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Improved finite element model updating of a full-scale steel bridge using sensitivity analysis

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ABSTRACT

There are many uncertainties related to existing bridges that are approaching or have exceeded their original design life. Lifetime extension analysis of bridges should be based on validated numerical models that can be effectively established. This paper presents a new procedure to obtain an optimal solution from sensitivity-based model updating with respect to an improvement in the modal properties, such as the natural frequencies and mode shapes, based on realistic parameter values. The procedure combines variations in the ratios of overdetermined systems with different definitions of local parameter bounds in a structured approach using a sensitivity analysis. The feasibility of the procedure is demonstrated in an experimental case study. Model updating is performed on a full-scale steel bridge using the natural frequencies and modal assurance criterion (MAC) numbers, where the numerical model is established by considering general uncertainties and model simplifications to reduce the model complexity. From the optimal solution for the case study considered, an improvement in modal parameters is obtained with highly reliable parameter values. The proposed procedure can be applied to similar case studies, irrespective of the structure under consideration and the corresponding parameterisation to be made, to effectively obtain a validated numerical model.

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Experimental study; finite element model updating; lifetime extension; parameter bounds; sensitivity method; steel bridge; structural health monitoring

1. Introduction

There are increasing demands on existing infrastructure with respect to traffic loads and intensity. Many highway and railway bridges are still in use despite that they are approaching or have exceeded their original design life. Although many uncertainties related to ageing, deterioration and damage accumulation are present in these bridges, lifetime extension is the preferred option to ensure continuous operation. Considering the requirements for precise numerical models in lifetime extension analyses of bridges, analyses should be carried out using validated models that adequately represent the current state given inherent uncertainties present in these structures.

Structural health monitoring (SHM) systems can provide updated information regarding the current state of a bridge condition. SHM, defined as the process of implementing an automated and online strategy for damage detection in a structure (Farrar & Worden, 2007), can be utilised for lifetime extension purposes. There are two main approaches in SHM: model-based and data-based (Barthorpe, 2010; Farrar & Worden, 2012). The model-based approach is an inverse problem, where a numerical model of the structure is established, and the relation to changes in the measured data from the structure to changes in the numerical model are investigated. The data-based approach relies on the use of machine learning for the identification of damage and ideally requires training data for all considered structural states, healthy and damaged, which can be challenging for bridges in service. However, the effective use of a numerical model can be made in a hybrid approach, which takes principles from both the model-based and data-based approaches into consideration by integrating a numerical model, experimental data and machine learning. In the SHM approaches where a numerical model is utilised, a validated numerical model is inevitable.

Finite element (FE) model updating is the process of calibrating the parameters of a numerical FE model based on vibration test data, where the aim of model updating is to reduce the discrepancy between the numerical model and available measurement data (Friswell & Mottershead, 1995). Model updating is essential for obtaining a validated numerical model. A validated numerical model can reduce model uncertainty in a reliability framework to improve the estimation of the remaining service life, where model uncertainty is quantified by the stress ratio between the structure (actual stress) and the numerical model (estimated stress). Furthermore, a validated numerical model can increase the accuracy of predictions in analysis related to the (1) structural response to the type of loads other than that used in the vibration test, (2) structural system behaviour in a

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different frequency range or in degrees of freedom (DOFs) different from those used in the model updating process and, (3) effects of structural modifications and structural damage (Mottershead, Link, & Friswell, 2011).

For the latter, several case studies are performed with respect to SHM and damage detection based on model updating (Alkayem, Cao, Zhang, Bayat, & Su, 2018; Bakir, Reynders, & De Roeck, 2007; Doebling, Farrar, & Prime, 1998; Reynders, De Roeck, Bakir, & Sauvage, 2007; Sohn et al., 2004; Teughels & De Roeck, 2004, 2005). Comprehensive reviews of model updating techniques and relevant methods are available in the literature (Link, 1999; Mottershead et al., 2011; Mottershead & Friswell, 1993; Sehgal & Kumar, 2016; Simoen, De Roeck, & Lombaert, 2015). With the increasing establishment of SHM systems on bridge structures and the considerable improvement in numerical models that can be obtained from model updating, applications on several case studies are reported in the literature. These studies include applications on highway and railway bridges (Deng & Cai, 2010; Feng & Feng, 2015; Frøseth, Rönnquist, & Øiseth, 2016; Jaishi & Ren, 2005; Ribeiro, Calçada, Delgado, Brehm, & Zabel, 2012; Sanayei, Phelps, Sipple, Bell, & Brenner, 2012; Schlune, Plos, & Gylltoft, 2009; Zordan, Briseghella, & Liu, 2014), footbridges (Naranjo-Pérez, Jiménez-Alonso, Pavic, & Sáez, 2020; Pavic, Hartley, & Waldron, 1998), cable-stay bridges (Asgari, Osman, & Adnan, 2013; Benedettini & Gentile, 2011; Ding & Li, 2008; Zárate & Caicedo, 2008; Zhang, Chang, & Chang, 2001; Zhong, Zong, Niu, Liu, & Zheng, 2016; Zhu, Xu, & Xiao, 2015), suspension and floating bridges (Hong, Ubertini, & Betti, 2011; Merce, Doz, de Brito, Macdonald, & Friswell, 2007; Petersen & Øiseth, 2017, 2019), and relevant test structures (Sanayei, Khaloo, Gul, & Catbas, 2015; Zapico, González, Friswell, Taylor, & Crewe, 2003; Zhang, Chang, & Chang, 2000).

Of the many model updating applications on bridges, several different approaches can be found. Sensitivity-based model updating considering parameterised models is a preferred method for full-scale bridges (Petersen & Øiseth, 2017). Model updating can provide large improvements in the modal properties such as the natural frequencies and mode shapes. However, it is still a requirement that the modelling errors are minimised and that the improvements are based on reasonable parameter values to consider the model validated. In sensitivity-based model updating, an overdetermined system should be considered, allowing for a unique solution to be obtained (Mottershead et al., 2011). Depending on (1) the overdetermined system and (2) the constraints enforced on the parameters of the numerical model, large variations in parameter values can render improved modal properties, irrespective of the type of model parameterisation. Constraining the parameters is necessary when dealing with large models. Furthermore, the overdetermined system depends on the model parameterisation and available modes from the system identification. Therefore, several choices can be made for how overdetermined the system should be and the size of the constraints to enforce on the parameters, or how these should be combined in the model updating. These choices require careful consideration and a structured approach in the model updating process. There are no studies in the literature where this problem has been addressed or fully considered in model updating of bridges.

Model updating should be performed by considering a detailed numerical model, to a level different from a conventional numerical model, to adequately represent the geometric and structural form (Brownjohn, Xia, Hao, & Xia, 2001; Brownjohn & Xia, 2000). However, there is a trade-off between a validated detailed numerical model in good agreement with measurements and a validated numerical model being computationally efficient for numerical simulations. For many engineering considerations, it is desirable to effectively obtain a validated numerical model that can be considered for several analysis purposes where the complexity of the model is left to a minimum but is still in acceptable agreement with measurements. Overall, the goal of model updating is to obtain improved modal properties based on reasonable and realistic parameter values. With the increased demand for validated models in lifetime extension analysis and a large number of ageing bridges, a procedure irrespective of the model parameterisation is needed to effectively establish validated models based on model updating.

This paper investigates the effects of using a sensitivity analysis for improved model updating. A new procedure based on a structured approach is proposed to obtain an optimal solution from sensitivity-based model updating with respect to an improvement in the modal properties combined with reasonable parameter values. The procedure is demonstrated on a full-scale steel bridge, a case study representative of many bridges still in service. The paper is organised in three parts. In the first part, the theory of the model updating framework is presented, including the theory of local parameter bounds to be included in the optimisation algorithm. The implementation of the theoretical framework using ABAQUS and Python is made available (Svendsen, 2020). The second part of the paper presents the experimental case study and outlines the proposed procedure. A numerical model is established and parameterised considering general uncertainties and model simplifications, where the model simplifications are introduced to reduce complexity. The effects from a sensitivity analysis are investigated by considering different ratios of overdetermined systems combined with two definitions of local parameter bounds. The results based on the optimal solution from the sensitivity analysis are presented. The final part presents a discussion of the proposed procedure. Based on the presented work, general recommendations are made with respect to the applicability to similar bridges in service.

2. Finite element model updating theory

2.1. General theoretical framework

The sensitivity method is used for performing the model updating. The main theoretical framework implemented is presented in the following section according to

(Mottershead et al., 2011), with a similar notation. It is assumed that q measured outputs are available and the model is considered to be parameterised in p parameters. In general, the number of output measurements should be larger than the number of parameters in the model, i.e. $q > p$, yielding an overdetermined system with a unique solution. In this study, an overdetermined system is considered using both the identified measured natural frequencies and the modal assurance criterion (MAC) numbers as the objective for the calibration of parameters in the numerical model. The model updating is performed by perturbation analysis.

The sensitivity method is based on a linearisation of the difference between the measured and analytically predicted outputs:

$$\boldsymbol{\varepsilon}_z = \mathbf{z}_m - \mathbf{z}(\boldsymbol{\theta}) \tag{1}$$

where $\mathbf{z}_m$ is the measured output and $\mathbf{z}(\boldsymbol{\theta})$ is the analytically predicted output as a function of the vector of parameters, $\boldsymbol{\theta}$. By reformulating the analytically predicted output, this becomes

$$\boldsymbol{\varepsilon}_z \approx \left(\mathbf{z}_m - \left(\mathbf{z}(\boldsymbol{\theta}_i) + \mathbf{G}_{i|\boldsymbol{\theta}=\boldsymbol{\theta}_i}\Delta\boldsymbol{\theta}_i\right) = \mathbf{z}_m - \mathbf{z}(\boldsymbol{\theta}_i) - \mathbf{G}_{i|\boldsymbol{\theta}=\boldsymbol{\theta}_i}\Delta\boldsymbol{\theta}_i\right) \tag{2}$$

The final form of the system equation is given as

$$\boldsymbol{\varepsilon}_z \approx \mathbf{r}_i - \mathbf{G}_{i|\boldsymbol{\theta}=\boldsymbol{\theta}_i}\Delta\boldsymbol{\theta}_i \tag{3}$$

where $\mathbf{r}_i = \mathbf{z}_m - \mathbf{z}(\boldsymbol{\theta}_i)$ is the residual, $\mathbf{G}_{i|\boldsymbol{\theta}=\boldsymbol{\theta}_i}$ is the sensitivity matrix and $\Delta\boldsymbol{\theta}_i$ is the parameter increment vector. The index i denotes the point of linearisation occurring at each iteration. The linear system, described in Equation (3), is established for q measured outputs (representing the rows) and p parameters (representing the columns) and is scaled:

$$\begin{bmatrix} \frac{\varepsilon_1}{z_{0,1}} \\ \vdots \\ \frac{\varepsilon_q}{z_{0,q}} \end{bmatrix} = \begin{bmatrix} \frac{r_1}{z_{0,1}} \\ \vdots \\ \frac{r_q}{z_{0,q}} \end{bmatrix} - \begin{bmatrix} \frac{\partial z_1}{\partial \theta_1}\frac{\theta_{0,1}}{z_{0,1}} & \cdots & \frac{\partial z_1}{\partial \theta_p}\frac{\theta_{0,p}}{z_{0,1}} \\ \vdots & \ddots & \vdots \\ \frac{\partial z_q}{\partial \theta_1}\frac{\theta_{0,1}}{z_{0,q}} & \cdots & \frac{\partial z_q}{\partial \theta_p}\frac{\theta_{0,p}}{z_{0,q}} \end{bmatrix} \begin{bmatrix} \frac{\Delta\theta_1}{\theta_{0,1}} \\ \vdots \\ \frac{\Delta\theta_p}{\theta_{0,p}} \end{bmatrix} \tag{4}$$

The frequencies are represented in the upper half of the sensitivity matrix, whereas the MAC numbers are represented in the lower half. The subscript zero denotes the scaling factors; i.e. θ_0 is the initial parameter value, and z_0 is the initial output value. The initial output value is taken as the analytically predicted value for the frequencies obtained from the initial numerical model and is 1 for the MAC numbers. The advantage of scaling is particularly to avoid large numerical values in the sensitivity matrix, which reduces potential ill-conditioning or matrix singularity. The terms in the sensitivity matrix can be established using an analytical approach or using numerical approximations by the perturbation procedure. For the latter,

$$\begin{aligned} \partial z_q &= z_{i,q}^{pert} - z_{i,q} \\ \partial \theta_p &= \theta_{i,p}^{pert} - \theta_{i,p} \end{aligned} \tag{5}$$

where the perturbed value is indicated with a superscript. The goal is to minimise the objective function, defined as

$$J(\Delta\boldsymbol{\theta}_i) = \boldsymbol{\varepsilon}_z^T \mathbf{W}_\varepsilon \boldsymbol{\varepsilon}_z \tag{6}$$

where $\mathbf{W}_\varepsilon$ is the symmetric weighting matrix. The weighting matrix is established as a diagonal and normalised matrix taking into consideration both natural frequencies and MAC numbers. In evaluating the minimisation, the objective function is reformulated as a weighted sum of the normalised residual squared:

$$J(\Delta\boldsymbol{\theta}_i)^* = \sum_{j=1}^{q} [\mathbf{W}_\varepsilon]_{j,j} \left(\frac{z_{m,j} - z_{i,j}}{z_{0,j}}\right)^2 \tag{7}$$

For the overdetermined system, the objective function defined in Equation (6) is minimised with respect to $\Delta\boldsymbol{\theta}_i$ at each iteration to give an improved parameter estimate of $\Delta\boldsymbol{\theta}_i$. The model is then updated to give

$$\boldsymbol{\theta}_{i+1} = \boldsymbol{\theta}_i + \Delta\boldsymbol{\theta}_i \tag{8}$$

Significant changes in parameters can occur during the minimisation, particularly during the first iteration steps. Hence, the parameters are constrained by establishing bounds in the minimisation problem to obtain a model improvement with reasonable changes in the defined parameters.

2.2. Local parameter bounds

The parameters are constrained by implementing lower and upper bounds in the minimisation problem, i.e.

$$\boldsymbol{\theta}_{\min} \le \boldsymbol{\theta}_{i+1} \le \boldsymbol{\theta}_{\max} \tag{9}$$

Introducing Equation (8) and rearranging, the bounds at iteration, i, become

$$\boldsymbol{\theta}_{i,\min} \le \boldsymbol{\theta}_i + \Delta\boldsymbol{\theta}_i \le \boldsymbol{\theta}_{i,\max} \tag{10}$$

$$\boldsymbol{\theta}_{i,\min} - \boldsymbol{\theta}_i \le \Delta\boldsymbol{\theta}_i \le \boldsymbol{\theta}_{i,\max} - \boldsymbol{\theta}_i \tag{11}$$

Considering the sensitivity matrix, the bounds must be scaled accordingly. The final scaled lower and upper bounds to be used in the minimisation become:

$$\begin{aligned} \Delta\boldsymbol{\theta}_{i,\min} &= \frac{\boldsymbol{\theta}_{i,\min} - \boldsymbol{\theta}_i}{\boldsymbol{\theta}_0} \\ \Delta\boldsymbol{\theta}_{i,\max} &= \frac{\boldsymbol{\theta}_{i,\max} - \boldsymbol{\theta}_i}{\boldsymbol{\theta}_0} \end{aligned} \tag{12}$$

The objective function can be minimised by solving the linear least squares problem with the defined bounds on the parameters. The bounds defined in Equation (12) are established as lower and upper allowable limits on the parameters per iteration and are referred to as local bounds.

2.3. Global parameter bounds

Global bounds are considered as the final lower and upper allowable limits for the parameters. The global bounds ensure that the parameters always attain values that are within a reasonable range from an engineering point of view. Ideally, these limits are never exceeded during the iterations in the model updating process. The local bounds mainly ensure that the parameter step is not too large in

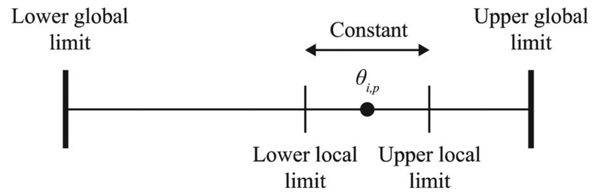

Figure 1. Relation between the local and global bounds.

each iteration. These limits are established as a percentage of the given parameter value in the current iteration. The lower and upper local bounds have the same percentwise change for each iteration step. Local bounds 'move' together with the updated parameter values within the global bounds. Figure 1 shows the relation between the local and global bounds.

2.4. Optimisation

The optimisation problem to be solved is, on a general form,

$$\min_{\mathbf{x}} \frac{1}{2}\|\mathbf{A}\mathbf{x}-\mathbf{b}\|_2^2,\ lb \leq \mathbf{x} \leq ub \tag{13}$$

where $\mathbf{A}$ is the design matrix, $\mathbf{b}$ is the target vector and $\mathbf{x}$ is the vector of parameters to be optimised subject to the lower and upper bounds, lb and ub, respectively. The subscript 2 denotes the Euclidean norm. By introducing Equation (3) into Equation (6), utilising that the weighting matrix is diagonal and thus symmetric, the objective function can be written as

$$J(\Delta\boldsymbol{\theta}) = \|-\mathbf{W}^{1/2}\mathbf{G}_i\Delta\boldsymbol{\theta}_i + \mathbf{W}^{1/2}\mathbf{r}_i\|_2^2 \tag{14}$$

with $\mathbf{A} = -\mathbf{W}^{1/2}\mathbf{G}_i$, $\mathbf{x} = \Delta\boldsymbol{\theta}_i$ and $\mathbf{b} = -\mathbf{W}^{1/2}\mathbf{r}_i$. Note that $\mathbf{A}$ is a q-by-p matrix and $\mathbf{b}$ is a vector of q elements. The subscript ε from the weighting matrix is removed for brevity. The bounds defined in Equation (12) are used directly as lower and upper bounds.

Equation (14) is solved using the *scipy.optimize.lsq_linear* function in Python (Virtanen et al., 2020). The optimisation problem defined in Equation (14) is convex. Hence, the found minimum of the bounded linear least squares problem is expected to be global. However, the final solution of the model updating can depend on the initial parameter values and the parameter bounds. Consequently, the model updating can converge to a local minimum and there is as such no guarantee of convergence to a global minimum.

Constraining parameters can be useful when dealing with complex models parametrised in a fair number of parameters, although at the cost of finding an optimal theoretical solution. Constraining parameters are mainly introduced to avoid large and non-realistic parameter changes. The constraints can provide some numerical stability since the unconstrained iterative optimisation based on first-order gradients sometimes can take too large steps, possibly stepping out of the area of interest. A similar effect can be obtained by applying a small amount of regularisation to the objective function defined in Equation (6), see (Mottershead et al., 2011).

2.5. Mode identification

A mode match index (MMI) is introduced to ensure the identification of correct modes during model updating (Simoen et al., 2015):

$$MMI = (1-\gamma)MAC_{m,n} - \gamma\frac{|f_m - f_n|}{f_m} \tag{15}$$

where γ is a value between 0 and 1 that provides the weighting to be considered between the MAC numbers and natural frequencies, f. The subscripts m and n are denoted for the measured and numerical modes, respectively. Furthermore, the MAC number is defined as (Allemang, 2003; Allemang & Brown, 1982)

$$MAC_{m,n} = \frac{\left|\boldsymbol{\Phi}_m^T\boldsymbol{\Phi}_n\right|^2}{\left(\boldsymbol{\Phi}_m^T\boldsymbol{\Phi}_m\right)\left(\boldsymbol{\Phi}_n^T\boldsymbol{\Phi}_n\right)} \tag{16}$$

where $\boldsymbol{\Phi}_m$ and $\boldsymbol{\Phi}_n$ are the measured and numerical mode shape vectors, respectively, and the superscript T denotes the transpose.

The MMI is also utilised as an indicator to measure the overall performance of the model updating results. The order of modes changes during the model updating, particularly for systems with closely spaced modes. Hence, an equal weighting obtained by setting $\gamma = 0.5$ is considered effective for the MMI.

2.6. Implementation of the theoretical framework

The theoretical framework is implemented using Python version 3.7.2, including SciPy version 1.3.2 (Virtanen et al., 2020), in combination with ABAQUS (Dassault Systémes Simulia Corp., 2014). The implementation is validated through a numerical case study and is made available (Svendsen, 2020).

3. Experimental case study

3.1. Bridge description

The Hell Bridge Test Arena, shown in Figure 2, is an open-deck steel riveted truss bridge with a main span of 35 m and width of 4.5 m. The bridge was formerly in operation as a train bridge for more than 100 years before it was taken out of service and moved to concrete foundations on land. The bridge serves as a full-scale laboratory for research and development for damage detection and SHM (Svendsen et al., 2020).

All cross sections, connections and details of the bridge were originally made using steel plates connected by rivets. The bridge has no upper lateral bracing, i.e. no lateral stiffening connected to the top girder of the bridge walls. Hence, the bridge cross section is formed as a U-section. The lateral bracing system is located below the bridge deck and provides a stiffening of the bridge in the lateral direction. The bridge deck structural system is made of longitudinal stringers connected to transverse girders with double angle connections.

Figure 2. Hell Bridge Test Arena.

3.2. Experimental study and system identification

Figure 3 shows an overview of the bridge, including the sensor locations used in the experimental study. The results obtained from ambient vibrations considering wind only are used. Data from 18 triaxial accelerometers were sampled at 400 Hz. The data were detrended, then low-pass filtered using an 8th order Butterworth filter with a cut-off frequency at 40 Hz and resampled to 100 Hz before it was used for analysis. A 30 min long time series was selected as the basis for performing the system identification. The wind was in the range of 5-8 m/s with wind gusts up to 12 m/s during the measurement period.

System identification was performed using the frequency domain decomposition (FDD) method (Brincker, Zhang, & Andersen, 2001). A Welch average was used for estimating the power spectral density. The first three singular values of the acceleration response spectrum and the modes identified are shown in Figure 4. All peaks in the acceleration response spectrum are evaluated, but only modes corresponding with the initial numerical model are used for FE model updating purposes. Altogether, 21 modes are established: 13 global modes and 8 semi-global modes. The global modes are related to modes in the lateral, vertical, torsional and longitudinal directions, whereas the semi-global modes are related to modes including mainly the bridge walls and to some extent the bridge deck. Modes 1-4 and 13-21 are global, whereas modes 5-12 are semi-global. No local modes identified are considered. The identified natural frequencies are given in Table 6. Closely spaced modes are observed in the system, specifically related to the higher vertical and torsional modes. However, the modes established in the system identification are generally considered well separated.

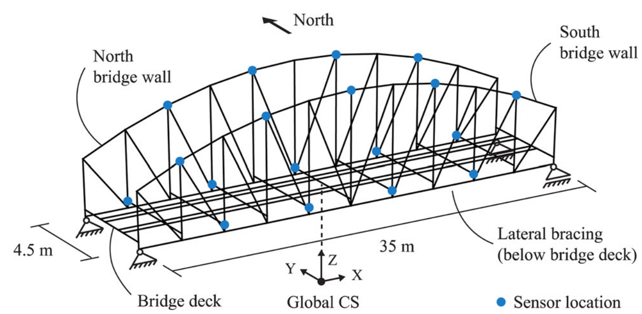

Figure 3. Overview of the bridge, including the sensor locations.

3.3. Model updating procedure

In model updating considering experimental case studies, there are several choices that can be made for how overdetermined the system is and the size of the constraints to enforce on the parameters. These choices depend on how the model is parameterised and the number of outputs available. Such considerations affect the results in the model updating and should be included in a sensitivity analysis. The following procedure is established as a structured approach to obtain an improved model updating:

1. Model parameterisation and definition of global parameter bounds. A numerical model is established, and model parameterisation is performed by considering general uncertainties and model simplifications.
2. Definitions of local parameter bounds. Two definitions of local parameter bounds are defined: rigid and semi-rigid. These definitions represent the different sizes of the constraints to enforce on the parameters.
3. Considerations of the overdetermined system ratios. The overdetermined system ratios are based on the model parameterisation and available outputs from the system identification.
4. Establishing underlying assumptions. The underlying assumptions are needed as common criteria for the analysis cases defined in the sensitivity analysis. These assumptions include considerations of the model quality assessment, the weighting of updating modes, the maximum number of iterations to perform and other assumptions.

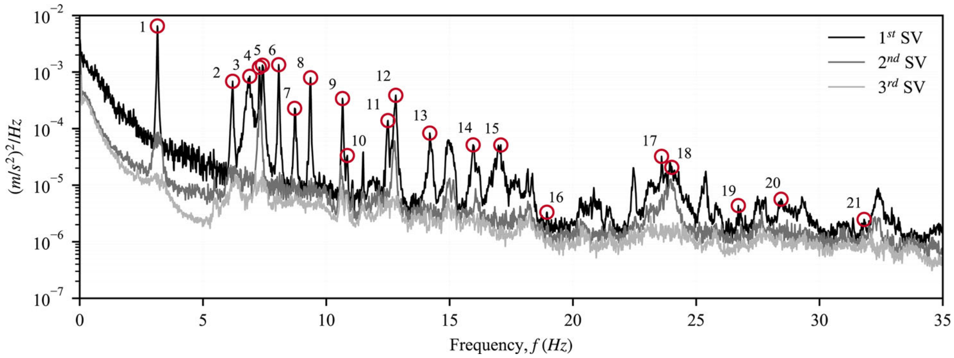

Figure 4. The first three singular values of the acceleration response spectrum. The identified modes are highlighted and numbered.

From bullet points 2 − 4, the effects of the sensitivity analysis can be investigated based on a given model parameterisation. The overdetermined system ratio is defined as

$$r_{os} = \frac{q}{p}, r_{os} > 1.0 \quad (17)$$

where q is the number of measured outputs and p is the number of parameters. Furthermore, the requirement for the overdetermined system is that $r_{os} > 1.0$. A low overdetermined system ratio implies updating on a small number of modes, whereas a high overdetermined system ratio implies updating on a large number of modes for the system considered.

Model quality assessment is an important consideration for the underlying assumptions. Determining the model quality requires the use of control modes, i.e. modes that are not used to update the parameters (Friswell & Mottershead, 1995). The quality of the underlying model is thus indicated by the correlation between the results obtained for the control modes of the updated model and the measurements.

The model updating procedure is included to depend on both the natural frequencies and MAC numbers as the modal properties of the structural system. It should be noted that performing model updating on large structures, often rendering complex models with high parameterisation, using natural frequencies only, can result in a significant improvement in natural frequencies but no improvement or even a decrease in the MAC numbers. Including both natural frequencies and MAC numbers in the model updating is advantageous for several reasons: it can preferably improve but most importantly avoid a decrease in MAC numbers; it ensures stability in the model updating procedure through improved mode identification; and it ensures more representative parameter values in the final updated model.

4. Finite element model and updating parameters

4.1. Finite element model

The numerical model is established using the FE software ABAQUS (Dassault Systémes Simulia Corp., 2014). The main structure of the bridge is included in the model, which consists of four major parts: two vertical walls, including wall diagonals and wall stiffeners, the bridge deck and the lateral bracing system. Secondary steel and non-structural items are represented as lumped point masses on the bridge deck to ensure proper mass distribution. Specifications provided by technical drawings and site inspections are used as the basis for constructing the model. Figure 5 shows the numerical model.

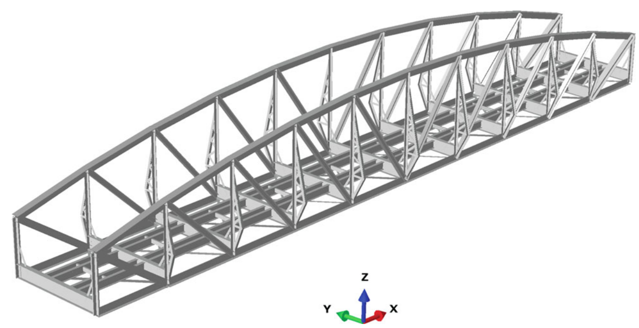

Figure 5. Numerical model.

A beam element model representation is established using two-node Timoshenko linear beam elements (B31) for the main structure and two-node connector elements (CONN3D2) for connections between beam elements of the main bridge parts. Three different connection types are utilised with different DOFs activated. The connection types account for local geometry and particularly joint details such as gusset plate design. The bridge is modelled as simply supported with pinned boundary conditions on one end (global translational x, y and z-direction constrained) and rolled boundary conditions on the other end (global translational x-direction partly constrained by spring elements and global translational y and z-direction constrained). The model is divided into 3035 elements, with a total of 8906 nodes and 15590 DOFs. Altogether, the model is established using a straightforward modelling procedure with several simplifications included based on engineering judgement.

4.2. Updating parameters

The updating parameters are based on the understanding of the local and global structural behaviour of the bridge. Parameters are chosen to mainly account for (1) modelling inaccuracies, including model simplifications, such as general uncertainties related to modelling and differences in the

Table 1. Parameters used in the model updating, including global lower and upper bounds.

Parameter	Type	Location	Reference value, $\boldsymbol{\theta}_0$	Global lower bound, $\boldsymbol{\theta}_{\min}$	Global upper bound, $\boldsymbol{\theta}_{\max}$	Unit
ρ_{steel}	Density	Main steel structure	7850	7065	8635	kg/m^3
ρ_{wvl}	Density	Wall verticals, lower	7850	6280	15700	kg/m^3
E_{lb}	Stiffness	Lateral bracing	2.10E + 11	1.68E + 11	2.52E + 11	N/m^2
E_{bd}	Stiffness	Bridge deck	2.10E + 11	1.47E + 11	2.73E + 11	N/m^2
E_{wg}	Stiffness	Wall girders, top and bottom	2.10E + 11	1.68E + 11	2.52E + 11	N/m^2
E_{wd}	Stiffness	Wall diagonals	2.10E + 11	1.68E + 11	2.52E + 11	N/m^2
E_{wvu}	Stiffness	Wall verticals, upper	2.10E + 11	1.68E + 11	2.52E + 11	N/m^2
E_{wvl}	Stiffness	Wall verticals, lower	1.05E + 12	2.10E + 11	2.10E + 12	N/m^2
m_{bd}	Mass	Substructure, bridge deck	18500	9250	37000	kg
k_x	Spring stiffness	End support BC	1.00E + 06	1.00E + 05	1.00E + 08	N/m

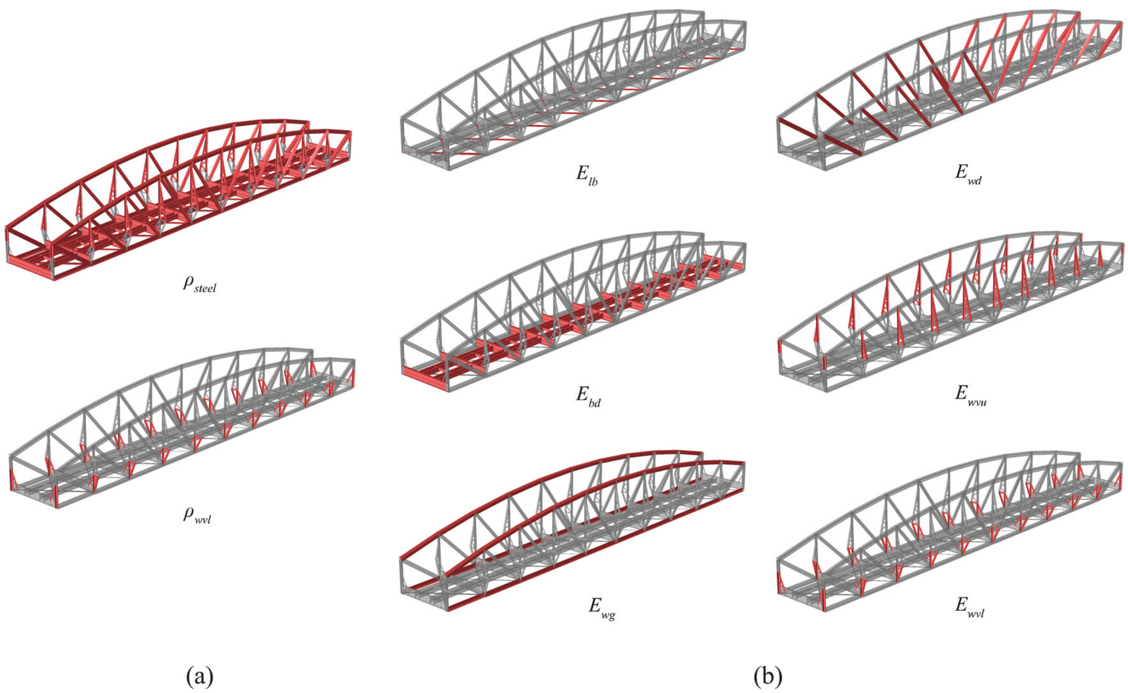

Figure 6. Parameterisation of the numerical model considering the material properties. The specific parameterised area is highlighted in red. (a) Density parameters. (b) Stiffness parameters.

geometry of the model compared to the real structure, and (2) uncertainties in the bridge structural properties. The parameters represent regions of the structure where modelling inaccuracies and general uncertainties might cause discrepancies in the predictions. In this study, four different parameter types are chosen: density, stiffness, mass and spring stiffness. The parameter types are related to the material properties of the bridge, mass of non-structural items and boundary conditions.

The numerical model is parameterised in a total of 10 parameters. Table 1 summarises the parameters used in the model updating, together with the global upper and lower parameter bounds. Engineering judgement is required to set the bounds, particularly for complex cases where large uncertainties are inherent in the parameters. Two parameters related to the density are included. The density of steel, ρ_{steel}, is introduced to account for any uncertainty in the mass of the structure. This parameter is valid for all the main steel, and consequently, the global and semi-global modes of the bridge are sensitive to this parameter. The

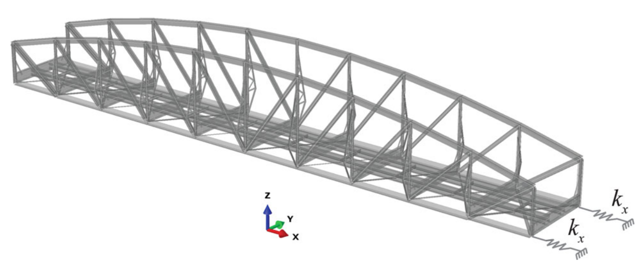

Figure 7. Parameterisation of the numerical model considering the spring stiffness parameter.

lower part of the wall verticals is originally designed with complex plate geometry but is simplified in the numerical model using dummy beam elements with increased stiffness. The density of the lower part of the wall verticals, ρ_{wvl}, is included to account for the underestimation of the mass and is as such expected to increase. This parameter mainly influences the global modes of the structure.

Six stiffness parameters are included in the parameterisation. The stiffness of the lateral bracing, E_{lb}, is included mainly to account for the simplifications introduced in the numerical model, i.e. continuous beam element modelling. The stiffness of the bridge deck, E_{bd}, is included to account for both the uncertainties and model simplifications. A reasonable engineering simplification is to exclude the substructure on the bridge deck from the model, i.e. the rails and wooden sleepers. This substructure is connected to the bridge deck structural system by steel hooks between the sleepers and the top flange of the longitudinal stringers. The substructure represents some additional stiffness to the bridge deck, which is initially not considered. This stiffness is highly difficult to estimate due to the large variability observed in the remaining functionality of the connections within the substructure and the connections of the substructure to the bridge deck.

The profiles constituting the truss beams of the bridge are made of riveted plates. For the top and bottom wall girders particularly, the profiles are tapered in parts of the beam lengths and especially towards the joints for strengthening purposes. The profiles are represented as equivalent beam element profiles in the numerical model, and any uncertainty with this representation is taken into account by the stiffness of the top and bottom wall girders, E_{wg}. The stiffness of the wall diagonals, E_{wd}, is included in the model due to the uncertainty of the joint flexibility in the upper and lower parts of the diagonals. Diagonals are connected to the wall joints using gusset plate details. The rotational out-of-plane stiffness of these diagonals are released in both ends in the numerical model to account for these details. Although this is considered a common engineering simplification, the true joint stiffness is represented somewhere in the middle of a full release and no release.

The stiffness of the upper and lower wall verticals, i.e. E_{wvu} and E_{wvl}, respectively, represent uncertainties related to model simplifications. The upper wall verticals are well represented in the model; however, the lower wall verticals are represented by dummy beam elements with estimated stiffness since secondary steel is excluded. Since these two parts of the wall verticals are connected, the relatively high uncertainty in the lower part may affect the upper part. Consequently, either an increase or decrease in parameter values is expected to occur for both. All stiffness parameters are sensitive to both the global and semi-global modes of the structure. Figure 6 shows the parameterisation of the numerical model with respect to the density and stiffness parameters.

The mass of the bridge deck substructure, m_{bd}, is introduced to account for the mass estimation error. This parameter is mostly sensitive to the global modes. Furthermore, there is a high degree of general uncertainty related to the spring stiffness, k_x, representing the roller boundary conditions. No information is available on how much functionality remains in the boundary conditions with respect to friction. Hence, it is important to include this parameter, although it has the least influence on the modes of all parameters included. Figure 7 shows the parameterisation of the numerical model with respect to the spring stiffness parameter. The sensitivity of all the updating parameters on the natural frequencies and MAC numbers is shown in Figure 9.

Table 2. Average values of the frequency error, MAC and MMI before model updating.

Modes	Δf_{error}	MAC	MMI
10	4.22%	0.86	0.41
12	4.85%	0.83	0.39
14	5.24%	0.77	0.36
15	5.45%	0.77	0.36
16	5.39%	0.77	0.36
17	5.11%	0.78	0.36
All	5.22%	0.72	0.33

In addition to the abovementioned, additional uncertainties inherent in the model parameterisation of the chosen parameter types are considered. First, all secondary steel and structural details are excluded or represented as mass in the numerical model. Second, all joints in the bridge are riveted. However, the flexibility of these joints is prone to high uncertainty based on operational wear during the bridge service life. An imprecision in the rivet connections and a deviation in the intended behaviour of individual rivets caused by damage result in unwanted joint flexibility and the possibility of nonlinear behaviour during loading. Third, unwanted joint behaviour and damage in the structural details of the bridge, particularly in the bridge deck, is likely caused by fatigue damage, which is common in these types of bridges (Haghani, Al-Emrani, & Heshmati, 2012). Fourth, effective beam lengths comprise uncertainty. Last, the material properties of steel that is more than 100 years old comprise uncertainty. Notably, there is a systematic error due to the difference between the measurements and the numerical model caused by meshing. Altogether, these uncertainties are also taken into consideration through the model parameterisation. Several of the uncertainties mentioned are difficult to quantify and thus represent in a numerical model, resulting in the need for introducing model simplifications.

5. Sensitivity analysis

5.1. Basis for evaluation

Table 2 shows the average values of the frequency error, MAC and MMI considering the different number of modes before model updating. Table 2 shows that for all 21 modes, the average frequency error is 5.22% with an average MAC and MMI of 0.72 and 0.33, respectively. It is also observed that the average values of the frequency error, MAC and MMI are better for 10 and 12 modes than the average values when more modes are considered. This observation clearly indicates that the lower modes of the initial numerical model compare better with the measured modes from the system identification than the higher modes.

5.2. Underlying assumptions for the sensitivity analysis

The sensitivity analysis is performed by considering different sets of definitions for the local parameter bounds and ratios of overdetermined systems. Two definitions for the parameter bounds are considered: rigid (R) and semi-rigid (SR). The semi-rigid definition provides less constraints on the parameters than the rigid definition. However, both definitions ensure that overly large steps are avoided in each iteration. The scaled local lower and upper parameter bounds for the rigid and semi-rigid definitions, together with the scaled global bounds, are summarised in Table 3.

Altogether, 12 analysis cases are included by considering six different ratios and two definitions of the parameter bounds. A set of underlying assumptions are included as a basis. First, both natural frequencies and MAC numbers are used consistently in the model updating. Second, model updating is performed by considering model quality assessment. Two considerations are made when choosing control modes: modes 11 and 12 are chosen for general model quality assessment, whereas higher end modes are chosen as additional indicators of model performance with respect to the structural response outside of the measurement frequency range. Hence, for all cases, the model updating is performed by including the lowest modes and excluding the control modes from the updating algorithm. The largest number of updating modes is 17, leaving a minimum of 4 control modes. Third, the weighting, $\mathbf{W}_{\varepsilon}$, is set equal for all considered cases. All modes are considered equally important and consequently given equal weighting. Natural frequencies are prioritised and weighted 2/3 per mode, whereas MAC numbers are weighted 1/3 per mode. Fourth, a maximum of 8 iterations are used in each case. If a global bound is exceeded, then the parameter value is set equal to the limit of this bound before the minimisation is carried out. To avoid model updating with parameters exceeding their global bounds, the analysis is terminated if the global bound of a parameter is exceeded two consecutive times. Last, to improve mode identification both during the perturbation analysis and during the iterations, local numerical modes are filtered out before performing the mode matching. A total of 250 modes are extracted in each numerical analysis. Combining the filtering of the local numerical modes with the MMI in the model updating process is effective, particularly for numerical model representations resulting in many local modes.

5.3. Results

The results obtained from the sensitivity analysis are summarised in Tables 4 and 5 for the rigid and semi-rigid definitions, respectively. Average values of the frequency error, MAC and MMI in addition to the change in the objective function, $J(\Delta\theta)^*$, and the number of iterations used are shown for the 12 cases considered. The results obtained are based on iterations in the updating algorithm until a fair stabilisation of the objective function is reached.

From Tables 4 and 5, it is clearly seen that the choice of definition for the parameter bounds affects the results. An evaluation of the results is performed by considering both the decrease in the objective function and the overall results in the modal properties, i.e. the average frequency error, MAC and MMI. The decrease in the objective function is based on the updating modes only. Hence, to fully assess the model quality, the evaluation of the results is mainly based on the overall modal properties considering all modes, which takes into consideration both the updating modes and the control modes.

Two general observations are made. First, analysis cases using a semi-rigid definition generally provide better results for the modal properties than cases using a rigid definition. By considering all modes, the average frequency errors obtained for analysis cases using the semi-rigid definition are all lower than for cases using the rigid definition, except when considering the case $r_{os} = 2.8$. Furthermore, there is little variation in the average MAC and MMI considering all modes for the 12 cases considered, ranging from 0.71 to 0.77 for MAC and 0.33 to 0.36 for MMI. Second, a large variability in the results can be obtained considering a specific overdetermined system ratio but using different parameter bounds definitions. This is particularly observed in the case $r_{os} = 3.2$, where the results obtained for the MAC and MMI are similar, considering all modes for the rigid and semi-rigid definitions. However, a large difference in the results is obtained considering the average frequency error. For this analysis case, using the rigid definition provides practically no improvement in the average frequency error, whereas using the semi-rigid definition provides the best improvement of all 12 cases considered, compared to the initial numerical model.

For the cases $r_{os} = 2.0$, $r_{os} = 3.0$, $r_{os} = 3.2$ and $r_{os} = 3.4$, the best results are obtained using the semi-rigid definition when considering the overall results in the modal properties. Similarly, for the cases $r_{os} = 2.4$ and $r_{os} = 2.8$, the best results are obtained using the rigid definition. Furthermore, a majority of the overdetermined system ratios obtain a larger decrease in the objective function for the semi-rigid definition compared to the corresponding rigid definition. For the cases rendering the best results, the significance of the overdetermined system ratio is seemingly small considering the improvement in the modal properties of the updated models. As such, improved results by evaluating the modal properties only can be obtained with variations in the overdetermined system ratio and different parameter bound definitions. However, the change in parameter values for the updated models is of importance. Many of the parameters affect the system in a similar way, and several combinations of parameters can solve the optimisation problem. Hence, the choice of final system overdetermined ratio to use should be based on the improvement in the overall modal parameters combined with how the parameter values are changed in the model updating.

The ratio of the parameter values obtained for the analysis cases with the best results are shown in Figure 8. From this figure, Tables 4 and 5, it is observed that a large variability in updated parameter values can be obtained despite fairly similar results in the modal properties. Parameters that are expected to obtain small changes are the density of steel, ρ_{steel}, and the stiffness of the wall girders and wall diagonals, i.e. E_{wg} and E_{wd}, respectively. All cases obtain reasonable results with respect to these parameters. The density of the lower wall verticals, ρ_{wvl}, and mass of the bridge deck, m_{bd}, can both increase or decrease and can cancel each other out in the optimisation. However, a large

Table 3. Scaled local parameter bounds for the rigid and semi-rigid definitions.

Parameter p	Reference value θ_0	Global lower bound (scaled)[a] $\theta_{min, scaled}$	Global upper bound (scaled)[a] $\theta_{max, scaled}$	Rigid (R) Local lower allowable change[b] $\Delta\theta_{i, min, scaled}$	Rigid (R) Local upper allowable change[b] $\Delta\theta_{i, max, scaled}$	Semi-rigid (SR) Local lower allowable change[b] $\Delta\theta_{i, min, scaled}$	Semi-rigid (SR) Local upper allowable change[b] $\Delta\theta_{i, max, scaled}$
ρ_{steel}	7850	0.90	1.10	−3%	3%	−3%	3%
ρ_{wvl}	7850	0.80	2.00	−10%	10%	−15%	15%
E_{lb}	2.10E + 11	0.80	1.20	−5%	5%	−10%	10%
E_{bd}	2.10E + 11	0.70	1.30	−5%	5%	−10%	10%
E_{wg}	2.10E + 11	0.80	1.20	−5%	5%	−10%	10%
E_{wd}	2.10E + 11	0.80	1.20	−5%	5%	−10%	10%
E_{wvu}	2.10E + 11	0.80	1.20	−3.5%	3.5%	−5%	5%
E_{wvl}	1.05E + 12	0.20	2.00	−10%	10%	−15%	15%
m_{bd}	18500	0.50	2.00	−15%	15%	−25%	25%
k_x	1.00E + 06	0.10	100.00	−50%	400%	−50%	900%

[a]Scale factor of the specific parameter reference value.
[b]Allowable change of the specific parameter value in the current iteration, unless a global bound is reached.

Table 4. Results from the sensitivity analysis for the rigid (R) parameter bounds definition.

Modes	Ratio r_{os}	Δf_{error} (%) Updating modes	Δf_{error} (%) All modes	MAC Updating modes	MAC All modes	MMI Updating modes	MMI All modes	Change in J*	No. of iterations
10	2.0	4.38%	5.21%	0.89	0.74	0.42	0.34	−23.5%	3
12	2.4	4.27%	4.25%	0.88	0.76	0.42	0.36	−49.9%	6
14	2.8	5.59%	4.82%	0.81	0.73	0.38	0.34	−36.7%	3
15	3.0	5.34%	5.63%	0.84	0.77	0.39	0.36	−50.4%	7
16	3.2	4.66%	5.21%	0.81	0.74	0.38	0.35	−37.4%	8
17	3.4	4.51%	4.46%	0.80	0.74	0.38	0.35	−15.3%	3

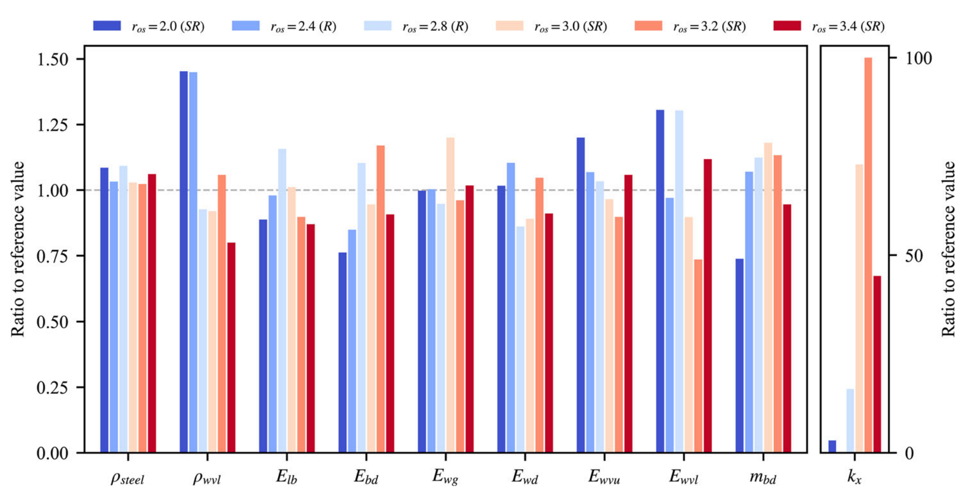

Figure 8. Ratio of parameter values.

reduction in both parameters is unlikely, which excludes the case $r_{os} = 3.4(SR)$. The bridge deck stiffness, E_{bd}, can be used as a control parameter. Based on prior discussions, an increase in this updated parameter value is expected. Two cases obtain an increase in the bridge deck stiffness: $r_{os} = 3.2(SR)$ and $r_{os} = 2.8(R)$. For the latter, the other parameter values obtained are also acceptable; however, the overall results in the modal properties are not satisfactory. The remaining cases, i.e. $r_{os} = 2.0(SR)$, $r_{os} = 2.4(R)$ and $r_{os} = 3.0(SR)$, all result in a decrease in the bridge deck stiffness. Furthermore, less realistic values for the other parameters are obtained for these cases than for the case $r_{os} = 3.2(SR)$.

From the sensitivity analysis, the analysis case $r_{os} = 3.2(SR)$ clearly provides the most reasonable parameter values. Moreover, this case also renders the best results of the modal properties, particularly considering the natural frequencies that were weighted higher than the MAC numbers in the model updating. It is, however, observed that this case has the smallest decrease in the objective function of all cases for the semi-rigid definition, and it has less decrease in the objective function than the corresponding analysis case for the rigid definition. This discrepancy is due to two low MAC values from modes 4 and 6 that penalise the objective function result. However, it has little effect on the average MAC result and is thus not reflected in the results presented in Table 5. A further evaluation of the results for this case is provided in the following section.

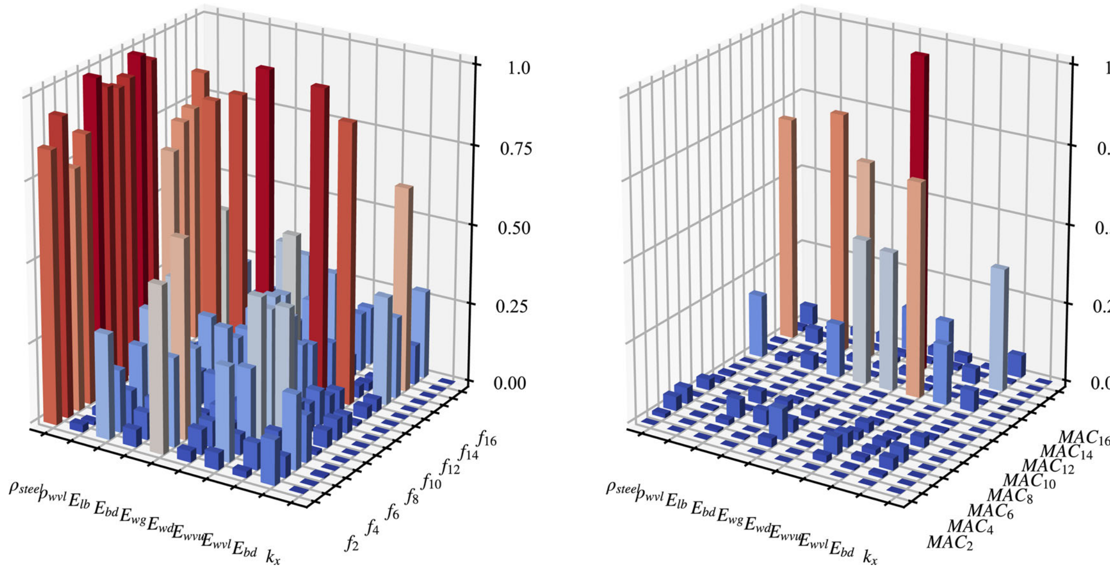

Figure 9. Normalised sensitivity plots of the frequencies (left) and MAC numbers (right) for the updating modes. The plots are normalised with respect to the frequencies and MAC individually.

Table 5. Results from the sensitivity analysis for the semi-rigid (SR) parameter bounds definition.

		Δf_{error} (%)		MAC		MMI			
Modes	Ratio r_{os}	Updating modes	All modes	Updating modes	All modes	Updating modes	All modes	Change in J*	No. of iterations
10	2.0	3.76%	4.24%	0.91	0.74	0.44	0.35	−43.7%	7
12	2.4	3.60%	3.85%	0.86	0.72	0.41	0.34	−45.6%	7
14	2.8	4.81%	5.12%	0.82	0.71	0.39	0.33	−38.3%	5
15	3.0	4.76%	4.07%	0.83	0.75	0.39	0.35	−51.5%	3
16	3.2	3.89%	3.84%	0.80	0.75	0.38	0.35	−30.8%	6
17	3.4	3.88%	3.99%	0.82	0.75	0.39	0.35	−42.8%	7

Table 6. Natural frequencies, MAC and MMI for the initial and updated model.

	Frequency, f(Hz)					MAC			MMI	
Mode	Measured	Initial	Error	Updated	Error	Initial	Updated	Change	Initial	Updated
1	3.15	3.23	2.63%	3.25	3.13%	0.99	0.99	0.00	0.48	0.48
2	6.20	6.55	5.60%	6.22	0.25%	0.89	0.89	0.00	0.42	0.45
3	6.90	6.38	−7.55%	6.62	−4.04%	0.78	0.93	0.15	0.35	0.44
4	7.29	8.17	12.17%	7.78	6.68%	0.77	0.60	−0.17	0.32	0.26
5	7.41	7.35	−0.76%	7.03	−5.09%	0.92	0.82	−0.10	0.46	0.38
6	8.07	7.86	−2.60%	7.51	−6.92%	0.85	0.66	−0.19	0.41	0.29
7	8.72	9.15	4.98%	8.71	−0.08%	0.92	0.91	−0.01	0.44	0.45
8	9.36	9.49	1.33%	9.03	−3.60%	0.92	0.91	−0.01	0.45	0.44
9	10.66	10.73	0.67%	10.45	−1.90%	0.92	0.93	0.01	0.46	0.45
10	10.85	10.43	−3.91%	10.08	−7.15%	0.66	0.65	−0.01	0.31	0.29
11	12.50	13.62	8.99%	12.35	−1.20%	0.38	0.65	0.27	0.15	0.32
12	12.82	13.25	3.39%	12.69	−1.02%	0.61	0.79	0.17	0.29	0.39
13	14.20	15.18	6.93%	14.43	1.63%	0.47	0.76	0.29	0.20	0.37
14	15.96	17.39	9.02%	16.47	3.22%	0.86	0.85	−0.02	0.39	0.41
15	17.08	16.30	−4.55%	15.97	−6.46%	0.45	0.63	0.18	0.20	0.28
16	18.95	20.96	10.66%	20.08	6.01%	0.40	0.59	0.19	0.15	0.26
17	23.60	25.56	8.34%	24.64	4.42%	0.79	0.77	−0.02	0.35	0.36
18	24.01	25.08	4.47%	24.40	1.60%	0.79	0.87	0.08	0.37	0.43
19	26.71	26.89	0.70%	29.01	8.60%	0.82	0.74	−0.08	0.40	0.33
20	28.44	30.46	7.09%	29.47	3.62%	0.37	0.35	−0.02	0.15	0.16
21	31.81	32.83	3.18%	33.09	4.01%	0.53	0.40	−0.13	0.25	0.18

6. Model updating results

6.1. Parameter sensitivities and weighting

For the analysis case $r_{os} = 3.2(SR)$, 16 modes are used, resulting in a total of 32 outputs. The remaining 5 modes are used as control modes for the assessment of the model quality. Two small changes in the parameter bounds have been implemented for the analysis case compared to Table 3. The stiffness of the lateral bracing, E_{lb}, is decreased from 10% to 5%, and the upper bound of the spring stiffness is decreased from 900% to 400%. These changes have minor effects on the results.

Normalised sensitivity plots of the natural frequencies and MAC numbers with respect to the updating parameters are shown in Figure 9. The normalised sensitivity plots illustrate how the parameters influence the natural frequencies and MAC numbers of the modes used in the model updating. The sensitivity plots change for each iteration in the model updating process. In Figure 9, the sensitivity plots for the initial model are shown. It is observed that all parameters influence both the natural frequencies and MAC numbers, except for k_x, which is shown to have a minor influence. Although this parameter has little influence in the initial part of the model updating, all parameters are included in the model updating process.

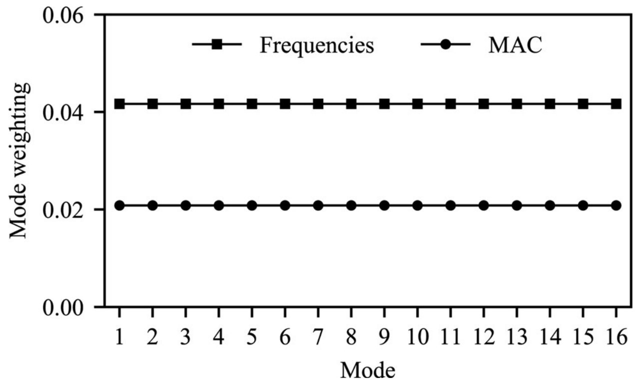

Figure 10. Weighting of the modes in the model updating.

The weighting implemented for the updating modes is shown in Figure 10. The weighting is given as 0.04167 and 0.02083 for the natural frequencies and MAC numbers per mode, respectively, and sum to 1 by considering the outputs of all modes.

6.2. Model updating results

Altogether, six iterations in the analysis are performed. The final value of the objective function decreased from 0.0302 to 0.0209, resulting in a decrease of 30.8%. The average absolute frequency errors for the initial and updated models are shown in Figure 11. The dashed horizontal lines represent the average absolute frequency error considering all modes for the initial and updated model, which is decreased from 5.22% to 3.84%. By considering the updating modes

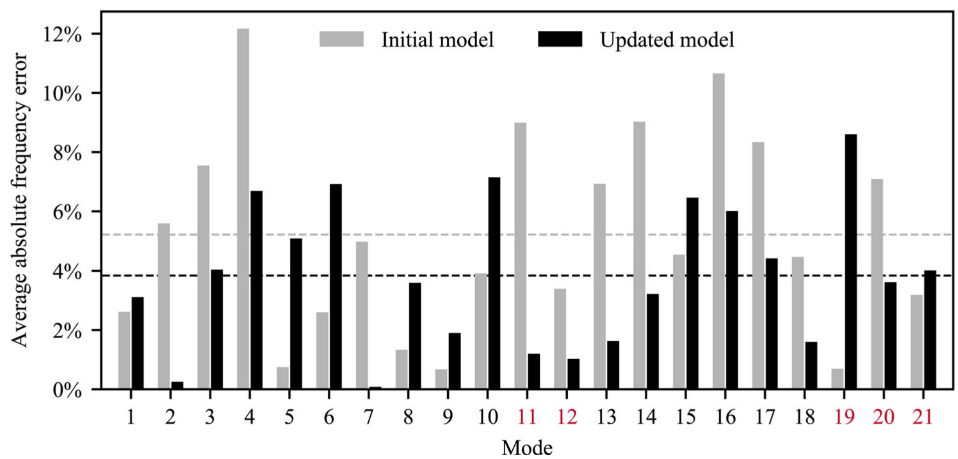

Figure 11. Average absolute frequency error for the initial and updated model. Control modes are highlighted in red.

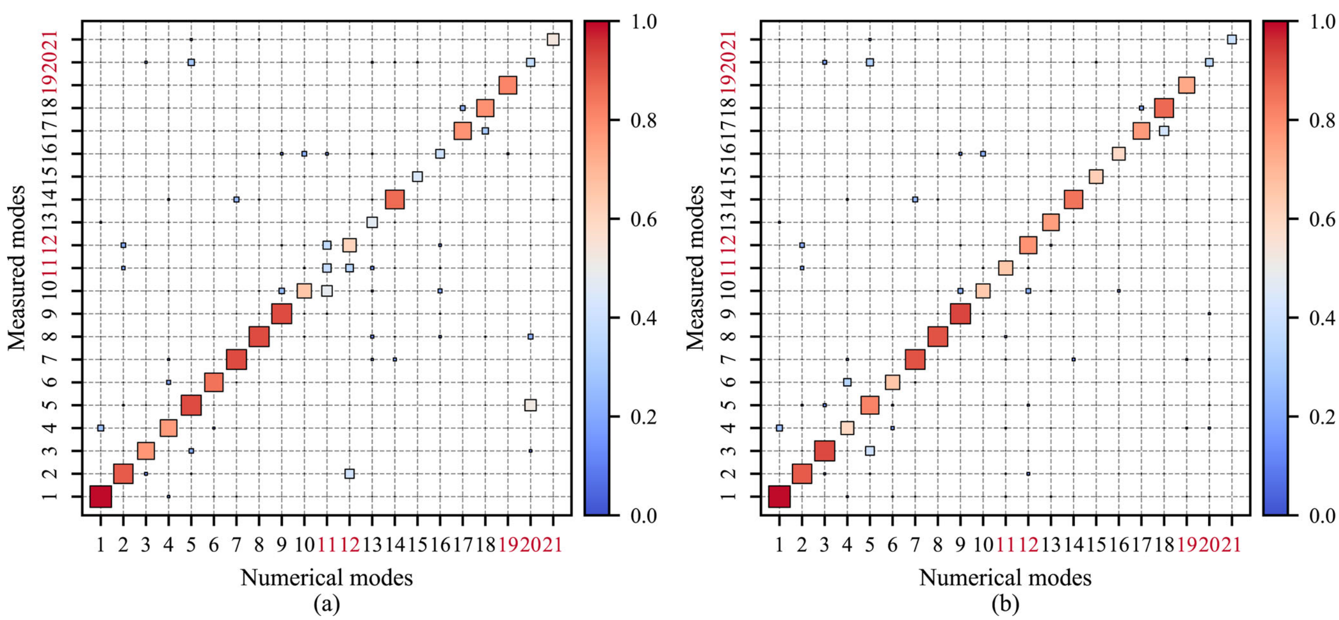

Figure 12. MAC numbers between the measured and numerical modes with control modes highlighted in red. (a) Initial model. (b) Updated model.

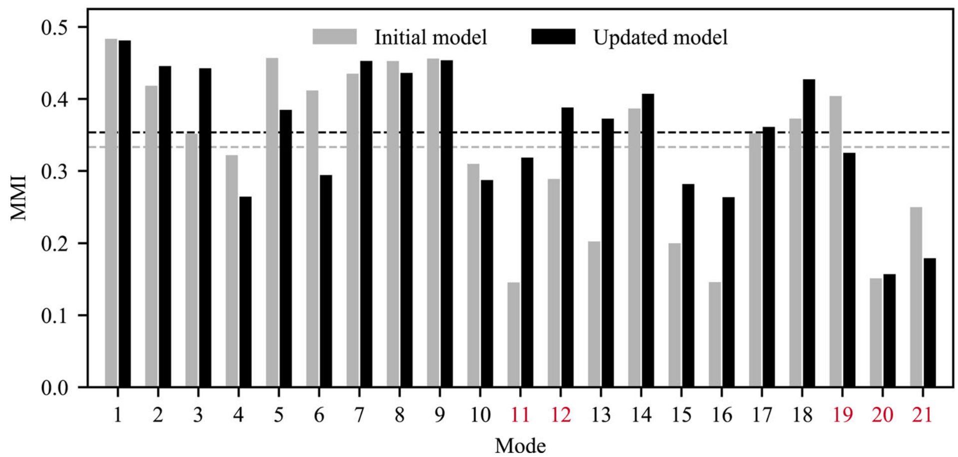

Figure 13. MMI for the initial and updated model. Control modes are highlighted in red.

Table 7. Parameter values from the initial and updated model, including change.

Parameter	Reference value	Updated value	Change	Percentage change
ρ_{steel}	7850	8032	182	2.3%
ρ_{wvl}	7850	8305	455	5.8%
E_{lb}	2.10E + 11	1.89E + 11	−2.14E + 10	−10.2%
E_{bd}	2.10E + 11	2.46E + 11	3.57E + 10	17.0%
E_{wg}	2.10E + 11	2.02E + 11	−8.20E + 09	−3.9%
E_{wd}	2.10E + 11	2.20E + 11	9.87E + 09	4.7%
E_{wvu}	2.10E + 11	1.89E + 11	−2.14E + 10	−10.2%
E_{wvl}	1.05E + 12	7.72E + 11	−2.78E + 11	−26.5%
m_{bd}	18500	20963	2463	13.3%
k_x	1.00E + 06	1.00E + 08	9.90E + 07	9900.0%

only, a decrease in the average absolute frequency error from 5.38% to 3.89% is obtained, whereas a decrease from 4.67% to 3.69% is obtained by considering the control modes only.

Figure 12 shows the MAC numbers between the measured and numerical modes for the initial and updated models. In general, the average MAC increased from 0.72 to 0.75 for all modes, whereas it increased from 0.77 to 0.80 and from 0.54 to 0.58 for the updating modes and control modes, respectively. An improvement in the MAC numbers is particularly seen for higher modes, i.e. from modes 11 to 21.

The MMI is used to assess the overall performance of the modal properties in the model updating. An identical match in the natural frequency and MAC between the numerical model and the measurements results in the maximum MMI value of 0.5. The MMIs for the initial and updated models are shown in Figure 13. The dashed horizontal lines represent the average MMI considering all modes for the initial and updated model, which is increased from 0.33 to 0.35. By considering the updating modes and control modes separately, an increase in the MMI from 0.36 to 0.38 and from 0.25 to 0.27 is obtained, respectively. It should be noted that the MMI is generally higher for the lower modes, indicating a larger difficulty in obtaining a good correspondence between measurements and numerical models for the higher modes.

Detailed results for the natural frequencies, MAC numbers and MMI before and after the model updating are summarised in Table 6. In summary, the results show that good improvement is obtained from the model updating when considering all modes. The best improvement is obtained when considering the reduction in the average absolute frequency error, as expected. A fair improvement is obtained in the MAC numbers, which in general are less sensitive than the natural frequencies.

An improvement in both the average absolute frequency error and MAC is obtained for the control modes, which indicates good model quality. By considering the overall assessment of the modal properties shown by the MMI, altogether 16 modes improve or exhibit no or a negligible decrease (i.e. less than or equal to 0.02 decrease). The best improvement is obtained for the higher modes; for modes 11 to 21, improvement is obtained in all modes, except for modes 19 and 21. Less improvement is obtained in the lower modes, i.e. modes 1 to 10, where 3 modes improve and 4 modes demonstrate no or a negligible decrease. Although all modes are considered equally important in this study, obtaining improvement in the higher modes is a good result considering the overall assessment.

The results of the updated parameter values, including the change from the initial parameter values, are summarised in Table 7. For the density parameters, a modest change in the parameter values is obtained. The density of the steel, ρ_{steel}, highly influences all modes, and an increase of 2.3% is obtained. This is a reasonable change, considering the material property itself but also considering all details of the structure such as the rivets and plates not being included in the numerical model. The density of the lower part of the wall verticals, ρ_{wvl}, increased 5.8%. This increase is highly relevant since this part of the structure consists of steel plates but is simplified in the numerical model using dummy beam elements.

For the stiffness parameters, reasonable changes in the parameter values are obtained. The stiffness of the lateral bracing, E_{lb}, decreased by 10.2%. This reduction can be attributed to the continuous beam element modelling made; the pin connections in all bracing cross points are excluded in the numerical model. An increase in the bridge deck stiffness, E_{bd}, of 17.0% is obtained. This increase is as expected due to the engineering simplification made of excluding the substructure of rails and wooden sleepers on the bridge deck, adding stiffness that initially is not taken into consideration. The wall girder stiffness, E_{wg}, obtained a decrease of 3.9%, well within reasonable changes. This change is attributed to simplifications from representing the model with equivalent beam element profiles. The stiffness of the wall diagonals, E_{wd}, obtained an increase of 4.7%. This increase in stiffness is reasonable considering that the rotational out-of-plane stiffness of all diagonals are released in both ends in the numerical model by accounting for the gusset plate details.

The largest changes in stiffness values are obtained for the lower and upper vertical wall stiffeners, with a decrease of 10.2% for the upper part, E_{wvu}, and a decrease of 26.5% for the lower part, E_{wvl}. The stiffness parameter for the lower part has the largest uncertainty of all stiffness parameters, and the initial assumptions based on estimation were clearly too stiff. However, the change is within the defined global limit. Furthermore, this change clearly also affects the stiffness of the upper part, and it is thus reasonable to reduce the stiffness of the upper part when a reduction in the lower part is obtained.

The mass of the bridge deck substructure is increased by 13.3%, well within the limits defined. Uncertainty is inherent in the estimation of this mass, especially related to the density of the wooden sleepers and the total mass of structural details such as the rivets, bolts and steel hooks. The largest change is observed for the spring stiffness parameter, k_x, which reaches its upper global limit. There are two main explanations for this change: first, there is a high uncertainty in this parameter; and second, this is the parameter with the least influence on the natural frequencies and MAC numbers, and consequently, large changes are required to influence the modal properties. The global limits could be extended; however, it is unlikely that the boundary conditions that were originally designed as rollers with little or no friction in the longitudinal direction completely lost their function. As such, this parameter change is accepted, but caution should be taken in accepting this as the definite result.

In summary, a fair improvement in the modal properties is obtained from model updating. The improvement is obtained based on the development of parameter values that are highly realistic and generally accepted. A further improvement can as such only be obtained with a different parameterisation of the model.

7. Discussion

From the sensitivity analysis, the following general observations are made:

- Semi-rigid local bounds on the parameters are preferred over rigid local bounds. Less constraints on the parameters, however, within reasonable values, allow for larger parameter adjustments in each iteration. This is more likely to ensure convergence towards an optimal solution rather than a suboptimal solution, although the solution may be based on a local minimum of the objective function. Moreover, including both local and global parameter bounds increases the control of the parameters in the updating process.
- Using a low overdetermined ratio, by updating on a few modes, improved the modal properties of all modes on average; however, it did not provide the best improvement in the modal parameters or reasonable parameter values. Using a high overdetermined system ratio yielded reasonable parameter values and the best improvement in the modal properties considering all modes on average, particularly for the higher modes of the structure.
- By considering the proposed procedure in a structured approach, allowing for an extensive number of analysis cases to be evaluated, the optimal solution for the model updating with respect to an improvement in modal properties is established with high confidence. Further improvement in modal properties would require a different model parameterisation.
- A large variability in the parameter values can be obtained when considering different combinations of overdetermined system ratios and rigidity of parameter bounds, leading to adequate results in terms of improved modal properties. These effects clearly demonstrate that care should be taken in allowing the model updating algorithm to decide upon the final parameter values without any predefined expectation or knowledge of what the final updated parameter values should be. These effects also demonstrate the importance of the sensitivity analysis for improved model updating results.

Many uncertainties are present for bridges that are approaching or have exceeded their initial design life. The numerical model is established using a straightforward modelling procedure with several simplifications included based on engineering judgement. As a result, there is a certain expectation on the outcome of the parameter values from the model updating, which strongly depends on the model parameterisation. For the considered case study, the validated model is intended for hybrid SHM using machine learning for detecting relevant damages of existing steel bridges that are approaching or have exceeded their original design life. As such, the reduced complexity of the validated model is beneficial for the large number of numerical simulations to be performed.

In general, the goal is to obtain an updated model with the best possible improvement in the modal properties combined with the most reasonable and realistic parameter values. Improved modal properties and realistic parameter values depend on the overdetermined system ratio and rigidity of the local parameter bounds. From the sensitivity analysis, cases with a high overdetermined system ratio combined with a semi-rigid parameter bounds definition provided the best results. Although a high overdetermined system ratio was found to provide the best results in this case study, it does not necessarily need to be valid for other case studies. The overdetermined system ratio strongly depends on the number of parameters for the model considered and the number of modes that are available from the system identification. As such, a generalisation of the overdetermined system ratio to use cannot be made since this is highly system specific. Nevertheless, it is recommended to start with a high overdetermined system ratio in the model updating process, as this can ensure a good improvement in modal properties combined with acceptable parameter values, in addition to obtaining an updated model that is likely to be improved over a wide frequency range. For parameter bounds, a semi-rigid definition is generally preferred over a rigid definition. Establishing parameter bounds definitions depends on the type of parameters and the uncertainty associated with these, the number of parameters and model parameterisation. Furthermore, setting parameter bounds requires engineering judgement to a large extent.

As such, for structures with similar applications as presented in this study, where higher modes are relevant and a wide frequency range in the modal properties are of interest for future applications, it is recommended to include a structured approach using a sensitivity analysis by combining the assessment of high overdetermined system ratios with a corresponding general semi-rigid definition for parameter bounds in model updating. Furthermore, it is advised to include a verification of the final numerical model using other results such as strain, if such data are available, for increased model validation purposes.

8. Conclusions

This paper presented a procedure to obtain an optimal solution from sensitivity-based model updating with respect to an improvement in the modal properties, such as the natural frequencies and mode shapes, combined with realistic parameter values. The procedure consists of performing a sensitivity analysis, which considers variations in the overdetermined system ratios combined with local parameter bounds definitions, in a structured approach.

An experimental study and system identification of a full-scale steel bridge identified 21 modes to be used in the model updating process. The numerical model was parameterised in a total of 10 parameters taking into consideration general uncertainties and model simplifications to obtain a model with reduced complexity. Sensitivity-based model updating was performed based on the natural frequencies and MAC numbers, and the effects from the sensitivity analysis were investigated. The effects showed that considering a high overdetermined system ratio with a corresponding general semi-rigid definition for parameter bounds provides an optimal solution for the model updating with respect to the improvement in modal properties based on realistic and acceptable parameters. From the optimal solution, the average absolute frequency error decreased from 5.22% to 3.84%, and the MAC numbers improved from 0.72 to 0.75 considering all modes, including the control modes. By considering the uncertainties inherent in the structure and the subsequent establishment of the numerical model with model simplifications, the results obtained are in acceptable agreement with the measurements.

The main limitation of the procedure presented is the need for an adequate number of modes established from the system identification to be included in the model updating. Depending on the case study considered, many analysis cases may be required to find the optimal solution with respect to improved modal properties combined with reasonable parameters. Nevertheless, the procedure presented in this paper demonstrates that an optimal solution can be effectively established. The procedure can be applied to similar case studies, irrespective of the structure under consideration and the corresponding parameterisation to be made. Furthermore, the procedure is applicable to case studies for model updating in a wide frequency range where the numerical model is parameterised in a fair number of parameters and an adequate number of modes are available from the system identification. Through the experimental case study, it is demonstrated that for an existing bridge with considerable uncertainties, a numerical model with several simplifications can be established, and a subsequent validated model with acceptable improvement from the model updating can be achieved.

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