Communication

Axion Electrodynamics and the Axionic Casimir Effect

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Abstract: A general scheme for axion electrodynamics is given, in which a surrounding medium of constant permittivity and permeability is assumed. Then, as an application, we provide simple numerical estimates for the electromagnetic current density produced by the electrically neutral time-dependent axions $a = a(t)$ in a strong magnetic field. As is known, the assumption $a = a(t)$ is common under astrophysical conditions. In the third part of the paper, we consider the implications by instead assuming an axion amplitude $a(z)$ depending on one coordinate z only. If such an axion field is contained within two large metal plates, one obtains an axion-generated splitting of the eigenmodes for the dispersion relation. These modes yield equal, though opposite, contributions to the pressure on the plates. We calculate the magnitude of the splitting effect in a simple one-dimensional model.

1. Introduction

Pseudoscalar axions of amplitude $a = a(x)$ (x meaning spacetime) are hypothetical particles that are one of the leading candidates for dark matter. If they can be found experimentally, this would mean an important step forward in our understanding of the universe's composition and development. These axions are believed to be all-pervading, hardly interacting with ordinary matter at all, and they are "cold" in the sense that they are moving with nonrelativistic velocity, $v \sim 10^{-3}c$.

The range of the axion mass m_a is assumed to extend over a few decades of moderate μeV/c^2. These particles may have originated very early in the universe's history, approximately during inflationary times. The existence of them was suggested by Helen Quinn and Roberto Peccei in 1977, in connection with the strong charge-parity (CP) problem in quantum chromodynamics (QCD), and the subject has since attracted considerable interest. Some recent references to axion electrodynamics are [1–13].

As the axions are present everywhere, it should also be possible to detect them under terrestrial conditions, at least in principle. In astrophysical contexts, it is common to assume that they are spatially uniform, $\nabla a = 0$, but vary periodically in time with frequency ω_a. A specific suggestion about how to detect axions on the Earth was presented in [1–3] (the haloscope approach) regarding looking for the resonance between the natural electromagnetic oscillations in a long plasma cylinder and those from the axion field. A strong magnetic field in the axial z direction was applied. Some extra measures were necessary, in order to obtain a cylinder sufficiently 'dilute' to make the electromagnetic oscillation frequencies low enough to permit the resonance condition (on the order of 100 GHz).

To begin in the next section, we present a brief overview of the axion-electrodynamic field in the presence of extraneous charges and currents. We allow for a uniform dielectric background with constant permittivity and permeability. Then, in Section 3, we give simple numerical estimates of the axion-generated longitudinal current in the plasma haloscope [1–3] assuming, as mentioned, that $a = a(t)$. These effects are very small, but nontrivial, as they show the existence of electric currents generated by charge-free particles in interaction with a magnetic field.

In Section 4, we consider the opposite extreme, namely an axial field constant in time but dependent on one spatial coordinate only, $a = a(z)$, in the region between two parallel large metal plates. The dispersion relation shows that there occurs an axion-induced splitting of one of the branches, so that there are two neighboring modes. One mode leads to a weak repulsive Casimir pressure, the other mode reverses the pressure direction. We calculate this effect, making use of scalar electrodynamics in a simple one-dimensional case.

2. Basics of Axion Electrodynamics Dielectric Environment

The fundamental process is the interaction between a pseudoscalar axion and two photons [5]. The Lagrangian equation describing the electromagnetic field in interaction with the axion field is

$$\mathcal{L} = -\frac{1}{4} F_{\alpha\beta} H^{\alpha\beta} - \frac{1}{4} g_\gamma \frac{\alpha}{\pi} \frac{1}{f_a} a(x) F_{\alpha\beta} \tilde{H}^{\alpha\beta}. \tag{1}$$

Here, g_γ is a model-dependent constant of order unity; for definiteness, we adopt the value $g_\gamma = 0.36$, which follows from the DFS model [4,14]. Further, α is the usual fine structure constant, and f_a is the axion decay constant whose value is only insufficiently known; it is often assumed that $f_a \sim 10^{12}$ GeV. We assume an isotropic and homogeneous dielectric background, with constant permittivity ε and permeability μ. When the medium is at rest, the constitutive relations are $\mathbf{D} = \varepsilon\mathbf{E}, \mathbf{B} = \mu\mathbf{H}$. In macroscopic electrodynamics, there are two field tensors, $F_{\alpha\beta}$ and $H_{\alpha\beta}$, where the latter describes the dielectric response to the fields. We will use the metric convention $g_{00} = -1$.

The quantity multiplying the axion $a(x)$ is, thus, the product of the electromagnetic field tensor $F_{\alpha\beta}$ and the dual of the response tensor, $\tilde{H}^{\alpha\beta} = \frac{1}{2}\varepsilon^{\alpha\beta\gamma\delta} H_{\gamma\delta}$, with $\varepsilon^{0123} = 1$. We will use the real metric with $g_{00} = -1$. It is convenient to provide the expressions for the field tensors explicitly,

$$F_{\alpha\beta} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & B_z & -B_y \\ E_y & -B_z & 0 & B_x \\ E_z & B_y & -B_x & 0 \end{pmatrix}, \tag{2}$$

$$\tilde{H}^{\alpha\beta} = \begin{pmatrix} 0 & H_x & H_y & H_z \\ -H_x & 0 & -D_z & D_y \\ -H_y & D_z & 0 & -D_x \\ -H_z & -D_y & D_x & 0 \end{pmatrix}. \tag{3}$$

Thus, $F_{\alpha\beta} F^{\alpha\beta} = 2(B^2 - E^2)$, $F_{\alpha\beta} \tilde{H}^{\alpha\beta} = -4\,\mathbf{E} \cdot \mathbf{H}$. The pseudoscalar nature of the interaction is apparent from the last expression. The definitions of $F_{\alpha\beta}$ and $H_{\alpha\beta}$ are covariant; they hold in any inertial system.

With the combined coupling constant $g_{a\gamma\gamma}$ defined as

$$g_{a\gamma\gamma} = g_\gamma \frac{\alpha}{\pi} \frac{1}{f_a}, \tag{4}$$

we, thus, have, for the last term in the Lagrangian (1),

$$\mathcal{L}_{a\gamma\gamma} = g_{a\gamma\gamma} a(x)\,\mathbf{E} \cdot \mathbf{B}. \tag{5}$$

Based upon expression (1), the extended Maxwell equations take the following form,

$$\nabla \cdot \mathbf{D} = \rho - g_{a\gamma\gamma}\mathbf{H} \cdot \nabla a, \tag{6}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \dot{\mathbf{D}} + g_{a\gamma\gamma}\dot{a}\mathbf{H} + g_{a\gamma\gamma}\nabla a \times \mathbf{E}, \tag{7}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{8}$$

$$\nabla \times \mathbf{E} = -\dot{\mathbf{B}}. \tag{9}$$

Here, $(\rho, \mathbf{J})$ are the usual electromagnetic charge and current densities. The equations are thus far general; there are no restrictions on the spacetime variation of $a(x)$. The equations are again covariant, with respect to the shift of the inertial system.

3. Axion-Generated Electric Current in a Strong Magnetic Field

We now put $\rho = \mathbf{J} = 0$, and consider a geometrical setup essentially as the haloscope model [1–3], whereby a strong static magnetic field $\mathbf{B}_0$ acts in the vertical z direction. The dimension in the z direction is assumed to be infinite, while the dimensions in the other directions form a cylinder of radius R. It is now natural to employ SI units, whereby the dimension of the axion $a(t)$ becomes J (joules).

The generalized Maxwell equations given above reduce to their conventional form, except for Ampère's equation, which becomes modified to

$$\nabla \times \mathbf{H} = \frac{g_{a\gamma\gamma}}{c\mu} \dot{a} \mathbf{B_0}. \tag{10}$$

Here, we have taken into account that the term containing $\mathbf{B}_0$ is the dominant term on the right hand side. The equation allows us to regard the right hand side as an axion-generated electric current density, $\mathbf{J}_{\text{axion}}$, and we consider it on the same footing as the ordinary current density, which was called $\mathbf{J}$ above.

Now, we write the time dependence of the axion as $a(t) = a_0 e^{-i\omega_a t}$ with a_0 a constant. As mentioned above, the axion velocity is small, $v/c \sim 10^{-3}$, and thus the frequency ω_a becomes proportional to the mass, $\hbar\omega_a = m_a c^2$. In our numerical estimates, we will assume $m_a c^2 = 10\,\mu\text{eV}$ as a typical value. This means that $\omega_a = 1.52 \times 10^{10}\,\text{rad/s}$. This is a low value, thus, justifying the picture of the axion as a classical oscillating field.

Regarding the amplitude of a_0, we may, following the notation of [15] express $a(t)$ in terms of the angle $\theta(t)$ characterizing the QCD vacuum state,

$$a(t) = f_a \theta(t). \tag{11}$$

Taking the axion field to be real, $a(t) = a_0 \cos \omega_a t$, and similarly $\theta(t) = \theta_0 \cos \omega_a t$, we have, for the amplitudes, $a_0 = f_a \theta_0$. The magnitude of the axion current density can, thus, be written as (replacing the permeability with μ_0 for simplicity)

$$J_{\text{axion}}(t) = \frac{g_{a\gamma\gamma}}{c\mu_0} \dot{a}(t) B_0 = -\left(\frac{g_\gamma}{c\mu_0} \frac{\alpha}{\pi} \omega_a B_0 \right) \theta_0 \sin \omega_a t. \tag{12}$$

Neither the axion amplitude a_0 nor the axion decay constant f_a occur in this expression; the essential quantity being only their ratio $a_0/f_a = \theta_0$. Experimental information, such as that coming from the limits on the electric dipole moment for the neutron [16], indicates that the value of θ_0 is very small. We quote the explicit result given in [17]

$$\theta_0 \sim 3 \times 10^{-19}. \tag{13}$$

We will here consider θ_0 as a free parameter, without assigning a numerical value to it. Inserting the values already mentioned, $g_\gamma = 0.36$, $B_0 = 10\,\text{T}$, $\omega_a = 1.52 \times 10^{10}\,\text{rad/s}$, we obtain

$$J_{\text{axion}}(t) = -3.37 \times 10^5 \times \theta_0 \sin \omega_a t \quad [\text{A/m}^2]. \tag{14}$$

Let us go one step further in this direction, by exploiting that the local axion energy density is approximately $0.45\,\text{GeV/cm}^3$ [2]. Equating this to $(m_a c^2)N$ with $m_a c^2 = 10\,\mu\text{eV}$ and N the number density of axions, we obtain

$$N = 4.5 \times 10^{19}\,\text{m}^{-3}. \tag{15}$$

This makes it possible to introduce a fictitious effective electric charge e_{eff} per axion. We can write

$$e_{\text{eff}}(Nm_a)v = J_{\text{axion}},\tag{16}$$

whereby, with $v \sim 10^{-3}c$, we obtain the estimate

$$e_{\text{eff}} \sim 10^{21} \times \theta_0,\tag{17}$$

with dimension C (coulomb). This is a physically a huge number, even with $\theta_0 = 10^{-19}$. Let us, therefore, recall the background for this calculation: there are reasonable parameters behind the axion current density (14), and there is common agreement regarding the axion energy density being around $0.45\,\text{GeV}/\text{cm}^3$. The axion number density (15) also appears reasonable. It is, thus, an open question whether the expression (17) has a physical meaning; the very idea of associating axions with a fictitous electric charge may be untenable. For the effective charge to be of the same order of magnitude as the electron charge, the value of θ_0 would have to be many orders of magnitude smaller than commonly assumed.

4. Spatially Varing Axion and Casimir-Like Effect

We will now investigate a typical case where the axion field a is constant in time but varies with position. For definiteness we adopt the usual geometric setup characteristic for Casimir investigations, namely two large and parallel metal plates separated by a gap L. We assume a zero temperature. In the region between the plates, we assume that $a(z)$ increases linearly with respect to the direction z orthogonal to the plates,

$$a(z) = \frac{a_0 z}{L}, \quad 0 < z < L,\tag{18}$$

where a_0 is the fixed axion value at the plate $z = L$. Outside the plates, we assume for definiteness that the values of a are constant: $a = 0$ for $z < 0$ and $a = a_0$ for $z > a$.

First, let us manipulate the generalized Maxwell equation above to obtain the field equations for the electric and magnetic fields (now in the Heaviside–Lorentz system of units again),

$$\nabla^2 \mathbf{E} - \varepsilon\mu\ddot{\mathbf{E}} = \frac{1}{\varepsilon}\nabla\rho + \mu\dot{\mathbf{J}} + g_{a\gamma\gamma}\frac{\partial}{\partial t}[\dot{a}\mathbf{B} + \mu\nabla a \times \mathbf{E}],\tag{19}$$

$$\nabla^2 \mathbf{H} - \varepsilon\mu\ddot{\mathbf{H}} = -\nabla \times \mathbf{J} - g_{a\gamma\gamma}\nabla\times[\dot{a}\mathbf{H} + \nabla a \times \mathbf{E}].\tag{20}$$

These equations can be simplified if we omit second order derivatives of the axion, which means time derivatives $\ddot{a}$, space derivatives $\partial_i\partial_j a$, as well as the mixed $\partial_i\dot{a}$. Certain manipulations then give us the reduced field equations

$$\nabla^2 \mathbf{E} - \varepsilon\mu\ddot{\mathbf{E}} = \frac{1}{\varepsilon}\nabla\rho + \mu\dot{\mathbf{J}} + g_{a\gamma\gamma}[\dot{a}\dot{\mathbf{B}} + \mu\nabla a \times \dot{\mathbf{E}}],\tag{21}$$

$$\nabla^2 \mathbf{H} - \varepsilon\mu\ddot{\mathbf{H}} = -\nabla \times \mathbf{J} - g_{a\gamma\gamma}\left[\dot{a}\nabla \times \mathbf{H} + \frac{\rho}{\varepsilon}\nabla a - [(\nabla a) \cdot \nabla]\mathbf{E}\right]\tag{22}$$

Now, we put $\rho = \mathbf{J} = 0$, and observe the condition (18) on the axion field. Equations (21) and (22) reduce to

$$\nabla^2 \mathbf{E} - \varepsilon\mu\ddot{\mathbf{E}} = g_{a\gamma\gamma}\frac{\mu a_0}{L}\hat{\mathbf{z}} \times \dot{\mathbf{E}},\tag{23}$$

$$\nabla^2 \mathbf{H} - \varepsilon\mu\ddot{\mathbf{H}} = g_{a\gamma\gamma}\frac{a_0}{L}\partial_z\mathbf{E}.\tag{24}$$

Going over to Fourier space, with $\mathbf{E} = \mathbf{E}_0 \exp[i(\mathbf{k} \cdot \mathbf{r} - \omega t)]$, we obtain, from Equation (23), the component equations

$$(-\mathbf{k}^2 + \varepsilon\mu\omega^2)E_x - g_{a\gamma\gamma}\frac{\mu a_0}{L}(i\omega)E_y = 0,\tag{25}$$

$$(-\mathbf{k}^2 + \varepsilon\mu\omega^2)E_y + g_{a\gamma\gamma}\frac{\mu a_0}{L}(i\omega)E_x = 0, \tag{26}$$

$$(-\mathbf{k}^2 + \varepsilon\mu\omega^2)E_z = 0. \tag{27}$$

These equations show that there are two dispersive branches. The first, following from Equation (27), is the common branch in axion-free electrodynamics,

$$|\mathbf{k}| = \sqrt{\varepsilon\mu}\,\omega, \quad k_z = \frac{\pi n}{L}, \quad n = 1, 2, 3... \tag{28}$$

The second branch follows from Equations (25) and (26) as

$$\mathbf{k}^2 = \varepsilon\mu\omega^2 \pm g_{a\gamma\gamma}\frac{\mu a_0\omega}{L}. \tag{29}$$

This branch is, thus, composed of two modes, lying very close to the first mode above. For a given ω, there are in all three different values of $|\mathbf{k}|$. As $g_{a\gamma\gamma}$ is very small, we may replace ω with $|\mathbf{k}|/\sqrt{\varepsilon\mu}$ in the last term in the last equation and solve with respect to ω,

$$\omega = \frac{1}{\sqrt{\varepsilon\mu}}\left[|\mathbf{k}| \pm g_{a\gamma\gamma}\frac{a_0}{2L}\sqrt{\frac{\mu}{\varepsilon}}\right], \tag{30}$$

neglecting the terms of order $g_{a\gamma\gamma}^2$. This kind of splitting of one of the branches into two slightly separated modes is encountered also in the analogous formalisms given in [4,5].

Let us calculate the zero-point energy $\mathcal{E}$ of the field, considering the second branch (30) only, since this is the primary interest. We will consider scalar electrodynamics, meaning that the vector nature of the photons is accounted for but not their spin. At temperature $T = 0$, the energy is $\frac{1}{2}\sum\omega$. We write the energy in the form

$$\mathcal{E} = \frac{1}{2\sqrt{\varepsilon\mu}}\sum_{n=1}^{\infty}\left[\int\frac{d^2k_{\perp}}{(2\pi)^2}\sqrt{k_{\perp}^2 + \frac{\pi^2 n^2}{L^2}} \pm \frac{\pi\beta}{L}\right], \tag{31}$$

where we have defined β as

$$\beta = g_{a\gamma\gamma}\frac{a_0}{2\pi}\sqrt{\frac{\mu}{\varepsilon}}. \tag{32}$$

For the small axion-related part of the energy, we omitted the continuous part involving $\mathbf{k}_{\perp}$.

The first term in the expression (31) can be evaluated using dimensional regularization (for instance, Ref. [18]). Replacing the transverse spatial dimension with a general d, we can write the first term, called $\mathcal{E}_I$, as

$$\mathcal{E}_I = \frac{1}{2\sqrt{\varepsilon\mu}}\sum_{n=1}^{\infty}\int\frac{d^d k_{\perp}}{(2\pi)^d}\int_0^{\infty}\frac{dt}{t}t^{-1/2}\exp\left[-t\left(k_{\perp}^2 + \frac{\pi^2 n^2}{L^2}\right)\right]\frac{1}{\Gamma(-\frac{1}{2})}, \tag{33}$$

where Γ is the gamma function with $\Gamma(-1/2) = -2\sqrt{\pi}$. We employed the Schwinger proper time representation of the square root. We integrate over $\mathbf{k}_{\perp}$,

$$\int\frac{d^d k_{\perp}}{(2\pi)^d}e^{-tk_{\perp}^2} = \frac{t^{-d/2}}{(4\pi)^{d/2}}, \tag{34}$$

so that

$$\mathcal{E}_I = -\frac{1}{4\sqrt{\pi\varepsilon\mu}}\frac{1}{(4\pi)^{d/2}}\sum_n\int_0^{\infty}\frac{dt}{t}t^{-1/2-d/2}\exp\left(-\frac{t\pi^2 n^2}{L^2}\right). \tag{35}$$

The sum over n can now be evaluated,

$$\mathcal{E}_I = -\frac{1}{4\sqrt{\pi\varepsilon\mu}}\frac{1}{(4\pi)^{d/2}}\left(\frac{\pi}{L}\right)^{d+1}\Gamma\left(-\frac{d+1}{2}\right)\zeta(-d-1), \tag{36}$$

where ζ is the Riemann zeta function.

We can now take into account the reflection property

$$\Gamma\left(\frac{z}{2}\right)\zeta(z)\pi^{-z/2} = \Gamma\left(\frac{1-z}{2}\right)\zeta(1-z)\pi^{(z-1)/2}, \tag{37}$$

to obtain

$$\mathcal{E}_I = -\frac{1}{2^{d+2}\pi^{d/2+1}}\frac{1}{\sqrt{\varepsilon\mu}}\frac{1}{L^{d+1}}\Gamma(1+\frac{d}{2})\zeta(2+d). \tag{38}$$

Substituting $d = 2$ and using $\zeta(4) = \pi^4/90$, we obtain for the total zero-point energy

$$\mathcal{E} = \frac{1}{\sqrt{\varepsilon\mu}}\left[-\frac{\pi^2}{1440}\frac{1}{L^3} \pm \frac{\pi\beta}{L}\zeta(0)\right]. \tag{39}$$

The last term is evidently the small correction from the axions propagating in the z direction. We will regularize the term simply by using the analytically continued zeta function, as this recipe has turned out to be effective and correct under the usual physical conditions in spite of a lack of mathematical rigor. Thus, we substitute $\zeta(0) = -1/2$, and obtain

$$\mathcal{E} = \frac{1}{\sqrt{\varepsilon\mu}}\left[-\frac{\pi^2}{1440}\frac{1}{L^3} \mp \frac{\pi\beta}{2L}\right]. \tag{40}$$

This is the total Casimir energy as it is dependent on the gap L. The Casimir pressure on the plates follows as $P = -\partial\mathcal{E}/\partial L$, and is attractive.

Of main interest, however, is the contribution from particles (photons and axions) moving in the transverse direction z. We call this the Casimir energy $\mathcal{E}_C$. From Equation (31), we see that this amounts to extracting the terms

$$\mathcal{E}_C = \frac{1}{2\sqrt{\varepsilon\mu}}\frac{\pi}{L}\sum_{n=0}^{\infty}(n\pm\beta). \tag{41}$$

This brings us to the Hurwitz zeta function, originally defined as

$$\zeta_H(s,a) = \sum_{n=0}^{\infty}(n+a)^{-s}, \quad (0 < a < 1, \quad \Re s > 1). \tag{42}$$

This function often turns up in Casimir-like problems (for instance, [19–21]). The function has a simple pole at $s = 1$. When $\Re s$ differs from unity, the function is analytically continued to the complex plane. For practical purposes, one needs only the property

$$\zeta_H(-1,a) = -\frac{1}{2}\left(a^2 - a + \frac{1}{6}\right). \tag{43}$$

Thus, we obtain, when omitting the small β^2 term,

$$\mathcal{E}_C = \frac{1}{4\sqrt{\varepsilon\mu}}\left(-\frac{\pi}{6L} \pm \frac{\pi\beta}{L}\right). \tag{44}$$

The first term in this expression comes from the scalar photons propagating in the z direction (the transverse oscillations of a closed uniform string of length L has a Casimir energy of $-\pi/(6L)$ [21]). The second term is the axionic contribution. Recall from Equation (32) that β is independent of L. As for the L dependence, the Casimir energies for the one-dimensional electrodynamic and the axion parts behave similarly, as one would expect.

In the above equations, the upper and lower signs match each other. In Equation (44), the small increase of the Casimir energy because of the axions comes from the particular mode in the dispersion relation (30) that is superluminal (meaning that the group velocity

is larger than $1/\sqrt{\varepsilon\mu}$). This mode corresponds to a weak repulsive Casimir force. The other mode corresponds to a weak attractive force.

We examined the two closely separated modes individually. These modes are physically real, contributing with equal though opposite contributions to the pressure on the plates. In a standard Casimir setup in which only the total pressure is measured, this axionic contribution will, thus, level out. There might be other cases in the future, however, where these small effects from the modes could be measurable. The axion-generated eigenmode splitting is of basic physical interest.

5. Conclusions

The current information from astrophysics, implying $a = a(t)$, indicates that the axions are slowly moving particles in a relativistic sense. In a strong magnetic field, as dealt with briefly in Section 3, the axions give rise to a very weak fluctuating electric current, parallel to the magnetic field. From a physical viewpoint this is quite striking, as an electric current flowing in a medium with a complex refractive index necessarily leads to energy dissipation, and, in our case, the axions are without electric charge.

In Section 4, we investigated the effects from time-independent but spatially varying axions in a standard Casimir configuration between two parallel plates. A zero temperature was assumed. Our formalism was limited to scalar electrodynamics. An important point from a physical viewpoint is the axion-generated splitting of the eigenmodes, resulting in two closely lying modes contributing to the Casimir pressure with equal magnitudes, but of opposite sign. One mode is superluminal corresponding to a weak repulsive pressure, while the other mode is subluminal and corresponds to a weak attractive pressure.

Data Availability Statement: Samples of the compounds are available from the authors.

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