If a single discount rate is applied for all projects without accounting for specific features of the individual project, it may result in incorrect valuation and poor decision-making (Fedorov et al. 2021).

each CAPEX's GBM (Fedorov et al. 2022), such that

$$\epsilon_\theta = \rho_{\theta\xi^{oil}} \cdot \epsilon_{\xi^{oil}} + \sqrt{1-\rho^2_{\theta\xi^{oil}}} \cdot \epsilon. \tag{32}$$

3.4.1 Model Calibration

There are seven parameters (κ, σ_χ, μ_ξ, σ_ξ, $\rho_{\chi\xi}$, λ_χ and λ_ξ) in the O&G price models that must be estimated, in addition to two initial conditions χ_0 and ξ_0. Since these parameters are usually not observable in the commodity markets, we estimate them by using the Kalman filter[14] (Kálmán 1960). The Kalman filter[15] generates an updated (posterior) prediction of a state vector's mean and covariance at time t, conditional on all information available up to and including time $t-1$ (Goodwin 2013). If historical oil prices (P_t) are considered as the measurement, then because of Equation 23, the Kalman filter can produce estimates of ξ_t, which in turn can be used to estimate the parameters in Equation 24. For a wider coverage of the Kalman filter, we refer the reader to Harvey (1989), Hamilton (1994) and West & Harrison (1996). We implement the Kalman filter in the same manner as Schwartz & Smith (2000), Goodwin (2013) and Fedorov et al. (2021) in order to calibrate these parameters, with the results presented in Section 4.3.

3.5 Solution Approaches for ROV Problems

As explained in Section 1.1, E&P investments with high costs under uncertainty have a big potential monetary value in managerial flexibility (Cortazar & Schwartz 1998, Jafarizadeh & Bratvold 2009*a*, Soares & Baltazar 2010, Fedorov et al. 2021). Since the classical DCF approach does not capture such flexibility, we follow a real options approach (ROA) instead. This approach has proved itself as a better method as it incorporates flexibility and the value of information into the valuation (Paddock et al. 1988, Chorn & Carr 1997). The field operator is assumed to be able to reevaluate the investment decision once a year in order to consider the market conditions. The investment decision consists of an irreversible investment cost, CAPEX, which occurs when the investment is made. The payoff corresponds to all future discounted cash flows generated by the project, including revenues, OPEX, tariff, and abandonment costs. In light of ROA, the investment decision of an oil field development can thus be seen as a Bermuda call option[16]. The comparison of the oil field development and the financial option is summarized in Table 2.

Table 2: Oil field development as a call option (Adapted from (McDonald 2013)).

(Financial) Option Terms	Project Terms
Call Option	Oil Field Investment
Strike Price	CAPEX
Underlying Asset Price	Present Value of Project
Expiration	Lifetime of the Field

The field operator optimizes the investment decision by choosing once a year whether to exercise the project or wait, based on the then-current state of the O&G market and the CAPEX. By waiting to exercise the option, the decision-maker potentially loses immediate payoffs, but gains more information regarding the uncertainties affecting the decision. This flexibility allows the decision owner to delay the option exercise until the O&G prices increase to favorable levels. If the market conditions never become favorable, the decision-maker might have avoided substantial losses due to the high investment costs associated with oil field developments.

There are three main solution methods to solve option valuation problems; the partial differential equations (PDE), the dynamic programming approach, and the simulation approach (Schwartz 2013). The first solution method directly solves the PDE that stems from most option pricing problems[17]. The dynamic programming approach assumes two possible future outcomes in each stage of a multi-staged period, and the option value can be calculated as the present value of all the probability-weighted outcomes[18]. Boyle (1977) demonstrated that simulation methods could obtain numerical solutions for option valuation problems, specifically by using the Monte Carlo method by Hammersley & Handscomb (1964). Simulation methods rely on generating random samples to achieve numerical results and are applicable for problems that cannot be solved due to the interference of a random variable. However, initially, the prevailing view was that simulation methods were not applicable to American-style valuation problems because these typically generate paths forward in time, while the optimal exercise policies are determined backward (Broadie &

[14] The Kalman filter has been widely applied in finance to estimate state variables of commodity price models, see e.g., Schwartz (1997), Schwartz & Smith (2000), Manoliu & Tompaidis (2002), Sørensen (2002), among others.

[15] One drawback of the Kalman filter is the missing-data problem. Since the Kalman filter normally assumes a complete panel data set, which is often not the case in financial markets, it disregards data and causes a loss of information. As a result, other procedures have also been proposed, see e.g., Sørensen (2002), Cortazar & Schwartz (2003), Cortazar et al. (2003), and Jafarizadeh & Bratvold (2012).

[16] In contrast to a standard American option, Bermuda options are restricted only to allow early exercise at predetermined discrete points in time (in this case once a year).

[17] The most well-known model of this kind is the Black-Scholes-Merton model (also referred to as the Black-Scholes model), which was originally developed by Black & Scholes (1973) and Merton (1973) for European call options.

[18] The most prominent methodology within this category is the Binomial option pricing model, originally proposed by Sharpe (1978) and later formalized by Cox et al. (1979) and Rendleman & Bartter (1979).

Glasserman 2004, Jafarizadeh & Bratvold 2009*b*). This view changed as several simulation methods started incorporating dynamic programming approaches to cope with this problem. Moreover, simulation methods gained popularity because the traditional approaches to value financial options become inadequate for ROV when there are too many state variables (Schwartz 2013).

Longstaff & Schwartz (2001) proposed a simulation approach that approximates the value of American options[19] by directly focusing on the conditional expected payoff, which became known as the least-squares Monte Carlo (LSM) model[20]. The model generates a large number of price paths of the underlying asset and calculates the option's payoff for each path. Then the payoffs are discounted back and averaged, resulting in the option value. This approach has received much attention in the finance literature and is considered state-of-the-art (Nadarajah et al. 2017). It is intuitive, transparent, flexible, easily implemented, and computationally efficient, as well as it allows the state variables to follow general stochastic processes because they can be simulated. To this end, we apply a LSM simulation approach to solve the real options valuation problem.

In our model, we first compute the expected yearly cash flows of the oil field investment by combining simulated production- and cost profiles, as well as O&G prices. Several sets of cash flows are generated forward, where each set corresponds to the simulated cash flows for when the investment decision is made. These cash flows serve as the main input for the LSM algorithm, which compares the estimated value of investing now with the estimated value from continuation at each time step (year). Since the option can be exercised at any time step until maturity, the model is required to work backwards from the last decision point in order to determine the optimal decision. It is, however, not legitimate to use the knowledge of future payoffs on a given simulation path to decide to exercise on a given time step. We resolve this by adopting the technique recommended by Longstaff & Schwartz (2001), who use least-square-regression. The fitted value of this regression is an efficient unbiased estimate of the conditional expectation function and allows accurate estimation of the stopping rule for the option. This technique allows for additional risk factors that affect the expected continuation values (Willigers 2009), which in our case are oil price, gas price, and CAPEX. Only in-the-money paths are included in the regression as this results in better estimations of the conditional expectation function in the region where exercise is relevant (Longstaff & Schwartz 2001).

In contrast to American options in the financial markets, where the payoff of the underlying is observable, the immediate investment payoffs of the oil field development are not available. This might lead to suboptimal investment strategies because the regression is biased. This issue is handled by Jafarizadeh & Bratvold (2009*b*), who extends the original LSM approach by regressing both the continuation values *and* the immediate investment payoffs separately on the oil price from the previous year. This implies that the real option exercise is triggered if the fitted value of the payoff regression is larger than the fitted value of the continuation value regression, given that the fitted payoff is positive.

4 Case Study

The model has been developed and tested using data from a real case provided by NPD. The case is presented in Section 4.1. Sensitive details are left out for confidentiality reasons, including selected values and axes in several figures. In Section 4.2, we present the results of the integrated optimization model that calculates the production profiles. In Section 4.3, we present the estimated parameter values for the price processes together with simulation results. In Section 4.4, we elaborate on the cost models that we use for the case study.

4.1 Case Description

In the case we study, a field operator holds a license for an undeveloped O&G reservoir on the NCS. It is not economically viable to develop a standalone facility for the field. However, there are two host facilities located nearby, *Host A* and *Host B*, as illustrated in Figure 6. The field operator wants to decide whether a tieback to any of these is profitable, and if so *which* host is most profitable. Furthermore, the field operator wants to assess the optimal investment timing.

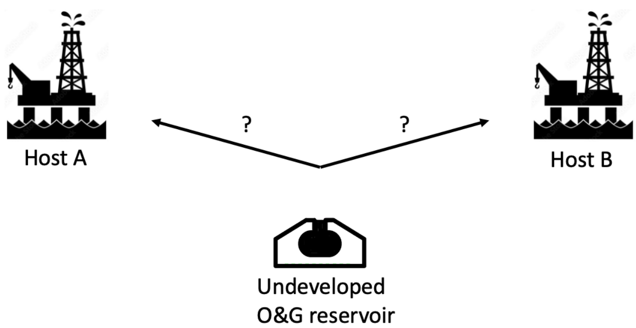


Figure 6: Illustration of the decision situation.

[19] Some notable investigations on the pricing of American options by simulation techniques include: Bossaerts (1989), Tilley (1993), Barraquand & Martineau (1995), Raymar & Zwecher (1997), Broadie et al. (1997), Boyle et al. (1997), Broadie & Glasserman (1997*a*), Broadie & Glasserman (1997*b*), Averbukh (1997), Carr, Peter (1998), Garcia (1999), Andersen (2000) and Ibanez & Zapatero (2004), Broadie & Glasserman (2004) and Glasserman (2004).

[20] Although this approach is usually credited Longstaff & Schwartz (2001), it was also suggested by Carriere (1996) and Tsitsiklis & Van Roy (1999).

The field and each host have a given capacity that limits the production rate. Based on these capacities, production profiles can be calculated for both hosts, which we present in Section 4.2. The production rates differ between the two hosts, as they are dependent on the spare capacity at the host facility. In addition, the hosts have different time horizons, as indicated in Figure 5. The investment decision can be made from Year 1, and the construction phase starts the same year as the investment is made. The construction and ramp-up phases take 4 and 5 years for Host A and Host B, respectively, and are assumed to be fixed regardless of the year of investment. The first production starts in the last year of construction, being respectively 3 and 4 years after investment for Host A and Host B. Host A is planned to be shut down in year 16, while Host B is planned to shut down in year 11. In our analyses below, we will investigate scenarios where the lifetime of Host B can be extended.

In line with Norwegian Petroleum Directorate (2020), we apply a discount rate of 7%. We use the same rate for both NPV and ROA to have a fair comparison. As mentioned in Section 1.1, the risk-free rate should be applied for ROA, but it can be argued whether the applied discount rate is too high to be considered risk-free[21]. Another appropriate choice of the risk-free rate would be the 10 year Norwegian government bond, corresponding to approximately 3%[22]. Since the main focus of the present paper is to provide good qualitative analyses of the valuation techniques, we prioritize using the discount rate of 7%.

4.2 Production Profiles

As explained in Section 3.3, the production profiles aim to maximize the project's NPV by producing the maximum feasible amount of a field's content of oil and gas. The initial year of the project is Year 0, while investment is possible from Year 1 onwards, when the simulated CAPEX and O&G price paths have reached their first step. The maximum recovery factor R is set to 0.6, as used in comparable model calculations.

In terms of the presentation of this case study, we intentionally disguise a number of parameter values, including capacities and production, in order to not expose commercially sensitive data. Figure 7 and Figure 8 illustrate the oil production profiles of tieback to Host A and Host B, assuming an immediate exercise. The green striped line and the yellow striped line represent the field potential and the host spare capacity, respectively. The black line represents the produced oil, while the brown line represents the accumulated produced oil.

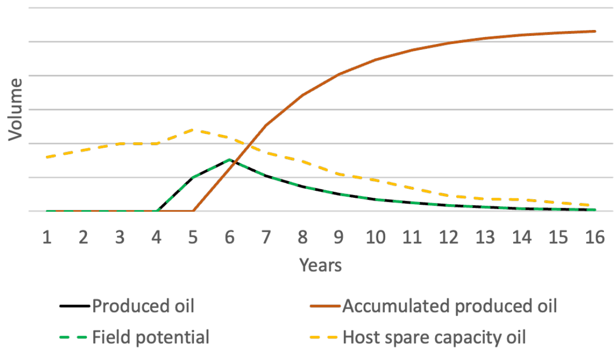


Figure 7: Oil production for Host A in the base case.

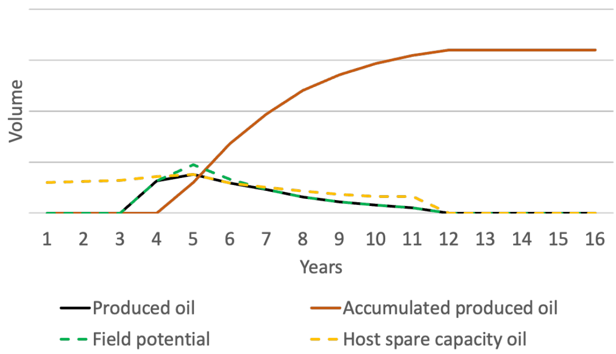


Figure 8: Oil production for Host B in the base case.

Normally, the spare capacity of a host increases due to the depletion of its original fields. However, the spare capacity for oil is declining for both hosts, which is explained by a gradual shutdown plan. The field po-

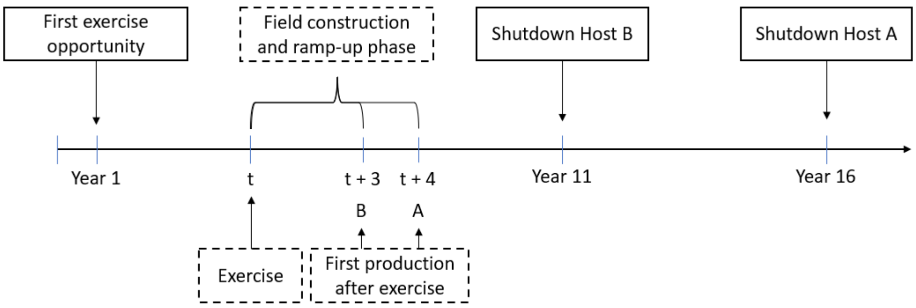


Figure 5: Time horizon of Host A and Host B.

[21]The determination of risk-free discount rate for ROV has received much attention in the literature, see e.g., Brandão & Dyer (2005), Smith (2005), Brandão et al. (2005), Laughton et al. (2008), and Jafarizadeh & Bratvold (2015). Although it is concluded that the risk-free rate should be used for the ROA, there is no common conclusion to how this value should be determined.

[22]The value is retrieved from `http://www.worldgovernmentbonds.com/bond-historical-data/norway/10-years/`.

tential for both hosts first increases rapidly, before it slowly declines due to the O&G pressure drop. The fact that the produced oil matches the field potential during the whole lifetime of Host A in Figure 7, indicates that it is solely the field potential that limits the maximum production of petroleum from the field. For Host B, on the other hand, the spare capacity influences the production, further decreasing its oil production deficit compared to Host A. In the case of gas production, it is essentially the field potential that restricts the production for both hosts, rather than the spare host capacity. Therefore, we omit the gas equivalents of figures Figure 7 and Figure 8.

Figure 9 presents the field's production profiles of oil (solid lines) and gas (striped lines) with tieback to Host A (blue lines) and Host B (orange lines). When comparing the oil production profiles, Host A is able to produce more oil initially, resulting in a quicker decline in production. As mentioned above, the oil production at Host B, is restricted by the spare capacity at the host. Therefore, it will maintain a more stable production over time, resulting in exceeding the yearly oil production at Host A around Year 7. Nevertheless, the oil production profile of Host A is favorable because: (1) more oil is produced initially, which is more valuable due to the time value of money, and; (2) more oil is produced in total because of longer lifetime. The tieback to Host B produces more gas the first year, but slightly less the remaining years of its lifetime. Regardless, Host A is able to produce significantly more gas because it has a longer lifetime. Hence, we can conclude that Host A provides the most attractive production profile for investment in Year 1, mainly because O&G are produced over more years than Host B.

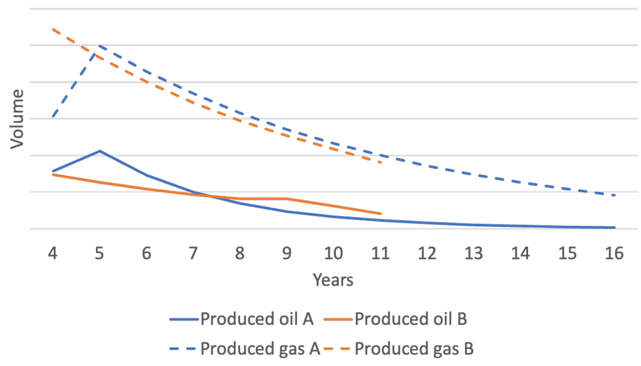


Figure 9: Production profiles of Host A and B.

As the hosts' capacity constraints do not limit the oil production sufficiently to affect the production, the conclusion for the base case in terms of choice between hosts is rather obvious. In Section 5.2, we perform a sensitivity analysis to gain insight into the optimal choice of a host from the perspective of a field operator. Therefore, we modify the production constraints such that the production profiles of the two host choices are more resembling and a clear choice of host is not evident.

4.3 Oil and Gas Price Simulations

The estimated price process parameters for both the O&G price simulations are retrieved from Thomas & Bratvold (2015), where Kalman filter and maximum likelihood estimation were applied for calibration, and are shown in Table 3.

Table 3: Oil and gas price model parameters (rounded to the nearest second decimal).

Parameter	Oil	Gas
ξ_0	4.26	4.80
χ_0	0.00	0.00
σ_ξ	0.22	0.25
σ_χ	0.47	0.75
λ_χ	-0.08	-0.07
μ_ξ^*	-0.02	-0.05
κ	0.50	0.91
$\rho_{\xi\chi}$	-0.71	-0.63
$\rho_{\chi^{gas}\chi^{oil}}$	0.64	

Given the parameter values stated in Table 3, the initial O&G prices are set equal to[23]:

$$p_0^{oil} = e^{\xi_0^{oil}+\chi_0^{oil}} = e^{4.26+0.00} = 70.81,$$

and;

$$p_0^{gas} = e^{\xi_0^{gas}+\chi_0^{gas}} \cdot 0.13$$
$$= e^{4.80+0.00} \cdot 0.13 = 15.80.$$

Figure 10 and Figure 11 show the results from the O&G price simulations, which are done in accordance with Section 3.4. The solid grey lines represent historical O&G prices, while the future expected O&G prices correspond to the solid green and blue lines, respectively. In addition, the confidence bands corresponding to the 90^{th} and 10^{th} quantile, are given with the green and blue striped lines. These are the barriers of where the 10% highest and lowest oil prices are found, while the area between the striped lines shows where 80% of the predictions of the O&G prices lay. This area is increasing with time as it becomes possible for prices to diverge more from expectations far into the future. The grey striped line represents one example of the 15,000 simulated price paths[24] that are used for our valuation procedure.

[23] We have converted the gas from p/therm (as received by NPD) to USD/BTU, which gives a factor of approximately 0.13 (see `https://ngc.equinor.com/Home/Price`).

[24] 15,000 simulations proved to be computationally reasonable and produce a stable result that deviated insignificantly throughout several code-run executions.

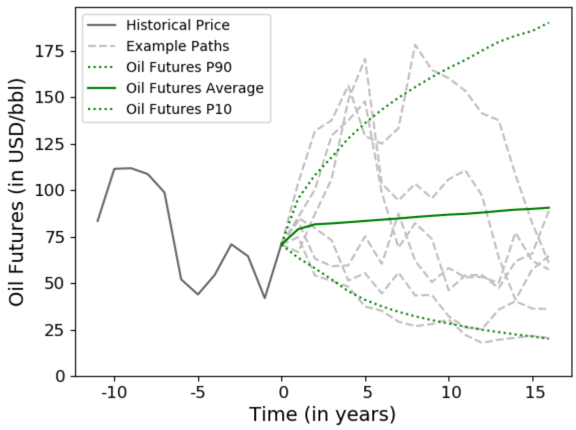


Figure 10: Historical and future brent crude oil prices with confidence bands.

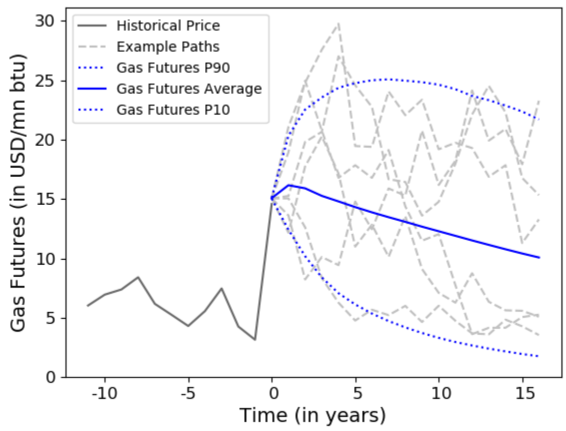


Figure 11: Historical and future natural gas liquids (NGL) prices with confidence bands.

4.4 Costs

4.4.1 OPEX and Tariff

For OPEX and Tariff, we have developed cost models that resemble the actual data provided by NPD. For confidentiality reasons, the real costs have been modified so that the models do not generate the exact numbers of the real case. The parameters used in the cost models are presented in Table 4. In this case study, we assume that the OPEX and tariff parameters are the same for Host A and Host B, but we will later relax this assumption in the sensitivity analysis we are conducting. Since OPEX and tariffs depend on the production volume, they will be different for the two hosts.

Table 4: OPEX and tariff model parameters.

Parameter	α	β_0	β_1	β_2
OPEX	8.0	-	1.0	0.0
Tariff	35.0	1.0	0.1	0.0

4.4.2 CAPEX and ABEX

In our case study, the CAPEX for tieback to Host B is approximately 20% higher than for Host A, but will vary in time as we model it as a GBM. We apply the same parameter values for Equation 7 as in Fedorov et al. (2022), and set the drift rate, μ_θ, equal to 2% and the volatility, σ_θ, equal to 10%. The results from the CAPEX simulations are presented in Figure 12, which shows the expected CAPEX and confidence bands for Host A and Host B during their lifetimes. The CAPEX will generally increase with time due to the drift rate.

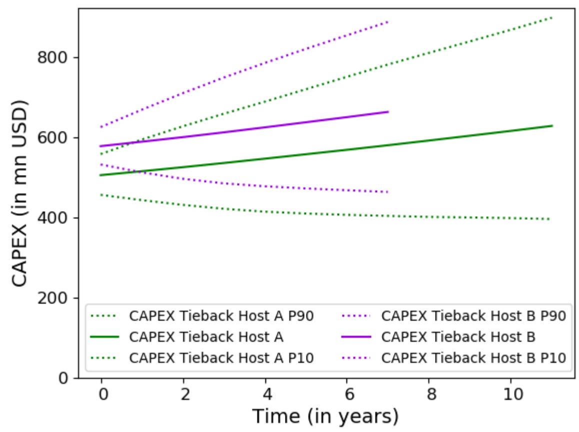


Figure 12: Future expected CAPEX with confidence bands.

As Fedorov et al. (2022) conclude, the correlation coefficient for oil price and CAPEX is important to define properly. Willigers (2009) identified that the rig rental rates in the North Sea correlate with the oil price with a coefficient of 0.87 with a one-year delay. Our CAPEX costs include additional elements with less sensitivity toward the oil price. Hence we decrease the correlation coefficient to 0.7, as proposed by Fedorov et al. (2022). In order to achieve an experienced correlation of 0.7 for the simulated data, we set $\rho_{\theta\xi^{oil}}$ equal to 0.92 in Equation 32. In order to illustrate the effect of the correlation, we have simulated three different oil price paths (solid lines) and CAPEX paths (striped lines) in Figure 13. There is a significant relation between the oil price and CAPEX, with one year lag for the latter.

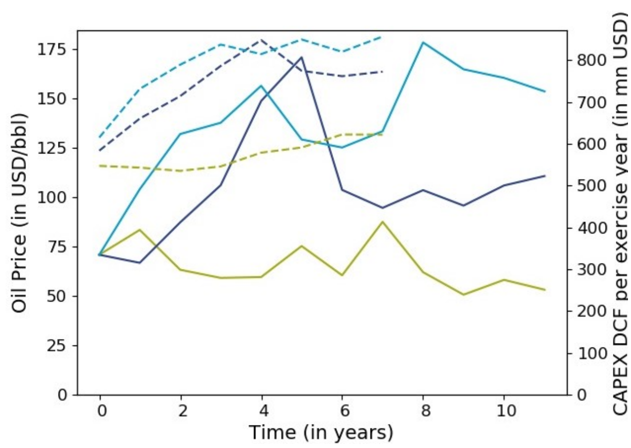


Figure 13: A selection of paths for future oil prices and CAPEX (dashed) for Host A.

In our case study, the ABEX for Host B is approximately 10% higher than for Host A. The values are remained fixed during the whole lifetime of the hosts and almost considered negligible in comparison to other costs due to their initial low values and many years of discounting.

5 Results

We now present the results of our analysis. Section 5.1 shows the results of applying our methodology to the base case. Section 5.2 presents sensitivity analyses to better understand the main drivers of the selection of hosts. Finally, in Section 5.3, we analyze under which conditions timing flexibility with regards to the investment decision in the field is most valuable.

5.1 Base Case Results

For the main decisions of the field operator, indicated in the decision flowchart of Figure 2, we conclude the following:

1. Yes, the investment is profitable.
2. Tieback to Host A is the optimal choice.
3. Immediate investment in Year 1 is optimal.

Table 5 states the project values from the perspective of the field operator for tieback developments to Host A and Host B, respectively. The NPV approach values the tieback development to Host A at 455.6 mn USD and Host B at 273.1 mn USD, while ROA values the same tieback development to Host A at 484.5 mn USD and Host B at 330.5 mn USD. Thus, the results show that tieback to Host A is the preferred choice by a great margin according to both valuation techniques. Lower costs, larger spare capacity, and longer lifetime, are the main reasons why tieback to Host A is significantly more profitable than Host B from the field operator's perspective. The project value is higher when quantifying timing flexibility in both cases, mainly because of two reasons. Firstly and most importantly, substantial losses can be avoided by choosing not to invest if the market conditions are expected to be unfavorable during the project's lifetime. Secondly, the field operator can optimize the timing of investment to exploit upside potential when the project is in-the-money. For the case of Host A, the resulting project value from the ROA is 6.3% higher than using a NPV analysis. For Host B the difference is 21%. We will later explain why the difference is higher for Host B.

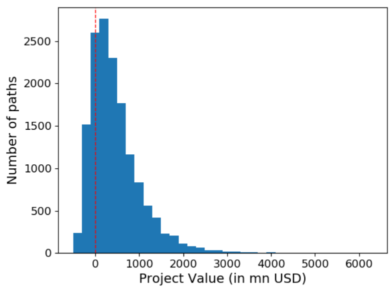
Table 5: Base case results (in mn USD).

	Host A	Host B
NPV	455.6	273.1
ROA	484.5 (+6.3%)	330.5 (+21.0%)

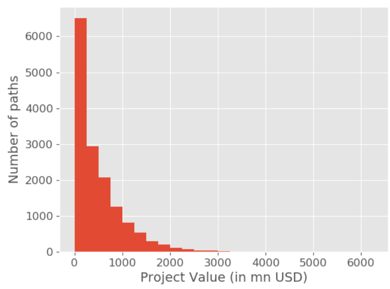
Figure 14 presents different histograms related to the distributions of outcomes from the perspective of the field operator. Figure 14a and Figure 14d show the distribution of project values by using the NPV approach to valuate tieback to each host, i.e., the number of simulations that resulted in project values within the different intervals. Figure 14b and Figure 14e show the corresponding distribution of project values by using the ROA to valuate tieback to each host. Figure 14c and Figure 14f show the distribution of the optimal timing of investment to each host according to ROA. As seen in Figure 14a and Figure 14d, there is a portion of the simulations that result in negative NPVs, showcasing the riskiness of the oil field development we are examining. These results occur due to unfavorable market prices. On the contrary, no project values are negative in Figure 14b and Figure 14e. Since ROA considers managerial flexibility, the project is not exercised until the market environment indicates that it is profitable. Sometimes, the market conditions never improve sufficiently, so the project is left unexercised, thereby avoiding substantial losses for the field operator.

It is interesting to see that the relative value of flexibility is significantly higher for Host B. This is mainly due to the ability to avoid investments that never become profitable. This is best explained by comparing the results in Figure 14c and Figure 14f. The majority of the simulations indicate that immediate exercise is most profitable for both tieback alternatives. However, a significant amount of simulations indicate never to invest in the project as it appears unprofitable throughout the whole lifetime. In contrast to tieback to Host A, where 18.3% of the cases are left unexercised, as much as 33.1% are left unexercised for Host B. Since a larger portion of cases would have resulted in negative NPV for tieback to Host B, considering the option to wait with investment and potentially not invest at all adds more value to the project than it does for tieback to Host A. Only 4.8% of the cases for Host A and 0.1% for Host B suggest exercising later than Year 1. This fact implies that the value of waiting with investment for better market conditions is negligible for our base case, in particular for tieback to Host B. Extracting and selling the O&G as quickly as possible is incentified by the time value of money, and the case study's production profiles with declining host spare capacity further demotivate postponing of the investment.

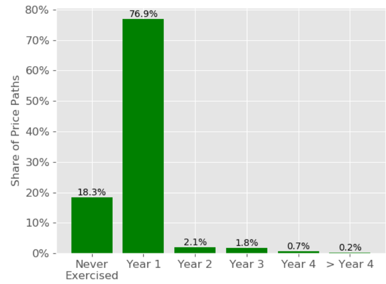
We now perform a sensitivity analysis on the correlation between the oil price and the CAPEX, and how it affects the project value. This analysis is interesting regarding modeling choice for evaluations of investment decisions in the O&G industry. Figure 15 shows the NPV (dashed lines) and ROV (solid lines) for tieback to each host as a function of the correlation coefficient of oil price and CAPEX. The NPV is more or less independent of the correlation coefficient value, but the ROV tends to decrease as the coefficient increases. The reason is that the oil price and CAPEX contribute in different directions when it comes to the profitability of the project. With a strong positive correlation, the two factors will to a larger extent cancel each other out with respect to the project value. Thus, the project



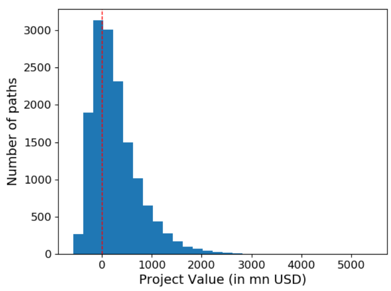
(a) NPV for tieback to Host A.



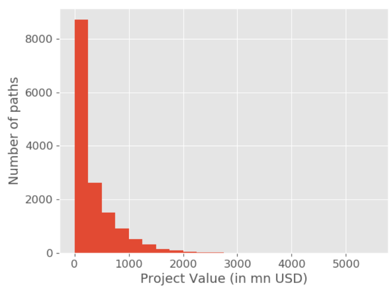
(b) ROV for tieback to Host A.



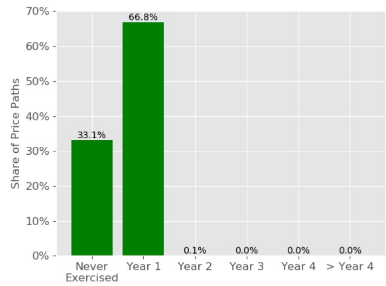
(c) Optimal investment timing for tieback to Host A.



(d) NPV for tieback to Host B.



(e) ROV for tieback to Host B.



(f) Optimal investment timing for tieback to Host B.

Figure 14: Distributions of project values and optimal investment timing for both hosts by NPV and ROV.

value becomes more stable as it will be less affected by changes in the oil price. However, when there is a strong negative correlation, the oil price and CAPEX will both contribute to the project value in the same direction, leading to either relatively larger profits or losses. This phenomenon resembles the characteristics of the option price when the volatility of the underlying asset increases, which according to option theory adds more value to the project due to managerial flexibility. As a result, larger movements of the project value due to a strong negative correlation will make it more attractive to wait to invest. The value of this flexibility is captured by ROV, which is why the relative difference between the two valuation techniques increases as the correlation coefficient becomes more negative.

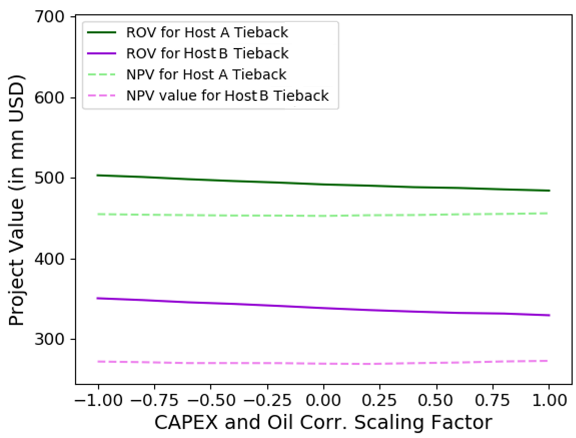


Figure 15: Field operator's project value as a function of the correlation coefficient between CAPEX and oil price.

5.2 The Main Drivers of Host Selection

We now aim to identify the main drivers for host selection. Host owners could have several reasons to take measures to become more attractive for tieback. For instance, the decommissioning cost of the production facility is often significant, thus any additional production that can delay this cost is beneficial for the host owner. Furthermore, if the potential of finding undiscovered oil fields near the existing host facility is considered high, it could be important for the host owner to retain production at the facility in order to make some profits (although less than initially), while pending further exploration. To achieve this, the host must offer sufficiently increased profits for the field operator to be preferred over alternative tieback hosts. At the same time, the costs of the measure(s) taken must be lower than the expected payoff from the host owner's perspective. In this section, we will focus on three specific actions the owner of Host B could take to become the optimal choice for tieback: reduce CAPEX, increase spare host capacity and extend its lifetime. For the figures in this section, the solid lines represent the project's ROV as a function of different key factors, while the dashed lines represent the corresponding for NPV. Green lines represent Host A tieback and purple lines Host B tieback.

The reduction of CAPEX is the first measure investigated. CAPEX for oil field developments are high and thus constitutes one of the strongest drivers of the project's profitability. If reducing the CAPEX of a tieback is possible, it could very likely change the optimal choice of host. However, it is strongly dependent on each specific case how much CAPEX reduction that is required to achieve a different outcome. While the CAPEX for tieback to Host A is kept fixed, we alter the yearly CAPEX for tieback to Host B between 100% to 50% of its initial value. Figure 16 shows the field operator's project values as functions of the scaling of CAPEX for tieback to Host B. The results suggest that CAPEX for Host B tieback would have to be reduced by 33% and 36% given NPV and ROV, respectively, in order for Host B to present the optimal choice of host. The amount of CAPEX that the host owner is able to reduce is case dependent. A significant part of the field operator's CAPEX is coverage of host facility modifications. In our case study, this is assumed to be the only part of the field operators' CAPEX that the host owner would be able to influence, which amounts to 31.7%. This means that even if all modification costs were covered by Host B, it would not be sufficient to become a more attractive tieback alternative than Host A. Moreover, covering such a large portion of the CAPEX would anyway make the tieback development unprofitable from the perspective of the host owner because the expenses would not be covered by the tariffs. Hence, it will likely not be a preferred action for Host B.

Similar to the outcome of the base case, the project values are higher for ROV than for NPV. By taking into account managerial flexibility, Host A is considered more attractive than Host B for a broad range of CAPEX reduction for Host B's tieback. The required reduction of CAPEX is larger for ROV, because the benefit of lower CAPEX for Host B must outweigh the relatively larger benefits of flexibility identified for Host A in this case. The value of flexibility is represented by the difference between the solid and striped line for each host tieback.

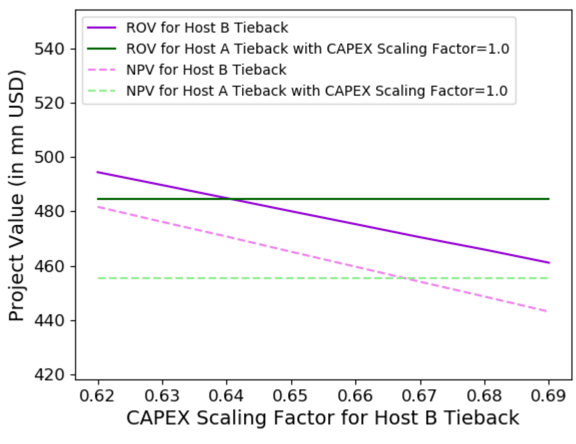


Figure 16: Field operator's project value as a function of Host B's CAPEX.

The second measure Host B could take to become the preferred choice is increasing its spare capacity. Specifically, we analyze the effect of altering the spare capacity profile of Host B up to 250% of the initial profile set in the base case, keeping the spare capacity of Host A fixed. As the host owner's profit in the base case amounts to approximately 20 mn USD, it is required that the spare capacity expansion costs less than this in order for this action to be attractive to implement in the perspective of Host B, unless it has other incentives as we have previously explained. Figure 17 shows the field operator's project values as functions of the scaling factor of the spare capacity of Host B. The results show that, in our case study, increasing the spare capacity of Host B alone would never make Host B more attractive for tieback than Host A. The project value for Host B tieback increases significantly when scaling the spare capacity up to 150% of its initial levels, but stagnates when increased above this level. The explanation is that, above this point, the field potential becomes the limiting factor, and any further spare capacity expansion is indifferent to the field operator's profits. To put the measure of the spare capacity increase in perspective of different project environments, we examine how an increase in spare host capacity affects the optimal choice of host if the field potential is significantly increased. This increase can occur, for example, due to higher than expected reservoir performance. We repeat the analysis performed in Figure 17, now doubling the field production potential. Figure 18 shows that, under these conditions, the tieback to host B becomes an optimal choice if the host capacity can be increased by at least. This showcases how spare capacity, as a measure to increase tieback attractiveness, has a larger impact on fields with large field potential.

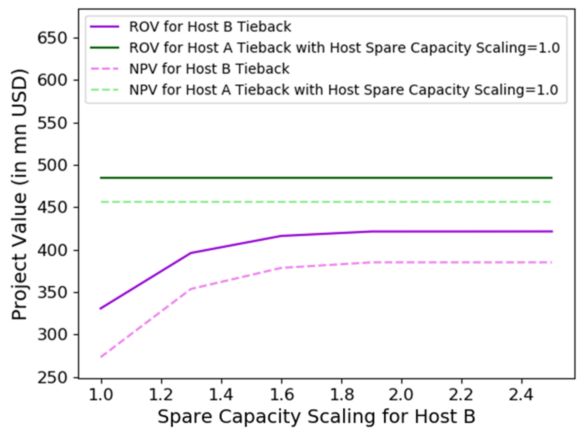


Figure 17: Field operator's project value as a function of Host B's spare capacity.

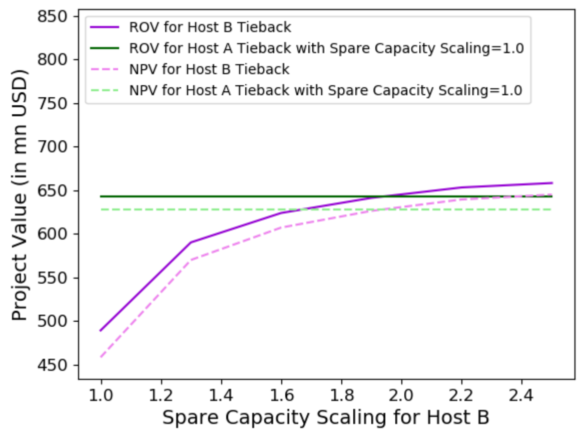


Figure 18: Field operator's project value as a function of Host B's spare capacity (with doubled field potential).

The third and last measure we analyze is extending the lifetime of Host B. For this analysis, the host owner is assumed to be able to extend the lifetime of the platform from 11 years to 22 years. The spare host capacity is assumed to remain at the same level as for Year 11 during the extended lifetime. The lifetime of Host A is assumed to remain fixed at 16 years in order to make the results comparable. Figure 19 shows the project values in the field operator's perspective as functions of the lifetime of Host B. The results suggest that extending the lifetime of Host B alone does not have a sufficient effect to make a tieback to Host B more valuable than to Host A. The project value of Host B tieback increases steadily until Year 16. After this point, further extension of the host lifetime is not beneficial due to the depletion of the field, which is not able to generate enough revenues to compensate for the tariffs levied by the host.

However, if we look at the same case, but with altered tariff parameters for Host B, a switch in optimal tieback host selection is feasible. Tariffs could be customized in numerous different ways as they are subject to contract negotiations between the field operator and the host owner. These negotiations could be conducted with the objective of making the production facility more attractive for tieback while maintaining profitability for both parts. We change the tariff parameters for Host B to zero fixed tariff cost, α, and increase the variable component β_1 from 1.0 mn to 8.0 mn USD/mn bbl produced. Figure 20 shows the field operator's project values as functions of lifetime for Host B, with the altered tariff parameters. Host A remains with the original tariff parameters and lifetime as in the base case. With this transition to an exclusively variable tariff scheme, the field operator's preferences change already as Host B's lifetime is increased by two years. This applies to both valuation methods.

Figure 21 and Figure 22 show the *host owner's* potential project values for a tieback as functions of Host B's lifetime with original and altered tariff parameters, respectively. The altered tariff schemes will give a total tariff cost roughly equal to the original scheme's total cost in a 12-year lifetime, but significantly lower in the later years when production is low due to the lower variable costs. Since the tariff costs correspond to the revenues for the host owner, accounting for the field operator's flexibility will actually reduce the revenues for the host owner. This is because the flexibility is exploited to maximize profits, and hence also keeping costs at a minimum. ROV evaluates the host owner's profits as lower than NPV because: (1) when the project is left unexercised, it implies lost revenues

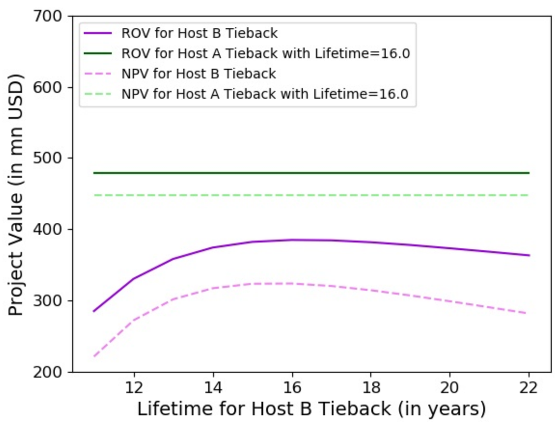


Figure 19: Field operator's project value as a function of Host B's lifetime (with base case tariff schemes).

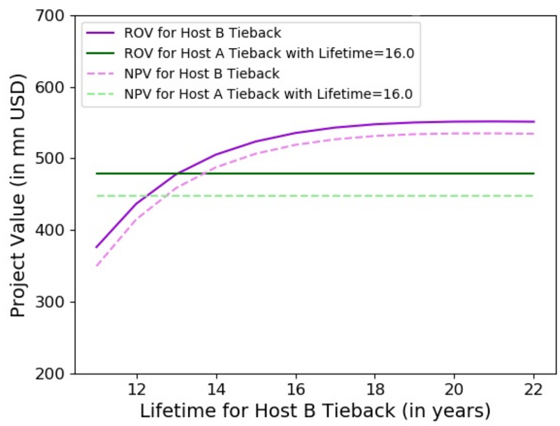


Figure 20: Field operator's project value as a function of Host B's lifetime (with altered tariff schemes).

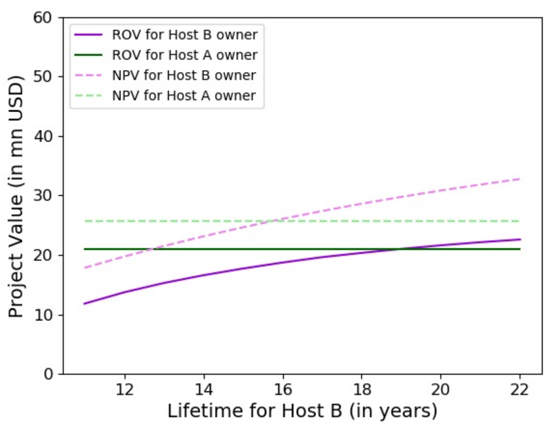


Figure 21: Host owner's project value as a function of Host B's lifetime (with base case tariff schemes).

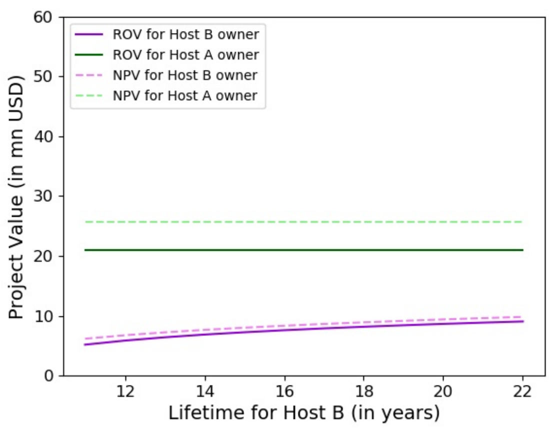


Figure 22: Host owner's project value as a function of Host B's lifetime (with altered tariff schemes).

for the host owner, and (2); when postponing the investment decision, revenues become lower than they would have been if the investment was made immediately due to the time value of money.

Evidently, the revenues for the host owner increase stronger with the original tariff parameters due to the fixed yearly costs, while the altered tariff scheme gives only a slight increase. As Host B never becomes optimal for tieback with the original tariff parameters, the altered tariff parameters would be preferable as the host actually could be selected for tieback if the lifetime is extended by two years or more. Regardless, the change of tariff parameters and a 2-year lifetime extension would not be relevant for the owner of Host B if the costs of these measures exceed the profits potentially gained. Another interesting observation made by comparing Figure 21 and Figure 22 is that the value of flexibility in the case of Host B tieback is significantly reduced with the altered tariff parameters. Variable tariff costs make the profitability less uncertain for the field operator, as it could be seen as a function of produced quantity and revenue. On the contrary, high fixed tariff components represent a risk in terms of the uncertainty regarding whether the revenue cash flows will be high enough to cover these recurring costs. ROA incorporates the managerial flexibility to reflect the response to this risk by timing the investment optimally, or avoiding it if coverage of the tariff never seems feasible.

To summarize the findings from Section 5.2, the main drivers of host selection seem to be heavily dependent on the specific case. The results from our case indicate which factors that potentially could drive a change in optimal host selection. Neither spare capacity increase nor extended lifetime is sufficient measures to change optimal host selection alone. However, a spare capacity increase is suggested as a feasible measure if the field has a larger field potential. An extended lifetime is also feasible if the tariff contract is changed to be more variable than fixed. On the other side, extending the lifetime with high fixed tariffs at the later stages of the field lifetime appears as an unattractive option for the field operator due to low production volumes in later stages. The CAPEX reduction could in theory be a measure that effectively could switch optimal host selection. But in our case study, a switch is still not attractive even if Host B covers *all* modification costs, and the amount would anyway be way above the potential profits gained from a tieback.

5.3 The Value of Timing Flexibility

In this subsection, we present sensitivity analyses that help explain for which marginal oil field investment environments the value of flexibility is higher. As mentioned in Section 2, this translates to which project environments that have larger differences between ROV and NPV. The value of flexibility increases as the probability of a project value being negative increases, since losses might be avoided by actions from the management. If the project already seems profitable, management might be in a position to postpone investments to even more favorable conditions. ROA is able to reflect the value of this flexibility that the management holds in real life, while the NPV approach is likely to underestimate the project value as the flexibility is not accounted for. All analyses are conducted considering a tieback to Host A from the case study, and we investigate both subsurface and market environments by evaluating the following key factors: initial O&G in place, the field potential, and O&G price volatility. In addition, we evaluate the effects of altering CAPEX, ABEX, and tariff schemes. Unless otherwise specified, for the figures in this section, the solid lines represent the field operator's project values by ROV for Host A tieback as a function of different key factors, while the dashed lines represent the corresponding for NPV.

The subsurface uncertainty is one of the main concerns for field operators when dealing with oil field development projects. The uncertainty in early field property estimations is even larger for marginal oil fields as less data are gathered for these estimations. We now conduct a sensitivity analysis of the initial oil and gas in place, altering it between 50% and 150% of initial estimates. The results are illustrated in Figure 23, where the project values are functions of the initial oil and gas in place.

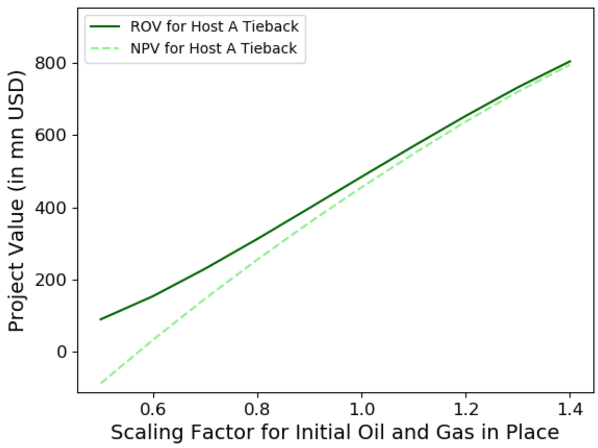


Figure 23: Field operator's project value as a function of the initial O&G in place in the field.

The results suggest that the project value increases as the initial oil and gas in place increases because larger reservoir volumes imply larger revenues and profits. The increase applies to both valuation methods, and their values converge as the initial oil and gas in place increases. The reason is that flexibility is less important to take into account when the project becomes more profitable and mitigating downside risk becomes less relevant. Equivalently, NPV and ROV diverge as the initial oil and gas in place decreases, hence suggesting a higher value of flexibility for marginal field developments. The reason is that smaller volumes reduce the profitability of the development project such that it might become unprofitable. With the option of waiting-to-invest, the field operator can leave the project unexercised if the market conditions do not justify investment in such a small field. This could be viewed as a partly hedge against the downside risk of the investment, which is why the ROV does not become negative in the figure. However, we emphasize that a perfect hedge is rarely possible for real options, in contrast to financial options, mainly due to private risks (e.g., reservoir uncertainty) that are not possible to hedge (Fedorov et al. 2021). The consequence of making a decision only based on NPV in this case might be that the field operator find the oil field too risky and leave the field unexploited. However, this could be a wrong conclusion as the oil field could potentially provide substantial value if the flexibility to wait is accounted for. This highlights how ROA could act as a valuable approach to gain insights into marginal oil fields' profitability.

Moreover, we have investigated how the value of flexibility develops if the field operator alters its field potential. We perform this analysis by altering the field potential between 50% and 150% of initial value. Figure 24 shows the project values as functions of the field potential. The results suggest that by increasing the field potential, more petroleum is extracted early, which in general increases the project value due to time value of money. However, the increase decays as the host capacity or the reservoir pressure becomes the limiting factor(s). In other words, it does not matter how much petroleum can be extracted by the field's wells each year if most of the oil in the field is already depleted or the host does not have the spare capacity to handle it. Regarding the value of flexibility, we see a similar tendency as in the previous sensitivity analyses: as the project becomes more unprofitable with lower field potential, there is more difference between the two valuation methods.

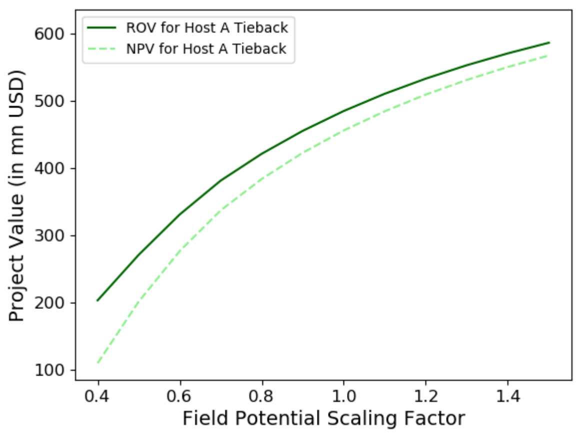


Figure 24: Field operator's project value as a function of the field potential.

Another important concern for the field operator is the market environment because the profitability of E&P investments are highly dependent on the O&G prices. As described in Section 2, O&G are among the most volatile commodities, and are furthermore subject to macroeconomic events such as economic recessions and expansions. We conduct a sensitivity analysis on the market uncertainty by altering both the long and short-term volatility parameters between 50% and 200% of the originally calibrated values. Figure 25 shows the project values as functions of the oil price volatility. The results suggest that the project value increases together with the oil price volatility. As the oil price is assumed and modeled not to reach negative values, increased volatility will also 'bias' the oil price paths to a higher average level as there does not exist any upper bound for the price. This leads to increased average revenues and profits for both valuation methods.

The value of flexibility decreases when the volatility is smaller because the market becomes more static and less uncertain. On the other side, and in line with option theory, increased volatility of the underlying asset implies a greater option value. This is because it is more likely to benefit from the timing flexibility when the O&G prices move significantly (McDonald 2013). This is better illustrated in Figure 26, which shows the optimal exercise timing as oil price volatility changes. This highlights another key advantage of ROA as it is able to capture the value of the opportunity to exploit the upside potential of investment decisions.

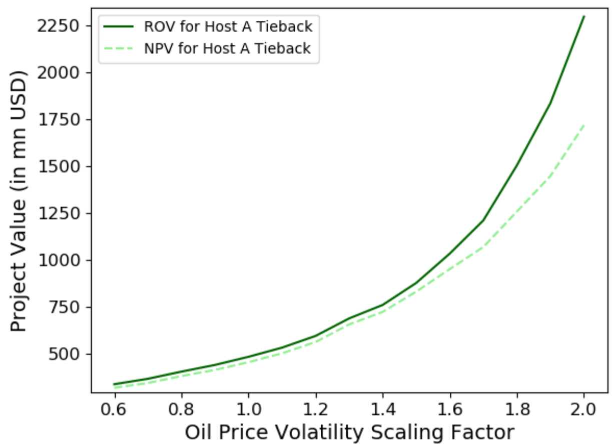


Figure 25: Field operator's project value as a function of the oil price volatility.

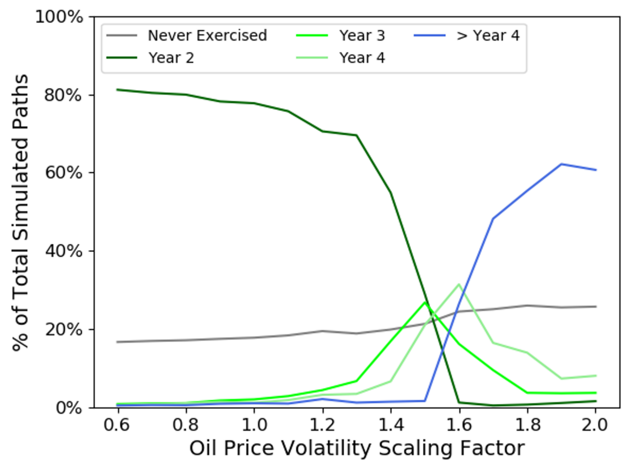


Figure 26: Optimal exercise timing for Host A with altered oil price volatility.

The optimal timing of exercising the investment decision tends to become later as the oil price volatility increases. For instance, exercising later than Year 4 is optimal for only 5% of the simulation paths when using the original volatility parameters, but it amounts to 60% of the simulations when the volatility is doubled. The reason is that there exists implicit *insurance* for the option holder by holding the option instead of exercising it (McDonald 2013). The insurance arises due to the fact that the present value of the project can fall below the investment cost at the end of the lifetime. The value of the insurance increases with higher volatility, which as explained favors waiting to exercise. The same sensitivity analyses were conducted on the gas price volatility, which showed the same results. However, the difference between NPV and ROV was larger for the oil than the gas price volatility analysis, due to the relatively higher significance of oil production for our field case.

While the sensitivity analyses above investigate increased risks regarding exogenous uncertainties (subsurface and market), we also conduct a sensitivity analysis on capital costs. These costs are often estimated by the field operator, but might diverge significantly from its forecasts due to unforeseen additional costs. CAPEX represents an expensive irreversible cost for the oil field development project, and might potentially jeopardize the profitability of the project if it becomes too large compared to the ensuing cash flows after investment. Figure 27 shows the project values as functions of the initial CAPEX cash flows varied from 100% to 200% of initial estimates.

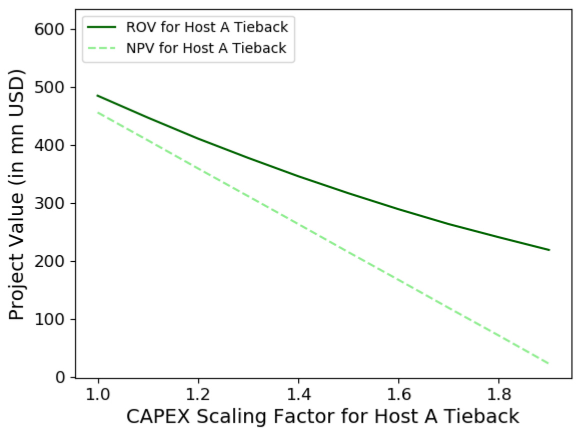


Figure 27: Field operator's project value as a function of CAPEX.

As expected, the project value decreases as the CAPEX increases. Moreover, the difference between the two valuation methods appears more clearly for higher CAPEX. As previously mentioned, this is a result of the increased risks for negative project value that higher CAPEX brings to the investment. Managerial flexibility adds value to the field because it can mitigate downside risks by delaying exercise if the conditions are unfavorable, similarly to the analysis of the field potential and intial oil and gas in place.

We have also conducted a sensitivity analysis of ABEX by altering it between 100% and 400% of the initial estimates, which resulted in an insignificant changes in the difference between the NPV and ROV from the field operator's perspective. The reason is that the value of ABEX is quite small in comparison to other cash flows, and that the value of ABEX is further depreciated due to many years of discounting. Hence, uncertainty regarding the field operator's ABEX has a rather insignificant impact on the project value, and it does not constitute a factor of the project that can be much exploited by managerial flexibility with the investment characteristics of our case study.

To summarize the findings from Section 5.3, the value of timing flexibility is indicated to be highest for the following project environments: (1) marginal initial O&G in place; (2) low field potential; (3) high market (O&G) price volatility; (4) high CAPEX, and; (5) tariff contracts with high fixed components. If one or several of these characteristics hold for a project environment, ROA is an appropriate methodology to evaluate the value of flexibility. This could avoid discarding O&G investments that by first sight appear unprofitable, but that still might be successful if the management considerately employs its available real options.

6 Extension Section: Optimal Allocation of Several Tieback Developments

In this extension section, we evaluate the same problem as before, but for a larger area with several potential tieback developments. The ROV model that is established in the present paper only considers the tieback development from one field to several hosts, which is considered a local optimum in the sense that it is most optimal for the field operator alone. However, this optimum does not necessarily constitute a part of the global optimal solution, which can be considered the total value extracted from the subsea in a certain area. Therefore, instead of considering the optimal choice of a host for *one* field operator, we hereby consider the optimal allocation of *all* fields to hosts within a given area. This problem is, in particular, relevant for large E&P companies (e.g., Equinor) that hold multiple licenses and operatorships as they seek to optimize their oil field portfolio. In addition, it is relevant for regulators like NPD, as their objective is to contribute to 'the greatest possible values from oil and gas activities to the Norwegian society, through efficient and responsible resource management' (Norwegian Petroleum Directorate n.d.*a*). The purposes of this extension section are thus to (1) give an introduction to the problem of optimal allocation of several tieback developments, (2) discuss scenarios where the optimal solution for a field operator might conflict with the optimal solution for the Norwegian society, and (3) demonstrate how our proposed methodology serves as a fundamental building block of a comprehensive optimization model in order to evaluate the portfolio of tieback developments. To achieve this, we establish a simplified optimization model in Section 6.1, which we apply to a synthetic case given in Section 6.2. We discuss the results in Section 6.3 and give final remarks in Section 6.4.

6.1 Model Description

From the perspective of a field operator, the optimal selection of host corresponds to the host to which the tieback development provides the highest field value (previously referred to as the project value). However, for regulators or large E&P companies, whose interest is to maximize the total value of all their fields within a given area, there are more factors to consider when allocating oil fields and hosts. Such considerations include prolonging the lifetime of a host due to potential discoveries in the future that are located nearby, which would have made that particular host important for future O&G production. This consideration is complicated because it is not easy to evaluate the potential of undiscovered oil fields, but it is highly relevant nonetheless. Another consideration is to avoid unexploited oil fields, which could have been developed profitably if the allocation had been better. For instance, small satellite fields often have a limited set of profitable tieback options. Therefore the relevant hosts should not be assigned to other fields without considering the consequences for the satellite field at the same time.

We now describe a simplified optimization model to solve the aforementioned allocation problem. We assume there are I oil fields with q_i oil in place, and J host facilities with c_j spare capacity. The decision variable, x_{ij}, states whether tieback from oil field i to host j is an optimal solution. The $FieldValue_{ij}$ corresponds to the field operator's project value of connecting host i to facility j, taking into account all associated development and operational costs.

$$\max_{x} \quad \sum_{i=1}^{I}\sum_{j=1}^{J} FieldValue_{ij} * x_{ij} \tag{33}$$

$$\sum_{i=1}^{I} q_i * x_{ij} \leq c_j, \quad \forall j \tag{34}$$

$$\sum_{j=1}^{J} x_{ij} \leq 1, \quad \forall i \tag{35}$$

$$x_{ij} \in \{0,1\}, \quad \forall i,j \tag{36}$$

The objective of the regulator or the company that seeks to solve the allocation problem is to maximize the total value of field developments by taking into account all possible combinations between fields and hosts, as defined in Equation 33. The maximization function is constrained such that the capacity in each host is not exceeded, as defined in Equation 34. Additionally, no more than the initial oil in place can be extracted, as defined in Equation 35, and it is assumed that a field cannot split its potential between several hosts, as defined in Equation 36. The model can easily be adapted to specific cases by adding more constraints, which we will demonstrate later.

6.2 Case Study

We employ the proposed optimization model on a synthetic case. The model is implemented in Excel and solved by the add-in function Solver, which uses the Levenberg-Marquardt algorithm for non-linear optimization problems. We assume a given area consisting of five host facilities (A-E)[25] and five undeveloped oil fields (1-5), which are located as illustrated in Figure 28. Each host facility has some amount of spare capacity, and each oil field has a certain amount of oil in it, which is given in Table 6. For simplicity reasons, we ignore the field potential constraint, which means we are not determining the production rate each year. Instead, one can consider the spare host capacity as an accumulated capacity constraint over the whole lifetime of the project. If the amount of oil in the field exceeds the spare capacity of the host, some of the oil will not be extracted within the project's lifetime. The unit of measurements has been excluded from the table as the numbers have no reference to realistic data, but they highlight the same problems that arise in the real world due to bottlenecks in spare capacities at hosts. We assume that the hosts are located in mature production areas, such that if they are not chosen for tiebacks for any of the undeveloped fields, they will be shut down shortly.

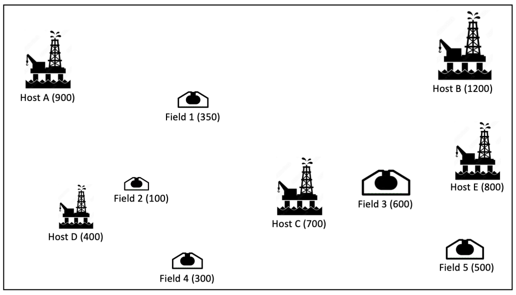


Figure 28: Illustration of case study with existing host facilities and new oil discoveries within a given area (capacities in parenthesis).

Table 6: Spare host capacities and amount of oil in fields.

Host	Capacity	Oil Field	Amount of Oil
A	900	1	350
B	1200	2	100
C	700	3	600
D	400	4	300
E	800	5	500

[25]We emphasize that host facilities A and B in the current extension section do not correspond to Host A and Host B from Section 4.

The field values for all technically feasible tieback developments between fields and hosts must be calculated by an underlying valuation model. As we have discussed in Section 5, there is substantial value in timing flexibility, which is captured by ROV and not NPV. For this reason, we propose to run the ROV model that we developed in Section 3. The ROV model takes into account the different tariffs, OPEX, CAPEX, and ABEX associated with each potential tieback between an oil field and a host facility when evaluating the field value. We emphasize that the established ROV model was not run in this synthetic case as it would be too time-consuming to generate the necessary amount of realistic data. Instead, the field values are considered functions of the amount of oil in the field and the distance between the oil field and the host. We apply the logic that bigger fields provide larger revenues and that longer distances imply higher costs. Table 7 shows the results of the field values for all possible (technically feasible) tiebacks.

Table 7: Field values for all possible tiebacks.

	A	**B**	**C**	**D**	**E**
1	587	276	611	472	304
2	82	-714	96	142	-466
3	168	204	766	273	1054
4	-34	-442	428	446	302
5	-256	-188	120	-18	358

The specific field values from Table 7 provide valuable insight for field operators. For instance, Field 3 and Field 5 should independently see choose Host E for tieback as it maximizes their field value. The problem that occurs is that Host E only has a spare capacity of 800, which is not sufficient to warrant production from both fields (600+500=1100). In these cases, it is useful to have a decision tool that selects the optimal tieback between one of the mentioned fields and Host E, considering the global problem.

6.3 Results and Discussion

We now investigate and compare three different scenarios, which highlight different considerations that are important to take into account when dealing with a portfolio of tieback developments.

6.3.1 Scenario 1: Base Case

Scenario 1 presents the base case with no further assumptions than those presented in Section 6.2. Table 8 states the optimal tieback developments for Scenario 1, with corresponding field values from the perspective of the field operators and the total area value. The results are further illustrated in Figure 29. The results suggest that the total area value corresponds to 2349. Furthermore, the field operators of Fields 2, 3, and 4 should invest in tiebacks to their preferred hosts. Field operators 1 and 5 must select their second best option for tieback in order for the total area value to become optimal. Even though these alternative options still indicate profitable solutions, the field operators lose profits of 24 for Field 1 and 238 for Field 5. This raises an important but difficult question of how to incentivize the field operators to select other hosts that are less profitable from a regulator's perspective. In this context, the sensitivity analysis we performed of the main drivers of host selection in Section 5.2 proves itself particularly useful.

Table 8: Optimal tieback developments for Scenario 1 with corresponding field values.

1 → A	**2 → D**	**3 → E**	**4 → D**	**5 → C**	**Total**
587	142	1054	446	120	**2349**

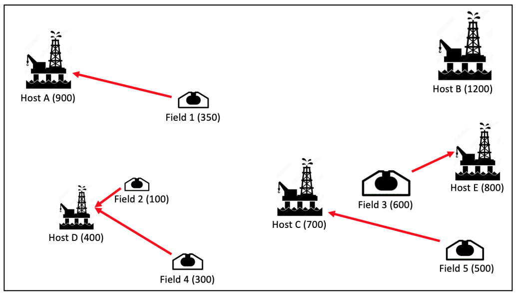


Figure 29: Optimal allocation for Scenario 1.

There are several interesting findings emerging from the results. Firstly, despite the fact that Host B has the largest spare capacity and a connection to Field 1 or Field 3 would lead to positive field values, the results suggest that it should be shut down. The reason is that there exist other hosts with sufficient capacity to produce from these fields that are more profitable because the hosts are located closer. Field D, on the other hand, has sufficient capacity to produce from Field 2 and Field 4 at the same time. This finding illustrates instances where it is more advantageous to allocate more oil fields to one specific host, than to share the fields more evenly between hosts. As mentioned previously, Field 3 and Field 5 have higher profitability when connected to Host E, but the total area value increases by 50 if Field 3 is allocated to Host E and Field 5 to Host C. This results show how useful the proposed optimization model can be as it maximizes the overall value to society or the company.

However, the latter finding might seem counter-intuitive as the longer tieback is suggested as the optimal solution[26], but there are several reasonable explanations for this outcome. Since there are many areas of negotiation associated with a tieback from an oil field to a production facility, different tariff schemes or cost-

[26] As explained in Section 2, there are more technical challenges as the length of the tieback development increases, e.g., long flowline often has more flow assurance issues and likely ends in less recovery due to larger pressure drop.

sharing can lead to the longer tieback being preferred. For example, if Host C considers it very important to continue its production with a tieback in order to postpone its own abandonment, then it may offer more favorable contract conditions so that it becomes more profitable for the field operator than other tieback options. Besides changing the cost structures, it could also be that extending the lifetime, spare capacity, or a combination of these would change the preferences of the field operator. This scenario also highlights the importance of the sensitivity analyses we performed in Section 5.2, as they identify the main drivers of host selection. This insight can be used by regulators or large E&P companies for recommendations or decisions regarding how specific host facilities can become more favorable in comparison to others. We highlight this aspect in Scenario 2.

6.3.2 Scenario 2: Large Potential in Undiscovered Oil Fields Near Host B

In Scenario 2, we assume that there is a great potential for undiscovered oil fields around Host B. Based on the subjective opinion of experts, the values of the undiscovered fields are so high that Host B should not be shut down at any cost. Hence, the production in Host B must be retained until these undiscovered fields are studied more closely. As mentioned previously, we could investigate how Host B can alter its tariff schemes, cover the modification cost for the field operator, extend the lifetime or free up spare capacity in order to become more favorable for field operators. By doing so, the field operator would choose a tieback development to Host B itself because it becomes the most profitable option. In this scenario, however, we assume that Host B was not able to become the preferred choice despite making such changes. Instead, we manually restrict the model to allocate at least one field to Host B, given by

$$\sum_{i=1}^{I} x_{iB} \geq 1. \tag{37}$$

Table 9 states the optimal tieback developments for Scenario 2, with corresponding field values from the perspective of the field operators, and the total area value. The results are further illustrated in Figure 30.

The results suggest that the total area value corresponds to 2065, which is 284 lower than for Scenario 1. Adding constraints to the model will always lead to less or equal total area value, but it does not take into account the value of the undiscovered fields. This implies that the subjective value of undiscovered fields must be at least 284 in order to change the optimal allocation from Scenario 1. Hence, the results present a specific value of undiscovered oil fields that experts can compare with their subjective estimates. For instance, if they valued the undiscovered oil fields to 200, it would be most optimal to shut down Host B. Another interesting finding is that the optimal allocation in this scenario includes tieback from Field 5 to Host B, which has a negative field value (-188). Even though there exist tieback developments for Field 5 that are economical viable (Host C and Host E), these hosts have other tieback options that provide higher field values. Field operator 5 would not agree to make an investment that loses money without being compensated. If we assume that the field operator is willing to connect to Host B if it results in a profit of 50 (a total compensation of 238), then the model suggests that the undiscovered fields around Host B must be valued at least the sum of the difference between total area value for Scenario 1 and Scenario 2, and the compensation for this scenario to be optimal. This corresponds to a value of 522 (284+238).

Table 9: Optimal tieback developments for Scenario 2 with corresponding field values.

1 → C	**2 → D**	**3 → E**	**4 → D**	**5 → B**	**Total**
611	142	1054	446	-188	**2065**

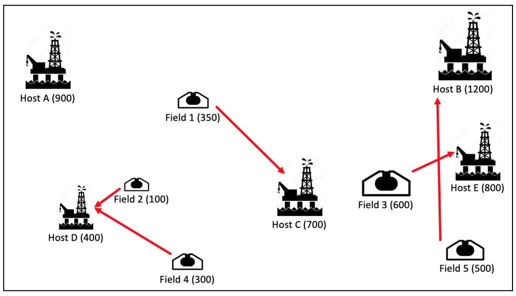


Figure 30: Optimal allocation for Scenario 2.

6.3.3 Scenario 3: Large Potential in Undiscovered Oil Fields Near Host B and Profitability Constraint

It could be discussed whether it is realistic to allow oil fields to become negatively valued as no field operator would proceed with such investment. However, it would have provided the highest total portfolio value. In scenario 3, we still assume great potential exists in undiscovered oil fields around Host B, but we do not allow tieback developments that are part of the optimal solution to become negative. We manage this by adding an additional constraint given by:

$$\sum_{j=1}^{J} FieldValues_j * x_{ij} \geq 0, \quad \forall i, \tag{38}$$

as well as keeping Equation 37 from Scenario 2. Table 10 states the optimal tieback developments for Scenario 3, with corresponding field values from the perspective of the field operators and the total area value. The results are further illustrated in Figure 31.

Table 10: Optimal tieback developments for Scenario 3 with corresponding field values.

$1 \rightarrow$ B	$2 \rightarrow$ D	$3 \rightarrow$ E	$4 \rightarrow$ D	$5 \rightarrow$ C	Total
276	142	1054	446	120	**2038**

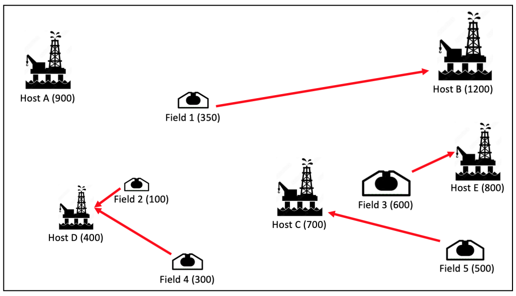


Figure 31: Optimal allocation for Scenario 3.

The results suggest that the total area value corresponds to 2038, which is 27 lower than for Scenario 2 and 311 lower than for Scenario 1. By not allowing any field values to become less than zero, the results suggest switching the tiebacks so that Field 1 is connected to Host C and Field 5 is connected to Host B. As expected, the total portfolio value decreases as we add more constraints. However, it only decreases by 27 compared to Scenario 2, which is almost negligible. In this scenario, the undiscovered field must be valued at least 311 to be more optimal than Scenario 1. Albeit Scenario 3 results in slightly less total portfolio value, it is probably more desirable than Scenario 2 because there is no need to compensate any field operators, and there is less risk associated with the undiscovered oil fields. In this scenario, the operator of Field 5 earns more than in Scenario 2, at the expense of the operator of Field 1, who earns significantly less (but still a substantial amount).

6.4 Final Remarks

In this extended section, we have presented an optimization model for a portfolio of a larger set of tieback developments, which we employed on a synthetic case and investigated three different scenarios. The ROV model we established in Section 3 can serve as an important building block in this optimization model, as it could generate the field value for each tieback development that can be used by the optimization model to maximize total area value on a larger area. The extended model reveals valuable insight regarding the optimal allocation of tieback developments from oil fields to hosts. It is demonstrated how easily the model can be adjusted in order to take into account specific assumptions, e.g., avoiding a host from being shut down due to potentially valuable undiscovered oil fields nearby. Moreover, the model can provide a specific minimum value of undiscovered oil fields around a given host in order to avoid that host from being shut down.

We emphasize that the proposed optimization model has several simplifications. Firstly, it only considers economic value when solving these kinds of problems and disregards other important considerations such as health, safety, the environment, and other users of the sea, which are other essential considerations in E&P projects. We suggest that the model is applied in combination with industry experts when determining the optimal allocation in order to include all considerations, also those that are difficult to quantify. Secondly, the proposed methodology only considers the accumulated spare capacity at the host facility and the amount of oil in place (see Table 6), but as described in Section 3.3 there are more constraints associated with production. Additionally, as production constraints often vary with time, it calls for a more dynamic way to handle these. Thirdly, in contrast to our ROV model, it is not able to incorporate uncertainties or managerial flexibility on the portfolio level. It is interesting for further research to account for the timing aspect of the portfolio value and not just for each individual combination.

Despite its limitations, the model still serves well for illustration purposes. Firstly, we have raised awareness of an emerging challenge in the O&G industry regarding the allocation of larger tieback development portfolios. This problem calls for novel methodologies that combine optimization and managerial flexibility to maximize the value of subsea resources for the Norwegian society. Secondly, we also presented scenarios where the interests of a field operator might conflict with what is considered most optimal *overall*. Although we briefly discussed how host facilities could change their tariff schemes, cost-sharing, lifetime, spare capacity, or a combination of these to become the preferred choice by field operators, it would be interesting to investigate these factors more thoroughly from the perspective of host facility owners and regulators. Thirdly, we have demonstrated how our proposed financial model can be used in a wider context and for larger and more complex problem-solving. The optimization method for production profiles that we incorporated in our model is important to handle the aforementioned limitation of the current portfolio optimization model, as it is dynamic and handles all the relevant constraints for extracting petroleum from reservoirs. Furthermore, the final field values that our model calculates are the most important values that the portfolio optimization model uses to determine the optimal allocation. Therefore, it is essential that the field values are realistic and robust in order for the portfolio optimization model to be reliable itself. We consider the ROV model that we have established to be a prominent choice of an underlying model.

7 Conclusion

This paper presents a novel methodology to evaluate optimal tieback developments for marginal oil fields with timing flexibility. Firstly, we used optimization techniques to maximize the production profiles based on spare host capacity, initial O&G in place, and the field potential. We followed a ROA, which we solved using the LSM framework in order to capture the value of timing flexibility. The future O&G prices were modeled by two-factor stochastic price processes, and both gas prices and CAPEX were modeled to correlate with the oil price.

The proposed methodology was applied to a real case study on the NCS, where a tieback from one marginal oil field to two alternative host platforms was considered. The methodology provides a basis to develop a tool for decision-makers in the petroleum industry by: (1) economically assessing the value of a marginal oil field, (2) determining the optimal choice of hosts, and (3) optimizing the timing of investment. The main findings suggest that marginal field developments carry large upside potential, which can be identified by our methodology because it takes managerial flexibility into account. Furthermore, we performed several sensitivity analyses in order to determine what drives the optimal choice of hosts and under which project environments the timing of flexibility increases. The results suggest that no factors alone were able to change the optimal choice of host since Host A was evidently much more attractive. However, altering the parameters of the tariff scheme in combination with extending the lifetime of the host could change the preferences of the field operator. We also identified that the value of timing flexibility increases as the profitability of the project decreases or the uncertainty of the investment increases. As marginal oil field developments often are characterized by relatively low profitability and prominent uncertainties, managerial flexibility is usually of high importance. Hence, ROV proves itself as a better valuation method as it allows us to capture the value of flexibility, while NPV tends to underestimate such investments.

Future work may be aimed at incorporating technical uncertainty in the valuation model in order to become more realistic. One way of handling technical uncertainty could be combining ROA and decision analysis. It would also be interesting to analyze the same problem with other types of flexibility, e.g., temporarily shutting down production. Furthermore, future work may focus on the problem described in Section 6, which calls for novel methodologies to determine the optimal allocation of a larger portfolio of tieback developments.

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Bibliography

Abbott, P., D'Souza, R., Solberg, I. & Eriksen, K. (1995), 'Evaluating Deepwater Development Concepts', *Journal of Petroleum Technology* **47**(04), 314–321.
URL: *https://doi.org/10.2118/28679-PA*

Allais, M. (1956), 'Évaluation des perspectives économiques de la recherche minière sur de grands espaces - application au Sahara Algérien', *Revue de l'Industrie Minérale* pp. 329–284.

Amram, M. & Kulatilaka, N. (1998), Real Options: Managing Strategic Investment in an Uncertain World.

Andersen, T. J. (2000), 'Real Options Analysis in Strategic Decision Making: An applied approach in a dual options framework', *Journal of Applied Management Studies* **9**(2), 235–255.
URL: *https://doi.org/10.1080/713674366*

Armstrong, M., Galli, A., Bailey, W. & Couet, B. (2004), 'Incorporating technical uncertainty in real option valuation of oil projects', *Journal of Petroleum Science and Engineering* **44**, 67–82.

Arrow, K. J. & Fisher, A. C. (1974), 'Environmental Preservation, Uncertainty, and Irreversibility', *The Quarterly Journal of Economics* **88**(2), 312–319.
URL: *http://www.jstor.org/stable/1883074*

Asche, F., Dahl, R. E. & Øglend, A. (2013), The volatility of oil and gas prices, Working paper, University of Stavanger.

Askari, H. & Krichene, N. (2010), 'An oil demand and supply model incorporating monetary policy', *Energy* **35**(5), 2013–2021.
URL: *https://EconPapers.repec.org/RePEc:eee:energy:v:35:y:2010:i:5:p:2013-2021*

Averbukh, V. (1997), Pricing American Options Using Monte Carlo Simulation, Ph.d. dissertation, Cornell University.

Bacati, F., Arcangeletti, G., Breuskin, B., Giolo, R., Muguerra, P., Radicioni, A., Shaiek, S. & Sibaud, E. (2020), How Technology Enables A Lower Cost for Subsea Tiebacks.

Baker, H. K., Dutta, S. & Saadi, S. (2010), 'Management Views on Real Options in Capital Budgeting', *Journal of Applied Finance* .

Ball, I. (2006), Challenges of Long Satellite Tiebacks, Vol. All Days of *OTC Offshore Technology Conference*. OTC-18405-MS.
URL: *https://doi.org/10.4043/18405-MS*

Barraquand, J. & Martineau, D. (1995), 'Numerical Valuation of High Dimensional Multivariate American Securities', *Journal of Financial and Quantitative Analysis* **30**(3), 383–405.
URL: *https://econpapers.repec.org/article/cupjfinqa/v_3a30_3ay_3a1995_3ai_3a03_3ap_3a383-405_5f00.htm*

Bernanke, B. (1983), Non-Monetary Effects of the Financial Crisis in the Propagation of the Great Depression, NBER Working Papers 1054, National Bureau of Economic Research, Inc.
URL: *https://EconPapers.repec.org/RePEc:nbr:nberwo:1054*

Bessembinder, H., Coughenour, J. F., Seguin, P. J. & Smoller, M. M. (1995), 'Mean Reversion in Equilibrium Asset Prices: Evidence from the Futures Term Structure', *Journal of Finance* **50**(1), 361–75.
URL: *https://EconPapers.repec.org/RePEc:bla:jfinan:v:50:y:1995:i:1:p:361-75*

Bjerksund, P. & Ekern, S. (1990), 'Managing investment opportunities under price uncertainty: from last chance to wait and see strategies', *Financial Management* **19**(3), 65–83.

Black, F. & Scholes, M. (1973), 'The Pricing of Options and Corporate Liabilities', *Journal of Political Economy* **81**(3), 637–54.
URL: *https://EconPapers.repec.org/RePEc:ucp:jpolec:v:81:y:1973:i:3:p:637-54*

Block, S. (2007), 'Are "Real Options" Actually Used in the Real World?', *The Engineering Economist* **52**(3), 255–267.
URL: *https://doi.org/10.1080/00137910701503910*

Bodie, Z. & Rosansky, V. I. (1980), 'Risk and Return in Commodity Futures', *Financial Analysts Journal* **36**(3), 27–39.
URL: *https://doi.org/10.2469/faj.v36.n3.27*

Bossaerts, P. (1989), Simulation Estimators of Optimal Early Exercise, Working paper, Camegie-Mellon University.

Boyle, P., Broadie, M. & Glasserman, P. (1997), 'Monte Carlo methods for security pricing', *Journal of Economic Dynamics and Control* **21**(8), 1267–1321. Computational financial modelling.

Boyle, P. P. (1977), 'Options: A Monte Carlo approach', *Journal of Financial Economics* **4**(3), 323–338.
URL: *https://EconPapers.repec.org/RePEc:eee:jfinec:v:4:y:1977:i:3:p:323-338*

Brandão, L. & Dyer, J. (2005), 'Decision Analysis and Real Options: A Discrete Time Approach to Real Option Valuation', *Annals OR* **135**, 21–39.

Brandão, L., Dyer, J. & Hahn, W. (2005), 'Response to Comments on Brandão et al. (2005)', *Decision Analysis* **2**, 103–109.

Brealey, R., Myers, S., Allen, F. & Mohanty, P. (2012), Corporate finance, Vol. Corporate finance of *SPE Hydrocarbon Economics and Evaluation Symposium*. SPE-37965-MS.

Brennan, M. & Schwartz, E. S. (1985), 'Evaluating Natural Resource Investments', *The Journal of Business* **58**(2), 135–57.
URL: *https://EconPapers.repec.org/RePEc:ucp:jnlbus:v:58:y:1985:i:2:p:135-57*

Broadie, M. & Glasserman, P. (1997*a*), Monte Carlo Methods for Pricing High-Dimensional American Options: An Overview:, Working paper, Columbia University.

Broadie, M. & Glasserman, P. (1997*b*), 'Pricing American-Style Securities Using Simulation', *Journal of Economic Dynamics and Control* **21**(8-9), 1323–52.

Broadie, M. & Glasserman, P. (2004), 'A Sotchastic Mesh Method for Pricing High-Dimensional American Options', *Journal of Computational Finance* **7**, 35–72.

Broadie, M., Glasserman, P. & Jain, G. (1997), 'Enhanced Monte Carlo Estimates for American Option Prices', *Journal of Derivatives* **5**(1), 25–44.

Brown, S. & Yucel, M. (2008), 'What Drives Natural Gas Prices?', *The Energy Journal* **Volume 29**(Number 2), 45–60.
URL: *https://EconPapers.repec.org/RePEc:aen:journl:2008v29-02-a03*

Cárdenas, C. (2017), Offshore Exploratory Drilling Campaigns During Low Oil Price Period: Maximizing Value Creation from Flexibility.

Carr, Peter (1998), 'Randomization and the american put', *The Review of Financial Studies* **11**(3), 597–626.

Carriere, J. F. (1996), 'Valuation of the early-exercise price for options using simulations and nonparametric regression', *Insurance: Mathematics and Economics* **19**(1), 19–30.
URL: *https://EconPapers.repec.org/RePEc:eee:insuma:v:19:y:1996:i:1:p:19-30*

Carter, C., Rausser, G. & Schmitz, A. (1983), 'Efficient Asset Portfolios and the Theory of Normal Backwardation', *Journal of Political Economy* **91**(2), 319–31.
URL: *https://EconPapers.repec.org/RePEc:ucp:jpolec:v:91:y:1983:i:2:p:319-31*

Castro, I. J. & Singh, P. (2019), Applying an integrated real options approach to oil field development: Valuing new information through appraisal under market and reservoir risk, Master's thesis, NTNU, Trondheim, Norway.

Chorn, L. & Carr, P. (1997), The Value of Purchasing Information to Reduce Risk in Capital Investment Projects, Vol. All Days of *SPE Hydrocarbon Economics and Evaluation Symposium*. SPE-37948-MS.
URL: *https://doi.org/10.2118/37948-MS*

Chorn, L. & Croft, M. (2000), 'Resolving Reservoir Uncertainty To Create Value', *Journal of Petroleum Technology* **52**.

Chorn, L. & Shokhor, S. (2006), 'Real options for risk management in petroleum development investments', *Energy Economics* **28**(4), 489–505.
URL: *https://EconPapers.repec.org/RePEc:eee:eneeco:v:28:y:2006:i:4:p:489-505*

Cortazar, G., Kovacevic, I. & Schwartz, E. S. (2013), Commodity and Asset Pricing Models: An Integration, Working Paper 19167, National Bureau of Economic Research.
URL: *http://www.nber.org/papers/w19167*

Cortazar, G. & Naranjo, L. (2006), 'An N-factor Gaussian model of oil futures prices', *Journal of Futures Markets* **26**(3), 243–268.
URL: *https://EconPapers.repec.org/RePEc:wly:jfutmk:v:26:y:2006:i:3:p:243-268*

Cortazar, G., Schwartz, E. & Naranjo, L. (2003), 'Term Structure Estimation in Low-Frequency Transaction Markets: A Kalman Filter Approach with Incomplete Panel-Data', *Anderson Graduate School of Management, UCLA, University of California at Los Angeles, Anderson Graduate School of Management* .

Cortazar, G. & Schwartz, E. S. (1994), 'The Valuation of Commodity Contingent Claims', *Journal of Derivatives* **1**, 27–35.

Cortazar, G. & Schwartz, E. S. (1998), 'Monte Carlo evaluation model of an undeveloped oil field', *Journal of Energy Finance & Development* **3**(1), 73–84.
URL: *https://www.sciencedirect.com/science/article/pii/S1085744399800696*

Cortazar, G. & Schwartz, E. S. (2003), 'Implementing a stochastic model for oil futures prices', *Energy Economics* **25**(3), 215–238.
URL: *https://EconPapers.repec.org/RePEc:eee:eneeco:v:25:y:2003:i:3:p:215-238*

Costa Lima, G. A. & Suslick, S. B. (2006), 'Estimating the volatility of mining projects considering price and operating cost uncertainties', *Resources Policy* **31**(2), 86–94.
URL: *https://EconPapers.repec.org/RePEc:eee:jrpoli:v:31:y:2006:i:2:p:86-94*

Cox, J. C., Ingersoll, J. E. & Ross, S. A. (1985), 'An Intertemporal General Equilibrium Model of Asset Prices', *Econometrica* **53**(2), 363–384.
URL: *http://www.jstor.org/stable/1911241*

Cox, J. C., Ross, S. & Rubinstein, M. (1979), 'Option pricing: A simplified approach', *Journal of Financial Economics* **7**(3), 229–263.
URL: *https://EconPapers.repec.org/RePEc:eee:jfinec:v:7:y:1979:i:3:p:229-263*

Cukierman, A. (1980), 'The Effects of Uncertainty on Investment under Risk Neutrality with Endogenous Information', *Journal of Political Economy* **88**(3), 462–75.
URL: *https://EconPapers.repec.org/RePEc:ucp:jpolec:v:88:y:1980:i:3:p:462-75*

Davalath, J. & Wiles, D. (2017), Significant Cost Reduction of Subsea Boosting Systems by Innovative Technologies, Vol. Day 3 Wed, May 03, 2017 of *OTC Offshore Technology Conference.* D031S033R006.
URL: *https://doi.org/10.4043/27639-MS*

Dias, M. (1997), The Timing of Investment in E&P: Uncertainty, Irreversibility, Learning, and Strategic Consideration, Vol. All Days of *SPE Hydrocarbon Economics and Evaluation Symposium.* SPE-37949-MS.
URL: *https://doi.org/10.2118/37949-MS*

Dias, M. (2004), 'Valuation of exploration and production assets: An overview of real options models', *Journal of Petroleum Science and Engineering* **44**, 93–114.

Dias, M. A. G. (2002), Investment in Information in Petroleum, Real Options and Revelation.

Dixit, A. K. & Pindyck, R. S. (1994), *Investment under Uncertainty*, Princeton University Press, Princeton, New Jersey.

Drew, L. (1967), 'Grid-drilling exploration and its application to the search for petroleum', *Economic Geology* **62**, 692–710.

Dusak, K. (1973), 'Futures Trading and Investor Returns: An Investigation of Commodity Market Risk Premiums', *Journal of Political Economy* **81**(6), 1387–1406.
URL: *https://EconPapers.repec.org/RePEc:ucp:jpolec:v:81:y:1973:i:6:p:1387-1406*

Elmerskog, C. (2016), Co-developing Johan Castberg and Alta/Gohta: a real options approach, Master's thesis, Nord universitet, Bodø, Norway.

Fedorov, S., Hagspiel, V. & Lerdahl, T. (2021), 'Real options approach for a staged field development with optional wells', *Journal of Petroleum Science and Engineering* **205**, 108837.

Fedorov, S., Hagspiel, V., Lerdahl, T., Idris, A. & Ghahfarokhi, A. J. (2020), Staged Development of a Marginal Oil Field Under Technical and Market Uncertainty: The Olympus Case, Vol. Day 3 Wed, October 28, 2020 of *SPE Annual Technical Conference and Exhibition.* D031S037R002.
URL: *https://doi.org/10.2118/201300-MS*

Fedorov, S., Nait Amar, M., Hagspiel, V. & Lerdahl, T. (2022), 'Sequential Production of Two Oil Fields With an Option to Switch'.

Fleten, S.-E., Gunnerud, V., Hem, Ø. D. & Svendsen, A. (2011), 'Real Option Valuation of Offshore Petroleum Field Tie-Ins', *Microeconomics: Production* .

Forskrift om andres bruk av innretninger (2005), *Forskrift om andres bruk av innretninger (FOR-2005-12-20-1625)*, Lovdata.
URL: *https://lovdata.no/dokument/SF/forskrift/2005-12-20-1625*

Galli, A., Armstrong, M. & Jehl, B. (1999), ' Comparison of three methods for evaluating oil projects', *Journal of Petroleum Technology* pp. 44–49.

Galli, A., Jung, T., Armstrong, M. & Lhote, O. (2001), Real Option Evaluation of a Satellite Field in the North Sea, Vol. All Days of *SPE Annual Technical Conference and Exhibition.* SPE-71410-MS.
URL: *https://doi.org/10.2118/71410-MS*

Garcia, D. (1999), A Monte Carlo Method for Pricing American Options, Working paper, University of California at Berkeley.

Gassert, M., Calabrese, V., Paccagnella, G., Fazio, A., Galisai, N., Argento, F., Castellitto, T., Magi, S., Citi, G., Infusino, L. & Rollo, F. (2019), Deepwater Opportunities Extra Long Oil Tiebacks Developments, Vol. Day 1 Mon, May 06, 2019 of *OTC Offshore Technology Conference.* D011S012R006.
URL: *https://doi.org/10.4043/29265-MS*

Gibson, R. & Schwartz, E. S. (1990), 'Stochastic Convenience Yield and the Pricing of Oil Contingent Claims', *Journal of Finance* **45**(3), 959–76.
URL: *https://EconPapers.repec.org/RePEc:bla:jfinan:v:45:y:1990:i:3:p:959-76*

Glasserman, P. (2004), *Monte Carlo Methods in Financial Engineering*, Springer-Verlag, New York.

Goodwin, D. (2013), Schwartz-Smith Two-Factor Model in the Copper Market: before and after the New Market Dynamics, Master's thesis, Lund University.

Hamilton, J. (1994), *Time Series Analysis*, Princeton University Press.

Hammersley, J. M. & Handscomb, D. C. (1964), *Monte Carlo Methods*, Springer, Dordrecht.

Harbaugh, J. W. (1984), ' Quantitative estimation of petroleum prospect outcome probabilities: an overview of procedures', *Marine and Petroleum Geology* .

Harris, D. (1990), *Mineral exploration decisions: A guide to economic analysis and modeling*, Wiley, New York.

Harvey, A. (1989), *Forecasting, Structural Time Series Models and the Kalman Filter*, Cambridge University Press.
URL: *https://EconPapers.repec.org/RePEc:cup:cbooks:9780521405737*

Heng, E., Ronalds, B. & Edwards, T. (2000), The Deepwater Subsea Tieback Option for Future Oil and Gas Production in Australia, Vol. All Days of *SPE Asia Pacific Oil and Gas Conference and Exhibition*. SPE-64485-MS.
URL: *https://doi.org/10.2118/64485-MS*

Henry, C. (1974), 'Investment Decisions Under Uncertainty: The "Irreversibility Effect"', *The American Economic Review* **64**(6), 1006–1012.
URL: *http://www.jstor.org/stable/1815248*

Horn, A., Kjærland, F., Molnár, P. & Steen, B. W. (2015), 'The use of real option theory in Scandinavia's largest companies', *International Review of Financial Analysis* **41**(C), 74–81.
URL: *https://EconPapers.repec.org/RePEc:eee:finana:v:41:y:2015:i:c:p:74-81*

Husy, S. (2011), Marginal Fields: Technology enables profitability/Marginal Fields and their Challenges, Vol. All Days of *OTC Offshore Technology Conference*. OTC-21382-MS.
URL: *https://doi.org/10.4043/21382-MS*

Ibanez, A. & Zapatero, F. (2004), 'Monte Carlo Valuation of American Options through Computation of the Optimal Exercise Frontier', *Journal of Financial and Quantitative Analysis* **39**(2), 253–275.
URL: *https://EconPapers.repec.org/RePEc:cup:jfinqa:v:39:y:2004:i:02:p:253-275_00*

Jafarizadeh, B. & Bratvold, R. (2009*a*), 'Real Options Analysis in Petroleum Exploration and Production: A New Paradigm in Investment Analysis', *SPE Economics & Management* .

Jafarizadeh, B. & Bratvold, R. (2009*b*), 'Taking Real Options Into the Real World: Asset Valuation Through Option Simulation'.

Jafarizadeh, B. & Bratvold, R. (2012), 'Two-Factor Oil-Price Model and Real Option Valuation: An Example of Oilfield Abandonment', *SPE Economics & Management* **4**, 158–170.

Jafarizadeh, B. & Bratvold, R. (2015), 'Oil and gas exploration valuation and the value of waiting', *Engineering Economist* **60**(4), 245–262.

Kálmán, R. E. (1960), A new approach to linear filtering and prediction problems" transaction of the asme journal of basic.

Karimaie, H., Pourmohammadi, S., Nyrønning, H. H. & Reeds, J. (2018), Feasibility Study and Development Concepts for Barents Sea Oil Discoveries, Vol. Day 3 Thu, March 22, 2018 of *Offshore Technology Conference Asia*. D032S004R016.
URL: *https://doi.org/10.4043/28498-MS*

Laine, J. (1997), Option Valuation of Field Development Projects, Vol. All Days of *SPE Hydrocarbon Economics and Evaluation Symposium*. SPE-37965-MS.
URL: *https://doi.org/10.2118/37965-MS*

Laughton, D. G. (1998), 'The Potential for Use of Modern Asset Pricing Methods for Upstream Petroleum Project Evaluation: Introductory Remarks', *The Energy Journal* **19**(1).
URL: *https://EconPapers.repec.org/RePEc:aen:journl:1998v19-01-a01*

Laughton, D. G. & Jacoby, H. D. (1993), 'Reversion, Timing Options, and Long-Term Decision-Making', *Financial Management* **22**(3).
URL: *https://EconPapers.repec.org/RePEc:fma:fmanag:laughton*

Laughton, D., Guerrero, R. & Lessard, D. (2008), 'Real Asset Valuation: A Back-to-basics Approach', *Journal of Applied Corporate Finance* **20**, 46–65.

Lazo, J., Aurélio, M., Pacheco, C. & Vellasco, M. (2003), 'Real option decision rules for oil field development under market uncertainty using genetic algorithms and monte carlo simulation'.

Lei, G., Silva, T. L. & Stanko, M. (2021), A Mathematical Programming Approach to Determine Optimal Tieback Development Concepts of Two Offshore Oil Fields in the Barents Sea, Vol. All Days of *International Ocean and Polar Engineering Conference*. ISOPE-I-21-1283.

Lin, J., de Weck, O., de Neufville, R. & Yue, H. (2013), 'Enhancing the value of offshore developments with flexible subsea tiebacks', *Journal of Petroleum Science and Engineering* **102**, 73–83.

Longstaff, F. A. & Schwartz, E. S. (1992), 'A Two-Factor Interest Rate Model And Contingent Claims Valuation', *The Journal of Fixed Income* **2**(3), 16–23.
URL: *https://jfi.pm-research.com/content/2/3/16*

Longstaff, F. & Schwartz, E. (2001), 'Valuing American Options by Simulation: A Simple Least-Squares Approach', *Review of Financial Studies* **14**, 113–47.

Lund, M. W. (1999), 'Real Options in Offshore Oil Field Development Projects', *Annals of Operations Research* **99**(1-4), 325–349.

Lund, M. W. (2000), 'Valuing Flexibility in Offshore Petroleum Projects', *Annals of Operations Research* **99**(1-4), 325–349.

Majd, S. & Pindyck, R. S. (1987), 'Time to build, option value, and investment decisions', *Journal of Financial Economics* **18**(1), 7–27.
URL: *https://www.sciencedirect.com/science/article/pii/0304405X87900596*

Manoliu, M. & Tompaidis, S. (2002), 'Energy futures prices: term structure models with Kalman filter estimation', *Applied Mathematical Finance* **9**(1), 21–43.
URL: *https://EconPapers.repec.org/RePEc:taf:apmtfi:v:9:y:2002:i:1:p:21-43*

McDonald, R. L. (2013), *Derivatives Markets*, 3 edn, Pearson.

McDonald, R. L. & Siegel, D. R. (1985), 'Investment and the Valuation of Firms When There Is an Option to Shut Down', *International Economic Review* **26**(2), 331–49.
URL: *https://EconPapers.repec.org/RePEc:ier:iecrev:v:26:y:1985:i:2:p:331-49*

Merton, R. (1973), 'Theory of Rational Option Pricing', *Bell Journal of Economics* **4**(1), 141–183.
URL: *https://EconPapers.repec.org/RePEc:rje:bellje:v:4:y:1973:i:spring:p:141-183*

Mikalsen, K. & Loper, C. (2019), Un-Locking Subsea Reserves Through a System-Based Approach for Tie-Back Solutions, Vol. Day 2 Tue, May 07, 2019 of *OTC Offshore Technology Conference*. D021S023R002.
URL: *https://doi.org/10.4043/29647-MS*

Moel, A. & Tufano, P. (2002), 'When Are Real Options Exercised? An Empirical Study of Mine Closings', *The Review of Financial Studies* **15**(1), 35–64.
URL: *http://www.jstor.org/stable/2696798*

Myers, S. C. (1977), 'Determinants of corporate borrowing', *Journal of Financial Economics* **5**(2), 147–175.
URL: *https://www.sciencedirect.com/science/article/pii/0304405X77900150*

Nadarajah, S., Margot, F. & Secomandi, N. (2017), 'Comparison of least squares Monte Carlo methods with applications to energy real options', *European Journal of Operational Research* **256**(1), 196–204.
URL: *https://www.sciencedirect.com/science/article/pii/S0377221716304404*

Norwegian Petroleum Directorate (2020), 'Resource Report'.
URL: *https://www.npd.no/en/facts/publications/reports/resource-report/resource-report-2020/*

Norwegian Petroleum Directorate (2021), 'Discoveries'.
URL: *https://www.norskpetroleum.no/en/facts/discoveries/*

Norwegian Petroleum Directorate (n.d.*a*), 'About Us'.
URL: *https://www.npd.no/en/about-us/*

Norwegian Petroleum Directorate (n.d.*b*), 'Eksport av olje og gass'.
URL: *https://www.norskpetroleum.no/rammeverk/rammevilkarpetroleumshistorie/*

Norwegian Petroleum Directorate (n.d.*c*), 'Production licence'.
URL: *https://www.npd.no/en/facts/production-licences/*

O'Dell, M. & Lamers, E. (2005), 'Subsurface Uncertainty Management and Development Optimization in the Harweel Cluster, South Oman', *SPE Reservoir Evaluation & Engineering - SPE RESERV EVAL ENG* **8**, 164–168.

Paddock, J. L., Siegel, D. R. & Smith, J. (1988), 'Option Valuation of Claims on Real Assets: The Case of Offshore Petroleum Leases', *The Quarterly Journal of Economics* **103**(3), 479–508.
URL: *https://EconPapers.repec.org/RePEc:oup:qjecon:v:103:y:1988:i:3:p:479-508.*

Parra-Sanchez, C. (2010), A life cycle optimization approach to hydrocarbon recovery, Master's thesis, University of Texas, Austin, USA.

Quigg, L. (1993), 'Empirical Testing of Real Option-Pricing Models', *Journal of Finance* **48**(2), 621–40.
URL: *https://EconPapers.repec.org/RePEc:bla:jfinan:v:48:y:1993:i:2:p:621-40*

Rajaratnam, S., Bedrossian, A., Roy, A., Lacerda, N., Cherkaoui, S., Giraudbit, S. & Gitahy, L. (2020), Development of Detailed Reeling Analysis for Electrically Heat Traced Flowline EHTF System, Vol. Day 2 Tue, May 05, 2020 of *OTC Offshore Technology Conference*. D021S016R005.
URL: *https://doi.org/10.4043/30700-MS*

Raymar, S. & Zwecher, M. (1997), 'A Monte Carlo Valuation of American Call Options on the Maximum of Several Stocks', *The Journal of Derivatives* **5**.

Rendleman, R. J. & Bartter, B. J. (1979), 'Two-State Option Pricing', *Journal of Finance* **34**(5), 1093–1110.
URL: *https://EconPapers.repec.org/RePEc:bla:jfinan:v:34:y:1979:i:5:p:1093-1110*

Ross, S. A. (1997), 'Hedging long run commitments: Exercises in incomplete market pricing', *Economic Notes by Banca Monte* **26**, 99–132.

Rystad Energy (2021), Utslippseffekten av produksjonskutt på norsk sokkel, Environmental report.
URL: *https://www.norskoljeoggass.no/globalassets/dokumenter/miljo/miljorapporter/utslippseffekten-av-produksjonskutt-pa-norsk-sokkel_rystad-energy.pdf*

Rønning, A., Haugsgjerd, J. & Rogstad, R. (2021), Real Options Approach to Determine Optimal Tieback Development Concepts for Marginal Oil Fields, Project thesis, NTNU, Trondheim, Norway.

Sandahl, G. & Sjögren, S. (2003), 'Capital budgeting methods among Sweden's largest groups of companies. The state of the art and a comparison with earlier studies', *International Journal of Production Economics* **84**, 51–69.

Schwartz, E. (2013), 'The Real Options Approach to Valuation: Challenges and Opportunities', *Latin american journal of economics* **50**, 163–177.

Schwartz, E. S. (1997), 'The Stochastic Behavior of Commodity Prices: Implications for Valuation and Hedging', *Journal of Finance* **52**(3), 923–73.
URL: *https://EconPapers.repec.org/RePEc:bla:jfinan:v:52:y:1997:i:3:p:923-73*

Schwartz, E. S. & Smith, J. E. (2000), 'Short-Term Variations and Long-Term Dynamics in Commodity Prices', *Management Science* **46**, 893–911.

Sharpe, W. F. (1978), *Investments*, Prentice-Hall, Hoboken, New Jersey.

Siegel, D., Smith, J. & Paddock, J. (1987), 'Valuing offshore oil properties with option pricing models', *Midland Corporate Finance Journal* **5**, 22–30.

Smith, J. E. (2005), 'Alternative Approaches for Solving Real-Options Problems: (Comment on Brandão et al. 2005)', *Decis. Anal.* **2**, 89–102.

Smith, J. E. & McCardle, K. F. (1999), 'Options in the Real World: Lessons Learned in Evaluating Oil and Gas Investments', *Oper. Res.* **47**, 1–15.

Smith, J. & Nau, R. (1995), 'Valuing Risky Projects: Option Pricing Theory and Decision Analysis', *Management Science* **41**, 795–816.

Soares, J. & Baltazar, D. (2010), 'Evaluation of real options in an oil field', *International Conference on Mathematical and Computational Methods in Science and Engineering - Proceedings* .

Stoisits, R. F., Bashagour, H. M. & Su, C. G. (2010), 'Application of Integrated Production and Reservoir Modeling to Optimize Deepwater Development'. SPE-131621-MS.
URL: *https://doi.org/10.2118/131621-MS*

Sørensen, C. (2002), 'Modeling Seasonality in Agricultural Commodity Futures', *Journal of Futures Markets* **22**, 393–426.

Thomas, P. & Bratvold, R. B. (2015), A Real Options Approach to the Gas Blowdown Decision, Vol. Day 1 Mon, September 28, 2015 of *SPE Annual Technical Conference and Exhibition.* D011S007R007.
URL: *https://doi.org/10.2118/174868-MS*

Tilley, J. A. (1993), 'Valuing American options in a path simulation model', *Transactions of the Society of Actuaries* **45**, 83–104.

Tourinho., O. A. F. (1979), The Option Value of Reserves of Natural Resources, Research Program in Finance Working Papers 94, University of California at Berkeley.
URL: *https://ideas.repec.org/p/ucb/calbrf/94.html*

Trigeorgis, L. (1996), Real Options: Managerial Flexibility and Strategy in Resource Allocation.

Trolle, A. B. & Schwartz, E. S. (2006), A General Stochastic Volatility Model for the Pricing and Forecasting of Interest Rate Derivatives, Working Paper 12337, National Bureau of Economic Research.
URL: *http://www.nber.org/papers/w12337*

Tsitsiklis, J. N. & Van Roy, B. (1999), 'Optimal stopping of Markov processes: Hilbert space theory, approximation algorithms, and an application to pricing high-dimensional financial derivatives', *IEEE Transactions on Automatic Control* **44**, 1840–1851.

U.S. Energy Information Administration (n.d.), 'WHAT DRIVES CRUDE OIL PRICES?'.
URL: *https://www.eia.gov/finance/markets/crudeoil/spot_prices.php*

Vielliard, C., Hester, K. C., Roccaforte, F., Di Lullo, A. G., Assecondi, L., Elkhafif, H. H., Ewis, A., Sabbagh, S., Solheim, H. & Lupeau, A. (2019), Real-Time Subsea Hydrate Management in the World's Longest Subsea Tieback, Vol. Day 2 Tue, May 07, 2019 of *OTC Offshore Technology Conference.* D021S017R002.
URL: *https://doi.org/10.4043/29232-MS*

Villar, J. & Joutz, F. (2006), 'The relationship between crude oil and natural gas prices', *Energy information administration, office of oil and gas* pp. 1–43.

West, M. & Harrison, J. (1996), *Bayesian Forecasting and Dynamic Models*, 2 edn, Springer-Verlag, New York.

Wiersema, U. (2008), *Brownian Motion Calculus*, John Wiley & Sons, West Sussex, UK.

Wiles, D. P., Widjaja, E. & Davalath, J. (2019), Subsea Long-Distance Tie-Back, Vol. Day 2 Tue, May 07, 2019 of *OTC Offshore Technology Conference.* D021S023R006.
URL: *https://doi.org/10.4043/29319-MS*

Willigers, B. & Bratvold, R. (2008), 'Valuing Oil and Gas Options by Least Squares Monte Carlo Simulation', *SPE Projects, Facilities & Construction* **4**.

Willigers, B. J. A. (2009), Enhanced Economic Modeling by Correlated Stochastic Models of E&P Costs and Hydrocarbon Prices: The Limitations of Fixed Price Decks and the Versatility of Least-Squares Monte Carlo Simulation, Vol. All Days of *SPE Europec featured at EAGE Conference and Exhibition.* SPE-121442-MS.
URL: *https://doi.org/10.2118/121442-MS*

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