Mixed-Integer Formulation of Unit Commitment problem for power systems: Focus on start-up cost

Mutaz Tuffaha
Department of Engineering Cybernetics
Faculty of Information Technology,
Mathematics and Electrical Engineering
Norwegian University of Science and Technology
NO-7491 Trondheim, Norway
Emial: mutaz.Tuffaha@itk.ntnu.no

Jan Tommy Gravdahl
Department of Engineering Cybernetics
Faculty of Information Technology,
Mathematics and Electrical Engineering
Norwegian University of Science and Technology
NO-7491 Trondheim, Norway
Email: jan.tommy.gravdahl@itk.ntnu.no

***Abstract*—In this work, the Mixed-Integer (MIP) formulation for unit commitment problem (UC) for power systems is discussed. A new formulation for the start-up cost is suggested as well. This new formulation of the start-up cost exploits the transformation of the conditional statements into inequalities that comprise linear combination of binary variables. Solutions of the suggested optimization problem were obtained. A comparison between these solutions and those of a strategy common in literature is held to show that the new strategy gives same results with less number of constraints and tighter capture of the start-up cost.**

Nomenclature:

Parameters

$c_j(p_j(t))$	Fuel consumption cost of unit j in time slot t.
$c_j^u(t)$	Start-up cost of unit j in time slot t.
cc_j	Maximum start-up cost when the unit is cold.
D_j	Time period that unit j is required (scheduled) to be off at the start of the planning horizon.
$D(t)$	Total demand in time slot t.
DT_j	Minimum period required for unit j to be off before it can be turned on.
hc_j	Minimum start-up cost when the unit is still hot.
j	Unit index.
J	Number of generators.
ND_j	Number of discrete steps of the start-up cost function of unit j.
$\overline{P}_j$	Maximum output power of unit j.
$\underline{P}_j$	Minimum output power of unit j.
$R(t)$	Spinning reserve at time slot t.
RD_j	Maximum ramp-down rate for unit j.
RU_j	Maximum ramp-up rate for unit j.
SD_j	Maximum shut-down rate for unit j.
SU_j	Maximum start-up rate for unit j.
t	Time slot (period) index.
T	Length of planning horizon in time slots.
U_j	Time period that unit j is required (scheduled) to be on at the start of the planning horizon.
UT_j	Minimum period required for unit j to be on before it can be turned off.
$\overline{\tau}_j$	Minimum time period required for unit j to completely cool down.
$\underline{\tau}_j$	Maximum time period for which unit j is considered hot.

Variables

$p_j(t)$	Power generated by unit j in time slot t.
$\overline{p}_j(t)$	Maximum available power produced by unit j in time slot t.
$u_j(t)$	Binary variable to indicate the status of unit j in time slot t.
$\alpha_j(t)$	Binary variable to indicate the start-up of unit j in time slot t.
$\beta_j(t)$	Binary variable to indicate the shut-down of unit j in time slot t.

I. INTRODUCTION

Unit commitment (UC) (see [1]) is the problem of finding (scheduling) the optimal number of generating units that must be activated to meet the total demand in a power system, and their output power levels. Basically, UC problem is an optimization problem that aims to minimize the operational cost, mainly, fuel consumption, start-up cost and shut-down cost, subject to constraints that guarantee appropriate working conditions. This paper considers thermal units that are fueled by any kind of fossil fuel. UC problem can be formulated by using different approaches; *neural networks*, or *Mixed-Integer Problem* (MIP). In this paper, the latter is considered. MIP concerns problems that involve integer or binary variables, which are not allowed to take fractional values. The existence of such variables make the problem non-linear and probably non-convex even if the objective function and constraints are linear. This type of problems is usually referred to as mixed-integer linear problem (MILP). The complexity of such problems increases with the number of the integer variables, and of course, with the number of constraints. Needless to say, if the objective function is non-linear, the problem will be more complicated. However, many numerical methods have been suggested in literature to solve such problems. The authors in [7], mention several numerical methods. Namely, *Cutting plane*, *Decomposition*, *Logic-based*, *Branch and Bound* (BB), and *Outer Approximation* (OA). Of these methods, BB seems to be the more efficient method [7]. But this does not mean that various methods can not be combined. For example, the CPLEX solver has the ability to combine the cutting plane method with BB to reduce the feasible region.

Spurred by the progress in the technology of processors and computers, many authors used the MIP formulation for the UC problem for on-land power systems (see e.g. [1], [6],

[10], and [4]). In such formulations, a binary variable is used to indicate the status of each generator. A cost function which represents the operational costs is suggested. Then, the problem is formulated as a minimization of the cost function subject to some constraints. Such constraints are suggested to ensure optimum operation of the generating units, as will be explained later.
In this work, the MIP formulation of the UC problem is presented. Besides, an alternative strategy to take the start-up costs in considerations is proposed. A comparison between the new strategy and the strategy proposed in [1] and [2] is done and presented. In the next section, the mathematical formulation of the problem is introduced, including the objective function and the constraints. In the third section, the comparison is presented based on the numerical solutions of the optimization problems formulated by the two strategies. In the last section, some conclusions are emphasized.

II. MATHEMATICAL FORMULATION

First of all, let the set of all generators be denoted by $\mathscr{J}$, such that $\mathscr{J} = \{1,...,J\}$. Let, also, the set of the time indexes that span the planning horizon be $\mathscr{T}$, such that $\mathscr{T} = \{1,...,T\}$.

A. Assumptions and Constraints

The following assumptions are used to specify the constraints required for the optimization problem:

1) A binary variable, $u_j(t)$, is assigned to each generator j in each time slot t such that:
$$u_j(t) = \begin{cases} 1 & \text{if unit } j \text{ is on during slot } t. \\ 0 & \text{otherwise} \end{cases} \quad (1)$$
2) Two binary variables, $\alpha_j(t)$ and $\beta_j(t)$, are assigned to each generator in each time slot to express that the unit is turned on or off according to:
$$\alpha_j(t) = \begin{cases} 1 & \text{if unit } j \text{ is turned on in slot } t \\ 0 & \text{otherwise.} \end{cases} \quad (2)$$
$$\beta_j(t) = \begin{cases} 1 & \text{if unit } j \text{ is turned off in slot } t \\ 0 & \text{otherwise.} \end{cases} \quad (3)$$
Note that the switching variables $\alpha_j(t)$ and $\beta_j(t)$ can be expressed by using the binary variable $u_j(t)$ as follows:
$$\alpha_j(t) = \left\lfloor \frac{u_j(t) - u_j(t-1) + 1}{2} \right\rfloor \quad (4a)$$
$$\beta_j(t) = \left\lceil \frac{u_j(t) - u_j(t-1) - 1}{2} \right\rceil, \quad (4b)$$
where $\lfloor\ \rfloor$ and $\lceil\ \rceil$ denote floor and ceiling values, respectively.
3) **Power generation**: The power generated by each generator is bounded from above and below according to manufacturer recommendations, that is, $\underline{P}_j \leq p_j(t) \leq \overline{P}_j$, $\forall j \in \mathscr{J}$, $\forall t \in \mathscr{T}$. The binary variable $u(t)$ is, usually, included in this inequality, that is to say ([1], [4], and [6]):
$$\underline{P}_j u_j(t) \leq p_j(t) \leq \overline{p}_j(t) \leq \overline{P}_j u_j(t), \quad (5)$$
thus, if $u(t)$ is zero, then so is $p(t)$. The variable $\overline{p}_j(t)$ is another decision variable that determines the maximum available output power of unit j during time slot t. Note that $\overline{p}_j(t)$ is not necessarily the same as $\overline{P}_j$.
4) **Power balance**: The power generated by all generators must be sufficient for the total demand in each time slot. The total demand, $D(t)$, during time slot t is assumed known. So, ([1], [4], and [6]):
$$\sum_{j \in \mathscr{J}} p_j(t) \geq D(t). \quad (6)$$
5) **Spinning reserves**: This is the redundant capacity that can be activated upon request within certain time. The working generators must also be able to provide this reserve, so ([1], [4], and [6]):
$$\sum_{j \in \mathscr{J}} \overline{p}_j(t) \geq D(t) + R(t), \quad (7)$$
where $R(t)$ denotes the reserve capacity, and is usually given as a percentage of the total demand.
6) **Ramping**: The change (up or down) of the output power level of a generating unit during successive time slots. The rapid change of the output level will "lead to the rotor fatigue and shorten the operational lives of generating units [9]". So, limitations on such changes must be put to ensure longer operational lives and less need for maintenance. In order to take such limitations into account, the following inequalities are used to constrain ramp-up and ramp-down, respectively ([1], and [6]):
$$p_j(t) - p_j(t-1) \leq RU_j u_j(t-1) + SU_j \alpha_j(t) \quad (8a)$$
$$p_j(t-1) - p_j(t) \leq RD_j u_j(t) + SD_j \beta_j(t). \quad (8b)$$
7) **Uptime and Downtime**: The minimum time each generating unit should stay on (or off) before being shut down (or started up). So ([1], and [6]):
$$\sum_{i=t}^{t+UT_j-1} u_j(i) \geq UT_j \alpha(t),$$
$$\forall t \in \{L_j+1,...,T-UT_j+1\} \quad (9a)$$
$$\sum_{i=t}^{t+DT_j-1} (1-u_j(i)) \geq DT_j \beta(t),$$
$$\forall t \in \{F_j+1,...,T-DT_j+1\} \quad (9b)$$
$$\sum_{i=t}^{T} (u_j(i) - \alpha_j(i)) \geq 0,$$
$$\forall t \in \{T-UT_j+1,...,T\} \quad (9c)$$
$$\sum_{i=t}^{T} (1 - u_j(i) - \beta_j(i)) \geq 0,$$
$$\forall t \in \{T-DT_j+2,...,T\} \quad (9d)$$
$$\sum_{i=1}^{F_j} u_j(i) = 0 \quad (9e)$$
$$\sum_{i=1}^{L_j} u_j(i) = L_j, \quad (9f)$$

where, in all inequalities, $j \in \mathscr{J}$. $F_j = \min\{T, D_j\}$, and $L_j = \min\{T, U_j\}$. Where D_j and U_j denote the time period that unit j is scheduled to be off and on, respectively, at the start of the planning horizon based on the solutions for the previous planning horizon.

8) **Logical constraints**: To ensure that $\alpha_j(t) = 1$ only when the unit is scheduled to be switched on in slot t (i.e., $u_j(t-1) = 0$ and $u_j(t) = 1$), and $\beta_j(t) = 1$ only when the unit is scheduled to be switched off in slot t (i.e., $u_j(t-1) = 1$ and $u_j(t) = 0$), the authors in [1] suggest the following constraint:

$$u_j(t-1) - u_j(t) + \alpha_j(t) - \beta_j(t) = 0, \quad \forall t \in \mathscr{T}, \forall j \in \mathscr{J}. \tag{10}$$

B. Cost function

The cost function to be optimized can be given as:

$$\min_{p_j(.),u_j(.)} \sum_{t \in \mathscr{T}} \sum_{j \in \mathscr{J}} (c_j(p(t)) + c_j^u(t)), \tag{11}$$

which comprises two parts:

1) Fuel consumption $c_j(p(t))$: The fuel consumption is supposed to be a function of the generated power of each unit and it is usually approximated by a quadratic function [1], [2]. For MIP, it is easier to consider it as a piece-wise linear function [1]. The total fuel consumption of all generators in time slot t is the sum of the fuel consumption of all generators, which can be written as:

$$F_C(t) = \sum_{j \in \mathscr{J}} c_j(p_j(t)). \tag{12}$$

2) Start-up cost $c_j^u(t)$: The start-up cost depends on the time the unit has been left inactive. Because the colder the thermal engine gets, the more fuel and time it needs to warm up [5]. In fact, modeling the start-up cost has been discussed in many treatises. The start-up cost is usually modeled as an exponential function of the time the unit has been inactive. The authors in [8] suggested the following model:

$$c_j^u(\tilde{t}) = hc_j + (cc_j - hc_j)(1 - e^{(-\tilde{t}/\tau)}), \tag{13}$$

where $\tilde{t}$ here is the continuous time, and τ is a factor to determine how fast the function converges to the final value. When using this model of the start-up cost, two problems arise; the non-linearity of the function, and counting the time slots during which the unit has been inactive. To solve the first problem, the authors in [10] approximated the nonlinear exponential problem by a linear function bounded from below and from above, that is:

$$c_j^u(\tilde{t}) = hc_j + \frac{cc_j - hc_j}{\overline{\tau}_j - \underline{\tau}_j}(x_j(\tilde{t}) - \underline{\tau}_j), \quad \forall \underline{\tau}_j \leq x_j(\tilde{t}) \leq \overline{\tau}_j, \tag{14}$$

where $x_j(\tilde{t})$ denotes the number of time slots during which unit j has been off up to continuous time $\tilde{t}$. The variable $x_j(\tilde{t})$ is, however, not easy to determine (see [10] for more details). In [5], the non-linear exponential function (13) was discretized into ND discrete steps, and the start-up costs were formulated as:

$$c_j^u = \max_{ND_j=0,..,ND_j^*} a_j^{ND_j}(u_j(t) - \sum_{i=1}^{ND_j} u_j(t-i)), \tag{15}$$

where $a_j^{ND_j}$ are cost coefficients, and ND_j^* is the number of the discrete steps (or time slots) that partitions the maximum time needed for the unit to cool down ($\overline{\tau}_j$). Based on this assumption, other authors, (e.g., [1], and [6]) suggested modeling the start-up cost by a decision variable bounded by the following constraints:

$$c_j^u(t) \geq K_j^k \Big[u_j(t) - \sum_{i=1}^{k} u_j(t-i)\Big], \quad \forall j \in \mathscr{J}, \ \forall t \in \mathscr{T}, \ \forall k \in \{1, ..., ND_j\} \tag{16a}$$

$$c_j^u(t) \geq 0, \ \forall j \in \mathscr{J}, \ \forall t \in \mathscr{T}, \tag{16b}$$

where the discrete start-up steps K_j^k were assumed to take two values only; hc_j for $t \leq DT_j + \underline{\tau}_j$, and cc_j for $DT_j + \underline{\tau}_j < t \leq ND_j$. The approach above is clever, specially, if the discrete steps K_j^k are taken to cover the whole range of the start-up cost. However, the inequality in (16a) includes ND_j more inequalities, because for each time slot t, and unit j, the step index k should take values in $\{1, .., ND_j\}$. To illustrate, the inequality in (16a) is, actually, rewritten as:

$$\begin{aligned} c_j^u(t) &\geq K_j^1 \Big[u_j(t) - u_j(t-1)\Big] \\ c_j^u(t) &\geq K_j^2 \Big[u_j(t) - u_j(t-1) - u_j(t-2)\Big] \\ &\vdots \\ c_j^u(t) &\geq K_j^{ND_j} \Big[u_j(t) - \sum_{i=1}^{ND_j} u_j(t-i)\Big], \end{aligned} \tag{17}$$

for each time slot t, and unit j. This means that, we will have $ND_j \times J \times T$ more constraints to describe the start-up cost, even if the start-up cost steps take two values only. Actually, making the discrete steps K_j^k take two values or more will not change the number of constraints.

The start-up cost depends on the time the unit has been inactive, as mentioned earlier. Note that the uptime and downtime constraints in (9) ensures that unit j will stay active after being turned on for at least UT_j. Similarly, if unit j is turned off it will stay inactive for at least DT_j. So, certainly, if the start-up status variable $\alpha_j(t)$ is chosen to be one, then unit j has been down for at least DT_j. We propose to use this, as will be shown, to estimate the time period during which the unit has been inactive. However, the values of DT_j and ND_j play a crucial role here. So, two cases must be discussed:

1) When $DT_j \geq ND_j$.
2) When $DT_j < ND_j$.

In the first case, the time needed for the unit to fully cool down (ND_j) -which is equal to the time steps of the discretized exponential function in (13)- is less than the time period the unit should stay inactive after being turned off. That is to say, the unit will stay inactive till it cools down completely. So, it makes sense to penalize the start-up status binary variable by the maximum start-up cost cc_j. Thus:

$$c_j^u(t) \geq \alpha_j(t) cc_j. \quad (18)$$

Compared to the start-up cost formulated in (16), the formulation in (18) looks simpler and more intuitive. Besides, it will reduce the number of constraints.

In the second case, the situation becomes more complicated because it is not easy to count the time periods during which the unit has been inactive. We propose to do this by using *propsitional calculus* presented in [7] to express conditional statements by linear combination of binary variables. We can begin with the first conditional state as follows:

$$[\alpha_j(t)=1] \wedge [\alpha_j(t-DT)=0] \longrightarrow c_j^u(t) \geq K_j^{DT+1}, \quad (19)$$

where $\wedge$, and $\longrightarrow$ denote the logical operations "**AND**", and "**IF**", respectively. When the conditions in the statement above are satisfied, then unit j has been inactive for at least $DT+1$ time slots. So, the start-up cost is, necessarily, greater than the start-up cost when the unit is turned off for $DT+1$ time slots. Similarly, we can write:

$$\begin{aligned} &[\alpha_j(t)=1] \wedge [\alpha_j(t-DT_j)=0] \\ &\quad \wedge [\alpha_j(t-DT_j-1)=0] \longrightarrow c_j^u(t) \geq K_j^{DT_j+2} \end{aligned} \quad (20a)$$

$$\vdots$$

$$\begin{aligned} &[\alpha_j(t)=1] \bigwedge_{i=DT_j}^{ND_j-1} [\alpha_j(t-i)=0] \\ &\qquad \longrightarrow c_j^u(t) \geq cc_j. \end{aligned} \quad (20b)$$

Now let $\gamma_j^1(t)$ be a binary variable which is equivalent to the statement $[\alpha_j(t)=1] \wedge [\alpha_j(t-DT_j)=0]$, such that $\gamma_j^1(t)$ is one when the statement is true and zero when the statement is false. Then, according to [7], the binary variable γ_j^1 can be determined by the following constraints:

$$\begin{aligned} -\alpha_j(t) + \gamma_j^1(t) &\leq 0 \\ -(1-\alpha_j(t-DT_j)) + \gamma_j^1(t) &\leq 0 \\ \alpha_j(t) + (1-\alpha_j(t-DT_j)) - \gamma_j^1(t) &\leq 1. \end{aligned} \quad (21)$$

Then, the conditional statement in (19) can be replaced by:

$$c_j^u(t) \geq \gamma_j^1(t)(K_j^{DT_j+1} - K_j^{DT_j}) + \alpha_j(t) K_j^{DT_j}, \quad (22)$$

because, when $\gamma_j^1(t)=1$, then $\alpha_j(t)=1$. So, the start-up cost must be greater than or equal to $K_j^{DT_j+1}$ as stated before. While, when $\gamma_j^1(t)=0$, then $\alpha_j(t)$ could be either zero (so the start-up cost is zero), or one (so the start-up cost is $K_j^{DT_j}$ only). Analogously, let γ_j^2 be a binary variable corresponding to the statement $[\gamma_j^1=1] \wedge [\alpha_j(t-DT-1)=0]$. And, in general, let $\gamma_j^i(t)$ be equivalent to the statement $[\gamma_j^{i-1}=1] \wedge [\alpha_j(t-DT-i+1)=0]$. Then, the start-up cost can be formulated as:

$$\begin{aligned} (ND_j - DT_j) c_j^u(t) \geq \; & \textstyle\sum_{i=1}^{ND_j-DT_j} \gamma_j^i(t) K_j^{DT+i} \\ & + (1-\gamma_j^i(t)) hc_j, \\ \forall t \geq ND_j, \forall j \in \mathscr{J}, & \end{aligned} \quad (23)$$

subject to the constraints:

$$\begin{aligned} -\alpha_j(t) + \gamma_j^{ND_j-DT_j}(t) &\leq 0 \\ \sum_{i=1}^{ND_j-DT_j} [\alpha_j(t-DT-i+1) - 1 + \gamma_j^i(t)] &\leq 0 \\ \alpha_j(t) - \sum_{i=1}^{ND_j-DT_j} [\alpha_j(t-DT-i+1) - 1 & \\ + \gamma_j^i(t)] &\leq (ND_j - DT_j), \\ \forall\ t \geq ND_j,\ \forall\ j \in \mathscr{J}&, \end{aligned} \quad (24)$$

where the above constraints are obtained after summing the constraints in (21) for all γ_j variables. While, for $t < ND_j$, ND_j is replaced with t in equations (24) and (23). One problem with this formulation is that the time index in (23) is forced to start from DT_j. This means that this strategy neglects the start-up cost for times below DT_j. But this can be compensated for by adding the term in (18) to the cost function for the time slots less than DT_j, that is:

$$c_j^u(t) \left\{ \begin{array}{lll} = & \alpha_j(t) cc_j, & t \leq DT_j \\ \geq & \sum_{i=1}^{ND_j-DT_j} \gamma_j^i(t) K_j^{DT+i} & \\ & + (1-\gamma_j^i(t)) hc_j, & t > DT_j \end{array} \right\}. \quad (25)$$

Besides, formulating the start-up costs as in (23) with constraints in (24) will add to the complexity of the optimization problem because it will increase number of the decision variables. In fact, the number of decision variables will increase by the number of the variables γ_j^i. While, the number of constraints is less compared to the constraints in (16). However, if the difference between ND_j and DT_j is not large, then the increase of the number of constraints will not be drastic compared to the constraints in (16). Here, we need only three constraints for each $c_j^u(t)$, while in (16), we need ND_j constraints as mentioned earlier.

The merit of this new formulation of the start-up cost manifests itself clearly when the discrete start-up cost steps K_j^i are approximated by few steps instead of taking all ND_j steps. For example, if the steps K_i are assumed to take two values only (which is usually the case) as follows:

$$K_j^t = \left\{ \begin{array}{ll} hc_j, & t \leq (ND_j/2) \\ cc_j, & t > (ND_j/2) \end{array} \right. , \quad (26)$$

then the number of the extra binary variables γ_j^i required for this formulation will be equal to $(ND_j/2) - DT_j$ if $(ND_j/2) > DT_j$. Whereas, if $(ND_j/2) \leq DT_j$ then no extra variables are needed.

Table I
THE SPECIFICATIONS OF THE GENERATION UNITS USED IN NUMERICAL SOLUTIONS [2]

	Type I	Type II	Type III
$\overline{P}[MW]$	455	130	85
$\underline{P}[MW]$	150	20	25
RD $(RU)[MWh]$	225	50	60
SD $(SD)[MWh]$	150	20	25
$DT^{(}UT)[h]$	8	5	3
$ND[h]$	14	12	3
$\overline{\tau}[h]$	5	4	2
hc[\$/h]	4500	560	260
cc[\$/h]	9000	1120	520
a[\$/MW^2h]	0.00048	0.002	0.00079
b[\$/MWh]	16.19	16.6	27.74
c[\$/h]	1000	700	480

III. NUMERICAL RESULTS

The planning horizon was assumed to be 24 hours divided into 24 time slots each of length 1 hour. Three different types of power generation units were assumed based on the specifications given in [1], and [2]. The specifications of the three types are listed in Table I. Note that the three types of the power generation units used differ according to DT_j and ND_j. For type I, $DT > (ND/2)$. For type II, $DT < (ND/2)$. While, for type III, $DT = ND$. The specific fuel consumption function $c_j(p(t))$ was assumed quadratic of the form $a_j p_j(t)^2 + bp_j(t) + c$, with the coefficients a_j, b_j, and c_j are as listed in Table I. Then, the quadratic function was expressed as a piecewise linear function. The optimization problem in (11) subject to the constraints given in (5), (6), (7), (8), (9), and (10) was solved twice for two different examples:

1) Example I: 14 power generating units were assumed, 4 of type I , 6 of type II, and 4 of type III.
2) Example II: 60 units were assumed, 20 of type I, 25 of type II, and 15 of Type III.

The demands were assumed arbitrarily, and they are listed in Table II for the two examples. The spinning reserves were assumed to be 5% of the total demand. In both examples, the discrete start-up cost steps K_j^t were assumed to take two values only, as in (26). Then, each case was solved twice; once with the start-up cost as in (16); the other, the proposed strategy in (25) with the constraints in (24) was used. IBM ILOG CPLEX Optimization Studio V12.5 was used. The codes were written in Optimization Programming Language (OPL). This program was chosen for its simplicity in constructing the problem.

Before discussing the solutions of the optimization problem for all the cases, important results of the proposed strategy can be noted without the need for calculations. Those results can be drawn from the following two cases. First, if unit j was scheduled to be off for D_j time slots at the beginning of the planning horizon, then the start-up cost would be equal to hc_j, according to the strategy in (16), if $D_j < DT_j + \underline{\tau}_j$. And this is not accurate, because scheduling unit j to be off for D_j time slots at the beginning of the planning horizon, infers that this decision was made in the previous planning horizon (Specially, if the optimization problem is solved on-line). Hence, according to the constraints in (9), unit j should stay off for at least DT_j. So, the start-up cost should not be equal to hc_j. The second case concerns the small power generation units, that do not take too long to cool down completely. That is to say, the units for which the difference $ND_j - \underline{\tau}_j$ is not too large. In that case, the start-up cost can not be captured as cc_j according to the constraints in (16), because K_j^t was assumed to take the value cc_j when $t > \underline{\tau}_j + DT_j$, as mentioned earlier. These two cases make the start-up cost less, which in turn, makes the decision of turning on a unit easier. While, in the proposed strategy, the start-up cost is usually higher, which in turn make the decision of turning on a unit harder to take, or at least, more highly penalized.

Table III
COMPARISON BETWEEN THE SOLUTIONS OF THE OPTIMIZATION PROBLEM BY THE TWO STRATEGIES IN EQUATIONS (16) AND (25) FOR EXAMPLE I

	Model in (16)	Model in (25)
Constraints	5472	3366
Variables	2339	2461
Binary var.	994	1116
Objective	\$902,835.6	\$912,344.6
Time [Sec]	2	2

Table IV
COMPARISON BETWEEN THE SOLUTIONS OF THE OPTIMIZATION PROBLEM BY THE TWO STRATEGIES IN EQUATIONS (16) AND (25) FOR EXAMPLE II

	Model in (16)	Model in (25)
Constraints	23700	14265
Variables	10036	10531
Binary var.	4275	4770
Objective	\$4,205,487	\$4,257,331
Time [Sec]	8	7

A comparison between the proposed strategy and the one in (16) is presented for the two cases in Table III, and IV. The solutions of the two problems were almost identical regarding the binary variables and scheduled power levels. The discrepancy seen in the objective function results from the difference of the start-up cost as explained earlier. The times required for computations in the two formulations are not of considerable difference. However, the difference is in favor of the proposed strategy. The number of constraints are less in the proposed strategy.

IV. CONCLUSIONS

In this paper, MIP formulation of the unit commitment problem for power systems were presented. A new strategy for considering the start-up cost was suggested based on counting the time periods for which the unit has been inactive. The

Table II
TOTAL DEMAND ASSUMED: EXAMPLE I, AND EXAMPLE II

Time slot	1	2	3	4	5	6	7	8
Demand, example I [MW]	1313	1189	1138	1324	1511	1731	1566	1832
Demand, example II[MW]	6206	5621	5381	6261	7141	8181	7405	8661
Time slot	9	10	11	12	13	14	15	16
Demand, example I [MW]	1765	2125	2357	2616	1714	1630	1613	1452
Demand, example II[MW]	8342	10046	11143	12367	9583	10046	8822	8822
Time slot	17	18	19	20	21	22	23	24
Demand, example I [MW]	1232	1155	1144	1177	2027	2125	1866	1866
Demand, example II [MW]	8103	7703	7623	6864	5824	5460	5408	5564

proposed strategy exploits the propositinal calculus by which, conditional statements can be transformed into inequalities of linear combination of binary variables. Two variables were depended on to formulate the proposed strategy, the time required for the unit to cool down completely, and the time it must stay down after being turned off. The optimization problem was solved by using well-known formulation used in literature (e.g., [1]), and by the proposed strategy. The results showed that the suggested formulation requires less number of constraints, and it gives a tighter capture of the start-up cost.

ACKNOWELEDGMENT

The authors would like to thank the Norwegian Research Council and SINTEF Fisheries and Aquaculture for sponsoring this work as part of the ImproVEDO-project. We want also to thank Professor Tor Arne Johansen for the discussions and feedback leading up to this paper.

REFERENCES

[1] J. Ostrowski, M. F. Anjos, and A. Vannelli, "Tight Mixed Integer Linear Programming Formulations for the Unit Commitment Problem", *IEEE Trans. Power systems*, vol. 27, No. 1, Feb. 2012, pp. 39-46.

[2] M. Carrión, and J. M. Arroyo, "A Computationally Efficient Mixed-Integer Linear Formulation for the Thermal Unit Commitment Problem", *IEEE Trans. on power systems*, vol. 21, No. 3,Aug. 2006, pp. 1371-1378.

[3] L. T. Biegler, and I. E. Grossmann. "Retrospective on optimization",*Computers and Chemical Engineering 28*, 2004, pp. 1169-1192.

[4] S. Takriti, and J. R. Birge, "Using Integer programming to refine Lagrangian-based Unit commitment solutions", *IEEE Trans. Power systems*, vol. 15, no. 1, Feb. 2000, pp. 151-156.

[5] M. P. Nowak and W. Römisch, "Stochastic Lagrangian Relaxation applied to Power Scheduling in a Hydro-Thermal System under Uncertainty", *Ann. Oper. Res.*, vol.100, 2000, pp. 251-272.

[6] J. M. Arroyo, and A. J. Conejo, "Optimal Response of a Thermal Unit to an Electricity Spot Market", *IEEE Trans. POWER SYSTEMS*, vol. 15, No. 3, Aug. 2000, pp. 1098-1104.

[7] A. Bemporad, and M. Morari, "Control of systems integrating logic, dynamics, and constraints", *Automatica 35*, 1999, pp. 407-427.

[8] R. Zhu, C. Fu, and S. Rahman, "Network programming technique for unit commitment", *Electrical Power and Energy Systems*, vol. 17, No. 2, 1995, pp. 123-127.

[9] C. Wang and S. M. Shahidehpour, "Ramp-Rate limits in Unit Commitment and Economic dispatch incorporating rotor fatigue effect", *IEEE Transactions on Power Systems*, vol. 9, No. 3, Aug. 1994, pp. 1539-1545

[10] X. Guan,P. B. Luh, H. Yan, and J. A. Amalfi, "An optimization-based method for unit commitment", *Electric Power and Energy Systems*, vol.14, No.1, Feb.1992, pp.9-17.