Master's thesis

NTNU
Norwegian University of Science and Technology
Faculty of Economics and Management
NTNU Business School

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Trading based on the dual-beta model: Evidence from the Norwegian Stock Market

Master's thesis in Economics and Business administration
Supervisor: Becker, Denis M.

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Sammendrag

Tradisjonelle finansielle målinger av risiko er begrenset til variansbaserte metoder, og det vanligste måleverktøyet er beta. Et problem med tradisjonell beta-beregning er at den vektes likt til både opp- og nedside variansen. Å bryte ned beta i opp- og nedside gir investorer muligheten til å mer intelligent bygge risiko inn i en portefølje. Tidligere studier viser at en dual-beta portefølje gir svært gode resultater for det amerikanske aksjemarkedet (Chong, 2022) (Guy, 2014). Hensikten med denne oppgaven er å utvikle en investeringsstrategi ved hjelp av opp- og nedside betaestimater, og se om denne strategien fungerer på det norske akjsemarkedet.

For å analysere problemstillingen bruker vi markedsdata fra 30 ulike selskaper på Oslo børs fra 1. april 2002 til 1. april 2022. Benchmarken vi skal teste den tosidig-beta porteføljen mot er OSEBX (Oslo Børs Hovedinndeks). For å undersøke nærmere om tosidig beta porteføljen er en konkurransedyktig investeringstrategi har vi også inkludert en jevnt vektet portefølje bestående av alle de 30 valgte aksjene og en portefølje basert på Markowitz porteføljeoptimeringsteori. Tosidig beta porteføljens ytelse vil først bli visuelt vurdert over økonomiske syskluser og deretter bli vurdert ved hjelp av Fama og French sin fem-faktor modell med momentfaktor. For analyse vil vi først ta i bruk en tosidig beta portefølje med $\beta^+ > 1$ (oppside beta) og $\beta^- < 1$ (nedside beta) basert på daglig aksjedata (i likhet med tidligere forskning i det amerikanske aksjemarkedet). Vi vil også analysere effekten av å justere betaterkslene og endre avkastningsintervallene fra daglig til ukentlig og månedlig.

Hovedfunnene fra analysen viser til at en tosidig beta portefølje med $\beta^+ > 1$ og $\beta^- < 1$ basert på daglig avkastningsintervall presterer dårligere enn benchmarken, mye grunnet lavt antall investeringer. Med justerte betaterskler og med månedtlig aksjedata ser vi at tosidig beta porteføljen overgår benchmarken for hele perioden, samt for de fleste underperioder. Den jevnt vektede porteføljen, som kan sees på som vell diversifisert, presterer på linje med disse justerte beta-porteføljene. Dette styrker teorien om et effesient marked. Et utvalg av stort sett kjente selskaper som med viten har overlevd hele analyseperioden, kan være en grunn til de gode resultatene til både den jevnt vektede porteføljen og den tosidige beta-porteføljen. Våre funn tyder på at en tosidig beta portefølje kan fungere på det norske markedet, men med usikkerhet om den skaper noe mer ekstraordinær avkastning enn en diverisifsert portefølje.

Abstract

Traditional financial risk measurements are limited to variance-based methods, and the most common measurement tool is beta. A problem with traditional beta calculation is that it is weighted equally to both upside and downside variance. The ability to break down the beta to upside and downside gives investors the ability to build risk into a portfolio more intelligently. Earlier studies show that a dual-beta portfolio performs very well for the US stock market (Chong, 2022) (Guy, 2014). This thesis aims to develop an investment strategy using upside- and downside beta estimates and sees if this strategy performs well in the Norwegian stock market.

To analyse the problem, we use market data from 30 different companies on the Oslo Stock Exchange from April 1 2002, to April 1 2022. We will test the dual-beta portfolio against OSEBX (Oslo Børs Main Index). We have included an equally weighted portfolio consisting of all 30 selected stocks and a portfolio based on Markowitz's portfolio optimisation theory to investigate further whether the dual-beta portfolio is a competitive investment strategy. The performance of the dual-beta portfolio will first be visually assessed over financial cycles and then assessed using Fama and French's five-factor model with momentum factor. For analysis, we will first use a dual-beta portfolio with $\beta^+ > 1$ (upside beta) and $\beta^- < 1$ (downside beta) based on daily stock data (similar to previous research in the US stock market). We will also analyse adjusting the beta thresholds and changing the return intervals from daily to weekly and monthly.

The main findings from the analysis indicate that a dual-beta portfolio with $\beta^+ > 1$ and $\beta^- < 1$ based on daily return interval performs worse than the benchmark, mainly as a result from too few investments. With adjusted beta thresholds and monthly stock data, the dual-beta portfolio exceeds the benchmark for the entire period and most sub-periods. The equally weighted portfolio, which is considered to be well-diversified, performs in line with the adjusted beta portfolios. These results strengthen the market efficiency theory. A selection of mostly well known companies with the knowledge of their undisputed survival throughout the whole analysis period, may be a reason for the good results shown by the equally weighted- and adjusted beta-portfolio. Our findings suggest that a dual-beta portfolio may work in the Norwegian market but with uncertainty in its capability to outperform a well-diversified portfolio.

Preface

This Master thesis is written as a part of our Master of Science in Economics and Business administration, with a major in Business Analytics. Our motivation for this thesis came from our interest in the financial market combined with data analysis. First, we would like to thank our supervisor Denis M. Becker for excellent guidance and constructive feedback throughout the process. We would also like to thank Stein Frydenberg for answering our questions regarding factor modelling.

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1 Introduction

This thesis will investigate how the use of upside and downside beta in portfolio theory affects portfolio selection. In 1952, Markowitz established a method where risk-averse investors could generate an efficient portfolio with an optimal balance between expected returns and acceptable risk (Markowitz, 1952). He defined risk as variance or standard deviation and thus formulated a quadratic optimisation problem. In later studies and practice in finance, several different ways have been proposed to define risk when trading stocks (Pindyck, 1983). Our analysis will study the effect of splitting risk into upside and downside.

It can be assumed that investors are more sensitive to losses than it appears (Tversky & Kahneman, 1991). This analysis aims to create a more sensitive portfolio to upswings and less sensitive to downswings. If it is possible to isolate stocks with these characteristics, portfolios that better capture an investor's risk tolerance and reduce volatility can be created. To solve this, we will use the dual-beta model. The dual-beta model tests the relation between beta and realised returns by segregating market returns into up – and down-market returns.

Numerous studies use the Capital Asset Pricing Model (CAPM) beta statistics to estimate asset returns. Much less research has been published regarding portfolio theory's upside- and downside beta. To the best of our knowledge, Chong (2022) and Guy (2014) are some of the few studies that have analysed this topic. Both Chong (2022) and Guy (2014) articles look at the US stock market. Their findings show that the dual-beta portfolio outperforms the benchmarks. Our thesis is different because we want to investigate if this trading strategy can work in the Norwegian market. In addition to our analysis, we will evaluate the trading strategy with different upside and downside beta thresholds. We will also study the impact of change in upside and downside beta values created with daily, weekly and monthly data.

This master thesis is divided into six parts. Part 2 presents theory for the thesis, where we look at modern portfolio theory, traditional financial theory of risk, upside and downside betas and the Fama-French five-factor model. In part 3, the data are presented using descriptive statistics. Part 4 deals with the empirical methods that have been chosen. In part 5, we present the analysis results using the dual-beta model. Finally, there is a conclusion in part 6, where we consider the main findings from the analysis.

2 Theory

2.1 Modern Portfolio Theory

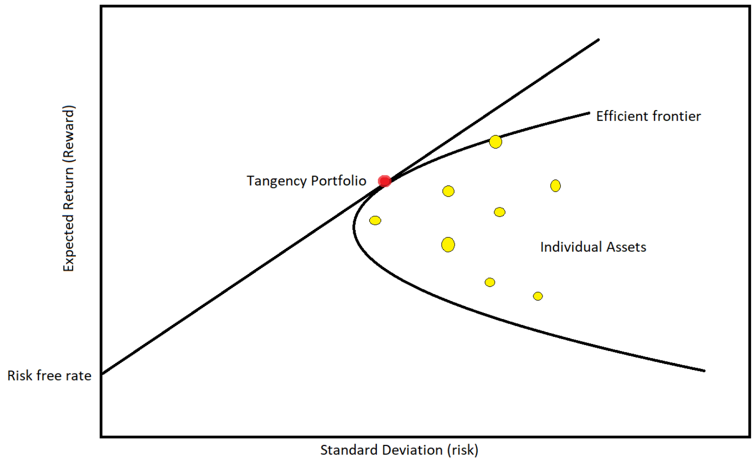
Markowitz portfolio optimization

Harry Markowitz presented in 1952 his theory on optimal portfolio selection, which laid the foundation for what is now referred to as Modern Portfolio Theory (MPT). Essentially, MPT is an investment framework for selecting and constructing a portfolio to maximise expected returns of the portfolio while minimising the portfolio risk (Fabozzi et al., 2002). Initially, Markowitz's portfolio theory gained relatively little interest, but in time have become a strongly adopted theory by financial investors (Fabozzi et al., 2002). Later, the theory was further developed, mainly by William Sharpe, related to developing the Capital Asset Pricing Model (CAPM) and the risk-adjusted measurement, the Sharpe ratio. Although investors typically use the theory to increase their return while keeping risk at an optimal level, it can also create an optimal portfolio based on the investor's willingness to take risks. The optimal portfolio combinations, with different expected returns and risks, align on the efficient frontier. An essential factor of understanding portfolio risk is that the total risk of a portfolio not only consists of the sum of the risk of the assets. Instead the total risk is affected by the correlation between assets, meaning how they depend on each other.

The efficient frontier

The efficient frontier represents the optimal amount of securities in a portfolio with the maximum expected return with a given level of risk. A trait of all combinations of portfolios on the frontier is that they are mean-variance-efficient (Mangram, 2013). The efficient frontier is usually presented in a graph showing all possible portfolio combinations with expected return on the y-axis and risk / standard deviation on the x-axis. The optimal portfolios then appear in a curve above the majority of the portfolio combinations. The optimal portfolio with the highest Sharpe ratio is located where the capital allocation line(CAL) is drawn from the risk-free rate and intercepts the frontier like a tangent. The Sharpe ratio is equivalent to the slope of the CAL.

Figure 1: The efficient frontier



Note: The figure shows possible combinations of all assets in yellow. The efficient frontier is formed around the possible combinations. The highest Sharpe ratio portfolio is marked in red, the CAL is drawn as a tangent on the efficient frontier from the risk free rate. Risk on x-axis and reward on y-axis.

Sharpe ratio

Sharpe argued in 1966 that when evaluating a mutual fund's performance, there should be a common way to analyse a fund's performance based on the amount of risk it has taken upon itself (Sharpe, 1966). The risk-reward dilemma is essential in understanding the Modern Portfolio Theory, by finding an efficient portfolio with the highest reward compared to the amount of risk taken. Since investors have different relationships to risk, would it have been great with a performance measurement that is indifferent to the investor's kind of risk relationship. Sharpe presented the Sharpe ratio, including the expected rate of return, risk-free rate and the predicted variability of risk, given by the standard deviation of return. The Sharpe ratio gives the relationship between the excess return, from the expected return minus the risk-free rate, divided by the predicted risk. A greater value means a higher excess return compared to the risk taken and is ultimately a better performance no matter what kind of risk relationship the investor wanted.

Efficient market model

A fundamental assumption of the security markets states that security prices fully reflect all available information (Fama, 1970). Eugene F. Fama presented in 1970 a paper reviewing the theoretical and empirical literature on the efficient markets model. The theory suggests that all prices are "fair" to the extent that any security information is already considered in making the price. Fama (1970) stated that a market in which prices always fully reflect available information is called "efficient". If this assumption holds, there would be no point in analysing security markets and attempting to beat the market. The empirical research regarding the efficient market model typically distinguish particular subsets of available information, and makes it possible to falsify the model on different levels. The first subset is called the weak form, and the information of interest is just past price histories. The next level is called semi-strong form and concerns the speed of price adjustment to other publicly available information (e.g., announcements of the stock split, annual reports, new security issues, etc.). Strong form suggests that all information is reflected in the price, including the case of an individual obtaining monopolistic access to some information and still not receiving higher expected trading profits than others. The stronger the level, the harder it is for the hypothesis to be approved. This thesis tests the efficient market model in its weak form, since we only include historical price data in our strategy.

2.2 Traditional Financial Theory of Risk

Any investment involves risk. Taking risk in the stock market means accepting that the market fluctuates daily and in some periods more than others. Risk is defined as the chance that an outcome or investment's actual gains will differ from an expected outcome or return (Chen, 2020). In general, the greater the risk an investor is willing to take, the greater the potential return.

One of the basic principles in modern portfolio theory is that volatility decreases when the number of assets increases. This is called diversification and means that you spread investments to avoid unreasonably high risk from one asset. There are two types of risk: systematic (market-related) and unsystematic (company-specific), which make up the portfolio's risk. Systematic risk is the risk related to movements in the market and not a single asset. Typical factors that affect the systematic risk are macroeconomic factors such as inflation and interest rates.

Unsystematic risk means the risk that is unique to a particular asset. This risk can be diversified away by investing in more assets. Since the individual asset's unsystematic risk is independent of each other, will the unsystematic risk converge to zero when more assets are added to the portfolio. Studies show that only a slight increase in the number of invested stocks will reduce the unsystematic risk considerably. Fisher and Lorie (1970) found that by holding 32 random stocks, the portfolio has reduced 95 % of the unsystematic risk and has therefore become a rule of thumb when reviewing the diversification of a portfolio.

Nevertheless, risk can also be defined in other ways than as variance or standard deviation. For example, in portfolio theory, risk can be defined by portfolio loss, where the investor determines the limit for the maximum loss that can be accepted (Chen, 2020). Given investors' risk aversion levels, differentiating between upside and downside risk may improve portfolio construction methodology (Guy, 2014). Upside risk is the stock return volatility in periods where the benchmark (for this thesis, OSEBX) returns are positive, and downside risk is stock return volatility where the benchmark returns are negative. To capture these new risks, beta is used.

2.3 Beta: Upside and Downside Betas

Beta tells how a stock moves compared to the rest of the market/a benchmark. In this case, the market is defined as the Oslo stock exchange main index (OSEBX). An asset is considered riskier if the asset has a higher level of beta and vice versa. For example, if a stock has a beta of 1,10, for every 1 percent move in the benchmark, the stock will move 1,10 percent. High beta stocks are commonly labelled as those with historically more volatility and, therefore, more perceived risk.

There are some criticisms of beta as a predictive tool in the stock market. First, it uses historical data because it looks backwards when calculating it. The past is not necessarily a guide to the future and could be dangerous to use. Another problem with beta is that there are different ways to calculate it. Should it be calculated using two, five or ten years of historical data? Furthermore, what kind of return interval gives the most precise measures? Another weakness is that beta only looks at stock risk relative to what the rest of the market is doing. For example, it does not give information about a specific sector. A final issue with traditional beta calculation is that it is equally weighted to both upside and downside variance. A stock with more volatility during market downturns versus one with more volatility during market upswings can theoretically generate similar betas (Guy, 2014).

It would be ideal for an investor if a stock had significant positive and low negative sensitivity. To complete this, we introduce upside and downside betas. Upside beta looks at a stock's behaviour when the benchmark return is positive. Therefore, upside beta is calculated on periods when the market has an upward trend (market return greater than zero). In other words, upside beta measures stock sensitivity to market raises. As an investor, you should pick stocks with high upside beta, which can help profit during a market upswing. Downside beta is the stock beta measured for periods when the benchmark return is negative. As an investor, you should try to construct a portfolio by minimising downside beta, which can help maintain value in times of market decline.

2.4 Return interval

One crucial choice when analysing financial data is to choose the frequency of the return intervals. When estimating the beta coefficients, the results may vary greatly when changing the return interval from daily to weekly or monthly, while the estimation period remains the same. According to Hawawini (1983), the shift in beta value is mainly caused by the intertemporal relationship between the daily returns of individual stocks and the market. This means some stocks may move the day after the majority of the market, and some may lead the movement (Hawawini, 1983). Previous research has shown that the leading cause of this intertemporal cross-correlation is friction in the trading process, causing a delay between the security prices and new information (Cohen et al., 1980). Using the daily changes in stock return in the beta estimating may cause imprecise results if some of the securities often lag or stay in front of the general market. Expanding the return interval will remove this daily lagging effect. The greater the expansion, the less lagging effect occurs. When using weekly data, most of the intertemporal cross-correlation is removed. Expanding to monthly return intervals will remove the daily lagging effect.

Hawawini's research also found a more straightforward way of predicting the shift in beta when changing the return interval. When the return interval expands, securities with a high market value of shares outstanding (MVSO) relative to the market average will experience a decreasing beta shift. Likewise, securities with low MVSO may experience an increasing beta value. Concluding that when using daily return intervals, securities with high market value may appear riskier than they are. Likewise, securities with low market value may appear less risky (Hawawini, 1983). How this impacts both upside and downside beta is uncertain, but the effects may reveal themselves in the analysis.

2.5 Fama-French five-factor model with momentum factor

A different approach for understanding portfolio performance is by using a factor model. When testing the portfolio against a simple one-factor model, such as CAPM (Sharpe, 1964), (Treynor, 1962), (Lintner, 1969) (Mossin, 1966), the portfolio is tested for its risk and alpha against the market premium. One portfolio may seem beneficial and return a significant alpha, but the model may not tell us the whole picture. What if the investor took upon himself more risk than detected by the model and simply invested in a more profitable part of the market. By adding more factors, one may achieve the portfolio's actual performance. Fama and French presented 1993 their three-factor model, adjusted later by (Carhart, 1997), adding the momentum factor. The model was later superseded by Fama and French's [2015] five-factor model, still missing the momentum factor. By asserting the similar research by (Chong, 2022), our preferred and implemented model includes the Fama and French five-factor model incorporated with a momentum factor.

Fama and French (1993) concluded their research that factors such as size (measured by market equity) and value (measured by B/M (Book to market ratio)) succeed in giving additional explaining power to the market model. Therefore, their three-factor model included the original market excess return factor and the difference between the returns on diversified portfolios of small and big stocks - SMB, and high and low B/M stocks – HML. Carhart (1997) later proposed the momentum factor – MOM, asserting a relationship between the winners (losers) and excessive positive (negative) market return. Fama and French (2015) later added the difference between stocks with robust and weak profitability – RMW, and stocks of low and high investment firms, meaning conservative minus aggressive investment – CMA.

The Fama and French factors used in the regression are generously provided by Kenneth French and mimic the European market (French, 2022). Later tests by Fama and French (2017) found that the investment factor CMA is redundant in the European market, which may result in no significant effect of this factor on the portfolio. When analysing Norwegian stocks, the Fama-French five factor model fails to explain all variation in expected returns, but the model still has explanatory value (Bakken, 2019). As found by Bakken (2019), the model can be expected to give an explanatory degree (R^2) between 32% and 90%.

3 Data

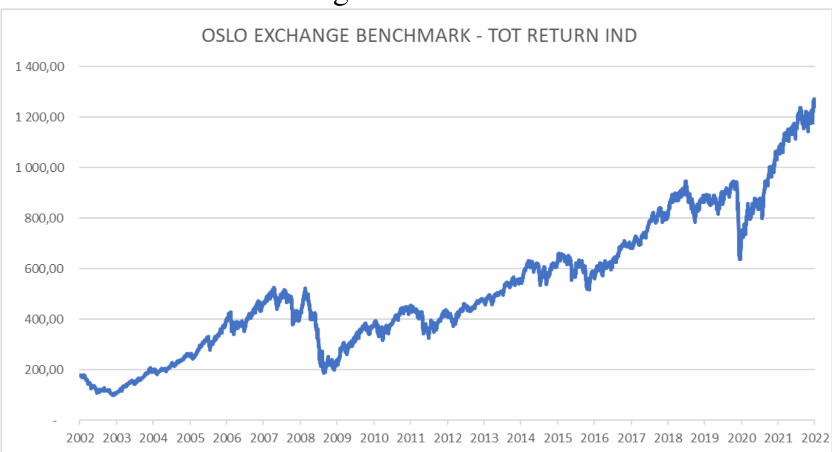
We use market data of thirty different stocks exchanged on the Oslo stock exchange from April 1, 2002 until April 1, 2022. We have downloaded daily, weekly and monthly historical data from the Thomson Reuters datastream (EIKON) for the chosen stocks. The stocks we have chosen to analyse are selected with the following criteria, listed on the Oslo stock exchange in the entire relevant period to cope with survivorship bias, and liquid stocks to reflect an investor's opportunities to buy and sell. We have selected 30 stocks to represent everyday investors' opportunities in diversifying their portfolios and therefore looked for companies with differences in size and sectors. A portfolio with several stocks can guard against volatility by investing in sectors with negative correlations.

The OSEBX (Oslo Stock Exchange Index) has two main tasks in our thesis. First, the OSEBX will be used as a proxy for the market return when estimating the beta-coefficients for each stock. OSEBX will later be used as a benchmark to test against the dual-beta model. OSEBX represents the largest and most liquid companies on the Oslo Stock exchange. The index is based on a free-float market capitalisation method to ensure liquidity. The number of companies on the index will vary with time, as of 2022, the index is composed of 69 companies, being revised semi-annually (Euronext, 2022).

Data preparation, data handling and data analysis are performed in Excel. The data consisted of several variables that were unnecessary for our analysis. Therefore, the variables open, high, low, close, and volume have been removed. The only variable's left are therefore: date and adjusted close prices for all the selected companies for the portfolio. The adjusted close price takes account of dividend payout and stock splits. With this variable, we get the actual progress of an investor's returns.

Figure 2 shows the OSEBX. It reflects the Norwegian stock market cycles from 2002 to 2022. The index had a slight downturn in the first year after a strong upturn leading up to The Great Recession starting in 2007. The Great Recession devastated the worlds financial markets, with the OSEBX going from over 500 points to below 200 points. We see a steadily upwards trend with normal shorter downturns following The Great Recession. This period lasted up until the COVID-19 virus was discovered in late 2019. On March 11, 2020, World Health Organization declared COVID-19 as a global pandemic (WHO, 2020). During this period, the Oslo stock exchange experienced its most significant price drop in one day since 2008. In three weeks, the exchange lost three and a half years' worth of market growth (Ghaderi et al., 2020). After the fear of the pandemic had settled, the stock markets experienced a rally of stock growth, starting already in late March 2020. The rally lasted until the turbulent conflict between Russia and Ukraine, but had little impact on the Norwegian stock market. Compared to Chong (2022), we have selected the same business cycles to evaluate the portfolios.

Figure 2: OSEBX



Note: The figure shows price history of Oslo Stock Exchange Benchmark Index from 2002 - 2022.

Table 1 shows descriptive statistics for the 30 companies used in our analysis. The first two columns show the company names and tickers. Column three shows the annual return for each company. Column four shows the annual standard deviation. Column five shows the companies' market value in millions in their local currency (NOK) as of May, 2022. The last column shows which market sector the companies have their primary business operating in.

Table 1: Descriptive statistics

Company name	Ticker	Ann. return	Std. deviation	Market Value	Industry Group
SAS	SAS.ST	-18,89 %	0,643	6 831,52	Airlines
NORSK HYDRO	NHY.OL	9,15 %	0,339	144 085,10	Aluminum
ABG SUNDAL COLLIER HOLDING	ABGA.OL	13,81 %	0,299	3 154,00	Asset Mngr, Custodian
DNB BANK	DNB.OL	11,18 %	0,293	274 026,90	Banks
SPAREBANK 1 SR-BANK	SRBNK.OL	11,55 %	0,256	28 925,44	Banks
SPAREBANK 1 SMN	MING.OL	15,16 %	0,233	16 125,68	Banks
ATEA	ATEA.OL	16,35 %	0,339	12 025,09	Computer Services
AF GRUPPEN	AFGA.OL	25,29 %	0,201	19 047,36	Construction
VEIDEKKE	VEI.OL	16,54 %	0,283	15 250,06	Construction
SCHIBSTED A	SCHA.OL	10,24 %	0,394	17 664,17	Consumer Digital Svs
KONGSBERG GRUPPEN	KOG.OL	21,52 %	0,296	70 925,25	Divers. Industrials
BONHEUR	BONHR.OL	12,23 %	0,344	14 056,79	Divers. Industrials
MOWI	MOWI.OL	23,32 %	0,455	126 743,90	Farming, Fishing
LEROY SEAFOOD GROUP	LSG.OL	24,39 %	0,351	49 419,39	Farming, Fishing
CARASENT	CARAC.OL	14,50 %	0,751	1 423,02	Health Care Services
EQUINOR	EQNR.OL	11,66 %	0,243	1 048 487,00	Integrated Oil & Gas
STOREBRAND	STB.OL	5,89 %	0,416	37 012,27	Life Insurance
TOMRA SYSTEMS	TOM.OL	20,04 %	0,307	49 734,73	Machinery: Industrial
HEXAGON COMPOSITES	HEX.OL	16,88 %	0,538	6 371,18	Machinery: Specialty
FRONTLINE	FRO.OL	-9,30 %	0,578	15 600,65	Marine Transportation
STOLT-NIELSEN	SNI.OL	3,30 %	0,325	9 515,96	Marine Transportation
SUBSEA 7	SUBC.OL	4,57 %	0,376	25 158,00	Oil Equipment & Svs
TGS	TGS.OL	8,94 %	0,390	17 510,48	Oil Equipment & Svs
PGS	PGS.OL	-14,42 %	0,600	2 595,77	Oil Equipment & Svs
SOLSTAD OFFSHORE	SOFF.OL	-29,04 %	0,677	1 383,46	Oil Equipment & Svs
SCANA	SCANA.OL	-29,89 %	0,639	479,97	Oil Equipment & Svs
DOF	DOF.OL	-25,43 %	0,605	458,86	Oil Equipment & Svs
DNO	DNO.OL	10,90 %	0,580	13 841,39	Oil: Crude Producers
PHOTOCURE	PHO.OL	8,14 %	0,468	2 824,45	Pharmaceuticals
TELENOR	TEL.OL	9,25 %	0,245	178 291,00	Telecom. Services

Note: The table shows descriptive statistics: company name, ticker, annual return(2002-2022), annual standard deviation(2002-2022), market value in millions of local currency(2022) and primary industry group for all 30 companies.

4 Methodology

4.1 Investment strategy

We have used the dual-beta model, which extends the Capital Asset Pricing Model (CAPM). The main difference between the dual-beta model and CAPM is that the dual-beta model splits positive and negative market returns. The dual-beta model can be expressed as:

$$(R_j - R_f)_t = \alpha_j^+ D + \beta_j^+ (R_m^+ - R_f)_t D + \alpha_j^- (1-D) + \beta_j^- (R_m^- - R_f)_t (1-D) + \varepsilon_t \quad (1)$$

The formula splits the returns using a dummy variable, D. The dummy variable will be 1 when the market return, R_m, is positive and zero when negative. The α's is the coefficient of the model. We will extract the betas in a regression between the market return and return of each asset. Without asymmetry in beta, the dual-beta model is identical to the CAPM.

The model invests in all stocks which meets both beta-thresholds, which is $\beta^+ > 1$ (upside beta) and $\beta^- < 1$ (downside beta) for our primary strategy. The model will weigh the stocks that meet the criteria equally.

4.2 Portfolio construction/rebalancning

To test the theory with upside and downside beta, we created a portfolio of 30 stocks from the Oslo Stock Exchange. Further, trailing three years upside and downside betas were calculated beginning with the period from 2002 to 2005, making the first investment in 2005. Upside periods were defined as days when the OSEBX return was positive and vice versa for downside periods. The portfolio is revised quarterly. Finally, the return of each period from April 1, 2005, until April 1, 2022, is chain-linked for analysis.

When the model had zero stockholdings, our model assumed to hold cash. We want to create a portfolio which can maintain value in times of market decline and still reap profit in the good times.

4.3 Weekly and monthly beta calculation

By changing the data values used for calculating the dual-beta-values from daily to weekly and monthly, we are dealing with a possible problem of lagged effects of market swings. For example, when the benchmark index is down one day, the corresponding effect on the stock may show the next day or visa-versa.
Therefore, the portfolio is revised according to beta values calculated with respectively daily, weekly and monthly stock prices.

4.4 Sharpe ratio

The Sharpe ratio indicates how well the portfolio performs compared to the rate of return on a risk-free investment. In our analysis, we will use the Sharpe ratio to measure the portfolio's performance to help understand the realised return of the investment compared to its risk. We will also use the Sharpe ratio as our maximisation goal in Markowitz portfolio optimisation. It is designed to calculate the risk-adjusted return. The calculation is as follows:

$$Sharpe\ ratio = (R_p - R_f)/\sigma_p \tag{2}$$

Where R_p is the return of the portfolio, R_f is the risk-free rate, and σ_p stands for the standard deviation of the portfolio return. The Sharpe ratio can be used to calculate past performance or expected performance by calculating expected returns, risk free rate and standard deviation. We will mainly use it to calculate past performance and to study the effect of splitting risk into upside and downside in previous years. A portfolio investing only in Norwegian government bonds would have a Sharpe ratio of 0. The higher the Sharpe ratio, the more attractive a portfolio is.

4.5 Equally weighted portfolio

The efficient market theory by Fama (1970) tells us that a well-diversified portfolio will have excluded all unsystematic risk and, therefore, perform similarly to the rest of the market. By equally weighing our 30 chosen stocks, we achieve a portfolio large enough to be considered well-diverisified by Fisher and Lorie (1970) rule of thumb, and should perform somewhat similar to the market - OSEBX. Later in this paper, the equally weighted portfolio is referred to as EW.

$$R^{EW} = \sum_{i}^{N} R_i \quad \times \quad 1/N \tag{3}$$

4.6 Markowitz portfolio optimization

Markowitz's portfolio theory assumes that an investor is risk-averse and will only take on increased risk if compensated with a higher expected return. The expected return of a portfolio is shown in equation [4], where R_p is the portfolio's return, R_i is the return on asset i and w_i is the corresponding weight of the asset.

$$E(R_p) = \sum_{i=1}^{N} w_i E(R_i) \tag{4}$$

$$\sigma_p^2 = \sum_{i=1}^{N} w_i \sigma_i^2 + \sum_{i}^{N} \sum_{j \neq i}^{N} w_i w_j \sigma_i \sigma_j p_{ij} \tag{5}$$

Equation [5] shows the variance of the portfolio where σ_i is the sample standard deviation of the periodic returns on an asset and p_{ij} is the correlation coefficient between the returns on assets i and j. Based on this equation, an investor can reduce the portfolio risk by holding combinations of securities that are not perfectly positively correlated, $p_{ij} = 1$ (Markowitz, 1952).

$$maximize \quad SR(w_1, ..., w_N) = \frac{\sum_{i=1}^{N} w_i \cdot \ \mu_i - r_f}{\sqrt{\sum_{i=1}^{N} \sum_{j=1}^{N} \sigma_{ij} \cdot w_i \cdot w_j}}$$

$$Subject \quad to \quad \sum_{i=1}^{N} w_i = 1 \tag{6}$$

Equation 6 shows Markowitz optimization problem, where $SR(w_1, ..., w_N)$ is the Sharpe ratio that depends on the portfolio weights. The sum of weights must be equal to 1. The calculation for the optimised portfolio is done by trailing three years of data, with the portfolio being revised quarterly. The calculations are done with the help of Excel solver.

4.7 Fama-French five-factor model with momentum

The goal of the model output is to test the portfolio's performance for statistical significance. We estimate the sensitivity of each portfolio's excess return to movements in the factors. We write the FF five-factor with each portfolio i in time period t as:

$$r_{it} - r_{ft} = \alpha_i + \beta_i(r_{mt} - r_{ft}) + s_i SMB_t + h_i HML_t + rw_i RMW_t + c_i CMA_t + m_i MOM_t + \varepsilon_{it} \tag{7}$$

The factors are as described earlier, $r_{mt} - r_{ft}$ is the market excess return, SMB is small minus big stocks, HML is high minus low B/M stocks, RMW is robust minus weak, CMA is conservative minus aggressive investment, MOM is the momentum factor, and ε_{it} is the zero-mean residual.

The coefficients β_i, s_i, h_i, rw_i and c_i are the factor loadings, also called beta values, for the five factors. The intercept α_i shows the excess variation in expected returns not picked up by the factors. If the factors capture all variation, alpha equals zero. A significant positive alpha is often looked upon as a sign of an investor's skills in beating the market and factors.

Although a model may be significant, "a p-value does not provide a good measure of evidence regarding a model or hypothesis" (Harvey, 2017). We incorporate the portfolio return decomposition (PRD (Israel & Ross, 2017)) and the mean value decomposition (MVD (Holgersson et al., 2014)). This shows the factors respectively marginal and relative contribution to the portfolio's excess return $(R_j - R_f)$, as a test for economic significance and a supplement to the statistical significance. PRD is calculated for each factor as

$$PRD = \beta \times r_{\beta i} \tag{8}$$

Here, beta is the factors coefficient, and $r_{\beta i}$ is the factors mean return in the time period, also called the factors market premium.
MVD is calculated for each factor as

$$MVD = \frac{PRD_{\beta}}{R_j - R_f} \tag{9}$$

Here, PRD is the previously calculated PRD for each factor and $R_j - R_f$ is the portfolio's excess return. Dividing PRD with the excess return shows the relative contribution.

5 Analysis

5.1 Portfolio comparison

Table 2 shows summary statistics for the entire period and the sub periods divided into five parts. The different periods are the First period which extends from April 1, 2005, until November 30, 2007, and the Second period - The Great Recession, which extends from December 1, 2007, until June 30, 2009. The third period extends from July 31, 2009, until January 1, 2020, the Fourth period - the COVID-19 recession extends from January 20, 2020, until April 30, 2020, and the Fifth period extends from May 4, 2020, until March 1, 2022. This division is made based on significant events/crises in the world. We want to construct a portfolio that can help maintain value in times of market decline and therefore choose to divide the periods as mentioned.

Dual-beta represents the dual-beta portfolio where the model invests in all stocks which meet both $\beta^+ > 1$ (Upside beta) and $\beta^- < 1$ (Downside beta), and OSEBX is the benchmark we will test the dual-beta model against. To compare with the dual-beta model, we have also included two other portfolios to better answer how our strategy performs in the Norwegian market. We have included EW, which represents a separate portfolio where the 30 selected companies are weighted equally throughout the period, and a portfolio based on Markowitz's portfolio theory.

The annual rate of return shows the average annual return within the representative period. Annual volatility shows the average annual volatility within the representative period. Maximum and minimum show the highest and lowest returns on the portfolios (by return interval). The Sharpe ratio indicates how well the portfolio performs compared to the rate of return on a risk-free investment. We have calculated the Sharpe ratio based on the average risk-free investment inside the representative period. Correlation shows how the portfolios are related to the benchmark (OSEBX). OSEBX will therefore correlate equally to 1 throughout the analysis.

The number of stocks shows the average number of stocks the portfolios have invested in inside the period. The sample shows the number of observations within the period of the different portfolios. The operation of Markowitz optimisation with 3-year trailing data over 17 years demands a lot of data power; we have therefore used monthly returns to simplify the processing. Therefore, we will see the lowest sample values for the Markowitz portfolio, as it is based on monthly data while the other portfolios are based on daily data.

For the whole period, dual-beta had the lowest annual rate of return of - 8,8 %. OSEBX and Markowitz had annual returns of respectively 9,7 % and 13,3 %. EW had the highest annual rate of return of 15,7 %. Our portfolio (Dual-beta) had the highest annual volatility of 35 % and the lowest Sharpe ratio of – 0,265. This means that our portfolio is the least profitable and the riskiest. For all periods, dual-beta had an average of investing in 1,42 stocks, which means that very few companies meet both the upside- and downside beta requirements.

In period 1, the dual-beta portfolio is the least profitable and the riskiest, with an annual rate of return of 24 % and annual volatility of 29,48 %. For this period, EW had the highest annual rate of return of 41 %. During this period, the dual-beta model invest in 2,28 stocks on average. The benchmark had an annual rate of return of 29 % during this period, which is slightly better than the dual-beta portfolio.

During The Great Recession in 2008-2009, all four portfolios had weak results. During this period, the Markowitz portfolio had the weakest result, with an annual rate of return of – 37,6 % and annual volatility of 41,93 %. This was a challenging period for the financial world, something that the portfolios naturally are characterised by. The portfolio that had the most negligible loss during this period was EW, with an annual rate of return of – 19 %. One of the reasons why the dual-beta portfolio performed poorly in this period could be that almost no stocks meet the beta requirements.

During The Great Recession, the average stocks for dual- beta was only 1,51. At the same time, it is not sure that it would have helped to be invested in several stocks since it, as mentioned earlier, was a very tough period.

The period after The Great Recession (period 3) extends over 10,5 years. This is a bad period for the dual-beta portfolio, with an annual rate of return of -18 % and annual volatility of 33,53 %. Within this period, the Markowitz portfolio had the highest annual rate of return of 18,8 %. During The Great Recession, dual-beta was invested in very few companies, which affected the result. On average, dual-beta was invested in 0,87 stocks during this period, an even lower average than during The Great Recession (1,51). We see positive returns for all portfolios except for the dual-beta portfolio for this period. This is critical for our portfolio because it extends over so many years.

The COVID-19 pandemic has created both a global health crisis and an economic crisis. This substantially impacted the Covid-19 recession, where all portfolios except the dual-beta portfolio suffered losses. The dual-beta portfolio had an annual rate of return of 11 %. During this period, the portfolio was on average invested in 1,41 stocks, which means that the model managed to find 1-2 stocks that performed well during this downturn. We see relatively significant losses for the other portfolios. The benchmark and EW had an annual rate of returns of – 42 % and – 49 %, respectively.

Our last period in the analysis (period 5) has positive figures for the portfolios. Also, during this period, the dual-beta portfolio performed worst, with an annual rate of return of 24 %. The EW portfolio had the highest annual rate of return of 38 %. Throughout three of five periods, the trend is that the dual-beta portfolio performs worst. The results may indicate that the beta requirements we set are somewhat strict. As mentioned, the portfolio invests in very few companies, and in many quarters the model chooses to invest in zero companies and assumes to hold cash instead.

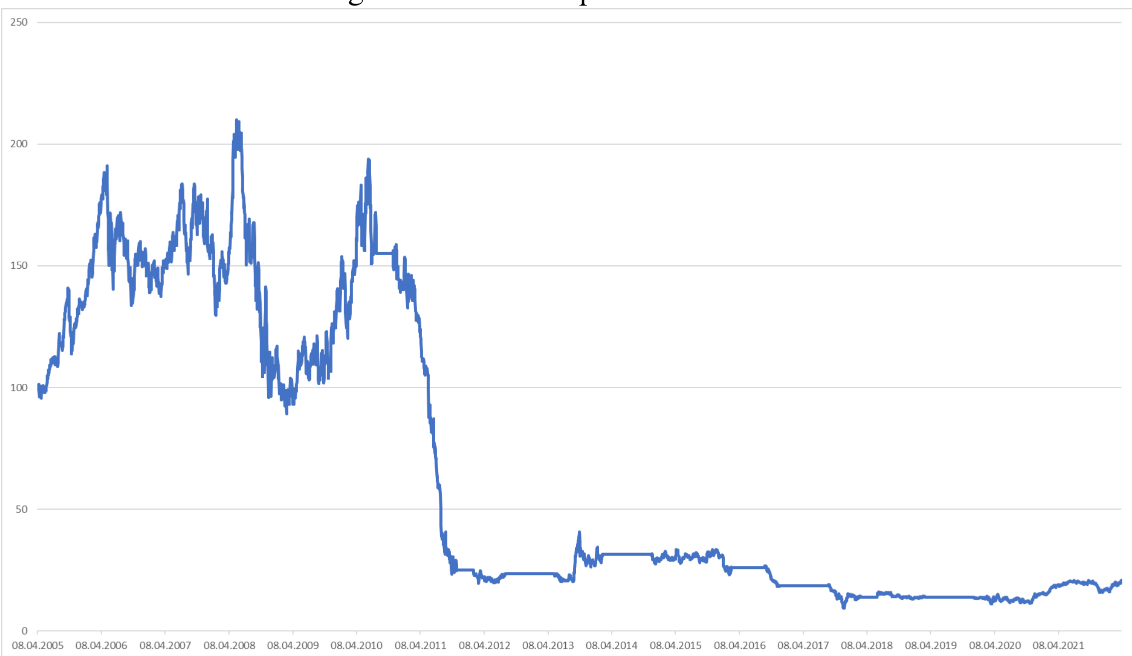
Table 2: Summary statistics by economic cycle

	Dual-beta	OSEBX	EW	Markowitz
Whole period (04/01/05-03/01/22)				
Ann. Return	-8,8 %	9,7 %	15,7 %	13,3 %
Annual volatility	35 %	23 %	21 %	25 %
Maximum	18,2 %	10,7 %	7,5 %	23,3 %
Minimum	-11,6 %	-9,9 %	-9,4 %	-29,2 %
Sharpe Ratio	- 0,26	0,40	0,73	0,52
Correlation	0,612	1,000	0,894	0,740
Number of stocks	1,42		30	7,96
Sample	4432	4432	4432	204
First period (04/01/05-11/01/07)				
Ann. Return	24 %	29 %	41 %	39,7 %
Annual volatility	29,48 %	21,00 %	18,05 %	20,02 %
Maximum	9,2 %	7,1 %	5,5 %	13 %
Minimum	-7,9 %	-5,7 %	-5,1 %	-10 %
Sharpe Ratio	0,74	1,31	2,15	1,96
Correlation	0,832	1,00	0,919	0,731
Number of stocks	2,28		30	9,375
Sample	670	670	670	31
Second period - The Great Recession (12/03/07-06/01/09)				
Ann. Return	-24 %	-29 %	-19 %	-37,6 %
Annual volatility	51,19 %	46,44 %	35,95 %	41,93 %
Maximum	11,4 %	10,7 %	7,5 %	22 %
Minimum	-11,6 %	-9,9 %	-9,2 %	-29 %
Sharpe Ratio	- 0,48	- 0,64	- 0,55	- 0,90
Correlation	0,887	1,000	0,936	0,844
Number of stocks	1,51		30	6,4
Sample	391	391	391	19
Third period (06/01/09-01/02/20)				
Ann. Return	-18 %	12 %	16 %	18,8 %
Annual volatility	33,53 %	18,15 %	17,62 %	22,88 %
Maximum	18,2 %	6,4 %	6,2 %	23 %
Minimum	-11,0 %	-6,0 %	-6,9 %	-21 %
Sharpe Ratio	-0,55	0,64	0,88	0,82
Correlation	0,440	1,000	0,877	0,670
Number of stocks	0,87		30	7,8
Sample	2741	2741	2741	127
Fourth period - COVID-19 recession (01/02/20-04/30/20)				
Ann. Return	11 %	-42 %	-49 %	-32,9 %
Annual volatility	53,65 %	37,90 %	42,80 %	42,75 %
Maximum	11,2 %	5,6 %	6,0 %	13 %
Minimum	-7,8 %	-8,8 %	-9,4 %	-21 %
Sharpe Ratio	0,19	-1,12	-1,16	-0,77
Correlation	0,537	1,000	0,943	0,905
Number of stocks	1,41		30	9,5
Sample	87	87	87	5
Fifth period (05/04/20-03/01/22)				
Ann. Return	24 %	29 %	38 %	24,6 %
Annual volatility	28,65 %	16,47 %	19,78 %	23,46 %
Maximum	6,5 %	4,4 %	5,0 %	13 %
Minimum	-5,8 %	-3,6 %	-4,4 %	-12 %
Sharpe Ratio	0,83	1,79	1,92	1,05
Correlation	0,592	1,000	0,857	0,551
Number of stocks	3,22		30	7,5
Sample	477	477	477	23

Note: The table shows performance measurements in each period. Dual-beta portfolio with beta thresholds of 1, beta-values calculated with daily return intervals.

That the dual-beta portfolio performed so poorly is somewhat surprising after seeing how well the same investment strategy worked in the US market (Chong, 2022) (Guy, 2014). Therefore, we tried to investigate the cause of the weak results in more detail. In figure 3 we see that the portfolio plunged approx - three years after The Great Recession, which strongly affected the analysis results. We saw positive returns for all portfolios within the third period apart from the dual-beta portfolio, which had a return of -18 %. Our model invested in a stock with fragile results during a short period of time, resulting in a capitulation of our portfolio. This stock had an upside – and downside beta value of around 1. If the beta thresholds had been adjusted slightly, the investments might never happen, or other stocks could have been included to prevent such a downfall.

Figure 3: Dual-beta performance



Note: The figure shows the historic performance of dual-beta portfolio with beta thresholds of 1, and beta-values calculated using daily return intervals.

5.2 Beta approach

In this part of the analysis, we will present table 3 showing results with different upside- and downside beta requirements. Table 4 is an extension of table 3 and shows the performance of the different beta portfolios in a simpler way. As mentioned in the portfolio comparison part, the results may indicate that the beta requirements are somewhat strict. Therefore, we now want to present some dual-beta portfolios with different beta thresholds to investigate whether this changes the results. In table 3, we present four different combinations of beta requirements for the portfolio: $\beta^{\pm} = 1$ (as in portfolio comparison, called dual-beta), $\beta^{+} > 0,7$ and $\beta^{-} < 1,3$ (called "kindest"), $\beta^{+} > 0,7$ and $\beta^{-} < 1$ (called "kind up"), and $\beta^{+} > 1$ and $\beta^{-} < 1,3$ (called "kind down"). In addition, we have included the benchmark (OSEBX) in the table.

For the whole period, "kindest" had the highest annual rate of return of 14,9 %. "Kindest" gets the best Sharpe ratio value, at 0,58. On the other hand, OSEBX is the least risky, with an annual volatility of 23 %. "Kindest" did not surprisingly invest in most stocks on average (approx. 11 stocks). "Kind up" invested in approximately 6, and "kind down" invested in approximately 5 stocks. As expected, if the requirements for upside- and downside beta become "kinder", the model invests in more stocks. This is good for diversification and avoids unreasonably high risk from one asset. One of the basic principles in modern portfolio theory is that volatility decreases when the number of assets increases, which emerges clearly in this table. All the new portfolios with adjusted beta values outperform the benchmark for the whole period regarding annual returns. Dual-beta invested in 1,42 stocks on average and has the highest volatility of 34,9 %.

In the first period, "kind up" had the highest annual rate of return of 43,7 %. This was a reasonable period for all the beta models, where all portfolios achieved an annual rate of return above 23 %. During the Great Recession (second period), we see negative results. When it comes to the annual rate of returns, all the portfolios perform relatively equally. However, we see significant differences in the number of stocks the models invest in. For example, "kindest" invests in 8,72 stocks and dual-beta invests in 1,51 stocks. Nevertheless, there is only a 3,6 % difference between the portfolio's rate of return. In other words, this had little effect on the result during this period.

Within period 3, we see positive figures for all portfolios except dual-beta, which had an annual rate of return of – 18,4 %. Dual-beta invested in 0,87 stocks on average and has the highest volatility of 33,5 %. Furthermore, the portfolio has a maximum return of 18,2 % and a minimum of – 11 %, so we see a massive return gap. "Kindest" performs best in this period, with an annual rate of return of 13,7 %, followed by "kind up" with 13,6 %.

We see significant differences between the portfolios during the Covid-19 recession (fourth period). Dual-beta had an annual rate of return of 11 %. We see negative results for the benchmark, "kind up" and "kindest". It was "kind down", which achieved the highest annual rate of return of 12,9 %. Our last period in the analysis (fifth period) shows positive figures for all portfolios. Once again, we see that dual-beta has the lowest annual rate of return of 23,7 %. The highest rate of return was for "kind up", with an annual rate of return of 42,2 %.

Table 3: Beta- Kinder Thresholds

	Dual-beta	Kind up	Kindest	Kind down	OSEBX
Whole period (04/01/05-03/01/22)					
Annual Rate of return	-8,8 %	14,3 %	14,9 %	14,0 %	9,7 %
Annual volatility	34,9 %	24,2 %	24,9 %	31,1 %	23,1 %
Maximum	18,2 %	9,0 %	10,0 %	11,2 %	10,7 %
Minimum	-11,6 %	-10,4 %	-10,5 %	-11,4 %	-9,9 %
Sharpe Ratio	- 0,26	0,57	0,58	0,44	0,40
Correlation	0,612	0,786	0,898	0,878	1,000
Number of stocks	1,42	6,03	10,64	4,90	
Sample	4432	4432	4432	4432	4432
First period (04/01/05-11/01/07)					
Annual Rate of return	23,6 %	43,7 %	36,4 %	39,0 %	29,1 %
Annual volatility	29,5 %	20,9 %	21,3 %	28,6 %	21,0 %
Maximum	9,2 %	6,1 %	7,3 %	10,1 %	7,1 %
Minimum	-7,9 %	-5,9 %	-5,7 %	-8,2 %	-5,7 %
Sharpe Ratio	0,74	2,01	1,63	1,30	1,31
Correlation	0,832	0,870	0,912	0,864	1,000
Number of stocks	2,28	5,67	7,99	3,58	
Sample	670	670	670	670	670
Second period - The Great Recession (12/03/07-06/01/09)					
Annual Rate of return	-23,8 %	-30,2 %	-20,2 %	-10,7 %	-29,0 %
Annual volatility	51,2 %	42,8 %	46,1 %	56,3 %	46,4 %
Maximum	11,4 %	9,0 %	10,0 %	11,2 %	10,7 %
Minimum	-11,6 %	-10,4 %	-10,5 %	-11,2 %	-9,9 %
Sharpe Ratio	- 0,48	- 0,72	- 0,45	- 0,20	- 0,64
Correlation	0,887	0,877	0,937	0,920	1,000
Number of stocks	1,51	5,99	8,72	3,89	
Sample	391	391	391	391	391
Third period (06/01/09-01/02/20)					
Annual Rate of return	-18,4 %	13,6 %	13,7 %	8,1 %	11,9 %
Annual volatility	33,5 %	20,4 %	21,0 %	27,0 %	18,2 %
Maximum	18,2 %	8,8 %	7,0 %	9,5 %	6,4 %
Minimum	-11,0 %	-6,6 %	-6,6 %	-11,4 %	-6,0 %
Sharpe Ratio	- 0,55	0,66	0,65	0,29	0,64
Correlation	0,440	0,737	0,885	0,860	1,000
Number of stocks	0,87	5,94	11,22	5,04	
Sample	2741	2741	2741	2741	2741
Fourth period - COVID-19 recession (01/02/20-04/30/20)					
Annual Rate of return	11,0 %	-23,9 %	-21,5 %	12,9 %	-41,6 %
Annual volatility	53,7 %	39,7 %	40,3 %	44,6 %	37,9 %
Maximum	11,2 %	6,5 %	5,9 %	7,0 %	5,6 %
Minimum	-7,8 %	-5,7 %	-7,4 %	-9,8 %	-8,8 %
Sharpe Ratio	0,19	- 0,62	- 0,55	0,27	- 1,12
Correlation	0,537	0,780	0,923	0,899	1,000
Number of stocks	1,41	7,17	12,17	4,83	
Sample	87	87	87	87	87
Fifth period (05/04/20-03/01/22)					
Annual Rate of return	23,7 %	42,2 %	40,9 %	38,6 %	29,4 %
Annual volatility	28,7 %	22,5 %	20,6 %	22,0 %	16,5 %
Maximum	6,5 %	7,3 %	4,3 %	7,5 %	4,4 %
Minimum	-5,8 %	-5,3 %	-6,9 %	-4,3 %	-3,6 %
Sharpe Ratio	0,83	1,88	1,99	1,76	1,79
Correlation	0,592	0,600	0,767	0,836	1,000
Number of stocks	3,22	7,00	12,52	6,80	
Sample	477	477	477	477	477

Note: The table shows performance measurements in each period. Beta-values calculated with daily return intervals. Dual-beta portfolio with beta thresholds of 1.
"Kind up": $\beta^+ > 0{,}7 \quad \& \quad \beta^- < 1$
"Kindest": $\beta^+ > 0{,}7 \quad \& \quad \beta^- < 1{,}3$
"Kind down": $\beta^+ > 1 \quad \& \quad \beta^- < 1{,}3$.

Beta score

In table 4, we set up a scorecard to show which beta composition performs best. The ranking system gives points from 1 to 5, where 1 means the portfolio that has performed best (marked with a solid green colour), while 5 means the portfolio that has performed worst (marked with a solid red colour). For the annual rate of return and Sharpe ratio, the highest value will rank 1, while for annual volatility, the lowest value will rank 1.

Looking at the scoreboard (table 4) for the annual rate of return, "kindest" is the portfolio with the lowest total score (12). Therefore, data may indicate that accepting a higher downside beta and lower upside beta is suitable for the return. At the same time, "kindest" has the third-highest total volatility score. The downside beta is the risk associated with loss. Therefore, it is expected that a portfolio with a higher downside beta will have relatively high risk. "Kind up" seems to be a better choice when it comes to risk. The scoreboard for volatility shows that this portfolio is the least risky of the dual-beta portfolios, only outperformed by the benchmark. "Kind up" also has the best score for the Sharpe ratio and the third-best score when it comes to the annual rate of return.

The scoreboard (table 4) brings out the weak results for dual-beta, which we have seen signs of throughout the analysis. In the annual rate of return, dual-beta performs worst in the whole period and 3 of 5 periods. In addition to weak returns, this portfolio also seems to be the riskiest, with the highest volatility score. Dual-beta has the worst volatility score throughout the whole period and in 4/5 periods. Not surprisingly, this also leads to weak scores regarding the Sharpe ratio. One thing worth considering from this analysis is that 3/4 dual-beta portfolios perform better than the benchmark regarding the annual rate of return. This suggests that the dual-beta model as an investment strategy could work for the Norwegian market, but with different beta requirements than have been used in the literature (Chong, 2022) (Guy, 2014).

Table 4: Betascore

	Dual-beta	Kind up	Kindest	Kind down	OSEBX
Annual Rate of return					
Whole period	5	2	1	3	4
First period	5	1	3	2	4
The Great Recession	3	5	2	1	4
Third period	5	2	1	4	3
COVID-19 recession	2	4	3	1	5
Fifth period	5	1	2	3	4
Annual Volatility					
Whole period	5	2	3	4	1
First period	5	1	3	4	2
The Great Recession	4	1	2	5	3
Third period	5	2	3	4	1
COVID-19 recession	5	2	3	4	1
Fifth period	5	4	2	3	1
Sharpe Ratio					
Whole period	5	2	1	3	4
First period	5	1	2	4	3
The Great Recession	3	1	4	5	2
Third period	5	1	2	4	3
COVID-19 recession	2	4	3	1	5
Fifth period	5	2	1	4	3

Note: The table shows the performance of different beta-portfolios compared to OSEBX. Performance measures are annual rate of return, annual volatility and Sharpe ratio, sorted by financial cycle. Where 1 is the best and 5 is the worst.

5.3 Return intervals

Table 6 shows us the performance of dual-beta portfolios created by calculating the beta values with weekly and monthly stock prices. All three portfolios invest in all stocks which meets both $\beta^{+} > 1$ (Upside beta) and $\beta^{-} < 1$ (Downside beta). The only factor separating the portfolios here is changing the stock data from daily to weekly and monthly. In addition, we have included the benchmark (OSEBX) and the EW portfolio in the table.

For the whole period, EW had the highest annual rate of return of 15,7 %. The dual-beta portfolio with the highest return was the portfolio with monthly data, with an annual rate of return of 14,3 %. We see the third-best results for the benchmark with an annual rate of return of 9,7 %, followed by the portfolio with weekly data of 4,9 %. This means that the original portfolio with daily data performs worst (annual rate of return of -8,8 %). The portfolio with monthly data invested in most stocks on average per quarter (6 stocks). This analysis shows that several stocks manage to achieve the beta requirement by using monthly or weekly data versus daily data. The results can change drastically only by switching to a different return interval.

For the second, third- and fifth period, the portfolio with monthly data had the highest annual rate of return of the dual-beta portfolios. Monthly data are commonly used for analysing periods over many years or even decades. In our analysis, some periods have a short time horizon, such as period four which only extends over two months. We should be careful not to draw conclusions based on monthly data in such periods. For monthly data, we should instead focus on how the portfolio has been managed within the periods with a longer time horizon, such as the whole period and the third period. During the third period, the dual-beta portfolio with monthly data had the second-highest annual rate of return of 12,1 %.

When we compare the daily portfolio with the weekly and monthly portfolios, we notice bad results for the portfolio with daily data during the third period. For the third period, the dual-beta portfolio with daily data had an annual rate of return of –18,4 % versus 9 % and 12,1 % for weekly and monthly, respectively. For this period, the portfolio based on daily data invested in less than one stock on average. This was when the portfolio invested in the fewest stocks, which seemed to affect the result to a damaging extent.

We see several stocks with drastic shifts in beta-value when expanding the return interval. The average upside beta value rose by 0,186 when changing from daily to monthly return interval (shown in table 5), which is quite a lot considering the strict beta thresholds . The average downside-beta value rose by 0,08. Which is less than the upside beta. Considering these changes, the model would have more stocks that fit the upside threshold and fewer stocks that fit the downside threshold.

Table 5: Average beta-values

Return interval	Upside	Downside
Daily	0,820	0,946
Monthly	1,007	1,033
Difference	0,186	0,087

Note: The table shows the average beta value of all 30 companies.

Table 6: Weekly-Monthly-Ew

	Daily	Weekly	Monthly	EW	OSEBX
Whole period (04/01/05-03/01/22)					
Annual Rate of return	-8,8 %	4,9 %	14,3 %	15,7 %	9,7 %
Annual volatility	34,9 %	32,0 %	29,0 %	21,1 %	23,1 %
Maximum	18,2 %	22,2 %	48,3 %	7,5 %	10,7 %
Minimum	-11,6 %	-23,0 %	-20,2 %	-9,4 %	-9,9 %
Sharpe Ratio	-0,26	0,14	0,49	0,73	0,40
Correlation	-0,03	0,66	0,65	0,89	1,00
Number of stocks	1,42	3,51	6,02	30,00	
Sample	4432	884	204	4432	4432
First period (04/01/05-11/01/07)					
Annual Rate of return	23,6 %	20,9 %	23,3 %	40,6 %	29,1 %
Annual volatility	29,5 %	26,2 %	25,5 %	18,1 %	21,0 %
Maximum	9,2 %	8,6 %	16,5 %	5,5 %	7,1 %
Minimum	-7,9 %	-9,4 %	-11,5 %	-5,1 %	-5,7 %
Sharpe Ratio	0,74	0,76	0,90	2,15	1,31
Correlation	0,83	0,80	0,79	0,92	1,00
Number of stocks	2,28	1,86	6,52	30	
Sample	670	136	31	670	670
Second period - The Great Recession (12/03/07-06/01/09)					
Annual Rate of return	-23,8 %	-35,8 %	-9,1 %	-19,2 %	-29,0 %
Annual volatility	51,2 %	46,5 %	36,8 %	35,9 %	46,4 %
Maximum	11,4 %	22,2 %	24,2 %	7,5 %	10,7 %
Minimum	-11,6 %	-17,4 %	-17,9 %	-9,2 %	-9,9 %
Sharpe Ratio	-0,48	-0,78	-0,25	-0,55	-0,64
Correlation	0,89	0,86	0,82	0,94	1,00
Number of stocks	1,51	2,82	8,42	30	
Sample	391	79	19	391	391
Third period (06/01/09-01/02/20)					
Annual Rate of return	-18,4 %	9,0 %	12,1 %	15,6 %	11,9 %
Annual volatility	33,5 %	28,3 %	24,0 %	17,6 %	18,2 %
Maximum	18,2 %	20,6 %	22,7 %	6,2 %	6,4 %
Minimum	-11,0 %	-23,0 %	-18,9 %	-6,9 %	-6,0 %
Sharpe Ratio	-0,55	0,31	0,51	0,88	0,64
Correlation	0,44	0,59	0,63	0,88	1,00
Number of stocks	0,87	3,86	5,72	30	
Sample	2741	554	127	2741	2741
Fourth period - COVID-19 recession (01/02/20-04/30/20)					
Annual Rate of return	11,0 %	-58,1 %	-26,3 %	-48,8 %	-41,6 %
Annual volatility	53,7 %	51,1 %	25,6 %	42,8 %	37,9 %
Maximum	11,2 %	12,8 %	7,5 %	6,0 %	5,6 %
Minimum	-7,8 %	-17,8 %	-13,0 %	-9,4 %	-8,8 %
Sharpe Ratio	0,19	-1,15	-1,03	-1,16	-1,12
Correlation	0,54	0,86	0,83	0,94	1,00
Number of stocks	1,41	4,24	6,60	30	
Sample	87	17	5	87	87
Fifth period (05/04/20-03/01/22)					
Annual Rate of return	23,7 %	25,4 %	49,8 %	37,9 %	29,4 %
Annual volatility	28,7 %	39,5 %	46,3 %	19,8 %	16,5 %
Maximum	6,5 %	18,7 %	48,3 %	5,0 %	4,4 %
Minimum	-5,8 %	-21,9 %	-20,2 %	-4,4 %	-3,6 %
Sharpe Ratio	0,83	0,64	1,07	1,92	1,79
Correlation	0,59	0,41	0,50	0,86	1,00
Number of stocks	3,22	4,39	5,13	30	
Sample	477	97	23	477	477

Note: The table shows performance measurements in each period. Dual-beta portfolios with beta thresholds of 1, beta-values calculated with respectively daily, weekly and monthly return intervals.

5.4 Data table

This part of the analysis will present three data tables elaborating the performance of a dual-beta portfolio made up with beta thresholds between 0,5 and 1,5, using monthly return intervals.

Annual Rate of Return

In table 7, we have created an overview of annual returns for different upside- and downside beta compositions for the whole period. The table covers all beta values from 0,5 to 1,5, for both upside and downside. We do this to see which beta compositions perform well and which do not. The best results for annual returns are marked with a solid green colour. Then, the green colour will decrease and go towards yellow at weaker returns. The worst returns are marked in red.

From looking at table 7, it seems that investing if the downside beta is less than 0,5 and the upside beta greater than 0,5 and up until 1 gives good returns. We see the highest annual rate of return of 29,7 % for downside beta < 0,5 and upside beta > 0,9. It seems that it is good for the returns to be strict on the downside beta and, at the same time, not be too strict on the upside beta. When the value of the upside beta creeps above 1, we see that the returns decrease as the beta increases. It is natural to think that this is related to the number of stocks the model invests in. As we have seen earlier in the analysis, too strict beta requirements will lead the model to invest in very few or no stocks. At the same time, we see that the return slowly decreases when the upside beta creeps below 0,9. This means that choosing a too low upside beta will not be optimal for the return either.

We see the lowest annual rate of return of 5,45 % for downside beta < 0,9 and upside beta > 1,5. Downside beta between 0,8 – 1 and upside beta higher than 1,2 stand overall for weak returns. Only accepting high upside beta values and widely accepting relatively high downside beta values is not suitable for returns. If we increase the downside beta even more, this will give better returns. But this is again related to the number of stocks the model invests in.

We see a collection of the highest annual returns between upside beta 0,5 – 0,9 and downside beta 0,5 – 0,6. Downside beta is the risk associated with loss, and here it is clear that the model performs best if we are strict on the downside beta requirements. When it comes to upside beta, it is good to allow a lower beta value than the market (upside beta =1). This means that the model invests in several stocks suitable for allocation risk and gives higher returns.

Table 7: Annual Rate of Return

Downside beta - invest if downbeta<x	Upside beta - invest if upbeta>x:										
Beta treshold	0,5	0,6	0,7	0,8	0,9	1	1,1	1,2	1,3	1,4	1,5
0,5	23,50 %	25,96 %	25,72 %	27,60 %	29,70 %	21,71 %	19,47 %	14,43 %	13,92 %	13,39 %	13,01 %
0,6	20,41 %	22,05 %	23,79 %	23,94 %	24,85 %	17,64 %	16,91 %	14,84 %	18,27 %	17,14 %	15,83 %
0,7	15,88 %	17,20 %	18,23 %	21,34 %	22,06 %	16,23 %	16,41 %	13,91 %	16,03 %	14,78 %	13,07 %
0,8	15,97 %	17,16 %	18,07 %	18,92 %	20,29 %	13,80 %	13,63 %	9,59 %	11,65 %	10,38 %	7,57 %
0,9	14,19 %	15,70 %	15,66 %	16,76 %	18,40 %	14,14 %	13,94 %	8,52 %	9,03 %	8,58 %	5,45 %
1	15,31 %	16,41 %	15,86 %	16,58 %	17,73 %	14,33 %	14,03 %	6,86 %	7,73 %	7,28 %	6,91 %
1,1	15,69 %	17,38 %	17,27 %	17,84 %	18,70 %	16,88 %	17,22 %	11,11 %	12,97 %	13,82 %	12,71 %
1,2	15,58 %	16,85 %	16,35 %	16,33 %	17,22 %	15,77 %	16,10 %	11,39 %	14,01 %	15,50 %	13,62 %
1,3	16,46 %	17,78 %	17,52 %	17,73 %	19,46 %	18,18 %	18,66 %	13,66 %	15,62 %	17,94 %	17,65 %
1,4	16,02 %	17,30 %	16,80 %	16,71 %	17,95 %	16,90 %	17,39 %	12,45 %	13,99 %	16,45 %	17,33 %
1,5	15,64 %	17,00 %	16,32 %	16,26 %	17,80 %	16,19 %	17,08 %	14,09 %	13,31 %	15,16 %	17,21 %

Note: The table shows annual rate of return for portfolios made with beta thresholds from 0,5 to 1,5, and beta-values calculated with monthly return intervals. Downside beta threshold on left and upside beta threshold above.

Average Number of Stocks

To better understand the return values above, we have set up table 8, which shows an overview of the number of stocks the model invests in for the different beta compositions. As expected, the model invests in most stocks when the beta requirements are at their "kindest". This means upside beta > 0,5 and downside beta < 1,5, and the model invests in 15,7 stocks on average per quarter for this beta composition. Looking back at the same beta composition in table 7, we see that it gives an annual return of 15,64 %, so this is a good alternative but not an optimal solution. As mentioned, we see the best annual rate of return for upside beta > 0,9 and downside beta < 0,5 (29,70 %). For this beta composition, the model invests in 3,27 stocks on average per quarter throughout the period.

Chong (2022) looks at the dual-beta model as an investment strategy in the U.S. market, and his findings show that the dual-beta portfolio outperforms the market. In his analysis, the model invested an average of 3,9 stocks per quarter throughout the period. This may indicate that investing in between 3-4 stocks is optimal for a strategy such as a dual-beta portfolio (the optimal solution in our case is to invest in 3,27 stocks on average).

Table 8: Number of stocks

		Upside beta - invest if beta >x										
	Beta Threshold	0,5	0,6	0,7	0,8	0,9	1	1,1	1,2	1,3	1,4	1,5
Downside beta - invest if beta <x	0,5	4,65073	4,12390	3,75512	3,46634	3,27122	2,96098	2,64878	2,26439	2,00098	1,80780	1,65366
	0,6	5,46829	4,90244	4,49268	4,04976	3,83512	3,44683	3,07610	2,63122	2,26634	2,03415	1,78634
	0,7	6,43415	5,75122	5,28293	4,76000	4,43805	3,89171	3,48195	2,95902	2,55122	2,29951	1,96976
	0,8	7,59024	6,84878	6,32195	5,58049	5,16293	4,53854	3,98732	3,28683	2,74439	2,47317	2,08488
	0,9	9,01951	8,16098	7,43902	6,65854	6,08488	5,38244	4,75317	3,95512	3,29171	2,86244	2,41951
	1	10,09268	9,15610	8,33659	7,51707	6,78537	6,02439	5,33659	4,45854	3,71707	3,26829	2,72780
	1,1	11,32195	10,30732	9,33171	8,33659	7,48780	6,60976	5,90244	4,90732	3,99024	3,50244	2,94244
	1,2	12,73171	11,65854	10,50244	9,31220	8,20976	7,23415	6,40976	5,37561	4,34146	3,79512	3,23512
	1,3	13,76585	12,61463	11,36098	10,09268	8,92683	7,85366	7,00488	5,85366	4,72195	4,13659	3,47220
	1,4	14,73171	13,54146	12,28780	10,90244	9,65854	8,50732	7,56098	6,37073	5,17561	4,53171	3,80195
	1,5	15,71707	14,48780	13,15610	11,65366	10,35122	9,15610	8,07317	6,86341	5,60976	4,90732	4,13854

Note: The table shows average number of stocks for portfolios made with beta thresholds from 0,5 to 1,5, and beta-values calculated with monthly return intervals. Downside beta threshold on left and upside beta threshold above.

Performance Measured by Sharpe Ratio

In table 9, we have created an overview of Sharpe ratios for different upside- and downside beta compositions. As in tables 7 and 8, we still use monthly stock prices. We do this to see if there is any connection between which beta values perform well concerning the rate of return on a risk-free investment. The table shows that having an upside beta threshold between 0,5 - 0,9 gives the best results for Sharpe ratio. We see a collection of the highest Sharpe ratios on the left side of the table. This means with an upside beta between 0,5 - 0,9 and downside beta between 0,5 -1,5. We see the highest Sharpe ratio of 0,73 for upside beta > 0,9 and downside beta < 0,6. We know from earlier that only investing in stocks with high upside beta may result in a low rate of return. The Sharpe Ratio tells us that it also includes more risk, with most of the right side of table 9 coloured in red.

Table 9: Sharpe ratio

		Upside beta - invest if beta >x										
	Beta Threshold	0,5	0,6	0,7	0,8	0,9	1	1,1	1,2	1,3	1,4	1,5
Downside beta - invest if beta <x	0,5	0,617095	0,66669	0,633588	0,678567	0,69407	0,495934	0,421731	0,326301	0,322785	0,299828	0,288683
	0,6	0,661397	0,693474	0,711248	0,720809	0,730537	0,471779	0,423538	0,371226	0,412178	0,383759	0,355159
	0,7	0,536364	0,564872	0,573586	0,667878	0,6411	0,431061	0,41423	0,337355	0,352143	0,323066	0,271419
	0,8	0,592138	0,612118	0,617085	0,63124	0,647042	0,420337	0,388072	0,249144	0,262755	0,233771	0,158609
	0,9	0,553117	0,598062	0,573658	0,602991	0,632635	0,477542	0,435516	0,235206	0,226971	0,208868	0,123302
	1	0,606561	0,635806	0,591187	0,607201	0,613136	0,483305	0,43927	0,188527	0,192435	0,175535	0,155387
	1,1	0,612133	0,658821	0,628804	0,635562	0,632618	0,56099	0,536133	0,307362	0,337463	0,347683	0,293764
	1,2	0,633357	0,670204	0,627654	0,607403	0,603714	0,541698	0,515279	0,334687	0,380185	0,406583	0,330082
	1,3	0,688406	0,724847	0,688105	0,674605	0,706777	0,653068	0,628058	0,419179	0,43915	0,48627	0,447083
	1,4	0,674228	0,709503	0,66402	0,640668	0,662595	0,617645	0,598847	0,389759	0,399806	0,452768	0,442837
	1,5	0,668826	0,703686	0,653497	0,631421	0,668975	0,600667	0,610482	0,473964	0,424404	0,457624	0,480982

Note: The table shows Sharpe ratio for portfolios made with beta thresholds from 0,5 to 1,5, and beta-values calculated with monthly return intervals. Downside beta threshold on left and upside beta threshold above.

5.5 Fama-French five-factor model

In this section, we will analyse the results from the Fama-French five-factor model with momentum factor, and look for statistical and economical significance. The dual-beta portfolio analysed has beta thresholds of 1 and beta values calculated with monthly return intervals. In table 10, we see that some significant factors are sources of added value for the portfolios. The market factor is statistically significant in all cases with a marginal contribution (PRD) of between 4,4 % and 4,6 %, resulting in a relative contribution of between 26 % and 45,7 %. Making it, not surprisingly, one of the main drivers of portfolio returns in all cases. The SMB and MOM factors are not statistically significant in half of the portfolios and have little economic value.

HML factor is statistically significant in all cases but contributes little economically value. It contributes to a relative effect of between -6,5 % and -11,1 %, with the most effect on OSEBX and the equally weighted portfolio. According to Israel and Ross (2017), "an exposure that is small but reliable means one can expect (with greater certainty) that it will affect the portfolio, but only in a small way.", which is our case with the HML factor.

The RMW factor is statistically significant for EW and OSEBX, explaining up to 45 % of the excess return on OSEBX. This means the EW and OSEBX behave like portfolios with positive exposure to stocks with high profitability. A factor exposure that is both economically meaningful and statistically significant can be counted on to affect a portfolio in a big way (Israel & Ross, 2017). We see the same significant exposure for the dual-beta and Markowitz portfolios but no statistical significance, meaning the factor could have a large impact, but with a high degree of uncertainty (Israel & Ross, 2017).

The CMA factor is of little economic value, primarily because of its low monthly mean return. It has a negative relationship with all portfolios, meaning all portfolios might be exposed to stocks with an aggressive investment strategy. The Markowitz portfolio and OSEBX are statistically significant at a 1 % significance level.

The R^2 tells us that the factor model explains more of the excess return on EW and OSEBX than the dual-beta and Markowitz portfolios. This is natural considering they have a larger amount of stocks and, therefore, a larger degree of diversification, resembling the overall market trends. The dual-beta and Markowitz portfolios are portfolios created by trading strategies. A lower R^2 indicates greater stock selectivity (Amihud & Goyenko, 2013). Therefore, it is natural with a more compact portfolio with a lower amount of stocks.

Most importantly, none of the portfolios achieves a significant alpha. Naturally, OSEBX has the smallest alpha with only 4,8 % in relative contribution. We see a higher alpha for all other portfolios, topped by the dual-beta portfolio with an alpha value contributing to 59,8 % of the economic value. Considering the alpha is not statistically significant, the dual-beta portfolio may be the result from a profitable trading strategy, but with a high degree of uncertainty (Israel & Ross, 2017).

Table 10: Fama-French five factor model with momentum factor

	FF6	Dual-beta			EW			Markowitz			OSEBX		
	Monthly mean return	Coeff.	PRD	MVD	Coeff.	PRD	MVD	Coeff.	PRD	MVD	Coeff.	PRD	MVD
Ann. Alpha		0,838	10,1 %	59,8%	0,495	5,9 %	43,2 %	0,371	4,4 %	29,6 %	0,040	0,5 %	4,8 %
MKT-RF	0,54 %	0,670	4,4 %	26,0 %**	0,703	4,6 %	33,4%**	0,678	4,4 %	29,5%**	0,698	4,6 %	45,7%**
SMB	0,16 %	0,304	0,6 %	3,4 %	0,368	0,7 %	5,0%*	0,477	0,9 %	6,0%*	-0,165	-0,3 %	-3,1 %
HML	-0,10 %	0,883	-1,1 %	-6,5%*	0,779	-1,0 %	-7,1%**	0,789	-1,0 %	-6,6%*	0,885	-1,1 %	-11,1%**
RMW	0,36 %	0,922	4,0 %	23,8 %	0,818	3,5 %	25,8%**	0,771	3,3 %	22,3 %	1,035	4,5 %	45,1%**
CMA	-0,06 %	-0,506	0,4 %	2,1 %	-0,536	0,4 %	2,8 %	-1,400	1,0 %	6,6%**	-0,949	0,7 %	6,8%**
MOM	0,75 %	-0,160	-1,4 %	-8,5 %	-0,048	-0,4 %	-3,1 %	0,211	1,9 %	12,6 %	0,132	1,2 %	11,8 %
Ann. Rj-Rf			16,8 %	100,0 %		13,8 %	100,0 %		15,0 %	100,0 %		10,0 %	100,0 %
Adj. R^2		0,3242			0,5752			0,4202			0,5811		

*Note: *P-value<0,05, **P-value<0,01*
FF6: Fama-French five factor model with momentum factor
PRD: Annual portfolio return decomposition (Israel & Ross, 2017)
MVD: Mean value decomposition (Holgersson et al., 2014)

6 Conclusion

This thesis aimed to develop an investment strategy using upside- and downside beta estimates and see if this strategy works in the Norwegian stock market. Based on earlier studies (Chong, 2022)(Guy, 2014), we used a dual-beta portfolio with $\beta^+ > 1$ and $\beta^- < 1$ based on daily return intervals. It was revealed that the dual-beta portfolio performed poorly, and it had a negative annual return for the entire period. However, it is essential to mention that these beta requirements became too strict for the Norwegian market, which resulted in few investments.

Further analysis indicates that a dual-beta portfolio can perform better than the benchmark, but with adjusted beta values and return intervals. We saw that "kinder" beta values will on average invest in more stocks, which gives significantly better returns. We have seen that monthly data performs better than daily data. When it comes to daily data, there is a shortcoming: it does not invest in enough stocks, making it dependent on just a small amount of investments to secure the return. Both upside- and downside beta increases when expanding the return interval, especially the upside beta. This resulted in more stocks making the beta threshold while eliminating the intertemporal cross-correlation. Table 7 shows a connection between sorting stocks with a relatively high upside beta (Higher than 0,5) and a considerable low downside beta (Lower than 0,7). This suggest that a strategy with protection against downfall would perform well. Another "point" shown in table 7 and table 8 shows the connection between the amounts of stocks invested and actual return. A larger amount of stocks, results in great excess returns, similar to EW, making the exact beta thresholds less important. These tables shows the downfall of investing in a considerable small amount of stocks(2-4), possibly resulting in deficient returns.

We found little evidence of a significant positive alpha when assessing our trading strategy's statistical and economic significance. Therefore it is not reasonable to assume that the dual-beta strategy would be the better option in all cases.

Yes, the dual-beta performed better than OSEBX in this period. However, considering both EW and Markowitz portfolios also beat OSEBX, one could suggest that the reason for these performances is based on the preliminary stock-picking before our analysis. Our stock selection is biased of the fact that we know for sure that all companies have survived the 20 years.

Future research could include transaction cost since it is not embedded in this analysis. Considering the equally weighted portfolio performed just as well as dual-beta, the added transaction cost would further decrease the excess return of dual-beta compared to EW. It could also be interesting to adopt the strategy with more stocks, including stocks that have gone off the exchange and new stocks that have been enlisted in later days.

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