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# Stochastic local flexibility market design, bidding, and dispatch for distribution grid operations

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ABSTRACT

In order to unlock the flexibility potential of energy consumers and prosumers, the development of market mechanisms for flexibility planning and procurement is necessary. The authors propose a stochastic local flexibility market to solve grid issues such as voltage deviations and grid congestion in a distribution grid. Their proposed solution includes activation of flexibility assets at the consumers' premises, using a stochastic local flexibility market design. They consider a pooled local flexibility market design under demand uncertainty and stochastic bidding process. Optimization models are used to determine flexibility demand and supply bids by the distribution system operator and the aggregator respectively. A stochastic AC-optimal power model to determine flexibility demand and a two-stage stochastic model to supply flexibility are implemented to simulate a stochastic local flexibility market. This allows to determine stochastic flexibility supply bid curves, and optimum flexibility supply dispatch. The analysis shows that the cost of grid operations is reduced when the system uses the local flexibility market. The proposed methodology is applicable for local flexibility market designs aiming to use potential end-user flexibility for grid operations.


## 1. Introduction

Electrification of sectors in the economy is not only beneficial for the power system, but also introduces the need for more demand-side flexibility at the distribution grid level in order to ensure grid security. The concept of *flexibility* in power systems relates to their ability to respond to sudden changes in power consumption and generation [1]. By using demand-side flexibility assets, such as load shifting or load curtailment, it is possible to address some grid problems in real-time [2]. In some cases of grid problems, this requires aggregating local flexibility resources [3] to ensure security of supply. An optimal utilization of flexible electricity resources in an efficient market design could address grid challenges and contribute to a deferral of costly grid investments [4]. One option would be to solve grid problems by using market pricing (indirect control). Another option would be to control flexible assets directly [5]. A centralized control of the flexibility assets might pose problems in terms of technical management of a large amount of resources by a single central planner. In this respect, we propose the utilization of a Local Flexibility Market (LFM) to solve grid problems using a market based mechanism between a group of agents, each one in charge of the management of different portions of the grid.

In general, three market players are considered in LFM research, according to Ref. [6]: consumers/the aggregator, the Distribution System Operator (DSO), and Balance Responsible Parties (BRPs). According to Ref. [7], the three main operational processes of an LFM are contracting and bidding, activation, and market settlement processes. In this paper, we first discuss how, via an aggregator, a number of consumers can provide flexibility from a portfolio of flexible assets in a market. Second, we discuss how the buyers of flexibility, in our case a single DSO, bid their flexibility need in the market. After the market is cleared, and the price and volumes are settled, the agreed-upon flexibility is provided by using load shifting, curtailment, and batteries. Thereafter, the optimal power flows, and load shedding if needed, are scheduled by the DSO in order to minimize system costs and to meet the demand for power in the local system.

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The authors in Ref. [7] indicate there are four layers of local flexibility trading such as market, control, ICT, and power grid. Our research considers power market and grid layers by addressing flexibility trading and its usage in grid operations. It is assumed that these two layers are communicating with each other. [8] describe a framework for flexibility management in electricity systems by considering techno institutional, economic, and operational elements in addition to flexibility service type. Our LFM design in this paper is based on the responsibilities of prosumers, markets, and price incentives. Our LFM provides trading and pricing of flexibility services according to timing, volume, and direction (ramp up/down).

[9] present the objectives and services of an LFM. According to them, the primary objective of an LFM is to support the trade of end-user flexibility for the benefit of the DSO's grid operations. According to Ref. [10], the congestion management, the voltage/reactive power, and the controlled islanding are solved via LFM. Furthermore, the cost of flexibility for congestion management is discussed by Ref. [11], based on the real-time activation of flexibility. According to Ref. [12], the DSO should make sure that the required flexibility is continuously available throughout the operational process. Such situations might be affected by short-term uncertainties [12]. According to Ref. [13] there is a research gap related to the usage and design benefits of LFMs. Although [13] investigates the efficient integration of renewable resources in the power system with LFM, the authors do not involve the grid operations in their research. In our paper we consider congestion management and voltage corrections under uncertainty, with a suggestion for a market design for LFM based on the paradigm of stochastic market clearing.

Stochastic dispatch and bidding strategies for reducing operational costs have been investigated in the literature. For example, [14] argue that demand and supply uncertainty can be addressed by using stochastic dispatch and clearing. Morales et al. [15] investigated a two-stage stochastic model for dispatch in a pooled design. Bjørndal et al. [16] consider an energy-only market with load uncertainty and flexibility costs for a stochastic dispatch mechanism and compared it with a myopic model (two-stage). In our research, we have designed a stochastic dispatch and bidding mechanism with deterministic cost parameters, influenced by Refs. [16,17], and [18]. The authors in Ref. [19] described market mechanisms based on systemic frameworks such as flexibility markets, local energy markets, hybrid local energy and flexibility markets. According to their paper, we propose a flexibility market.

Our paper has convex (the aggregator) and non-convex (the DSO) market participant models. The behavior of our LFM participants are non-strategic (price taker), stochastic, and aligned with ID market. Our power system has AC-OPF constraints and the cost minimization corresponds to social welfare maximization. According to the authors knowledge, there is not an another study that designs a LFM with these conditions.

The main contributions of our research, presented in this paper, are as follows:

- We present a stochastic LFM design in which flexibility is used for distribution grid operations to supplement an ID market for power.
- We explain the nature of stochastic flexibility bids with deterministic cost parameters in an LFM.
- We demonstrate a stochastic LFM design which leads to cost reduction compared with a situation where the DSO only use load shedding to resolve problems in rid operations.

The remaining part of this paper is organized as follows. In Section 2 we present the stochastic LFM design, system architecture, and bidding process. Section 3 provides the mathematical models. In Section 4 we describe the case study, grid problems, and present the results of our research, which are then discussed in Section 5. The main conclusions are provided in Section 6.

## 2. Market design

The design of an LFM must address the grid topology, timing aspects, and the heterogeneity of flexibility technologies. In this paper, we present our design for a pooled market, including an aggregator that bids on behalf of flexibility providers, and a single buyer, the DSO. The approach can easily be widened to include more buyers and sellers in the market place.

### 2.1. Bidding process details

In the pooled LFM design, we assume perfect competition, where each market participant is a price-taker that does not act strategically. For the aggregator, this means that the objective is to provide stochastic bids with the aim of minimizing the expected cost of the flexibility supply by using the available demand-side assets. For the DSO, the aim is to minimize the system cost of meeting demand in the network (including the option to shed load at the cost of Value Of Lost Load (VoLL)).

The uncertainty structure of bidding is two-stage stochastic optimization, as illustrated in Fig. 1 [20]. The bidding process is modeled as a two-stage stochastic problem due to the presence of flexibility assets that need to be considered over the entire time-span. Until time $t_{10}$, the parameters are deterministic and therefore both they and the bids have the same values for each scenario (the reason for this particular choice is discussed by Ref. [21] for the same case study studied under direct control). After $t_{10}$ up to $t_{24}$, the red filled-in circles in Fig. 1 represent scenario realizations that are uncertain when seen from time periods until $t_{10}$. In the suggested pooled market design, the aggregator bids stochastically into the LFM to establish a flexibility supply curve for each scenario and each time period. At period $t_{11}$, the second stage—during which uncertainty is resolved—starts, and the scenario-dependent demand for each customer becomes known.

Although we have stochastic power demand, the model (AC-OPF) to be solved by the DSO at each time period and in each scenario is deterministic. This is because the DSO always balances the system in real time using the flexibility procured and the option to shed at VoLL, but otherwise does not have any flexibility or storage option. This leads to a one-period deterministic problem for each time period and scenario.

During the stochastic bidding process, the aggregator needs to know the individual costs of flexibility assets in order to determine bid prices (i.e., the marginal costs of providing flexibility after scheduling flexibility assets and consumption). This bidding process, based on marginal costs, establishes a *flexibility supply curve* in the LFM under conditions of perfect competition. These flexibility cost parameters are deterministic, but the load in the different scenarios is stochastic, as is the demand for flexibility in the LFM.

At this time, the DSO examines how much flexibility is needed in the LFM to solve voltage drops and grid congestion issues with minimum costs. The DSO has a perfect foresight of the grid status and load in the buses. In the bidding phase, the DSO does not know where flexibility will be provided in the grid, it just signals an aggregated demand to the market. After the LFM market is cleared, the different consumers' flexibility supply is dispatched by the aggregator and communicated to the DSO.

If the cost of flexibility supply (i.e., the LFM price) is higher than the VoLL, or if the flexibility supply is insufficient to solve the grid

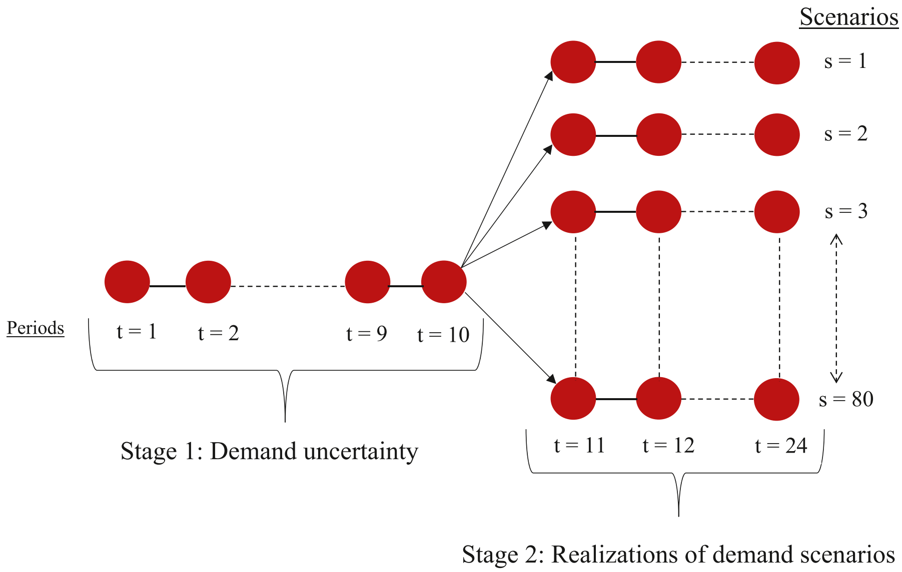



**Fig. 1.** Uncertainty structure and stages of the aggregator model.

problem, the DSO will apply load shedding instead. This could also happen as a consequence of the dispatch, as the grid location of flexibility is not known when bids are made.

### 2.2. *The stochastic LFM design and process*

In our proposed design for LFM, the customers are the flexibility providers, but they are represented in the market by an aggregator. We assume perfect competition for our proposed pooled LFM design. In this LFM, the flexibility supply bids are priced at the marginal cost (similar to balancing markets [4]). The interaction between the power customers (the flexibility providers), the aggregator, and the DSO in the pooled market is summarized in Fig. 2.

Every sixth period, the customers buy power from the grid at Intraday (ID) price. The delivery of the power is determined by the consumers' choice over the six periods until the next intraday trade possibility. It should be noted that when this is done, demand is known for the five periods, due to the uncertainty structure. In all periods, the aggregator can sell flexibility and the DSO can buy flexibility.

The DSO sees a set of stochastic scenarios of the active power demand for every customer and location in the grid. The bidding is done under uncertainty, so the DSO presents the flexibility demand bid for any time period in the form of a discrete probability distribution, with flexibility demand represented for each of the scenarios in every time period. The load used in these scenarios is

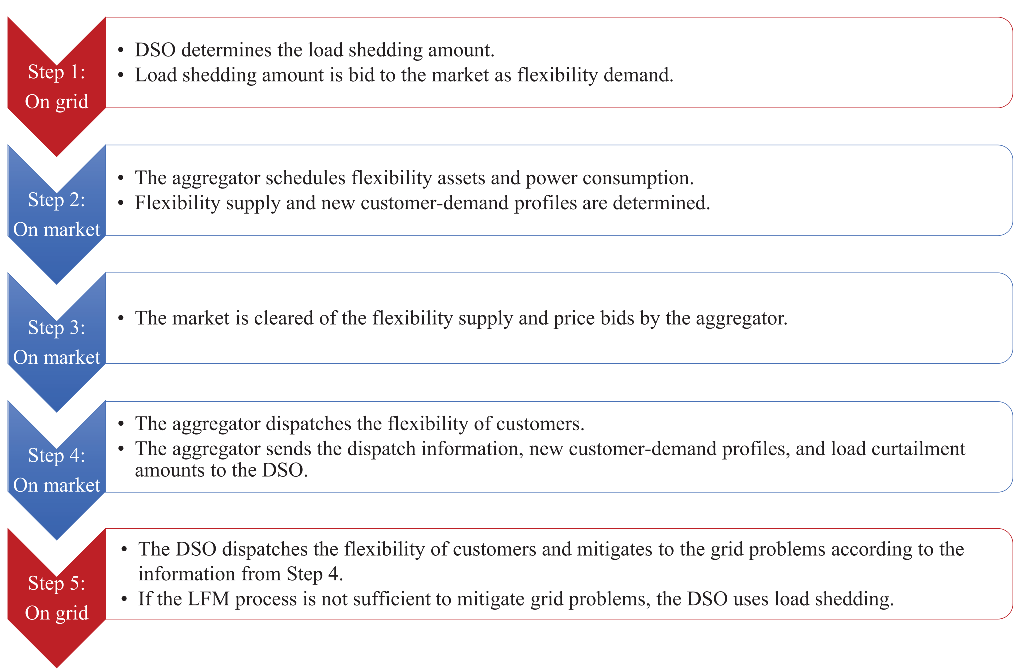



**Fig. 2.** The stochastic LFM design and process.

before any demand-side actions are taken or battery scheduling is done.

The aggregator sees the same information as the DSO, without knowing the grid topology. However, as part of its bidding process the aggregator will perform optimal scheduling of batteries, load shifting, and curtailment in order to provide flexibility at an expected minimum cost over the whole horizon. The aggregator provides scenario-dependent flexibility supply bids that consist of the price needed to meet each scenario's flexibility demand. Only *active power* is traded in the LFM, but reactive power is considered by the DSO when solving the AC optimal power flow (AC-OPF) problems to calculate the demand for flexibility.

The purpose of the flexibility bids and the stochastic dispatch is to enable market clearing that equalizes the flexibility demand by the DSO to the flexibility supply by the aggregator in every scenario at every time period. The DSO's objective is to avoid load shedding due to congestion or voltage drops. As the LFM is pooled, the DSO cannot know or control which customer provides the dispatched flexibility; rather, the decision is up to the aggregator.

In short, the steps in the bidding, market clearing, and dispatch process for a specific time period are as follows.

Step 1. The DSO determines the amount of load shedding for active and reactive power before flexibility trade for each scenario and period, by using a deterministic AC-OPF model. After solving the AC-OPF problem, the DSO calculates flexibility demand according to the active power load shedding amount and bids this flexibility demand to the LFM for each scenario and time period.

Step 2. By considering the demand in each scenario and period, as well as the probabilities of the scenarios, the aggregator schedules flexibility assets, determines the new demand level of each customer, and bids a price-quantity pair as a scenario-dependent flexibility bid to the LFM for each period and scenario.

Step 3. For each scenario and period, the market is cleared so that the demand for flexibility is equal to supply. In each scenario, this results in a flexibility price. Prices in the LFM are the marginal costs of flexibility provision.

Step 4. For each scenario and period with a flexibility supply requirement and price, flexibility of customers are dispatched by the aggregator according to the schedule. The aggregator then communicates the dispatch to the DSO as the provided flexibility service at the cleared price, and provides information about the consumers' new demand level.

Step 5. By considering new demand levels after flexibility procurement, the DSO solves the new OPF. If new demand levels after the dispatch of flexibility do not resolve the congestion or voltage problems, load shedding may still be needed. This may also be because the flexibility has not been dispatched to the locations in the grid where it is needed. When load shedding is used, the DSO sells back purchased ID power to the main grid in order to compensate for reduced demand compared with the volume bought by the aggregator.

## 3. Mathematical models and equations

In this section we describe three used models for flexibility demand determination by the DSO, for flexibility supply and LFM prices determination by the aggregator, and for final stochastic dispatch by the DSO. The first subsection 3.1 presents the AC-OPF formulations used by the DSO for determining how much flexibility is needed in the operation of the system. The second subsection 3.2 presents a two-stage stochastic aggregator model to schedule the flexibility supply from consumers and the corresponding bidding and market clearing in the LFM. The third and final subsection 3.3 presents the DSO's final power flow optimization in which dispatched flexibility is included and load shedding is used as the last resort. The nomenclature of mathematical models are provided in Table 2.

### 3.1. Model 1: The DSO's calculation of the flexibility demand

To determine how much flexibility the DSO needs, we use a non-linear AC-OPF model with load shedding. While consumers buy power from the ID market, the DSO estimates how that will lead to congestion and voltage problems.

We assume perfect competition and let the DSO minimize the system cost. At this stage, the DSO does not consider the available consumer flexibility, but rather considers the different households' original demand, excluding the operation of batteries, load shifting, and curtailment. The DSO solves the model for each period and scenario in order to estimate flexibility demand based on the need for load shedding. The shed volumes are then used as bids for buying flexibility in the LFM. The aim is that the aggregator can provide flexibility at a lower cost than VoLL. The equations in the following subsections present the mathematical model used by the DSO for each of the scenarios and periods.

#### 3.1.1. Load balance constraints

Equations (1) and (2) satisfy the active ($L^p_{i,t,s}$) and reactive power ($L^q_{i,t,s}$) demand at each bus by purchasing from the transmission grid, $P_{g,t,s}$ and $Q_{g,t,s}$, and by load shedding, $P^{shed}_{i,t,s}$ and $Q^{shed}_{i,t,s}$,

$$\sum_{j\in J} AF_{i,j,t,s} = \sum_{g\in G_i} P_{g,t,s} - L^p_{i,t,s} + P^{shed}_{i,t,s} \tag{1}$$

$$\sum_{j\in J} RF_{i,j,t,s} = \sum_{g\in G_i} Q_{g,t,s} - L^q_{i,t,s} + Q^{shed}_{i,t,s} \tag{2}$$

#### 3.1.2. Allocation constraint

The allocation constraint in equation (3) outlines the purchases of active power/electricity from the ID market via the transmission grid according to consumer demands. The purchase is done at every *sixth* period, but it is allocated to be used in *every* period. The ID purchases take place in periods $(t_1, t_6, t_{11}, t_{16}, t_{21})\in\mathcal{T}^1$ which we call operational periods while allocation to demand is done in all periods (from $t_1$ to $t_{24}$), which we call balancing periods.

The allocation process and interaction with the ID market and the LFM is illustrated in Fig. 3. The aggregator purchases power from the transmission grid at ID prices (large circles in Fig. 3). Purchased power is allocated to customers and the LFM is cleared (filled-in circles).

It is possible to buy electricity, $\Omega^{DSO}_{g,t,s}$, from the ID market in every operational period $t\in\mathcal{T}^1$ and it can be consumed ($P_{g,t,s}$) in every period, $t\in\mathcal{T}^1\cup\mathcal{T}^2$ (allocation). More specifically, the purchase/consumption relation is modeled as

$$\Omega^{DSO}_{g,t^1,s} = P_{g,t^1,s} + \sum_{t^2\in\mathcal{T}^2_{t^1}} P_{g,t^2,s}, \qquad t^1\in\mathcal{T}^1 \tag{3}$$

with $\mathcal{T}^2_{t_1}$ represents the balancing periods in which flexibility services can be bought, but only previously purchased ID power from the operational period $t_1$ is available, if not already consumed.

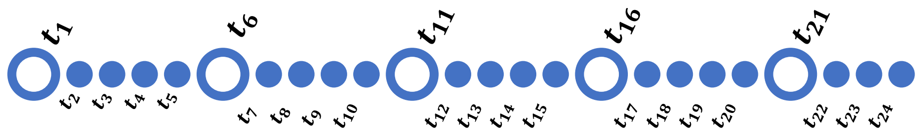



**Fig. 3.** ID and LFM alignment.

*3.1.3. Grid congestion constraint*

Equation (4) models the grid power flow limitation:

$$AF^2_{i,j,t,s} + RF^2_{i,j,t,s} \le S^2_{i,j} \tag{4}$$

where $S$ represents the installed capacity of the line.

*3.1.4. Power flow constraints*

AC power flow constraints enforce the active power balance (equation (5)) and reactive power balance (equation (6)) at each bus in the distribution grid.

$$AF_{i,j,t,s} = V^2_{i,t,s} Y_{i,j,s} cos\theta_{j,i,s} - V_{i,t,s} V_{j,t,s} Y_{i,j,s} cos(\delta_{i,t,s} - \delta_{j,t,s} + \theta_{j,i,s}) \tag{5}$$

$$RF_{i,j,t,s} = V^2_{i,t,s} Y_{i,j,s} sin\theta_{j,i,s} - V_{i,t,s} V_{j,t,s} Y_{i,j,s} sin(\delta_{i,t,s} - \delta_{j,t,s} + \theta_{j,i,s}) - \frac{bV^2_{i,t,s}}{2} \tag{6}$$

*3.1.5. Load shedding equations*

Equation (7) is used to keep the power factor constant at the bus where the load shedding happens.

$$Q^{shed}_{i,t,s} = P^{shed}_{i,t,s} \cdot \tan(\theta_i) \tag{7}$$

*3.1.6. Voltage magnitude limit*

Equation (8) gives magnitude limits for voltage

$$\underline{V} \le V_{i,t,s} \le \bar{V} \tag{8}$$

*3.1.7. The objective function of the DSO model*

The objective function (equation (9)) that is minimized under every scenario $s \in \mathcal{S}$ is defined by the total cost of the DSO's grid operations (OF1), considering both purchases of power (by the aggregator) and load shedding (by the DSO) in order to meet system demand. The cost of power purchases from the main grid is given by the ID market price, whereas the cost of load shedding is VoLL (EUR 3000/MWh[1]). It should be noted that this does not consider the use of flexibility on the consumer side, as the purpose is to identify the flexibility demand from the system's perspective. Based on this assumption, there exists a joint multivariate distribution for all the consumer demands that the DSO, the aggregator, and the consumers see, which is the best available demand prediction. This is an approximation, as the consumers and the aggregator may have their own incentives to use demand-side flexibility, such as the ID price.

$$\text{minimize OF1} = \sum_{t^1 \in \mathcal{T}^1} \sum_{g \in \mathcal{G}} B_{t^1} \cdot \Omega^{DSO}_{g,t^1,s} + \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} P^{shed}_{i,t,s} \cdot \text{VoLL} \tag{9}$$

*3.1.8. Flexibility demand bids to the LFM*

After Model 1 is solved by the DSO and calculating the active and reactive power shedding amounts from equations (1), (2) and (9), the DSO bids the required flexibility amount, $D_{i,t,s}$, to the LFM as *active power* for each time period and scenario.

The index of consumers ($c \in \mathcal{C}$) in the aggregator model is mapped to the index of buses ($i \in \mathcal{I}$) in the DSO model. Each household represents a different bus in the distribution grid topology, as illustrated in Fig. 4, but not all buses corresponds households ($\mathcal{I} \rightarrow \mathcal{C}$ and $\mathcal{C} \subset \mathcal{I}$). The demand for flexibility is transmitted to the LFM as pooled (i.e., $\sum_{i \in \mathcal{I}} D_{i,t,s}$ as post-calculation) without considering grid topology.

*3.2. Model 2: The aggregator and flexibility supply bids*

The aggregator formulates a two-stage stochastic program under uncertainty to schedule the use of flexible resources for all consumers, and provides aggregated (over the consumers) flexibility bid curves (active power) for each scenario and period. The scheduling process calculates the new demand level of each customer according to the flexibility supply bid. While the DSO can solve the grid problems as single-period single-scenario problems, the aggregator must solve the whole problem jointly as a stochastic program because the periods and scenarios are interlinked by using storage and load shifting.

*3.2.1. Demand-side and storage-side flexibility balance*

When considering the DSO flexibility demand and power prices, the main aim is to schedule the flexibility supply to minimize total system costs. Load balance equations are used to calculate the purchase from the ID market and scheduling of each customer's assets in order to define new demand levels after load shifting, curtailment, and battery scheduling.

Equation (10) expresses the purchases of active power from the ID market in every sixth period and allocation to consumer in every period, in the same way as in equation (3) and Fig. 3, where it is estimated by the DSO:

$$\Omega^{agg}_{c,t^1,s} = \rho_{c,t^1,s} + \sum_{t^2 \in \mathcal{T}^2_{t^1}} \rho_{c,t^2,s} \tag{10}$$

where $t^1 \in \mathcal{T}^1$.

The difference between Model 1 and 2 is that the DSO does not consider the available flexibility to the consumers, whereas the aggregator does, as will be shown in the following equations (equations (11)–(13) and (15)). For every customer, the aggregator schedules flexibility assets in order to use demand-side flexibility in every scenario and period, and to determine new customer demand

[1] The number that we used for VoLL is based on Nord Pool's day ahead maximum price cap, as it is described in https://www.nordpoolgroup.com/en/message-center-container/newsroom/exchange-message-list/2013/Q4/No-692013—New-minimum-and-maximum-price-caps-in-NOK-from-29-December-/

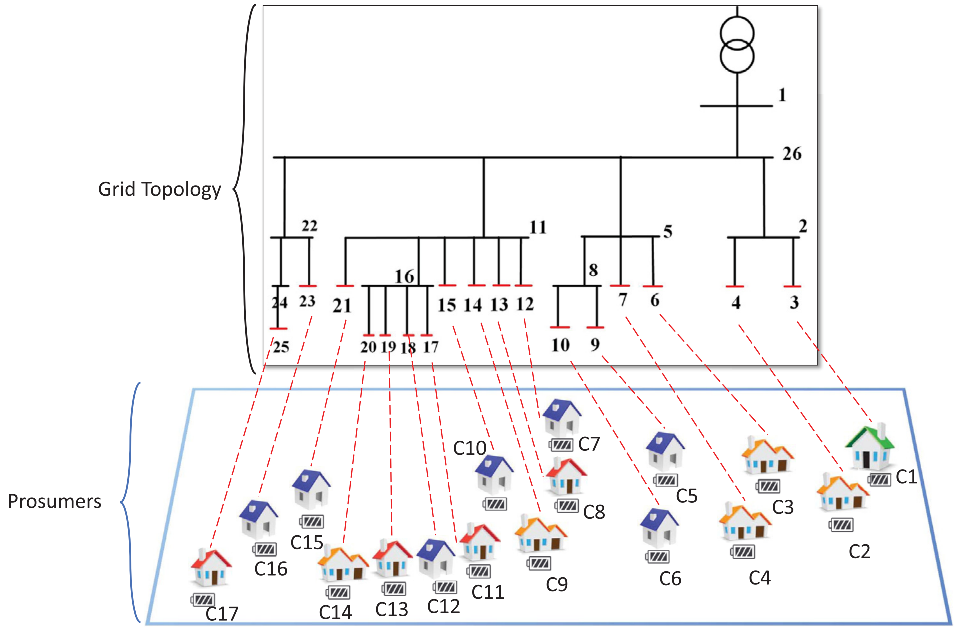



**Fig. 4.** Grid topology and market participants presentations.

levels ($L^{new}_{c,t,s}$) when supplying the flexibility, as modeled in equation (11). This action corresponds to Step 2 in subsection 2.2 and includes original load $L_{c,t,s}$, net battery discharge ($\rho^{dis}_{c,t,s} - \rho^{chr}_{c,t,s}$), curtailment $\varrho^{curt}_{c,t,s}$ and load shifting out of the time period $t$, $\rho^{shift}_{c,t,s}$.

$$L_{c,t,s} - \left(\left(\rho^{dis}_{c,t,s} - \rho^{chr}_{c,t,s}\right) - \varrho^{curt}_{c,t,s} - \rho^{shift}_{c,t,s}\right) = L^{new}_{c,t,s} \tag{11}$$

where $t \in \mathcal{T}$ and

$$0 \leq \varrho^{curt}_{c,t,s} \leq L_{c,t,s} \tag{12}$$

In equation (13), $\rho^{A}_{c,t,s}$ represents the volume of power flexibility for accommodating the DSO's flexibility request after scheduling the assets of consumers and determining new demand levels. Equation (13) is used to ensure that the new demand level after the aggregator has scheduled the flexibility assets is either equal to or lower than the old demand level (i.e., the demand before shifting, curtailing, and battery usage) during congested hours, for each scenario. It should be noted that when flexibility supply is negative, it will correspond to the periods when batteries are charged or load is increased in the shifting process. These are periods and scenarios without flexibility demand from the DSO.

$$L^{new}_{c,t,s} + \rho^{A}_{c,t,s} = L_{c,t,s} \tag{13}$$

Equation (14) establishes the supply-demand balance in the ID market for the new demand levels.

$$L^{new}_{c,t,s} = \rho_{c,t,s} \tag{14}$$

### 3.2.2. Flexibility supply-demand balance in the LFM

The flexibility balance equation (equation (15)) calculates the amount of flexibility supplied by the aggregator to meet the DSO's flexibility demand at each period and scenario where flexibility demand $D_{i,t,s}$ exists.

$$\sum_{c \in \mathcal{C}} \rho^{A}_{c,t,s} \geq \sum_{i \in \mathcal{I}} D_{i,t,s} : \quad \delta^{A}_{t,s} \quad \text{if} \quad \sum_{i \in \mathcal{I}} D_{i,t,s} > 0 \tag{15}$$

where $\sum_{i \in \mathcal{I}} D_{i,t,s}$ is the pooled demand of flexibility from the DSO, which is obtained as a result of solving the previous problem (Model 1).

The dual variable ($\delta^{A}_{t,s}$) of equation (15) measures *the marginal cost of flexibility provision* (Step 3 in Subsection 2.2). The aggregator's flexibility bid to the pooled market is for each scenario and time period in which the price ($\delta^{A}_{t,s}$) is combined with the volume $\sum_{c \in \mathcal{C}} \rho^{A}_{c,t,s}$.

### 3.2.3. Import power limit from the main grid

Equation (16) keeps the purchase from the main grid under the installed capacity of the transformer connecting the distribution grid to the main grid. The value $S_{i,j}$ shows the capacity of only one line, the line between Low-voltage (LV) and Medium-voltage (MV) grid (between buses 1 and 26),

$$\sum_{c \in \mathcal{C}} \rho_{c,t,s} \leq S_{i,j} \tag{16}$$

### 3.2.4. Intertemporal constraints relating to batteries

Equation (17) calculates the state of charge for batteries, whereas equations (18)–(20) calculate the capacity of batteries, and charging and discharging limits, respectively.

$$\Psi_{c,t,s} = \Psi_{c,t-1,s} + E^{chr} \rho^{chr}_{c,t,s} - \frac{\rho^{dis}_{c,t,s}}{E^{dis}}, \qquad s \in S \qquad c \in C \tag{17}$$

where $t \in T$.

$$\underline{\Psi} \leq \Psi_{c,t,s} \leq \bar{\Psi} \tag{18}$$

$$0 \le \rho^{chr}_{c,t,s} \le H \cdot \bar{\Psi} \tag{19}$$

$$0 \le \rho^{dis}_{c,t,s} \le H \cdot \bar{\Psi} \tag{20}$$

### 3.2.5. Load shifting

Load shifting is modeled using four equations to make it convex and piecewise linear using breakpoints $k \in \mathcal{K}$. Equation (21), represents x-axis values (amount of power) in the load shifting cost function, whereas the function row, equation (22), represents the y-axis (non-linear cost function). The data for x-axis values, $\sigma_k$, and variable costs for the function's slope, $VC_k$, are taken from Ref. [21] as are other load shifting, battery, and load curtailment parameters. Equation (23) is used for the convex combination of breakpoints. As the cost function is convex, two neighboring breakpoints will be used by design, making the approximation as close as possible. Equation (24) restricts the usage of load shifting; that there cannot be any load shifting outside a pre-specified time interval. Equation (25) emphasizes that within a specified time interval the total load allocated in the different periods needs to be equal to the total load withdrawn from the other periods.

$$\text{minimize OF2} = \sum_{t^1_m \in \mathcal{T}^1_m} \left( B_{t^1_m} \cdot \Omega^{agg}_{t^1_m} \right) + \sum_{c \in \mathcal{C}} \sum_{t_m \in \mathcal{T}_m} \left( \Gamma_{c,t_m} + \pi^{shift}_{c,t_m} + C^{curt} \varrho^{curt}_{c,t_m} \right) + \sum_{s \in S} P_s \left( \sum_{c \in \mathcal{C}} \sum_{t^1_n \in \mathcal{T}^1_n} \left( B_{t^1_n} \cdot \Omega^{agg}_{c,t^1_n,s} \right) + \sum_{c \in \mathcal{C}} \sum_{t_n \in T_n} \left( \Gamma_{c,t_n,s} + \pi^{shift}_{c,t_n,s} + C^{curt} \varrho^{curt}_{c,t_n,s} \right) \right) \tag{27}$$

$$\rho^{\text{shift}}_{c,t,s} = \sum_{k \in \mathcal{K}} \lambda_{c,t,s,k} L_{c,t,s} \sigma_k \quad t^{down} \le t \le t^{up} \tag{21}$$

$$\pi^{\text{shift}}_{c,t,s} = \sum_{k \in \mathcal{K}} \lambda_{c,t,s,k} L_{c,t,s} \sigma_k VC_k \quad t^{down} \le t \le t^{up} \tag{22}$$

$$\sum_{k \in \mathcal{K}} \lambda_{c,t,s,k} = 1, \quad 0 \le \lambda_{c,t,s,k} \le 1 \quad t^{down} \le t \le t^{up} \tag{23}$$

$$\rho^{shift}_{c,t,s} = 0, \quad t \le t^{down}, t \ge t^{up} \tag{24}$$

and

$$\sum_{t \in \mathcal{T} \cap [t^{down}, t^{up}]} \left( \rho^{shift}_{c,t_n} + \rho^{shift}_{c,t_m,s} \right) = 0 \tag{25}$$

### 3.2.6. The cost of discharging battery

To assign a cost to battery usage, we consider a cost coefficient associated with the battery discharge, while we assume that battery charge is done at no cost. In equation (26), we multiply the amount of discharge (MWh) by the fixed cost of battery discharge, EUR 0.140/MWh. The cost of the battery discharge is taken from Ref. [22],

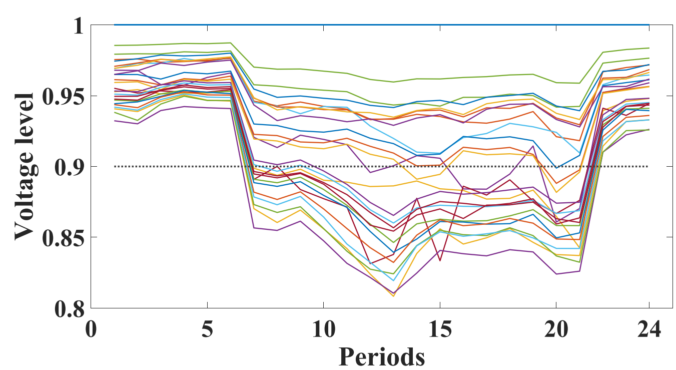
$$\Gamma_{c,t,s} = \rho^{dis}_{c,t,s} \cdot \text{IC} \tag{26}$$

### 3.2.7. Non-anticipativity constraints

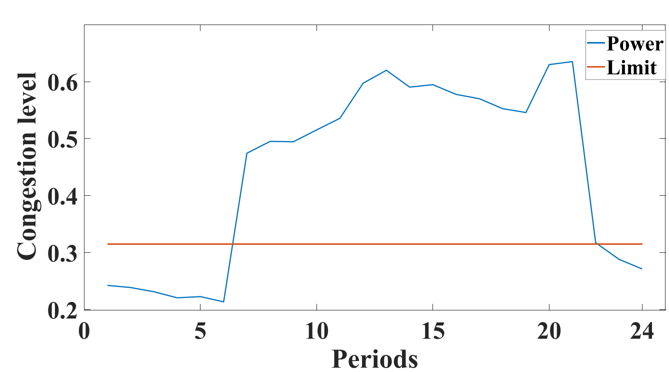
The aggregator model is two-stages. Therefore, non-anticipativity constraints are needed to keep first-stage variables at the same values for all scenarios [23] in the first stage. The first-stage variables in the aggregator model are all variables up to and including period $t_{10}$.

### 3.2.8. The objective function of the aggregator

The aim of the aggregator is to minimize equation (27), which defines the cost of operations for the aggregator (OF2). The first term denotes the purchase from the main grid; the second element is the sum of battery usage cost, the load shifting cost, and the load curtailment cost. The third and fourth elements represent the same costs at the second stage.




(a) Voltage levels of each bus (MVA). Each line represents a bus.




(b) Congestion level (MWh).

**Fig. 5.** Voltage and congestion problems.

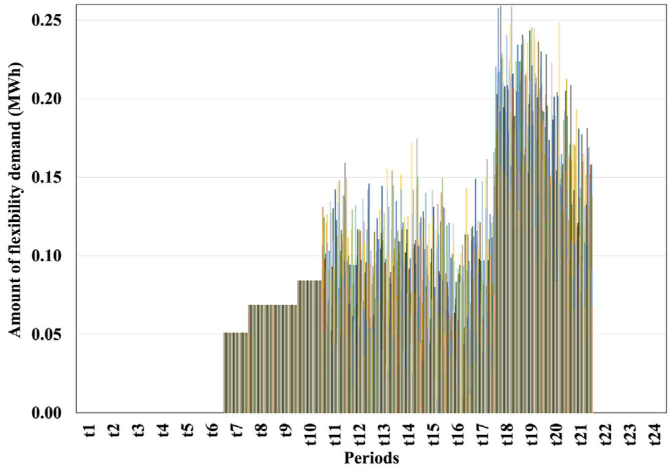



**Fig. 6.** Flexibility demand by the DSO as active power (MWh), aggregated per customer (MWh). Each color represents a scenario.

### 3.3. Model 3: The DSO's final dispatch of the flexibility

The formulation of the stochastic dispatch is mainly the same as in the AC-OPF model presented in subsection 3.1, except for the load balance equations. This corresponds to Step 4 in subsection 2.2. It should be noted that the new customer demand levels need to be represented as both active power and reactive power, hence $L^{new_p}_{c,t,s} = L^{new}_{c,t,s}$ and the reactive power is calculated in equation (28) as follows

$$L^{new_q}_{c,t,s} = L^{new_p}_{c,t,s} \cdot \tan(\theta_i) \quad c \in \mathcal{C} \tag{28}$$

The new demand level levels, $L^{new_p}_{c,t,s}$ and $L^{new_q}_{c,t,s}$, of each consumer of each consumer are mapped into different nodes $i \in \mathcal{I}$: $(\mathcal{C} \rightarrow \mathcal{I})$ and then into the respective active and reactive power as follows (see also Fig. 4 in subsection 4.1). An important detail here is that although pricing of the flexibility supply is done in a pooled market, the information about the new demand levels from individual consumers are shared with the DSO by the aggregator (Step 4 in Subsection 2.2).

Equations (29) and (30) model the load balance constraints of the DSO, considering the flexibility services from the LFM and the new demand levels ($L^{new_p}_{i,t,s}$ and $L^{new_q}_{i,t,s}$) (Step 5 in Subsection 2.2).

$$\sum_{j \in J} AF_{i,j,t,s} = \sum_{g \in G_i} P_{i,g,t,s} - L^{new_p}_{i,t,s} + P^{shed}_{i,t,s} \tag{29}$$

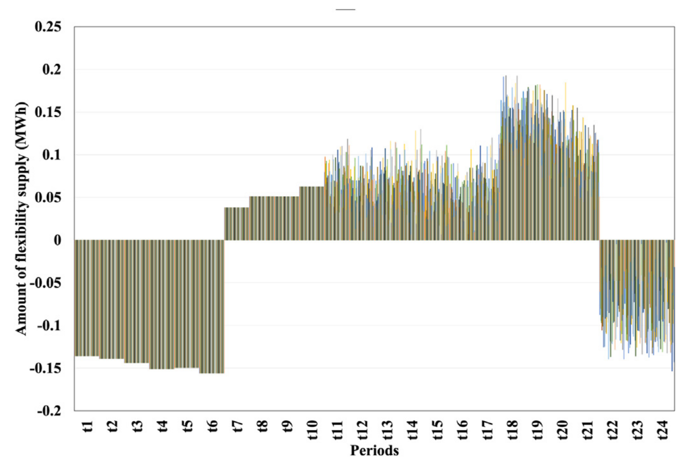



**Fig. 8.** Flexibility supply from the aggregator in the pooled market as active power, aggregated for all consumers (MWh). Each color represents a scenario.

$$\sum_{j \in J} RF_{i,j,t,s} = \sum_{g \in G_i} Q_{i,g,t,s} - L^{new_q}_{i,t,s} + Q^{shed}_{i,t,s} \tag{30}$$

#### 3.3.1. The objective function for the DSO's dispatch

The objective function in equation (31) (OF3) aims to minimize the cost of electricity traded in the transmission grid by the DSO and the aggregator at the ID price, in addition to the cost of load shedding.

$$\text{minimize OF3} = \sum_{t^1 \in T^1} \sum_{g \in G} \left( B_{t_1} \cdot \Omega_{g,t^1,s} \right) + \sum_{t \in T} \sum_{i \in I} \left( P^{shed}_{i,t,s} \cdot \text{VoLL} \right) \tag{31}$$

## 4. Case study and results

The case study includes real-life data with extensive analysis and solution proposals from a day with coercive conditions. First, we explain the grid structure and our consumer data. Second, we go through the steps of bidding, market clearing, and dispatch, as described above in subsections 3.1, 3.2, and 3.3.

### 4.1. Grid structure and consumer data

Our case study is a distribution system in the Norway-Hvaler municipality of Viken County in southern Norway, and the data were recorded in January 2016. The Hvaler area comprises small

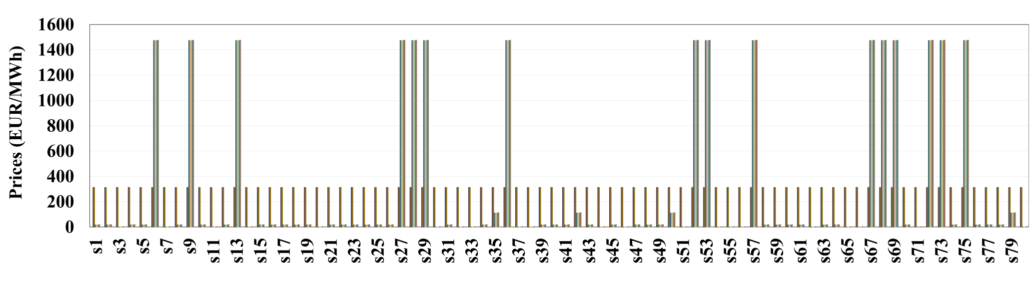



**Fig. 7.** LFM prices in the pooled market (EUR/MWh). Each color represents a period.

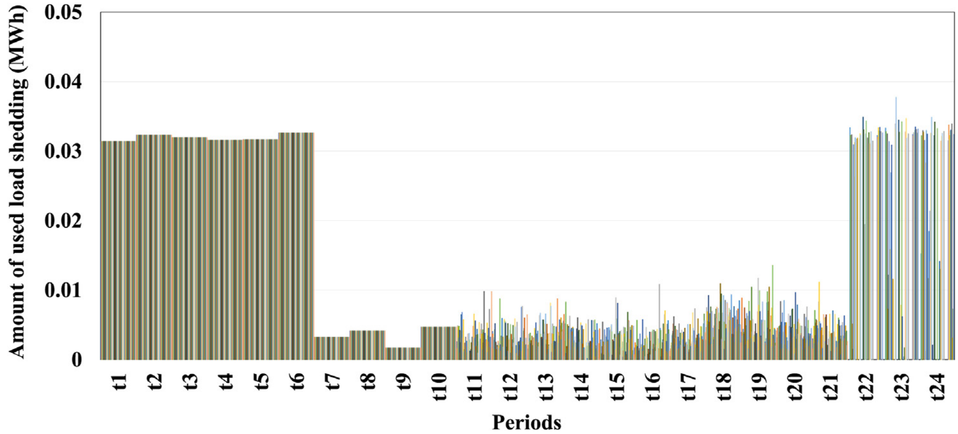



**Fig. 9.** Used load shedding by the DSO as active power (MWh). Each color represents a scenario.

**Table 1**
Price-quantity pairs (EUR/kWh-kW) for bid curves at each scenario per period.

| | s1 | s2 | s3 | s4 | s5 | s6 | s7 | s8 | s9 | s10 |
|---|---|---|---|---|---|---|---|---|---|---|
| **t7** | (312.74, 37.9) | (312.74, 37.9) | (312.74, 37.9) | (312.74, 37.9) | (312.74, 37.9) | (312.74, 37.9) | (312.74, 37.9) | (312.74, 37.9) | (312.74, 37.9) | (312.74, 37.9) |
| **t8** | (312.74, 50.9) | (312.74, 50.9) | (312.74, 50.9) | (312.74, 50.9) | (312.74, 50.9) | (312.74, 50.9) | (312.74, 50.9) | (312.74, 50.9) | (312.74, 50.9) | (312.74, 50.9) |
| **t9** | (312.74, 50.8) | (312.74, 50.8) | (312.74, 50.8) | (312.74, 50.8) | (312.74, 50.8) | (312.74, 50.8) | (312.74, 50.8) | (312.74, 50.8) | (312.74, 50.8) | (312.74, 50.8) |
| **t10** | (312.74, 62.4) | (312.74, 62.4) | (312.74, 62.4) | (312.74, 62.4) | (312.74, 62.4) | (312.74, 62.4) | (312.74, 62.4) | (312.74, 62.4) | (312.74, 62.4) | (312.74, 62.4) |
| **t11** | (16.64, 79.4) | (16.64, 97.3) | (0.7026, 52.3) | (16.64, 89.4) | (16.64, 65.8) | (1473.51, 92.5) | (0.7026, 58.6) | (16.64, 72.9) | (1473.51, 38.6) | (16.64, 67.8) |
| **t12** | (16.64, 71.8) | (16.64, 66.4) | (0.7026, 36.7) | (16.64, 82.7) | (16.64, 70.4) | (1473.51, 74.2) | (0.7026, 51.2) | (16.64, 32.8) | (1473.51, 69.9) | (16.64, 46.7) |
| **t13** | (16.64, 37.2) | (16.64, 42.4) | (0.7026, 60.7) | (16.64, 38.1) | (16.64, 69.3) | (1473.51, 65.1) | (0.7026, 45.6) | (16.64, 39.5) | (1473.51, 46.7) | (16.64, 85.5) |
| **t14** | (16.64, 100) | (16.64, 55.7) | (0.7026, 52.4) | (16.64, 39.2) | (16.64, 71.6) | (1473.51, 85.9) | (0.7026, 40.8) | (16.64, 64.3) | (1473.51, 81) | (16.64, 75.8) |
| **t15** | (16.64, 92.4) | (16.64, 82.6) | (0.7026, 1.8) | (16.64, 51.9) | (16.64, 25.3) | (1473.51, 27.4) | (0.7026, 34.5) | (16.64, 63.2) | (1473.51, 95.2) | (16.64, 92.2) |
| **t16** | (17, 24.3) | (17, 65.8) | (1.0626, 1.6) | (17, 45.4) | (17, 48.5) | (1473.87, 88.6) | (1.0626, 61.6) | (17, 59.2) | (1473.87, 55.5) | (17, 43.8) |
| **t17** | (17, 11.1) | (17, 39.2) | (1.0626, 51.8) | (17, 22.7) | (17, 67.3) | (1473.87, 87) | (1.0626, 81.4) | (17, 56.3) | (1473.87, 87.9) | (17, 64) |
| **t18** | (17, 107.3) | (17, 112.5) | (1.0626, 125) | (17, 121.2) | (17, 163.6) | (1473.87, 133.1) | (1.0626, 113) | (17, 117.8) | (1473.87, 150.7) | (17, 143.5) |
| **t19** | (17, 166.1) | (17, 113.6) | (1.0626, 135.5) | (17, 113.3) | (17, 156.8) | (1473.87, 166.4) | (1.0626, 123.2) | (17, 161.6) | (1473.87, 174.1) | (17, 134.3) |
| **t20** | (17, 142.3) | (17, 75) | (1.0626, 92.1) | (17, 122.4) | (17, 111.7) | (1473.87, 135.2) | (1.0626, 71.8) | (17, 150.6) | (1473.87, 169.4) | (17, 118.2) |
| **t21** | (20, 50.9) | (20, 91.2) | (4.0626, 76.8) | (20, 121.4) | (20, 109.7) | (1476.87, 126.8) | (4.0626, 72.9) | (20, 122) | (1476.87, 154.8) | (20, 119.1) |

| | s11 | s12 | s13 | s14 | s15 | s16 | s17 | s18 | s19 | s20 |
|---|---|---|---|---|---|---|---|---|---|---|
| **t7** | (312.74, 37.9) | (312.74, 37.9) | (312.74, 37.9) | (312.74, 37.9) | (312.74, 37.9) | (312.74, 37.9) | (312.74, 37.9) | (312.74, 37.9) | (312.74, 37.9) | (312.74, 37.9) |
| **t8** | (312.74, 50.9) | (312.74, 50.9) | (312.74, 50.9) | (312.74, 50.9) | (312.74, 50.9) | (312.74, 50.9) | (312.74, 50.9) | (312.74, 50.9) | (312.74, 50.9) | (312.74, 50.9) |
| **t9** | (312.74, 50.8) | (312.74, 50.8) | (312.74, 50.8) | (312.74, 50.8) | (312.74, 50.8) | (312.74, 50.8) | (312.74, 50.8) | (312.74, 50.8) | (312.74, 50.8) | (312.74, 50.8) |
| **t10** | (312.74, 62.4) | (312.74, 62.4) | (312.74, 62.4) | (312.74, 62.4) | (312.74, 62.4) | (312.74, 62.4) | (312.74, 62.4) | (312.74, 62.4) | (312.74, 62.4) | (312.74, 62.4) |
| **t11** | (0, 75.2) | (16.64, 58.9) | (1473.51, 67.3) | (0.7026, 89.8) | (16.64, 74.5) | (16.64, 93.8) | (16.64, 80.4) | (16.64, 51.4) | (16.64, 42.3) | (0.7026, 50.5) |
| **t12** | (0, 60.7) | (16.64, 51.7) | (1473.51, 69.8) | (0.7026, 59.2) | (16.64, 53.5) | (16.64, 86.1) | (16.64, 53.8) | (16.64, 95.9) | (16.64, 45.8) | (0.7026, 12.6) |
| **t13** | (0, 50.3) | (16.64, 35.6) | (1473.51, 54.1) | (0.7026, 42.5) | (16.64, 70.7) | (16.64, 67) | (16.64, 76.3) | (16.64, 65.7) | (16.64, 91.9) | (0.7026, 52.4) |
| **t14** | (0, 34.4) | (16.64, 74.9) | (1473.51, 80.9) | (0.7026, 36.1) | (16.64, 69.2) | (16.64, 112.4) | (16.64, 105.6) | (16.64, 64.2) | (16.64, 57.4) | (0.7026, 86.7) |
| **t15** | (0, 23.8) | (16.64, 21.9) | (1473.51, 77.2) | (0.7026, 44.7) | (16.64, 57.9) | (16.64, 38.1) | (16.64, 104.1) | (16.64, 59.6) | (16.64, 59.7) | (0.7026, 56.1) |
| **t16** | (0.36, 28.1) | (17, 47) | (1473.87, 89.7) | (1.0626, 27.2) | (17, 43.6) | (17, 26.5) | (17, 36.8) | (17, 45.8) | (17, 73.2) | (1.0626, 46.8) |
| **t17** | (0.36, 42.4) | (17, 55.4) | (1473.87, 83.2) | (1.0626, 76.5) | (17, 71.1) | (17, 61.4) | (17, 88.6) | (17, 26.5) | (17, 110.6) | (1.0626, 23.8) |
| **t18** | (0.36, 93.4) | (17, 160.7) | (1473.87, 191.3) | (1.0626, 110.1) | (17, 156.2) | (17, 134) | (17, 161.4) | (17, 126.3) | (17, 142.4) | (1.0626, 117.7) |
| **t19** | (0.36, 87) | (17, 178.7) | (1473.87, 177.1) | (1.0626, 116.8) | (17, 176.7) | (17, 118.6) | (17, 112.5) | (17, 121.9) | (17, 121.5) | (1.0626, 124.8) |
| **t20** | (0.36, 89.1) | (17, 145.3) | (1473.87, 137.3) | (1.0626, 90) | (17, 129.2) | (17, 98.2) | (17, 121) | (17, 90.4) | (17, 129.1) | (1.0626, 112.1) |
| **t21** | (3.36, 82.7) | (20, 98.5) | (1476.87, 81.9) | (4.0626, 77.1) | (20, 100.2) | (20, 127.3) | (20, 63.7) | (20, 47.8) | (20, 105.3) | (4.0626, 49.4) |

islands. The area has a population of 4000, but during holidays the population increases up to 40,000. The case study data were recorded in a single day, with coercive conditions for the grid. Most of the consumers in the grid are commercial buildings, family houses, and Norwegian second homes [24]. The mentioned ID market prices are ELSPOT prices from Nord Pool for the same period as for the demand data.

The case grid is a 22 kV and 230 V radial structure, as shown in Fig. 4. There are 26 buses in the grid, and 17 end users. We assume that the lines have sufficient capacity to feed end users, with the exception of the transformer between buses 1 and 26, they are connection to the transmission grid. The transformer has a capacity of 0.3085 MVA (for active power) and might be congested during peak load periods. End users and the aggregator have flexibility assets, such as load curtailment, load shifting, and batteries. Every grid member has a battery with 14 kW capacity without inverters.

To include uncertainty, we generate 80 scenarios for the demand data by using a *forecast-based moment-matching* scenario generation algorithm [25,26]. For details of this process, see Ref. [21].

We use CONOPT for Non-linear programming (NLP) and CPLEX for Linear Programming (LP) problems as solvers, and our models are implemented in GAMS using a computer with an Intel(R) Core(TM) i7-7500U processor at 2.70 GHz and 16 GB RAM. The total run time for the NLP model is less than 5 min, whereas for the LP model it is 30 s.

**Table 2**
Nomenclature.

| | |
|---|---|
| **Abbreviations** | |
| LFM | Local Flexibility Market |
| ID | Intraday |
| LP | Linear Programming |
| NLP | Non-linear Programming |
| AC-OPF | AC-Optimal Power Flow |
| DSO | Distribution System Operator |
| BRPs | Balance Responsible Parties |
| VoLL | Value of Lost Load |
| LV | Low-voltage |
| MV | Medium-voltage |
| **Sets** | |
| $c \in \mathcal{C}$ | Set of consumers indexed with $c$ and $\mathcal{C} \subset \mathcal{I}$ |
| $t \in \mathcal{T}$ | Set of periods with index $t$ |
| $t_m \in \mathcal{T}_m$ | Set of periods for first-stage, $m = \{1, \ldots, 10\}$ |
| $t_n \in \mathcal{T}_n$ | Set of periods for second-stage, $n = \{11, \ldots, 24\}$ |
| $\mathcal{T}_m \cup \mathcal{T}_n = \mathcal{T}$ | |
| $t^1 \in \mathcal{T}^1$ | Set of ID periods with index $t^1$ |
| $t^2 \in \mathcal{T}^2$ | Set of LFM market periods with index $t^2$ |
| $\mathcal{T}^1 \cup \mathcal{T}^2 = \mathcal{T}$ and $\mathcal{T}^1 \cap \mathcal{T}^2 = \varnothing$ | |
| $k \in \mathcal{K}$ | Index for break points in load shifting cost function (decision maker defined) |
| $s \in \mathcal{S}$ | Set of scenarios, index $s$ |
| $i \in \mathcal{I}$ | Set of buses in network with index $i$ |
| $j \in \mathcal{J}$ | Set of buses in network with index $j$ |
| $g \in \mathcal{G}$ | Set of generators with index $g$ |
| **Parameters** | |
| $B_{t_m^1}$, $B_{t_n^1}$ | Deterministic price at ID market |
| $H$ | Multiplying parameter for minimum battery capacity |
| $\overline{\Psi}$ and $\underline{\Psi}$ | Maximum and minimum capacities of batteries |
| $VC_k$ | Variable cost for load shifting |
| $IC$ | Battery investment cost |
| $L_{c,s,t}$ | Stochastic power demand |
| $L_{c,t}$ | Deterministic power demand |
| $E^{chr}$ and $E^{dis}$ | Charging and discharging efficiency coefficients of a battery |
| $P_s$ | Probability of a scenario (%) |
| $L_{c,t,s}^{shed}$ | Amount of load shedding demanded by DSO per customer |
| $\sigma_k$ | The percentage of demand in correspondence of breakpoint $k$ |
| $S_{i,j}$ | Line capacity limit of the distribution grid as active power (0.3085 MVA) |
| $C^{curt}$ | Cost of load curtailment (1500 EUR/MWh) |
| VoLL | Value of lost load (3000 EUR/MWh) |
| $L_{i,t}^{new_p}$, $L_{i,t}^{new_q}$ | Active and reactive new demand |
| $\varrho_{c,t,s}^{curt_p}$ | The amount of active load curtailment |
| **Variables** | |
| $\Omega_{g,t^1}^{agg}$ | The amount of total active power purchase by the aggregator from ID market |
| $\Omega_{g,t^1}^{DSO}$ | The estimated amount of total power purchase need by the DSO from ID market |
| $\Omega_{g,t^1}$ | The amount of net power purchase by the system from ID market |
| $\rho_{c,t,s}$ | The purchase from the ID market |
| $\rho_{t,s}^{shift}$ | The amount of shifted load |
| $\rho_{c,t,s}^{chr}$, $\rho_{c,t,s}^{dis}$ | Charging and discharging amount of battery |
| $D_{i,t,s}$ | Flexibility demand by the DSO |
| $\varrho_{c,t}^{curt}$ | The amount of load curtailment by the aggregator |
| $\Psi_{c,t,s}$ | State of charge for batteries at period $t$ |
| $\Gamma_{c,t,s}$ | Battery discharge cost |
| $\lambda_{c,t,s,k}$ | Continuous variable between 0 and 1 |
| $\pi_{c,t,s}^{\text{shift}}$ | Cost of load shifting |
| $\rho_{c,t,s}^{A}$ | Amount of flexibility supply for DSO's request |
| $\delta_{t,s}^{A}$ | Dual value, the marginal cost of flexibility provision for pooled market |
| $AF_{i,j,t}$, $RF_{i,j,t}$ | Active and reactive power flow between nodes $i$ and $j$ |
| $L_{i,t}^{p}$, $L_{i,t}^{q}$ | Active and reactive demand from bus $i$ |
| $V_{i,j,t}$ | Voltage magnitude |
| $P_{i,t}^{shed}$, $Q_{i,t}^{shed}$ | Amount of active and reactive power shedding |
| $P_{i,g,t}$, $Q_{i,g,t}$ | Active and reactive of scheduled production from a generator |

**Table 2** (*continued*)

| | |
|---|---|
| $Y_{s,ij}$ | Impedance value in AC-OPF model |
| $\delta_{i,t}$, $\theta_{s,i,j,t}$ | Voltage angles between buses $i$ and $j$ |
| $L^{new}_{c,t,s}$ | The new demand profile for a customer after scheduling |
| OF1 | Objective function result of Model 1 |
| OF2 | Objective function result of Model 2 |
| OF3 | Objective function result of Model 3 |

### 4.2. Grid problems and analysis

In order to use the potential demand-side flexibility, we use the system data where the electricity demand increases significantly in a sample day, with coercive conditions for the grid. In our case study with consumer data, we observe voltage profiles (Fig. 5a) that are under the feasibility threshold ($\approx$0.80 p.u. at the lowest), with grid congestion (Fig. 5b) that blocks the transfer from the transmission grid. We use MATPOWER developed by Ref. [27] to perform power flow calculations.

To estimate the need for flexibility under the voltage drop and grid congestion problems shown in Fig. 5, the DSO initiates its AC-OPF model to determine how much flexibility is needed to keep the system within the normal range of operation (voltage within the range 0.9 p.u. and 1.1 p.u. and no grid congestion). The load shedding amount, according to the equations from subsection 3.1, represents the flexibility requested by the DSO that is bid to the LFM as demand.

In Fig. 6 we observe the flexibility demand profile, where occurs mainly between $t_6$ and $t_{21}$.

### 4.3. The aggregator's perspective

The aggregator schedules flexibility assets such as load shifting, load curtailment, and batteries in order to meet load and flexibility demand. The scheduling results for these flexibility assets for the case study are presented in Appendix A.

The LFM prices are varied in order to clear flexibility supply and demand ranges between 4 EUR/MWh to 1500 EUR/MWh. Although it is possible to see lower prices than 1500 EUR/MWh for some scenarios (Fig. 7), the results show that the aggregator often uses load curtailment as the marginal asset to supply flexibility, as illustrated in Figure A.14 in Appendix A.

Fig. 8, shows the flexibility provision from the aggregator in all scenarios and periods. It should be noted that this represents a stochastic market clearing, as in each period the dispatched supply depends on the scenario-dependent demand.

### 4.4. The DSO final dispatch

The DSO uses the new load from the aggregator's dispatch schedule for each period and scenario. The exception is the curtailed volumes, for which the DSO is free to decide whether or not they will be curtailed. The DSO knows the price of flexibility in each scenario and time period, as well as the new load for each consumer (bus).

It should be borne in mind that the main aim of the LFM is to reduce the usage of the load shedding by the DSO and to obtain cost-efficient solutions to grid problems. The flexibility supply by the aggregator's customer portfolio is equal to the flexibility demand by the DSO, but it is the aggregator, not the DSO, that decides on the location of the flexibility supply. As a consequence, the DSO still may need to use load shedding if the flexibility provided in the buses does not resolve all issues (see Fig. 9).

The load shedding decision by the DSO is followed by reselling the same amount to the ID market to cancel out that part of the aggregator's ID buying. The income from this trade is paid by the DSO to the aggregator.

## 5. Discussions

In this section we discuss the nature of the stochastic bids, the cost-efficiency of using LFM, and the location of flexibility.

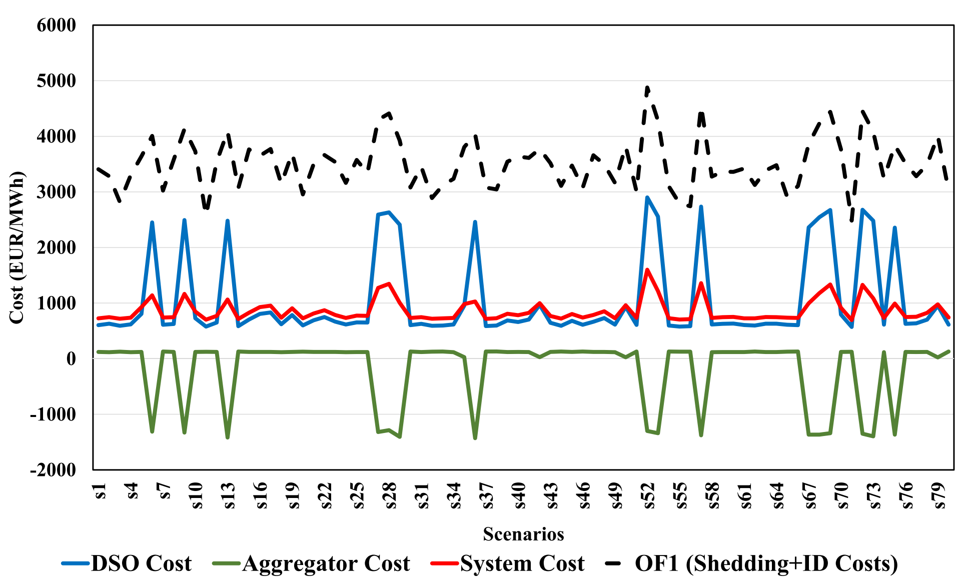



**Fig. 10.** Cost profiles of the system and LFM participants -lines are valid only in the scenario points.

### 5.1. The stochastic bids

The cost parameters of our flexibility assets are deterministic. However, the aggregated flexibility cost varies depending on which assets are available within the flexibility portfolio of the aggregator in the different time periods and scenarios. Hence, we observe same LFM prices for different flexibility supply amounts at each scenario in Table 1.

In general, the curtailment or shifting of small amounts with a high number of prosumers is more cost-efficient than dispatching all the flexibility from one prosumer. This can be explained by the disutility curve used to calculate the cost of load shifting [21] with increasing marginal cost. When the number of customers providing flexibility increases, the cost of flexibility (bid price) will decrease. If the same amount of flexibility is provided by a single consumer, the marginal cost will increase. In our case study, we observe that the LFM prices increase for some scenarios when we get closer to the end of the operational period ($t_{24}$), due to the limited number of flexibility providers.

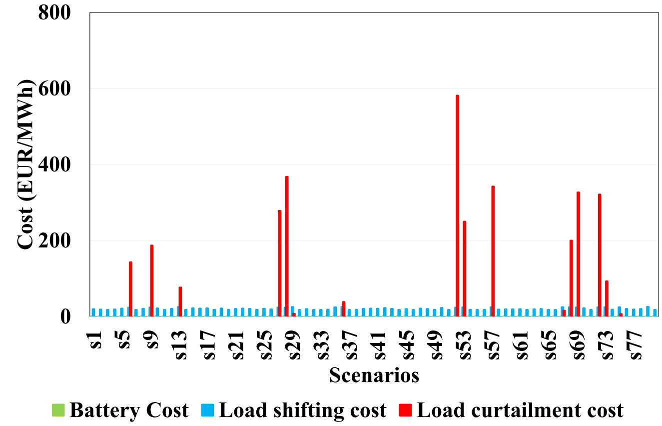
### 5.2. The cost-efficiency of the LFM

The cost-efficiency of our stochastic LFM is measured by considering the cost of the aggregator, the DSO, and the system. These costs are compared with the case without LFM, where only load shedding is available at the cost of VoLL. We separate this into the DSO cost, the aggregator cost, and the system cost.

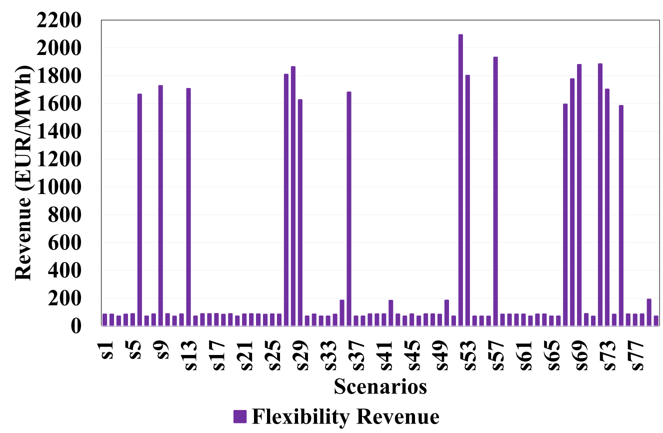
*The DSO cost* includes the load shedding cost from OF3 and the revenue payment for flexibility supply to the aggregator by the DSO (flexibility supply multiplied by LFM price). *The aggregator cost* includes the aggregator's ID market purchase (the net trading with ID market from OF3), battery, load shifting, revenues from flexibility supply, and load curtailment costs from OF2. *The system cost* includes the net trading with ID market from OF3 and load shedding cost from OF3, in addition to the flexibility cost from OF2 (load curtailment, shifting, and battery costs). All these costs are illustrated in Fig. 10.

The cost efficiency of the LFM usage becomes prominent, as shown by the comparison between the system cost and load shedding cost in Fig. 10. All cost profiles, especially the system cost (red line), are lower than the only load shedding usage cost (dashed black line). For the majority of the scenarios, the DSO cost (blue line) is lower than the system cost, except for scenarios when there is load curtailment usage. Accordingly, the social benefit of using LFM is illustrated in Fig. 10 as the area between load shedding cost and the system cost.

Fig. 11 shows the flexibility revenue (blue line) and cost of using each flexibility asset in the aggregator's portfolio (red, green, and orange areas) for the aggregator's cost/revenue profiles. The *flexibility revenue* is defined as the revenue payment for flexibility supply by the DSO and the revenue from the repayment of ID market trades due to load shedding in OF3 (load shedding amount multiplied with ID price). In every scenario, the flexibility revenue exceeds the overall flexibility cost (summation of load shifting, load curtailment, and battery discharge costs). Especially in scenarios with load curtailment usage, such as scenario 6, most of the flexibility is provided by the load shifting. Even the load curtailment is a more expensive choice, as the load shifting and batteries are insufficient to supply all flexibility. However, for the aggregator, the load curtailment is the marginal choice for flexibility supply and it decides the price. Hence, the flexibility revenue of the aggregator (flexibility supply multiplied by the marginal cost) exceeds the flexibility cost because all flexibility supply is priced according to the marginal cost. When the aggregator has no other flexibility options available, it activates the expensive resource and that resource sets the LFM price.




(a) Costs of flexibility assets (EUR/MWh).




(b) Revenues from flexibility (EUR/MWh).

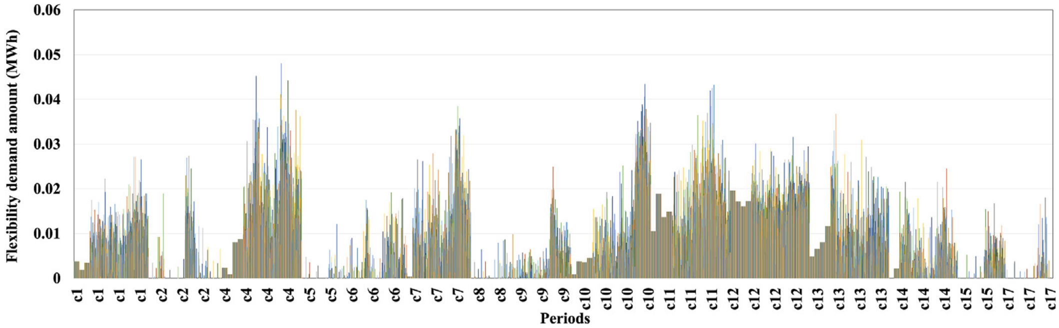
**Fig. 11.** Cost and revenue profiles of the aggregator's portfolio -aggregated for all periods.

Thus, the usage of the LFM to mitigate the grid problems decreases the system cost up by to 40% in scenarios without load curtailment. In scenarios with load curtailment, the cost-efficiency is up to 30%. The usage of a local flexibility market is efficient for solutions to grid problems too, as it is cost-efficient for all participants in the LFM.

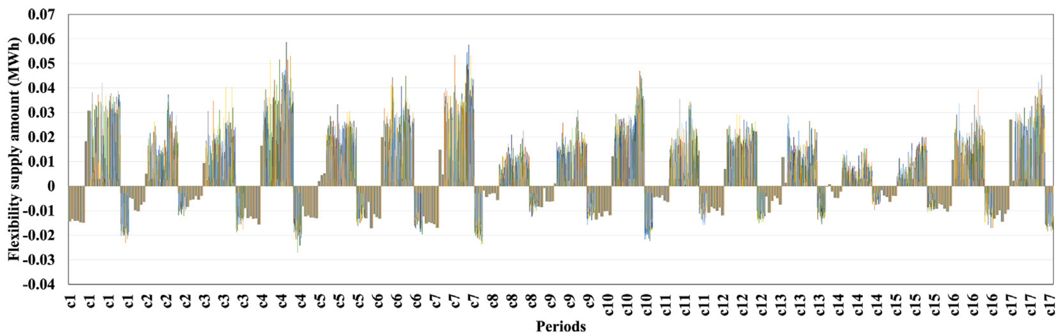
### 5.3. The location of flexibility

In the case study, the aggregator supplies flexibility for grid operations in real time but still we observe the usage of the load shedding by the DSO at the VoLL. In this regard, the location of a flexibility asset is important. As shown in Fig. 12a and b, the flexibility demand of the DSO is compared with the flexibility supply of the aggregator from each customer for each scenario. We observe that the overall flexibility supply from the aggregator meets the overall flexibility demand of the DSO in volume for each scenario. However, the location of the flexibility supply (i.e., the customer who supplies the flexibility in the aggregator's portfolio) does not meet with the DSO's location (bus) requirement. Hence, we observe the load shedding in Fig. 9 (Fig. 4 can show how to convert consumer index $c$ to bus index $i$ at the x-axis of Fig. 12).

Thus, a pooled LFM could mitigate grid problems and supply all the needed flexibility demand in the right periods for each scenario, but to provide more effective and cost-efficient solutions, the




**(a)** Flexibility demand from the DSO per customer as active power (MWh).




**(b)** Flexibility supply from the aggregator per customer as active power (MWh).

**Fig. 12.** Comparison of flexibility in supply-demand locations in the case study. Each color represents a scenario.

spatiality of flexibility suppliers needs to be considered. The location-specific problems, such as voltage drop, ideally need to be addressed where they occur on the grid. An approach with direct control of the flexibility (e.g., Ref. [21]), could provide more cost-efficient solution based on bilateral contracts. However, this would have the drawback that there would not be an established market, and price formation would not be clear.

## 6. Conclusion and recommendation

In this paper we have presented the results of our research on an optimal LFM design for grid operations under demand uncertainty. Our primary contribution in this research is the novel LFM design and the optimization models supporting it. We provide a model to calculate the flexibility requirement of the DSO, as well as an aggregator biding model and a model for the DSO final dispatch. A radial distribution grid with a deterministic AC-OPF model is used to determine the flexibility demand for efficient grid operations. As the flexibility supplier for the pooled LFM, an aggregator is modeled with a two-stage stochastic model for bidding and scheduling with stochastic dispatch to clear the LFM.

The usage of a stochastic LFM provides efficient mitigation of grid problems. With a stochastic LFM design, we achieved up to 40% more cost-efficient solutions than a system with only load shedding (without LFM) for grid operations such as congestion and voltage management. The improvement was achieved by scheduling flexibility products such as load curtailment, load shifting, and batteries.

Our results suggest that it is possible to mitigate grid congestion problems by using a pooled LFM, but for the voltage problem, the DSO or the LFM needs to address the locations of the flexibility assets. A single-phase balanced equivalent model is considered to simulate the distribution network. An unbalanced network may be considered in future work. Strategic decision making and multiple aggregators in LFM would be natural extensions of our research.

In this LFM design, the aggregator supplied flexibility with correct timing according to the stochastic demand distribution of the DSO. However, the spatiality of the flexibility resource is important because voltage problems are location-specific on the grid. For this reason, the DSO could not avoid load shedding. A direct control approach with bilateral contracts could avoid the problem, but it would have the disadvantage that a market-based price formation would not exist. An area for future research would be how to include spatiality in a pooled LFM market design. Due to the non-convexity of the OPF model, also large scale global solution methods could provide an interesting area for future research.

## Credit roles

Güray Kara: Conceptualization, Data curation, Modeling, Software, Formal analysis, Methodology, Investigation, Visualization, Validation, Writing – original draft. Paolo Pisciella: Conceptualization, Modeling, Software, Formal analysis, Methodology, Validation, Writing – review & editing. Asgeir Tomasgard: Conceptualization, Modeling, Methodology, Writing – review & editing, Supervision. Hossein Farahmand: Conceptualization, Methodology, Writing – review & editing, Supervision. Pedro

Crespo Del Granado: Conceptualization, Methodology, Writing — review & editing.

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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## Appendix A. Scheduling results of the aggregator model

The load shifting assets are scheduled to supply in LFM and minimize costs. The results are presented in Figure A.13. Load curtailment is an expensive asset. According to the results presented in Figure A.14, the load curtailment is needed especially when there is high flexibility demand. We assume batteries are already charged at the initial period and they return to their initial stage of charge at the end of operational period ($t_{24}$). The results are illustrated in Figures A.15 and A.16. The results of power purchase from the main grid are illustrated in Figure A.17 and limited according to equation (16).

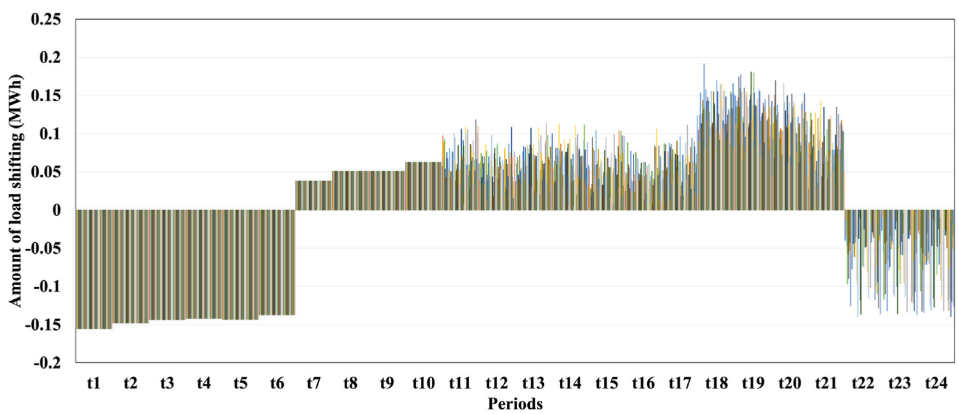


**Fig. A13.** Load shifting by the aggregator as active power. Results are aggregated for customers. Each color represents a scenario.

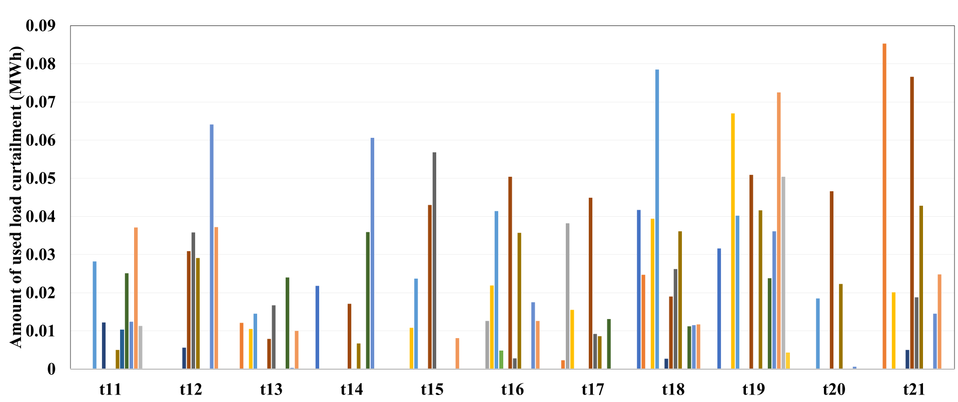


**Fig. A14.** Load curtailment by the aggregator as active power. Results are aggregated for customers. Each color represents a scenario.

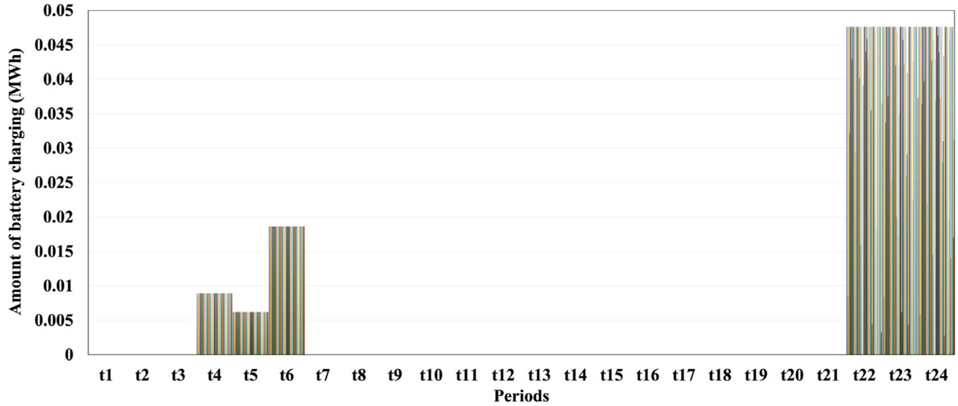


**Fig. A15.** Battery charging pattern of the aggregator as active power. Each color represents a scenario.

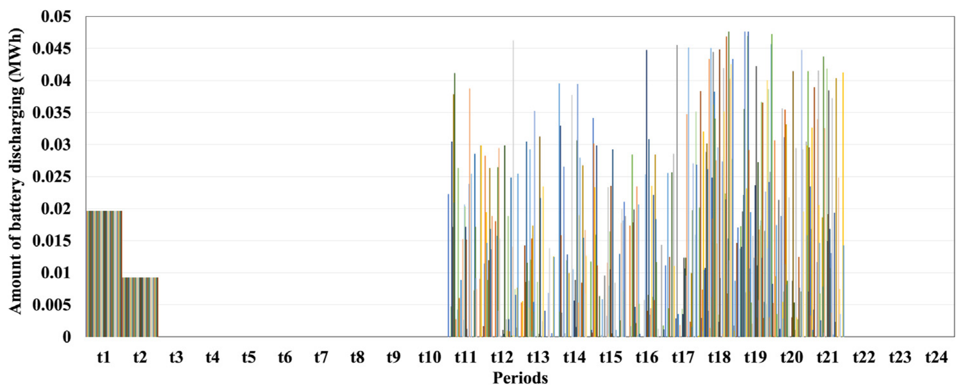


**Fig. A16.** Battery discharging pattern of the aggregator as active power. Each color represents a scenario.

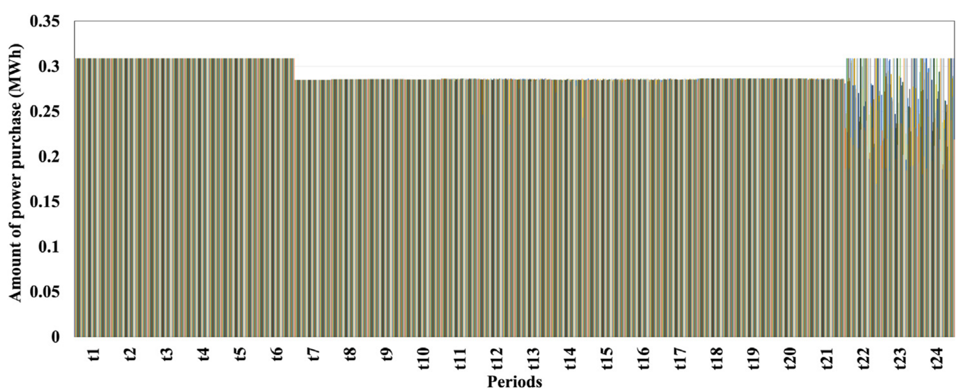


**Fig. A17.** Power purchasing pattern of the aggregator as active power. Each color represents a scenario.

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