Improved Determination of Heights Using a Conversion Surface by Combining Gravimetric Quasi/Geoid and GPS-levelling Height Differences

H. Nahavandchi, A. Soltanpour

Norwegian University of Science and Technology, Division of Geomatics, 7491Trondheim, Norway. Hossein.nahavandchi@ntnu.no Ali.soltanpour@ntnu.no

ABSTRACT

The quasi/geoid (quasigeoid/geoid) can be determined from the Global Positioning System (GPS) ellipsoidal height and the normal/orthometric heights derived from levelling (GPS-levelling). In this study a gravimetric quasigeoid and GPS-levelling height differences are combined to develop a new surface, suitable for "levelling" by GPS. This new surface provides better conversion of GPS ellipsoidal heights to the national normal heights. Different combining procedures, a four-parameter solution, linear and cubic splines interpolations, as well as the least-squares collocation method were investigated and compared over entire Norway. More than 1700 GPS-levelling stations were used in this study. The combined surface provides significant accuracy improvement for the normal height transformation of GPS height data, as demonstrated by the post-fitting residuals. The best solution, based on the least-squares collocation, provided a conversion surface for the transformation of GPS heights into normal height in Norway with an accuracy of about 5 cm.

Keywords. GPS-levelling, quasi/geoid, normal/orthometric heights, Splines, least-squares collocation

1 Introduction

The wide spread use of GPS for precise height determination has established quasi/geoid determination as a practically relevant product of physical geodesy. The geoid is the equipotential surface of the Earth's gravity field, which most closely corresponds to the mean sea level, and is commonly used as the zero surfaces for topographic elevations. Quasigeoid (Heiskanen and Moritz, 1967) is often used to approximate geoid. A gravimetric quasigeoid/geoid (quasi/geoid) is then to be used by a surveyor to transform GPS ellipsoidal heights into normal/orthometric heights above the mean sea level. However, the gravimetric quasi/geoid does not exactly coincide with the height datum used for the normal/orthometric heights. This is due to a combination of the approximations used in the gravimetric quasi/geoid computation, systematic errors in heights and gravity data, and the exact definition of the height system. The departure of the height datum from the equipotential quasi/geoid is a few decimeters in Scandinavia (Nahavandchi and Sjöberg 1998). Moreover, long wavelength discrepancies of approximately 1 m exist in the gravimetric quasi/geoid, when compared with GPS and the levelling heights (see e.g., Forsberg et al. 1996). The practical problem this presents is that the gravimetric quasi/geoid is not suited for the direct determination of normal/orthometric heights by GPS. Currently, a surveyor using GPS and a gravimetric quasi/geoid must apply further data reductions in order to make his/her elevations compatible with the height datum (see e.g. Featherstone et al. 1998; Smith and Roman 2001). This is particularly problematic for real-time GPS positioning, since surveyors must post-process the height data, which results in an increased survey cost. For instance, to determine the height difference of two points separated by 50 km takes several days by conventional spirit levelling, whereas it will take only a few hours with

GPS and the conversion (combined quasi/geoid) surface, such as the one produced in this study. It is thus expected that this can result in significant cost savings. Therefore, it is logical to provide a surface that specifically defines the separation of the vertical datum from the reference ellipsoid used by GPS.

For many years GPS and levelling data have been used to empirically verify gravimetric quasi/geoid solutions (e.g. Sideris et al. 1992; Nahavandchi 1998; Smith and Small 1999; Nahavandchi and Sjöberg 2001). Many studies have also been carried out to combining a gravimetric quasi/geoid and GPS-levelling data (Jiang and Duquenne 1996, Kotsakis and Sideris 1999, Denker et al. 2000, Featherstone 2001, Iliffe et al. 2003, Duquenne et al. 2004, Nahavandchi and Soltanpour 2004a, b).

In this study, GPS-levelling data are used as an additional source of quasi/geoid information, specifically to improve the determination of normal/orthometric heights from GPS by means of a new conversion (combined) surface, supplemented with a case study in Norway. This combined surface is a hybrid model meaning that it incorporates the gravimetric information of a gravimetric quasi/geoid model as well as the information obtained through GPS measurements on levelling benchmarks. Under an ideal situation (perfect gravimetric quasi/geoid), both quasi/geoid (GPS-levelling and gravimetric) would be identical, subject only to an offset (zero undulation), e.g., due to the vertical datum and/or the geopotential scale factor assumed for gravimetric quasi/geoid. In Norway, heights are referred to the NN1954 Norwegian height system, which is based on precise levelling, conducted during 1916-1954 and adjusted in 1956 (Lysaker 2003), but the height system remain ambiguous. The above inconsistency also makes it necessary to find an interim solution for height determination with GPS using a gravimetric quasi/geoid model to transform ellipsoidal heights to normal/orthometric height. It does not always yield results that are compatible with the local vertical datum (e.g., Featherstone et al. 1998). To improve this transformation, the gravimetric quasi/geoid model can be fitted to the GPS-levelling data. The new hybrid surface (importantly which is no longer the classical quasi/geoid) can then be used to give a more direct height transformation. Until the NN1954 vertical datum and/or quasi/geoid models are refined, the use of this interim solution is necessary.

2 Combining of gravimetric and GPS –levelling quasi/geoid

Several methods have been proposed to combine a gravimetric quasi/geoid with the quasi/geoidal heights determined at a set of levelled GPS benchmarks. This is due to the interests in determination of a new surface suitable for GPS-levelling. This means that after a combining procedure a new combined surface (which is not equipotential anymore) will be obtained. . The idea is that the new surface, after combining, can then be used in a GPS-levelling height determination. After that, the transformation of the GPS-heights to national heights (normal or orthometric heights) will be more straightforward. Combining methods must model the global behavior as well as local variations of both quasi/geoid data.

2.1 Four-parameter Regression Model

It is common to use the GPS-levelling data to validate, in an absolute sense, a gravimetric quasi/geoid model. This is usually done at a number of discrete points covering the area of interest and using the formulas below:

$$N_{\text{Geometric}} = h - H \tag{1}$$

or

$$\zeta_{\text{Geometric}} = h - H^N \tag{2}$$

where H is orthometric height, H^N is (Molodensky) normal height (Heiskanen and Moritz 1967), h is the GPS-derived ellipsoidal height, N is the geoid and ζ is the quasigeoid. The quasi/geoid derived from Eqs. (1) and (2) is called here the "geometric quasi/geoid". Furthermore, we attempt to minimize the offsets between the gravimetric and geometric quasi/geoidal heights by introducing a four-parameter regression model. In practice the usual four-parameter model of (Heiskanen and Moritz 1967)

$$\Delta N = N(\zeta)_{\text{Geometric}} - N(\zeta)_{\text{Gravimetric}} = \cos\phi\cos\lambda\Delta x + \cos\varphi\sin\lambda\Delta y + \sin\varphi\Delta z + RS \qquad (3)$$

is used for the datum transformation, where Δx, Δy and Δz correspond to the coordinate origin (geocenter) offset and RS could be interpreted as a scale factor. This transformation represents a very useful regression formula, which may be used for combining a regional gravimetric quasi/geoid with a set of GPS-levelling derived quasi/geoidal heights. However, it should be noted that the parameters resulting from such a regression model will not necessarily be the "true" coordinate origin offset, as long as long-wavelength geoid errors, often seen as tilt, are absorbed into these parameters and the quasi/geoid coverage is small. Furthermore, this type of transformation also demands a sufficient number of GPS-levelling stations, which should also cover evenly the study area.

2.2 Spline Interpolation

Spline interpolation uses piecewise continuous polynomials (linear, cubic, quadratic...), passing through each of the data points. So, there is a separate curve or line for each interval. Given $n+1$ pairs of data points (x_i, y_i), $i = 1, \ldots\ldots, n$, a piecewise polynomial $S(x)$ can be found so that it is composed of polynomials $S_i(x)$ and $S_i(x)$ are polynomials with $S(x_i) = y_i$. The $S_i(x)$ can be a different degree polynomial. The linear splines can be written as:

$$S_i(x) = a_i + b_i(x - x_i),\ i = 0,1,\ldots\ldots,n-1 \quad (4)$$

where a_i and b_i are the unknown coefficients. In total there are *2n* unknowns in the linear case. The conditions necessary to solve the unknowns could be written as:

$$S_i(x_i) = y_i \quad (5)$$

$$S_i(x_{i+1}) = y_{i+1} \quad (6)$$

In case of the cubic splines we have:

$$S_i(x) = a_i(x - x_i)^3 + b_i(x - x_i)^2 + c_i(x - x_i) + d_i,\ i = 0,1,\ldots\ldots,n \quad (7)$$

Since there are *n* intervals and four coefficients for each polynomial piece, a total of *4n* parameters is required to define the spline S(x) in a cubic case. Therefore, *4n* independent conditions should be found to fix them. Two conditions for each interval are derived from the requirement that the cubic polynomial matches the values of the table at both ends of the interval [See Eqs. (5) and (6)]. Note that these conditions result in a piecewise continuous function. *2n* more conditions are still needed. Since it is desired to make the interpolation as smooth as possible, we require that the first and second derivatives be also continuous. Then:

$$S'_{i-1}(x_i) = S'_i(x_i) \quad (8)$$

$$S''_{i-1}(x_i) = S''_i(x_i) \quad (9)$$

Cubic splines are popular because they are easy to implement and produce a curve that appears to be seamless. A straight polynomial interpolation of evenly spaced data tends to build up distortions near the edges of the table. Cubic splines avoid this problem, but they are only piecewise continuous, meaning that a sufficiently high derivative (i.e., the third one) is discontinuous. So, if an application is sensitive to the smoothness of derivatives higher than the second degree, then the cubic splines may not be the best choice.

2.3 Least-squares Collocation

The popular least-squares collocation (Moritz 1980a) has also been used to produce the new conversion surface in this study. The differences ΔN, between the gravimetric and the GPS-levelling quasi/geoids at a set of levelled GPS points are considered known. These differences then can be split up into two parts:

$$\Delta N_i = s_i + n_i \tag{10}$$

From a statistical point of view, s_i represents the signal at the GPS-levelling stations that represents the correlated part of ΔN_i, while n_i is the noise, which is the uncorrelated part. The correlated part of the differences is e.g., due to the gravity, GPS and levelling systematic errors, while the uncorrelated part contains measurement accidental errors and it may also contain some unexpected vertical motions of benchmarks and GPS antennae. The signal and noise parts can be computed by the least-squares collocation:

$$s = C_{s\Delta N} C_{\Delta N \Delta N}^{-1} \Delta N \tag{11}$$

where $C_{s\Delta N}$ is the covariance matrix between the signal and the GPS-levelling height differences and $C_{\Delta N\Delta N}^{-1}$ is inverse of the covariance matrix of the GPS-levelling height differences. When evaluating Eq. (11) , a large noise n reveals a large error at a particular levelled GPS point, whereas a large signals s suggest errors of gravity survey used to compute the gravimetric quasi/geoid. One important condition of the collocation solution is that the sought signal must constitute a stationary random function. This should be investigated and if the gravimetric and geometric quasi/geoid differences are not stationary, a de-trending procedure must be applied. Again, residuals, after such de-trending, have some information signal s and noise n.

Covariance matrices of Eq. (11) are not generally available. These covariance matrices (C) can be determined empirically from data (Moritz 1980a). In this study it is assumed that the covariances between points are isotropic. This means that the covariance depends only on the distance r between the points and not on the direction. The ΔN data, after removing trend if necessary, are used as the input data into the covariance function determination using the following formula:

$$C(r) = \frac{1}{n} \sum_{(i,j)} \Delta N_i \Delta N_j \qquad (12)$$

The summation above is over n (i, j) pairs. In the next step, an exponential covariance function is used for fitting to the empirical covariance function using following formula:

$$C(r) = C(0) \exp\left(\frac{-r \ln 2}{\xi} \right) \qquad (13)$$

where $C(0)$ *is* the signal variance and ξ is the correlation length, i.e.., the value of the argument r for which $C(r)$ has decreased to half of its value at r=0 (Moritz 1980a). The covariance between points must now be computed by fitting $C(0)$ and ξ *of* [Eq. (13)] to empirically determined covariance values from the data by using [Eq. (11)], which also yields $C(0)$ when $i=j$.

3 Numerical Investigations

The case study region of Norway was chosen principally as the funded project includes the determination of different quasi/geoid and hybrid models over Norway. Therefore, in this study, a new gravimetric quasigeoid model which employs the least-squares modification of Stokes's kernel (Sjöberg 1984) was firstly computed. The GGM01S global gravity model of GRACE mission (Tapley et al. 2003) was used as a source of the long-wavelength quasigeoid information. Over 230,000 gravity anomaly data points, obtained from the Norwegian Mapping Authority (Solheim and Omang, 2004, personal computations), were used in the gravimetric quasigeoid computation along with the 1-km Digital Elevation Model (DEM) of National Geophysical Data Center (NGDC) (http://www.ngdc.noaa.gov/mgg/topo/globe.html). This DEM was used for topographic corrections. For further details in the gravimetric quasigeoid computations see Nahavandchi et al. (2004) and Nahavandchi (2004). A software package in FORTRAN was developed for creation of the quasigeoid model over Norway and the improvement of this surface. Figure 1 shows the gravimetric quasigeoid model of Norway, which is based on the Geodetic Reference System 1980 (GRS80) reference ellipsoid (Moritz 1980b).

Furthermore, 392 GPS-levelling stations, distributed over Norway, were used for combining the gravimetric and GPS-levelling quasigeoids. Figure 2 depicts the locations of these stations. These 392 stations include the GPS-levelling database that is usually used for the combining procedure in Norway (see e.g. Solheim 2000). They cover whole Norway. Other group of GPS-levelling stations, altogether 1333 points (none of them are in the first group of 392 points), is usually used for the validation of the combination procedure (hybrid model). Figure 3 shows the location of the 1333 GPS stations. The levelled heights refer to the NN1954 local vertical datum (Lysaker 2003). The height system used in NN1954 is rather ambiguous (Lysaker 2003), but will be assumed to be a normal height system (ibid.). Finally, the GPS ellipsoidal heights in Norway are referred to the EUREF89 datum. In this study, the variance of all these observations will be assumed zero, which is not true but there is not any reliable variance information at present. Also, since we seek a combined hybrid surface that makes $\zeta_{\mathrm{Geometric}} - h + H^N = 0$, which is enforced by assuming zero variance. The gravimetric quasigeoid heights were bilinearly interpolated to the GPS-levelling points.

Figure 4 shows the differences between the geometric and gravimetric quasigeoids at the 1333 stations before combining (see also Table 1). The gravimetric quasigeoid model is then combined with the 392 GPS-levelling quasigeoid heights by using the four-parameter regression formula. The statistics of the post-fit residuals from the four-parameter fitting at the 1333 validation stations are presented in Table 1.

After the four-parameter combination (Eq. 3), the RMS of the 1333 GPS-levelling stations is ±25.8 cm. The residuals vary from -88.5 to 62.8 cm. For most of Norway (about 80%) absolute values of the residuals are within 40 cm. Figure 5 demonstrates these residuals at the 1333 test points. The systematic differences between the gravimetric and geometric quasigeoid models are due to long wavelength errors in the geopotential model, the land uplift and computational effects.

In the next step the splines (both linear and cubic) [see Eqs. (4)- (9)] were also applied to the same datasets to combine the gravimetric and geometric quasigeoid models. Again 392 GPS-levelling data was used for the combining and the second group of the 1333 GPS-levelling stations was used for the verification only. The statistics of the residuals, before and after combining are summarized in Table 1.

The results show that the conversion surfaces (both linear and cubic) obtained by the spline interpolation provides better agreement with the geometric quasigeoid, about 3 times better when compared to the previous four-parameter regression model. This may be due to the fact that the splines fit well locally. Furthermore, the cubic splines interpolation provides, somewhat better results compared to the linear splines. It is due to the continuous first and second derivatives of the cubic splines. So, fitting properties are smoother than in the case of the linear splines. This means that, unlike for linear splines, there are no breaks of the surface smoothness when using the cubic splines.

Figures 6 and 7 depict the residuals after applying the linear and cubic spline combining models, respectively. Smaller residuals are observed in the post-fitting residuals for the cubic splines than is the case for the linear ones.

Finally, a histogram for the combined residuals obtained with the cubic spline interpolation technique is plotted in Figure 8. High tendency to the normal distribution of the post-fit residuals around the "0-mean" is clearly shown in this histogram.

In the last step the least-squares collocation method was used to model the differences between the gravimetric and GPS-levelling quasigeoids. Since the least-squares collocation requires unbiased, stationary data, a de-trending procedure was applied to the input data of the quasi/geoid differences ΔN. A planar surface was removed from the data before the covariance functions were determined.

The differences, after the removal of a trend, were then used for the determination of an empirical covariance table by using Eq. (12). The 4 km interval was used for the bin classification of the distances between the data points. Next, an exponential covariance function Eq. (13) was fitted to the above, empirically determined, covariance values. The fitted covariance function and the empirically determined values are plotted in Figure 9. The variance C(0) = $(\pm 26\ \text{cm})^2$ and the correlation length of 53.705 km were determined by this fitting process.

The same group of 1333 GPS-levelling stations (not used in the combining process) was used for the validation of the conversion surface results. Applying the least-squares collocation solution provides smaller noise-level than the other solutions. The standard deviation of the residuals decreased down to ±4.7 cm, with a maximum and minimum value of 15.9 cm and -15.6 cm, respectively. Discrepancies, after applying the least-squares collocation method are plotted in Figure 10. The last figure is the new combined surface, plotted in Figure 11. This surface is derived using the least-squares collocation method and it provided the least noise level after combining. This hybrid model encompasses all gravimetric information of the gravimetric quasigeoid model of Figure 1 as well as the vertical datum information of GPS-levelling benchmarks. This surface was built to support the direct conversion of the ellipsoidal heights into the vertical datum heights.

4 Conclusions

Producing a conversion surface, this is optimised specifically for the determination of the Norwegian Height Datum (NHD) heights from GPS height observations, by using a gravimetric quasigeoid, combined with accurate GPS and NHD height data. This was the main goal of this study. It enables us to further enhance the computed quasigeoid model over Norway. This procedure helps to improve a determination of the NHD heights for GPS users, especially for real-time applications, since there is no need to post-process the results to account for the differences between the gravimetric quasigeoid and the NHD. This will be of most beneficial to surveying, mapping and exploration applications in Norway, since these rely heavily upon real-time GPS techniques.

Different combining process of the geometric and gravimetric quasigeoid, the four-parameter, linear and cubic splines as well as the least-squares collocation methods was studied. The simple four-parameter model removed long wavelength quasigeoid errors, but it was unable to fit the quasigeoid locally to GPS-levelling data. Using splines provided continuous surfaces, which were locally fitted to data. Linear and cubic spline interpolation techniques have shown significant improvement of post-fit residuals. Smaller residuals and their smoothness were observed for the cubic spline method in this study.

The best combining procedure of the gravimetric quasigeoid with the GPS-levelling data was the least-squares collocation method. This procedure guarantees that the height reference of the classical levelling is practically unchanged. The careful statistical separation of the correlated and uncorrelated parts of the differences between the gravimetric and GPS-levelling quasigeoids guarantees the precision of this procedure.

The procedure used for combining the gravimetric quasi/geoid models to GPS-levelling data assumes that both the GPS and the levelling are without errors. This is not true, but the purpose of computing the conversion surfaces as height reference surfaces is not to get a high precision quasi/geoid model. What is computed is a combined reference surface which gives the height in a system which as closely as possible coincides with the national height datum. Provided that the GPS measurements are made over a not too long time span then it is not necessary to have a model of the land uplift (e.g. Ekman 1989) because the combining procedure will implicitly solve for and remove its effect. Large parts of the corrections in the conversion surface are due to long wavelength errors in the geopotential, the land uplift and computational effects. In fact, if the quasi/geoid model, the GPS and the leveling were without errors then the correction terms would simply be the land uplift at least to a first order approximation when disregarding the corresponding change in the quasi/geoid (see Solheim 2000).

Acknowledgements. The authors are greatly indebted to Dr. J. Kouba and an unknown reviewer for patiently reading the paper and for the valuable comments. The authors would like to thank Dag Solheim from Norwegian National Mapping Authority and Ove Omang from Norwegian University of Life and Science for providing gravity anomaly and GPS-levelling data over Norway. This work is a part of a project funded by the Norwegian Research Council grant, numbered 147618/V30, under the independent research program at the Division of Science and Technology.

References

Denker H, Torge W, Wenzel H-G, Ihde J, Schirmer U (2000) Investigation of different methods for the combination of gravity and GPS/levelling data, in: Schwarz K-P (ed.) *Geodesy Beyond 2000: The Challenges of the First Decade*, Springer, Berlin Heidelberg New York, pp 137-142

Duquenne H, Everaerts M, Lambot P (2004) Merging a gravimetric model of the geoid with GPS/levelling data: an example in Belgium. Proceedings IAG International Symposium Gravity, Geoid and Space Missions GGSM 2004. August 30th to September 3rd. Porto (CD-ROM)

Ekman M (1989) Impacts of geodynamic phenomena on systems for height and gravity, Bulletin Geodesique. 63: 281–296

Featherstone WE (2001) Absolute and relative testing of gravimetric geoid models using Global Positioning System and orthometric height data, *Computers and Geosciences*, 27(7): 807-814.

Featherstone WE, Dentith MC, Kirby JF (1998) Strategies for the accurate determination of orthometric heights from GPS. Survey Review 34:278-296.

Forsberg R, Kaminskis J, Solheim D (1996) Geoid of the Nordic and Baltic region from gravimery and satellite altimetry. Proceedings of International Symposium on Gravity, Geoid and Marine geodesy (GRAGEOMAR), Tokyo, IAG Symposium Vol. 117, pp 540-547. Berlin: Springer-Verlag.

Heiskanen WA, Moritz H (1967) Physical geodesy. W.H. Freeman and Co., San Francisco

Iliffe JC, Ziebart M, Cross PA, Forsberg R, Strykowski G, Tscherning CC (2003) OGSM02: a new model for converting GPS-derived heights to local height datums in great Britain and Ireland. Survey Review 37(290): 276-293

Jiang Z, Duquenne H (1996) On the combined adjuctment of a gravimetrically determined geoid and GPS levelling stations, Journal of Geodesy 70(8): 505-514.

Kotsakis J, Sideris MG (1999) On the adjustment of combined GPS/levelling/geoid networks. Journal of Geodesy 73: 412-421.

Lysaker D (2003) An evaluation of the Norwegian height system NN1954, different gravity corrections and assumptions of the adjustment, MSc thesis, Norwegian University of Life and Science, Oslo

Moritz H (1980a) Advanced Physical Geodesy, F. Wichmann Verlag, Karlsruhe.

Mortiz H (1980b) Geodetic reference system 1980. The Geodesist's Handbook vol 54, No 3

Nahavandchi H (1998) Precise Gravimetric-GPS geoid determination with improved topographic corrections applied over Sweden. Royal Institute of Technology, Division of Geodesy. PhD Dissertation, Rep. 1050, Stockholm.

Nahavandchi H, Sjöberg LE (1998) Unification of vertical datums by GPS and gravimetric geoid models using modified Stokes’s formula. Journal of Marine Geodesy 21: 261-273.

Nahavandchi H, Sjöberg LE (2001) Precise geoid determination over Sweden by the Stokes-Helmert method and improved topographic corrections. Journal of Geodesy 75: 74-88.

Nahavandchi H (2004) The quest for a precise geoidal height model. Kart og Plan 1:46-56.

Nahavandchi H, Soltanpour A (2004a) Adjustment of the surface of the gravimetric geoidal height model between constrained GPS-levelling stations. Proceedings IAG International Symposium Gravity, Geoid and Space Missions GGSM 2004. August 30th to September 3rd. Porto (CD-ROM)

Nahavandchi H, Soltanpour A (2004b) An attempt to define a new height datum in Norway. The Geodesy and Hydrography Days 2004, 4-5 November, Sandnes, Norway

Nahavandchi H, Soltanpour A, Nyrnes E (2004) A New Gravimetric Geoidal Height Model over Norway Computed by the Least-Squares Modification Parameters. Presented at GGSM 2004, Porto, Porugal. August 30th September 3rd.

Sjöberg LE (1984) Least squares modification of Stokes's and Vening Meinez' formulas by accounting for errors of truncation, potential coefficients and gravity data, Department of Geodesy, University of Uppsala, No. 27, Uppsals, Sweden.

Sideris MG, Mainville A, Forsberg R (1992) Geoid testing using GPS and leveling. Australian Journal of Geodesy, Photogrammetry and Surveying 57:62-77.

Smith DA, Small HJ (1999) The CARIB97 high-resolution geoid height model for the Caribbean Sea. Journal of Geodesy 73: 1-9.

Smith DA, Roman DR (2001) GEOID99 and G99SSS: 1-arc-minute geoid models for the United States. Journal of Geodesy 75: 469-490

Solheim D. (2000) A New Height Reference Surface for Norway. Symposium of the IAG Subcommission for Europe (EUREF), Tromso 22 – 24 June.

Tapley BD, Chambers DP, Bettadpur B, Ries JC (2003) Large scale ocean circulation from the GRACE GGM01 Geoid. Geophysical Research Letters 30:2163–2166.

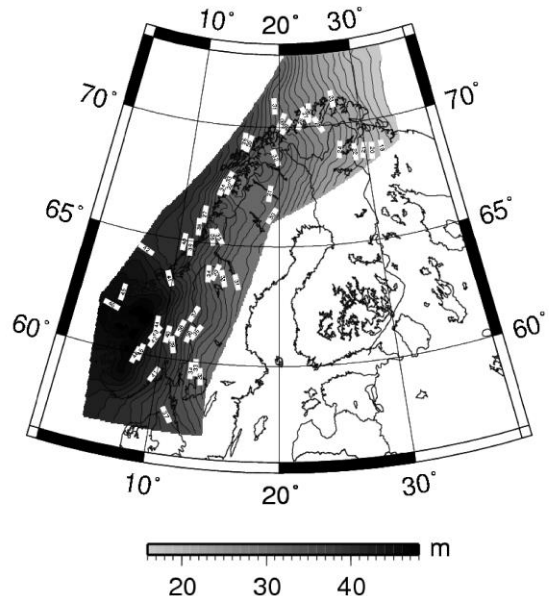

Fig 1 Quasigeoid model of Norway computed by the least-squares modification parameters.

GRS80 is the reference ellipsoid.

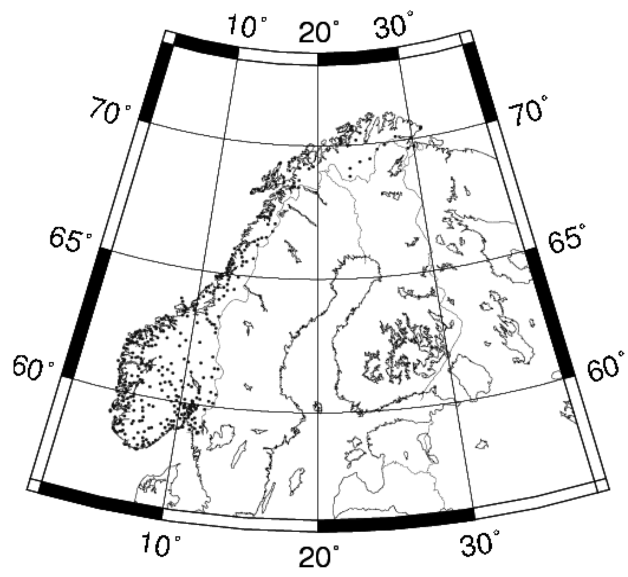

Fig 2 Spatial distribution of the 392 GPS-levelling stations used in the combining process.

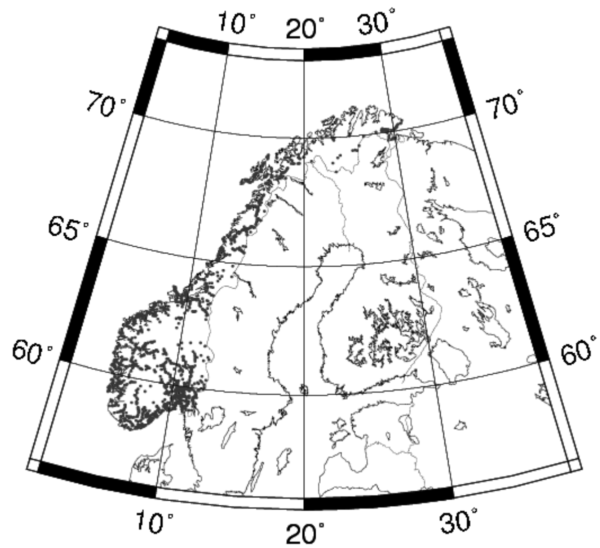

Fig 3 Spatial distribution of the 1333 GPS-levelling stations used for the validation process.

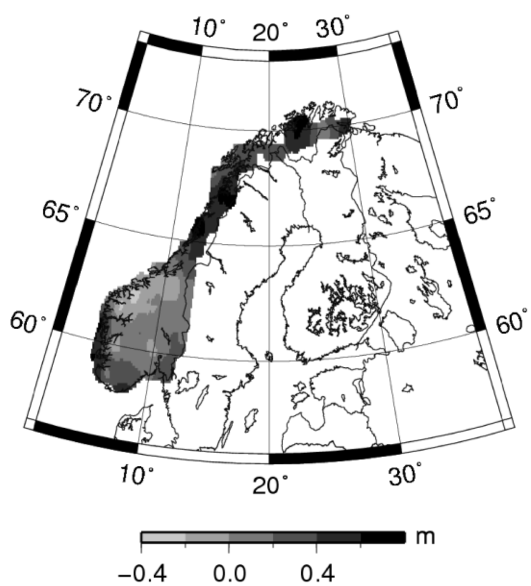

Fig. 4 Discrepancies between the gravimetric and geometric quasigeoids on the 1333 GPS-levelling stations over Norway.

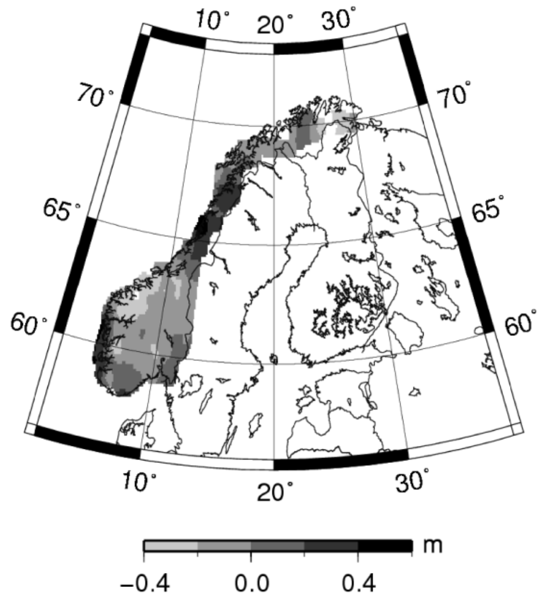

Fig. 5 Post-fitting residuals on the 1333 GPS-levelling stations after applying the four-parameter model.

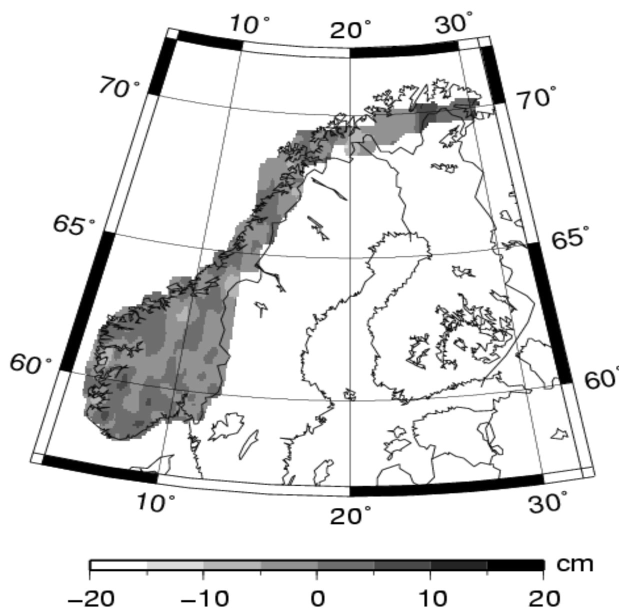

Fig 6 Post-fitting residuals on the 1333 GPS-levelling stations after applying the linear spline model.

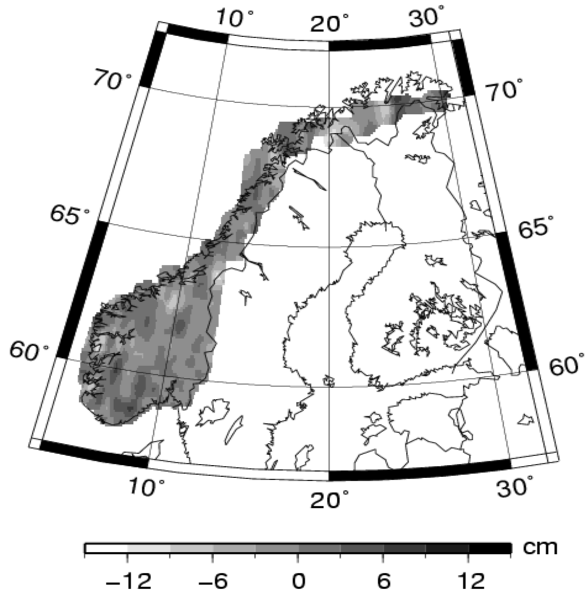

Fig. 7 Post-fitting residuals on the 1333 GPS-levelling stations after applying the cubic spline model.

.

.

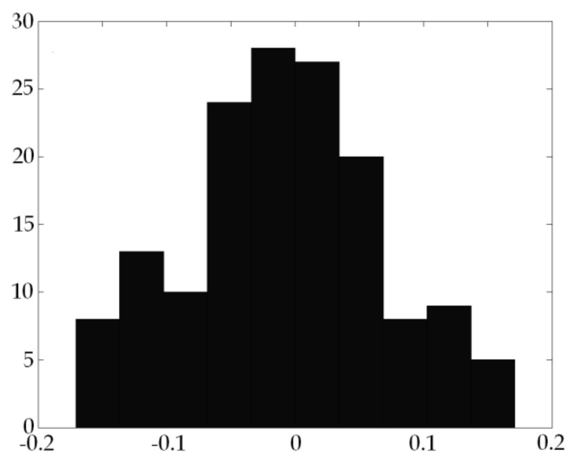

Fig. 8 Histogram of the residuals after applying the cubic spline interpolation.

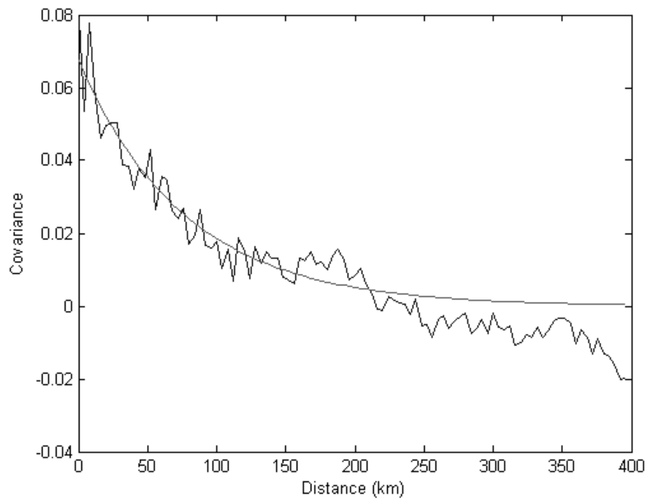

Fig 9 Empirical covariance function and best fitted exponential covariance function in m^2.

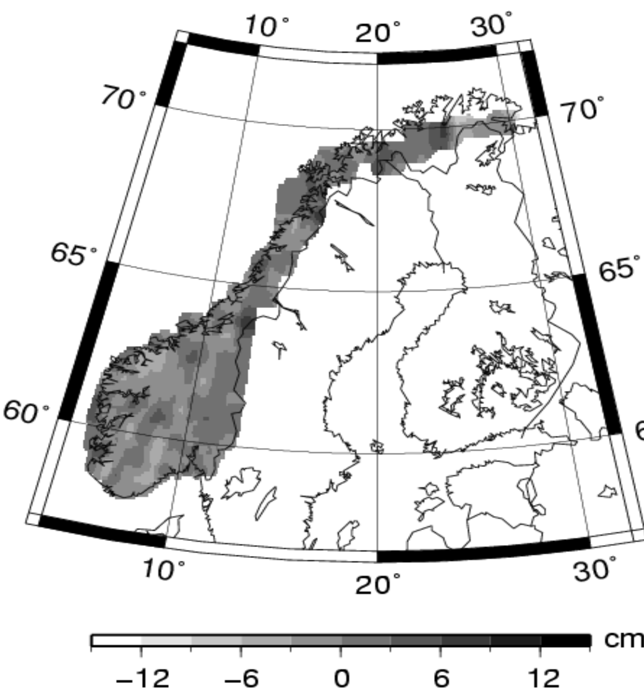

Fig 10 Post-fitting residuals on the 1333 GPS-levelling stations after applying the least-squares collocation model.

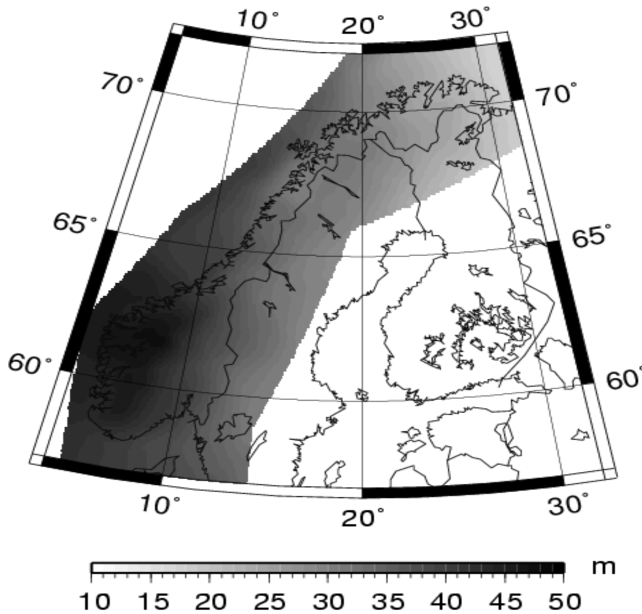

Fig 11 Final conversion (combined) surface computed as the new height reference surface in Norway.

Table 1. Validation Statistics for the hybrid surface at 1333 GPS-levelling stations using different combining procedures (m)

	Before Combination	After Combination (Four-parameter)	After Combination (Linear Splines)	After Combination (Cubic Splines)	After Combination (Least-Squares Collocation)
Max	0.879	0.628	0.192	0.171	0.159
Min	-0.615	-0.885	-0.218	-0.171	-0.156
Mean	0.238	0.000	0.000	0.000	0.000
Std	**0.279**	**0.258**	**0.078**	**0.076**	**0.047**
RMS	**0.366**	**0.258**	**0.078**	**0.076**	**0.047**