Master's thesis

NTNU
Norwegian University of Science and Technology
Faculty of Economics and Management
Dept. of Industrial Economics and Technology Management

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Optimizing the Design of Charter Contracts for Installation Vessels at Offshore Wind Farms using Branch-and-Price

Master's thesis in Industrial Economics and Technology Management
Supervisor: Magnus Stålhane
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Kunnskap for en bedre verden

Preface

This Master's Thesis concludes our Master of Science degree with a specialization in Managerial Economics and Operations Research at the Norwegian University of Science and Technology (NTNU). The research and writing of the thesis took place during the spring of 2022. The thesis is a continuation of the work conducted in our specialization project in Managerial Economics and Operations Research (TIØ4500) during the fall of 2021.

We would like to thank our supervisor Professor in Operations Research Magnus Stålhane at NTNU for valuable feedback and insightful discussions throughout the months working on this thesis. We are also grateful to Troels Martin Range, Simen Tung Vadseth, and Gaute Messel Nafstad for valuable guidance on configuration and implementation of the solution method. Additionally, we would like to thank Jose Jorge Garcia Agis at Ulstein International and Frederik C. Andersen at Clarksons Platou who have supported the project with industry-specific knowledge of the problem addressed in this thesis.

June, 2022

Marie Tyssen Bruu & Lea Bakkevig Thorsen

Summary

A global transition to low carbon and renewable energy is happening now, with offshore wind at the center of the revolution. However, challenges related to the installation of offshore wind turbines must be solved to realize the full potential of offshore wind in a sustainable future and to ensure profitability. Today, the number of eligible installation vessels is scarce, and vessel day rates make a significant contribution to the installation cost and wind farm life cycle cost. The problem studied in this thesis is that of designing charter contracts for installation vessels at offshore wind farms. The goal is to minimize both the charter costs and project duration of the installation phase, while considering uncertainty in the weather conditions and complying with many real-life restrictions. More specifically, the mix of vessels that should be chartered, the start and end date of these vessels' charter periods, as well as which options to include in their contracts must be decided on. To achieve this, operational schedules that include information on which activities the chartered vessels are to perform, and when, are considered.

A two-stage stochastic mathematical formulation for the design of charter contracts is presented. As the formulation proves to be difficult to solve for realistic test instances using a commercial solver, we propose a Dantzig-Wolfe reformulation of the mathematical formulation that combines schedules into a feasible project plan in order to design the contracts. The schedules are generated by a number of subproblems formulated as shortest path problems with resource constraints on acyclic networks of nodes, and we present two labeling algorithms to solve these. A branch-and-price algorithm with the extension of a primal heuristic is implemented to solve the reformulated problem. A similar problem with all the considered aspects has to the best of our knowledge not previously been investigated in published research. Furthermore, branch-and-price is an approach scarcely used to solve stochastic problems and the research on this topic is limited.

We find that the branch-and-price algorithm succeeds at finding much better dual bounds compared with a commercial mixed integer programming solver. Computational results reveal that our method finds up to 700% better dual bounds. This is mainly due to the improved linear programming relaxation from the Dantzig-Wolfe reformulation. Further, the implemented primal heuristic is able to exploit the generated schedules to find good integer feasible solutions, resulting in lower optimality gaps for larger test instances.

Sammendrag

Verdenssamfunnet tar sikte på å bli et lavkarbonsamfunn basert på fornybare energikilder innen kort tid. Havvind har vist potensiale til å kunne spille en sentral rolle i denne overgangen, men det er både tekniske og økonomiske utfordringer som må løses før dette kan realiseres fullt ut. En av utfordringene er at det i dagens marked er svært få fartøy som kan bidra i installasjonsfasen av en offshore vindpark. Dette resulterer i høye leiekostnader som gir et betydelig bidrag til de totale installasjonskostnadene og vindparkens livssykluskostnad. I denne masteravhandlingen studerer vi et optimeringsproblem knyttet til leiekontraktsbestemmelser for disse fartøyene under værusikkerhet, med sikte på å minimere både kostnader og varigheten av installasjonsfasen. Mer spesifikt, er målet å bestemme hvilke typer fartøy som skal leies inn, start- og slutt dato for deres leieperiode, samt om det skal inkluderes kontraktuelle opsjoner. For å oppnå dette ser vi blant annet på operasjonelle tidsplaner med scheduling av aktiviteter for hvert fartøy.

Vi presenterer to ulike matematiske formuleringer av problemet. Først presenteres en to-stegs stokastisk modell som viser seg å være vanskelig å løse for realistiske test instanser ved bruk av en kommersiell solver. Derfor har vi gjort en Dantzig-Wolfe reformulering av modellen og utviklet en branch-and-price algoritme med en enkel primal heuristikk som løsningsmetode. I den reformulerte modellen kombineres tidsplaner for hvert enkelt fartøy til en gyldig prosjektplan for fullføring av hele vindparken. Subproblemene som benyttes i branch-and-price metoden løses som korteste-vei-problemer med ressursbegrensninger på et asyklisk nettverk, og vi presenterer to ulike labeling algoritmer for å løse disse. Et lignende problem med tilsvarende detaljnivå har så vidt vi vet ikke blitt studert i tidligere publisert litteratur. Videre skiller vårt arbeid seg fra eksisterende forskning ved at det er lite utbredt å benytte branch-and-price for å løse stokastiske modeller.

Våre resultater viser at ved å løse den dekomponerte modellen med branch-and-price lykkes vi med å oppnå bedre duale grenser enn de som ble funnet med kommersiell solver for den opprinnelige to-stegs modellen. Vår dekomponeringsmetode finner opp til 700 % bedre duale grenser. Dette skyldes hovedsakelig at den reformulerte modellen har en sterkere LP-relaksering. Ved hjelp av primalheurisitkken er vi også i stand til å oppnå et lavere optimalitetsgap på større testinstanser.

Table of Contents

List of Figures

List of Tables

Acronyms

B&B Branch-and-Bound

B&P Branch-and-Price

CLV Cable Laying Vessel

GBF Gravity-Based Foundation

H_S Significant Wave Height

IP Integer Programming

JUB Jack-Up Barge

LP Linear Programming

MFRP Maritime Fleet Renewal Problems

MFSMP Maritime Fleet Size and Mix Problems

MIP Mixed Integer Program

O&G Oil & Gas

O&M Operations & Maintenance

OWT Offshore Wind Turbine

REF Resource Extension Functions

RMP Restricted Master Problem

ROV Remotely Operated Vehicle

SA Simulated Annealing

SPPRC Shortest Path Problem with Resource Constraints

VNS Variable Neighbourhood Search

VRP Vehicle Routing Problem

WTIV Wind Turbine Installation Vessel

Chapter 1

Introduction

Wind energy is vital to reach climate targets and to secure an affordable energy supply. By 2030, the European Union (EU) will invest almost 20 billion Euros in the wind energy market. Offshore wind will be a big part of this, and 60% of the investments will be aimed at offshore wind specifically (Wu et al. 2019). Wind is an energy source that is abundantly available and offshore wind farms can be installed and operated while respecting nature and other users of the sea (NorthWind 2021). Today, offshore wind provides only a small fraction of the global electricity supply, but it is set to expand significantly in the coming decade (Lee and Zhao 2021). The European Commission predicts that there will be 300 GW installed offshore wind capacity in the EU by 2050 (NorthWind 2021). However, success will depend on the development of cost-competitive solutions.

To enable the predicted increase in new offshore wind farm installations, the necessary annual installation rate is about 28 GW/year by 2030 and about 45 GW/year by 2050 (Jagite 2022). Challenges related to installation and maintenance of turbines must be addressed to realize the full potential of the wind energy sector in a sustainable future. Today, the number of eligible installation vessels is scarce, and charter costs make a significant contribution to the installation cost and wind farm life cycle cost. Thus, developing optimized installation strategies for offshore wind farms is of interest to enable the massive deployment of future offshore wind farms and to reduce costs.

The problem studied in this thesis is that of designing charter contracts for installation vessels at offshore wind farms. A heterogeneous fleet of specialized vessels is needed to perform the installation of offshore wind turbines. Charter contracts for these vessels are entered into years in advance of the planned installation process. The vessels' ability to perform installation activities is highly weather dependent, and as strategic charter decisions must be made without knowledge of weather forecasts for the time of installation, the design of charter contracts is prone to uncertainty and risk. To hedge against the need to charter extra vessels in the event of delays, charter contracts usually include options to extend the contract period. The contract design problem comprises decisions on which mix of installation vessels that should be chartered, the start and end date of these vessels' charter period, and which options to include in their contracts. To decide this, we generate operational schedules for the chartered vessels which specify when a vessel is to perform certain installation activities. The optimal charter contracts minimize both charter costs and end date of the installation process under the uncertainty of weather conditions.

In this thesis, a two-stage stochastic mathematical formulation for design of charter contracts is presented. The problem of designing charter contracts under the uncertainty of weather conditions is complex, and models with the necessary level of detail to describe a realistic wind farm installation project are difficult to solve within reasonable computational time. The purpose of this thesis is to develop an improved solution method for the problem. For this reason, we apply a Dantzig-Wolfe reformulation of the problem that combines schedules into a feasible project plan in order to design the contracts. The schedules are generated by a number of subproblems formulated as Shortest Path Problem with Resource Constraints (SPPRC). We then develop a Branch-and-Price (B&P) algorithm with the extension of a primal heuristic algorithm to solve the contract design problem. Testing revealed that by introducing a Dantzig-Wolfe reformulation of the model, a tighter Linear Programming (LP) relaxation of the problem is obtained. Further, the implemented B&P algorithm requires fewer Branch-and-Bound (B&B) nodes than the commercial solver in order to obtain dual bounds of higher quality.

A similar problem with all the considered aspects has to the best of our knowledge not previously been investigated. The existing literature on optimization of offshore wind farm installation is scarce, and the majority does not consider the fleet size and mix aspect of the problem. Our contribution is that we provide stochastic mathematical formulations, that consider weather uncertainty and the use of multiple vessels for the installation of an offshore wind farm. We also include the aspect of contractual options, which to our knowledge has not been done in any published research papers. Furthermore, B&P is an approach scarcely used to solve stochastic problems and the research on this topic is limited.

The remainder of this thesis is organized as follows. In Chapter 2 we provide a general introduction to the offshore wind farm installation process. Based on this, a problem description is formulated in Chapter 3. Further, a review of scientific papers on relevant problems is provided in Chapter 4. A two-stage stochastic formulation for the design of charter contracts for installation vessels at offshore wind farms is presented in Chapter 5, while a decomposed formulation for the problem is presented in Chapter 6. Chapter 7 gives a detailed description of how we apply B&P to solve the problem. In Chapter 8 we explain the input data and introduce problem instances that form the basis for testing the performance of our models. Computational results are presented and analyzed in Chapter 9. We conclude and present final remarks in Chapter 10. Finally, we point out interesting directions for future research on the problem in Chapter 11.

Chapter 2

Background

In this chapter, we provide a general introduction to offshore wind farm installation. The content is to a large extent the same as in our earlier work, Bruu and Thorsen (2021). First, we introduce the concept and main characteristics of an offshore wind farm in Section 2.1. In Section 2.2 we provide a description of the offshore wind supply chain, with emphasis on the main offshore installation activities and concepts. Common installation vessels are further described in Section 2.3. Finally, we describe relevant aspects of vessel charter contracts and how weather affects the installation process in Section 2.4 and Section 2.5, respectively.

2.1 Anatomy of Offshore Wind Farms

An offshore wind farm is a power plant located on the continental shelf that generates electricity by exploiting the natural movement of air. A systematic overview of the main components of a wind farm is presented in Figure 2.1.

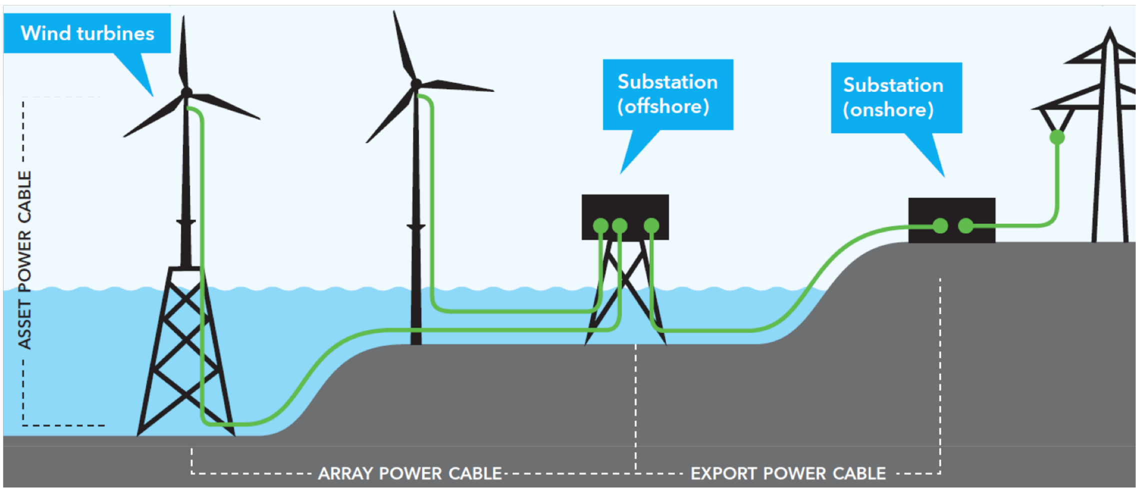


Figure 2.1: The main components of an offshore wind farm (DNV-GL 2018, p. 11).

The most important components of the plant are the wind turbines: generators that convert wind power into electric power (Kyriakopoulos 2021). The number of turbines per wind farm can vary from only a couple to more than 100. In 2019 the median number of connected turbines per new wind farm connected to the European grid was 43 (Walsh et al. 2020). The turbines are linked with array cables that connect them to an offshore substation. At the substation, the voltage is increased before the power is transmitted to onshore facilities through export cables (Ng and Ran 2016).

2.2 Offshore Installation Activities

Figure 2.2 illustrates the supply chain for offshore wind farms. Prior to installation, the components must be manufactured. Depending on the distance from the manufacturing port to the offshore site, finished components are transported either directly to the offshore location or via an intermediate port facility, known as marshalling or staging port. Once all turbines have been installed and connected to the grid, the Operations & Maintenance (O&M) phase starts. This phase lasts for the entire life span of the wind turbines, approximately 20 to 25 years. The last step in the supply chain is decommissioning.

Figure 2.2: The supply chain for offshore wind farms.

During the installation phase, the installation vessels operate in cycles. First, the vessel is loaded with components, then the vessel sails to the offshore site where all components on board are installed before the vessel returns to port to load a new set of components.

The main offshore installation activities to be performed during the installation of an offshore wind farm are foundation installation, cable installation and turbine installation. In addition to these main activities, one or more offshore substation must also be installed. Substation installation is a heavy lift operation that can be done in parallel with the other activities (BVG Associates 2019) and is not further discussed in this thesis. Due to varying site conditions and the continuous development of sub-structures and turbine sizes, there is no current standard for how to conduct these installation activities (Jiang 2021a). In the following sub-sections, a brief overview of common methods for each activity is described. Installation methods for floating wind turbines are out of scope for this thesis and have been omitted.

2.2.1 Foundation Installation

The first component to be installed for an Offshore Wind Turbine (OWT) is the foundation. Common foundation structures for bottom-fixed OWTs are monopile, Gravity-Based Foundation (GBF), and jacket. Schematic drawings of these foundations are shown in Figure 2.3.

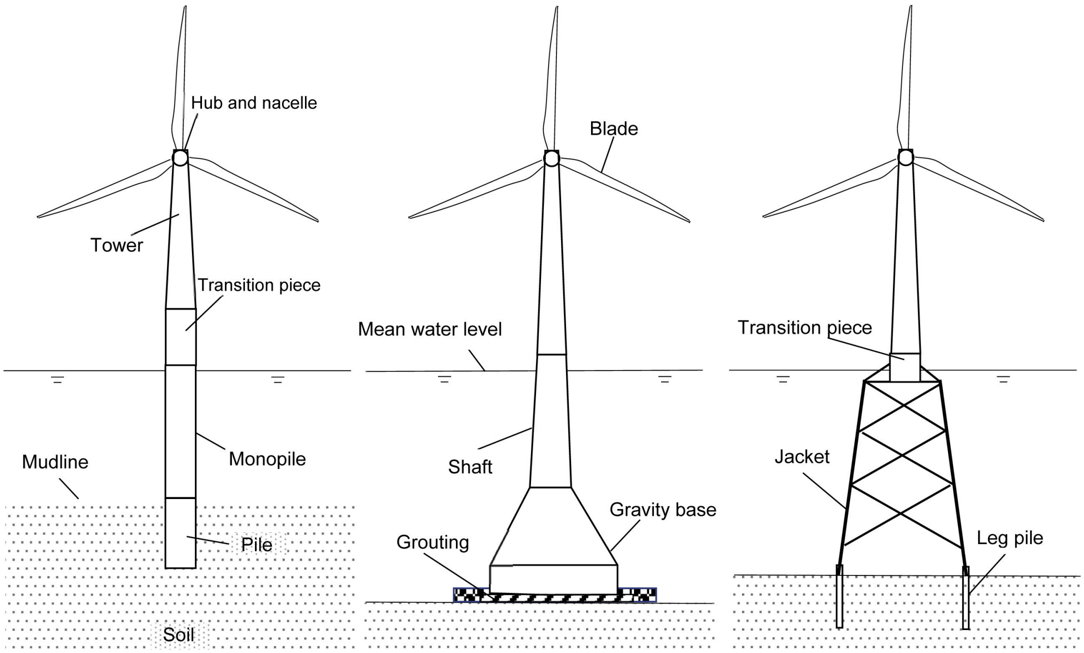


Figure 2.3: Illustration of monopile, gravity based and jacket foundations (Jiang 2021a).

Of these three foundation structures, the monopile is most used, followed by jackets (Sánchez et al. 2019). The installation method differs based on which foundation type is used. Choice of foundation depends on water depth and seabed characteristics. Table 2.1 is adapted from Sánchez et al. (2019) and provides some of the most accepted seabed depth ranges for which the separate foundations are found suitable. However, in addition to depth, other factors such as experience and resource availability also affect the preferred solution (Sánchez et al. 2019).

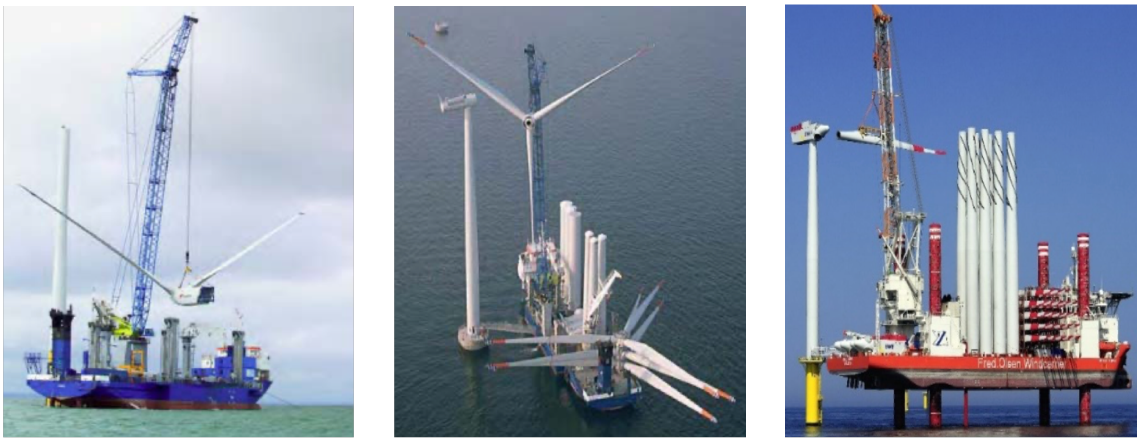
Table 2.1: Seabed depth limitations for different foundation concepts (Sánchez et al. 2019).

Foundation	Ashuri and Zaaijer, 2007	DNV, 2013	Iberdrola, 2017
GBF	0-10 m	0-25 m	0-30 m
Monopile	0-30 m	0-25 m	0-15 m
Jacket	>20 m	20-50 m	>30 m

2.2.2 Turbine Installation

Once the foundation is in place, installation of the actual wind turbine can begin. A turbine consists of four main components: tower, nacelle, hub and blades. The three main assembly strategies used for installation of OWTs are bunny ear, full rotor star and separate parts installation (Vis and Ursavas 2016, BVG Associates 2019). For bunny ear and full rotor star installation, parts of the assembly take place at port, reducing the number of offshore operations. Despite requiring the highest number of offshore operations, the separate parts method is the current preferred practice (BVG Associates 2019). This might be because an increased number of pre-assembled pieces may prevent efficient use of deck space on the transport vessel as well as it will require cranes with larger lifting capacities (Jiang 2021a).

Figure 2.4 shows images of each of the methods mentioned above used in practice. The images also illustrate the use of deck space and show jack-up vessels that are commonly used for turbine installation due to the need for a stable platform for the tall lifting operations (BVG Associates 2019).



(a) Bunny ear　**(b)** Rotor star　**(c)** Separate parts

Figure 2.4: Different turbine assembly strategies (Jiang 2021b).

2.2.3 Cable Installation

As explained earlier, array cables connect the OWTs to an offshore substation, and export cable enables transmission of power from the substation to onshore facilities. The installation of these cables involves three main activities: cable laying, cable burial and cable pull-in. Cable pull-in refers to the connection of the cable to either a turbine, substation, or shore (BVG Associates 2019). According to our interviews with Ulstein and Clarkson Platou, it is common to install the array cables during foundation installation, while installation of export cables can be considered as an independent operation.

There are two main strategies used for cable installation: simultaneous lay and burial, and post-lay burial (BVG Associates 2019, Ng and Ran 2016). In the former, laying and burial operations are coupled and performed by a single vessel. In the latter, laying and burial is a two-stage process, performed by separate vessels (BVG Associates 2019).

2.3 Vessel Concepts

Various offshore vessels are involved during the installation phase of an offshore wind farm. Other than the choice of technology and assembly strategies, the selection of vessels for each activity is based on market availability, day rates and turbine sizes (Jiang 2021a). A summary of common vessel concepts is provided in Table 2.2. The table is based on Jiang (2021a), Ahn et al. (2017) and BVG Associates (2019). The day rates listed in the table do not necessarily represent the present market and should therefore only be considered as estimates.

Previously the fleet for foundation installation and turbine installation overlapped, but due to increasing component sizes and weights, specialized vessels for the different operations are entering the market, resulting in a diverging fleet for these operations (BVG Associates 2019). Whilst the most critical vessel characteristic for foundations installation vessels is the lifting capacity in terms of weight, the most critical requirement for installing turbine components is the crane's lifting height capabilities. Thus, foundation installations are increasingly being performed by floating heavy lift vessels, while specially designed jack-up vessels with higher cranes and longer legs, Wind Turbine Installation Vessel (WTIV), are used to install turbine components (Interview with Ulstein, Appendix A).

2.4 Charter Contracts

Charter contracts for vessels are usually entered two to four years in advance of the planned installation process. The main elements of the contract are cost agreements and duration of the charter period, and the contract parties are the charterer and the shipowner. All information in this section is based on an interview with Clarksons Platou (Appendix B).

There are two main types of contracts used for offshore wind farm installation: time charter contracts and fixed price contracts. Of these, time charter contracts are most common. In time charter contracts the charge is based on day rates. When entering the contract, the charterer agrees to lease the vessel (including crew) for a fixed number of days. Additionally, the time charter contract may include options for the charterer to extend the contract. This can be desirable in case of delays. Besides day rates, a time charter contract includes a fixed project cost. This cost includes costs for mobilization and engineering work. Variable costs such as fuel and harbor dues are not included in a time charter contract. In fixed-price contracts the contract parties agree on a fixed price for the shipowner to perform some task, for instance to install a given number of OWT foundations.

In time charter contracts, day rates are influenced by the length of the charter period. To secure revenue, shipowners aim to make the fixed charter period as long as possible. Shorter or more flexible time charters may thus lead to higher day rates. Flexibility in the form of options is always in favor of the charterer. A contract can contain several options of different lengths and day rates that can be exercised individually. The charterer must give reasonable notice if they want to exercise the agreed upon options, and if the option is exercised, the charterer commits to paying day rates for the entire option period.

Table 2.2: Vessel concepts used for offshore wind farm installation.

Vessels	**Characteristics**	**Day rates (USD)**
Tugboat	Used for towing non self-propelled barges or floating foundations	1 000 - 5 000
Crane barge	Crane capacity 1 000 - 4 000 tons	80 000 - 100 000
Jack-up barge	Self-elevating Seabed depth limitations Not self-propelled Crane capacity 200 - 1 300 tons	100 000 - 180 000
Cargo barge	Large deck area for cargo Not self-propelled	30 000 - 50 000
Monohull heavy lift vessel	Loading and lifting of heavy objects (e.g. OWT foundations)	250 000
Wind turbine installation vessel	Purpose-built jack-up vessel Self-elevating Seabed depth limitations Crane capacity 800 - 1 500 tons	150 000 - 250 000
Semi-submersible vessel	Large loading and lifting capacity Grat lifting height Crane capacity 2 000 - 20 000 tons	280 000 - 500 000
Rock-dumping vessel	Dumping rocks for seabed preparations, scour protection or cable burial	20 000 - 40 000
Cable-laying vessel	Cable installation (and burial) Equipped with ROV Cable carousel with capacity up to 7 000 tons	115 000
Cable burial vessel	Used for post-lay cable burial Equipped wit various burial tools	120 000

2.5 Weather Impacts

Weather plays a critical role in the planning and execution of installation activities for OWTs. Due to safety considerations and vessel capabilities, each offshore operation can only be performed under certain limits for wave height and wind speed. If an operation cannot be performed due to the current weather conditions, this is known as a "weather delay". Weather delays are common in offshore operations and make up a considerable cost as vessel day rates must be paid regardless of whether or not the vessel can operate due to weather conditions (BVG Associates 2019; Ng and Ran 2016).

Table 2.3 is adapted from Paterson et al. (2018) and shows upper limits for wind speed and wave height as well as estimated time spent for different installation activities per OWT. However, these limits may be a bit conservative. For instance, a report by BVG Associates (2019) states that the operating limits have increased and that the current maximum wind speed for blade installation is 13 m/s at hub height. Activities such as loading and unloading of turbine components in port and transshipment could also be added to the list of activities affected by weather, this has been done in Rippel et al. (2019). In the review by Rippel et al. (2019) it is also revealed that the installation of the separate turbine parts have different limits, for instance, installation of turbine towers is less sensitive to wind speed than installation of turbine blades.

Table 2.3: Weather limits for installation activities (Paterson et al. 2018).

Activity	Weather limits		Duration (h/OWT)
	Wind Speed (m/s)	Wave Height (m)	
Foundation	12	2	48
Transition Piece	12	2	24
Turbine	8	2	24.5
Scour Protection	15	2.5	14.4
Cable Installation	15	1.5	31.7
Cable Burial	12	3	36

In general, the weather is harsher during winter. Hence, to minimize risk of weather delays, it is common to plan for all installation activities to be performed in the summer months. This means the vessel chartering market is prone to seasonality: high demand during the summer months, followed by excess capacity in the winter. These market dynamics influence the charter prices, and if installation activities are planned during the winter months, lower day rates can be obtained. However, this price reduction must be seen in combination with the higher expectancy of weather delays.

Chapter 3

Problem Description

The problem studied in this thesis is that of designing charter contracts for installation vessels at offshore wind farms. The goal is to design a charter strategy that minimizes both the overall charter costs and the completion time of the wind farm. Moreover, a feasible strategy respects the operational restrictions elaborated below.

A wind farm consists of a predefined number of identical OWTs. The installation of an OWT requires the following activities: foundation installation, cable installation and turbine installation. As described in Section 2.2, the nature and details of these activities depend on installation strategy and foundation design. In addition to the installation activities, activities such as loading components onto a vessel in port and sailing between port and the offshore site must be considered. Some activities must be performed in a given order, for instance, the use of a monopile foundation requires a transition piece to be in place before the turbine is installed. Moreover, only one activity can be performed for a given OWT at a given time.

A specialized fleet of vessels is needed to perform the installation activities. The vessels have different capacity and capability limitations and are often specialized to perform certain activities. All vessels have upper limits on loading and lifting capacity in terms of weight. Loading capacity is also determined by component sizes and available deck area for storing them on board the vessel. Additionally, the installation activities have upper limits for at which wave height and wind speed they can be performed, which restricts when a vessel can perform specific activities.

Due to operational limitations, the installation of an offshore wind farm is highly influenced by uncertain weather conditions. As a result of this it is common with weather delays. Contractual agreements for vessel chartering are made years in advance of the installation phase. Due to the long planning horizon and uncertainty related to weather conditions, options for the charterer to extend the contract for a specific number of days may be included in the charter contract. Options are included to hedge against the need to charter extra vessels in the event of weather delays. Then, if it becomes necessary to extend the charter period for certain vessels in order to complete the planned activities arises, the options can be exercised at a predefined cost.

All charter contracts include the following two cost components: variable costs and fixed costs. The variable costs include daily charter rates and operating costs, whilst the fixed costs include ship expenses, insurance, depreciation and overhead costs. Of the two variable cost terms, charter rates are dominant and are thus the most important to consider. Additionally, as described above, the charter contract may include options. The total cost related to options also consists of two parts; an inclusion cost and an exercising cost. The inclusion cost is paid regardless of whether the option is used or not, whereas the exercising cost is only paid if the option is exercised.

Once the wind farm is installed and connected to the electricity grid, it starts to generate income through production of electricity. If the installation phase is prolonged unnecessary, the excess installation time can be regarded as an increased installation cost as it causes loss of income due to loss of electricity production time. Hence, completion time of the wind farm is of importance.

In summary, a charter strategy for vessels used to install an offshore wind farm should minimize both charter costs and the end date of the installation phase. The decisions to be made are which vessels to charter, the start and end date of their charter period, as well as what options to include in the contracts. In order to evaluate the cost and effectiveness of a given strategy, the expected installation costs and end date must be evaluated. This involves assignment of activities to vessels in accordance with operational limitations, and to provide installation schedules for the vessels that respect precedence requirements in the installation sequence. Lastly, one must decide whether or not the options included should be exercised.

Chapter 4

Literature Review

In this chapter, we provide a review of existing literature that is relevant for the problem related to the design of charter contracts for installation vessels at offshore wind farms. Firstly, we explain the search strategy in Section 4.1. The focus of Section 4.2 is literature that directly addresses the planning problem for offshore wind farm installation, while Section 4.3 discusses literature on the more general maritime fleet size and mix problem. The content in these sections is to a large extent based on our earlier work (Bruu and Thorsen 2021), but with added consideration for solution methods. In Section 4.4 we review relevant literature on the application of B&P as a solution method for stochastic problems. Finally, we summarize the contribution of this thesis in Section 4.5.

4.1 Search Strategy

The first part of the literature review is to a large extent based on cited articles in Rippel et al. (2019), who provide a review of existing research in planning approaches for installation of offshore wind farms. To the best of our knowledge, the literature on optimization of offshore wind farm installation is limited. Therefore, as the installation of offshore wind farms requires a fleet of different types of specialized vessels, we have included a review of literature on the Maritime Fleet Size and Mix Problems (MFSMP). The MFSMP aims to find the necessary number and type of vessels required to meet a service demand, usually with the objective of minimizing costs. Many MFSMPs also include scheduling decisions, which is highly relevant for our problem. When reviewing literature om MFSMP, the main focus was given to stochastic formulations and solution methods for these. Literature on the use of B&P for stochastic problems is also reviewed. Here, the main focus is given to articles studying problems that resemble certain aspects of our problem, such as scheduling decisions and two-stage stochastic formulations.

The primary search engine used in the literature search was Google Scholar, a freely accessible search engine for academic papers. Google Scholar ranks publications based on where they are published, their author(s), and how often and recently it has been cited. This ranking was used to confine our search. Our search words are presented in Table 4.1.

Table 4.1: Summary of search words used in the literature search.

Offshore Wind Farm Installation	**MFSMP**	**Branch-and-Price**
Stochastic	Stochastic	Stochastic
Weather uncertainty	Weather uncertainty	Dantzig-Wolfe
Planning/scheduling	Offshore wind	Planning/scheduling

4.2 Offshore Wind Farm Installation Problems

Operations research is applied within the field of installation of offshore wind farms for scheduling of installation activities and to find optimal fleet configurations. Ait Alla et al. (2013), Barlow et al. (2018), Irawan et al. (2017), Scholz-Reiter et al. (2011, 2010) and Ursavas (2017) present (mixed) linear integer programs for this purpose. Common modelling assumptions in these papers are that all OWTs are assumed to be in the same location, that necessary components are available at the harbor, and that a predefined installation sequence must be obeyed for each turbine. Different approaches are used to enforce the predefined installation sequence. While Barlow et al. (2018) use an approach based on project-scheduling, Scholz-Reiter et al. (2011) and Ursavas (2017) combine typical multi-period production and job-shop formulations to ensure precedence requirements for the installation activities.

Most of the reviewed papers consider the influence of weather conditions on the vessels' ability to perform installation activities, but very few capture its uncertainty. Ait Alla et al. (2013), Irawan et al. (2017), Scholz-Reiter et al. (2011) and Ursavas (2017) use discrete weather categories, such as good, medium and bad to model weather conditions. Amongst these, only Ursavas (2017) capture the uncertain nature of weather conditions through a two-stage stochastic model, where the weather conditions are realized in the second stage, and the first stage consists of charter decisions. The most common approach, which is used in the remaining of the mentioned papers, is to pre-generate a weather scenario which is given as deterministic input to the model. The generation of this weather scenario is handled in different ways. For instance, Ait Alla et al. (2013) use weather data based on forecasts from the last 50 years, while Irawan et al. (2017) use an algorithm to randomly generate daily weather conditions for a year.

Barlow et al. (2018) handle weather effects through a mixed-method approach that comprises a simulation algorithm and an optimization model. They use simulation to assign activities to vessels, while the optimization model is used to find a schedule minimizing the total duration of the installation project given the simulated assignment of activities. The simulation model explores the impact of starting operations at different months throughout the year based on historical weather data. Their method was successfully applied to a case study with 120 turbines.

A common modelling choice for offshore wind farm installation problems is to use predefined loading sets. This is done in Ait Alla et al. (2013), Scholz-Reiter et al. (2011) and Ursavas (2017). A loading set consists of a specific number of the different OWT components. Prior to each time a vessel leaves port, one of the predefined loading sets must be selected, and its components loaded on board. The chosen loading set then defines which activities the vessel can perform on its next voyage. Irawan et al. (2017) also use predefined configurations of what a vessel should be loaded with to perform certain activities, but the authors do not explicitly call these configurations for loading sets. Another common modelling feature is the use of discrete time periods, which is done in both Ait Alla et al. (2013) and Irawan et al. (2017). Scholz-Reiter et al. (2011) and Ursavas (2017) use discretization of time in their weather modelling, but vessel operations are not limited by this discretization.

Two differentiating features of the existing literature are the number of vessels considered in the models, and the measure of the objective function. Scholz-Reiter et al. (2011) and Ursavas (2017) aim to minimize the duration of the project and consider only a single installation vessel that can perform two activities: installation of sub-structures and installation of top-structures. Barlow et al. (2018) also present an objective related to minimization of project duration, but similar to Ait Alla et al. (2013) and Irawan et al. (2017), their model considers multiple vessels and the fact that different vessels have different capabilities in terms of which activities they can perform. Contrary to Scholz-Reiter et al. (2011), who state that they consider minimization of the installation time to be proportional to the installation costs, Irawan et al. (2017) considers these objectives to be conflicting and accounts for this by introducing a bi-objective model. Ait Alla et al. (2013) only minimize installation costs in their objective.

The different takes on the objective function can to some extent be seen in relation to the number of vessels considered in the models. With only one installation vessel, cost and project duration may be contemplated as proportional, but with multiple vessels this is no longer the case. Hiring many vessels over a shorter time period may allow for faster completion of the wind farm, but due to fixed vessel costs and limited availability this can be more costly than hiring fewer vessels to complete the wind farm over a longer time period.

Some papers solely focus on the installation of the top-structure and try to identify the best turbine installation strategy by resolving the problem for different methods. Backe and Haugland (2017) and Sarker and Faiz (2017) have formulated mathematical models for this purpose. While Backe and Haugland (2017) also consider decisions regarding which vessels and ports to use to minimize installation costs under different strategies, Sarker and Faiz (2017) have predefined a homogeneous fleet of jack-up vessels and focus on time minimization. A weakness of the latter is that weather impacts are not accounted for. In Backe and Haugland (2017), the weather is considered in a deterministic manner based on historic data. Furthermore, the model of Sarker and Faiz (2017) is formulated as an algorithm rather than an optimization problem and does not consider scheduling.

The installation of an offshore wind farm is a complex problem, and the proposed models are often complicated and hard to solve to optimality for realistic problem instances. Scholz-Reiter et al. (2011) solve their single vessel model for a test instance with 12 turbines and three weather categories in less than a minute. Nonetheless, the authors point out that increasing the number of turbines and weather categories increases the solution time and ultimately makes the model unsolvable. Backe and Haugland (2017), Irawan et al. (2017) and Ursavas (2017) face similar problems with their models. To handle larger and more realistically sized test instances with more vessels, time periods and turbines, Irawan et al. (2017) propose metaheuristic approaches using Variable Neighbourhood Search (VNS) and Simulated Annealing (SA). Their study concludes that the metaheuritic approaches generally outperform the exact method used in CPLEX in terms of computing time, and that VNS seems to be more efficient than SA. To handle a larger number of weather scenarios in their model, Ursavas (2017) develops a Benders based decomposition method to solve linear relaxations of the model and uses B&B to find integer solutions. The model is successfully applied to two real-life projects in the North Sea.

Several simulation models have also been proposed to investigate aspects related to installation of offshore wind farms. Lange et al. (2012) develop a tripartite tool consisting of an input tool, a simulation tool and an evaluation tool which can be applied to several parts of the offshore wind supply chain. Due to the wide scope of their decision tool, the installation process is simplified compared to other models solely focusing on the installation process. Beinke et al. (2017) suggests a discrete-event and agent-based simulation model aimed at examining the potential of a joint use of resources in the installation phase of offshore wind energy, whilst Muhabie et al. (2018) has implemented a discrete-event simulation approach to investigate the effect of stochastic parameters on different installation procedures.

4.3 Maritime Fleet Size and Mix Problems

Pantuso et al. (2014) present a literature survey for the MFSMP. They distinguish between single-period MFSMP, which comprises problems that aim to find a fleet of vessels that is to remain unchanged over time, and multi-period MFSMP, also referred to as the Maritime Fleet Renewal Problems (MFRP), which comprises problems that seek adjustments to an existing fleet to accommodate market changes. Single-period MFSMPs are further categorized as either strategic or short-term. Strategic MFSMPs usually involve building, purchasing, sale and/or scrapping of ships. The problem studied in this report is closest related to what Pantuso et al. (2014) refer to as the short-term MFSMP, dealing with chartering and deployment of vessels in a constant fleet.

4.3.1 Offshore Wind Farm Operations and Maintenance

The MFSMP has been applied within several different maritime industries, but within the offshore wind supply chain existing research mainly revolves around O&M. The offshore wind farm O&M problems aim to identify a cost-efficient fleet of vessels, and sometimes helicopters, to support O&M operations. There are several similarities to the installation problem: the operations are weather dependent, different vessel concepts are required for different types of activities, and vessel scheduling and charter periods must be considered. However, the nature of the problem differs from the installation problem due to uncertainty about which task will be necessary to perform and the use of offshore depot bases. Furthermore, while installation is a one-time event in the life span of an offshore wind farm, O&M is a reoccurring planning problem, and acquisition of vessels on a permanent basis is more relevant.

As for offshore wind farm installation, the influence of weather conditions is relevant for the vessels' ability to perform O&M activities. Gundegjerde et al. (2015), Gutierrez-Alcoba et al. (2019) and Stålhane et al. (2021, 2019, 2016) capture the uncertain nature of weather conditions through stochastic mathematical formulations for the offshore wind farm O&M problem. Most of the mentioned papers have formulated two-stage models, where the first stage comprises vessel charter and acquisition decisions. The second stage consists of deployment decisions and takes place after weather is revealed and demand for maintenance (occurred failures) is known. An exception is Stålhane et al. (2021) who propose a dual-level stochastic model, that considers both long-term strategic uncertainty and short-term operational uncertainty in a single optimization model. The other exception is Gundegjerde et al. (2015) who formulate a three-stage stochastic model. The additional stage is an intermediate stage, where one is allowed to charter in extra vessels. Their results showed that the use of a stochastic model gave significant value compared to previous deterministic formulations.

The stochastic mathematical formulations of the MFSMP are solved using a variety of solution methods. Gundegjerde et al. (2015) transform their three-stage formulation into a scenario tree node-based deterministic equivalent before solving their model in a commercial Mixed Integer Program (MIP) solver and were able to solve test instances of realistic size. Stålhane et al. (2016) also solve their model using a commercial MIP solver, while Stålhane et al. (2021) propose an ad hoc integer L-shaped method with customized optimality cuts as a solution method to their problem. The results of Stålhane et al. (2021) show that the proposed method outperforms solving the deterministic equivalent using a commercial MIP solver.

Both Gutierrez-Alcoba et al. (2019) and Stålhane et al. (2019) apply a priori generation of possible operational patterns for each vessel. Gutierrez-Alcoba et al. (2019) solve their model using two different methods. First, they present a deterministic equivalent of a two-stage stochastic MILP based on a priori generation of possible operational patterns and solve it using a commercial solver. Gutierrez-Alcoba et al. (2019) argue that, due to anticipation, a drawback of the deterministic MILP formulation is that costs are underestimated compared to what can be achieved in practice under incomplete information. A heuristic that simulates the practical scheduling of O&M activities is also presented, with the aim of providing a better cost estimate. In the heuristic, no anticipation of weather conditions and failures are considered. Only information on weather and failure events at the beginning of each shift, when the scheduling decisions are made, are considered. Their results show a slight increase in costs when the heuristic is used. Furthermore, it is observed that the heuristic allows for more slack in the scheduling, increasing the ability to react to random events, and indicating that in practice, the best vessel plan contains more vessels than that predicted by an a priori information model. Stålhane et al. (2019) solve their model using a heuristic algorithm to generate a subset of the feasible patterns, and performs an ad hoc Dantzig–Wolfe decomposition, where parts of the second stage problem remain in the master problem. The heuristic uses a labeling algorithm based on a SPPRC to generate patterns.

4.3.2 Other Industries

Most of the research on MFSMPs is related to shipping, a business renowned for its volatility. Nonetheless, at the time Pantuso et al. (2014) conducted their survey, there were only a handful of studies including uncertain elements for the MFSMP. Among these, Meng and Wang (2010) model a short-term MFSMP for container shipping involving chartering and deployment decisions, where they used chance constraints to tackle uncertain shipment demand. Meng et al. (2012) extended the problem to include transshipment and handle uncertainty through a two-stage stochastic Integer Programming (IP) model.

Others consider uncertain elements without stochastic model formulations. Fagerholt et al. (2010) combine optimization and simulation to solve a strategic MFSMP for tramp shipping, where they consider uncertainty in timing and quantity of cargoes. Crary et al. (2002) solve their mixed integer program with different realizations of random input parameters to find the best performing fleet of combat ships for the US Navy. Alvarez et al. (2011) present a robust MIP model for a bulk shipping MFRP, with random variations in selling and purchasing prices for ships, as well as charter rates.

Pantuso et al. (2016) try to evaluate if better results are obtained by using two-stage stochastic programming compared to a deterministic model with average values. The MFRP of liner shipping is used as a case study. Uncertain elements in this problem are related to the shipping market and include ship purchasing and selling prices, charter rates, and demand. Pantuso et al. (2016) found that the stochastic model valued flexibility in the sense that it awaited sale of ships longer than the deterministic model. The deterministic model did not value the option of keeping an extra vessel in the fleet "just in case" it could be needed in some future scenario, resulting in higher expected costs, compared to the stochastic solution. Their conclusion is that the stochastic model performs better, which is consistent with what Bakkehaug et al. (2014) found in their multi-stage stochastic approach to a similar problem, and what Arslan and Papageorgiou (2017) found when they studied a tactical MFSMP for bulk shipping.

Another industry where the MFSMP has been applied, is the offshore Oil & Gas (O&G) supply. As for O&M, problems related to O&G supply share several similarities with our problem. A contribution to stochastic consideration within MFSMPs related to O&G is Shyshou et al. (2010), who studied a problem related to offshore anchor handling operations. Their aim was to decide how many vessels to charter on long term contracts, and for how long, versus the need to supply the fleet with expensive spot charters in times with an excess requirement for anchor handling operations, caused by earlier weather delays. The trade-off between chartering vessels on long term contracts versus spot charters is closely related to the offshore wind farm installation problem. To handle weather uncertainty, Shyshou et al. (2010) use a discrete event simulation framework.

Fagerholt and Lindstad (2000) study a short-term MFSMP concerning weekly scheduling for offshore supply vessels and present a deterministic IP model based on pre-generated voyages. The underlying problem is similar to the Vehicle Routing Problem (VRP) with multiple vehicles and time windows. Post analysis is used to evaluate the robustness of the solution. A similar problem is studied by Halvorsen-Weare et al. (2012), who try to make it more realistic by including aspects such as spread of departure times from the depot, capacity constraints for the depot and limitation on duration of voyages. Halvorsen-Weare and Fagerholt (2011) further extend the supply vessel problem by combining the deterministic model from Halvorsen-Weare et al. (2012) with simulation to ensure a more robust solution with respect to weather uncertainty. Their results show that the inclusion of robustness criteria gives a lower predicted cost compared to the deterministic model where no such criterion is considered.

The MFSMPs are commonly solved using either simulation (Arslan and Papageorgiou 2017; Shyshou et al. 2010), commercial solvers for MIPs (Alvarez et al. 2011; Crary et al. 2002; Meng and Wang 2010; Pantuso et al. 2016), heuristic approaches (Bakkehaug et al. 2014), or combinations of these methods (Fagerholt et al. 2010). An exception is Meng et al. (2012), who propose an algorithm that integrates sample average approximation with dual decomposition and Lagrangian relaxation approach to solve their model.

4.4 Branch-and-Price for Stochastic Problems

B&P (Barnhart et al. 1998) is an exact solution method that combines a B&B search with column generation. The use of B&P for solving stochastic integer (or mixed-integer) programs is relatively new. Although consideration of uncertainty is limited in offshore research, stochastic programming is widely used in operations research in other industries. Further, a variety of applications, such as those arising in scheduling, routing, location and production planning, lead to integer and combinatorial optimization models, which is relevant to our problem.

Christiansen and Lysgaard (2007), Yuan et al. (2015), McKinnon and Yu (2016), Wang et al. (2020) and Yanıkoğlu and Yavuz (2022) implement B&P algorithms for solving stochastic scheduling and routing problems within different industries. Yuan et al. (2015) study a problem related to the daily scheduling and routing of caregivers in home health care. They consider stochastic service times and skill requirements, where caregivers can care for specific patients. This is similar to our problem, where vessels can perform specific activities and the time it takes to perform these activities is stochastic due to weather dependencies. Wang et al. (2020) and Yanıkoğlu and Yavuz (2022) model scheduling decisions under uncertainty. Wang et al. (2020) study a problem related to scheduling of earth observation satellites under stochastic impact of clouds, while Yanıkoğlu and Yavuz (2022) study a problem of machine scheduling with sequence-dependent setup times. Christiansen and Lysgaard (2007) formulate a two-stage stochastic model for the capacitated vehicle routing problem with stochastic demand. Similarly, McKinnon and Yu (2016) formulate a multi-stage stochastic model for a single commodity distribution problem under uncertain demand for a fleet of ships.

The B&P algorithm is also used to solve capacitated lot sizing problems and multistage stochastic capacity planning problems by Lulli and Sen (2004), Singh et al. (2009) and Zhang et al. (2020). Although location and production planning applications differ from the problem studied in this thesis, they still have similarities. For instance, in the context of the stochastic bin-packing problem studied in Zhang et al. (2020), the bins can correspond to vessels that are available for a given period, while the items can represent activities, and the item sizes can represent the time it takes to perform an activity. Silva and Wood (2006) survey B&P for two-stage stochastic MIPs, including routing, scheduling and the general allocation problem. They also implement B&P algorithms for a stochastic facility-location problem and report that the B&P method can be significantly faster than solving the original problem using a classical B&B approach.

The B&P algorithm is applicable to the deterministic equivalent of both two-stage and multi-stage stochastic MIPs. Then the original stochastic problem is decomposed into a collection of deterministic subproblems, which are linked together by a master problem. Often, the resulting master problem and/or subproblems can reflect well-studied optimization problems. For instance, Yuan et al. (2015) formulate their master problem as a classical set partitioning problem that combines routes for caregivers into feasible schedules, and Wang et al. (2020) formulate their problem into a set packing master problem, that combines schedules for satellite orbits to maximize profits of successful observations. Also McKinnon and Yu (2016) formulate their master problems as a set partitioning problem, but with extra inventory constraints to accommodate storage limits at ports. The common ground for McKinnon and Yu (2016) and that of Yuan et al. (2015), is that they both study routing problems. Christiansen and Lysgaard (2007) also study a routing problem, but in their master problem the partitioning constraints are changed into covering constraints in order to obtain a smaller dual solution space. A set partitioning master problem is also formulated in Silva and Wood (2006).

In several of the reviewed papers the subproblems are versions of the shortest path problem, and labeling algorithms based on dynamic programming are presented to solve them (Christiansen and Lysgaard 2007; Yuan et al. 2015; McKinnon and Yu 2016). While the subproblems in Christiansen and Lysgaard (2007) and Yuan et al. (2015) concerns routing of vehicles, the subproblems in McKinnon and Yu (2016) are ship routing problems. However, not all subproblems are routing problems. Yanıkoğlu and Yavuz (2022) solve one subproblem per machine, and Wang et al. (2020) solve one subproblem per satellite orbit, formulated as a constrained longest weighted path planning problem. In Silva and Wood (2006), the subproblems are stochastic and are formulated as multi-dimensional knapsack problems, solved by B&B with explicit constraint branching.

When implementing a B&P algorithm different algorithmic design choices must be made, including decisions on search and branching strategies in the enumeration tree. Yanıkoğlu and Yavuz (2022) demonstrate that a search strategy that prioritizes improvement of the dual bound yields the best computational performance. Hence, they choose to adopt a best-first search. However, a best-first strategy may require a long time to obtain feasible solutions, which can be problematic for computationally demanding problems, as the search may terminate before finding an acceptable feasible solution. Thus, McKinnon and Yu (2016) suggest to combine a best-first strategy with a depth-first strategy, where a depth-first search is used in the beginning to find feasible solutions early in the search, before moving on to a best-first strategy to produce better bounds and prove optimality.

Yuan et al. (2015) use a best-first search strategy where they branch on mandatory and forbidden arcs, which is standard for the VRP. Christiansen and Lysgaard (2007) develop a more advanced branching scheme for their routing problem, based on accumulated expected demand. If a customer is visited by two paths, with different expected demands, a threshold is set between these expectation values. Then one branch is created where paths with expected demand larger than this threshold are forbidden, and another branch is created where paths with lower expected demand than the threshold are forbidden. Their computational results show that the strategy based on accumulated expected demand is able to solve more problem instances and tends to search fewer nodes before reaching optimum than traditional branching on single flow variables.

Lulli and Sen (2004) use the common branching strategy to find a fractional variable that should be integer in a feasible solution, and make two branches, one where the variable is required to be smaller or equal to the fractional value rounded down to the nearest integer, and one where the variable must be greater or equal to the fractional value rounded up to the nearest integer. Adopting a similar scheme, Yanıkoğlu and Yavuz (2022) select to branch on the variable with the largest integer infeasibility, that is, the integer variable with the fractional value closest to 0.5. Zhang et al. (2020) use a pair-based branching strategy that determines whether or not two particular items are allocated in the same bin.

Generally, the authors of the reviewed articles found that B&P algorithms outperformed other exact solution methods due to stronger LP relaxation. However, as the size of the test instances increase, the computational effort increases. McKinnon and Yu (2016) find that this is highly dependent on subproblems complexity, and their computational results show that around 75–94% of the computational time to solve the problem is used to solve the ship subproblems.

Algorithmic extensions aimed at accelerating the search in the B&P method can allow for larger problems to be solved. A common strategy is to use heuristic approaches, both to initialize the column generation algorithm, and throughout the search. Yuan et al. (2015) expand their B&P algorithm to include both a greedy heuristic and a variable neighbourhood descent. They use the greedy heuristic to generate initial columns, and find that this heuristic is efficient for speeding up solution process at the root node of the search tree as it effectively reduces the objective value and stabilise the values of dual variables. Yanıkoğlu and Yavuz (2022) adapt a heuristic algorithm where they sort jobs on due dates and make assignments based on this, and find that this heuristic significantly improves the performance of their solution algorithm. Yuan et al. (2015) further extend their algorithm to return multiple columns in every iteration of the column generation algorithm, which is another common acceleration strategy for B&P algorithms. This is also done by Zhang et al. (2020) who investigate the choices between returning all improving columns and returning only the first improving column and found that their model performed best when selecting all columns. Finally, the problem structure of the decomposed model in which there typically is independence among all the subproblems makes the B&P method amenable to parallelization. This will speed up each iteration of the column generation algorithm and can therefore lead to faster convergence.

4.5 Our Contribution

Although the relevance of offshore wind energy has increased steadily, the research on optimization problems related to offshore wind farm installation is scarce. To our knowledge, only five published papers suggest (mixed) integer programming formulations of the installation planning problem. A summary of relevant aspects of the modelling approach in these papers compared to our formulations is given in Table 4.2. Additionally, the applied solution methods are stated.

The combination of a stochastic approach to handle weather effects and the possibility of including several vessels has, to our knowledge, not been investigated in optimization research for offshore wind farm installation. Additionally, we contribute to the research on applying B&P for stochastic problems, which is a scarcely investigated research area. Another feature that differentiates our thesis from previously published research on optimization of offshore wind farm installation, is the possibility of adding options to the charter contract of a vessel. This adds flexibility to the solutions and gives a more realistic model formulation as options are a common part of a real-life charter contracts.

Finally, the possibility of easily changing activities, and thus allowing for consideration of new technologies and installation procedures, has not been given much attention. Our model is expressed in a general manner, allowing users to define input parameters to fit their problem. This means that the model can be used to evaluate different installation strategies, including solutions for floating wind turbines and other future solutions.

Table 4.2: Comparison of our problem and relevant studies.

Paper	Multiple Vessels	Two-Stage Stochastic	Options
This thesis	✓	✓	✓
Scholz-Reiter et al. (2011)			
Irawan et al. (2017)	✓		
Ursavas (2017)		✓	
Ait Alla et al. (2013)	✓		
Barlow et al. 2018	✓		

Chapter 5

Mathematical Model

In this chapter, we present a mathematical formulation for the problem described in Chapter 3. The formulation is to a large extent based on the formulation presented by Bruu and Thorsen (2021). In Section 5.1 we describe the main characteristics of the mathematical model. The mathematical formulation is presented in Section 5.2. Finally, Section 5.3 describes parameter calculations while Section 5.4 addresses reduction and fixing of variables. The deterministic equivalent of the model is presented in Appendix C.

5.1 Description of the Model

We have formulated a stochastic and time discrete model for design of charter contracts for installation vessels at offshore wind farms. To determine the charter period for each vessel, operational schedules for each vessel are designed. The following sections describe our modeling choices and assumptions.

5.1.1 Stochastic Two-Stage

To account for uncertainty in weather conditions, the problem is modeled as a two-stage stochastic optimization problem. The first stage occurs two to four years in advance of the installation phase and includes decisions on which vessels to charter and for what time periods, as well as which options to include in their charter contracts. The second stage occurs after realization of the weather conditions and aims to assign the chartered vessels to the necessary installation activities and decide whether or not to exercise the included options. Through this stochastic approach, the optimal solution will reveal the charter strategy that gives the best expected objective value across all weather realizations. That is, the solution found may not be optimal for any isolated realization of the weather conditions, but it is the solution that is best on average, which is a good approach to handling the weather uncertainty.

Although weather conditions are realized from day to day during a real-life project, our choice of a two-stage model implicates the assumption that the weather for the entire planning horizon is revealed at the beginning of the installation phase. This is a reasonable assumption as contract costs are defined by the start of the charter period, regardless of whether or not the vessel begins to operate. Hence, it should never be beneficial not to use the first available weather window. For this reason, it is unlikely that realization of the weather through a multi-stage formulation would affect the scheduling decisions.

5.1.2 Time Discretization

The planning horizon is divided into a set of time periods of equal length, making the problem formulation time discrete. It is assumed that all durations in the model can be expressed as an integer number of time periods. Hence, all activities start at the beginning of a time period and finish at the end of a time period. In reality, the duration of each activity may not add up to a multiple of the length of a time period. In such cases, the duration is rounded to fit with the chosen time discretization.

5.1.3 Round Trips

The second-stage model is built on a vessel round trip concept inspired by the single-vessel models by Scholz-Reiter et al. (2011) and Ursavas (2017). A round trip, henceforth also referred to as a tour, consists of the following processes: (1) load the vessel in port, (2) sail to the offshore wind farm site, (3) perform installation activities, and (4) return to port. The start and end times of a tour are considered to be the first time period of loading in port, and the final time period of sailing back to port after performing the installation on site, respectively. A vessel can have multiple tours throughout its charter and option period, but these cannot overlap in time. For modeling purposes, an upper limit on the number of tours per vessel has been predefined by a set $\mathcal{I}_v$. The purpose of this set is to provide unique indices for each tour a vessel conducts, and its size should be set so that it does not restrict a vessel's possible schedules.

5.1.4 Installation Activities

The set of installation activities is defined so that each activity should be performed exactly once for each OWT. Furthermore, the set of installation activities is ordered. That is, the order of activities in the set should correspond to the installation sequence.

5.1.5 Loading Sets

Each vessel has a set of predefined loading sets that consists of a set of installation activities. Prior to each time a vessel leaves port, one of the predefined loading sets must be selected, and the resources required to perform its activities must be loaded on board. Hence, the chosen loading set defines which activities the vessel can perform on its next round trip. The resources required for a loading set cannot exceed the vessel's weight capacity, lifting capabilities, and available deck area. Further, a loading set cannot contain activities the vessel is unable to perform.

5.1.6 Unfinished OWTs

There is a predefined number of OWTs that should be installed. In some cases, it may be impossible to complete all installation activities within the planning horizon for all weather realizations. To avoid infeasibility in such cases, the model is allowed not to complete all OWTs against a penalty in the objective function.

5.1.7 Other Assumptions

We assume that a vessel only can be chartered once during the project period. Further, we do not consider travel times between turbine locations since these are negligible. Operational costs related to fuel consumption and port fees are disregarded in the model formulation as these are assumed to be negligible in comparison to the charter costs. We also assume that sailing and loading of vessels are operations that are not affected by weather conditions. Lastly, uncertainties beyond weather conditions, for instance, vessel and component availability, are not considered.

5.2 Mathematical Formulation

This section introduces the sets, parameters and decision variables used in the mathematical formulation, as well as objective functions and the constraints the models are subject to. The first stage model is presented in Section 5.2.1, whilst the second stage model is presented in Section 5.2.2.

5.2.1 First Stage Model

The first stage model consists of strategic decisions to be made before realization of the weather conditions, including which vessels to charter and which options to include in the charter contracts.

Definition of Sets

$\mathcal{V}$ - Set of vessels, indexed by v

$\mathcal{T}$ - Set of time periods, indexed by t

$\mathcal{O}$ - Set of options, indexed by o

Definition of Parameters

C_v^F - Fixed charter cost for vessel v

C_v^V - Variable charter cost for vessel v per time period in its fixed charter period

K_v - Minimum length of a tour for vessel v

P_{ov}^F - Price for including option o in the contract for vessel v

Definition of Variables:

s_{tv}^C - 1 if charter of vessel v starts in time period t, 0 otherwise

e_{tv}^C - 1 if charter of vessel v ends in time period t, 0 otherwise

α_v - 1 if vessel v is chartered in the project, 0 otherwise

β_{tv} - 1 if vessel v is on fixed charter in time period t, 0 otherwise

μ_{ov} - 1 if option o is included in the contract for vessel v, 0 otherwise

Objective

$$min \quad \sum_{v\in\mathcal{V}} C_v^F \alpha_v + \sum_{t\in\mathcal{T}}\sum_{v\in\mathcal{V}} C_v^V \beta_{tv} + \sum_{o\in\mathcal{O}}\sum_{v\in\mathcal{V}} P_{ov}^F \mu_{ov} + E_\xi[Q(\alpha_v, \beta_{tv}, \mu_{ov}, e_{tv}^C, \xi)] \qquad (5.1)$$

The costs in the first stage consist of fixed charter cost, variable charter cost and option inclusion price. As expressed in (5.1), the objective is to minimize these costs, as well as the expected value of the second stage objective, Q, which is further explained in Section 5.2.2.

Charter Constraints

$$\sum_{t \in \mathcal{T}} s^C_{tv} = \alpha_v, \qquad v \in \mathcal{V}, \quad (5.2)$$

$$\sum_{t \in \mathcal{T}} s^C_{tv} - \sum_{t \in \mathcal{T}} e^C_{tv} = 0, \qquad v \in \mathcal{V}, \quad (5.3)$$

$$\sum_{t \in \mathcal{T}} te^C_{tv} - \sum_{t \in \mathcal{T}} ts^C_{tv} \geq K_v \alpha_v, \qquad v \in \mathcal{V}. \quad (5.4)$$

Constraints (5.2) state that all vessels included in the fleet must have a start time for their fixed charter period. Further, constraints (5.3) ensure that if a vessel's fixed charter period starts, it must also end, while constraints (5.4) define the end of a vessel charter period so that it succeeds its start. K_v is a lower bound on the number of time periods vessel v is chartered, given that it is chartered.

$$\beta_{tv} = \beta_{(t-1)v} + s^C_{tv} - e^C_{(t-1)v}, \qquad t \in \mathcal{T} \setminus \{1\}, v \in \mathcal{V}, \quad (5.5)$$

$$s^C_{tv} = \beta_{tv}, \qquad t = 1, v \in \mathcal{V}, \quad (5.6)$$

$$\mu_{ov} \leq \alpha_v, \qquad o \in \mathcal{O}, v \in \mathcal{V}. \quad (5.7)$$

β_{tv} is set to one in the fixed charter period for each vessel by constraints (5.5) and (5.6) set. Constraints (5.7) ensure that options can only be included for vessels that have been chartered.

Binary Constraints

$$\alpha_v \in \{0, 1\}, \qquad t \in \mathcal{T}, v \in \mathcal{V}, \quad (5.8)$$

$$\beta_{tv} \in \{0, 1\}, \qquad t \in \mathcal{T}, v \in \mathcal{V}, \quad (5.9)$$

$$s^C_{tv} \in \{0, 1\}, \qquad t \in \mathcal{T}, v \in \mathcal{V}, \quad (5.10)$$

$$e^C_{tv} \in \{0, 1\}, \qquad t \in \mathcal{T}, v \in \mathcal{V}, \quad (5.11)$$

$$\mu_{ov} \in \{0, 1\}, \qquad o \in \mathcal{O}, v \in \mathcal{V}. \quad (5.12)$$

The first stage decision variables are made binary by constraints (5.8)-(5.12).

5.2.2 Second Stage Model

The second stage model consists of operational decisions to be made upon weather realization, i.e. scheduling of installation activities using the chartered vessels.

Definition of Sets

$\mathcal{V}$	-	Set of vessels, indexed by v
$\mathcal{T}$	-	Set of time periods, indexed by t
$\mathcal{A}$	-	Set of installation activities, indexed by a
$\mathcal{L}_v$	-	Set of loading sets for vessel v, indexed by l
$\mathcal{I}_v$	-	Set of tours for vessel v, indexed by i
$\mathcal{O}$	-	Set of options, indexed by o

Definition of Parameters

D^L_{lv}	-	Duration of loading loading set l for vessel v
D^S_v	-	Duration of sailing from port to site or vice versa for vessel v
L_o	-	Length of option o given in time periods
N^T	-	Total number of turbines to be installed
N^L_{al}	-	Number of times installation activity a can be performed if a vessel is loaded with loading set l
$M^1_{atv}(\xi)$	-	Big-M in the last round trip constraint for the combination of activity a, time period t and vessel v
M^2_{av}	-	Big-M used in first activity constraint for activity a and vessel v
M^3_v	-	Big-M used in third activity constraint for vessel v
P^E_{ov}	-	Price for exercising option o for vessel v
P^L	-	Penalty cost for lost income
P^U	-	Penalty cost for unfinished OWTs
$T_{atv}(\xi)$	-	Number of time periods it takes vessel v to complete activity a if it is started in time period t
e^C_{tv}	-	1 if charter of vessel v ends in time period t, 0 otherwise
α_v	-	1 if vessel v is chartered in the project, 0 otherwise
β_{tv}	-	1 if vessel v is on fixed charter in time period t, 0 otherwise
μ_{ov}	-	1 if option o is included in the contract for vessel v, 0 otherwise

Definition of Variables

e^P	-	Last time period of the project
u	-	Number of unfinished OWTs
s^T_{itv}	-	1 if the first period of tour i for vessel v is in time period t, 0 otherwise
s^A_{aitv}	-	1 if activity a starts in time period t in tour i for vessel v, 0 otherwise
δ_{ilv}	-	1 if loading set l is loaded onto vessel v for tour i, 0 otherwise
γ_{ov}	-	1 if option o is exercised for vessel v

Objective

In the second stage model the objective is to minimize costs and the final completion date based on the realization of the uncertain parameters.

$$Q(\alpha, \beta, \mu, e^C, \xi) = min \quad \sum_{o \in \mathcal{O}} \sum_{v \in \mathcal{V}} P^E_{ov} \gamma_{ov} + P^U u + P^L e^P \tag{5.13}$$

The objective (5.13) minimizes the cost of exercising options, the cost of unfinished OWTs and the cost of lost income. By minimizing cost of lost income the completion time of the wind farm is also minimized.

Round Trip Constraints

$$\sum_{t \in \mathcal{T}} s^T_{itv} \leq \alpha_v, \qquad v \in \mathcal{V}, i \in \mathcal{I}_v, \tag{5.14}$$

$$s^T_{itv} \leq \beta_{tv}, \qquad i = 1, v \in \mathcal{V}, \tag{5.15}$$

$$\sum_{t \in \mathcal{T}} s^T_{itv} \geq \sum_{t \in \mathcal{T}} s^T_{(i+1)tv}, \qquad v \in \mathcal{V}, i \in \mathcal{I}_v \setminus \{|\mathcal{I}_v|\}. \tag{5.16}$$

Constraints (5.14) state that each tour can only have one start time and that the vessel must be chartered to start a tour. Further, constraints (5.15) restricts the first tour for each vessel to start within the vessel's fixed charter period. By requiring that tours of lower indices should be utilized before higher indexed tours, constraints (5.16) reduce symmetry.

$$(t + T_{atv} + D^S_v) s^A_{aitv} \leq \sum_{t \in \mathcal{T}} t s^T_{(i+1)tv} + M^1_{atv} (1 - \sum_{t \in \mathcal{T}} s^T_{(i+1)tv}), \quad a \in \mathcal{A}, t \in \mathcal{T}, v \in \mathcal{V}, i \in \mathcal{I}_v \setminus \{|\mathcal{I}_v|\}, \tag{5.17}$$

Constraints (5.17) ensure that a vessel cannot start a new tour before its previous tour has ended.

Activity Constraints

$$\sum_{\tau \in \mathcal{T}} \tau s^T_{i\tau v} + \sum_{l \in \mathcal{L}} (D^L_{lv} + D^S_v)\delta_{ilv} \leq t s^A_{aitv} + M^2_v(1 - s^A_{aitv}), \qquad a \in \mathcal{A}, t \in \mathcal{T}, v \in \mathcal{V}, i \in \mathcal{I}_v. \tag{5.18}$$

Constraints (5.18) secure that the first activity during a tour does not start before the vessel has been loaded and sailed to site.

$$\sum_{a \in \mathcal{A}} \sum_{t'=t-T_{atv}+1}^{t} s^A_{ait'v} \leq 1, \qquad t \in \mathcal{T}, v \in \mathcal{V}, i \in \mathcal{I}_v, \tag{5.19}$$

$$\sum_{a \in \mathcal{A}} \sum_{t \in \mathcal{T}} s^A_{aitv} \leq M^3_v \sum_{t \in \mathcal{T}} s^T_{itv}, \qquad v \in \mathcal{V}, i \in \mathcal{I}_v, \tag{5.20}$$

$$\sum_{t \in \mathcal{T}} s^T_{itv} \leq \sum_{a \in \mathcal{A}} \sum_{t \in \mathcal{T}} s^A_{aitv}, \qquad v \in \mathcal{V}, i \in \mathcal{I}_v. \tag{5.21}$$

That a vessel can perform at most one activity in each time period, is ensured by constraints (5.19). Constraints (5.20) state that no installation activities can be scheduled to a tour that is not taking place, while constraints (5.21) ensure that no tours without scheduled activities are started.

Loading Constraints

$$\sum_{l \in \mathcal{L}_v} \delta_{ilv} \leq 1, \qquad v \in \mathcal{V}, i \in \mathcal{I}_v, \tag{5.22}$$

$$\sum_{t \in \mathcal{T}} s^A_{aitv} \leq \sum_{l \in \mathcal{L}_v} N^L_{al} \delta_{ilv}, \qquad a \in \mathcal{A}, v \in \mathcal{V}, i \in \mathcal{I}_v. \tag{5.23}$$

Constraints (5.22) restrict a vessel to load only one loading set for each tour, while constraints (5.23) ensure that the number of times a vessel performs an installation activity during a tour does not exceed the number defined by the loading set on board.

Project Plan Constraints

$$(t + T_{atv} + D^S_v - 1) s^A_{aitv} \leq \sum_{\tau \in \mathcal{T}} \tau e^C_{\tau v} + \sum_{o \in \mathcal{O}} L_o \gamma_{ov}, \qquad a \in \mathcal{A}, t \in \mathcal{T}, v \in \mathcal{V}, i \in \mathcal{I}_v, \tag{5.24}$$

$$\sum_{v \in \mathcal{V}} \sum_{i \in \mathcal{I}_v} \sum_{t'=1}^{t} s^A_{ait'v} \leq \sum_{v \in \mathcal{V}} \sum_{i \in \mathcal{I}_v} \sum_{t'=1}^{t-T_{(a-1)t'v}} s^A_{(a-1)it'v}, \qquad a \in \mathcal{A} \setminus \{1\}, t \in \mathcal{T}, \tag{5.25}$$

$$\sum_{t \in \mathcal{T}} \sum_{v \in \mathcal{V}} \sum_{i \in \mathcal{I}_v} s^A_{aitv} \geq N^T - u, \qquad a = |A|. \tag{5.26}$$

Constraints (5.24) ensure that activities are scheduled to take place within each vessel's charter period, consisting of both fixed charter period and option period. Precedence between installation activities according to the order of the set of installation activities is ensured by constrains (5.25). That each activity is performed at least the same number of times as number of turbines to be installed, is ensured by constraint (5.26). This constraint allows installation of more turbines than required. However, this will not be profitable in the optimal solution due to additional costs. As explained in Section 5.1.6, the variable u is included to allow for unfinished turbines in case the realized weather conditions make it impossible to complete all OWTs within the time horizon of the model.

$$e^P \geq (t + T_{atv} + D_v^S - 1)s_{aitv}^A, \qquad a \in \mathcal{A}, t \in \mathcal{T}, v \in \mathcal{V}, i \in \mathcal{I}_v, \quad (5.27)$$

$$\gamma_{ov} \leq \mu_{ov}, \qquad o \in \mathcal{O}, v \in \mathcal{V}. \quad (5.28)$$

The end date for the installation project is defined by constraints (5.27), which demands the value of e^P to be greater or equal to the final time period where an activity is performed. Constraints (5.28) forbid exercising options that are not included in a vessel's contract.

Binary and non-negativity constraints

$$e^P \geq 0, \text{ integer}, \quad (5.29)$$

$$u \geq 0, \text{ integer}, \quad (5.30)$$

$$\delta_{ilv} \in \{0, 1\}, \qquad v \in \mathcal{V}, l \in \mathcal{L}_v, i \in \mathcal{I}_v, \quad (5.31)$$

$$s_{itv}^T \in \{0, 1\}, \qquad t \in \mathcal{T}, v \in \mathcal{V}, i \in \mathcal{I}_v, \quad (5.32)$$

$$s_{aitv}^A \in \{0, 1\}, \qquad a \in \mathcal{A}, t \in \mathcal{T}, v \in \mathcal{V}, i \in \mathcal{I}_v, \quad (5.33)$$

$$\gamma_{ov} \in \{0, 1\} \qquad o \in \mathcal{O}, v \in \mathcal{V}. \quad (5.34)$$

Constraints (5.29) and (5.30) restrict the variables e^P and u to be non-negative and integer. The remaining variables are made binary by constraints (5.31) - (C.33).

5.3 Parameter Definitions

In this section we describe parameter definitions. First, we explain the definitions of the big-M parameters. Thereafter, we explain how we have defined the maximum number of tours $\mathcal{I}_v$, minimum length of tours K_v, and weather-dependent activity duration $T_{atv}(\xi)$. All of these parameters, except for $\mathcal{I}_v$, are calculated in the same manner as described in Bruu and Thorsen (2021).

5.3.1 Calculation of Big-M Parameters

To make the LP-relaxation of constraints (5.17) as tight as possible, M_v^1 can be set to the maximal value of the left hand side of the constraints, and can be calculated as shown in Equation (5.35).

$$M_{atv}^1(\xi) = t + T_{atv}(\xi) + D_v^S \tag{5.35}$$

M_{av}^2, appearing in constraints (5.18), can be set to the latest possible start time of an activity that allows for the activity to be completed and the vessel to sail back to port before the end of its charter and option period. Introducing D_a^A as the minimum number of time periods required to complete activity a, the parameter is calculated as shown in Equation (5.36).

$$M_{av}^2 = |T| - D_v^S - D_a^A \tag{5.36}$$

In constraints (5.20) the big-M parameter, M_v^3, can be set to the maximum size of the allowable loading sets for vessel v. Thus, M_v^3 can be calculated as shown in Equation (5.37).

$$M_v^3 = \max_{l \in \mathcal{L}_v} \{\sum_{a \in \mathcal{A}} N_{al}\}. \tag{5.37}$$

5.3.2 Number of Tours

The maximum number of tours can be found through a minimization problem for each vessel. Introducing x_l as the number of times loading set l is loaded onto the vessel, and A_v as the set of activities vessel v can perform, the minimization problem for each vessel can be formulated as the knapsack problem described by Equations (5.38) - (5.40).

$$\mathcal{I}_v \quad = \min \sum_{l \in \mathcal{L}_v} x_l, \tag{5.38}$$

$$s.t \quad \sum_{l \in \mathcal{L}_v} N_{al} x_l \geq N^T, \qquad a \in A_v, \tag{5.39}$$

$$x_l \geq 0, integer \qquad l \in \mathcal{L}_v. \tag{5.40}$$

The objective (5.38) minimizes the number of tours by minimizing the number of loading sets that are loaded onto the vessel. Constraints (5.39) are knapsack constraints that ensure that the vessel loads enough resources to complete the activities it can perform for all planned turbines. There may be more vessels that can perform the same activities, but worst case, a single vessel should perform the activity for all OWTs, and $\mathcal{I}_v$ must be large enough to allow this. The final constraints, (5.40), enforces non-negativity and integer requirements on the decision variables.

5.3.3 Minimum Length of Tours

To tighten the LP-relaxation of constraints (5.4), the parameter K_v should be set as high as possible. Based on the assumption that a charter contract must include a fixed charter period, we believe it is reasonable to assume that a vessel must perform at least one round trip within its fixed charter period, and that K_v can be set to the minimum length of a vessel trip. This can be calculated as shown in Equation (5.41).

$$K_v = 2D_v^S - 1 + \min_{l \in \mathcal{L} | F_{lv}=1} \{D_{lv}^L + \min\{\sum_{a \in \mathcal{A}} N_{al} D_a^A, \sum_{a \in \mathcal{A} | N_{al} \neq 0} N^T D_a^A\}\} \tag{5.41}$$

For real-life projects, the number of turbines is expected to be higher than in our test instances. If so, the parameter K_v may be calculated differently, as it is likely that one would like more than one round trip for each vessel within their fixed charter period.

5.3.4 Activity Duration Parameter

The parameter $T_{atv}(\xi)$ must be calculated for different realizations of ξ. This parameter states how many time periods it will take vessel v to complete activity a if it is started in time period t under the realized weather conditions ξ. $T_{atv}(\xi)$ depends on the duration of performing an activity (D_a^A), the weather limit for performing the activity with that vessel, and the realized weather in each time period and scenario.

Figure 5.1 illustrates how $T_{atv}(\xi)$ is calculated. The idea is that there must be enough subsequent time periods with wave height and wind speed below the limit for a given activity to complete the activity. If the wave height or wind speed in a given time period is above the weather limits for the activity, referred to as "bad weather" in the figure, the activity cannot be performed in this time period, adding on to the value of $T_{atv}(\xi)$.

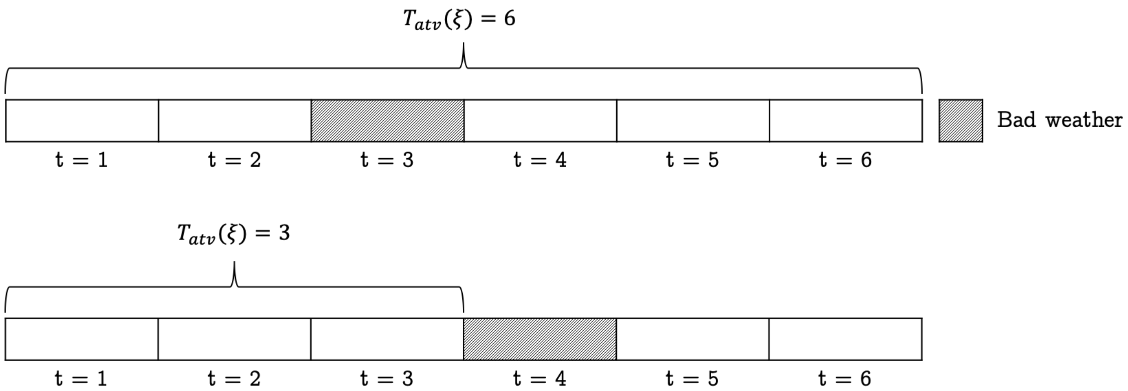


Figure 5.1: $T_{atv}(\xi)$ for an activity with $D_a^A = 3$ starting in time period $t = 1$.

5.4 Reduction and Fixing of Variables

The mathematical model has a large number of variables and constraints, making it demanding to solve. In order to reduce the number of indifferent solutions to the model and thus, the solution time, we have reduced and fixed some variables.

Firstly, variables s^A_{aitv} are only created if the corresponding parameter $T_{atv}(\xi)$ is of equal value as the duration of the activity a, that is, if $T_{atv}(\xi) = D^A_a$. The reason behind this is that it will never be preferable to choose a start time for an activity that leads to longer activity duration than necessary. Secondly, variables s^A_{aitv} are not created if they correspond to a parameter $T_{atv}(\xi)$ that leads the completion time of the activity to be after the final time period of the planning horizon. Finally, s^A_{aitv} variables that correspond to combinations of activities and vessels that are not compatible are not created.

We have applied fixing of variables for α_v, the binary variable indicating whether or not a vessel v is chartered. If only one of the vessels in the set of vessels is capable of performing a certain activity, the completion of the project is dependent on having that vessel included in the chartered fleet. For such vessels, we have fixated α_v to 1, indicating that the vessel must be included in the optimal fleet.

Chapter 6

Decomposed Model

Optimization models for practical problems tend to become complex and hard to solve with commercial solvers within a reasonable time. To solve such models, a common approach is to exploit the problem structure through a reformulation, resulting in a sequence of smaller and simpler problems that can be solved individually. Such methods are commonly called decomposition methods. In Section 6.1, we discuss how we can exploit the problem structure of the mathematical formulation presented in Chapter 5, henceforth referred to as the *original model*. Then we present a decomposed model formulation in Section 6.2.

6.1 Problem Structure

In the deterministic equivalent to the original model, we identify a primal block angular structure where the charter and project plan constraints constitute the set of linking constraints. The remaining constraints and variables can be isolated for each combination of a vessel v and a scenario s. This allows for $|\mathcal{V}| \cdot |\mathcal{S}|$ separate subproblems to be formed, where $\mathcal{S}$ represents the set of scenarios. These subproblems reflect scheduling problems over the entire planning horizon for combinations of a vessel and a scenario, whilst the linking constraints constitute the master problem and connect the schedules into a feasible project plan. Figure 6.1 illustrates the primal block angular structure of the original model. For problems with such structure, Dantzig-Wolfe has proven to be an effective decomposition method (Lundgren et al. 2010).

The primary motivation for applying decomposition techniques is to solve a sequence of smaller and easier problems instead of solving one large problem (Lundgren et al. 2010). Another benefit of using decomposition techniques is that we get a better description of the convex hull of the original problem (Vanderbeck 2000). A tighter LP-relaxation is obtained in a decomposed formulation since many of the (relaxed) integer and binary requirements from the original MIP are preserved in the subproblems of the reformulation.

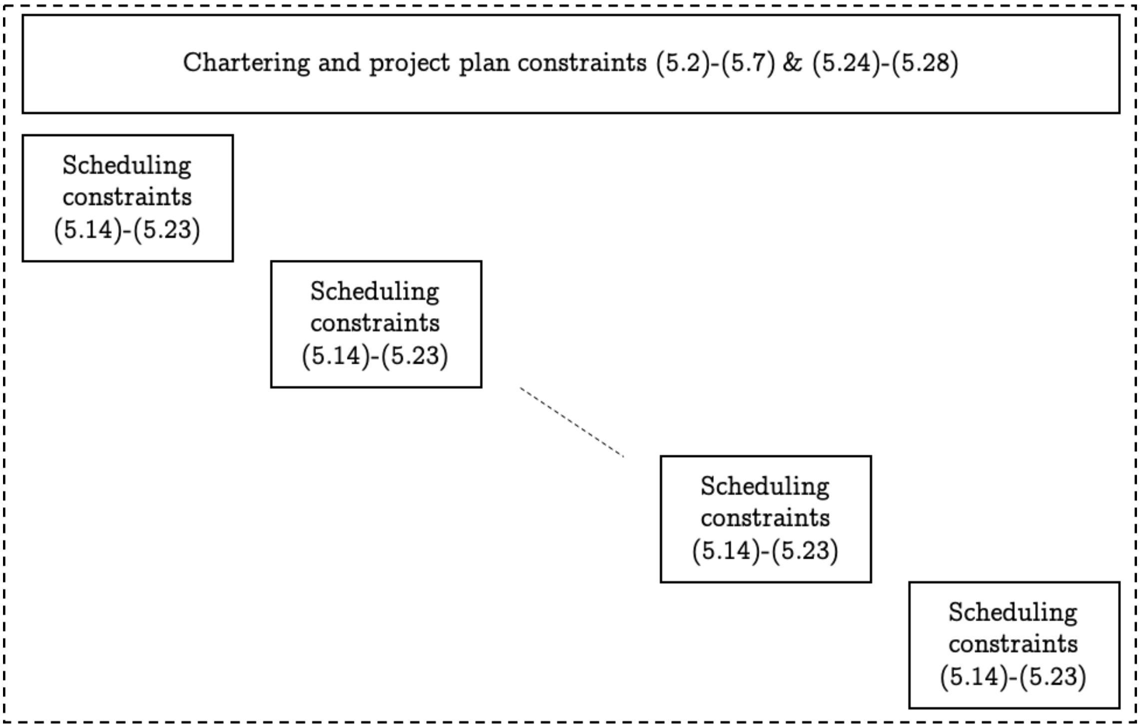


Figure 6.1: Primal block angular structure of the problem formulation with separate subproblems per vessel and scenario combination and linking constraints.

6.2 Reformulation of the Original Model

The decomposed model combines schedules into a fleet configuration that minimizes the expected fleet cost and project duration. The schedule combination must comply with the charter and project plan constraints from the original model. A prerequisite for the model is that all possible schedules are known.

A schedule describes the installation sequence performed by a vessel v in scenario s and provides the start and end time of the vessel's operational period. Figure 6.2 illustrates examples of feasible schedules. The letters L, S and A represent loading, sailing, and performing an installation activity, respectively. The weather is categorized as *bad* if the weather conditions exceed the operational weather limits for performing the activity.

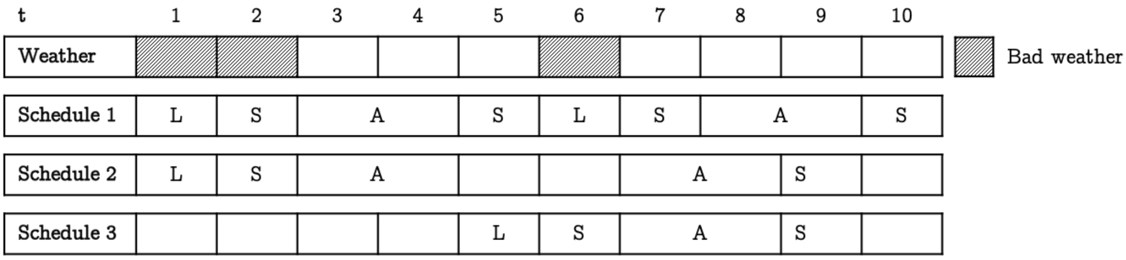


Figure 6.2: Examples of schedules with one installation activity.

We let $\mathcal{P}_{vs}$ represent the feasible schedules for vessel v in scenario s. A schedule $p \in \mathcal{P}_{vs}$ is described by the parameters

$$\text{Schedule } p = [T_p^{start}, T_p^{end}, B_{apt}],$$

where T_p^{start} and T_p^{end} represents the first and final time period of a schedule p, respectively. B_{apt} is a binary matrix where the value 1 indicates that an activity a ends in time period t in schedule p. The duration of an activity a is denoted by the parameter D_a^A.

The binary variables s_{tv}^C and e_{tv}^C of the original model are replaced by the integer variables s_v^C and e_v^C in the decomposed formulation. Further, a binary variable λ_p is defined for all $p \in \mathcal{P}_{vs}$. This variable indicates whether a schedule p is used or not. For remaining notation, see the deterministic equivalent of the original model (Appendix C). The mathematical formulation of the decomposed model follows.

Objective

$$\begin{aligned} \min \quad & \sum_{v\in\mathcal{V}} C_v^F \alpha_v + \sum_{v\in\mathcal{V}} C_v^V (e_v^C - s_v^C + \alpha_v) + \sum_{o\in\mathcal{O}} \sum_{v\in\mathcal{V}} P_{ov}^F \mu_{ov} \\ & + \sum_{s\in\mathcal{S}} \sum_{v\in\mathcal{V}} \sum_{o\in\mathcal{O}} p_s (P_{ov}^E \gamma_{ovs} + P^L e_s^P + P^U u_s) \end{aligned} \tag{6.1}$$

The objective corresponds to the objectives of the original model, and minimizes fixed and variable charter costs, the cost of including and exercising options, as well as penalties for long completion time, resulting in lost income, and unfinished turbines.

Constraints

$$\mu_{ov} \leq \alpha_v, \qquad o \in \mathcal{O}, v \in \mathcal{V}, \tag{6.2}$$

$$\gamma_{ovs} \leq \mu_{ov}, \qquad o \in \mathcal{O}, v \in \mathcal{V}, s \in \mathcal{S}, \tag{6.3}$$

$$\sum_{p\in\mathcal{P}_{vs}} T_p^{start} \lambda_p \geq s_v^C, \qquad v \in \mathcal{V}, s \in \mathcal{S}, \tag{6.4}$$

$$\sum_{p\in\mathcal{P}_{vs}} T_p^{end} \lambda_p \leq e_v^C + \sum_{o\in\mathcal{O}} L_o \gamma_{ovs}, \qquad v \in \mathcal{V}, s \in \mathcal{S}, \tag{6.5}$$

$$\sum_{v\in\mathcal{V}} \sum_{p\in\mathcal{P}_{vs}} \sum_{t'=1}^{t} B_{(a+1)pt'} \lambda_p \leq \sum_{v\in\mathcal{V}} \sum_{p\in\mathcal{P}_{vs}} \sum_{t'=1}^{t-D_{(a+1)}} B_{apt'} \lambda_p, \qquad a \in \mathcal{A} \setminus \{|\mathcal{A}|\}, t \in \mathcal{T}, s \in \mathcal{S}, \tag{6.6}$$

$$\sum_{p\in\mathcal{P}_{vs}} \lambda_p = \alpha_v, \qquad v \in \mathcal{V}, s \in \mathcal{S}, \tag{6.7}$$

$$\sum_{t \in \mathcal{T}} \sum_{v \in \mathcal{V}} \sum_{p \in \mathcal{P}_{vs}} B_{apt} \lambda_p \geq N^T - u_s, \qquad a = |\mathcal{A}|, s \in \mathcal{S}, \quad (6.8)$$

$$e_s^P \geq \sum_{p \in \mathcal{P}_{vs}} T_p^{end} \lambda_p, \qquad v \in \mathcal{V}, s \in \mathcal{S}. \quad (6.9)$$

Constraints (6.2) and (6.3) are identical to constraints (5.7) and (5.28) in the original formulation, and ensure that options can only be included for vessels that have been chartered, and that only included options can be exercised, respectively. Constraints (6.4)-(6.7) are included to ensure that schedules are combined to a feasible project plan. Constraints (6.4) guarantees that the assigned schedule for each vessel starts after the vessel is chartered, while constraints (6.5), adapted from constraints (5.24), secure that each vessel's charter doesn't end before its assigned schedule ends. Further, constraints (6.6) are adaptations of constraints (5.25) and ensure precedence between installation activities according to the order of the set of installation activities. Constraints (6.7) are convexity constraints and state that only one schedule can be performed by each chartered vessel. Finally, adaptations of constraints (5.26) and (5.27) are added trough constraints (6.8) and (6.9) respectively. Constraints (6.8) ensure that all turbines are either installed or penalized in the objective, while constraints (6.9) define the end date for the project.

Binary and non-negativity constraints

$$e_s^P \geq 0, \text{integer}, \qquad s \in \mathcal{S}, \quad (6.10)$$

$$u_s \geq 0, \text{integer}, \qquad s \in \mathcal{S}, \quad (6.11)$$

$$s_v^C \geq 0, \text{integer}, \qquad v \in \mathcal{V}, \quad (6.12)$$

$$e_v^C \geq 0, \text{integer}, \qquad v \in \mathcal{V}, \quad (6.13)$$

$$\alpha_v \in \{0, 1\}, \qquad t \in \mathcal{T}, v \in \mathcal{V}, \quad (6.14)$$

$$\gamma_{ovs} \in \{0, 1\}, \qquad o \in \mathcal{O}, v \in \mathcal{V}, s \in \mathcal{S}, \quad (6.15)$$

$$\mu_{ov} \in \{0, 1\} \qquad o \in \mathcal{O}, v \in \mathcal{V}, \quad (6.16)$$

$$\lambda_p \in \{0, 1\}, \qquad p \in \mathcal{P}_{vs}, v \in \mathcal{V}, s \in \mathcal{S}. \quad (6.17)$$

Constraints (6.10)-(6.13) restrict the variables e_s^P, u_s, s_v^C and e_v^C to be non-negative and integer. The remaining variables are made binary by constraints (6.14)-(6.17).

Chapter 7

A Branch-and-Price Algorithm for the Decomposed Model

A prerequisite for the decomposed model presented in Chapter 6 is that all possible schedules are known. For realistic problem instances, it will be impractical, or even impossible, to generate all schedules. To circumvent this problem, we propose to solve the model using B&P, where only a subset of all schedules is explicitly generated, while the remaining schedules are implicitly considered. We start by introducing an overview of the B&P methodology in Section 7.1. We then present how we solve the subproblems in Sections 7.2 - 7.5, including pricing of schedules, description of subproblems, and two different labeling algorithms. Lastly, we describe configurations for the B&P algorithm and acceleration techniques in Section 7.6 and Section 7.7, respectively.

7.1 Overview of the Solution Method

Dantzig-Wolfe decomposition, introduced by Dantzig and Wolfe (1960), is a method that combines a Dantzig-Wolfe reformulation of an optimization problem with a method to solve the decomposed problem. The original problem is reformulated into a master problem and a number of subproblems. For our problem, the model presented in Chapter 6 describes the master problem. Constraints related to scheduling of operations for each vessel and scenario combination from the original model are transferred to the subproblems.

To complete the Dantzig-Wolfe decomposition, a method for solving the reformulated problem is needed. We apply a B&B approach, where the linear relaxation of the reformulated problem is solved to obtain a lower bound at each node of the enumeration tree. Due to the potentially very large number of schedules that need to be generated to solve the problem, we apply a B&P algorithm where column generation is used to dynamically generate the necessary schedules in each B&B node.

In the B&P algorithm, an LP relaxation of the master problem that contains only a subset of schedules, referred to as the Restricted Master Problem (RMP), is solved to obtain primal and dual solutions to the problem. Dual information from the RMP solution is passed on to the subproblems, which are solved to find negative reduced cost schedules. The schedules from the subproblems solutions are then added to the current RMP and a new iteration is started by re-optimizing the RMP.

The B&P method alternates between the RMP and the subproblems until we are unable to generate a new schedule with negative reduced costs. Then, the optimal value of the current RMP provides a lower bound for the current B&B node. If this solution violates any of the integral requirements of the problem, and the bound of the node is better than the best known integer solution, two new nodes are created by branching on some properties of the problem. This process is repeated until all nodes are considered, and the best feasible solution is returned as the optimal solution to the problem. The solution process is illustrated in Figure 7.1. The shaded flowchart elements represent the column generation algorithm performed in each B&B node.

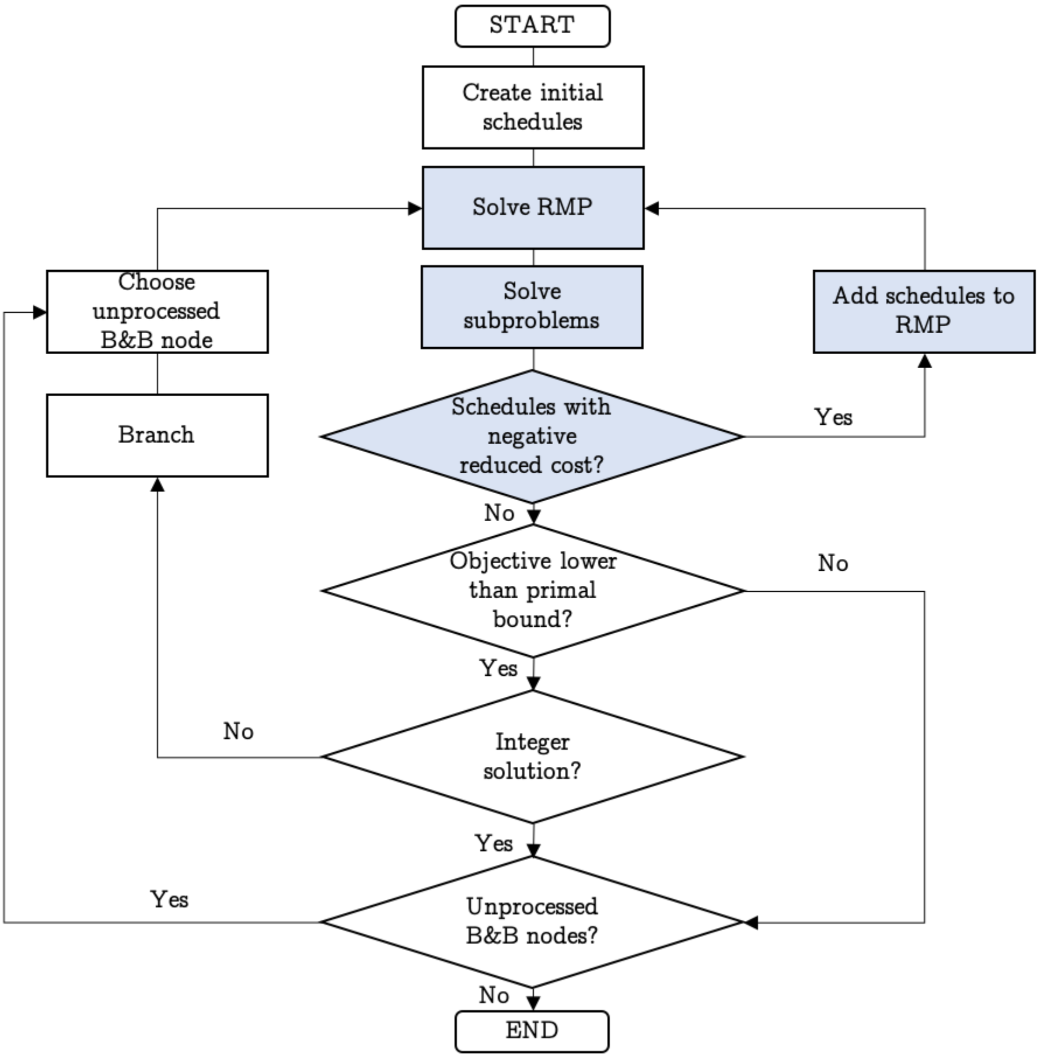


Figure 7.1: Flowchart of the solution procedure.

7.2 Pricing of Schedules

Let π^4, π^5, π^6, π^7, π^8_{at} and π^9 be the values of the dual variables associated with constraints (6.4), (6.5), (6.6), (6.7) (6.8) and (6.9) in the decomposed model, respectively. The reduced cost of a given schedule p can then be calculated as follows:

$$\bar{c}_p = \min \quad \sum_{t_2 \in \mathcal{T}} \sum_{a \in \mathcal{A} \setminus \{|\mathcal{A}|\}} \sum_{t_1 = t_2 + D_{(a+1)}}^{|\mathcal{T}|} \pi^8_{at_1} B_{at_2} - \sum_{t_2 \in \mathcal{T}} \sum_{a \in \mathcal{A} \setminus \{1\}} \sum_{t_1 = t_2}^{|\mathcal{T}|} \pi^8_{(a-1)t_1} B_{at_2}$$
$$-T^{start}\pi^4 - T^{end}(\pi^5 - \pi^7) - \pi^6 - \sum_{t \in \mathcal{T}} B_{|\mathcal{A}|t}\pi^9 \tag{7.1}$$

7.3 Subproblems

The subproblems needed to generate schedules for the RMP is presented in this section. The objective of the subproblems is to find the minimum priced schedule for the RPM. As described in Section 6.1, a series of $|\mathcal{V}| \cdot |\mathcal{S}|$ subproblems can be formulated. Each of these subproblems can be formulated as a SPPRC on an acyclic network of nodes.

The underlying network in each subproblem is defined by three different node types: loading nodes $\mathcal{N}^L$, activity nodes $\mathcal{N}^A$, and waiting nodes $\mathcal{N}^W$. The sets $\mathcal{N}^L$, $\mathcal{N}^A$ and $\mathcal{N}^W$ are defined by Equations (7.2) - (7.4).

$$\mathcal{N}^L = \mathcal{L}_v \times \mathcal{T} \tag{7.2}$$

$$\mathcal{N}^A = \mathcal{A}_v \times \mathcal{T} \tag{7.3}$$

$$\mathcal{N}^W = \{1\} \times \mathcal{T} \tag{7.4}$$

Additionally, a source node o and a sink node d are included at the beginning and the end of the network, respectively. The resulting network, $\mathcal{N} = \mathcal{N}^L \cup \mathcal{N}^A \cup \mathcal{N}^W \cup \{o, d\}$, is illustrated in Figure 7.2.

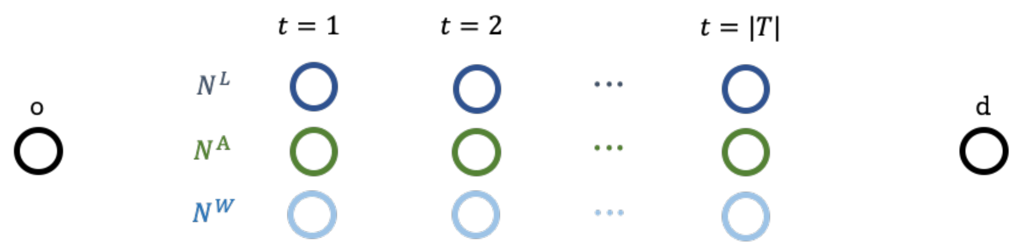


Figure 7.2: Illustration of node structure in the subproblems.

When a loading node is visited, the vessel is loaded with the corresponding loading set. Each loading set node represents the time period in which loading and sailing to site is completed, i.e., when the vessel is ready to start performing the activities corresponding to the content of the loading set. When an activity node is visited, the vessel must perform the corresponding installation activity once. The time index of the node represents the final time period of the installation operation, including consideration of weather effects. A waiting node represents doing nothing in a given time period. Such nodes are included to allow for delayed start of the operating period or waiting between installation activities due to weather conditions or precedence requirements on the installation sequence.

The set of nodes that are connected by arcs to a node $n \in \mathcal{N}$ depends on which of the subsets $\mathcal{N}^L$, $\mathcal{N}^A$, $\mathcal{N}^W$, n belongs to. For instance, there are no arcs connecting loading nodes to waiting nodes, as equivalent solutions can be obtained by visiting more waiting nodes prior to loading. Let $(i,t) \in \mathcal{N}$ and $(j,\tau) \in \mathcal{N}$, represent nodes with time indices $t \in \mathcal{T}$ and $\tau \in \mathcal{T}$, respectively, where i and j define which of the subsets $\mathcal{N}^L$, $\mathcal{N}^A$, or $\mathcal{N}^W$ the nodes belong to. The set of arcs, $((i,t),(j,\tau))$, in the network can then be described by Equation (7.5), where D^S, D^L and T_{at} correspond to the parameters D^S_v, D^L_v and $T_{atv}(\xi)$, respectively, as presented in Chapter 5, for the vessel and scenario realization that the subproblem corresponds to.

$$((i,t),(j,\tau)) : \begin{cases} i = o, j \in \mathcal{N}^L, & \tau = t + D^L + D^S \\ i \in \mathcal{N} \setminus \mathcal{N}^L, j \in \mathcal{N}^L, & \tau = t + D^L + 2D^S \\ i \in \mathcal{N} \setminus \mathcal{N}^L, j \in \mathcal{N}^W, & \tau = t + 1 \\ i \in \mathcal{N} \setminus \{o\}, j \in \mathcal{N}^A, & \tau = t + T_{a(t+1)} \\ i \in \mathcal{N}^A, j = d, & \tau = t + D^S \end{cases} \tag{7.5}$$

Some arcs can be deemed non-preferable based on weather conditions. Hence, the following arcs are eliminated from the network: (1) Arcs to activity nodes that are longer than the duration of the activity the node represents, and (2) arcs to loading nodes that are not connected to any arcs satisfying (1). These arcs can be removed because equivalent schedules can be obtained from paths that visit more waiting nodes before the respective activity or loading node is visited. The motivation for removing arcs is to make the labeling algorithm presented in the next chapter more effective by limiting the number of possible extensions.

7.4 An Activity Counting Labeling Algorithm

To solve the problems, an adaption of the labeling algorithm described in Irnich and Desaulniers (2005) is applied. The algorithm is described in Algorithm 1, where $\mathcal{U}$ is the set of unprocessed labels, which have not yet been extended, and $\mathcal{P}$ is the set of processed labels. The algorithm is initialized with $\mathcal{U} = \{L_0\}$, where L_0 represents a label at the source node, and $\mathcal{P} = \emptyset$. Let $\mathcal{L}_t$ denote the set of all non-dominated labels in $\mathcal{U} \cup \mathcal{P}$ that belong to a node with time index t. While there are unprocessed labels left in $\mathcal{U}$, the labeling algorithm invokes two procedures: a *path extension procedure* and a *dominance procedure*.

The path extension procedure replaces an element $L \in \mathcal{U}$ by all of its feasible extensions, (L, n), where $n \in \mathcal{N}$, before L is transferred to the set $\mathcal{P}$. To accelerate the overall labeling algorithm, the dominance procedure tries to reduce the sets $\mathcal{U}$ and $\mathcal{P}$ according to some criteria, further described in Section 7.4.4. This limits the number of necessary extension steps. To further enhance performance, the algorithm prioritizes processing labels corresponding to the lowest time indexed nodes. Let $\mathcal{D}$ represent the set of paths that correspond to labels at the sink node. Once $\mathcal{U} = \emptyset$, all paths in $\mathcal{D}$ are added to the restricted master problem.

Algorithm 1 Pseudocode for the labeling algorithm

```
𝒰 = {L_0}
while 𝒰 ≠ ∅ do
    L = remove first(𝒰)
    for each feasible extension L → L' do
        t = time(L')
        if no label in ℒ_t dominates L' then
            ℒ_t = ℒ_t ∪ {L'}
            if L' belongs to d then
                𝒟 = 𝒟 ∪ {L'}
            else
                𝒰 = 𝒰 ∪ {L'}
                for L̂ ∈ ℒ_t do
                    if L' dominates L̂ then
                        ℒ_t = ℒ_t \ {L̂}
                    end if
                end for
            end if
        end if
    end for
    𝒰 = 𝒰 \ {L}
    𝒫 = 𝒫 ∪ {L}
end while
Add all paths in 𝒟 with negative reduced cost to RMP
```

7.4.1 Labels

The following data is stored for each label:

- η, the node of the label
- ϕ, the predecessor label
- c, accumulated (reduced) costs
- $\vec{a}$, a vector of length $|\mathcal{A}|$ that represents the number of times each activity can be performed with the resources on board the vessel
- $\vec{f}$, a vector of length $|\mathcal{A}|$ that represents the number of times each installation activity has been completed so far along the path
- s, start status. Set to 1 the first time a non-waiting node is visited
- r, activity start status. Set to 1 the first time an activity node is visited.

Henceforth, the notation $\eta(L)$ is used to refer to the node of label L and similar notation is used for the rest of the resources, i.e., $\phi(L)$, $c(L)$, $\vec{a}(L)$, $\vec{f}(L)$, $s(L)$, and $r(L)$.

7.4.2 Initial Label

To initialize the algorithm described Section 7.4 an initial label, L_0, is required. We define L_0 as described in Equations (7.6) - (7.9). $\phi(L)$ is not defined as there is no predecessor label to the initial label.

$$\eta(L_0) = (o, 0), \tag{7.6}$$

$$c(L_0) = -\pi^6, \tag{7.7}$$

$$\vec{a}(L_0) = \vec{f}(L_0) = \vec{0}, \tag{7.8}$$

$$s(L_0) = r(L_0) = 0. \tag{7.9}$$

7.4.3 Label Extension

When a label L is extended along an arc $((i,t),(j,\tau))$, a new label L' at node $(j,\tau) \in \mathcal{N}$ is created. If $j \in \mathcal{N}^A$ the following resource constraints must be satisfied: (1) the vessel is loaded with the required resources to perform the activity, a, represented by node j, and (2) the total number of completed activities of type a cannot exceed the number of turbines to be installed throughout the project, N^T, after node (j,τ) is visited.

Given that the extension to L' is resource feasible, the information stored in the label is updated by the Resource Extension Functions (REF) (7.10) - (7.16). Equations (7.10) and (7.11) update the current node and the predecessor node, respectively. The accumulated reduced costs are updated as described described in Equation (7.12). Equation (7.13) updates the quantities of given activity resources on board a vessel, where $\vec{l}(j)$ represents the loading set corresponding to node $(j,\tau) \in \mathcal{N}^L$, while the the calculation of $\vec{f}(L')$ is shown in Equation (7.14). Finally, Equations (7.15) and (7.16) show how the start status and the activity start status are updated.

$$\eta(L') = (j,\tau) \tag{7.10}$$

$$\phi(L') = L \tag{7.11}$$

$$c(L') = c(L) - (\pi^5 - \pi^7)(\tau - t) + \begin{cases} \sum_{t'=\tau+D^A_{a+1}}^{|\mathcal{T}|} \pi^8_{at'}, & \text{if } j \in \mathcal{N}^A, a = 1 \\ -\sum_{t'=\tau}^{|\mathcal{T}|} \pi^8_{(a-1)t'} - \pi^9, & \text{if } j \in \mathcal{N}^A, a = |\mathcal{A}| \\ \sum_{t'=\tau+D^A_{a+1}}^{|\mathcal{T}|} \pi^8_{at'} - \sum_{t'=\tau}^{|\mathcal{T}|} \pi^8_{(a-1)t'}, & \text{if } j \in \mathcal{N}^A, a \neq 1, a \neq |\mathcal{A}| \\ -\pi^4(\tau - D^L - D^S + 1)(1 - s(L)), & \text{if } j \in \mathcal{N}^L, \\ 0, & \text{if } j \in \mathcal{N}^W \cup \{d\}. \end{cases} \tag{7.12}$$

$$\vec{a}(L') = \begin{cases} \vec{l}(j) & \text{if } j \in \mathcal{N}^L \\ \vec{a}(L) - \vec{e}_j & \text{if } j \in \mathcal{N}^A \\ \vec{a}(L) & \text{if } j \in \mathcal{N}^W \end{cases} \tag{7.13}$$

$$\vec{f}(L') = \begin{cases} \vec{f}(L) + \vec{e}_j & \text{if } j \in \mathcal{N}^A \\ \vec{f}(L) & \text{if } j \notin \mathcal{N}^A \end{cases} \tag{7.14}$$

$$s(L') = \begin{cases} 1, & \text{if } j \in \mathcal{N}^L, s(L) = 0 \\ s(L), & \text{otherwise} \end{cases} \tag{7.15}$$

$$r(L') = \begin{cases} 1 & \text{if } j \in \mathcal{N}^A, r(L) = 0 \\ r(L) & \text{otherwise} \end{cases} \tag{7.16}$$

7.4.4 Dominance Criterion

Let L_i and L_i' denote two partial paths from the source node to the node $(i,t) \in \mathcal{N}$, and let $\psi(L_i)$ represent the resource vector of L_i. Desaulniers et al. (1998) show that if all REFs along an arc $((i,t),(j,\tau))$ solely depend on $\psi(L_i)$, and all REFs are non-decreasing, a standard dominance criterion can be applied to eliminate labels. The criterion is that if $\psi(L_i) \leq \psi(L_i')$, then an extension of L_i' cannot lead to an optimal shortest path, and is dominated by L_i.

In the activity counting labeling algorithm the following must be satisfied for a label L_1 to dominate another label L_2:

1. $t(L_1) = t(L_2)$
2. $s(L_1) \geq s(L_2)$
3. $r(L_1) \geq r(L_2)$
4. $c(L_1) \leq c(L_2)$
5. $\vec{a}(L_1) \geq \vec{a}(L_2)$
6. $\vec{f}(L_1) \leq \vec{f}(L_2)$

In addition to this, for L_1 to dominate L_2, the set of nodes that can be extended to from $\eta(L_2)$ must also be possible to extend to from $\eta(L_1)$. Let X_t^L represent the set of nodes that can be extended to from a loading node with time index t, and let X_t^W and X_t^A represent similar sets for waiting- and activity nodes, respectively. Based on the arcs described in Section 7.3, the following relation between these sets can be established:

$$X_t^L \subset X_t^W \subset X_t^A \tag{7.17}$$

This implies that a label that belong to a loading node only can dominate labels belonging to other loading nodes, that labels belonging to waiting nodes only can dominate labels of other waiting nodes or loading nodes, while labels that belong to activity nodes may dominate labels that belong to both loading and waiting nodes.

7.5 A Tour Counting Labeling Algorithm

In the algorithm in Section 7.4, henceforth referred to as the *activity counting algorithm*, we limit the number of feasible paths by restricting the number of times each installation activity can be performed. Another alternative is to restrict the number of times a vessel can visit loading nodes in a feasible path, which is equivalent to restricting the number of round trips for a vessel. This is similar to our formulation of the second stage model in Chapter 5, where the size of the set $\mathcal{I}_v$ puts an upper bound on the number of round trips for vessel v. In this section we describe an alternative labeling algorithm for the subproblem SPPRC based on this concept, henceforth referred to as the *tour counting algorithm.* The two labeling algorithms are similar, but relevant adjustments are elaborated on below.

A label in the tour counting algorithm contains the same data as the label described in Section 7.4.3, except for $\vec{f}$, which is replaced by a new resource, w. The new resource counts the number of times loading nodes have been visited so far along the path, and is initialized as described in Equation (7.18). Its REF is described in Equation (7.19). All other resources are updated by the same REFs as in the activity counting labeling algorithm.

$$w(L_0) = 0 \tag{7.18}$$

$$w(L') = \begin{cases} w(L) + 1 & \text{if } j \in \mathcal{N}^L \\ w(L) & \text{otherwise} \end{cases} \tag{7.19}$$

The resource constraints that must be satisfied for feasible extensions in the tour counting algorithm are: (1) to extend to a node $(j, t) \in \mathcal{N}^A$, the vessel must be loaded with the required resources to perform the corresponding activity, and (2) w must be within its resource window, as described in Equation (7.20).

$$w \in [0, \mathcal{I}_v] \tag{7.20}$$

The dominance criterion consists of the same requirements as for the activity counting algorithm, with the exception of requirement 5 which is replaced by the requirement described in Equation (7.21).

$$w(L_1) \geq w(L_2) \tag{7.21}$$

As the new requirement requires fewer pairwise comparisons, the dominance criterion of the tour counting algorithm is less strict than that of the activity counting algorithm. Hence, we expect more labels to be removed by dominance in this algorithm, which theoretically should speed up the computational time of the subproblems. However, the increased speed comes at a cost. As we in the tour counting algorithm allow for generation of schedules where more turbines than necessary are installed, the dual bound obtained by solving the restricted master problem will become weaker.

7.6 Configurations for the Branch-and-Price Algorithm

To implement the B&P algorithm, different algorithmic design choices must be made. First, we describe how we initialize the column generation algorithm. Then we describe the search strategy, i.e., the order in which B&P nodes in the enumeration tree are explored. Then, the branching strategy, i.e., how the solution space is partitioned to produce new nodes, is defined. Finally, we define a termination criterion for the search.

7.6.1 Initial Schedules

In the root node of the enumeration tree there are no previously generated schedules. To initialize the column generation algorithm, empty schedules for each vessel and scenario combination are added to the RMP. An empty schedule is a schedule where no activities are performed, and that has $T_p^{start} = T_p^{end} = 0$.

7.6.2 Search Strategy

The choice of search strategy can have significant consequences on the computational time needed for the B&P algorithm to reach optimality. As the subproblems of the decomposed model are complex and solving them is very time consuming, we want to limit the number of B&B nodes that need to be solved to obtain the optimal solution. Further, as we perform a Dantzig-Wolfe reformulation of the original problem, we expect a tight gap between the solution in the root node of the enumeration tree and the optimal integer feasible solution. Since we further extend the B&P algorithm with a primal heuristic for finding good integer feasible solutions, which is introduced in Section 7.7, the focus in the enumeration tree is on improving the dual bound. Therefore, to decide which node to solve next, we employ a *best first* search strategy: we always investigate nodes with the most promising optimistic bounds first. The optimistic bound of an unsolved B&P node is defined by the objective value of its parent node.

7.6.3 Branching

As the shortest path labeling algorithm enforces integrality in the subproblems, the relevant variables to branch on are those found in the master problem of the decomposed model. We apply a hierarchical branching strategy, where the first stage decision variables of the original model are prioritized over the second stage variables. The u_s and e_s^P variables are not considered for branching, as all solutions where any u_s variable is larger than zero are considered infeasible, and e_s^P does not directly affect the first stage decisions of contractual agreements.

The first variables that are considered for branching are the binary α_v variables, which represent the decision of whether or not a vessel v is included in the chartered fleet. These variables are prioritized as they have a large numerical impact on the objective function. Another benefit of branching on α_v, is that in the 0-branch it will not be necessary to solve the subproblems related to vessel v, as it is not necessary to find new schedules for a vessel that is not chartered. The 1-branch for a α_v variable indicates that vessel v must be chartered, and hence should perform some operations. This implies that the initial empty schedule is infeasible in this branch, something we have imposed by giving it a penalty in the objective function. The non-integer α_v with the lowest v index is branched on first.

The next variables considered for branching are μ_{ov}. These variables represent the decisions of whether or not to include option o in the contract for vessel v, and also these variables have a considerable impact on the objective value. Similarly, to the branching strategy for α_v variables, the lowest indexed non-integer μ_{ov} is branched on first.

Next, the variables s_v^C and e_v^C are branched on, again the lowest indexed non-integer variables are prioritized. As these variables are non-binary, the branching must impose a lower or upper bound on them. Let s_v^{C*} represent a non-integer value of a s_v^C variable in the optimal node solution. Equations (7.22) and (7.23) show how the branching strategy creates a new lower or upper bound on this variable, respectively. Similar equations also apply for branching on the e_v^C variables.

$$s_v^C \geq \lfloor s_v^{C*} \rfloor + 1 \tag{7.22}$$

$$s_v^C \leq \lfloor s_v^{C*} \rfloor \tag{7.23}$$

Branching on s_v^C and e_v^C also affects the characteristics of feasible schedules in the subproblems for vessel v. A lower bound requirement for s_v^C, poses a lower bound on the start time of feasible schedules for vessel v, as described in Equation (7.24). Further, an upper bound for e_v^C, combined with μ_{ov}-branching decisions, affects the latest feasible end time for schedules for vessel v. This is described by Equation (7.25), where $\hat{\mu}_{ov}$ equals zero if the variable μ_{ov} earlier has been branched to zero, and otherwise equals 1. Schedules that break either of these bounds are forbidden in the master problem and are therefore not generated during column generation for the relevant B&B node.

$$T^{start} \geq \lfloor s_v^{C*} \rfloor + 1 \tag{7.24}$$

$$T^{end} \leq \lfloor e_v^{C*} \rfloor + \sum_{o \in \mathcal{O}} L_o \hat{\mu}_{ov} \tag{7.25}$$

Once all the first stage decision variables α_v, μ_{ov}, s_v^C, and e_v^C are integer, the second stage decision variables γ_{ovs} may be branched on. This is done in the same manner as for μ_{ov}.

Even when all of the α_v, μ_{ov} and γ_{ovs} take integer values, the schedule variables, λ_p, are not guaranteed to be integer. If the λ_p variables are non-integer for a subproblem, different schedules are mixed. To guarantee reasonable charter contracts, such solutions are not sufficient. Hence, we must apply a branching strategy that enforces integer values also for these variables. To branch on the λ_p variables one by one would be very inefficient, as there are many similar schedule opportunities in each subproblem. To affect a large range of these variables, we apply a branching strategy where upper and lower bounds are set on the parameters T^{start} and T^{end} for schedules in each subproblem. Schedules that break these bounds are forbidden in the master problem and are therefore not generated during column generation for the relevant B&P node.

The schedule start and end bounds can only be branched on for one subproblem at the time, and bounds for T^{start} parameters are branched on before bounds for T^{end}. To affect as many schedule variables as possible, the weighted average of the start times of the schedules used in the solution, denoted $\overline{T}_{vs}$, is calculated for each subproblem and rounded down to the nearest integer, as shown in Equation (7.26), where λ_p^* represents the value of λ_p in the node solution.

$$\overline{T}_{vs} \leq \left\lfloor \sum_{p \in \mathcal{P}_{vs}} T_p^{start} \lambda_p^* \right\rfloor \tag{7.26}$$

We choose to execute the branching for the subproblem where the distribution of schedule start times is most balanced around $\overline{T}_{vs}$. That is, the subproblem with the lowest difference, d_{vs}, in number of schedules with T^{start} before and after $\overline{T}_{vs}$ as expressed by Equations (7.27) - (7.29).

$$X_{vs} = \{p \in \mathcal{P}_{vs} | T_p^{start} > \overline{T}_{vs} \wedge \lambda_p^* > 0\} \tag{7.27}$$

$$Y_{vs} = \{p \in \mathcal{P}_{vs} | T_p^{start} < \overline{T}_{vs} \wedge \lambda_p^* > 0\} \tag{7.28}$$

$$d_{vs} = |(|X_{vs}| - |Y_{vs}|)| \tag{7.29}$$

In the first branch, the upper bound of T^{start} is set as described in Equation (7.30), while in the second branch the lower bound of T^{start} is set as described in Equation (7.31). Branching on bounds for T^{end} is done in the same manner.

$$T^{start} \leq \overline{T}_{vs} \tag{7.30}$$

$$T^{start} \geq \overline{T}_{vs} + 1 \tag{7.31}$$

7.6.4 Termination Criterion

The B&P search is terminated if the optimality gap is less than a predefined value ϵ, as expressed in Equation (7.32). The primal bound is the best found integer feasible solution, and the dual bound is an optimistic bound on the objective, determined by the lowest parent node objective to an unsolved B&P node.

$$\text{Optimality gap} = \frac{\text{Primal Bound} - \text{Dual Bound}}{\text{Primal Bound}} \leq \epsilon \tag{7.32}$$

The value of ϵ is set to 0.01%, which is a value commonly used in commercial solvers.

7.7 Acceleration Techniques

Algorithmic extensions aimed at accelerating the B&P algorithm have been implemented. First, we expand the B&P algorithm by a primal heuristic to more quickly find integer feasible solutions. A second extension is to add all schedules with negative reduced cost to the RMP in each iteration. We also extend the algorithm by applying partial pricing. Lastly, we include heuristic labeling for the subproblems.

7.7.1 Inclusion of a Primal Heuristic

To quickly find integer feasible solutions, we have included a primal heuristic into the B&P algorithm. After the column generation algorithm produces an LP solution in a B&B node, the RMP is re-solved with binary and integer requirements on the variables to obtain an integer feasible solution. The primal bound is updated if the integer feasible solution from the primal heuristic is better than the current best integer feasible solution. Following the primal heuristic, the B&P algorithm returns to assess whether or not to branch further, based on the node solution.

7.7.2 Generating Additional Schedules

A well known strategy for accelerating column generation is to return more than one negative reduced cost column to the master problem in each iteration of the column generation procedure. To add more than one columns is particularly easy when the subproblems are solved by dynamic programming, and generally decreases the number of necessary column generation iterations (Desrosiers and Lübbecke 2005; Kallehauge et al. 2005). Hence, in our implementation, we add all generated schedules with negative reduced cost to the master problem in each iteration.

7.7.3 Partial Pricing

When there are many subproblems it is often beneficial to consider only a few of them each time the pricing problem is called (Desrosiers and Lübbecke 2005). This is known as partial pricing. In our implementation, we solve the subproblems one by one. If a subproblem returns at least one schedule with negative reduced cost, the schedules are added to the RMP immediately, and the RMP is re-solved. Then the algorithm continues to the next subproblem and repeats the procedure until no new schedules with negative reduced costs are found in any of the subproblems.

Additionally, we inspect the dual values of a subproblem before it is solved. Subproblems with dual values that are unchanged from the previous iteration are not solved. The subproblem is also skipped, if the absolute values of its relevant duals sum up to zero.

7.7.4 Heuristic Labeling

Solving the labeling algorithm for the subproblems as described in Section 7.4 can be very time consuming. To accelerate the search for new feasible schedules, we have added a heuristic labeling algorithm. The heuristic labeling algorithm is almost the same as the exact labeling algorithm described in Section 7.4, except for the dominance criteria, which have been relaxed. The following criteria must be satisfied for a label L_1 to dominate another label L_2 in the heuristic labeling algorithm:

1. $t(L_1) = t(L_2)$
2. $s(L_1) \geq s(L_2)$
3. $r(L_1) \geq r(L_2)$
4. $c(L_1) \leq c(L_2)$
5. $\vec{a}(L_1) \geq \vec{a}(L_2)$
6. $\eta(L_1) \in N^L \rightarrow \eta(L_2) \in N^L$

When a B&P node is solved, the heuristic labeling algorithm is used until no new schedules are found for any of the subproblems. Then, the exact labeling algorithm is used to search for additional schedules. If the exact labeling algorithm identifies new schedules in a subproblem, the heuristic labeling algorithm is resumed. This alternation between the two algorithms continues until neither algorithm is able to find new schedules in any of the subproblems, and the optimal solution to the B&P node is found.

Chapter 8

Instance and Scenario Generation

In this chapter, we present the test instances that form the basis for testing the performance of our models. The chapter is to a large extent re-used from our project report (Bruu and Thorsen 2021). First, we present the selection of input data and generation of weather scenarios in Section 8.1. Then, penalty cost calculations are presented in Section 8.2. Finally, tabular summaries of the test instances are given in Section 8.3.

8.1 Input Data

In this section, we describe the data used to generate test instances. To generate realistic instances, the data is based on information given in Chapter 2, and interviews with Ulstein International and Clarkson Platou presented in Appendices A and B. Additionally, general knowledge from our studies within marine project engineering is applied.

8.1.1 Planning Horizon

As described in Section 2.5, it is common to plan for all installation activities to be performed during the summer months due to harsher weather conditions during the winter. Based on this, a planning horizon of four months, from April until the end of July, is applied in the test instances. The planning horizon is created with a time discretization of 12 hours, resulting in a total of 244 time periods.

8.1.2 Installation Activities

For testing purposes, installation of a monopile OWT is used as an example. The installation process then involves the following operations: installation of foundation and transition piece, turbine installation, scour protection and cable installation and burial.

Operations that are usually conducted by the same vessel are aggregated into a single activity. Hence, installation of foundation and transition piece are regarded as a single activity and referred to as *installation of foundation*. Further, we assume that a simultaneous lay and burial strategy is used for cable installation. The duration of this activity is set to three time periods, reflecting the duration of cable burial which is the most time consuming out of the two activities. Weather limits for what is referred to as "cable installation" in the table in Section 2.5 are used, as it is assumed that these reflect the limitations of a Cable Laying Vessel (CLV), which can perform both activities.

A summary of the activities and their duration and weather limits is presented Table 8.1. The activities are numbered from one to four, reflecting precedence requirements. Weather limits and duration of activities are based on data presented in Section 2.5. In addition to performing installation activities, the vessels must sail between port and the offshore site. The duration of sailing depends on the vessels' speed and the location of the wind farm. For testing purposes, we set the time of sailing each way to one time period.

Table 8.1: Input data for activities, including durations and weather limits.

No.	Activity	Duration (time periods)	Wind limit (m/s)	Wave limit (m)
1	Installation of foundation	6	12	2
2	Installation of cables	3	15	1.5
3	Scour protection	1	15	2.5
4	Installation of turbine	2	8	2

8.1.3 Vessels

To complete the installation activities, a vessel pool consisting of a CLV, Jack-Up Barge (JUB), Rock Dumping Vessel (RDV) and WTIV is included. The variable costs and installation capabilities for each vessel are shown in Table 8.2. The cost data is based on the day rates presented in Section 2.3, and our knowledge from studies within marine project engineering. The variable costs are recalculated to correspond to 12-hour time periods. As the Jack-Up Barge (JUB) is not self-propelled, the charter cost of a tug-boat is included in its variable cost.

Table 8.2: Input data for vessels, including costs and abilities.

Vessel	Variable Cost (USD/12h)	Fixed Cost (USD)	Installation activities
CLV	57 500	6 000	Cable laying and burial
JUB	71 500	6 000	Foundation
RDV	15 000	6 000	Scour protection
WTIV	100 000	6 000	Foundation; Turbine

8.1.4 Options

In real-life charter contracts for installation vessels, the possibility of adding flexibility to the contract through options is subject to negotiations and influences the overall cost of the contract. In this thesis, we apply a simplified approach to contractual options through inclusion and exercise prices for options, inspired by Voster and Kjelby (2020).

We define the exercise price of an option as the option length multiplied by day rates for the option period. We assume the day rates increase by 10% in the option period compared to normal day rates. Hence, the exercise price of an option can be expressed as in Equation (8.1).

$$\textit{exercise price} = 1.10 \cdot \textit{day rate} \cdot \textit{option length} \tag{8.1}$$

The inclusion price of an option is calculated as described in Equation (8.2), where the exercise price is multiplied by a factor γ. As options represent uncertain income, and hence, a risk for the shipowner, γ can be considered to represent a degree of "inconvenience" for the shipowner. For the purpose of testing, this factor is set to 20%.

$$\textit{inclusion price} = \gamma \cdot \textit{exercise price} \tag{8.2}$$

The possibility of including options for one or two weeks (or both), corresponding to 14 and 28 time periods, respectively, for each vessel is included. The prices for including and exercising these options are calculated as described above and presented in Table 8.3. Note that the option length is given in number of time periods.

Table 8.3: Input data for option prices.

Vessel	Option length	Inclusion price (kUSD)	Exercise price (kUSD)
CLV	14	177.1	885.5
	28	354.2	1771
JUB	14	220.22	1101.1
	28	440.45	2202.2
RDV	14	46.2	231
	28	92.4	462
WTIV	14	308	1540
	28	616	3080

8.1.5 Loading Sets

The vessels have predefined loading sets. The size of each loading set is based on space limitations on board the vessels, as well as the size of turbine components commonly used today. An overview of the loading sets, including their resources, corresponding activity, and the vessel the loading set belongs to, is shown in Table 8.4. The duration of loading a loading set onto a vessel is set to one time period.

Table 8.4: Input data for loading sets, including corresponding activity and vessel.

Loading set	Activity	Vessel
10 × cable	2	CLV
4 × foundation	1	JUB
5 × scour protection	3	Rock Dumping Vessel
6 × turbine	4	WTIV
6 × foundation	1	WTIV

8.1.6 Weather Data

To account for uncertain weather effects, the scheduling problems for each vessel are solved for different realizations of weather conditions. Different realizations of weather conditions over the span of the planning horizon will henceforth be referred to as different *scenarios*.

Weather data was made available by the FINO (Forschungsplattformen in Nord- und Ostsee) initiative, which was funded by the German Federal Ministry of Economic Affairs and Energy (BMWi) on the basis of a decision by the German Bundestag, organised by the Projekttraeger Juelich (PTJ) and coordinated by the German Federal Maritime and Hydrographic Agency (BSH) (FINO 2021).

Measurements of specific wave height and wind speed from the FINO1 research platform in the North Sea, collected from April 1st to July 31st for the years 2013 - 2020, were used to generate eight weather scenarios, one corresponding to each year. We assume that the weather scenarios are uniformly distributed, meaning that the probability of each weather scenario is set to $1/|S|$, where $|S|$ is the total number of scenarios considered in the model.

Our mathematical models require each time period to have a given weather state in each scenario. At FINO1, measurements of wind speed and wave height are made more frequent than every 12 hours. We assume that a vessel cannot operate operate through a 12-hour time period if the weather conditions at any point within this time period exceed its operational limits. Hence, in the generation of scenarios, the weather state in each time period is set to a conservative value equal to the maximum wind speed and wave height over the 12 hours of weather data.

8.2 Parameter Calculations

The penalty cost parameters used as weighting coefficients in the objective functions must be calculated before solving the model. Their calculations are explained in this section.

8.2.1 Penalty Cost for Lost Income

To minimize the project end date, a penalty cost term for lost income is added to the objective function, inspired by Hansen and Siljan (2017). A prolonged installation phase causes loss of electricity production and potential revenue for the electricity providers. Thus, the value of the penalty cost is represented by the lost revenues of not generating electricity.

How much energy the wind farm can produce per time period can be found by multiplying the capacity factor of each turbine by the total number of turbines in the wind farm and the turbine rating. The capacity factor represents, on average, how much energy is produced by a wind turbine and is given as a percentage of the turbine rating. To find revenues, the generated electricity per time period must be multiplied by some time interval and an electricity price, as expressed in Equation 8.4.

$$P^L = Price^{el} \cdot \underbrace{Capacity \cdot Rating \cdot nTurbines}_{\text{Generated electricity}} \cdot Interval \tag{8.3}$$

Based on values from offshore wind farms in the UK, a capacity factor of 40% is applied in this thesis (Energy Numbers 2022). Further, a turbine rating of 8 MW is used based on our knowledge from studies within marine project engineering. The energy price is calculated as the average of energy prices from 2017-2021 with data from Nord Pool (Nord Pool AS 2020), and converted to USD based on average exchange rates over the same time period (XE.com Inc. 2022). The applied time interval is 12 hours.

8.2.2 Penalty Cost for Unfinished OWTs

The penalty cost for unfinished turbines, P^U, must be set to a value that ensures that it does not become advantageous to not finish all planned turbines in the wind farm. Hence, the penalty must be set higher than the cost of completing the installation activities for each OWT. This cost will depend on vessel day rates and processing times for the installation activities. For the test instances in this report, the penalty cost is set as described in Equation (8.4), where $|T|$ is the number of time periods in the test instance and P^L is the penalty cost for lost income.

$$P^U = 10\ 000\ 000 + P^L \cdot |T| \tag{8.4}$$

8.3 Test Instances

In this section we present test instances for the problem of designing charter contract for installation vessels at offshore wind farms. The test instances are made based on the input data and weather scenarios presented earlier in this chapter.

The applicability of the model depends on the number of turbines it can be solved for, thus test instances with an increasing number of turbines have been made. Further, the more scenarios the model can handle, the better the uncertainty is accounted for. Therefore, we have also created test instances with an increasing number of scenarios. Tabular summaries of the test instances with number of turbines, scenarios, and activities, as well as available vessels, are presented in Table 8.5 and Table 8.6. All test instances span 244 time periods with a 12-hour time discretization.

Table 8.5: Test instances for increasing number of turbines.

Instance	Turbines	Scenarios	Activities	Vessels	Time Periods
T2	2	1	4	CLV, JUB, RDV, WTIW	244
T4	4	1	4	CLV, JUB, RDV, WTIW	244
T6	6	1	4	CLV, JUB, RDV, WTIW	244
T8	8	1	4	CLV, JUB, RDV, WTIW	244
T10	10	1	4	CLV, JUB, RDV, WTIW	244
T12	12	1	4	CLV, JUB, RDV, WTIW	244
T14	14	1	4	CLV, JUB, RDV, WTIW	244
T16	16	1	4	CLV, JUB, RDV, WTIW	244
T18	18	1	4	CLV, JUB, RDV, WTIW	244
T20	20	1	4	CLV, JUB, RDV, WTIW	244

Table 8.6: Test instances for increasing number of scenarios.

Instance	Turbines	Scenarios	Activities	Vessels	Time Periods
T2S2	2	2	4	CLV, JUB, RDV, WTIW	244
T2S4	2	4	4	CLV, JUB, RDV, WTIW	244
T2S6	2	6	4	CLV, JUB, RDV, WTIW	244
T2S8	2	8	4	CLV, JUB, RDV, WTIW	244
T4S2	4	2	4	CLV, JUB, RDV, WTIW	244
T4S4	4	4	4	CLV, JUB, RDV, WTIW	244
T4S6	4	6	4	CLV, JUB, RDV, WTIW	244
T4S8	4	8	4	CLV, JUB, RDV, WTIW	244
T6S2	6	2	4	CLV, JUB, RDV, WTIW	244
T6S4	6	4	4	CLV, JUB, RDV, WTIW	244
T6S6	6	6	4	CLV, JUB, RDV, WTIW	244
T6S8	6	8	4	CLV, JUB, RDV, WTIW	244
T8S2	8	2	4	CLV, JUB, RDV, WTIW	244
T8S4	8	4	4	CLV, JUB, RDV, WTIW	244
T8S6	8	6	4	CLV, JUB, RDV, WTIW	244
T8S8	8	8	4	CLV, JUB, RDV, WTIW	244
T10S2	10	2	4	CLV, JUB, RDV, WTIW	244
T10S4	10	4	4	CLV, JUB, RDV, WTIW	244
T10S6	10	6	4	CLV, JUB, RDV, WTIW	244
T12S2	12	2	4	CLV, JUB, RDV, WTIW	244
T12S4	12	4	4	CLV, JUB, RDV, WTIW	244
T12S6	12	6	4	CLV, JUB, RDV, WTIW	244
T14S2	14	2	4	CLV, JUB, RDV, WTIW	244
T14S4	14	4	4	CLV, JUB, RDV, WTIW	244
T16S2	16	2	4	CLV, JUB, RDV, WTIW	244
T16S4	16	4	4	CLV, JUB, RDV, WTIW	244
T18S2	18	2	4	CLV, JUB, RDV, WTIW	244
T18S4	18	4	4	CLV, JUB, RDV, WTIW	244
T18S6	18	6	4	CLV, JUB, RDV, WTIW	244
T20S2	20	2	4	CLV, JUB, RDV, WTIW	244
T20S4	20	4	4	CLV, JUB, RDV, WTIW	244
T20S6	20	6	4	CLV, JUB, RDV, WTIW	244

Chapter 9

Computational Study

In this chapter we study the effect of applying B&P to the problem of designing charter contracts for installation vessels at offshore wind farms through testing of the instances presented in Section 8.3. To provide a meaningful analysis, the results from solving the decomposed model using B&P are compared to results from solving the original model using a commercial MIP solver.

In Section 9.1, we compare the performance of the two labeling algorithms for solving the subproblems of the decomposed model. Then, we analyze how the computational performance of the implemented B&P algorithm changes with different instance characteristics. In Section 9.2 and Section 9.3 we study the effect of increasing the number of turbines and the number of scenarios, respectively, and evaluate the ability of our B&P algorithm to generate dual bounds and provide a tighter LP-relaxation.

The B&P algorithm is implemented in C++17, and combined with Gurobi Optimizer to solve the RMP. The original model is implemented in Mosel and solved using FICO Xpress with a single thread. Testing was conducted on a Lenovo NextScal nx360 M5 computer with the specifications presented in Table 9.1. For practical purposes, the maximum run time for each test instance was set to three hours. As the problem studied in this thesis is a strategic long-term planning problem, a longer run time could be considered acceptable if the models were to be used to provide decision support for real-life projects.

Table 9.1: Specifications on computer and solvers used for the computational study.

Processors	2× 2.3 GHz Intel E5-2670v3 12 core
RAM	64 GB
FICO Xpress	Version 8.9.0
Gurobi Optimizer	Version 9.5

9.1 Comparison of the Labeling Algorithms

To see how the performance of the B&P method is affected by the choice of labeling algorithm, we have run some test instances with both algorithms presented in Section 7.4 and Section 7.5. To cover different aspects of the model, random instances with varying number of turbines and scenarios were chosen.

Table 9.2 shows, for each test instance, the number of seconds it took to solve the root node (Root Time), the root node objective (Root Obj.), the final dual bound (Dual Bound), final optimality gap (Gap (%)), as well as the number of processed B&B nodes (Nodes) and number of generated schedules (Schedules) for the activity counting algorithm. The same result data for the tour counting algorithm is displayed in Table 9.3. Both tables show results after three hours of computational time.

Table 9.2: Results for the activity counting algorithm.

Instance	Root Time (s)	Root Obj.	Dual Bound	Gap (%)	Nodes	Schedules
T2S8	1 504	246 653	246 792	0.42	34	16 183
T4	356	524 007	524 338	0.00	4	6 244
T4S4	1 497	635 203	635 324	1.40	40	16 294
T8	985	1 974 083	1 976 299	27.49	76	16 700
T8S2	2 456	1 897 322	1 897 408	25.61	23	16 569
T10S4	7 276	2 579 733	2 579 733	21.90	3	34 343
T14S2	8 301	4 993 672	4 993 777	76.93	4	24 951
T20	2 898	35 807 532	35 808 024	73.90	19	18 484

Table 9.3: Results for the tour counting algorithm.

Instance	Root Time (s)	Root Obj.	Dual Bound	Gap (%)	Nodes	Schedules
T2S8	3 269	170 600	175 851	46.77	11	40 673
T4	381	415 222	420 850	19.81	162	24 239
T4S4	945	502 604	504 300	87.16	46	24 071
T8	648	1 418 583	1 421 587	95.06	57	29 916
T8S2	1 087	1 529 714	1 529 933	93.07	70	18 856
T10S4	2 151	2 298 952	2 299 003	96.18	19	29 479
T14S2	1 277	4 467 395	4 467 589	90.18	47	24 074
T20	806	35 805 726	35 811 862	73.90	29	32 184

There are positives and negatives for both algorithms. When we apply the tour counting algorithm, the root node solution is found considerably faster than when we apply the activity counting algorithm. This indicates that solving each node in the enumeration tree requires less computational time with the tour counting algorithm compared to the activity counting algorithm. This is further supported by the fact that more nodes are solved within the time limit. As a result of processing more B&B nodes, the tour counting algorithm also generates a significantly higher number of schedules. However, as this algorithm allows schedules where more OWTs than necessary are installed, the primal heuristic is not able to combine these schedules into a feasible installation sequence for the wind farm as a whole, indicated by the large optimality gaps.

If we look at the objective of the root node solution and the dual bounds, the activity counting algorithm performs better, as stronger bounds are found. As neither of the algorithms manage to improve the dual bound much from the root node solution, this is a valuable aspect of the activity counting algorithm. Moreover, the activity counting algorithm obtains significantly lower optimality gaps. This is because the primal heuristic is able to find better integer solutions based on the schedules generated by this algorithm. In the remainder of the computational study the activity counting algorithm is used.

9.2 Increasing the Number of Turbines

Table 9.4 shows, for each test instance, the total computational time in seconds (Time (s)), root node objective (Root Obj.), final dual bound (Dual Bound), final optimality gap (Gap (%)) and number of processed B&B nodes (Nodes) for the original model solved with FICO Xpress when the number of turbines is increased. The same result data is shown for the decomposed model solved with the B&P method in Table 9.5, with additional columns that show the number of schedules generated (Schedules) and the percentage of schedules that were generated by the heuristic labeling algorithm described in Section 7.7.4 (H (%)).

Table 9.4: Results from solving the original model with a commercial solver for instances with an increasing number of turbines.

Instance	Time (s)	Root Obj.	Dual Bound	Gap (%)	Nodes
T2	38	148 405	214 434	0.01	439
T4	22	398 143	525 377	0.00	25
T6	440	1 086 388	1 237 690	0.01	11 224
T8	10 800	761 060	2 000 608	0.11	230 652
T10	10 800	1 030 114	3 421 115	0.29	122 581
T12	10 800	1 769 153	4 298 728	8.76	54 220
T14	10 800	1 811 726	3 054 296	85.61	53 050
T16	10 800	2 100 427	3 778 077	85.61	53 850
T18	10 800	2 768 765	4 584 318	95.18	79 735
T20	10 800	1 828 331	4 447 344	97.69	72 673

Table 9.5: Results from solving the decomposed model with the B&P method for instances with an increasing number of turbines.

Instance	Time	Root Obj.	Dual Bound	Gap (%)	Nodes	Schedules	H (%)
T2	139	213 716	213 745	0.00	4	1 669	67
T4	780	524 007	524 338	0.00	4	6 244	81
T6	10 800	1 156 686	1 158 079	6.24	169	11 577	52
T8	10 800	1 974 083	1 976 299	27.49	76	16 700	49
T10	10 800	2 779 661	2 782 447	22.77	36	19 228	69
T12	10 800	4 217 365	4 219 482	7.51	20	17 536	69
T14	10 800	5 168 978	5 169 917	9.72	17	13 101	67
T16	10 800	6 161 164	6 161 355	84.61	25	16 977	72
T18	10 800	7 260 563	7 261 153	90.65	15	19 200	64
T20	10 800	35 807 532	35 808 024	73.90	19	18 484	65

For the smallest instances we observe that the original model, solved with a commercial solver, outperforms our B&P method. However, for instances with 12 or more turbines, the B&P method is able to obtain better dual and primal bounds, and lower optimality gaps.

There is a leap in optimality gap when the number of turbines is increased beyond 14 in the original model, and 16 in the decomposed model. A closer study of the results reveals that this is due to unfinished turbines in the primal solutions for these instances. There is also a large leap in the final dual bound and root node solution obtained for instance T20 compared to T18 with the B&P method. A closer look at the results for instance T20 revealed that there are 1.6 unfinished turbines in the root node solution. This means that it is not possible to install 20 or more OWTs with the given input data, and that better weather conditions, more vessels, or more time periods are needed to complete their installation. That the optimal solution has unfinished turbines can also explain the decrease in optimality gap for this instance compared to the gap obtained for T16 and T18.

For all the tested instances, the root node objective found by the B&P method is higher than that found for the original model by the commercial solver. This indicates that the decomposed model obtains a tighter LP-relaxation than the original model, as predicted. In Figure 9.1, we show the percentual difference in the root node objective found by the two methods. The difference is calculated as $(r^d - r^c)/r^c$, where r^d is the root node objective obtained by solving the decomposed model with the B&P method, and r^c is the root node objective found by the commercial solver for the original model. For all tested instances with eight or more turbines the improvement is more than 130%. As can be seen from the figure, especially the instances with many OWTs have a large improvement in root node objective.

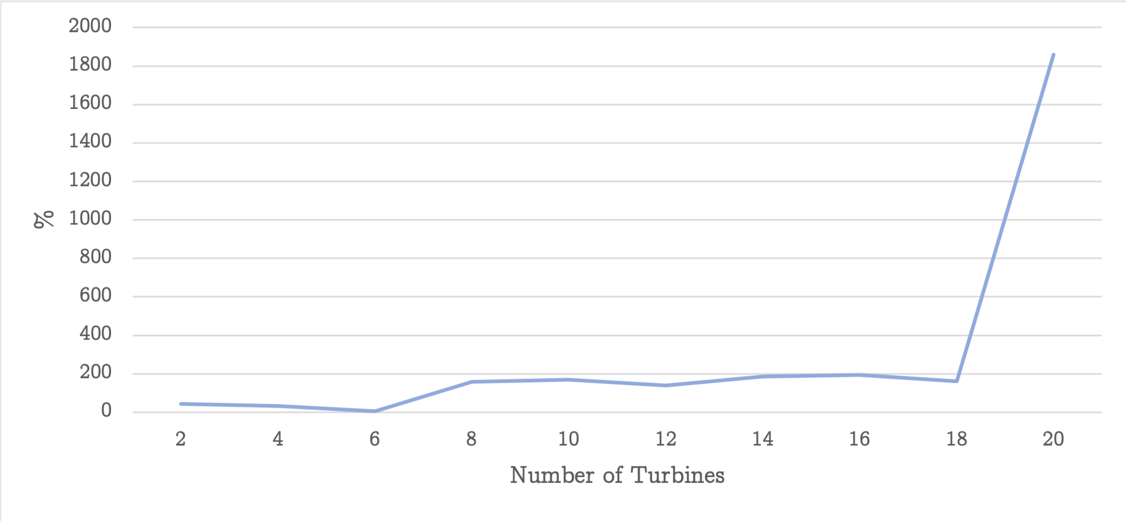


Figure 9.1: Percentual difference in the root node objective found by the B&P method and the commercial solver.

Figure 9.2 shows the percentual difference in the dual bounds obtained from the decomposed model and the original model. The difference is calculated as $(b^d - b^c)/b^c$, where b^d is the final dual bound obtained by solving the decomposed model with the B&P method, and b^c is the final dual bound obtained by the commercial solver for the original model. As can be seen from the figure, the dual bounds for the instances with 14 or more turbines are remarkably improved by the B&P method. The greatest bound improvement is found for instance T20, where we observe more than 700% increase. The very large increase in both dual bound and root node objective for test instance T20 is due to the stronger LP-relaxation of the decomposed formulation, which avoids solutions with unfinished OWTs. This is something that is not achieved by the LP-relaxation of the original model.

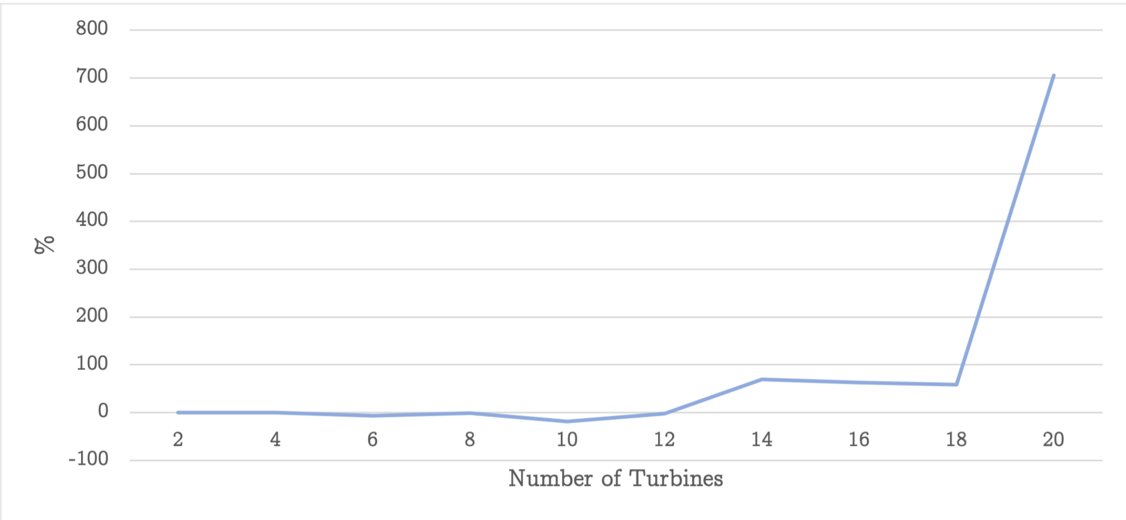


Figure 9.2: Percentual difference in dual bound obtained by the B&P method and the commercial solver.

We observe that there is very little improvement from the root node objective to the final dual bound obtained by the B&P method. This indicates that there is limited improvement in the objective value as the enumeration tree is expanded. Hence, the stronger bounds obtained by the B&P method, compared to the commercial solver, are mainly a result of the better root node solutions. Nonetheless, we also note that the B&P method requires a significantly lower number of nodes also for the instances that are solved to optimality with both methods. For the test instances T2, T4 and T6 the B&P method only processes 4, 4 and 169 nodes, respectively, while the commercial solver must process a total of 439, 25 and 11 224 nodes, respectively, to prove optimality. However, the B&P method uses a lot more computational time, indicating that it is very time consuming to solve each node. At closer inspection, we revealed that it is the subproblems that take up most of the computational time, and that especially the exact labeling algorithm is very time consuming.

There is no obvious trend in the number of generated schedules for the presented instances. However, the average percentage of heuristically generated schedules for the presented instances is only 65%. Thus, there is potential to reduce the computational time of the subproblems further by developing heuristics that are able to identify a larger share of these feasible schedules faster than the exact labeling algorithm.

9.3 Increasing the Number of Scenarios

Tables 9.6 and 9.7 display results for the original model solved with a commercial solver and the decomposed model solved with the B&P method, respectively, when the number of scenarios is increased. Both tables show the following data for each test instance: the total computational time in seconds (Time (s)), final dual bound (Dual Bound), final optimality gap (Gap (%)) and number of processed nodes (Nodes). Additionally, Table 9.7 shows the number of schedules generated during the B&P algorithm (Schedules). Testing of the original model was stopped when the optimality gap did not fall below 90% during the three-hour run time.

The data in Table 9.6 reveals that when the number of scenarios is increased, it becomes more difficult to solve the problem. Though there is some variation, the overall trend is that the dual bounds become weaker, and the optimality gaps increase. The conclusion that an increased number of scenarios complicates the problem can also be drawn based on the data in Table 9.7. However, unlike the results for the original model, the dual bounds from the decomposed model do not necessarily decrease. On the contrary, for both instances T18S4 and T20S4, the dual bound and optimality gap is strengthened compared to test instances T18S2 and T20S2, respectively.

Table 9.6: Results from solving the original model with a commercial solver for instances with multiple scenarios.

Instance	Time (s)	Dual Bound	Gap (%)	Nodes
T2S2	1 108	242 697	0.01	32 158
T2S4	10 800	195 425	48.46	54 003
T2S6	10 800	134 345	93.89	25 903
T2S8	10 800	117 017	80.51	9 269
T4S2	512	606 223	0.01	5 946
T4S4	10 800	433 396	88.64	162 910
T4S6	10 800	458 684	32.89	12 700
T4S8	10 800	382 292	51.78	6 319
T6S2	10 800	1 191 318	0.46	97 201
T6S4	10 800	846 127	32.27	9 974
T6S6	10 800	659 466	61.66	5 639
T6S8	10 800	659 060	81.27	2 292
T8S2	10 800	1 142 722	88.39	61 009
T8S4	10 800	1 215 970	87.67	13 383
T8S6	10 800	905 164	69.90	2 971
T8S8	10 800	684 807	85.53	3 623
T10S2	10 800	2 188 201	36.14	66 603
T10S4	10 800	1 454 911	94.11	10 978
T12S2	10 800	1 932 766	95.38	35 455

Table 9.7: Results from solving the decomposed model with the B&P method for instances with multiple scenarios.

Instance	Time (s)	Dual Bound	Gap (%)	Nodes	Schedules
T2S2	10 800	241 233	0.67	339	5 432
T2S4	10 800	259 206	0.16	101	11 800
T2S6	10 800	268 367	0.31	68	11 212
T2S8	10 800	246 792	0.42	34	16 183
T4S2	10 800	603 836	1.50	120	10 268
T4S4	10 800	635 324	1.40	40	16 294
T4S6	10 800	648 332	3.72	17	21 478
T4S8	10 800	588 211	4.19	8	30 998
T6S2	10 800	1 134 957	5.88	63	12 632
T6S4	10 800	1 115 245	4.36	17	20 656
T6S6	10 800	1 132 623	11.40	7	28 840
T6S8	10 800	1 044 808	11.07	3	35 404
T8S2	10 800	1 897 408	25.61	23	16 569
T8S4	10 800	1 780 836	21.48	8	25 733
T8S6	10 800	1 806 401	18.53	3	35 601
T8S8	10 800	-	100.00	0	46 981
T10S2	10 800	2 688 448	20.81	26	16 995
T10S4	10 800	2 579 733	21.90	3	34 343
T10S6	10 800	-	100.00	0	46 049
T12S2	10 800	3 900 422	13.75	8	24 553
T12S4	10 800	3 661 643	18.07	1	39 821
T12S6	10 800	-	100.00	0	56 071
T14S2	10 800	4 993 777	76.93	4	24 951
T14S4	10 800	-	100.00	0	45 471
T16S2	10 800	6 365 055	94.81	4	25 491
T16S4	10 800	-	100.00	0	29 395
T18S2	10 800	21 978 459	88.43	4	23 635
T18S4	10 800	23 346 642	85.87	1	27 701
T18S6	10 800	-	100.00	0	50 533
T20S2	10 800	55 793 312	81.49	8	26 119
T20S4	10 800	59 499 762	74.39	4	31 444
T20S6	10 800	-	100.00	0	45 583

As there were unfinished turbines in the root node solution for the single scenario instance with 20 turbines, we expect unfinished turbines in the root node solution also for T20S2 and T20S4. This explains why the dual bounds obtained by the decomposed method for these instances are very high compared to the rest. The same behavior is observed for T18S2 and T18S4, indicating that the additional scenarios have weather conditions that make it impossible to complete the installation of all 18 turbines.

Similarly as for when the number of turbines is increased, the results show that the decomposed model again outperforms the original model in terms of finding good dual bounds, and lower optimality gaps when multiple scenarios are introduced. Especially the dual bounds for the instances with six or more turbines show great improvement, but also the test instances with a lower number of turbines should be noted. For instance, the B&P method obtains a gap below 1% for all instances with two turbines, and a gap below 5% for all instances with four turbines. These results are very good compared to those obtained by the commercial solver, where the gaps for the same instances average around 50%. The only test instances for which the commercial solver obtains a lower gap than the B&P method within the time limit are T2S2, T4S2 and T6S2. Furthermore, there are some instances where the B&P method is not able to solve the root node within the time limit. However, based on the trend from the other results, we expect that if given more time, also the dual bound for these instances would be much stronger than those of the original model.

Another observation from the results in Table 9.7, is that there are fewer processed nodes and more generated schedules compared to the results for the single-scenario instances in Table 9.5. The decrease in number of processed nodes is natural, as when the number of scenarios is increased by one, the number of subproblems increases by the number of vessels in the test instance. For the instances tested here, this means four additional subproblems for each added scenario. Hence, more subproblems must be solved in each B&B node, which increases the computational time for each node. In addition to there being more subproblems, the increased number of schedules may be explained by the heuristic labeling algorithm. Based on a closer study of the results, we believe that this algorithm identifies more feasible schedules in the earlier iterations of the subproblems.

Overall, the results indicate that we are able to obtain good dual bounds with the B&P method, but that the primal bounds become weaker when the test instances increase in size, resulting in higher optimality gaps. Hence, to exploit the full potential of the method, a search for better primal solutions is necessary. As mentioned, we observe that the number of processed nodes is limited for the larger instances. If a greater number of nodes were to be processed, this would increase the likelihood of finding nodes with integer solutions, which can result in better primal bounds. Furthermore, with more nodes, more schedules will be generated, which may enhance the performance of our primal heuristic, which combines the existing schedules into the best possible integer solution.

Chapter 10

Concluding Remarks

In this thesis we have studied the design of charter contracts for installation vessels at offshore wind farms. The problem involved decisions on the mix of vessels that should be chartered, the start and end date of these vessels' charter periods, as well as which extension options to include in their contracts. We modeled the problem as a two-stage stochastic mixed integer programming problem, and applied a Dantzig-Wolfe reformulation to exploit the structure of the problem. Then, we developed a branch-and-price algorithm with the extension of a primal heuristic to solve the problem.

The branch-and-price method was tested on a series of test instances with a varying number of turbines and weather scenarios. The results were compared with results from solving the original stochastic formulation with a commercial solver, and revealed that our method outperforms the commercial solver in terms of obtaining better dual bounds and lower optimality gaps for most of the tested instances. The main reason why the branch-and-price method succeeds at obtaining better dual bounds, is because the LP-relaxation is much tighter. The lower optimality gaps are mainly a result of the better dual bounds, but also a result of a successful heuristic search for integer solutions.

We tested two different labeling algorithms for the subproblems, which were formulated as shortest path problems with resource constraints. One algorithm counted the number of completed installation activities of each type along a path, while the other only counted the number of round tours. From this we could observe a trade-off between the time it took to solve each subproblem, and the strength of the solutions obtained by the relaxed master problem. Counting of activities resulted in more complex subproblems due to an increased number of resources in the labeling algorithm compared to the tour counting labeling algorithm. However, the activity counting labeling algorithm gave a much stronger relaxed master problem, and better dual bounds were obtained even though a lower number of B&B nodes were processed within the predefined time limit with this algorithm.

The time limit during testing was set to three hours. As the design of charter contracts for offshore wind farm installation vessels is a strategic planning problem used for planning years in advance of when the installation of the wind farm is to take place, solution time is not a critical factor, and longer solution time may be accepted. Nonetheless, as only the smallest instances are solved to optimality, the solution method needs more refining before real managerial insight can be provided.

Chapter 11

Future Research

In this thesis, we apply a B&P algorithm to solve the problem of designing charter contracts for installation vessels at offshore wind farms. Results from the computational study indicate that the B&P algorithm succeeds at finding good dual bounds. However, efforts aimed at reducing computational time and enhancing computational performance should be considered. In Section 11.1 we identify potential directions for improving the current implementation of the B&P algorithm. Further, the models proposed in this thesis do not capture every aspect of the real-life problem. Relevant model extensions are suggested in Section 11.2 to increase the applicability of the models. However, new model extensions may further increase the computational complexity and should therefore not be considered before an improved solution method is implemented.

11.1 Improvements to the Solution Method

A potential improvement to the solution method is to use a heuristic to pre-generate non-empty schedules that the B&P algorithm can be initialized with. With initial schedules where activities are performed, better dual values will be obtained from the RMP, and the number of iterations needed in the column generation algorithm may be reduced. The subproblems are complex, and solving them takes up a high percentage of the total computational time. Hence, a decreased number of column generation iterations can speed up the convergence of the B&P algorithm.

Another way to reduce the computational effort spent on solving the subproblems, and thereby speeding up the overall solution method, is to apply a bidirectional labeling algorithm. In bi-directional search an initial label is set both at the start and the end of the network, and is extended forwards and backwards, respectively. The benefit of such an approach is that forward and backward labels do not need to be propagated through the entire network. Instead, the labels only need to be extended up to a half-way point.

Then, paths originating from different sides of the network can be combined into complete paths. This limits the overall number of labels created, and reduces the computational burden of the subproblems.

The computational study revealed that for larger instances, the proposed solution method is able to obtain much better dual bounds than those obtained by a commercial solver. However, the optimality gaps are still quite large, as the method is unable to enhance the primal bounds to the same degree. Hence, another possible improvement to the proposed solution method is to develop additional primal heuristics. Identification of high-quality primal bounds can help prune the enumeration tree, and by this reduce the computational effort required to find the optimal solution.

Another observation from our computational study is that there is little improvement in the objective value as the enumeration tree is expanded. This is a common problem for B&P methods, and a possible remedy can be to extend the method to a branch-price-and-cut method, where cutting planes are added to tighten the LP relaxation of the master problem. Another possibility is to investigate the use of other branching strategies. Depth-first search has the advantage of finding feasible solutions early, and can be combined with the best-first strategy by first using depth-first search to find an early integer solution and then switching to best-first search to produce better bounds. Additionally, a variety of heuristics can be implemented to ease the search in the enumeration tree. Possible extensions the current implementation of B&P is to include relaxation induced neighborhood search (RINS) and diving methods to obtain good primal solutions faster.

Finally, to exploit the problem structure in which we have independence among all the subproblems after the decomposition, an extension of solving the subproblems in parallel can be implemented. This will speed up each iteration of the column generation algorithm and can therefore lead to faster convergence.

11.2 Model Extensions

The planning problem related to installation of OWTs is complex. Although the model proposed in this report accounts for many aspects of a real-life installation problem, additional modifications can be implemented to make the model increasingly realistic. As of now, the optimization model only considers uncertainty related to offshore weather conditions. A possible extension to the model could be to include more uncertain parameters in the installation process. Wind turbine technology quickly evolves towards larger and more complex turbines; thus processing times provide a major source of uncertainty and could be considered. Other interesting aspects are breakdowns on logistic resources like vessels or vessel equipment, and transportation or supply chain delays.

A final interesting extension is the implementation of a feeder vessel concept in the model. Offshore installation vessels have high day rates. As a strategy to save costs, less expensive feeder vessels can be used to transfer wind turbine parts from port to the offshore site. Due to good infrastructure and availability of ports, it has so far been unnecessary to use feeder vessels in Europe. However, increasing turbine sizes and wind farms located further offshore is assumed to make this strategy relevant in the near future.

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Appendix

A Minutes, Ulstein International AS

Deputy managing director Jose Jorge Garcia Agis at Ulstein, September 29, 2021

Foundation Installation

Installation of foundations mainly requires the ability to perform heavy lifts. Historically, jack-up vessels have been used for this purpose. However, due to increasing foundation weights, the requirements for lifting operations are changing, and use of other vessel types are emerging. The increased foundation weight is related to increasing turbine sizes. Also, turbines are installed in deeper waters than earlier. Today, the most common vessels for foundation installation are heavy lifters and monohull vessels. Vessels from the Oil & Gas sector have been used, but specially adapted heavy lifters for the wind industry are entering the market. Semi-submersible vessels may also be used, and Ulstein has recently launched a new vessel concept, Alpha Lift, which can perform heavy lift crane operations with the main deck submerged in water.

Turbine Installation

While there is usually no need to perform high lifts for foundation installations, installation of turbine components requires vessels with the ability to perform higher and lighter lifts due to the increased turbine sizes. To meet these requirements, a vessels concept known as WTIV has been developed. These are specially adapted jack-up vessels for installation of wind turbines with longer support legs and greater crane height.

Cable Laying and Burial

Two types of cables are needed when installing offshore wind farms: inter-array cables and export cables. The inter-array cables connect the turbines internally at the wind farm. These cables are often laid before the turbines are installed. A vessel with an Remotely Operated Vehicle (ROV) will then be needed to connect the cables to the foundations, as the cables are too heavy for divers to perform these operations. Cable laying can be performed in parallel with other operations such as foundation installation. Today, the availability of vessels decides when the cable laying activities are performed. Export cable laying is a parallel project and can be performed independently of other operations at the wind farm.

Weather Effects

Weather conditions affect installation operations. Most vessels have an upper limit for the specific wave height that they can operate in. For instance, dynamically positioned monohull vessels cannot perform lifting activities for Significant Wave Height (H_S) greater than 2 meters. Semi-submersible vessels will be able to perform lifts at greater wave heights than this.

Jack-up vessels are sensitive to both wave heights and wind speed, depending on the activities to be performed. The vessels cannot jack up/down for H_S greater than 1.5 due to stress on the support legs. The time spent jacking up and down is approximately 6 hours for each direction. When the vessel is jacked up, its performance will no longer be dependent on H_S, but on wind speed. The vessel cannot be in a jacked-up position for wind speeds exceeding approximately 10 m/s. However, it should be noted that lifts of turbine parts at wind speeds higher than 10 m/s will normally not be performed regardless of vessel capabilities, as it becomes difficult to position components accurately due to movements in the crane.

Vessel Capacities

Turbine sizes increase with their ability to produce power. Turbines recently installed have been in the range of 6-8 MW. Most turbine installation vessels can carry six turbines of this size. In the near future, turbines in the size range 10-15 MW will enter the market, and the current fleet of installation vessels will only be able to carry 2-4 turbines of this size per trip. Normally, it takes one day to install each turbine. Hence, the increased turbine size will increase the number of trips needed between the offshore site and the port from approximately one trip per week to 2-3 trips per week.

B Minutes, Clarksons Platou AS

Managing Director for Offshore Renewables Frederik C. Andersen at Clarksons, October 15, 2021

Vessel Contracts

Charter contracts are entered two to four years in advance of the installation phase. This is to ensure availability of the required vessels. The charterer poses a request for a tender which specifies the requirements of the contract. Then all providers compete to make the winning tender offer that fulfils these requirements. In other words: the charterer can decide exactly what they want and can choose between several contractual offers satisfying their demands. Nonetheless, the requirements will affect the offering prices.

There are two main types of contracts used in offshore wind installation: fixed price contracts and time charter contracts. Time charter contracts are most common.

Fixed Price Contracts

In a fixed price contract, the contract parties agree upon a fixed price for a specified scope, e.g. to install 50 turbines. The price and time frame of the contract is defined by the offerer's estimates of how much time and money they will spend per turbine, with some margins added.

Time Charter Contracts

In a time charter contract the charterer agrees to pay a daily rate to rent the vessel for a predefined number of days. In addition to the predefined time period, the contract can include options to extend the contract duration if necessary. The options can be of varying number of days and can have separate day rates. Multiple options can be included in the same contract. If the charterer decides to exercise their option, they agree to pay for the entire option period.

Contractual options are always in favor of the charterer. The shipowner may demand higher day rates if more options are requested, however, if the price is too high he might not win the contract. The charterer must notify the shipowner in advance about whether or not they want to exercise their options. This is to minimize risk for the shipowner, so that they can plan and schedule vessels for upcoming projects. The length of the option affects how far in advance notification should be given. The exact option terms are an object of negotiation.

In addition to day rates the contract includes a "project cost". This cost can either be included by increasing the day rates, or by adding a fixed cost to the contract. The project cost covers special tasks the shipowner has to do to prepare the vessel for the contract. This may include engineering work, deck preparations and a mobilization fee to transport the vessel to the correct start location. Costs for vessel crew are included in the contract, but if the charterer wants to bring their own personnel, they have to cover their accommodation costs. The charterer must also pay separately for variable costs such as fuel and port fees.

The Role of Business Relations

The offshore business is largely based on relationships. The different parties are dependent on building trust and good relations to ensure future contracts and business opportunities.

Seasonality

It is common to plan for installation of offshore wind farms to take place in the summer due to better weather conditions. This makes the demand for installation vessels prone to seasonality, and the huge demand makes the day rates peak during the summer months. An interesting topic is thus to find ways to perform installation activities during winter, when there is a lot of free capacity and day rates are lower.

Installation Sequence

Today, the common installation sequence is to install array cables after foundations, but before turbine towers. Turbines are done lastly because they are very capital intensive, and it is convenient to connect the pre-installed cables immediately after installation. Array cable installation takes about two days per turbine. Installation timer per turbine is on average 2.5 days.

C Deterministic Equivalent of the Original Model

Sets

$\mathcal{V}$ - Set of vessels, indexed by v

$\mathcal{T}$ - Set of time periods, indexed by t

$\mathcal{A}$ - Set of installation activities, indexed by a

$\mathcal{L}_v$ - Set of loading sets for vessel v, indexed by l

$\mathcal{I}_v$ - Set of tours for vessel v, indexed by i

$\mathcal{O}$ - Set of extension options, indexed by o

$\mathcal{S}$ - Set of scenarios, indexed by s

Parameters

C_v^F - Fixed charter cost for vessel v

C_v^V - Variable charter cost for vessel v per time period in its fixed charter period

D_{lv}^L - Duration of loading loading set l for vessel v

D_v^S - Duration of sailing from port to site or vice versa for vessel v

F_{lv} - 1 if vessel v can carry loading set l, 0 otherwise

K_v - Minimum length of a tour with vessel v

L_o - Length of option o given in time periods

N^T - Total number of turbines to be installed

N_{al}^L - Number of times installation activity a can be performed by a vessel loaded with loading set l

M_{atvs}^1 - Big-M used in the first round trip constraint for the combination of activity a, time period t and vessel v under weather scenario s

M_{av}^2 - Big-M used in first activity constraint for activity a and vessel v

M_v^3 - Big-M used in third activity constraint for vessel v

P_{ov}^F - Price for including option o in the contract for vessel v

P_{ov}^E - Price for exercising option o for vessel v

P^L - Penalty cost for lost income

P^U - Penalty cost for unfinished OWTs

T_{atvs} - Number of time periods it takes vessel v to complete activity a if it is started in time period t under weather scenario s

p_s - Probability of realization of scenario s

Variables

α_v	-	1 if vessel v is chartered in the project, 0 otherwise
β_{tv}	-	1 if vessel v is on fixed charter in time period t, 0 otherwise
s^C_{tv}	-	1 if charter of vessel v starts in time period t, 0 otherwise
e^C_{tv}	-	1 if charter of vessel v ends in time period t, 0 otherwise
μ_{ov}	-	1 if option o is included in the contract for vessel v, 0 otherwise
e^P_s	-	Final time period of the installation process in scenario s
u_s	-	Number of unfinished OWTs in scenario s
s^T_{itvs}	-	1 if first period of tour i for vessel v is in time period t in scenario s, 0 otherwise
s^A_{aitvs}	-	1 if time period t is the first period of operation o in tour i for vessel v and installation activity a is performed in scenario s, 0 otherwise
δ_{ilvs}	-	1 if loading set l is loaded onto vessel v for tour i in scenario s, 0 otherwise
γ_{ovs}	-	1 if option o is exercised for vessel v in scenario s, 0 otherwise

Objective

$$\begin{aligned} \min \quad & \sum_{v\in\mathcal{V}} C^F_v \alpha_v + \sum_{t\in\mathcal{T}}\sum_{v\in\mathcal{V}} C^V_v \beta_{tv} + \sum_{o\in\mathcal{O}}\sum_{v\in\mathcal{V}} P^F_{ov}\mu_{ov} \\ & + \sum_{s\in\mathcal{S}}\sum_{v\in\mathcal{V}}\sum_{o\in\mathcal{O}} p_s(P^E_{ov}\gamma_{ovs} + P^L e^P_s + P^U u_s) \end{aligned} \tag{C.1}$$

Charter Constraints

$$\sum_{t\in\mathcal{T}} s^C_{tv} = \alpha_v, \qquad v \in \mathcal{V}, \tag{C.2}$$

$$\sum_{t\in\mathcal{T}} s^C_{tv} - \sum_{t\in\mathcal{T}} e^C_{tv} = 0, \qquad v \in \mathcal{V}, \tag{C.3}$$

$$\sum_{t\in\mathcal{T}} t e^C_{tv} - \sum_{t\in\mathcal{T}} t s^C_{tv} \geq K_v \alpha_v, \qquad v \in \mathcal{V}, \tag{C.4}$$

$$\beta_{tv} = \beta_{(t-1)v} + s^C_{tv} - e^C_{(t-1)v}, \qquad t \in \mathcal{T}\setminus\{1\}, v \in \mathcal{V}, \tag{C.5}$$

$$s^C_{tv} = \beta_{tv}, \qquad t = 1, v \in \mathcal{V}, \tag{C.6}$$

$$\mu_{ov} \leq \alpha_v, \qquad o \in \mathcal{O}, v \in \mathcal{V}. \tag{C.7}$$

Round Trip Constraints

$$\sum_{t\in\mathcal{T}} s^T_{itvs} \leq \alpha_v, \qquad v\in\mathcal{V}, i\in\mathcal{I}_v, s\in\mathcal{S}, \quad \text{(C.8)}$$

$$s^T_{itvs} \leq \beta_{tv}, \qquad v\in\mathcal{V}, i=1, s\in\mathcal{S}, \quad \text{(C.9)}$$

$$\sum_{t\in\mathcal{T}} s^T_{itvs} \geq \sum_{t\in\mathcal{T}} s^T_{(i+1)tvs}, \qquad v\in\mathcal{V}, i\in\mathcal{I}_v\setminus\{|\mathcal{I}_v|\}, s\in\mathcal{S}, \quad \text{(C.10)}$$

$$(t+T_{atvs}+D^S_v)s^A_{aitvs} \leq \sum_{t\in\mathcal{T}} ts^T_{(i+1)tvs} + M^1_{atvs}(1-\sum_{t\in\mathcal{T}} s^T_{(i+1)tvs}), \quad a\in\mathcal{A}, t\in\mathcal{T}, v\in\mathcal{V}, i\in\mathcal{I}_v\setminus\{|\mathcal{I}_v|\}, s\in\mathcal{S}. \quad \text{(C.11)}$$

Activity Constraints

$$\sum_{\tau\in\mathcal{T}} \tau s^T_{i\tau vs} + \sum_{l\in\mathcal{L}} (D^L_{lv}+D^S_v)\delta_{ilvs} \leq ts^A_{aitvs} + M^2_v(1-s^A_{aitvs}), \quad a\in\mathcal{A}, t\in\mathcal{T}, v\in\mathcal{V}, i\in\mathcal{I}_v, s\in\mathcal{S}, \quad \text{(C.12)}$$

$$\sum_{a\in\mathcal{A}} \sum_{t'=t-T_{atvs}+1}^{t} s^A_{ait'vs} \leq 1, \qquad t\in\mathcal{T}, v\in\mathcal{V}, i\in\mathcal{I}_v, s\in\mathcal{S}, \quad \text{(C.13)}$$

$$\sum_{a\in\mathcal{A}} \sum_{t\in\mathcal{T}} s^A_{aitvs} \leq M^3_v \sum_{t\in\mathcal{T}} s^T_{itvs} \qquad v\in\mathcal{V}, i\in\mathcal{I}_v, s\in\mathcal{S}, \quad \text{(C.14)}$$

$$\sum_{t\in\mathcal{T}} s^T_{itvs} \leq \sum_{a\in\mathcal{A}} \sum_{t\in\mathcal{T}} s^A_{aitvs}, \qquad v\in\mathcal{V}, i\in\mathcal{I}_v, s\in\mathcal{S}. \quad \text{(C.15)}$$

Loading Constraints

$$\sum_{l\in\mathcal{L}_v} \delta_{ilvs} \leq 1, \qquad v\in\mathcal{V}, i\in\mathcal{I}_v, s\in\mathcal{S}, \quad \text{(C.16)}$$

$$\sum_{t\in\mathcal{T}} s^A_{aitvs} \leq \sum_{l\in\mathcal{L}_v} N^L_{al}\delta_{ilvs}, \qquad a\in\mathcal{A}, v\in\mathcal{V}, i\in\mathcal{I}_v, s\in\mathcal{S}. \quad \text{(C.17)}$$

Project Plan Constraints

$$(t + T_{atvs} + D_v^S - 1)s_{aitvs}^A \leq \sum_{\tau \in \mathcal{T}} \tau e_{\tau v}^C + \sum_{o \in \mathcal{O}} L_o \gamma_{ovs}, \qquad a \in \mathcal{A}, t \in \mathcal{T}, v \in \mathcal{V}, i \in \mathcal{I}_v, s \in \mathcal{S}, \tag{C.18}$$

$$\sum_{v \in \mathcal{V}} \sum_{i \in \mathcal{I}_v} \sum_{t'=1}^{t} s_{ait'vs}^A \leq \sum_{v \in \mathcal{V}} \sum_{i \in \mathcal{I}_v} \sum_{t'=1}^{t-T_{(a-1)t'vs}} s_{(a-1)it'vs}^A, \qquad a \in \mathcal{A} \setminus \{1\}, t \in \mathcal{T}, s \in \mathcal{S}, \tag{C.19}$$

$$\sum_{t \in \mathcal{T}} \sum_{v \in \mathcal{V}} \sum_{i \in \mathcal{I}_v} s_{aitvs}^A \geq N^T - u_s, \qquad a = |\mathcal{A}|, s \in \mathcal{S}, \tag{C.20}$$

$$e_s^P \geq (t + T_{atvs} + D_v^S - 1)s_{aitvs}^A, \qquad a \in \mathcal{A}, t \in \mathcal{T}, v \in \mathcal{V}, i \in \mathcal{I}_v, s \in \mathcal{S}, \tag{C.21}$$

$$\gamma_{ovs} \leq \mu_{ov}, \qquad o \in \mathcal{O}, v \in \mathcal{V}, s \in \mathcal{S}. \tag{C.22}$$

Binary and non-negativity constraints

$$\alpha_v \in \{0, 1\}, \qquad t \in \mathcal{T}, v \in \mathcal{V}, \tag{C.23}$$

$$\beta_{tv} \in \{0, 1\}, \qquad t \in \mathcal{T}, v \in \mathcal{V}, \tag{C.24}$$

$$s_{tv}^C \in \{0, 1\}, \qquad t \in \mathcal{T}, v \in \mathcal{V}, \tag{C.25}$$

$$e_{tv}^C \in \{0, 1\}, \qquad t \in \mathcal{T}, v \in \mathcal{V}, \tag{C.26}$$

$$\mu_{ov} \in \{0, 1\}, \qquad o \in \mathcal{O}, v \in \mathcal{V}, \tag{C.27}$$

$$e_s^P \geq 0, \text{ integer}, \qquad s \in \mathcal{S}, \tag{C.28}$$

$$u_s \geq 0, \text{ integer}, \qquad s \in \mathcal{S}, \tag{C.29}$$

$$s_{itvs}^T \in \{0, 1\}, \qquad t \in \mathcal{T}, v \in \mathcal{V}, i \in \mathcal{I}_v, s \in \mathcal{S}, \tag{C.30}$$

$$s_{aitvs}^A \in \{0, 1\}, \qquad a \in \mathcal{A}, t \in \mathcal{T}, v \in \mathcal{V}, i \in \mathcal{I}_v, s \in \mathcal{S}, \tag{C.31}$$

$$\delta_{ilvs} \in \{0, 1\}, \qquad v \in \mathcal{V}, l \in \mathcal{L}_v, i \in \mathcal{I}_v, s \in \mathcal{S}, \tag{C.32}$$

$$\gamma_{ovs} \in \{0, 1\} \qquad o \in \mathcal{O}, v \in \mathcal{V}, s \in \mathcal{S}. \tag{C.33}$$

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