Energy Systems
https://doi.org/10.1007/s12667-022-00505-8

ORIGINAL PAPER

A real options analysis of existing green energy facilities: maintain or replace?

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Received: 14 October 2020 / Accepted: 3 February 2022

Abstract
We consider an operator of machinery with deteriorating efficiency, facing the problem of optimally timing of either a minor (maintenance) investment or a major (replacement) investment under price uncertainty. If a maintenance investment is chosen, the efficiency of the machinery will deteriorate more slowly, and replacing later is still possible. The optimal decision rule is expressed in the form of thresholds for long-run prices, indicating that it may be rational to wait to see which of the large and small investment is the better choice. We relate the setting to repowering of green energy facilities, such as hydropower plants and wind farms. Our analysis provides several managerial insights. We characterize the conditions that govern whether the smaller investment should be considered at all, and we quantify the effect of having a replacement option embedded in a maintenance option. Our analysis demonstrates that the large investment may get postponed significantly in expectation, which recognizes maintenance as a temporary alternative to replacement.

Keywords Green energy · Maintenance · Real options · Replacement

1 Introduction

Owners of assets with deteriorating performance often have a range of possible actions to choose from in order to increase future expected profits. Replacements are often needed, either when the asset suffers from severe deterioration, the operating requirements change, or when new technology is available. Determining the time of replacing existing assets involves economic analysis of various drivers, such as the current value of the existing asset, operation and maintenance costs, and the cost and value associated with replacing the asset with an improved one [4, 15, 22]. As an alternative to replacing, maintaining existing assets can extend the useful economic

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Published online: 03 March 2022

life and often incurs lower immediate costs. Moreover, similar to replacement, many maintenance tasks have discretion over timing, uncertain benefits, and irreversible costs. Both maintenance and replacement actions can therefore be viewed as exercising real options [11], as opposed to the traditional view, which casts maintenance to be performed until marginal benefits equal marginal costs [5].

The problem of determining maintenance and replacement schedules under price uncertainty is of particular importance for operators of existing green energy facilities, such as wind farms and hydropower plants. Many hydropower plants in the European Union, United States, and Canada were built in the early to mid twentieth century, and many of these plants suffer from inefficiencies in power production [13, 23]. Moreover, the number of wind turbines in service are expected to increase over the coming decades [38], making it increasingly important to assess the value of performance-enhancing activities with uncertain benefits. Particular focus has been given to the replacement option, which is the process of replacing existing machinery or equipment with new ones that have higher capacity and/or efficiency. We refer to this process as *repowering*. Repowering leads to increased energy or power output, which has a positive effect on future profits for the operator. Eventually, repowering will appear as an attractive project to undertake, but such an investment is costly. In practice, different maintenance investment possibilities can be undertaken to postpone repowering. As an example, maintenance investments in hydropower plants include rehabilitation of existing turbines through surface treatment and hard coating [17, 42], and changes in the prevailing operational pattern to mitigate the damaging effects already inflicted on the machinery.[1] Similar preventive tasks can be executed for wind turbines, where the upper cutoff point for high wind speeds is reduced to extend the economically useful life.

In this paper, we consider performance-enhancing activities, such as maintenance and replacement, as real options and analyze the range of flexibility that is offered by joint valuation of projects of different scales. We specify and analyze two mutually exclusive options: (i) A replace-only option, and (ii) a compound option where first maintenance is undertaken before replacement at a later point in time. The first investment alternative we consider is renewal/replacement. Upon renewal, the efficiency, or the profitability associated with the asset, is reset, while the market price is exogenous and unchanged. The second investment alternative we examine is a compound option where first maintenance can be undertaken, while keeping the replacement option alive. In our analysis, maintenance is an investment that reduces the efficiency deterioration rate and thus alters the drift of the underlying stochastic profit process. Our study provides a novel framework that incorporates the managerial flexibility which is present when an operator faces a deteriorating technical sub-system of existing facilities in the presence of uncertain market prices. In our framework, uncertain market prices are modelled as a Geometric Brownian motion

[1] The latter is often done in practice, where the operator lets go of potentially higher revenues, e.g. by reducing loads or starting the machinery less frequently, in order to maintain its facility and hence postpone a larger investment.

(GBM), while having a deterministically declining efficiency, which has a negative effect on the profit stream.

The paper is organized as follows. We review the relevant literature in Sect. 1.1. In Sect. 2 we present real option models. In Sect. 3 we characterize the optimal values, optimal policies and we analytically compute some comparative statics. In Sect. 4 we conduct numerical experiments, expanding the analytical work in Sect. 3. Concluding remarks are provided in Sect. 5.

1.1 Contributions in light of existing literature

Early contributions to the real options literature that focus on replacement decisions include Refs. [33], [11, Chapter 4], and [32], among others. Further developments in this literature can be divided into two main categories, namely, capital replacement of physical assets ([36, 44]), and asset renewals in general [1, 35].[2] The former focuses on minimizing losses incurred by having an imperfect component, whereas the latter studies the problem of maximizing the net profit by balancing the revenue from the component with its operational and maintenance costs. Our work fits into the latter category. Differently from Ref. [1], only the profitability of the firm's infrastructure resets upon renewal, while we assume that the market price is unchanged, which is similar to Ref. [35]. In contrast to Ref. [35], we assume the efficiency to be deterministic, which allows us to value different investment alternatives analytically. We contribute to this literature by having the possibility to postpone renewal by investing in a smaller maintenance project.

Maintenance policies have traditionally been studied from an industrial engineering perspective, where maintenance often is optimized with criteria such as reliability, availability, work safety, and maintenance cost [16]. A limitation of traditional maintenance optimization models is that they often do not take into account market uncertainty. Jin et al. [24] addresses this limitation and proposes a methodology based on option pricing theory for joint scheduling of production and preventive maintenance under uncertain demand. Although the maintenance option in our framework has different characteristics, we view maintenance in a similar light as Ref. [24], and consider maintenance as a real option. However, unlike Ref. [24], we study the interaction between a maintenance option and a replacement option when the associated profits are uncertain, which is new to the real options literature on asset management.

By focusing on several options, we contribute to the stream of literature that studies mutually exclusive options. More specifically, we complement the literature that studies the types of problems introduced by Ref. [9] where the investment policy is not merely a simple trigger strategy, but may instead be governed by an investment region that is no longer a connected set. Examples of works that have studied these types of problems include Bobtcheff and Villeneuve [6] who analyze mutually exclusive projects under

[2] The heuristic [1] employ, also used by Ref. [37], has been shown not to always lead to correct solutions, see e.g. Refs. [8, 27].

input price and output price uncertainty, and Adkins and Paxson [2] who analyze a model with stochastic price and deterministic declining output flow and implications on choices of exit and new technologies involving different flow rates. In line with the literature on mutually exclusive options, we analyze how features of our model affect investment triggers when correctly accounting for the full set of choices available.

An important feature of our model is that exercising the maintenance option entails a change in the drift of the underlying profit flow process, which is similar to Refs. [20, 21, 26]. In this sense, we contribute to the real options literature where the firm has a one-time opportunity to boost the profit rate. Kwon [26] consider a firm which has the opportunity to innovate an ageing product while facing a declining profit stream. At any point in time, the firm can choose to continue operations or exit. A predefined change in drift that boosts the stream of profits if the firm chooses to innovate. The main findings are that the threshold for exiting decreases in volatility and that the threshold for investing might decrease in volatility if the profit boost from investing is sufficiently large. Hagspiel et al. [21] extend the analysis and show monotonicity of exercise threshold in volatility numerically if the firm can choose the capacity when investing. Similar to this literature, we provide a comparative statics analysis. We examine how the maintenance option, which upon exercise reduces the deterioration rate, or equivalently, boosts the profit stream, affect investment triggers, waiting regions and expected hitting times.

Finally, our case study provides novel insights for real options applications in green energy. Existing literature on the topic include Linnerud [29] who find that the investment behavior of professional developers of hydropower projects is consistent with real options theory. Moreover, Rohlfs and Madlener [37] use a real option approach to value different technologies in the energy sector, including photovoltaics, wind, hydro, coal- and gas-fired power plants, among others, and Fleten et al. [14] focus on capacity choice and investment timing in a case where a local load is to be served, and only surplus power is sold in the market. From this perspective, our study fits in the growing literature on real option valuation in green energy, see, e.g. Ref. [7] or the survey by Ref. [25]. We specifically study mutually exclusive projects with applications in green energy. Other papers that have studied this include Siddiqui and Fleten [39] who consider a firm that may choose to deploy an existing green energy technology, or switch to an unconventional energy technology. Moreover, similar to us, Detemple and Kitapbayev [10] study mutually exclusive projects with different cost structures. In Ref. [10], both the cost and revenue of a project are stochastic, described by two distinct correlated geometric Brownian motions. Methodologically, our work is similar, but we focus on investment alternatives allowing to cope with the deteriorating efficiency of existing green energy facilities rather than cost uncertainty related to undertaking new projects. In contrast to our work, neither efficiency deterioration nor maintenance are considered.

2 Models

The operator can choose to undertake the following *projects*: Maintenance investment and replacement. Moreover, the firm can choose between the following *mutually exclusive options*:

1. Replace-only option: Replace the existing machinery by new machinery at a fixed cost I_R.
2. Compound option: First, invest in maintenance of existing machinery at a fixed cost $I_M < I_R$, followed by a replacement of the existing (maintained) machinery at cost I_R.[3]

The operator needs to carefully select which of these options to choose. The correct solution of this problem, joint valuation, takes the form of decision rules depending on thresholds for the price. We assume that the operator is price-taking and that investments are made instantaneously, meaning that there is no investment lag and no shutdown time associated with the execution of any of the projects.[4]

Cashflows from green energy facilities come from electricity generation. As variable operating costs for a green energy operator are typically very small, we consider variable costs to be negligible. We let the profit flow, $\pi(t)$, consist of three components: Electricity price $P(t)$, machinery efficiency $Q(t)$, and production quantity $R(t)$,

$$\pi(t) = P(t)Q(t)R(t). \tag{1}$$

We assume that the electricity price, $P(t)$, follows a GBM,

$$dP(t) = \alpha P(t)dt + \sigma P(t)dZ(t), \tag{2}$$

where α is the drift and $dZ(t)$ is the increment of a Wiener process. The volatility is denoted by $\sigma > 0$. The choice of a GBM is supported by Pindyck [34] who indicates that applying a GBM for the price of a commodity is an appropriate choice when considering long-term investments. Similarly, Fleten et al. [14] argue that although using a GBM to model price dynamics ignores short-term mean reversion in prices, the short-term mean reversion has a minor influence on long-term investment decisions. Alternatives to GBM for modeling electricity prices are discussed in Ref. [31].

The second component, the production quantity, is denoted by $R(t)$. The supply of green energy facilities, e.g. wind for wind farms or inflow to water reservoirs for hydro producers, are by nature stochastic. However, there is typically a very small memory effect in supply, meaning that this year's supply is a poor predictor of the next year's supply. Therefore, we consider the instantaneous production quantity to be deterministic and normalized to 1. Thus, the production quantity is given by

[3] Maintenance required to keep the machinery available on a day-to-day basis is not considered as a maintenance investment in our model. We consider any other activity that enhances the performance of existing machinery, such as e.g. surface treatment, coating of turbine blades, or actively protecting components by reducing maximum load in certain periods, as a maintenance investment. We account for the latter activity by allowing a certain fraction of profit to be lost.

[4] In practice these shutdown times vary with the project size, but we regard them as negligible in our analysis as they typically are short compared to expected project lifetimes.

$$R(t) = \begin{cases} 1 & t \le \tau_1, \\ 1 - k & \tau_1 < t \le \tau_2, \\ 1 & t > \tau_2, \end{cases} \tag{3}$$

where τ_1 is the time when a maintenance investment is undertaken, and τ_2 is the time when replacement is undertaken. These stopping times are unknown in advance. The parameter k captures the loss of revenues from changed operational pattern by undertaking the maintenance investment, where k determines the lost fraction of revenues between τ_1 and τ_2.[5]

The third component that determines the profits in our model is the efficiency of the machinery, $Q(t)$, which we define as

$$Q(t) = \begin{cases} Q_E e^{-\gamma_E t} & t \le \tau_1, \\ Q_E e^{-\gamma_M t} & \tau_1 < t \le \tau_2, \\ Q_R e^{-\gamma_E t} & t > \tau_2, \end{cases} \tag{4}$$

where γ_E is the efficiency deterioration rate of the existing and replaced machinery, and $\gamma_M < \gamma_E$ is the efficiency deterioration rate after maintenance. Parameters Q_E and Q_R are the initial efficiency of the existing and replaced machinery, respectively. By solving the differential equation in (2), using expressions for $R(t)$ and $Q(t)$ in (3) and (4), and inserting into (1), we obtain instantaneous profits using the original machinery, the original machinery after a maintenance investment, and using a replaced, i.e. new machinery, respectively,

$$\pi(t) = \begin{cases} Q_E p e^{(\alpha - \gamma_E - \frac{\sigma^2}{2})t + \sigma Z(t)} & t \le \tau_1, \\ Q_E p(1-k) e^{-\tau_1(\gamma_E - \gamma_M)} e^{(\alpha - \gamma_M - \frac{\sigma^2}{2})t + \sigma Z(t)} & \tau_1 < t \le \tau_2, \\ Q_R p e^{\tau_2 \gamma_E} e^{(\alpha - \gamma_E - \frac{\sigma^2}{2})t + \sigma Z(t)} & t > \tau_2, \end{cases}$$

where $P(0) = p$. The factor $e^{-\tau_1(\gamma_E - \gamma_M)}$ adjusts for the deterioration process before maintaining at time τ_1, and the factor $e^{\tau_2 \gamma_E}$ resets the deterioration process when the machinery is replaced at time τ_2. Figure 1 illustrates price simulations and corresponding profit flow simulations. In the upper panel, three price scenarios are generated from the GBM in (2), and the dashed line is the expected price. The dashed line in the second upper panel is the efficiency, $Q(t)$, and the second lower panel shows the production quantity $R(t)$. The profit flow, $\pi(t)$, in the bottom panel is the product of price, efficiency and production quantity. Lost profit from changed operations after maintenance, k, is apparent by the vertical downward shift at τ_1, seen in the production panel. Moreover, we observe that profits are expected to increase at a higher rate (dashed line) between the stopping times τ_1 and τ_2, because $\gamma_M < \gamma_E$.

[5] Changed operational pattern means that the operator chooses to deviate from the optimal production policy to reduce the deterioration rate of the machinery.

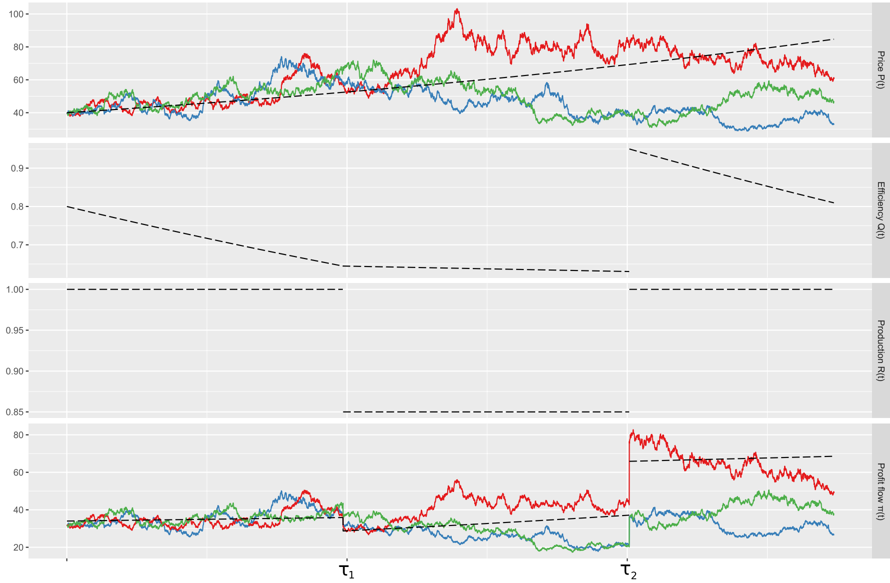


Fig. 1 Illustration of dynamics of prices and profits, with numerical values $P_0 = 40$, $Q_E = 0.80$, $Q_R = 0.95$, $\alpha = 0.025$, $\gamma_E = 0.04$, $\gamma_M = 0.004$. Colored lines in the upper panel represent price simulations, and the profit flow for each simulated price path is illustrated in the lower panel. Times τ_1 and τ_2 are times where maintenance and replacement takes place, respectively, and are unknown in advance

Using instantaneous profits, we can formulate the optimal stopping problem for an operator of machinery with deteriorating efficiency, facing the problem of optimally timing of the maintenance investment project or the replacement project. We first specify the optimal stopping problem for each of the alternatives, and then incorporate both in the same framework.

The optimal stopping problem for the replace-only option can be formulated as

$$\begin{aligned} F_R(p) = \sup_{\tau_2} \mathbb{E}\Bigg[\int_0^{\tau_2} e^{-\rho t} Q_E p e^{(\alpha - \gamma_E - \frac{\sigma^2}{2})t + \sigma Z(t)} \mathrm{d}t - I_R e^{-\rho \tau_2} \\ + \int_{\tau_2}^{\infty} e^{-\rho t} Q_R p e^{\tau_2 \gamma_E} e^{(\alpha - \gamma_E - \frac{\sigma^2}{2})t + \sigma Z(t)} \mathrm{d}t \, \Big| P(0) = p \Bigg]. \end{aligned} \tag{5}$$

We assume that the decision-maker discounts the future profit at a constant exogenous rate, $\rho > \alpha - \gamma_E$. This assumption ensures that it would never be optimal to delay exercise either of the options forever, as would be the case if expected growth exceeds the discount factor.

The optimal stopping problem for the compound option is given by

$$
\begin{aligned}
G(p) = \sup_{\tau_1,\tau_2>\tau_1} \mathbb{E}\Bigg[& \int_0^{\tau_1} e^{-\rho t} Q_E p e^{(\alpha-\gamma_E-\frac{\sigma^2}{2})t+\sigma Z(t)} \mathrm{d}t - I_M e^{-\rho\tau_1} \\
& + \int_{\tau_1}^{\tau_2} e^{-\rho t} Q_E p(1-k) e^{-\tau_1(\gamma_E-\gamma_M)} e^{(\alpha-\gamma_M-\frac{\sigma^2}{2})t+\sigma Z(t)} \mathrm{d}t - I_R e^{-\rho\tau_2} \\
& + \int_{\tau_2}^{\infty} e^{-\rho t} Q_R p e^{\tau_2\gamma_E} e^{(\alpha-\gamma_E-\frac{\sigma^2}{2})t+\sigma Z(t)} \Big| P(0)=p \Bigg].
\end{aligned} \tag{6}
$$

Note that in this problem, the maintenance action must be undertaken before renewal/replacement. The next problem takes into account that the maintenance action can be skipped:

$$
\begin{aligned}
H(p) = \sup_{\tau_1,\tau_2\geq\tau_1} \mathbb{E}\Bigg[& \int_0^{\tau_1} e^{-\rho t} Q_E p e^{(\alpha-\gamma_E-\frac{\sigma^2}{2})t+\sigma Z(t)} \mathrm{d}t - I_M e^{-\rho\tau_1} \mathbb{1}_{\{\tau_1<\tau_2\}} \\
& + \int_{\tau_1}^{\tau_2} e^{-\rho t} Q_E p(1-k) e^{-\tau_1(\gamma_E-\gamma_M)} e^{(\alpha-\gamma_M-\frac{\sigma^2}{2})t+\sigma Z(t)} \mathrm{d}t - I_R e^{-\rho\tau_2} \\
& + \int_{\tau_2}^{\infty} e^{-\rho t} Q_R p e^{\tau_2\gamma_E} e^{(\alpha-\gamma_E-\frac{\sigma^2}{2})t+\sigma Z(t)} \Big| P(0)=p \Bigg].
\end{aligned} \tag{7}
$$

The firm can either (1) choose $\tau_1 < \tau_2$ which corresponds to the compound option or (2) choose $\tau_1 = \tau_2$ which corresponds to the replace-only option. If $\tau_1 = \tau_2$ the second term in (7), which is the time spent using the machinery between the maintenance investment and replacement investment, becomes zero, and the firm does not have to pay any maintenance investment costs, which is ensured by the indicator function $\mathbb{1}_{\{\tau_1<\tau_2\}}$. The only difference between (6) and (7) is that the firm can choose $\tau_1 = \tau_2$ and replace directly without maintaining first.

3 Characterization of optimal policies and values

This section presents the solutions to (5), (6), and (7). We first analyze the replace-only option and the compound option separately, and then provide an analysis when both are considered in the same framework, as defined in (7). For ease of notation, we define $\mu_E = \rho - \alpha + \gamma_E$ and $\mu_M = \rho - \alpha + \gamma_M$. Similar to Ref. [1], we aim at identifying the economic conditions that trigger a renewal to restore the economic potential of the machinery. In their framework, the option to renew the asset appears attractive when the revenue stream from the existing one is low, similar to a put option. This is because exercising the renewal option entails the output revenue being restored to the original value. However, in our case, a renewal appears attractive when the profit associated with the existing machinery is high, i.e. we view the replacement option as a call option, as opposed to Ref. [1]. The difference is that, in our case, it is the price that drives the profitability of any investment alternative. Replacing when the price is low will not appear attractive as the firm has to pay a sunk cost, which will not be covered by the profit stream of the renewed machinery.

3.1 Replace-only option

Proposition 1 gives the value of the option to replace the machinery defined in (5).

Proposition 1 *It is optimal for the firm to replace its machinery as soon as $P(t)$ reaches the optimal threshold, given by*

$$p_R^* = \frac{\beta_E}{\beta_E - 1} \cdot \frac{\mu_E}{Q_R - Q_E} I_R, \tag{8}$$

where

$$\beta_E = \frac{1}{2} - \frac{\alpha - \gamma_E}{\sigma^2} + \sqrt{\left(\frac{\alpha - \gamma_E}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}}. \tag{9}$$

Thus, the value of the option to replace the existing machinery is given by

$$F_R(p) = \begin{cases} A_1 p^{\beta_E} + \frac{Q_E p}{\mu_E} & \text{if } p < p_R^*, \\ \frac{Q_R p}{\mu_E} - I_R & \text{if } p \geq p_R^*, \end{cases} \tag{10}$$

where

$$A_1 = \frac{I_R}{\beta_E - 1} \left[\frac{\beta_E - 1}{\beta_E} \cdot \frac{Q_R - Q_E}{\mu_E} \cdot \frac{1}{I_R} \right]^{\beta_E}. \tag{11}$$

This option represents a single investment opportunity, and therefore closely resembles the solution of a standard real option problem, e.g. as in Ref. [11]. The values in the continuation region, $p < p_R^*$, and in the stopping region, $p \geq p_R^*$ are given in (10). In the waiting region, the last term is the perpetual profit without any investment, whereas the first term represents the value of the option to improve the efficiency once the profit is large enough. When the replacement option is exercised, the producer operates with increased efficiency of the machinery, Q_R, and with a deterioration rate γ_E.

3.2 Compound option: maintenance before replacement

The second option, the compound option, is a constrained sequential option where the maintenance project needs to be undertaken before the replacement project. The optimal stopping problem is formulated in (6). We solve this problem backwards, where Proposition 2 gives the value of the replacement option, provided that the maintenance project already is undertaken.

Proposition 2 *With the maintenance investment option already exercised, it is optimal for the firm to replace its existing machinery as soon as $P(t)$, $t > \tau_1$, reaches the optimal threshold given by*

$$p^*_{M,R} = I_R \frac{\beta_M}{\beta_M - 1} \cdot \frac{\mu_E \mu_M}{Q_R \mu_M - (1-k) Q_E \mu_E}, \tag{12}$$

where

$$\beta_M = \frac{1}{2} - \frac{\alpha - \gamma_M}{\sigma^2} + \sqrt{\left(\frac{\alpha - \gamma_M}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}}. \tag{13}$$

Thus, the value of the option to replace the existing machinery, after having maintained it, is given by

$$G_R(p) = \begin{cases} B_2 p^{\beta_M} + \frac{(1-k)Q_E p}{\mu_M} & \text{if } p < p^*_{M,R}, \\ \frac{Q_R p}{\mu_E} - I_R & \text{if } p \geq p^*_{M,R}, \end{cases} \tag{14}$$

where

$$B_2 = \frac{I_R}{\beta_M - 1} \left[\frac{\beta_M - 1}{\beta_M} \cdot \frac{Q_R \mu_M - (1-k) Q_E \mu_E}{\mu_E \mu_M} \cdot \frac{1}{I_R} \right]^{\beta_M}. \tag{15}$$

The solution in Proposition 2 is very similar to the replace-only option, but in this case the profit flow in the continuation region $p < p^*_{M,R}$ is affected by the maintenance investment option being exercised beforehand. The value in the stopping region $p \geq p^*_{M,R}$ coincide with the perpetual revenues of the replace-only alternative in the stopping region in (10), i.e. the value when $p > p^*_R$. As the sequential investment alternative eventually will lead to a replacement of the machinery, the trade from going from a maintained state to a replaced state of the machinery must entail a net positive increase in the operating profits. If not, the option to replace after maintenance investment will have no value. This can be expressed as follows:

$$(1-k) Q_E \mu_E < Q_R \mu_M. \tag{16}$$

The value of the option to invest in the first stage, i.e. to undertake a maintenance investment on the existing machinery, is presented in Proposition 3.

Proposition 3 *It is optimal for the firm to invest in maintenance of its existing machinery as soon as $P(t), t < \tau_1$, reaches the optimal threshold p^*_M which implicitly solves the equation given by*

$$B_2 \frac{\beta_E - \beta_M}{\beta_E} {p^*_M}^{\beta_M} + \frac{\beta_E - 1}{\beta_E} \cdot \frac{Q_E\big((1-k)\mu_E - \mu_M\big)}{\mu_E \mu_M} p^*_M - I_M = 0, \tag{17}$$

where β_E and β_M are given in (9) and (13), respectively. Thus, the value of the option is given by

$$G_M(p) = \begin{cases} B_1 p^{\beta_E} + \frac{Q_E p}{\mu_E} & \text{if } p < p_M^*, \\ G_R(p) - I_M & \text{if } p \geq p_M^*, \end{cases} \tag{18}$$

where

$$B_1 = B_2 \frac{\beta_M}{\beta_E} p_M^{*\,\beta_M - \beta_E} + \frac{Q_E}{\beta_E} \cdot \frac{(1-k)\mu_E - \mu_M}{\mu_E \mu_M} p_M^{*\,1-\beta_E}, \tag{19}$$

and where $G_R(p)$ and B_2 is given by (14) *and* (15), *respectively.*

It is worth pointing out that the value in the stopping region (18) is not a linear function of profit, reflecting the fact that it is an option itself. This option value is given by the option value to replace after having maintained, which is the option value in (14). We also note that the solution to the characteristic equation, β_E, differs from β_M in Proposition 2 due to the change in degradation rate from the degradation rate in a maintained state, γ_M, to the degradation rate of the non-maintained state, or equivalently, replaced state, γ_E. Since β_M is governed by the smallest degradation rate, it follows that $\beta_E > \beta_M$.

3.3 Joint framework for the compound option and the replace-only option

We now compare the two investment alternatives to determine when it is optimal for the firm to choose to invest sequentially or simply replace the machinery. The problem is formulated in (7). The following proposition gives the condition for when the replace-only alternative always dominates the sequential investment alternative.

Proposition 4 *It will be optimal to replace directly if the following holds for all p*:

$$\frac{I_R}{\beta_E - 1} \left[\frac{\beta_E - 1}{\beta_E} \cdot \frac{Q_R - Q_E}{I_R \mu_E} p \right]^{\beta_E} - B_2 \beta_M p^{\beta_M} - Q_E \frac{(1-k)\mu_E - \mu_M}{\mu_E \mu_M} p + I_M \geq 0, \tag{20}$$

where B_2 is given by (15).

The proof of Proposition 4 is provided in A.3. To determine whether the replace-only alternative is the dominant choice, the option values for the different regions need to be taken into account. If the replace-only option has a higher value than the compound option in the region before any thresholds are reached, the replace-only option will always have the higher value. If Proposition 4 holds it will never be optimal for the producer to choose the sequential investment alternative, meaning that the threshold for replacement is given by Proposition 1, and the value of $H(p)$ defined in (7) coincides with $F_R(p)$ in (10). Relevant thresholds if Proposition 4 holds are illustrated in Fig. 2a.

Relevant thresholds when Proposition 4 does not hold are illustrated in Fig. 2b. The solution space is divided into four different regions, $(0, p_M^*)$, $[p_M^*, p_L^*)$, $[p_L^*, p_H^*)$, and $[p_H^*, \infty)$. The threshold p_M^* is given by (12), whereas the remaining thresholds

(a) Relevant triggers if Proposition 4 holds. Between 0 and p_R^*, it is optimal to wait (white). For higher values it is optimal to replace (light grey).

(b) Relevant triggers if Proposition 4 does not hold. Between 0 and p_M^*, and between p_L^* and p_H^* it is optimal to wait (white), between p_M^* and p_L^* it is optimal to perform the maintenance investment (dark grey), and for values greater than p_H^* it is optimal to replace (light grey).

Fig. 2 Continuation and stopping regions when considering the compound option and the replace-only option jointly. Relevant thresholds depend on whether Proposition 4 holds or not

will be defined below. In the first and the third regions, it is optimal to wait, whereas the second and fourth regions are stopping regions, where investments are undertaken immediately. We refer to these four regions as waiting, maintenance, inaction, and replacement regions, respectively.

The value of the option $H(p)$ can be written as

$$H(p) = \begin{cases} B_1 p^{\beta_E} + \frac{pQ_E}{\mu_E} & p < p_M^* \\ B_2 p_M^{\beta_M} + \frac{(1-k)Q_E p_M}{\mu_M} - I_M & p_M^* \le p < p_L^* \\ Cp^{\beta_E} + Dp^{\beta_E^-} + \frac{Q_E p}{\mu_E} & p_L^* \le p < p_H^* \\ \frac{Q_E p}{\mu_E} - I_R & p \ge p_H^* \end{cases} \tag{21}$$

In the first region, the waiting region $(0, p_M^*)$, the option value $H(p)$ coincides with the value of the option in the continuation region in (18). It is optimal to wait until the investment threshold p_M^* is reached, and then to perform the maintenance action. This is the same continuation region as in the standard model proposed by Ref. [11].

In the second region, the maintenance region $[p_M^*, p_L^*)$, it is optimal to perform the maintenance action immediately in order to reduce the degradation rate and retain the option to replace it. In this case, the value of H is given by the value of the option in the stopping region in (18), or equivalently, the difference between the value in the continuation region in (14) and the maintenance investment cost.

The third region, the inaction region $[p_L^*, p_H^*)$, is defined by two thresholds p_L^* and p_H^* that form an intermediate region of inaction around an indifference point. We show that the indifference point always is a part of the inaction region, where it is optimal for the operator to wait, in A.4. It follows that $H(p)$ on the interval $[p_L^*, p_H^*)$, is of the form $Cp^{\beta_E} + Dp^{\beta_E^-} + \frac{Q_E p}{\mu_E}$. The first two terms represent the value of waiting without having made any irreversible decisions yet. More specifically, the first term represents the option to replace directly if the price increases to p_H^*, whereas the second term represents the option to invest sequentially if the price decreases to p_L^*. The coefficients C and D, as well as the optimal stopping thresholds p_L^* and p_H^* can be found by solving the value matching and smooth pasting conditions. A feature that follows from the existence of the inaction region $[p_L^*, p_H^*)$ is that it can be optimal for the firm to undertake an investment even though the price falls. It is optimal to exercise the maintenance investment option when the price falls to p_L^*, because p_L^* is higher than p_M^*, above which it would be optimal to perform the maintenance action in the constrained sequential alternative. Moreover, it is too costly to wait until the price reaches the upper threshold p_H^* and then invest in replacement due to the time value of money. The prerogative to choose between two different projects, instead of being confined to either one of them, also increases the demand for information and creates an additional incentive to delay investment. Thus, in this particular region, it is optimal to delay the investment even though it would be optimal to invest if only the compound option was available.

In the fourth region, the replacement region $[p_H^*, \infty)$, it is optimal to replace immediately, and the option value coincides with the option value in the stopping region in (10).

3.4 Comparative statics

In this section, we focus on the degradation rate parameters, γ_E and γ_M. Comparative statics and relevant conditions are formalized in Propositions 5 and 6.

Proposition 5 *An increase in γ_E leads to an increase in the threshold for replacement before and after maintenance:* $\frac{\partial p_R^*}{\partial \gamma_E} > 0$ *and* $\frac{\partial p_{M,R}^*}{\partial \gamma_E} > 0$.

Proposition 6 *Under the condition in* (16), *an increase in γ_M and γ_E leads to a decrease in the threshold for replacement after maintenance and the threshold for maintenance, respectively*:

a. $\frac{\partial p_{M,R}^*}{\partial \gamma_M} < 0$

b. $\frac{\partial p_M^*}{\partial \gamma_E} < 0$ *if and only if* $K_2 p_M^* > K_1 I_M$, *where K_1 and K_2 are provided in* (77)–(78).

Extensive numerical testing shows that the condition in Proposition 6 is met for reasonable parameter values. These results complement existing analytical results on the impact of changes in the drift parameter of the underlying stochastic process. These results include Ref. [26], where the investment threshold is monotonic in the drift, and Ref. [21], where the return function is strictly increasing in the post-investment drift rate. A difference between our model and the above-mentioned studies is that we consider two performance-enhancing projects that can boost profit, while Refs. [21] and [26] consider one project that boosts profit and the possibility to exit operations. Therefore, our results show that monotonicity in investment thresholds, with respect to the drift, are preserved also in the situation when the firm instead has the option to invest in a larger project, as opposed to exit. In the next section, we analyze the sensitivity of the thresholds with respect to other model parameters numerically.

4 Numerical illustrations

In this section, we examine the implications of our model in a hydropower example. We analyze expected hitting times and study how our results are affected by changes in selected parameter values. Furthermore, we examine the conditions under which the optimal choice transitions from the dichotomous environment, i.e. if Proposition (4) does not hold, to when the replace-only choice is dominant over the entire state space, or vice versa.

4.1 Parameter choices

Our baseline parameter values are given within a Norwegian hydropower context. When considering the efficiency of the existing machinery, we consider a mid-life machinery that has experienced some efficiency decay but is still some time from reaching its economic lifetime. We set 0.91 as a baseline value. The efficiency of a new machinery reflects the state of the art for this technology. This parameter varies depending on the type of machinery and on how the machinery is designed to operate with different loads. According to Ref. [30], a suitable value for Q_R is 0.95, which also gives a realistic difference between Q_E and Q_R.[6].

The degradation rate for machinery in the hydropower industry is quite low compared to other energy generating industries. In the appraisal of applications from Norwegian hydropower producers, the regulator, the Norwegian Water Resources and Energy Directorate (NVE), uses a guiding degradation rate of 0.00087[7], which is a suitable choice as a baseline value. A suitable value for the post-maintenance parameter γ_M is significantly harder to find because of the lack of empirical studies on the subject. Thus, we opt for a value which gives an obvious reduction in the degradation rate so that the firm might be willing to perform a maintenance investment. Still, the reduction cannot be too large as this would mean that the machinery virtually does not degrade, which contradicts industry observations. With this in mind, we set the value for γ_M equal to 0.0005.

The investment costs, I_R and I_M, are highly dependent on the specific hydropower plant due to the high level of idiosyncracy. However, some general characteristics of the relationship between the two do exist. First, the value of I_M should be significantly lower than I_R. This is because of the difference in the physical characteristics of the two investments. A replacement requires a brand new machinery to be made, whereas a maintenance investment is a significantly less extensive procedure. Moreover, a replacement typically means that the plant is unavailable for a longer period compared to maintenance, which means that there is a higher cost associated with production loss. To quantify the suitable cost levels, we have consulted several experts on the area. Based on these discussions, and taking the limitations above into account, we have set I_R equal to 30 MNOK, I_M is set to 1.75 MNOK, and

[6] The numerical values for Q_E and Q_R are based on a Francis turbine subject to Norwegian weather and market conditions.

[7] https://www.nve.no/Media/5330/veileder-elsertifikater-ou_vannkraftverk_09-02-2017.pdf

Table 1 Baseline parameter values

Parameter description	Symbol	Baseline value
Discount rate	ρ	0.06
Starting efficiency of the existing machinery	Q_E	0.91
Starting efficiency of a new machinery	Q_R	0.95
Degradation rate of original machinery	γ_E	0.00087
Degradation rate of maintained machinery	γ_M	0.0005
Investment cost of replacement	I_R	30
Investment cost of maintenance	I_M	1.75
Fraction of profits lost to changed operations	k	0.005
Volatility of gross profit	σ	0.2
Growth rate of gross profit	α	0.025

fraction of profits lost to changed operations in the maintenance project, k, is set to 0.005. In the setting of a hydropower producer with storage reservoirs, changed operational pattern, e.g. using the machinery for production through periods with low prices to avoid unnecessary starts and stops, implies a relatively small change in the production schedule.

Estimation of parameters σ and α often demands an in-depth analysis of different economical and site-specific factors. We use the work of Ref. [3] as a basis for σ, and set α to reflect the expected rate of inflation. For the volatility, we choose $\sigma = 0.2$ as our baseline, whereas the drift rate, α, is set to 0.025. The discount rate for a given repowering project, ρ, can vary significantly, depending on the plant's risk characteristics and financing. Andersson et al. [3] argue for a discount rate of 7% on an investment in a setting similar to ours. However, in recent years, the discount rate has shown a downward trend [12]. A survey performed by a consulting and accounting firm [18] proposes a guiding discount rate of 5.75% for a levered hydropower firm. Their results were obtained by consulting incumbents in the Nordic hydropower industry. Since the latter study is more up to date, we choose a discount rate of 6%.

Table 1 summarizes the baseline parameter values which are used in the analysis.

After solving (5)–(7), the thresholds for maintenance, lower threshold for waiting, the threshold for replacing directly if options are valued separately, and the threshold for replacing directly if options are valued jointly for the baseline values are: $p_M^* = 32.1$, $p_L^* = 37.5$, $p_R^* = 69.5$, $p_H^* = 69.7$, respectively.

4.2 Value functions

Figure 3 indicates how far the replace-only option value, $F_R(p)$, and the compound option value, $G_M(p)$, are from the joint value of the options. In Fig. 3, the relative difference, $\frac{(H(p)-V(p))-(F_R(p)-V(p))}{H(p)-V(p)}$ and $\frac{(H(p)-V(p))-(G_M(p)-V(p))}{H(p)-V(p)}$ are represented by black and grey curves, respectively. We adjust the option values by $V(p) = \frac{Q_E p}{\mu_E}$, which is the profit generated by doing nothing. The value of doing nothing enters in all

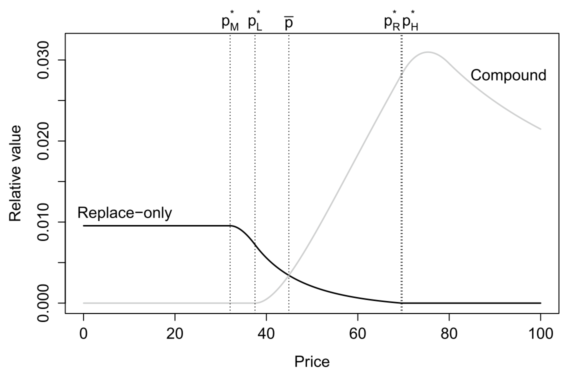


Fig. 3 Relative difference in value. The black line is $\frac{(H(p)-V(p))-(F_R(p)-V(p))}{H(p)-V(p)}$ and the grey line is $\frac{(H(p)-V(p))-(G_M(p)-V(p))}{H(p)-V(p)}$, where $V(p) = \frac{Q_E p}{\mu_E}$ is the profit stream if neither the replace-only nor the compound option is exercised. Price thresholds are plotted in vertical dotted lines

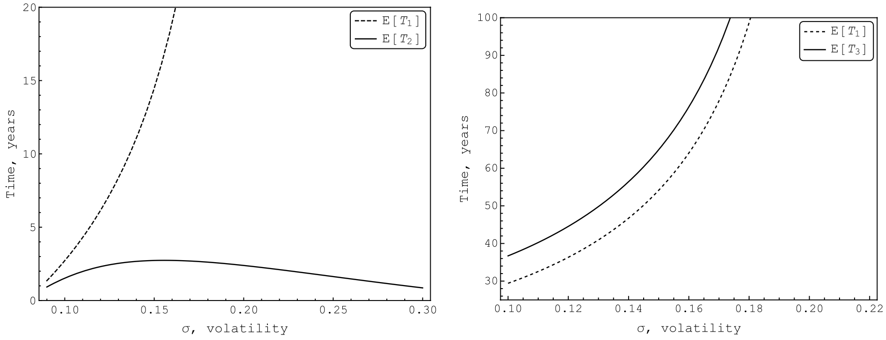
values, see (10), (18), and (21), and by adjusting for this value we can analyze the additional profit generated by considering the replace-only option and compound option, respectively. Several features can be observed. First, the compound option adds value in the region $p < p^*_H$. This can be seen by studying the black solid line in Fig. 3, showing a positive relative difference between the joint option value, $H(p)$, and the replace-only option value $F_R(p)$ in the region $p < p^*_H$. This highlights the added value of having a smaller investment project in the portfolio. Second, having the option to replace directly adds value to the compound option in the region from $p > p^*_L$. In this region, the relative difference between the joint value $H(p)$ and the compound option value $G_M(p)$ is positive. Third, separate valuation of the replace-only option and the compound option leads to suboptimal investment thresholds. At the point $\bar{p}$ we observe that the compound option value and the replace-option value are equal. Below this, the firm would maintain immediately, and above this point the firm would wait and replace if the price reaches p^*_R under separate valuations. Under joint valuation we find triggers $p^*_H > p^*_R$ and $p^*_L < \bar{p}$ for our base case parameter values.

4.3 Expected hitting times

Figure 4a shows the expected time to hit the threshold p^*_R, i.e. the threshold for replacement in the replace-only alternative, $\mathbb{E}[T_1]$, and the expected time to exit the inaction region, $\mathbb{E}[T_2]$, i.e. hitting either p^*_L from above or p^*_H from below, given a current price in between the thresholds.[8]

We observe that the expected time to exit the inaction region, for the current price $p = 50$, first increases in volatility and then decreases. This can be explained by the fact that both the replacement option and compound option become more valuable

[8] We follow the approach presented in Ref. [43], and calculate the expected times to hit the investment thresholds. In Fig. 4a and b, $\mathbb{E}[T_1] = \frac{1}{-\sigma^2/2+\alpha-\gamma_E} \ln\left(\frac{p^*_R}{p}\right)$, $\mathbb{E}[T_3] = \frac{1}{-\sigma^2/2+\alpha-\gamma_M} \ln\left(\frac{p^*_{M,R}}{p}\right)$, and the expected time to exit the inaction region is $\mathbb{E}[T_2] = \frac{1}{0.5\sigma^2-\alpha+\gamma_E}\left(\ln\left(\frac{p}{p^*_L}\right) - \ln\left(\frac{p^*_H}{p^*_L}\right)\left(1-(p/p^*_L)^{1-2(\alpha-\gamma_E)/\sigma^2}\right)/\left(1-(p^*_H/p^*_L)^{1-2(\alpha-\gamma_E)/\sigma^2}\right)\right)$.



(a) The expected time to hit the replace-only threshold (dashed line) and to exit the inaction region (solid line) for $p = 50$.

(b) The expected time to hit the replace-only threshold (dashed line) and to replace after having maintained (solid line) for $p = 30$.

Fig. 4 Expected hitting times as a function of volatility for the following parameter set: $\rho = 0.06$, $Q_E = 0.91$, $Q_R = 0.95$, $\gamma_E = 0.00087$, $\gamma_M = 0.0005$, $I_R = 30$, $I_M = 1.75$, $k = 0.005$, and $\alpha = 0.025$

as σ increases. This means that the probability of hitting the replacement threshold first decreases with σ for the initial price below this threshold, and the probability of hitting the maintenance threshold first increases with σ for the initial price above this threshold. Moreover, we observe that having the option to replace directly delays the expected time for the firm to act by 2.4 years for our baseline of $\sigma = 0.2$, compared to only having the compound option. In addition, if the firm does not have the option to maintain, the expected time until the investment is 80 years. Figure 4b shows the expected time to replace with and without the compound option as a function of volatility, at the current price $p = 30$. At this price, it is optimal to exercise the compound option immediately for values of σ between 0 and 0.20. Figure 4b shows that the expected time spent in the maintained state is $\mathbb{E}[T_3] = 65$ for $\sigma = 0.15$, while the expected time to replace, without the option to maintain, is $\mathbb{E}[T_1] = 54$. Hence, a large investment is expected to be delayed significantly with the replacement option embedded in the maintenance option, compared to only having the option to replace.

4.4 Sensitivity analysis of investment thresholds

As comparative statics for model parameters are difficult to obtain analytically, we perform the sensitivity analysis using numerical illustrations for reasonable parameter values. In the subsequent figures, we use dark grey and light gray shading to illustrate the stopping region for the compound option and the replace-only option, respectively. The investment threshold when the replace-only option is dominant over the entire state space is p_R^*. The maintenance region lies between p_M^* and p_L^*. The region above p_H^* is the replacement region.

We start by examining the effect of volatility, σ in Fig. 5.

As can be seen, the second inaction region $[p_L^*, p_H^*)$ increases in volatility. Interestingly, both p_L^* and p_H^* increase in volatility, until the replace-only alternative becomes dominant (around $\sigma = 0.34$). Thus, it can be optimal for the hydropower firm to undertake an investment even though the price falls. Guerra et al. [19] find

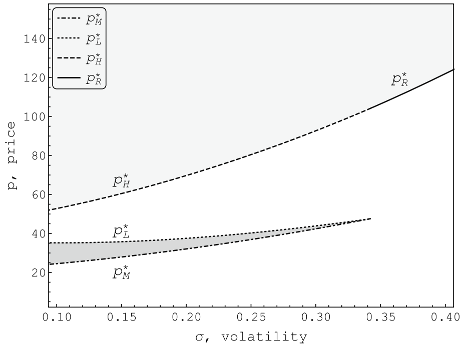


Fig. 5 The effect of varying volatility for the following parameter set: $\rho = 0.06$, $Q_E = 0.91$, $Q_R = 0.95$, $\gamma_E = 0.00087$, $\gamma_M = 0.0005$, $I_R = 30$, $I_M = 1.75$, $k = 0.005$, and $\alpha = 0.025$. The compound option is exercised in the dark grey region, the replace-only option is exercised in the light grey region, and white is the waiting region

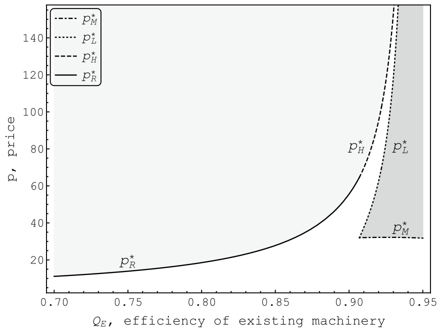
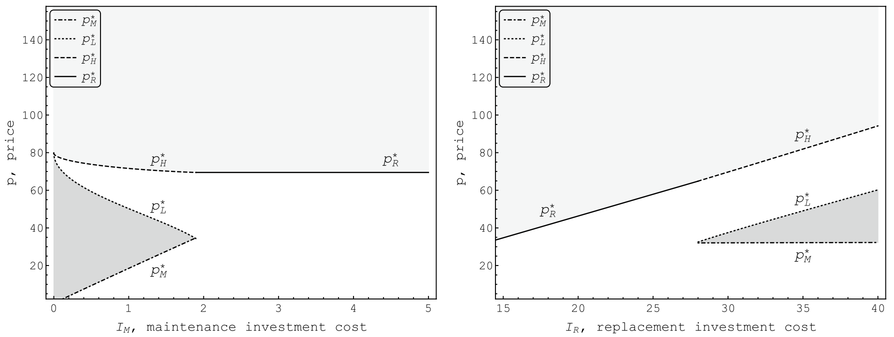


Fig. 6 The effect of varying pre-investment machinery efficiency for the following parameter set: $\rho = 0.06$, $\sigma = 0.20$, $Q_R = 0.95$, $\gamma_E = 0.00087$, $\gamma_M = 0.0005$, $I_R = 30$, $I_M = 1.75$, $k = 0.005$, and $\alpha = 0.025$. The compound option is exercised in the dark grey region, the replace-only option is exercised in the light grey region, and white is the waiting region

a similar behavior when considering mothballing and exit options. However, unlike in their analysis, where the lower threshold decreases in volatility, in our model, p_L^* increases in σ. This is due to the added value of the replacement option that is available after having exercised the maintenance option.

The effect of changing the initial efficiency of the existing machinery, Q_E, is shown in Fig. 6.

In contrast to the volatility, a lower efficiency of the machinery makes the replace-only option dominant. To understand this, note that if the efficiency is already low, the payoff from replacing and restarting the degradation process dominates that of the maintenance investment, which only slows down degradation, even though the cost is higher. In addition, we observe that when the dichotomous environment is prevailing, all thresholds except p_M^* experience a significant increase when Q_E approaches Q_R. This is because when the net benefit of replacing the machinery is smaller, the firm requires a drastically higher price level before it is profitable to replace. However, the same effect has little influence on the threshold to perform the maintenance investment, p_M^*. This can be explained by two contradicting incentives. On the one hand, the firm has an incentive to invest in maintenance earlier because reducing the degradation rate on a machinery with higher efficiency extends its economic lifetime more substantially, and hence delays the subsequent replacement. On the other hand, the threshold is indirectly affected by the replacement option through the implicit equation (17). This gives the hydropower producer an incentive to delay

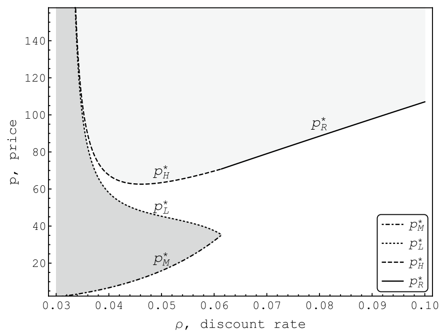


(a) For $I_R = 30$.

(b) For $I_M = 1.75$.

Fig. 7 The effect of varying investment costs for the following parameter set: $\rho = 0.06$, $\sigma = 0.20$, $Q_E = 0.91$, $Q_R = 0.95$, $\gamma_E = 0.00087$, $\gamma_M = 0.0005$, $k = 0.005$, and $\alpha = 0.025$. The compound option is exercised in the dark grey region, the replace-only option is exercised in the light grey region, and white is the waiting region

Fig. 8 The effect of varying discount rate for the following parameter set: $\sigma = 0.20$, $Q_E = 0.91$, $Q_R = 0.95$, $\gamma_E = 0.00087$, $\gamma_M = 0.0005$, $I_R = 30$, $I_M = 1.75$, $k = 0.005$, and $\alpha = 0.025$. The compound option is exercised in the dark grey region, the replace-only option is exercised in the light grey region, and white is the waiting region



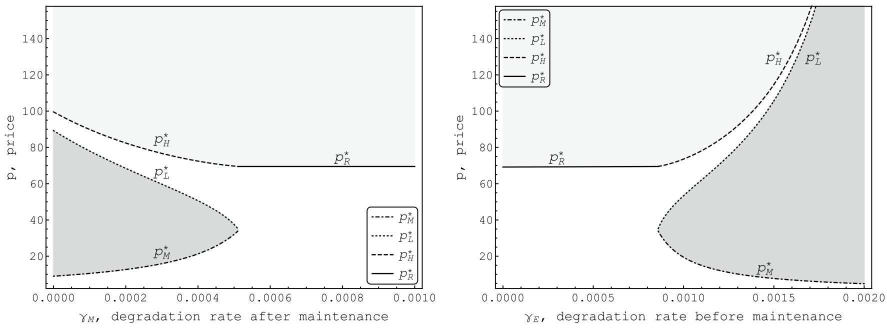
the investment because replacement is no longer as imminent with such a high efficiency of the initial machinery. The dominating effect is the former, which leads to a reduction in the threshold.

In Fig. 7a and b, the value of I_M and I_R vary, respectively.

Increasing the replacement cost I_R increases all thresholds. In addition, the replace-only alternative is dominating only for low values of I_R. This is because the value gained from the maintenance project before an eventual replacement is not high enough compared to directly replacing the machinery for low replacement cost. As the maintenance investment cost I_M increases, the inaction region becomes larger and the investment threshold p_M^* increases. The thresholds p_L^*, and p_H^*, however, decline with I_M. This is because for larger I_M, the replace-only option becomes more attractive.

Figure 8 shows the effect of changing the discount rate ρ.

We observe that increasing ρ effectively devalues the sequential investment, making the single investment choice dominant for higher values of ρ. A higher discount rate dampens the relative importance of the change in drift after the maintenance



(a) For $\gamma_E = 0.00087$. **(b)** For $\gamma_M = 0.0005$.

Fig. 9 The effect of varying degradation rate for the following parameter set: $\rho = 0.06$, $\sigma = 0.20$, $Q_E = 0.91$, $Q_R = 0.95$, $I_R = 30$, $I_M = 1.75$, $k = 0.005$, and $\alpha = 0.025$. The compound option is exercised in the dark grey region, the replace-only option is exercised in the light grey region, and white is the waiting region

investment, so the sequential alternative loses its attractiveness, and thus the gain of a lifetime extension is discounted too much to be a viable choice for the firm. Therefore, the maintenance investment region shrinks as a result of an increase in the threshold p_M^* and a decline of p_L^*. In the case of the replace-only threshold, however, it is not as clear-cut. In fact, the threshold p_H^* deceases for low values of ρ and increases for large values of ρ. This happens due to two opposing effects. On the one hand, the replacement option becomes more attractive than the maintenance option as ρ increases. On the other hand, however, increasing the cost of capital reduces the value of the expected future cash flows from replacement relative to the expected future cash flows from continuing current operations. Similar opposing effects for the discount rate have been found in the literature, e.g. Lavrutich [28] who found that the entry timing of a firm who considers entering a market with an active incumbent, is non-monotonic in discount rate. For large values of ρ the discounting effect dominates, and the firm is incentivized to replace the machinery earlier, making it the more valuable option. It is also worth mentioning that the waiting region $[0, p_M^*)$ and the inaction region $[p_L^*, p_H^*)$ are expanding with ρ. This is caused by an increased value in the option to invest in either of the two alternatives and hence increases the opportunity cost of investing immediately.

The effects when changing the degradation rates γ_E and γ_M are shown in Fig. 9a and b, respectively. These Figures supplement Propositions 5 and 6 by illustrating the investment thresholds based on a realistic set of parameter values for a hydropower operator. We observe that when γ_M is low, the dichotomous environment dominates, whereas for large γ_M the firm will choose to replace directly. This is because the more efficient the maintenance investment is, implying a smaller γ_M, the more valuable the maintenance option becomes. Varying γ_E has the opposite effect, there the dichotomous environment dominates when γ_E is large. These effects can be explained by the increased benefits of maintenance for low γ_M and large γ_E, which leads to an increase of the investment thresholds which define the inaction region. At the same time, p_M^* decreases drastically under the dichotomous regime. Both of these changes can be explained by the attractiveness of operating after the maintenance project when the relative difference between γ_E and γ_M escalates. By executing the maintenance project earlier, the benefit is reaped sooner and the time until a replacement is required is prolonged due to the decelerated degradation rate. When γ_E increases, the inaction region, $[p_L^*, p_H^*)$, shrinks, while the maintenance region, $[p_M^*, p_L^*)$, expands rapidly. Moreover, note that p_R^* is independent of γ_M, since this parameter only relates to the maintenance project in the sequential investment alternative. We also see that p_R^* is quite insensitive to changes in γ_E. This is most likely due to the drift rate being dominated by the profitability growth α.

4.5 Limitations

In this paper, we provide a novel perspective on managing assets with deteriorating performance by quantifying the effect of correctly accounting for the mutually exclusive mitigation options. We emphasize the real options perspective within the field of maintenance and renewal. To keep the model tractable, we make several assumptions, e.g., deterministic efficiency deterioration and Gaussian relative changes in long-term prices. However, it is valuable to extend the current framework to account for potential other real-world features, such as stochastic efficiency deterioration, or breakdown risk. This can be done by, for example, assuming that the degradation rate follows a GBM with negative drift, and that the breakdown risk is represented by a Poisson jump process or a gamma process [41]. Intuitively, the additional source of uncertainty in efficiency will make the option to wait more valuable in line with the standard real options theory. However, such extensions will require numerical solutions. Another interesting extension is to add other options to the investment portfolio, for example, sequential maintenance options. We demonstrate our framework on a hydropower example, and the framework allows to provide insights for investment decisions in other power generating industries, e.g., wind power. In order for the model to fit other particular industries, the numerical values will have to be adjusted. Compared to hydro-specific estimates, this will likely imply slightly higher values of the efficiency deterioration [40] and in the case of stochastic deterioration, also a higher volatility of the efficiency deterioration process in the case of wind energy.

5 Conclusions

This paper examines the decisions of a firm concerning a potential maintenance or replacement of machinery within a real options framework. We present a tractable model, applicable to general asset management, where we examine the conditions for when it is optimal to undertake investments, and possibly switch from the minor maintenance project to the major replacement project. The paper contributes to the literature on replacement options in mutually exclusive investment projects within the real options framework.

We find that there is a possibility that the investment region is dichotomous. That is, the investment region is no longer a connected set, similar to the findings in Refs. [9] and [19]. We demonstrate implications of our model by studying investment alternatives for a hydropower producer facing a deteriorating efficiency of its generation units. Our analysis shows that hydropower producers are likely to operate in an environment where the dichotomous investment environment is present. We further find that the dichotomous environment is more likely to be present when the maintenance option is valuable, and that the maintenance investment becomes preferable when the volatility, the discount rate, the maintenance investment cost, and the deterioration rate after maintenance are low, and when the replacement cost and the deterioration rate before maintenance are high. By analyzing expected hitting times, we find that that the replacement decision may be delayed significantly in expectation with the replacement option embedded in the maintenance option. Furthermore, for intermediate values of the current price, the expected time for the producer to act is first increasing in volatility, then decreasing, when the sequential investment alternative and replace-only alternative are valued jointly. However, if the alternatives are valued separately, the inaction region does not exist, and the producer would undertake maintenance investment immediately. This shows the importance of properly identifying the potential alternatives available in the project portfolio.

A Proof of Propositions

A.1 Proposition 2

The stopping value is given by

$$G_R(p) = \mathbb{E}\left[\int_0^\infty e^{-\rho t} Q_R p e^{-\left((\alpha-\gamma_E-\frac{\sigma^2}{2})t+\sigma Z(t)\right)} \mathrm{d}t\right] - I_R = \frac{Q_R p}{\mu_E} - I_R. \tag{22}$$

In the continuation region, the problem is almost the same as in Proposition 1. The differences between the two are that the initial condition and drift of the profit flow. Using parameters from the profit flow after maintenance at time τ_1, we obtain the following expression for the value of the replacement option in the continuation region,

$$G_R(p) = B_2 p^{\beta_M} + \frac{(1-k)Q_E p}{\mu_M}, \tag{23}$$

where

$$\beta_M = \frac{1}{2} - \frac{\alpha-\gamma_M}{\sigma^2} + \sqrt{\left(\frac{\alpha-\gamma_M}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}}. \tag{24}$$

To find the optimal stopping value, p_M^*, the value matching and smooth pasting conditions must be met. These are given by the following expressions:

Value matching:

$$B_2 p_M^{*\,\beta_M} + \frac{(1-k)Q_E p_M^*}{\mu_M} = \frac{Q_R p_M^*}{\mu_E} - I_R. \tag{25}$$

Smooth pasting:

$$B_2 \beta_M p_M^{*\,\beta_M - 1} + \frac{(1-k)Q_E}{\mu_M} = \frac{Q_R}{\mu_E}. \tag{26}$$

Solving these equations to find p_M^* and B_2, yields

$$p_M^* = \frac{\beta_M}{\beta_M - 1} \cdot \frac{\mu_E \mu_M}{Q_R \mu_M - (1-k)Q_E \mu_E} \cdot I_R, \tag{27}$$

$$B_2 = \frac{I_R}{\beta_M - 1}\left[\frac{\beta_M - 1}{\beta_M} \cdot \frac{Q_R \mu_M - (1-k)Q_E \mu_E}{\mu_E \mu_M} \cdot \frac{1}{I_R}\right]^{\beta_M}. \tag{28}$$

Thus, the value of the option to replace in the sequential alternative is given by

$$G_R(p) = \begin{cases} B_2 p^{\beta_M} + \frac{(1-k)Q_E p}{\mu_M} & \text{if } p < p_M^*, \\ \frac{Q_R p}{\mu_E} - I_R & \text{if } p \geq p_M^*. \end{cases} \tag{29}$$

A.2 Proposition 3

In the stopping region, one pays the investment cost to obtain the second option. Thus, the value of the option is given by

$$G_M(p) = B_2 p^{\beta_M} + \frac{(1-k)Q_E p}{\mu_M} - I_M. \tag{30}$$

In the continuation region, the Bellman equation must hold. This equation is given by

$$\rho G_M dt = \mathbb{E}[dG_M] + Q_E p_0 dt. \tag{31}$$

Solving this equation for the homogeneous and the particular solution yields the following expression for the option value:

$$G_M = B_1 p^{\beta_E} + \frac{Q_E p}{\mu_E}, \tag{32}$$

where

$$\beta_E = \frac{1}{2} - \frac{\alpha - \gamma_E}{\sigma^2} + \sqrt{\left(\frac{\alpha - \gamma_E}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}}. \tag{33}$$

At the investment threshold, p_M^*, the following value matching and smooth pasting conditions must hold:

Value matching:

$$B_1 {p_M^*}^{\beta_E} + \frac{Q_E p_M^*}{\mu_E} = B_2 {p_M^*}^{\beta_M} + \frac{(1-k)Q_E p_M^*}{\mu_M} - I_M. \tag{34}$$

Smooth pasting:

$$B_1 \beta_E {p_M^*}^{\beta_E - 1} + \frac{Q_E}{\mu_E} = B_2 \beta_M {p_M^*}^{\beta_M - 1} + \frac{(1-k)Q_E}{\mu_M}. \tag{35}$$

The expression for p_M^* cannot be solved analytically, but implicitly solves the following equation:

$$ {p_M^*}^{\beta_M} B_2 \frac{\beta_E - \beta_M}{\beta_E} + p_M^* \frac{\beta_E - 1}{\beta_E} \cdot \frac{Q_E((1-k)\mu_E - \mu_M)}{\mu_E \mu_M} - I_M = 0. \tag{36}$$

Given the value of p_M^*, one can calculate the value of B_1 as

$$B_1 = B_2 \frac{\beta_M}{\beta_E} \cdot {p_M^*}^{\beta_M - \beta_E} + \frac{Q_E}{\beta_E} \cdot \frac{(1-k)\mu_E - \mu_M}{\mu_E \cdot \mu_M} {p_M^*}^{1-\beta_E}. \tag{37}$$

Thus, the value of the option is given by

$$G_M(p_0) = \begin{cases} B_1 p^{\beta_E} + \frac{Q_E p}{\mu_E} & \text{if } p < p_M^*, \\ B_2 p^{\beta_M} + \frac{(1-k)Q_E p}{\mu_M} - I_M & \text{if } p \geq p_M^*. \end{cases} \tag{38}$$

A.2.1 Proof of Unique Solution for p_M^*

The implicit solution for p_M^* in the sequential alternative, given by (36), is of the following form:

$$\Psi(p_M^*) = A {p_M^*}^{\beta_M} + B p_M^* - C = 0. \tag{39}$$

To prove the existence of a unique solution for p_M^* we start by defining the domain of the above function, which is restricted to positive values only, i.e. $p_M^* \in [0, \infty >$. We also know that β_M is the positive root of the quadratic equation given by (13), and is thus greater than 1 (see [11]).

We can prove that the constants A, B and C are strictly positive. A consists of two terms, namely $(\frac{\beta_E - \beta_M}{\beta_E})$ and the constant B_2 defined by (15). First, we know that $\beta_E > \beta_M$ due to the fact that $\gamma_E > \gamma_M$. This means that $(\frac{\beta_E - \beta_M}{\beta_E})$ is always positive. In order for B_2 to be positive, we must assume that

$$Q_R \mu_M > (1-k) Q_E \mu_E. \tag{40}$$

This inequality signifies that the net benefit of replacing the machinery after first having upgraded it is positive. Combined, these two parts yield that A is always positive. Furthermore, to assure a positive B, we require that

$$(1 - k)\mu_E \geq \mu_M. \tag{41}$$

This is the same as assuming that the net benefit from upgrading the pre-existing machinery is either zero or strictly positive, which must be true, otherwise the option would have no intrinsic value. The last constant, C, represents the investment cost of upgrading and is by definition always strictly greater than zero. As we know that the constants are always positive, we can take the derivative of (39) to show that the function is monotonically increasing

$$\Psi'(p_M^*) = A\beta_M {p_M^*}^{\beta_M - 1} + B. \tag{42}$$

Since we have already confirmed that $\beta_M > 1$, this is a monotonically increasing function for $p_M^* \in [0, \infty>$. By applying the intermediate value theorem, we therefore know that (39) has a unique solution for p_M^*.

A.3 Proposition 4

We know that when all thresholds are reached, the value of the sequential option is I_M to the right relative to the option to replace directly. It is also known that the derivative of the option value in the stopping region of G_M is less than the derivative of the option value in the stopping region for F_R. From value matching and smooth pasting, we know that the values in the continuation regions will always converge towards the values in their respective stopping regions in terms of both values and derivatives. Using this, and the fact that the first derivatives of all option values in the continuation regions are strictly positive, it can be shown that the sequential option will first converge towards a less steep function and thereafter converge towards the right-shifted parallel line. It will therefore never cross the option value which converges towards the stopping value of the replace-only option.

Let us, therefore, consider the option values where both alternatives are in the first inaction region. In the case where F_R is more valuable, the following inequality will hold:

$$F_R^{\mathcal{C}} - G_M^{\mathcal{C}} \geq 0 \tag{43}$$

Inserting the relevant expressions from (10) and (18), yields

$$\left[A_1 p^{\beta_E} + \frac{pQ_E}{\mu_E}\right] - \left[B_1 p^{\beta_E} + \frac{pQ_E}{\mu_E}\right] = A_1 p^{\beta_E} - B_1 p^{\beta_E} \geq 0. \tag{44}$$

The inequality simplifies to

$$A_1 - B_1 \geq 0. \tag{45}$$

We now substitute these parameters by their expressions given in Eqs. (11) and (19)

$$\frac{I_R}{\beta_E - 1}\left[\frac{\beta_E - 1}{\beta_E} \cdot \frac{Q_R - Q_E}{\mu_E} \cdot \frac{1}{I_R}\right]^{\beta_E} - \left[B_2 \frac{\beta_M}{\beta_E} {p_M^*}^{\beta_M - \beta_E} + \frac{Q_E}{\beta_E} \cdot \frac{(1-k)\mu_E - \mu_M}{\mu_E \mu_M} {p_M^*}^{1-\beta_E}\right] \geq 0. \tag{46}$$

By reformulation,

$$I_R \frac{\beta_E}{\beta_E - 1}\left[\frac{\beta_E - 1}{\beta_E} \cdot \frac{Q_R - Q_E}{I_R \mu_E} p_M^*\right]^{\beta_E} - B_2 \beta_M {p_M^*}^{\beta_M} - Q_E \frac{(1-k)\mu_E - \mu_M}{\mu_E \mu_M} p_M^* \geq 0. \tag{47}$$

Thus, if (47) holds, replace only will be the dominant choice in the entire state space.

A.4 The indifference point

The indifference point, $\tilde{p}^*$, never belongs to any of the stopping regions and will always be located between p_L^* and p_H^*. We show this in this subsection. The point is implicitly given by the following equation:

$$B_2 \tilde{p}^{*\ \beta_M} + \frac{(1-k)Q_E\mu_E - Q_R\mu_M}{\mu_E\mu_M}\tilde{p}^* - \left(I_M - I_R\right) = 0, \tag{48}$$

where β_M and B_2 are given by (13) and (15), respectively. When contemplating investment, the firm will select the alternative which generates the highest net expected profit, given the current price p. The value of investment is therefore the highest stopping value of the two alternatives, i.e. $\max\{G_M^{\mathcal{S}}, F_R^{\mathcal{S}}\}$. When the two alternatives are equally valuable, it is called the indifference point. This point is given as the solution to

$$B_2 \widetilde{p}_0^{*\ \beta_M} + \frac{(1-k)Q_E\widetilde{p}^*}{\mu_M} - I_M = \frac{Q_R\widetilde{p}^*}{\mu_E} - I_R. \tag{49}$$

Rearranging, we get

$$B_2 \widetilde{p}^{*\ \beta_M} + \left[\frac{(1-k)Q_E\mu_E - Q_R\mu_M}{\mu_E \cdot \mu_M}\right]\widetilde{p}^* - (I_M - I_R) = 0. \tag{50}$$

For values of p below the indifference point, the value of the sequential option exceeds that of the replace-only option, and vice versa for values above the indifference point.

The intuition for why the indifference point never belongs to the stopping region is quite instructive. We start with the heuristic argument put forward by Ref. [11] to justify the smooth pasting condition. Suppose that the current profit is equal to the indifference point. Then, by waiting for a small time dt, the firm can observe the evolution of the profit without having to make any decisions. The intuitive idea is that by waiting a little longer, the firm can observe the next step of p and choose to invest on either side of $\widetilde{p}^*$. The resulting average pay-off is thus greater than the payoff obtained by investing at the indifference point itself since the payoff at this point is not differentiable. This is an implication that follows directly from Jensen's inequality, which states that, given a convex function, equally spaced changes in p_0 give rise to unequally spaced changes in $V(p_0)$. In particular, $V[\mathbb{E}(p)] \leq \mathbb{E}[V(p)]$. This remains true even though the average payoff must be discounted because it occurs at a later time dt. The reason is that, for a Brownian motion, the movements are proportional to $\sqrt{dt}$, which is valid for the expected payoff. However, the cost due to discounting is of magnitude dt, and thus when dt is small, the $\sqrt{dt}$-term dominates. The result is that the firm is better off by waiting for more information, which gives rise to an inaction region. Thus, whenever the inequality given by Proposition 4 does not hold, in contrast to Ref. [11], the stopping region is dichotomous, and the optimal investment decision is not governed by a simple trigger strategy.

A.5 Values for C, D, p_L^* and p_H^*

To find the values for C, D, p_L^* and p_H^*, value matching and smooth pasting conditions must be met at the two thresholds. The conditions at p_L^* are given by

Value matching:

$$Cp_L^{*\,\beta_E} + Dp_L^{*\,\beta_E^-} + \frac{Q_E p_L^*}{\mu_E} = \frac{Q_E(1-k)p_L^*}{\mu_M} + B_2 p_L^{*\,\beta_M} - I_M. \tag{51}$$

Smooth pasting:

$$\beta_E C p_L^{*\,\beta_E - 1} + \beta_E^- D p_L^{*\,\beta_E^- - 1} + \frac{Q_E}{\mu_E} = \frac{Q_E(1-k)}{\mu_M} + \beta_M B_2 p_L^{*\,\beta_M - 1}. \tag{52}$$

Rearranging (52), we get

$$C = \left[\frac{Q_E(1-k)}{\mu_M} + \beta_M B_2 p_L^{*\,\beta_M - 1} - \frac{Q_E}{\mu_E} - \beta_E^- D p_L^{*\,\beta_E^- - 1}\right] \frac{p_L^{*\,1-\beta_E}}{\beta_E}. \tag{53}$$

Inserting this into (51) and rearranging, yields

$$D = Q_E \frac{\beta_E - 1}{\beta_E - \beta_E^-} \cdot \frac{\mu_E(1-k) - \mu_M}{\mu_E \mu_M} p_L^{*\,1-\beta_E^-} + B_2 \frac{\beta_E - \beta_M}{\beta_E - \beta_E^-} p_L^{*\,\beta_M - \beta_E^-} - I_M \frac{\beta_E}{\beta_E - \beta_E^-} p_L^{*\,-\beta_E^-}. \tag{54}$$

By using the expression for D given by (54) in (53), we get

$$C = Q_E \frac{\beta_E^- - 1}{\beta_E^- - \beta_E} \cdot \frac{\mu_E(1-k) - \mu_M}{\mu_E \mu_M} p_L^{*\,1-\beta_E} + B_2 \frac{\beta_E^- - \beta_M}{\beta_E^- - \beta_E} p_L^{*\,1-\beta_E} - I_M \frac{\beta_E^-}{\beta_E^- - \beta_E} p_L^{*\,-\beta_E}. \tag{55}$$

On the other end of the interval, the conditions at p_H^* are given by

Value matching:

$$Cp_H^{*\,\beta_E} + Dp_H^{*\,\beta_E^-} + \frac{Q_E p_H^*}{\mu_E} = \frac{Q_R p_H^*}{\mu_E} - I_R. \tag{56}$$

Smooth pasting:

$$\beta_E C p_H^{*\,\beta_E - 1} + \beta_E^- D p_H^{*\,\beta_E^- - 1} + \frac{Q_E}{\mu_E} = \frac{Q_R}{\mu_E}. \tag{57}$$

Rearranging (57), we get

$$C = \left[\frac{Q_R - Q_E}{\mu_E} - \beta_E^- D p_H^{*\,\beta_E^- - 1} \right] \frac{p_H^{*\,1-\beta_E}}{\beta_E}. \tag{58}$$

Inserting this in (56) and solving for D, yields

$$D = \frac{\beta_E - 1}{\beta_E - \beta_E^-} \cdot \frac{Q_R - Q_E}{\mu_E} p_H^{*\,1-\beta_E^-} - I_R \frac{\beta_E}{\beta_E - \beta_E^-} p_H^{*\,-\beta_E^-}. \tag{59}$$

By using the expression for D given by (59) in (58), we get

$$C = \frac{\beta_E^- - 1}{\beta_E^- - \beta_E} \cdot \frac{Q_R - Q_E}{\mu_E} p_H^{*\,1-\beta_E} - I_R \frac{\beta_E^-}{\beta_E^- - \beta_E} p_H^{*\,-\beta_E}. \tag{60}$$

The expressions for C and D in both ends of the inaction region can be generalized by using the following expressions:

$$M_{i,j}(p) = \frac{\beta_i - 1}{\beta_i - \beta_j} \cdot \frac{Q_R - Q_E}{\mu_E} p^{1-\beta_j} - I_R \frac{\beta_i}{\beta_i - \beta_j} p^{-\beta_j}, \tag{61}$$

$$N_{i,j}(p) = Q_E \frac{\beta_i - 1}{\beta_i - \beta_j} \cdot \frac{\mu_E(1-k) - \mu_M}{\mu_E \mu_M} p^{1-\beta_j} + B_2 \frac{\beta_i - \beta_M}{\beta_i - \beta_j} p^{\beta_M - \beta_j} - I_M \frac{\beta_i}{\beta_i - \beta_j} p^{-\beta_j}. \tag{62}$$

By setting equal the two expressions for both constants, it is possible to rearrange the initial system to

For C:

$$N_{21}(p_L^*) = M_{21}(p_H^*). \tag{63}$$

For D:

$$N_{12}(p_L^*) = M_{12}(p_H^*). \tag{64}$$

These expressions can now be used to obtain the thresholds p_L^* and p_H^* by using a numerical solution procedure.

A.6 Proposition 5

We start with the first part. To prove this, we follow Ref. [28]. Taking the derivative of expression (8) with respect to γ_E gives

$$\frac{\partial p_R^*}{\partial \gamma_E} = \frac{I_R}{Q_R - Q_E} \frac{-\mu_E \frac{\partial \beta_E}{\partial \gamma_E} + \beta_E(\beta_E - 1)}{(\beta_E - 1)^2}, \tag{65}$$

where

$$\begin{aligned} \frac{\partial \beta_E}{\partial \gamma_E} &= -\frac{\frac{\alpha-\gamma_E}{\sigma^2} - \frac{1}{2}}{\sigma^2 \sqrt{(\frac{\alpha-\gamma_E}{\sigma^2} - \frac{1}{2})^2 + \frac{2\rho}{\sigma^2}}} + \frac{1}{\sigma^2} \\ &= \frac{\beta_E}{\sigma^2 \sqrt{(\frac{\alpha-\gamma_E}{\sigma^2} - \frac{1}{2})^2 + \frac{2\rho}{\sigma^2}}}. \end{aligned} \tag{66}$$

Rearranging and using the expression for β_E, $\sqrt{(\frac{\alpha-\gamma_E}{\sigma^2} - \frac{1}{2})^2 + \frac{2\rho}{\sigma^2}} = \sigma^2\left(\beta_E - \frac{1}{2} + \frac{\alpha-\gamma_E}{\sigma^2}\right)$, gives

$$\begin{aligned} \frac{\partial p_R^*}{\partial \gamma_E} &= \frac{I_R}{(Q_R - Q_E)}\left(\frac{\beta_E}{\beta_E - 1} - \frac{\mu_E \frac{\partial \beta_E}{\partial \gamma_E}}{(1-\beta_E)^2}\right) \\ &= \frac{I_R}{(Q_R - Q_E)}\left(\frac{\beta_E}{\beta_E - 1} - \frac{\mu_E \frac{\beta_E}{\sqrt{(\alpha-\gamma_E-\frac{\sigma}{2})^2+2\rho}}}{(1-\beta_E)^2}\right) \\ &= \frac{I_R \beta_E}{(Q_R - Q_E)}\left(\frac{(\beta_E - 1)(\sigma^2\beta_E - \frac{\sigma^2}{2} + \alpha - \gamma_E) - \mu_E}{(1-\beta_E)^2\sqrt{(\alpha - \gamma_E - \frac{\sigma}{2})^2 + 2\rho}}\right) \\ &= \frac{I_R \beta_E}{(Q_R - Q_E)}\left(\frac{\frac{\sigma^2}{2}\beta_E^2 + (\alpha - \gamma_E - \frac{\sigma^2}{2})\beta_E - \rho + \frac{1}{2}(\beta_E - 1)^2\sigma^2}{(1-\beta_E)^2\sqrt{(\alpha - \gamma_E - \frac{\sigma}{2})^2 + 2\rho}}\right) \\ &= \frac{I_R \beta_E \sigma^2}{2(Q_R - Q_E)(1-\beta_E)^2\sqrt{(\alpha - \gamma_E - \frac{\sigma}{2})^2 + 2\rho}}. \end{aligned} \tag{67}$$

Differentiating (12) with respect to γ_E gives

$$\frac{\partial p_{M,R}^*}{\partial \gamma_E} = \frac{\mu_M I_R Q_R \beta_M^2}{(\beta_M - 1)(Q_R\mu_M - (1-k)Q_E\mu_E)^2}, \tag{68}$$

which is greater than zero for all $\beta_M > 1$, which shows the second part of the proposition.

A.7 Proposition 6

Differentiating (12) with respect to γ_M gives

$$\frac{\partial p^*_{M,R}}{\partial \gamma_M} = I_R\mu_E\left(\frac{-\mu_M(Q_R\mu_M-(1-k)Q_E\mu_E)\frac{\partial\beta_M}{\gamma_M} - Q_E\mu_E(1-k)\beta_M(\beta_M-1)}{(Q_R\mu_M-(1-k)Q_E\mu_E)^2(\beta_M-1)^2}\right) < 0, \tag{69}$$

since $\frac{\partial\beta_M}{\gamma_M} > 0$, $\beta_M > 0$ and $Q_R\mu_M - (1-k)Q_E\mu_E > 0$ if the condition in (16) is met. For the maintenance threshold, let f denote the implicit equation (17). We have

$$0 = \frac{df}{d\gamma_E} = \frac{\partial f}{\partial \gamma_E} + \frac{\partial f}{\partial p^*_M}\frac{\partial p^*_M}{\partial \gamma_E}, \tag{70}$$

which implies

$$\frac{\partial p^*_M}{\partial \gamma_E} = -\frac{\frac{\partial f}{\partial \gamma_E}}{\frac{\partial f}{\partial p^*_M}}. \tag{71}$$

Computing the denominator

$$\frac{\partial f}{\partial p^*_M} = B_2\beta_M\frac{\beta_E-\beta_M}{\beta_E}p_M^{*\beta_M-1} + \frac{\beta_E-1}{\beta_E}Q_E\frac{\mu_E(1-k)-\mu_M}{\mu_E\mu_M}, \tag{72}$$

which is positive given the precondition in (16). We are left to show that $\frac{\partial f}{\partial \gamma_E}$ is positive when the condition holds. Differentiating f with respect to γ_E gives

$$\frac{\partial f}{\partial \gamma_E} = \frac{1}{\mu_E^2\beta_E^2}\big(a_1 + a_2 + a_3\big), \tag{73}$$

where

$$a_1 = \frac{\partial \beta_E}{\partial \gamma_E}\mu_E\left(\beta_M p_M^{*\beta_M}\mu_E B_2 + Q_E p^*_M\left(\frac{\mu_E(1-k)-\mu_M}{\mu_M}\right)\right),$$
$$a_2 = -\frac{\partial B_2}{\partial \gamma_E}p_M^{*\beta_M}\mu_E^2\big(\beta_E\beta_M - \beta_E^2\big),$$
$$a_3 = Q_E p^*_M(\beta_E^2 - \beta_E).$$

Differentiating B_2 with respect to γ_E gives

$$\begin{aligned}\frac{\partial B_2}{\partial \gamma_E} &= -\frac{Q_R}{\mu_E^2}\left(\frac{\beta_M-1}{\beta_M}\left(\frac{\mu_M Q_R-(1-k)Q_E\mu_E}{I_R\mu_E\mu_M}\right)\right)^{\beta_M-1}\\ &= -\frac{Q_R\beta_M\mu_M}{\mu_E(\mu_M Q_R-(1-k)Q_E\mu_E)}B_2\end{aligned} \tag{74}$$

Inserting for $\frac{\partial B_2}{\partial \gamma_E}$ and $\frac{\partial \beta_E}{\gamma_E}$ given by (66) into (73), and eliminating $B_2 p_M^{*\beta_M}$ using the implicit equation f gives

$$\frac{\partial f}{\partial \gamma_E} = K_0(K_2 p_M^* - K_1 I_M), \tag{75}$$

where

$$K_0 = \frac{1}{\mu_E \beta_E(\beta_E - \gamma_E)(\beta_E - \frac{1}{2} + \frac{\alpha - \gamma_E}{\sigma^2})}, \tag{76}$$

$$K_1 = \beta_M \beta_E \left(\left(\beta_E - \frac{1}{2} + \frac{\alpha - \gamma_E}{\sigma^2}\right)\left(\beta_E - \beta_M\right) C_2 - \frac{\mu_E^2}{\sigma^2} \right), \tag{77}$$

$$K_2 = \frac{\mu_E^2}{\sigma^2} C_1 \left(\beta_E - \beta_E \beta_M\right) + \left(\beta_E - \frac{1}{2} + \frac{\alpha - \gamma_E}{\sigma^2}\right)(\beta_E - 1)\left(\beta_E - \beta_M\right)\left(Q_E + C_1 C_2 \beta_M\right), \tag{78}$$

where

$$C_1 = \frac{Q_E\left((1-k)\mu_E - \mu_M\right)}{\mu_E \mu_M},$$
$$C_2 = \frac{Q_R \mu_E \mu_M}{\mu_M Q_R - (1-k) Q_E \mu_E}.$$

Hence, $\frac{\partial f}{\partial \gamma_E} > 0$ if and only if

$$K_2 p_M^* > K_1 I_M, \tag{79}$$

and combined with (71), (72), and (16), this implies $\frac{\partial p_M^*}{\partial \gamma_E} < 0$, which shows the second part of the proposition.

Acknowledgements We thank Roel Nagy for valuable discussions for computing comparative statics analytically. We gratefully acknowledge support from the Research Council of Norway through project 268093, and the research centers HydroCen, RCN No. 257588, and NTRANS, RCN No. 296205.

Funding Open access funding provided by NTNU Norwegian University of Science and Technology (incl St. Olavs Hospital - Trondheim University Hospital).

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