Optimization Strategy for Energy Allocation through Cooperative Storage Management

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***Abstract*—We propose a strategy to optimize energy utilization through battery management in a cooperative environment where households share access to a community-owned energy farm. The households are equipped with lossy rechargeable batteries, which exhibit a non-linear discharging behavior. To devise our strategy, we first design the battery discharging operation in each household, and then we optimize the energy allocation policy among participating users. Our proposed strategy seeks to minimize the collective energy expenditure, and accounts for time- and location-dependent electricity prices. Both the battery discharging operation and the energy allocation policy are designed by solving constrained optimization problems. Specifically, calculus of variations and optimal control theory are used to provide explicit solutions and determine closed-form performance estimates. Extensive simulations are presented to validate our analysis and evaluate the impact of different system parameters.**

***Index Terms*—Energy allocation, storage management, price sensitivity, optimization.**

I. Introduction

Internet of Things (IoT) technologies can improve the operation of the power grid [1], [2], and allow us to optimize the use of renewable energy (RE) in current generation systems [3], [4]. This in turn can help us to reduce carbon emissions and minimize operational costs. Proper RE management can also be used to reduce the peak-to-average power ratio in grid networks, thus making them more resilient [5], [6].

However, RE management is a challenging process, given, for example, the intermittency of sources such as solar irradiance and wind speed. Energy storage systems (ESSs) have been proposed as a means to enhance the utility of RE generation systems and combat their intermittency. ESSs can enhance the impact of RE generation by allowing users to schedule their grid energy consumption [7], [8], and take advantage of time-varying pricing to reduce their electricity bills [8]–[11]. ESSs can also benefit utilities, as distributed storage can be used in load balancing applications [12], [13].

In this paper, we propose an energy allocation strategy for cooperating households with access to a community-owned energy farm. Our proposed RE cooperation strategy seeks to minimize the energy expenditure incurred by participating households over a finite planning horizon. We assume a community of households with shared access to a farm, where RE is harvested and stored. Each household is equipped with a lossy battery which has a limited storage capacity and a non-linear discharging behavior. Unlike existing works, our battery model takes into account the non-linear relationship between the discharging rate and the battery's remaining charge. Energy management strategies that are aware of such non-linear behavior can lead to extended battery lifetimes [14] and higher cost savings [15].

To devise our strategy we divide the optimization problem into two subproblems. First, we optimize the battery discharging operation in each household, the amount of energy available to use is subject to causality constraints. Then, assuming optimized discharging operations in all households, we solve a constrained optimization problem to allocate, among participants, the total energy available at the generation facility.

The proposed energy allocation policy seeks to minimize the total energy cost incurred by all the participating households over the specified planning horizon. The strategy proposed is cooperative in nature, but the cost savings obtained can be allocated following different policies, e.g., in proportion to the households' investment share in the energy farm [16].

It is shown analytically that the proposed strategy can optimize both the discharging operation and the energy allocation policy across participants. Moreover, extensive simulations show agreement between analytical and numerical results. The analysis presented in this paper can be used to reduce the computational complexity of existing strategies based on techniques such as linear or dynamic programming, and assess the performance of the ESS while altering battery parameters such as nominal output power or efficiency rate.

Cost-minimization strategies leveraging shared ESSs have been proposed in [5], [7], [8], [13], [17], [18]. Some of these works have considered RE assets, e.g., [5] and [7].

Storage sharing strategies for utility maximization have been proposed in [9]. Similarly, strategies based on shared ESSs have been introduced in [12] and [19]. In [20] the authors discussed an energy trading system for users with shared access to an ESS. In most of these works, the shared asset is the storage capacity of the ESSs, not the energy stored throughout the planning horizon. Moreover, in most cases the ESSs have been modeled as linear devices.

Cooperative energy management has been studied in [18], [21]–[29]. The strategies proposed in [26], [27], [29] are

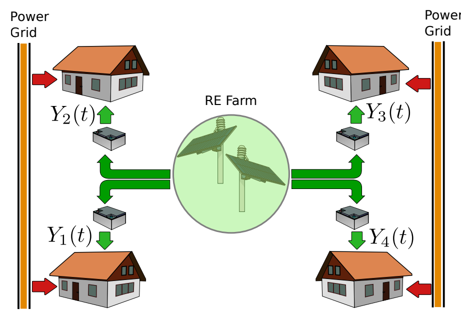

Fig. 1. Battery-equipped households with shared access to an energy farm.

meant to optimize the energy utilization in microgrids and buildings through energy sharing and storage management.

Unlike the works listed above, this paper studies the energy allocation problem by accounting for both centralized and distributed ESSs, as well as location- and time-dependent electricity prices. Moreover, the battery model used in this paper accounts for non-linear characteristics of the discharging operation, which allows for a higher performance and a longer battery lifespan [14]. Finally, the optimization techniques used in this paper lead to results in closed form, which can be used to derive an explicit performance metric and thus assess the achievable cost savings in terms of battery parameters such as nominal output power and efficiency rate. For a journal version of this paper, readers are referred to [30].

II. System Model

A. System Setup

We consider M grid-connected households with shared access to an energy farm. Fig. 1 illustrates this setup with $M = 4$. The energy available in the farm is allocated to the households at the beginning of the planning horizon, e.g., on a day-ahead basis.

B. Loads, Pricing Model, and Planning Horizon

The power demand at each household is denoted by $L_i(t)$, where t is the time index, and the planning horizon is $[0, T]$. For generality, we assume an arbitrary planning horizon, i.e., $T > 0$ can refer to hours or days. The electricity prices are denoted by $P_i(t) > 0,\ \forall\ t,\ i \in \{1, \ldots, M\}$. [1]

Let $Y_i(t)$ denote the power used up by the ith household from the ESS, then the cost incurred by the same household in $[0, T]$ is $\int_0^T P_i(t)\,[L_i(t) - Y_i(t)]\,dt$. And the total energy cost incurred by all the participating households in $[0, T]$ is

$$\mathsf{EC} = \sum_{i=1}^{M} \int_0^T P_i(t)\,[L_i(t) - Y_i(t)]\,dt, \tag{1}$$

where $Y_i(t) \le L_i(t),\ \forall\ t,\ \forall\ i$.

[1]Continuous-time pricing signals also account for discrete-time pricing schemes, such as hourly settlements. The signal $P_i(t)$ can be defined as a piecewise constant function to model scenarios in which prices remain constant over pre-defined periods and only change at specified points in time.

C. Energy Storage Systems (ESSs)

The following are the characteristics of the ESSs deployed across households.

- Dynamics of the ESSs: The energy available in the ESS at the ith household is denoted by $E_i(t)$, and satisfies:

$$E_i(t) = E_i(0) - \int_0^t X_i(\tau)d\tau, \tag{2}$$

where $X_i(t)$ is the power used up by the ith household before losses. In general, $X_i(t) \ge Y_i(t),\ \forall\ t,\ \forall\ i$, as losses are incurred during the discharging operation.

- Non-linear discharging model: Each ESS in the system is subject to discharging[2] losses, which are modeled after Peukert's law [31]. Specifically, the relationship between $X_i(t)$ and $Y_i(t)$ is stated as follows:

$$Y_i(t) = \min\left\{X_i(t), \Psi_i\left[\frac{X_i(t)}{\Psi_i}\right]^{\frac{1}{\alpha_i}}\right\}, \tag{3}$$

where $\Psi_i > 0$ and $\alpha_i > 1$ are, respectively, the rated output power, and the battery's efficiency rate (Peukert's exponent). From (3), it follows that $X_i(t) \ge Y_i(t),\ \forall\ i\ \forall\ t$.

- Let Θ_i denote the capacity of the ith ESS, hence:

$$0 \le E_i(t) \le \Theta_i, \forall\ t\ \in\ [0, T]. \tag{4}$$

III. Problem Formulation and Numerical Solution

A. Problem Formulation

Given the energy initially available at the farm, and denoted by $E(0)$, we want to design an energy allocation policy that minimizes the energy cost EC, as defined in (1). Therefore, the decision variables are $E_1(0), \ldots, E_M(0)$, and the optimization problem is cast as follows:

$$\text{P0:} \quad \min_{E_1(0),\ldots,E_M(0)} \quad \mathsf{EC}$$
$$\text{s.t. (1), (2), (4) and } \sum_{i=1}^{M} E_i(0) = E(0).$$

P0 is not a convex optimization problem because its objective is not a function, but a functional, and the inequality (4) states an infinite, and uncountable number of constraints.

B. Numerical Solution

In the following we show how P0 can be cast as a linear program by introducing discretization in time, and linearization to handle the relationship between $X_i(t)$ and $Y_i(t)$.

1) Discretization: In the discrete domain, P0 becomes:

$$\text{P0D:} \quad \min_{E_1(0),\ldots,E_M(0)} \quad \sum_{i=1}^{M} \Delta t \sum_{k=1}^{N} P_i(k\Delta t)\,[L_i(k\Delta t) - Y_i(k\Delta t)]$$
$$\text{s.t. } 0 \le E_i(k\Delta t)\ \le \Theta_i,\ \forall\ k,\ \forall\ i, \text{ and } \sum_{i=1}^{M} E_i(0) = E(0),$$

where we have replaced definite integrals with sums and continuous-time functions with their uniformly spaced samples. The sampling interval is $\Delta t > 0$, and the total number of samples in $[0, T]$ is $N > 0$.

[2]Charging losses are accounted for implicitly.

2) Linearization: The non-linear relationship between $X_i(t)$ and $Y_i(t)$ can be approximated by using a piece-wise linear function. Specifically,

$$Y_i(t) \approx \mathcal{F}_i\left[X_i(t)\right], \quad (5)$$

where $\mathcal{F}_i : \mathbb{R} \to \mathbb{R}$, is the point-wise minimum of a set of Q affine functions:

$$\mathcal{F}_i\left[x\right] = \min_{j\in\{1,\dots,Q\}} \zeta_{i,j}x + \omega_{i,j},$$

where parameters $\zeta_{i,j},\ \omega_{i,j}\ \in\ \mathbb{R}$ can be chosen to minimize the approximation error. The more linear segments are used, the more accurate is the approximation in the region $[\Psi_i, \infty)$.

3) Matrix Formulation: Once discretization and linearization are introduced, the following relaxation can be used to cast P0 as a standard linear program:

$$Y_i(t) \leq \mathcal{F}_i\left[X_i(t)\right]. \quad (6)$$

Constraints $0 \leq E_i(k\Delta t),\ \forall\ k$ can be written as follows:

$$\Delta t \begin{pmatrix} 1 & 0 & 0 & 0 & \dots & 0 \\ 1 & 1 & 0 & 0 & \dots & 0 \\ 1 & 1 & 1 & 0 & \dots & 0 \\ 1 & 1 & 1 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & 1 & \dots & 1 \end{pmatrix} \begin{pmatrix} X_i(\Delta t) \\ X_i(2\Delta t) \\ X_i(3\Delta t) \\ X_i(4\Delta t) \\ \vdots \\ X_i(N\Delta t) \end{pmatrix} \preceq \begin{pmatrix} E_i(0) \\ E_i(0) \\ E_i(0) \\ E_i(0) \\ \vdots \\ E_i(0) \end{pmatrix},$$

where $\preceq$ denotes element-wise inequality. Note that constraints $E_i(k\Delta t) \leq \Theta_i,\ \forall\ k$ can be ignored because the $E_i(t)$'s are non-increasing functions in $[0, T]$, as there are no charging operations once the energy allocation is completed.

With these considerations, P0D can be cast as a linear program. Finding accurate[3] solutions incurs high computational costs, which motivates us to explore alternative approaches to the problem.

IV. Proposed Solution

To obtain an analytical solution to our problem, we decompose P0 into two subproblems: We first design the optimal discharging operations across households, and then, we optimize the energy allocation policy. This strategy follows a master-slave decomposition approach [32].

A. Optimal Discharging Profiles

The first subproblem is formulated to optimize the trajectory $Y_i(t)$, and hence, it can be tackled by using variational techniques [33].

1) Formulation: To optimize $Y_1(t), \dots, Y_M(t)$ we formulate the following optimization problem:

$$\text{P1:} \quad \min_{Y_i(t)} \int_0^T P_i(t)\left[L_i(t) - Y_i(t)\right] dt$$
$$\text{s.t.} \quad 0 \leq E_i(t) \leq \Theta_i, \text{ and } Y_i(t) \leq L_i(t),\ \forall\ t,$$

where $E_i(t)$ is linked to $X_i(t)$ through (2), and $X_i(t)$ is related to $Y_i(t)$ through (3). By solving P1 we optimize $Y_i(t)$ in terms of $E_i(0)$.

[3]The accuracy of its solution is determined by the sampling interval Δt, and the linearization error incurred to approximate (3).

2) Simplifications: As P0, P1 is not a convex optimization problem. P1 also has an infinite number of constraints, as stated in $0 \leq E_i(t) \leq \Theta_i,$ and $Y_i(t) \leq L_i(t),\ \forall\ t$. We can solve P1 numerically. However, to obtain a more insightful result we introduce the following relaxations:

a) We relax (3) to:

$$Y_i(t) = \Psi_i\left[\frac{X_i(t)}{\Psi_i}\right]^{\frac{1}{\alpha_i}}, \quad (7)$$

which incurs an approximation error only when $X_i(t) < \Psi_i$. This error approaches 0, as $\alpha_i \to 1^+$. Practical values of α_i are normally in the range $[1.1, 1.3]$ [14].

b) We relax constraint $Y_i(t) \leq L_i(t)$ to $\int_0^T X_i(t)dt \leq \int_0^T L_i(t)dt$. This relaxation assumes households with the capability to flexibly utilize assigned RE during the planning period.

P1 can be written in terms of $X_i(t)$, and with the considerations explained above, it simplifies to:

$$\text{P2:} \quad \min_{X_i(t)} \int_0^T P_i(t)\left[L_i(t) - \Psi_i\left[\frac{X_i(t)}{\Psi_i}\right]^{\frac{1}{\alpha_i}}\right] dt$$
$$\text{s.t.} \quad \int_0^T X_i(t)dt = E_i(0).$$

In P2, we have substituted the constraint (2) with $\int_0^T X_i(t)dt = E_i(0)$, as no charging operations occur in $[0, T]$. Note that $E_i(t)$ is non-increasing in t, hence, it follows that $E_i(t) \leq \Theta_i\ \forall\ t$, and constraint $E_i(t) \geq 0$ implies $\int_0^T X_i(t)dt = E_i(0)$, as it is clearly suboptimal to use less energy than the one allocated to each household.

3) Solution: To solve P2 we use the Euler-Lagrange optimality condition [33]. Specifically, optimal $X_i(t)$ must satisfy the following differential equation:

$$P_i(t)\frac{\partial}{\partial X_i}\left[\Psi_i\left[\frac{X_i(t)}{\Psi_i}\right]^{\frac{1}{\alpha_i}} - \lambda X_i(t)\right] = 0, \quad (8)$$

where $\lambda \in \mathbb{R}$ is the Lagrange multiplier. Eq. (8) yields the following family of candidate solutions:

$$X_i^*(t) \triangleq \Psi_i\left[\frac{P_i(t)}{\lambda}\right]^{\frac{\alpha_i}{\alpha_i - 1}}, \quad (9)$$

where λ can be chosen to comply with $\int_0^T X_i(t)dt = E_i(0)$:

$$\lambda = \left[\frac{E_i(0)}{\Psi_i \int_0^T \left[P_i(t)\right]^{\frac{\alpha_i}{\alpha_i - 1}} dt}\right]^{\frac{1-\alpha_i}{\alpha_i}}. \quad (10)$$

Substituting (10) and (9) into (7) and (1) yields:

$$\mathsf{EC}^*$$

$$= \sum_{i=1}^{M}\left\{\int_0^T P_i(t)L_i(t)dt \right. \quad (11)$$
$$\left. - \left[\Psi_i \int_0^T \left[P_i(t)\right]^{\frac{\alpha_i}{\alpha_i - 1}} dt\right]^{\frac{\alpha_i - 1}{\alpha_i}} E_i(0)^{\frac{1}{\alpha_i}}\right\}.$$

B. Optimal Energy Allocation

The second subproblem is formulated to optimize the energy allocation across participants. Formally, we cast the following optimization problem to optimize $E_1(0), \ldots, E_M(0)$:

$$\text{P3:} \quad \min_{E_1(0),\ldots,E_M(0)} \quad \mathsf{EC}^*$$
$$\text{s.t.} \sum_{i=1}^{M} E_i(0) = E(0),\ E_i(0) \leq \Theta_i, \text{ and}$$
$$E_i(0) \leq \int_0^T L_i(t)dt \ \forall\ i.$$

P3 is convex if $\alpha_i > 1$, $\Psi_i > 0$, and $P_i(t) \geq 0\ \forall\ t\ \forall\ i$. Moreover, candidate solutions can be found by using the KKT conditions. When the following conditions are met: $E(0) < \int_0^T L_i(t)dt$, $\Theta_i \geq E(0)$, and $\alpha_i = \alpha\ \forall\ i$, a closed-form solution can be obtained:

$$E_i(0) = E_i^*(0) \triangleq \frac{\left(\frac{1}{\eta_i}\right)^{\frac{\alpha}{1-\alpha}}}{\sum_{j=1}^{M}\left(\frac{1}{\eta_j}\right)^{\frac{\alpha}{1-\alpha}}} E(0),\ \forall\ i, \tag{12}$$

where $\eta_i = \left[\Psi_i \int_0^T [P_i(t)]^{\frac{\alpha}{\alpha-1}}\, dt\right]^{\frac{\alpha-1}{\alpha}}$. As seen, the household with the largest $\left[\Psi_i \int_0^T [P_i(t)]^{\frac{\alpha}{\alpha-1}}\, dt\right]$ will take the largest share of energy. Hence, the household offered the highest electricity fees, and whose battery has the largest nominal output power, will take the largest share of $E(0)$. If the prices were the same across households, then the allocation criterion would be determined entirely by the nominal output power of each battery (Ψ_i).

V. Numerical Results

This section is divided into four parts. First, we illustrate the significance of the non-linear discharging model by comparing our results against those obtained with existing approaches based on linear ESS models. In the second part, we compare the solutions obtained with the proposed numerical and analytical methods. In the third part, we evaluate the proposed strategy in terms of battery parameters such as rated output power and efficiency. In the fourth part, we show the optimality of the proposed allocation policy by plotting the achievable cost savings against alternative policies. Simulation parameters are summarized in Table I.

To ease comparisons, we consider batteries with the same characteristics[4] across all households, i.e., $\Psi_i = \Psi$, $\alpha_i = \alpha,\ \forall\ i$. The performance metric used is cost savings CS, defined as the difference between the energy cost incurred when $E(0) = 0$, and the optimized energy cost (EC^*):

$$\mathsf{CS} = \sum_{i=1}^{M}\left\{\left[\Psi_i \int_0^T [P_i(t)]^{\frac{\alpha_i}{\alpha_i-1}}\, dt\right]^{\frac{\alpha_i-1}{\alpha_i}} E_i(0)^{\frac{1}{\alpha_i}}\right\}. \tag{13}$$

[4]Nominal output power and discharging efficiency rate.

TABLE I
SIMULATION SCENARIOS

Parameter	Value
$\{T,\ \Delta t,\ M,\ \Psi\}$	$\{1,\ 0.01,\ 2,\ 1\}$
$P_1(t)$	$\sin(7t) + 2$, or $\sim \mathcal{U}(0,1)$
$P_2(t)$	$\cos(7t) + 2$, or $\sim \mathcal{U}(0,1)$
$L_i(t)$	$\sim \mathcal{U}(0,1),\ \forall\ i \in \{1,\ldots,M\}$
Q	21

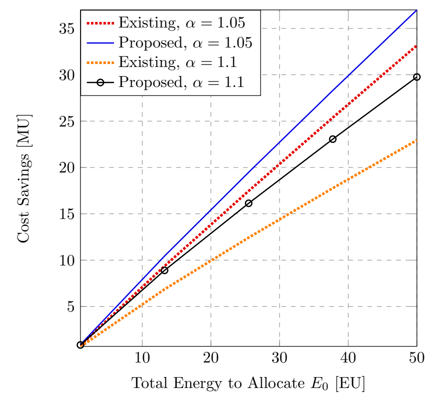

Fig. 2. Comparison with existing approaches based on linear ESS models. Performance loss increases with α.

A. Linear vs. Non-linear ESD model

We consider the scenario shown in Table I with random pricing signals, and plot the results obtained in Fig. 2. As seen, the proposed strategy outperforms existing solutions, in particular as α_i deviates from 1. As α_i increases, discharging losses are more significant, and ignoring them results in a more prominent performance degradation.

B. Numerical vs. Analytical Solution

We consider the simulation scenario summarized in Table I with deterministic pricing signals. We then plot the discharging profiles obtained with the numerical approach and the proposed strategy in Fig. 3. The accuracy of the solution obtained through discretization and linearization depends on the discretization step and the number of linear segments used to approximate (3). It is observed that the numerical approach is more accurate when the optimized value of $X_i(t)$ is below 10 [PU]. This follows because the number of linear segments used to approximate (3) is larger for $X_i(t) < 10\Psi_i$, as unitary nominal output power has been assumed. As seen, the discharging schedules track the pricing signals.

Now we consider the simulation scenario summarized in Table I, with random pricing signals uniformly distributed between 0 and 1. We then plot the average cost savings obtained with the proposed strategy and the numerical approach in Fig. 4. As seen, the two strategies achieve very similar performance across different values of $E(0)$.

C. Impact of Battery Parameters on Performance

We consider random pricing signals together with the simulation scenario in Table I, and plot the average cost savings obtained in Figs. 5 and 6, for different values of α

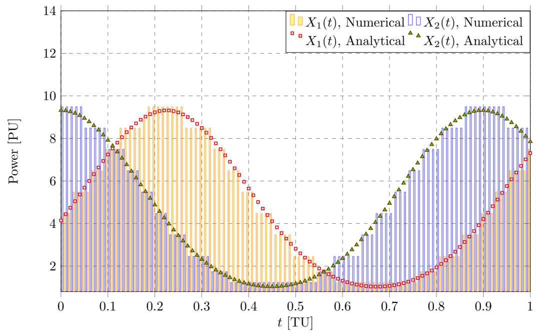

Fig. 3. Discharging schedules obtained numerically and analytically with $E(0) = 10$[EU]. Mismatch depends on discretization step and number of linear segments used to approximate Eq. (3).

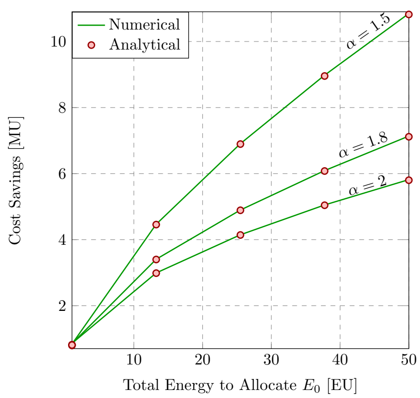

Fig. 4. The proposed strategy achieves nearly the same performance as the more computationally-expensive numerical approach.

and Ψ. As seen in Fig. 5, smaller α leads to higher cost savings. As α grows, the losses incurred in the discharging operation increase. In this scenario, the discharging power is above the battery's rated output Ψ for most of the planning horizon. When $X_i(t) > \Psi$ the power loss PL incurred in the discharging operation is given by $\mathsf{PL} = \Psi_i \left[\frac{X_i(t)}{\Psi_i}\right]^{\frac{1}{\alpha_i}} - X_i(t)$. Consequently, larger values of Ψ (i.e., Ψ_i) lead to smaller power loss and better performance. This is also shown in Fig. 6, where different values of Ψ have been considered.

D. Optimality of Proposed Energy Allocation Policy

We consider the simulation scenario summarized in Table I, with random pricing signals uniformly distributed between 0 and 1. Then, while enforcing $E_1(0) + E_2(0) = 10$, we plot in Fig. 7 the average cost savings obtained with the proposed strategy for different values of $E_1(0)$. As seen in Fig. 7, the highest performance is obtained when the energy allocation strategy is the one stated in (12). Again, we see that smaller values of α lead to better performance. Moreover, optimizing the energy allocation policy is more critical when $\Psi = 3$ and $\alpha = 2$. This follows because the concavity of CS increases with α.

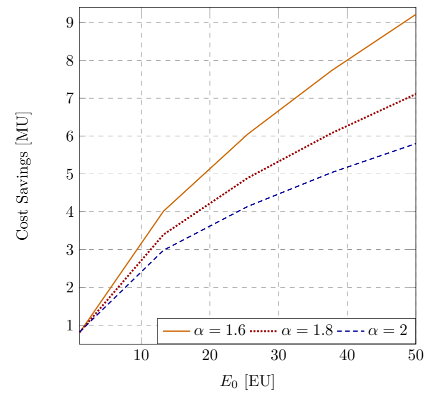

Fig. 5. Impact of battery parameters on performance with $\Psi = 1$. The closer is α to 1^+, the higher are the cost savings. The concavity of the curve follows from the losses incurred in the discharging operation.

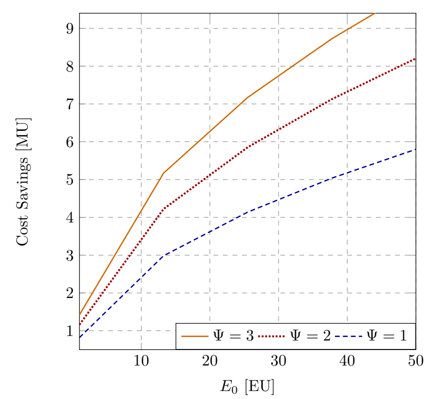

Fig. 6. Impact of battery parameters on performance with $\alpha = 2$. The larger is Ψ, the higher are the cost savings. Again, the concavity of the curve follows from the losses incurred in the discharging operation.

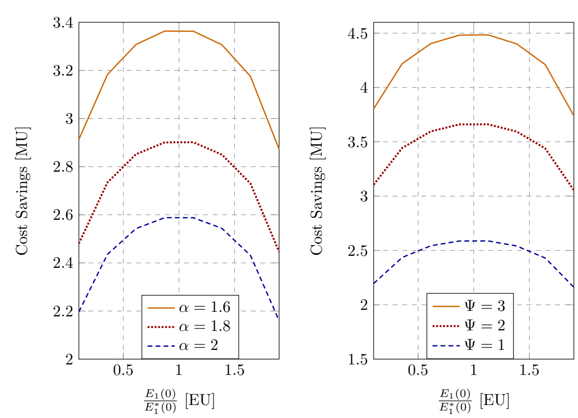

Fig. 7. Optimality of the proposed energy allocation policy. Larger Ψ and smaller α lead to better performance.

VI. CONCLUSIONS

We have proposed a policy for energy allocation across households with shared access to a renewable energy farm. Location- and time-dependent electricity prices have been considered for generality. The proposed strategy minimizes the collective energy expenditure incurred by a group of cooperating households over a finite planning horizon. The proposed optimization framework accounts for non-linear discharging losses across participating storage units.

We have used calculus of variations to solve a relaxed version of the optimization problem in closed form. The solution encompasses optimal discharging schedules and the corresponding energy allocation policy. By using these results we have derived a mathematical expression to estimate the cost savings achieved with the proposed strategy over a finite planning period.

Simulations showed that the proposed strategy achieves nearly the same performance as the more computationally-expensive numerical approach. We also assessed the performance of the proposed setup in terms of battery parameters such as efficiency rate and nominal output power. The results presented in this paper can be used to reduce the computational complexity of energy optimization strategies involving storage management. The performance estimate derived can be used to assess the potential of cooperative optimization in communities with shared energy generation infrastructure.

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