Skrifter från SMDF, Nr. 14

Bringing Nordic mathematics education into the future

Preceedings of Norma 20
The ninth Nordic Conference on
Mathematics Education
Oslo, 2021

Editors:
Nortvedt, G. A., Buchholtz, N. F., Fauskanger, J., Hreinsdóttir, F., Hähkiöniemi, M., Jessen, B. E., Kurvits, J., Liljekvist, Y., Misfeldt, M., Naalsund, M., Nilsen, H. K., Pálsdóttir, G., Portaankorva-Koivisto, P., Radišić, J. and Wernberg, A.

SMDF **S**vensk Förening för **M**atematik**D**idaktisk **F**orskning
Swedish Society for Research in Mathematics Education

Skrifter från Svensk Förening för MatematikDidaktisk Forskning, Nr 14

ISBN 978-91-984024-3-8
ISSN 1651-3274

SMDF

Svensk Förening för Matematikdidaktisk Forskning
c/o Nationellt Centrum för Matematikutbildning
Göteborgs universitet
Box 160
SE - 405 30 Göteborg

Swedish Society for Research in Mathematics Education

The report is published at http://matematikdidaktik.org

Students' perceptions and challenges regarding mathematics teaching cycle in practices of historical and philosophical aspects of mathematics course

Melih Turgut and Iveta Kohanová

Norwegian University of Science and Technology (NTNU), Trondheim, Norway;
melih.turgut@ntnu.no, iveta.kohanova@ntnu.no

This paper explores master students' perceptions and challenges when designing a mathematics lesson in the Historical and Philosophical Aspects of Mathematics course following the perspective of the mathematics teaching cycle. The sample consists of thirteen students who are enrolled in this course that is taught within a mathematics education master's program at a large Norwegian university. The data come from students' productions related to their practice assignment, post-classroom reflections, and group and individual interviews. The data are triangulated and analysed by the lens of the components of the mathematics teaching cycle. According to the findings, master students' major perception regarding the mathematics teaching cycle is that the hypothetical learning trajectory is a prediction of student thinking. Their main challenge is the lack of information regarding student pre-knowledge to express hypotheses and making the assessment.

Keywords: Mathematics teaching cycle, Hypothetical learning trajectory, Lesson plans to incorporate the history of mathematics, Master students.

Introduction

The professional development of preservice mathematics teachers (PMTs) has received particular attention from mathematics educators (Rowland, Huckstep, & Thwaites, 2005; Wilson, Mojica, & Confrey, 2013). The core content in this direction can be described with developing subject matter knowledge and pedagogical content knowledge (Ball, Thames & Phelps, 2008). Regarding these contents, some of the recent research in Scandinavia has focused on PMTs' pedagogical content knowledge occurring in teaching in practice schools. Munthe, Bjuland and Heldevold (2016) studied how to shift the focus of Norwegian preservice teachers when conducting field practice from surviving in the classroom to observing pupils' learning and making their learning visible. Hemmi and Ryve (2015) explored effective mathematics teaching as constructed in Finnish and Swedish teacher educators' discourses with PMTs. In this paper, we particularly focused on professional development by looking at the link between a master course and its teaching practice with a perspective of Mathematics Teaching Cycle (MTC) (Simon, 1995). Primarily, we trained 13 PMTs in a Historical and Philosophical Aspects of Mathematics (HPAM) course about MTC and its specific component, the notion of Hypothetical Learning Trajectory (HLT). Students referred to these notions to create their lesson content and to implement their lesson plans. We report here students' perceptions and challenges regarding MTC (as well as HLT) after they carried out their teaching in practice schools.

Theoretical framework and research question

The notion of MTC is elaborated by Simon (1995) in order to address the core role of a lesson and task *design* in mathematics teaching. Emphasis is given to HLTs in MTC, which mainly refers to

anticipated student learning steps under the teacher's formulation of an explicit learning goal and her careful plan for teaching activities. Here the mathematical knowledge for teaching (Ball, Thames & Phelps, 2008), both subject matter knowledge and pedagogical content knowledge, is of crucial importance. The teacher blends her subject matter knowledge with students' pre-knowledge and creates a plan for activities through hypothesized students' thinking. Therefore, HLT is here referring to the teacher's prediction regarding students' learning and thinking, such as which steps they would follow, which misconceptions could occur, and then which strategies could be followed. Simon (1995) puts forward HLTs and assessment of students' knowledge as the core parts of the mathematics teaching process. MTC and associated components are rendered in Figure 1.

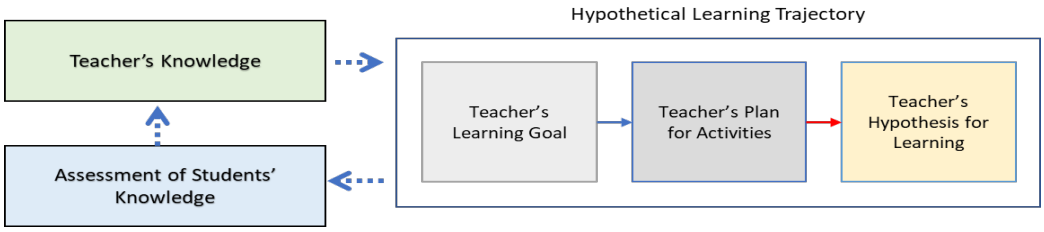

Figure 1. Mathematics Teaching Cycle (adopted from Simon, 1995, p. 136)

In the related literature, the notion of HLT has been receiving particular attention from several researchers. On the one hand, it has been considered to be a design heuristic in designing teaching-learning environments (Huang, Zhang, Chang, & Kimmins, 2019); on the other hand, it has been exploited as an educational tool for teachers' professional development on the development of knowledge of students' thinking (Wilson et al., 2013). For instance, Wilson et al. (2013) observed that the (hypothetical) learning trajectory perspective contributes to teachers' model-building skills regarding students' possible learning paths and it also contributes to teachers' understandings of mathematics itself. Similarly, in the present paper, we refer to the notion of MTC, particularly HLT, as a professional development tool to improve master students' task design skills and focus on the following research question: *What are the perceptions and challenges of master students in mathematics education regarding mathematics teaching cycle in the Historical and philosophical aspects of mathematics course*?

Research context and methods

This research is part of the pilot study entitled "Understanding student teachers' learning and development in the Master course of HPAM". The HPAM course is designed as a master-level course for PMTs under a two-fold aim. On the one hand, the aim is to provide information about historical, ontological and epistemological foundations for basic mathematical concepts and algorithms with a focus on algebra and geometry, as well as knowledge on what mathematics is, how its nature and methods developed, and what it constitutes today. On the other hand, it is aimed to develop PMTs' skills in transforming knowledge of the history of mathematics into didactical and pedagogical designs for teaching mathematics from 5th to 10th grade. Along the latter direction, the students (in groups) prepare pedagogical designs for field practice, they implement their designs and write formal practice reports, which is one of the compulsory tasks to pass the course.

Along with the pilot study, we observed that the students had issues in designing lesson plans and incorporating the history context meaningfully. To prepare the students better, two lecturers of the course (i.e., the authors of this paper) organized in autumn 2019 a workshop (lasting approximately

4 hours) two weeks before their school practice in two interrelated directions: the first is introducing students to the notions of MTC and HLT, and the second is discussing, contributing and exchanging ideas about students' initial designs for practice. In this way, we aimed to improve the PMTs' professional development in designing a lesson that incorporates the history of mathematics. After we introduced the notions of MTC and HLT with examples, the students worked in four groups for a while and focused on lesson (and practice) plans. They got feedback from the lecturers and then they finalized their plans and presented them. Thereafter, the students got final feedback from the lecturers and their peers and they continued to work on lesson plans. While updating their documents, they communicated with the lecturers a few times. Afterwards, they carried out the teaching and wrote reports about it as an academic text and uploaded them to the course's online portal. The timeline of the process is rendered in Figure 2.

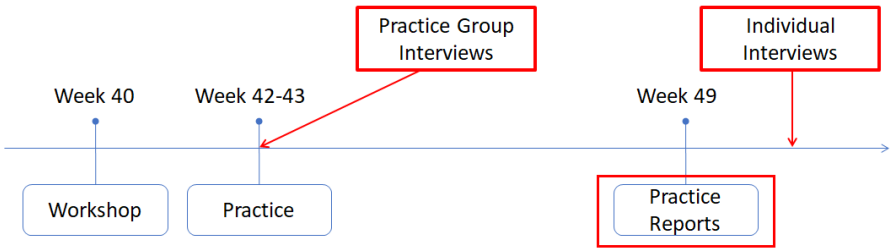

Figure 2. Timeline of research

The participants of the course were in their first year of a two years master program for grades 5-10 at the department of teacher education. The group consisted of 13 (5 females and 8 males) PMTs who all volunteered to be part of the research. Eleven of them started the master program straight after their bachelor studies; two (named D and L; both with ten-year teaching experience) were working as mathematics teachers at public schools and were back at the university to complete a master's degree. Students studied mathematics didactics four out of six semesters during their bachelor years and mathematics comprised 50% of their workload during these semesters. Those mathematics didactics courses are more focused on didactics and less on advancing the students' subject matter knowledge. However, they did not know the notions of MTC and HLT before the lecturers introduced them, as they were trained in their earlier study years to use the didactic relationship model (Bjørndal & Lieberg, 1978) and it's simpler form, known as "hva-hvordan-hvorfor" (what-how-why) scheme, for their lesson plan designs in previous practices.

The data come from practice groups interviews, students' practice reports, and individual interviews, whose time location is shown in Figure 2 (through red rectangles). Students arranged four practice groups (we call them G1 - G4) themselves and they implemented their teaching in these groups in four different practice schools. For the sake of anonymity, we coded students' names as follows: Group 1: A, B, C; Group 2: D, E, F; Group 3: G, H, I, J and Group 4: K, L, M. We conducted group interviews with two randomly selected practice groups (G1 and G3) and three individual interviews, where individuals are coming from G1, G2 and G3. All the interviews were audio-recorded and transcribed, and they lasted about 25 minutes. Practice group G1 interviewees were B and C, interviewees from G3 were only G, H and I, as not all the students were available. Because of their good communication skills, we invited students B and G for individual interviews, and we have included (due to the amount of teaching experience) student D as well. Practice groups interview question was: to what extent was the session about designing a mathematical lesson useful for you?

What did you learn and how did you use it in your teaching in the practice school? And individual interview questions were: (1) What do you think, is there any difference between the notions of HLT and MTC and the way you have been taught/requested to prepare a lesson by the practice office/teachers? (2) In your practice description, did you refer to HLT? Why? Explain… (3) Which part(s) was (were) hard for you to write the HLT? (4) Did your lesson plan work? What happened?

The practice report task included many points: (*a*) explain why your group has planned/chosen this teaching activity and your main aims for using this teaching activity in the classroom, (*b*) clearly explain your learning goal, classroom activities, hypotheses for learning - HLT, planned assessment of your practice, as well as how selected historical context should support students' learning and why? (*c*) give a short description of the class or the group of students, (*d*) report your findings, (*e*) give a reflection and evaluation of the teaching activity and the result of the work in the classroom. We analysed the data by following the steps of Braun and Clarke (2006); familiarisation of data, generation of initial codes, search for themes, review of themes, defining and naming themes and producing the report.

Findings

This section is divided into four subsections according to three different (data) sources and later we provide an overview regarding perceptions and challenges of MTC and (particularly) HLT. Because of page constraints, we briefly present dialogues.

Analysis of practice group interviews

The practice group interviews were conducted in practice schools and interviews started after the interviewer asked the question about lesson design. The master students, G, H and I (belonging to G3) immediately expressed their views regarding MTC and HLT that were already elaborated in the workshop. The following excerpts are drawn from their discussion:

I: … I didn't really understand what was the meaning of the planning, cause we know ... we have learned how to plan a lesson from before, so I didn't understand what was the difference in this way of planning...

G: … that was just more of a bit deeper into some things what we already knew ... maybe ... and so it wasn't really a lot of new information, I would say ...

It is obvious from the lines above, that I and G think that the notion of MTC is not completely different from their pre-knowledge on (and way of) planning of a mathematics lesson; it is planning but with not fully new information. At the same time, H pointed out that there were too many slides in the workshop, it could be a word document to work on since they are so experienced in preparing plans, it could be rather briefly summarised by giving just instructions through a list of steps. Thereafter, I and G also confirmed this notion. Later, H and I expressed that the notion of MTC is more or less the same with their way of planning (didactic relationship model), just the format is different. Regarding the other practice group (G1) interview, we observe that B (like C) encounters the notions of MTC and HLT for the first time and she thinks that the notions are useful '*because it makes me think more about what I want pupils to learn. What I expect them to learn*'. Further, B felt that the description of MTC during the workshop was a little bit '*jumpy*' and '*not like a specific list of things*', even though she found the PowerPoint file adequate.

Analysis of student practice reports

We analysed the participants' practice reports with a four-dimensional framework as ingredients of MTC. Table 1 presents our coding (i.e., precise, not precise, detailed and/or not clear) with our comments associated with each column.

Group No	Learning Goal	Plan of Activities	Hypotheses for Student Learning	Plan of Assessment
G1	not precise	detailed	-list of learning steps -some of the possible difficulties and how to solve them	detailed with several different techniques
G2	precise and thought through	detailed and ambitious	not formulated explicitly, but reference to it in the final reflection	not clear
G3	confusing, not precise	detailed	partial, not completed	not clear
G4	precise	detailed	vague	short without details

Table 1: An overview of students' practice reports regarding MTC

Table 1 shows that although the plan of activities appeared in each group report as adequate, the description of learning goals and formulating hypotheses for student learning seemed to be problematic. There was also a lack of a plan for assessment and its techniques in the students' practice reports.

Analysis of individual interviews

The first (individual) interview was conducted with D, who belongs to G2. Regarding the notion of HLT, it was new for him and in a situation when he was going to teach a topic which was also new for him, it was hard to know what to expect from the students, what difficulties they might have, what they will learn. As he says '*... I always think about the learning goals, how are they going to get to those learning goals …*'. Further, regarding hypothetical learning, he pointed out that he never wrote something similar before. Following this, he underlined that he could not imagine what would happen in the classroom in this practice assignment. He means that it was very hard to formulate students' anticipated learning steps because they for the first time considered such a task in the classroom: '*... But the hard thing was to discuss when we discussed actually what are the students going to learn. Because I haven't used any task like this in class. So I didn't know what to expect from the students to understand ...*'. Regarding assessment, he underlines the time issue to spare some time at the end of the lesson: '*I'm bad at evaluating what the students have learned. I think one of the reasons is the time or I don't put enough time at the end of the lesson to evaluate*'.

The second interview was with G, who belongs to G3. He perceived the concept of HLT as something very theoretical, but with a practical application: '*... it's pretty much just saying what do you think will happen … So /laughing/ that's ... hurdle … it's just kind of a fancy way to say it*'. He understands it as potential paths of students' learning (i.e., thinking): '*... If they manage this then we can go further*

and if not then … kind of like a programming language /laughing/ if-then-else …'. He emphasized the importance of classroom discussions, but on the other hand, he questioned the engagement of students. With better knowledge of students it would be easier to know what to expect from them: '*... is just about the relation, the teacher-student relationship, that builds up over time ... better you know the students, the easier it will be and more accurate it will be* (*HLT*)…'.

The third interview was with B, who belongs to G1. According to her, her (existing) lesson planning schema is parallel to the learning goal and the teacher's plan for activities of HLT. She finds that the whole HLT is an extended form of her schema because HLT has a (third) part where '*we have to project what the pupils might misunderstand or don't understand*'. Next, she clearly expresses the relation between her pre-knowledge about the what-how-why approach and learning goal and planning activities: '*... I have to do hva* (*what*)*, hvordan* (*how*)*, hvorfor* (*why*) *to make the learning goal and plan for the activities. I have to keep that in mind when I'm doing those two…*'. She considers the prediction of students' difficulties – a hypothesis for students' learning, as a very useful extension of her scheme '*this one ... takes it a little bit further ... I think I can just merge them and expand it ... it's kind of like an additional thing that I find very useful …*'. It was new for her, '*... it was a little bit different, we haven't done anything like this before …*', and a little bit difficult too, because her group did not know what students knew. In sum, she addresses that, in their group work, they mostly spoke about hypotheses for students' learning. She always plans how to assess students' learning: '*... it always has been part of the summarization*', but in this practice assignment they didn't have time for it, '*... when we came to class we didn't have as much time ... so it was just like a short summary ...*'.

Overview of perceptions and challenges

In order to articulate our findings, we overview major perceptions and major challenges (with participants' codes) observed in three previous sections through Table 2.

Perceptions	Challenges
– The notions of MTC and HLT are different ways of designing a lesson (G, H, I) – HLT is just formulating of learning goals and hypothesis for student learning (D) – HLT is a kind of fancy way of approaching teaching (G) – HLT is composing plans like programming: if the students manage to do or they cannot (G) – HLT is having backup plans at stake (B) – HLT is predicting what students will find difficult (B) – HLT is useful to project student view (and like to-do list) and thinking about their studentship (B)	– Time issue for assessing in the classroom (B, D) – Lack of a word document as a list of necessary steps to transform MTC and HLT into practice (B, G, H, I) – lack of information regarding students' pre-knowledge to express hypotheses step-by-step (D) and making an assessment (B) – Considering a completely new task in the classroom (D, G) – Focusing only on the HLT part, forgetting about assessment (B, D)

Table 2: An overview of perceptions and (interrelated) challenges regarding MTC and HLT

As seen from Table 2, MTC is not at the fore while perceptions for HLT appear more frequent. Also, we see that the participants' common perception of HLT is thinking and predicting students' thinking rather than considering all three components of HLT. Regarding challenges, PMTs addressed interrelated major points: time issue for assessing in the classroom, not having information about students, for example, what they know and what they don't know to formulate concrete hypothetical learning steps and a lack of a list of necessary steps for MTC. The latter seems to indicate that PMTs are looking for a recipe for how to plan a lesson rather than a heuristic (the MTC) that must be fleshed out by them for each topic and class.

Discussion

In this paper, we focused on students' perceptions and challenges regarding MTC proposed by Simon (1995). Our findings revealed that some of the participants think about MTC as something in a different form but with the same aim of planning a lesson. This conclusion could be due to students' experience coming from the "what-how-why approach" of the didactic relationship model (Bjørndal & Lieberg, 1978) that they referred to many times to prepare lesson plans with justifying each step. However, some of the master students perceive MTC and HLT notions as new, however, they addressed a particular focus on (only) writing hypotheses for students' learning not together with learning goals and/or plan for activities. The main resource for this phenomenon could be due to students' tendency to consider curriculum competencies as learning goals. Even though our research was short for a professional development program and with a small sample of PMTs, as similarly reported in Wilson et al. (2013), it could be said that inclusion of MTC in the HPAM course contributed to students' awareness regarding hypotheses for students' learning. Concerning this, however, participants reflected on our training with presentation tools pointing out that a description of MTC and HLT through PowerPoint, showing them examples and classroom work on a lesson plan with their discussion, is not enough for making sense of the core ideas. This is in line with Simon (1995) and Wilson et al. (2013) which address that the continual change regarding MTC takes more time than expected. Participants also address that the lack of students' pre-knowledge is a barrier for them to formulate hypotheses for students' learning, considering new task situations and making a good assessment. All conclusions together imply that PMTs lacked a well-developed subject matter knowledge and pedagogical content knowledge needed for a meaningful HLT for students whose prior knowledge is unknown to them. Bearing in mind these conclusions, we plan to follow MTC as a heuristic tool in the forthcoming semesters and focus on the development of PMTs teaching designs step by step. Regarding assessment, it is not at the fore, to participants, due to time issues. We speculate that formative assessment techniques could be key and heuristic tools as integration of MTC and HLT notions.

Acknowledgement

We would like to thank students who volunteered to participate in this work, as well as to our colleague Liping Ding who conducted group interviews. Special thanks go to the reviewers (particularly external reviewer) for making constructive suggestions that improved the paper.

References

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, *59*(5), 389–407.

Bjørndal, B., & Lieberg, S. (1978). *Nye veier i didaktikken? En innføring i didaktiske emner og begreper* [New Paths in Didactics? An Introduction to Didactic Topics and Concepts]. Oslo: Aschehoug.

Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology*, *3*, 77–101.

Hemmi, K. & Ryve, A. (2015). Effective mathematics teaching in Finnish and Swedish teacher education discourses. *Journal of Mathematics Teacher Education*, *18*(6), 501–521.

Huang, R., Zhang, Q., Chang, Y., & Kimmins, D. (2019). Developing students' ability to solve word problems through learning trajectory-base and variation task-informed instruction. *ZDM–Mathematics Education*, *51*(1), 169–181.

Munthe, E., Bjuland, R., & Helgevold, N. (2016). Lesson study in field practice: a time-lagged experiment in initial teacher education in Norway. *International Journal for Lesson and Learning Studies, 5*(2), 142–154.

Rowland, T., Huckstep, P., & Thwaites, A. (2005). Elementary teachers' mathematics subject knowledge: The knowledge quartet and the case of Naomi. *Journal of Mathematics Teacher Education*, *8*(3), 225–281.

Simon, M.A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education, 26*(2), 114–145.

Simon, M.A., & Tzur, R. (2004). Explicating the role of mathematical tasks in conceptual learning: An elaboration of the hypothetical learning trajectory. *Mathematical Thinking and Learning*, *6*(2), 91–104.

Wilson, P. H., Mojica, G. F., & Confrey, J. (2013). Learning trajectories in teacher education: Supporting teachers' understandings of students' mathematical thinking. *Journal of Mathematical Behavior, 32,* 103–121.