Nesset, E., Solli-Sæther, H. & Strand, Ø. *Bidrag innen kundeverdi og marked. Festskrift til Øyvind Helgesen.* Oslo: Universitetsforlaget.
DOI: https://doi.org/10.18261/9788215055596-2021-07

7. Utilizing structural models to evaluate probability of default for Norwegian stock-based firms

Bendik Persch Andersen, Mathilde Rundhaug and Petter Eilif de Lange

Abstract Structural credit default models are traditionally applied to publicly traded companies operating in highly liquid markets. In this study, we apply two option-based models to *non-listed, relatively illiquid, privately held, Norwegian companies*. By introducing sector-specific reorganization boundaries, we consider the observed asset-to-debt ratios at *actual* default for companies with illiquid tangible assets, providing convincing results for both models.

Keywords structural credit default models | reorganization boundaries | volatility measures

7.1 INTRODUCTION

Credit risk is, broadly speaking, the risk of loss due to default on contractual obligations. There are numerous models used for computing a firm's theoretical probability of default, which have been applied and validated for decades. Generically these models fall into one of two broad categories: structural models and reduced factor models. The majority of the models are primarily applied to publicly traded firms in highly liquid markets. The aim of this study is to develop a quantitative framework for calculating credit risk in the Norwegian market for relatively illiquid, privately held, stock-based firms. Our approach is of course applicable to any illiquid corporate market.

On the 27th of July 2018, *Nordic Credit Rating* (NCR) became a *European Securities Markets Authority* (ESMA)-registered rating agency, with the aim of providing credit ratings in the Nordic market, reflecting local risk factors. Its rating process consists of qualitative and quantitative analysis, reflecting both systematic and idiosyncratic risk factors. NCR aims at employing a combination of objective evaluation parameters while exploiting local market expertise. The quantitative framework for credit risk we develop in this chapter is intended to serve as an integral part of NCR's rating process. We compare the performance of two acknowledged structural credit default models, originally derived by Merton (1973) and Black and Cox (1976), when applied to empirical data on defaults in the Norwegian market of privately held stock-based firms, the majority of which are non-listed.

As far as we know, there are almost no other studies which apply structural default models to privately held, non-listed companies. Law and Roache (2015) examine the information content in estimated default rates of a variety of Chinese corporates, including non-listed companies, employing both structural and reduced form models. They conclude that structural credit models that estimate the stand-alone one-year probability of default can be usefully applied in China. They further find that these default probabilities have provided signals of increased financial stress for some firms, including the first onshore corporate bond default. They also find that these probabilities respond in intuitively and quantitatively sensible ways to changes in a firm's fundamentals, including profitability and balance sheet strength.

The main contributions of this chapter are the following: (i) the construction of sector-specific reorganization boundaries taking account of observed asset-to-debt ratios at actual default across sectors; (ii) estimating volatility measures for non-listed companies; and (iii) calibrating and implementing modified versions of the Merton (1973) and Black and Cox (1976) model on a unique data set comprising more than 100,000 non-listed Norwegian companies, obtaining empirical default frequencies, and finding that both models perform convincingly well. Utilizing empirical data, our framework yields empirical default frequencies which can be exploited in a number of ways, for example, calculating theoretical credit spreads.

In the literature, there are various definitions of *default*, but in essence, a firm is generally defined as bankrupt as soon as the value of its liabilities exceeds the value of the company, often expressed in terms of the market value of its assets. In reality, however, a firm is usually allowed to keep its operations going as long as it is able to meet its financial obligations. This implies that the actual value of the assets at default has often fallen short of the value of the debt by a considerable amount. In this study, we amply demonstrate the importance of including this characteristic when modeling the probability of default. Furthermore, we recognize that the asset-to-debt ratio at default, often referred to as the *reorganization boundary*, deviates substantially across a range of sectors.

Another crucial input in structural models is the *volatility* of the firm's asset value. For publicly traded firms, the volatility of outstanding shares can be estimated in numerous ways, usually from either daily stock returns or by calculating implied volatility from option prices on the underlying stock. The fact that we, in this study, are modeling credit risk for privately held firms implies that neither of these approaches is applicable. Instead, we derive firm-specific volatility estimates by incorporating both sector- and firm-specific measures, in an attempt to reflect both systematic and idiosyncratic risk factors.

The rest of this study is organized as follows: In section 7.2 we present a review of relevant literature on the topic of credit default models. Section 7.3 provides an overview of data and statistics incorporated in our research. Section 7.4 outlines the methods we have applied. In section 7.5 we present and discuss our findings. Finally, in section 7.6, we conclude on our findings and provide a brief discussion on further research. Three appendices provide further mathematical details on parameter estimations and relevant statistical tests. The appendices can be accessed via the following link: https://www.ntnu.no/documents/1265701259/1281473463/Appendices.de.lange_Rundhaug_Andersen_ProbabilityOfDefault.pdf/f37b4b4d-f797-ea50-d4e1-69f63cbbb2a5?t=1629188415731

7.2 LITERATURE REVIEW

For decades, a lot of attention in financial literature has been devoted to modeling default risk, which has resulted in a variety of models. There is, however, no general perception of a superior model for all purposes. To a large extent, this is due to the fact that risk factors vary substantially across both sectors and markets. Still, a rather limited number of models are certainly more applied than others. The majority of models fit into one of two generic categories: structural or reduced-form models. The structural models utilize theoretical differential equations describing the evolution of endogenous variables, whereas, for the reduced-form models, the system of equations is already solved for the endogenous variables. Arora, Bohn, and Zhu (2005) assess both the reduced-form approach and the structural modeling approach by investigating three acknowledged models on corporate default risk: the two structural models known as the *Merton* and *Vasicek-Kealhofer* (VK) models and the one reduced-form model known as the *Hull-White* model. Based on cross-sectional variations on credit default swap spreads, the robustness of the models, and their ability to predict default, they conclude that the structural models outperform the reduced-form model.

One of the most commonly used structural models is the previously mentioned Merton model (Merton, 1973). The model regards a firm's equity as a European call option on its underlying assets with a strike price equal to its liabilities and utilizes the Black and Scholes (1973) option pricing formula. The model is often considered a benchmark among credit default risk models, due to both its simplicity and its accuracy. Feldhutter and Schaefer (2018) argue that the model's ability to estimate default probabilities is adequate as long as a sufficiently long time horizon is considered. However, it has its limitations beyond the need for a long time horizon. The model is built on the assumption that a firm's liabilities consist of one zero-coupon bond only, maturing at the end of the considered period. This implies that one is only modeling the risk that the firm is not able to fulfill its financial obligations at maturity, neglecting the fact that a firm, in reality, can default at any point in time. Additionally, as a consequence of the assumption of a constant debt structure, it implicitly assumes a decrease in leverage over time because of the assumed expected growth rate $\mu > 0$ of the firm's assets. In most cases this is a highly unrealistic assumption (Breccia, 2012). The inconvenience of these limitations has led to several extensions of the model.

One extended model is the VK model, which is based on a perpetual barrier option, allowing for default at any point in time. In addition, the model allows for a richer firm structure compared to the assumption of one zero-coupon bond only in the Merton model. Another model, the Black and Cox (1976) model, is closer to the Merton model in its assumptions, also assuming one zero-coupon bond only, with maturity at the end of the considered time period. It does, however, model the firm's equity as a knock-out barrier option, allowing for default at any point in time up until maturity. In theory, this model should yield more realistic default measures than the Merton model, partly compensating for the Merton model's tendency to underestimate the actual default risk (Arora et al., 2005). In practice, however, the Black and Cox model is less applied than the Merton model, due to its complexity and also because it is unclear whether the introduction of a safety covenant does, in fact, contribute to improvements (Kovacova & Kollar, 2018).

Moody's, one of the largest credit rating agencies in the United States, has implemented a version of the VK model in order to produce expected default frequencies. This method, also known as the *Moody's Kealhofer-Merton-Vasicek* (MKMV) model, is based on the calculation of a distance-to-default measure to obtain firms' default probabilities. In general, many practitioners consider the distance-to-default strategy to be a good approach as part of a modeling framework, preferably combined with empirically inferred default frequencies. For financial institutions, however, Chan-Lau and Sy (2006) argue that the probability of default is better modeled by the use of *distance-to-capital*[1], as a consequence of the fact that these

1 The distance-to-default approach assumes that all equity capital can be used to fulfill financial obligations, which is not the case for financial institutions, primarily due to industry-specific capital requirements.

institutions are subject to unique structures and regulations, such as pre-default regulatory actions.

There are other modeling approaches which we do not consider in this study. For instance, Fruhwirth and Sogner (2006) employ the Jarrow/Turnbull continuous time reduced form model, estimating default intensities for German bank and corporate bond prices. They find, among other things, that a joint implicit estimation of default intensities and recovery rates is numerically unstable.

Over the last few decades, *machine learning* (ML) algorithms, or more generally AI models, have made their way into the field of applied finance, and many of these algorithms have been employed for credit default predictions with promising results. Chen and Guestrin (2016) provide a nice introduction to tree boosting algorithms, which are among the most popular for default predictions. We think that AI models in banking and finance will increase in importance going forward. At present, the main challenge with AI models is that it is difficult to explain the economic intuition behind the output of the models, that is, a lack of transparency. This is also the reason why we do not apply ML algorithms in this study. Since transparency is like a holy grail in the credit rating industry, credit rating agencies are not yet ready to fully incorporate these algorithms into their credit rating processes.

When modeling credit risk in the Norwegian market, liquidity is an important topic in as much as the Norwegian stock market is generally a relatively illiquid market. Modeling credit spreads on contingent convertible bonds issued by two Norwegian banks, de Lange, Stiberg, and Aamo (2019) emphasize the fact that liquidity issues are not captured by the Merton model. As we are modeling credit risk for non-listed privately held firms, it is crucially important that the model account for liquidity premiums. We account for this by introducing a sector-specific expected asset-to-debt ratio at default, also known as the *reorganization boundary* (Mora, 2012), as well as sector- and firm-specific volatilities. Arora et al. (2005) find empirical evidence showing that the liquidity premium is implicitly incorporated in the reorganization boundaries, which will be further discussed in subsequent sections.

In the literature as well as in practice, structural models are widely applied. The models are, however, primarily utilized in evaluating the evolution of risk for publicly traded companies, and almost exclusively in highly liquid markets. In our study, we are expanding existing research on this topic by applying both the Merton model and the more complex Black and Cox model on Norwegian privately held firms, which are neither publicly traded nor part of a highly liquid market. As noted above, apart from a study of Chinese companies, some of which were non-listed (Law & Roache, 2015), we are not aware of any former studies applying these models to non-listed, privately held companies. In the spirit of the MKMV approach, we combine theoretical values with empirical data, yielding a framework which not only is able to differentiate a firm's credit risk but also comprises empirically inferred risk measures for each firm. We are incorporating Norwegian market characteristics in the model parameters only, implying that our approach is also applicable to privately held firms outside of Norway.

7.3 DATA SOURCES

Utilizing empirical data is an essential part of this analysis, both for parameter estimation and for model evaluation. The majority of data applied in our analysis are contained in two distinct data sets, collected for different purposes. We perform relevant statistical tests on the data, which we describe successively.

7.3.1 Norwegian stock-based companies

The core of our analysis is the development of a structural credit default model, intended to form an integral part of the credit rating processes of NCR, a Nordic credit rating agency. More specifically, our model is calibrated on data comprising Norwegian stock-based firms, that is, privately held firms with limited liability known as *aksjeselskap*. A crucial part in the evaluation of this model is the acquisition of both reliable and sufficient company data. Sufficient data on privately held firms are often either difficult to retrieve or prohibitively expensive. We have, however, been able to obtain comprehensive data comprising both accounting and default data on all Norwegian stock-based firms for the five-year period 2014–2018[2]. The data have been collected from a database compiled by *Proff Forvalt*[3], initially encompasing 250,482 firms. However, in order to obtain reliable data, we omitted a considerable number of companies on the following criteria:

1. All companies that are not registered in the Norwegian register for value-added taxes (VAT) are omitted. Companies that are not registered have an annual VAT relevant revenue of less than NOK 50,000. This means that the companies are either inactive, start-ups or active within sectors that are exempt from VAT and would likely yield unreliable results.

2 01.01.2014–01.11.2018. Default events for November and December 2018 are for convenience considered negligible.

3 www.forvalt.no

2. All companies with less than two years of activity are omitted, that is, companies registered later than 01.01.2012. This is due to the fact that more than 55% of all companies either are deleted, are inactive or have defaulted within one year of registration (Statistics Norway, 2018). In this study, we are only concerned with probabilities of default for well-established companies; including start-up companies could possibly yield misleading results.
3. Companies with an asset value of less than NOK 500,000 are omitted, under the same argument as stated for criteria (2).
4. Companies operating within sectors in which there are no companies listed on the *Oslo Stock Exchange* (OSE) are omitted, as listed firms play a vital part in our estimations of sector-specific parameters. Additionally, companies within the financial sector are also left out of this analysis. This is because the financial sector is subject to a set of unique regulations, such as capital requirements, implying that the sector should be evaluated separately (Bharath & Shumway, 2008; Chan-Lau & Sy, 2006).

After eliminating companies according to the above criteria, we were left with a set of 101,257 firms, out of which 3,060 had defaulted over the time period 01.01.2014–01.11.2018. Relevant figures for these companies are debt and asset value from the end of 2013. For defaulted companies, we have also collected debt and asset values at default, which will be utilized in the estimations of the expected asset-to-debt ratio at default described in section 7.4.2. Furthermore, we are utilizing the firms' NACE[4] codes to separate them by sector. Individual firms within this data set are hereafter referred to as firm f being part of the set of privately held firms F, that is, $f \in F$. On the basis of this data set, we are building a framework for modeling the five-year probability of default.

7.3.2 Companies listed on the Oslo Stock Exchange

Several crucial parameters are considered in the calculation of the theoretical probability of default, including the volatility of the firm's assets. As we further discuss below, the volatility of privately held firms cannot be estimated using traditional methods for volatility estimation. Instead, our firm-specific volatility estimates are partly based on sector-specific figures, in an attempt to capture some of the systematic risk. These sector-specific volatility estimates are in turn based on long-term

4 Nomenclature of Economic Activities (NACE) is the European classification of economic activities

volatility predictions for companies listed on the OSE. For this purpose, we have collected adjusted closing prices[5] for publicly traded companies, covering the period 01.01.2011–31.12.2013, from which we will derive sector-specific estimates for the five subsequent years.[6] Preferably, we would have obtained data for a longer time period than three years in order to fully capture long-run characteristics, but the choice of time span was the result of a trade-off between the number of listed companies included and the length of the interval, simply because the listed firms are a non-static pool. The data have been collected from Yahoo Finance and include a total of 96 publicly traded companies, operating within 11 sectors. The companies within this data set are hereafter referred to as listed companies l being part of the set of listed companies L, that is, $l \in L$.

7.3.3 Sector overview

The firms, both privately held and publicly traded, are divided into the following sectors presented in Table 7.1, for which we will derive unique sector-specific parameters.

Table 7.1: Overview of both listed and privately held firms within each of the 11 sectors. The first column depicts the different sectors represented on the OSE. The second column shows the number of listed firms within each sector, whereas the two rightmost columns depict the total number of privately held firms and how many of these have defaulted, respectively.

Sector	$\text{Listed}_{\in F}$	$\text{Private}_{\in F}$	$\text{Defaults}_{\in F}$
A – Seafood	9	1884	26
B – Energy	31	580	10
C – Industry	7	7059	291
D – Supply	1	804	9
E – Water industry	2	442	7
F – Entrepreneur	2	14656	661
G – Retail	2	31741	1454
H – Transport	10	4754	121
J – Communication and IT	21	16841	295
L – Real Estate	6	21699	176
Q – Health	5	797	10
	96	101257	3060

5 The daily closing prices, adjusted for any dividends, new stock offerings and stock splits.

6 01.01.2014–31.12.2018

7.4 METHODOLOGY

The core of this analysis is to calculate probabilities of default on the set of Norwegian stock-based firms. We perform these calculations utilizing the two models described in the following sections, testing whichever performs better. Both models are frequently applied to listed companies, either by utilizing implied volatilities in the presence of liquid option markets or by applying various volatility models to the individual firm's stock returns. In this study, however, we are building a framework for forecasting the probabilities of default for privately held firms, implying that the future volatility of each firm cannot be estimated by using option prices or stock returns. Instead, we are applying sector- and firm-specific measures for the estimates, which we further describe in subsequent sections.

7.4.1 Default models

7.4.1.1 Distance-to-default

The first default model is based on the Merton model, calculating the distance to default, hereafter referred to as the DD model. The foundation of this model is the consideration of the firm's equity as a call option on the underlying unobservable value of the company, with a strike price equal to the face value of the firm's debt (Bharath & Shumway, 2008). In the literature, the concept of the unobservable value of the company is used interchangeably with the concept of the equally unobservable value of firm assets, including intangible assets. It has no practical significance for our analysis whatsoever, whether one tries to estimate the unobservable value of the company or the equally unobservable value of the company's assets. The model rests on the following assumptions:

1. Each company's asset value follows a Geometric Brownian Motion process, given by $dA(t)/A(t) = (\mu - \delta)dt + \sigma dZ(t)$, where μ is the expected rate of return, δ is the dividend, σ is the volatility of the firm's assets and dZ is a Wiener Process. In our analysis, we are not including ex ante beliefs on future growth rates for either individual firms or sectors, and both μ and δ are naively set to zero for all firms.
2. The debt structure of individual firms is assumed to be constant through the time interval $[t, T]$, consisting of one zero-coupon bond only maturing at time T.

The Black-Scholes-Merton formula (equation 1), which expresses the value of the firm's equity, thus applies. In this formula, and throughout the rest of our analysis, *A* is the firm's asset value, *D* is its debt value, *r* is the risk-free interest rate, *σ* is the volatility of the firm's assets and *N* is the normal cumulative distribution.

$$E = AN\left(d_1\right) - e^{-rT} DN\left(d_2\right) \tag{1}$$

where

$$d_1 = \frac{ln\left(\frac{A}{D}\right) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$

and

$$d_2 = d_1 - \sigma\sqrt{T}$$

In other words, the equity value *E* of the firm is expressed as the firm's asset value in excess of the properly discounted face value of debt. The d_1 term expresses the assets' distance to the value of the debt, relative to its volatility, that is, measured in units of standard deviations. Moody's KMV approach utilizes this measure as a way of categorizing firms' credit risks by their respective distances to default and combines this measure with a substantial amount of empirical data to obtain an empirically inferred default frequency. To some extent, we apply the same approach in this study. However, in order to evaluate the DD model against the *Black and Cox* (BC) model presented below, we need a measure for the theoretical probability of default. The first model assumption, that the asset value follows a Geometric Brownian Motion process, means that the incremental changes of asset values are normally distributed. Based on this assumption, the probability of default is given by the normal cumulative distribution as:

$$PD_{DD} = N\left(-DD\right) \tag{2}$$

where

$$DD = \frac{ln\left(\frac{A}{K}\right) + \left(\mu - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} \tag{3}$$

In the above equation, *K* is the preassigned asset value at default, which consists of current debt value and the reorganization boundary derived in section 7.4.2.2.

7.4.1.2 Black and Cox

One limitation of the Merton model is the fact that it only calculates the probability of default exactly at maturity. An extension to Merton's model, and an attempt to address this limitation, is the BC model (Black & Cox, 1976), built on the foundation of a knock-out barrier option model. This is a first-time-passage model, not only calculating the probability that the company will not default at maturity, but also accounting for the risk that the firm might default over the time interval $[t,T]$ prior to maturity, by introducing a safety covenant. A safety covenant permits bondholders to force bankruptcy if certain contractual conditions are met. Black and Cox modeled such a safety covenant as an exogeneous time-dependent reorganization boundary. The probability that the asset value exceeds the value of the debt at maturity, and has not reached its reorganization boundary in the meantime, is given by:

$$PS_{BC} = N(u_1) - \left(\frac{A}{Ke^{-\gamma(T-t)}}\right)^{1-\left(2(\mu-\delta-\gamma)/\sigma^2\right)} N(u_2) \tag{4}$$

where

$$u_1 = \frac{lnA - ln(D) + \left(\mu - \delta - \frac{1}{2}\sigma^2\right)(\tau - t)}{\sqrt{\sigma^2(\tau - t)}}$$

and

$$u_2 = \frac{2lnKe^{-\gamma(T-t)} - lnA - ln(D) + \left(\mu - \delta - \frac{1}{2}\sigma^2\right)(\tau - t)}{\sqrt{\sigma^2(\tau - t)}}$$

where γ is the discount rate on the safety covenant, τ the continuous time $\tau \in [t,T\rangle$, δ the continuous dividend yield, and K the preassigned asset value at default. The discount rate is assumed firm-independent and is set to 8% as a weighted average cost of capital. The choice of 8% as an appropriate discount rate is also confirmed by Koziol (2013).

The BC model comprises two main components: $N(u_1)$ denotes the probability of the asset value A not falling below some preassigned threshold, at time T. This term is equivalent to the complement[7] of the default probability of the Merton model and can be interpreted as the probability of not defaulting at maturity. The

7 $PD^c_{DD} = 1 - PD_{DD}$

second part includes the probability of the asset value not reaching the default barrier over the time interval $[t,T\rangle$, that is, the probability of not defaulting before maturity. The barrier is a time-dependent measure which at time τ is given by $K = e^{-\gamma(T-\tau)}$, which is simply a properly discounted value of the liabilities. The probability of default is thus given as $PD_{BC} = 1 - PS_{BC}$.

7.4.2 Sector-specific reorganization boundary

An important parameter in our analysis is the preassigned expected asset-to-debt ratio at default, hereafter referred to as the *reorganization boundary*. In theory, the market value of assets at default equals the value of debt. In reality, however, this need not to be the case since a firm is generally allowed to keep on going as long as it is able to meet its financial obligations in terms of periodic payments. When modeling the probability of default, the reorganization boundary is often set equal to the sum of short-term liabilities plus half the long-term liabilities (Bharath & Shumway, 2008). In terms of the asset-to-debt ratios, the reorganization boundary is often assumed to be sector independent and is set equal for all firms.

We believe, however, that the observed default boundaries may vary across sectors, which was also found by Mora (2012) and Altman and Kishore (1996). The reason for this belief is partly the fact that certain sectors tend to be more asset-heavy than others. This means that there are differences in the underlying book value of the firm's tangible assets as a fraction of the value of total assets. In addition to the book value of the tangible assets, the liquidity of asset markets is likely to affect the creditor's desire to file a motion for default. Compare, for instance, the Norwegian shipping and real estate markets over the last decade. While the Norwegian real estate market has been highly liquid, the overall shipping market, and particularly the dry bulk market, has experienced historically low freight rates and weak liquidity conditions. In practice, this means that it has been undesirable for creditors to acquire a fleet consisting of several ships in an illiquid shipping market, compared to highly tradable real estate assets. Our belief is therefore that a sector-specific reorganization boundary and sector- and firm-specific volatilities would reflect some of these differences, yielding an implicitly given liquidity premium as well as capturing variations in bankruptcy costs.

7.4.2.1 Analysis of variance

Before estimating sector-specific reorganization boundaries, we wish to test whether our assumption of sector variations is reasonable. We do this by analyzing variances on the observed asset-to-debt ratios at default within each sector. A broadly used approach for this kind of analysis is a *one-way analysis of variance* (ANOVA). This approach, however, is based on the assumption of normally distributed data within each sector, a qualification which needs to be confirmed prior to the analysis. In appendix A.2, we show that both the Shapiro-Wilk test (Shapiro & Wilk, 1965) of approximate normality and the Q-Q normal plots indicate that the reorganization boundaries presumably are not normally distributed, thus the assumption of normality is not fulfilled. Instead, we apply the Kruskal-Wallis test (Kruskal & Wallis, 1952), which utilizes the ranks of the reorganization boundaries rather than the actual values. In appendix A.3 we show that the null hypothesis is rejected, meaning that our hypothesis of sector-specific reorganization boundaries is likely to hold, and we proceed with our computations.

7.4.2.2 Empirical Bayes method on reorganization boundaries

The sector-specific estimates of reorganization boundaries are based on empirical asset-to-debt ratios at default, for defaulted firms in the data set $f \in F$. Due to the data scarcity within certain sectors, the empirical average alone does not yield robust estimates. To reduce the consequences of data scarcity, we therefore apply the empirical Bayes method.

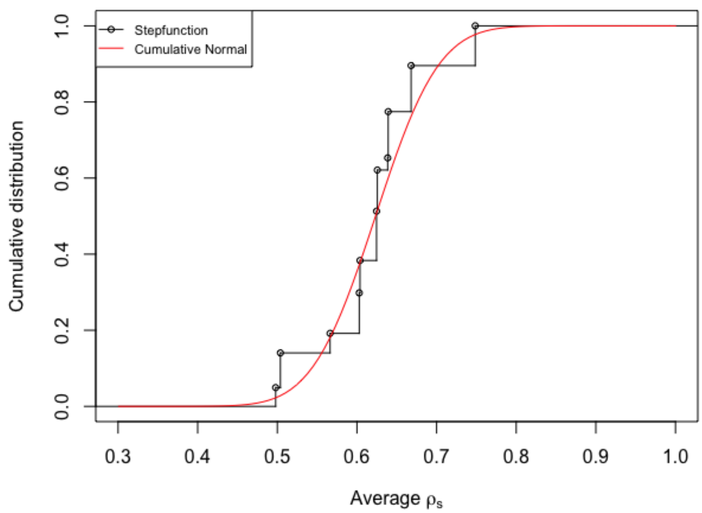

Figure 7.1: Mean reorganization boundaries for each of the sectors represented in a step function, with fitted cumulative normal distribution given by the red line. Each step represents the sample mean reorganization boundary found for each sector, with the step height equal to the multiplicative inverse of the sample standard deviation.

Our sector independent prior is based on the empirical means for all sectors. We obtain the prior distribution employing a step function with step height $h = 1/\rho_s^*$, where ρ_s^* is the sample standard deviation on the set of asset-to-debt ratios at default in sector *s*.

We then proceed to find the best fitted normal cumulative distribution, $\rho \sim N\left(m, \xi^2\right)$[8], using *mean squared error* (MSE), depicted in Figure 7.1.

Next, we have the likelihood function, which best describes the observed individual reorganization boundaries. Although the Shapiro-Wilk test concludes that the observed reorganization boundaries are not likely to be normally distributed, we still find the normal distribution to be the best fitting distribution. This finding is evident in Figure 7.2, showing the Q-Q plots for some of the best fitting distributions. Assuming that the reorganization boundaries are, in fact, approximately normally distributed, the likelihood function is given by the joint probability density functions for each observation $\boldsymbol{y} = \left(y_1, \ldots, y_n\right) \sim N\left(\rho, \zeta^2\right)$. Ultimately, this yields a posterior normal distribution, from which the mean parameter, ρ_S, serves as our estimate on the sector-specific reorganization boundary (see appendix A.4 for derivations) for sector *s*. This estimate is given by:

$$\rho_s = \frac{m\zeta^2 + n_s \bar{y}_s^{\,2}}{\zeta^2 + \frac{2}{s} n_s} = \frac{\frac{\frac{m}{2} + \frac{n_s \bar{y}}{\zeta^2}}{s}}{\frac{\frac{1}{2}}{s} + \frac{n_s}{\zeta^2}} \tag{5}$$

where n_S is the number of defaults and $_S$ is the standard deviation of the observed reorganization boundaries in sector *s*. As the number of observations within a sector increases, the estimate approaches the sample mean, whereas sectors with only a few observations lie closer to our prior belief. Ultimately this yields reorganization boundaries for firm *f* operating in sector *s*, given as $K = \rho_s D_f$ and $K\left(\tau\right) = \rho_s D_f e^{-\gamma(T-\tau)}$ for the DD and BC models, respectively. Notice that the reorganization boundary in the DD model is a static measure, whereas the BC model utilizes a dynamic discounted reorganization boundary.

8 Should not be confused with index description *s*.

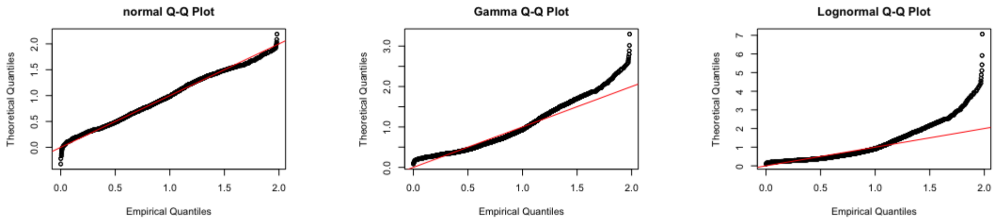

Figure 7.2: Q-Q plots on observed reorganization boundaries. As seen in the leftmost plot, the normal distribution serves as the best fit for the distribution of the reorganization boundaries, compared to the gamma distribution and the log-normal distribution, illustrated in the middle and rightmost plots, respectively.

7.4.3 Volatility estimation

A vital parameter in both of the default models considered here is the volatility of company assets, which has a substantial impact on default probabilities. As previously stated, asset volatilities for the companies in this analysis cannot be estimated by using either option prices or stock returns. Instead we will derive volatility estimates on the basis of sector specifics and firm size.

7.4.3.1 Sector-specific volatility estimate

Similar to the reorganization boundary, we assume that a firm's volatility is highly dependent on the sector in which it is operating. Empirical analysis by Ray and Tsay (2000) shows that the sector significantly affects the number of common long-range dependent components in volatility. To test whether sector dependency is present for the listed companies in our data set $l \in L$, we perform an analysis of variance. As we soon will demonstrate, the observed volatilities are non-normally distributed, and once again we apply the non-parametric Kruskal-Wallis test to verify our assumption of sector dependence. The results of this test are stated in appendix B.1, showing that the null hypothesis is rejected, and we proceed to derive a unique volatility estimate for each sector.

Our estimates are primarily based on volatility forecasts for each listed company operating within a specific sector. There are numerous methods for volatility forecasting, in particular for short-term forecasting. Our estimates, however, should represent our best predictions for a period of five years. An intuitive first approach would simply be using the long-run volatility, which is known to be relatively stable over time (Christoffersen, 2012). However, in terms of credit risk modeling, the long-run volatility has a severe limitation; it tends to underestimate the overall probability of default. The cause of this underestimation is the fact that the probability of default exponentially increases as a function of asset volatility.[9]

For instance, if a volatility peak occurs during the time period under study, the resulting increase in overall volatility is only partly captured when using long-run volatility. This feature is depicted in Figure 7.3, where the dashed line represents the trailing 30-day standard deviation on log-returns of a fictitious firm, and the solid line represents the trailing five-year probability of default, as calculated by the BC model, utilizing the current standard deviation measure. For a volatility of less than 0.15, the probability of default is close to zero. However, in periods of volatility peaks, the probability of default is substantial. Significant jumps in the firm's volatility lasting for only short periods of time would only contribute to a minor increase in average volatility, but would at the same time result in a considerable increase in the overall probability of default. This is visualized by the dashed-dotted and dotted horizontal lines, which represent the average[10] PD_{BC} (≈ 0.0111) and the PD_{BC} calculated using the average volatility (≈ 0.0030), respectively. This clearly demonstrates that the use of averaged volatility severely underestimates the overall probability of default, in the presence of volatility jumps.

In reality, however, the relation between asset volatility and default frequency is not necessarily as apparent as it may seem in Figure 7.3. The estimation of future volatility can be done in a variety of ways, and two of the most commonly used methods are the exponentially weighted moving average model (RiskMetrics, 1996), and the GARCH(1,1)[11] model (Bollerslev, 1996). Both models combine historical volatility and daily returns for estimating future volatility. In addition, the GARCH(1,1) model assumes that volatility is mean reverting, with a tendency to revert towards its long-run mean. However, none of these models fully capture the fact that a negative return yields a substantially greater credit risk, compared to a positive return of the same magnitude.

9 This feature applies to both the DD and the BC models.

10 The average value of the solid line in Figure 7.3.

11 Generalized Autoregressive Conditional Heteroskedasticity.

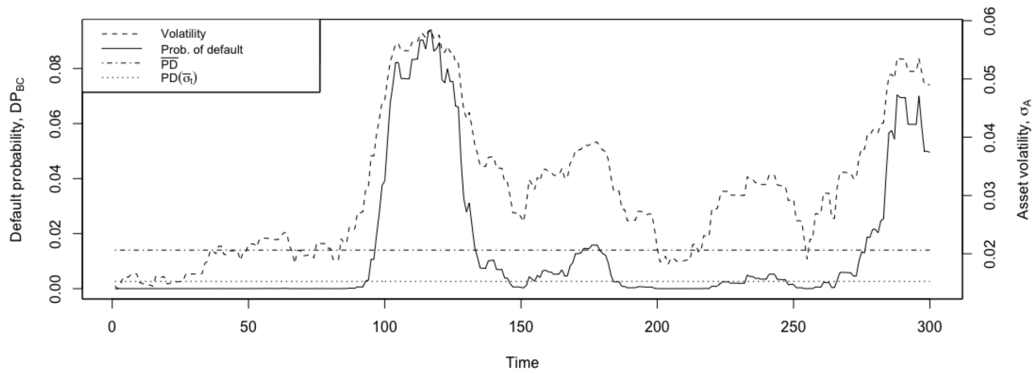

Figure 7.3: Real time BC probability of default and asset volatility, visualized by solid and dotted lines, respectively. The dashed-dotted and dotted horizontal lines, representing the average PD*BC* (≈ 0.0111) and the PD_{BC} calculated using the average volatility (≈0.0030), respectively, show that significant jumps in the firm's volatility lasting for only short periods of time would only contribute to a minor increase in average volatility, but would at the same time result in a considerable increase in the overall probability of default.

In order to fully capture the increased credit risk in case of a downturn, often referred to as the leverage effect, we apply the nonlinear GARCH (NGARCH) model given by:

$$\sigma_{l,t+1}^2 = \omega + \alpha\left(R_{l,t} - \theta\sigma_{l,t}\right)^2 + \beta\sigma_{l,t}^2 \tag{6}$$

where R_t is the daily log return, and the persistence parameters (α, β, θ) are estimated by maximum likelihood estimation (see appendix B.4). For each listed company, we compute the daily stock variance using the NGARCH model. Our forecast of asset volatility for listed firm l is then computed by naively leveraging the average equity volatility from equation 6, given by:[12]

$$\hat{\sigma}_l^A = \frac{A_l - D_l}{A_l}\sqrt{\frac{1}{N^2}\sum_{t=1}^{N}\sigma_{l,t}^2} \tag{7}$$

where N is the number of daily observations and A_l and D_l are the asset and debt values of listed firm l at time T, respectively.[13] The initial volatility estimate for all private firms f will be based on the volatility of listed companies operating within

12 We naively assume the observed stock volatility to be a representative measure for the volatility of the firm's equity.

13 We consider the time period 01.01.11–31.12.13, hence time T is 31.12.13.

the same sector. However, due to the limited amount of listed companies within certain sectors, we cannot simply utilize the sector average, $\bar{\sigma}_s^A$, as a credible estimate. Similar to computing the reorganization boundary, we apply the empirical Bayes method in computing our final volatility estimates. Karolyi (1993) and Darsinos and Satchell (2007) both assume the volatility parameter to be inverse-gamma distributed. However, when observing the volatilities of US-traded stocks, Ho, Lee, and Marsden (2011) find that the gamma distribution yields a substantially better fit than the inverse-gamma. Similarly, applied to our data, we see that the gamma distribution provides a significantly better fit than the inverse-gamma, as can be seen from Figure 7.4. We do, however, find that the log-normal distribution yields a slightly better fit than the gamma distribution, which is shown in Figure 7.4. We therefore assume that the individual volatilities are log-normally distributed.

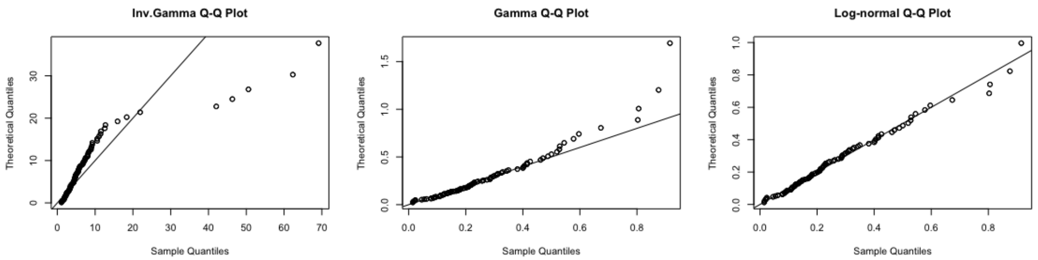

Figure 7.4: Quantile-quantile plots on volatility, showing that the log-normal distribution provides a slightly better fit than the gamma distribution and a significantly better fit than the inverse-gamma distribution.

Our prior on the volatility mean is normally distributed with hyperparameter $\hat{m}$ and $\hat{s}$, estimated by fitting the cumulative distribution to a step function on the sector averages. This is done in the same manner as described for the reorganization boundaries in subsection 7.4.2.2. In other words, the individual steps in the step function are the multiplicative inverse of the sample standard deviation of each sector. Based on the findings depicted in Figure 7.4, we assume that the observed volatilities in sector s, z_s are log-normally distributed with unknown mean parameter σ and sample precision parameter ξ. Our posterior belief on the sector-specific volatility is then given by:[14]

$$\hat{\sigma}_s^A = \frac{\hat{m}\hat{s} + n_s \xi \bar{z}_s}{\hat{s} + n_s \xi} \tag{8}$$

14 See appendix B.2 for derivations.

where n_s and $\overline{z}_s$ are the number and average of observed volatility measures in sector s.

7.4.3.2 Firm size effects on volatility

Despite the fact that empirical default frequencies (EDF) are found to be negatively correlated with firm size (Statistics Norway, 2018), firm size is not explicitly a factor of either the BC or the Merton models. Well aware of the fact that the models, and the DD model in particular, are widely used for credit rating purposes, we may assume that firm size is implicitly taken into account, presumably through both volatility and differences in the asset-to-debt ratios of small and large firms. Cheung and Lilian (1992) find that stock volatility is somewhat negatively correlated with firm size, which ultimately yields a higher expected probability of default for smaller firms. Unfortunately, the number of listed companies on the OSE is not sufficient to provide accurate estimates of firm size effects on volatility. Instead, we find an implicit firm size effect[15], given by the relation between the theoretical probability of default and the empirical default frequency.

We order our stock-based firms by their asset value from largest to smallest, before dividing the whole set of firms into 10 equally sized quantiles. Next, we find the average debt value, asset value, reorganization boundary and volatility[16] within each quantile. Using these values, we calculate the five-year probability of default using both default models and compare the results with the five-year empirical default frequency[17] in each quantile, depicted in the leftmost plot in Figure 7.5. Along the x-axis, we have $log(A/A_{max})$, where A is the asset value, and A_{max} the average asset value of the quantile containing the largest firms – quantile 1. Ultimately, for each quantile, we numerically derive the volatility that would have given a probability of default equal to the empirical default frequency and divide this figure by the original quantile volatility. As evident in the rightmost plot in Figure 7.5, the implied volatilities tend to be negatively correlated with firm size, and the slope of the linear regression serves as our estimated size effect on volatility. We employ this procedure for both the DD and the BC models, yielding slightly different size effects.

15 Although firm size does not necessarily affect volatility, we review the found relation as a firm size effect on volatility.

16 At this point, all firms have been assigned their sector-specific reorganization boundary and volatility estimate.

17 Empirical default frequency over the time period 31.12.2013–01.11.2018.

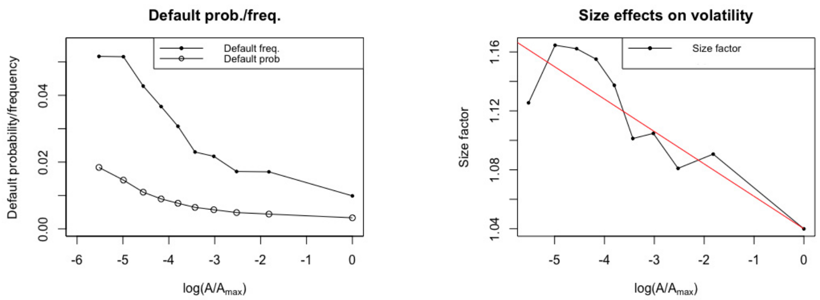

Figure 7.5: The leftmost plot shows the EDF and the probability of default using the DD model within each quantile. The rightmost plot shows the multiplicative factor on the volatility for each quantile.

The fact that we only have default data for one five-year period means that we are not able to test the size effect out-of-sample. In order to reduce over-fitting, we perform a fivefold cross-validation, testing how well the size effect found in separate training sets performs on an omitted test set (further description and results can be found in appendix B.3.1). In addition, we perform linear regressions on volatility as a function of market capitalization for traded companies on the NASDAQ stock exchange, testing whether it is reasonable to assume that our size component would also be found out-of-sample (appendix B.3.4).

Our final estimate for the asset volatility of a stock-based firm f is a product of sector-specific figures and firm size, expressed as:

$$\hat{\sigma}_f^A = \hat{\sigma}_s^A \left(1 + g\left(x_f\right)\right) where \quad \hat{\sigma}_f^A = \hat{\sigma}_s^A \left(1 + g\left(x_f\right)\right) for\, all \quad f \in F \tag{9}$$

where $\hat{\sigma}_s^A$ is the sector-specific volatility estimate from equation 8, $A_{s,max}$ is the asset value of the largest firm within the sector in which firm f is operating, and $g(x_f)$ is the firm-specific size component.

7.4.4 Model evaluation

In order to test how well the two models perform, we sort all firms $f \in F$ by their theoretical probability of default, and split the data set into 15 equally sized quantiles. Next, we compare the EDF within each quantile to the range of theoretical probabilities of default. For illustrative purposes, the leftmost plot in Figure 7.6 shows a visualization of these ranges as box plots, whereas the EDF is visualized by the solid line.

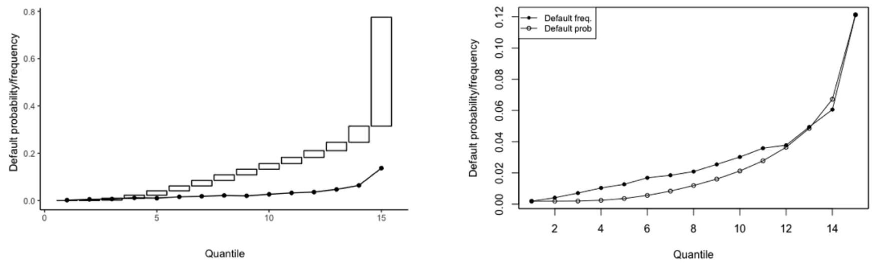

Figure 7.6: The leftmost plot illustrates the EDF and the ranges of theoretical probability of default within each sector. The rightmost plot compares the empirical result by the scaled theoretical results.

The deviation between the theoretical probability of default and the empirical default frequency, that is, comparing the probability ranges to the EDFs, is not necessarily a good performance measure. If sufficient data on defaulted companies are provided, the model's ability to correctly sort companies relative to each other is just as useful, simply because a theoretical model combined with empirical data is likely to outperform the theoretical model alone. This is essentially the approach of Moody's KMV model, where the distance-to-default measure is used for evaluation of firms, rather than their theoretical probability of default. In order to test this ability, we collect the median value[18] for each quantile and scale the distribution of probabilities, equalizing the theoretical probability of default and the empirical default frequency for quantiles 1 and 15. This is depicted in the rightmost plot in Figure 7.6. We then evaluate the performance of the models by deriving the mean squared prediction error (MSPE) on the EDF and the scaled probabilities of default.

7.4.4.1 Model specifications

In addition to the testing of how well the two models perform in comparison to one another, we also investigate the effects of our estimated parameters, namely, reorganization boundaries and volatilities. We do this by running both models with six distinctive parameter specifications each, hereafter referred to as model specifications (A)–(F), given as follows:

18 We use the median values instead of quantile averages, simply because outliers could have misleading effects on the results when using average values.

1. Sector-specific reorganization boundaries (eq. 5) and firm-specific volatility estimates, including both sector attributes and firm size effects (eq. 9). $\hat{\rho}_f = \hat{\rho}_s$, $\hat{\sigma}_f^A = \hat{\sigma}_f^A$
2. Sector-specific reorganization boundaries (eq. 5) and sector-specific volatility estimates, excluding firm size effects (eq. 8). $\hat{\rho}_f = \hat{\rho}_s$, $\hat{\sigma}_f^A = \hat{\sigma}_s^A$
3. Identical reorganization boundaries for all firms regardless of sector, and firm-specific volatility estimates, including both sector attributes and firm size effects (eq. 9). $\hat{\rho}_f = 0.6$, $\hat{\sigma}_f^A = \hat{\sigma}_f^A$
4. Identical reorganization boundaries for all firms regardless of sector, and sector-specific volatility estimates, excluding firm size effects (eq. 8). $\hat{\rho}_f = 0.6$, $\hat{\sigma}_f^A = \hat{\sigma}_s^A$
5. Sector-specific reorganization boundaries (eq. 5) and firm-independent volatility estimates, identical for all firms, found as the computed mean of all sectors. $\hat{\rho}_f = \hat{\rho}_s$, $\hat{\sigma}_f^A = 0.2$
6. Identical reorganization boundaries for all firms regardless of sector, and firm-independent volatility estimates, identical for all firms, found as the computed mean of all sectors. $\hat{\rho}_f = 0.6$, $\hat{\sigma}_f^A = 0.2$

These specifications will be successively highlighted throughout the results section.

7.5 RESULTS

In the following sections, we will outline the essence of our findings, obtained by the models and methods described in section 7.4. We will present our parameter estimates, as well as results from running both the DD and the BC models, with the set of parameter specifications presented in section 7.4.4.1. We successively discuss our findings and their implications as well as limitations in our research.

7.5.1 Sector-specific measures

Table 7.2 contains derived estimates for sector-specific reorganization boundaries and volatilities. Evidently, the estimated reorganization boundaries $\hat{\rho}_s$ do not deviate substantially across sectors, apart from the communication and IT sector, which has a noticeably low reorganization boundary. Libert and Beck (2016) found that companies within the technology sector have the smallest percentage of physical assets owned relative to the total asset value across all sectors, with an empirical average of about 10–15%. As pointed out in section 7.4.2, a firm is generally allowed to keep operations going as long as it is able to fulfill its periodic obligations towards its creditors. In other words, in case of a default, the amount of current assets is presumably negligible, and what is left for the creditors is highly dependent on the value of physical assets, which to some extent is reflected in our estimate for the communication and IT sector. On the contrary, Libert and Beck (2016) find the transportation sector to score rather high in terms of physical assets, with an average of about 60% of the total asset value. This corresponds well with our findings, as the sectors considered to be asset-heavy, including the transportation sector, tend to have a relatively high reorganization boundary. This observation reflects the fact that creditors are reluctant to force asset-light companies into bankruptcy, as these companies tend to provide relatively poor recovery rates, as opposed to asset-heavy companies. Moreover, the nature of asset-heavy companies, often involving considerable debt financing, makes these companies more likely to default on their payments at a high asset-to-debt ratio.

As opposed to the reorganization boundaries, the listed volatility estimates show rather significant variations across the 11 sectors. The seafood sector, which on the OSE is primarily represented by salmon farmers, was and still is undoubtedly subject to great price oscillation, partly due to seasonality as well as a high degree of idiosyncratic production risk (Oglend & Sikveland, 2018).

Table 7.2: Sector-specific figures, including posterior estimates on sector-specific reorganization boundary and volatility. Listed in the rightmost column is the asset value of the largest privately held firm within each sector, which is applied in the calculation of the firm size component in the volatility estimate. [1] In thousand NOK.

Sector	$Listed_{EF}$	$Private_{EF}$	$Defaults_{EF}$	$\hat{\rho}_s$	$\hat{\sigma}_s^A$	$A_{s,max}$ [1]
A – Seafood	9	1884	26	0,624	0.262	912402
B – Energy	31	580	10	0.595	0.207	983356
C – Industry	7	7059	291	0.625	0.160	996942
D – Supply	1	804	9	0.668	0.171	996180
E – Water industry	2	442	7	0.625	0.169	808152
F – Entrepreneur	2	14656	661	0.639	0.135	972927
G – Retail	2	31741	1454	0.603	0.180	997180
H – Transport	10	4754	121	0.664	0.229	998339
J – Communication and IT	21	16841	295	0.509	0.259	983230
L – Real Estate	6	21699	176	0.606	0.144	998585
Q – Health	5	797	10	0.605	0.228	794046
	96	101257	3060			

The communication and IT sector has also been assigned relatively high volatilities, partly counteracting the long theoretical distance-to-default implied by the low reorganization boundary observed at actual default. As opposed to firms within the seafood and the communication and IT sectors, firms within the real estate sector have been assigned remarkably low volatilities, most certainly reflecting the sentiment in the Norwegian real estate market over the last decade. The estimated parameters will shortly be further evaluated in terms of their actual effects on the probability of default.

7.5.2 Firm-size component in the volatility estimates

As described in section 7.4.3.2, we introduce firm size as a component in the firm-specific volatility estimates. We derived two unique relations, one for each of the two default models. The asset volatilities for firm $f \in F$ applied to the DD and BC models, respectively, are given by:

$$DD : \sigma^A_{f,DD} = \sigma^A_s \left(1 + 0.0022 x_f\right) \text{for all } f \in F \tag{10}$$

$$BC : \sigma^A_{f,BC} = \sigma^A_s \left(1 + 0.0020 x_f\right) \text{for all } f \in F \tag{11}$$

where the parentheses represent the size component $g(x_f)$ from section 7.4.3.2, where $x_f = log(A_f/A_{max})$ for firm f. The size effect is small for most of the firms in our data set, although the smallest companies are charged with a non-negligible volatility premium.

7.5.3 Probability of default

Figure 7.7 visualizes the results of both default models with the model specification (A), that is, applying both sector-specific reorganization boundary and firm-specific volatility. The plots clearly demonstrate that both models are convincingly accurate in their prediction of actual credit risks, empirically inferred in the 15 quantiles. However, as pointed out in section 7.4.4.1, we are not only concerned about how well the two models perform. Equally important for building our framework is evaluating effects of the estimated parameters.

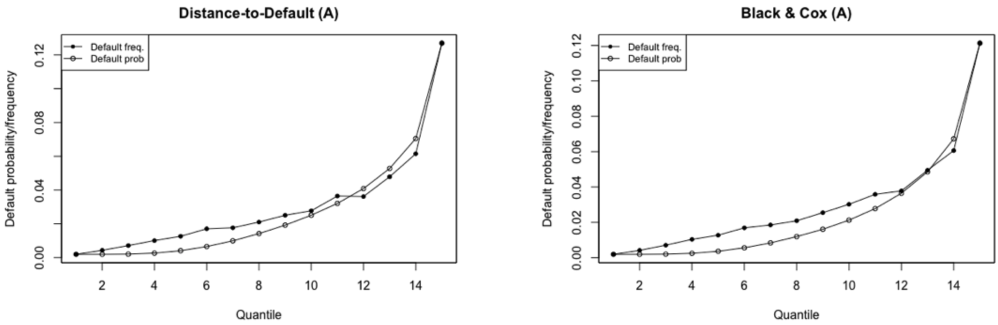

Figure 7.7: Empirical default frequency and scaled probabilities of default for both the DD and the BC models, both with model specification (A), utilizing the sector-specific reorganization boundary and firm-specific volatility estimate.

Table 7.3 contains the calculated MSPE, measuring the deviation between the EDFs and the scaled probabilities of default for both models, with the different model specifications (A)–(F). Plots and detailed figures for each specification can be found in appendices C.1 and C.2 for the DD and BC models, respectively. At first glance, both the estimated reorganization boundaries as well as the volatility estimates provide considerable improvements. However, the model specification that is in fact yielding the lowest MSPE, slightly better than the DD-(A), is the BC-(E)[19], suggesting that our sector-specific volatility estimates might not accurately enough capture the actual sector-specific risk. There may be several reasons for this. First of all, our estimates are based on historical data, which are not necessarily a good predictor for what is to come. Additionally, the estimates are based on a limited number of publicly traded stocks. Although we are searching to reduce the limitations of data scarcity by employing the empirical Bayes method, there is reason to believe that the estimates could be significantly improved by including more data, for example, volatility data from other Nordic stock markets such as the NASDAQ Nordic. Lastly, and presumably equally important, is the fact that the sector-specific estimates are applied to a whole range of industries within the sector, which may in fact not be subject to the same risk factors; hence the use of a sector-specific volatility estimate might be too crude.

19 Model specification (E) utilizes the estimates on reorganization boundaries, but a common volatility of 0.2 for all companies.

Table 7.3: MSPE of the EDF and the scaled probabilities of default for all model specifications. The results show that the BC-(E) model, with sector-specific reorganization boundary but common volatility measure, slightly outperforms the DD-(A) model, including both sector-specific reorganization boundary and volatility.

Specification	ρ	σ	MSPE-DD	MSPEBC
(A)	ρ_s	σ_f	$3.7608E^{-5}$	$5.1082E^{-5}$
(B)	ρ_s	σ_s	$8.2307E^{-5}$	$8.6120E^{-5}$
(C)	0.6	σ_f	$6.5657E^{-5}$	$9.8571E^{-5}$
(D)	0.6	σ_s	$1.1199E^{-4}$	$1.3026E^{-4}$
(E)	ρ_s	0.2	$1.6379E^{-4}$	$3.5362E^{-5}$
(F)	0.6	0.2	$1.9600E^{-4}$	$4.2539E^{-5}$

We do however find that the size component improves the performance in all cases, suggesting that this is indeed an effect that should be considered when modeling credit risk. Relatively small companies tend to be less solid in terms of asset-to-debt ratios. This is demonstrated by the increased theoretical PD for small companies, also without considering the size component on volatility estimates, depicted in the leftmost plot in Figure 7.5. However, the EDFs in our data set suggest that there is, in fact, an increased risk of default for small companies, even when adjusting for the capital structure. In the fivefold cross-validation in appendices B.3.2 and B.3.1, this is found to be a more or less uniform characteristic of the whole data set.

Furthermore, we see that the sector-specific reorganization boundaries are generating considerable improvements. Although the estimates on reorganization boundaries are derived in-sample, we believe these estimates reflect actual sector variations in such a way that they would also yield improvements out-of-sample. The reason for this belief is the large amount of data that the estimates are based on and also the fact that the estimates seem to reflect possession of physical assets – the estimated reorganization boundaries for typically asset-heavy sectors tend to be higher than for less asset-heavy sectors.

7.5.4 Interpretation of the overall results

Both models yield promising results on credit risk modeling in the Norwegian market for privately held firms. Despite its complexity compared to the DD model, the BC model does not outperform the DD model on a general basis, suggesting that it should be equally possible to build a reliable framework based on the simpler DD model. The fact that both models outperform the other with different parameter specifications – neither DD or BC is consistently better for all specifications – suggests that parameter estimates are equally or even more important than the actual choice of model. A huge effort should be made estimating parameters and calibrating the models in order to capture the characteristics of the particular markets under study.

7.5.4.1 A note on research limitations

We have amply demonstrated a suitable framework for modeling default probabilities for non-listed, privately owned Norwegian companies. Nonetheless, structural credit models face limitations, some of which have already been discussed above. Our research and conversely our findings were subject to one main challenge, data scarcity. This implies uncertainty in our parameter estimates. Also, we are unable to test the models out-of-sample. Preferably, the parameters should have been further calibrated in-sample, before being tested out-of-sample, for example, on similar data covering another five-year period. One might argue that we could have divided our data set into two separate and randomly drawn sets in order to perform in-sample and out-of-sample analysis. However, this approach would not have captured the fact that the majority of firms are subject to cyclical ups and downs, making default measures non-static, and that findings from the five-year period under study are not necessarily directly transferable to previous or future time periods.

The methods used for volatility estimation are mostly well known, although we make some adjustments in order to capture the actual credit risk inferred by empirical data, such as averaging the daily NGARCH estimates (equation 7). Instead of using only publicly traded data to capture the sentiment within a sector, another approach would presumably be to combine it with implied in-sample risk from data on defaulted firms. Once again, this approach requires more data for out-of-sample testing. The same goes for the model-specific size component on volatility (equations 10 and 11). Despite our attempt to circumvent overfitting by introducing K-fold cross-validation and comparison to the NASDAQ stock exchange, the stated relations must be considered as predictions on a complex and uncertain future.

7.6 CONCLUSION

In this study, we have presented a framework for modeling credit risk of non-listed, stock-based Norwegian firms, by combing structural models with empirical data, observing the characteristics of the Norwegian corporate market. As part of the computation of probability measures, we derive firm- and sector-specific estimates on reorganization boundaries and volatility, attempting to capture the real-life risk factors to which these firms are exposed. Conducting comprehensive parameter estimations, we have examined the accuracy of the Merton and Black and Cox credit default models, finding that both models perform convincingly well. Utilizing empirical data, our framework yields empirical default frequencies which can be exploited in a number of ways, for example, calculating theoretical credit spreads. Although the framework should benefit from incorporating more data, our research shows that a structural modeling approach is highly applicable to privately held firms. As noted in the introduction, there are other valid approaches for assessing the credit default risk of corporate entities which might yield equally good results, provided that the data those models require are available (i.e., explanatory variables). Many reduced-form models employ logit and/or probit functions in order to estimate theoretical default probabilities. We have not attempted this approach on the data set underlying this study. An advantage of our approach – structural models over reduced form models – is the small number of variables needed in the structural setup. Basically, we only need estimates of the value and volatility of firm assets and debt.

7.6.1 Further research

Our research could be extended in numerous ways. The first and presumably most obvious extension is using a broader data set. By running the models on similar data from other countries in the Nordic region, preferably over several time intervals, we may obtain better parameter estimates and stronger confidence in the overall results. Although we intentionally based our parameter estimates on Norwegian data exclusively, we could have extended our data set of listed companies to include companies listed on the NASDAQ Nordic stock exchange, which presumably would yield more robust volatility estimates. This is particularly true for sectors which are highly dependent on multinational factors.

The established framework is exclusively built on objective measures, eliminating the impact of any subjective beliefs. The firm-specific parameters *expected rate of return* and *dividend yield* were therefore naively set equal to zero for all firms. It is, however, reasonable to assume that it is possible to obtain even more realistic results by incorporating beliefs on expected growth measures, utilizing local expertise on the Nordic market. We could include such measures in our analysis by integrating the Q4 2013 consensus[20] on expected growth rates for each sector. Before doing this, however, it is crucial to clarify the impact of sector growth on the default frequencies within the specific sector. In addition, the historical performance of the consensus view, or any other measurable subjective assessment, should be tested. There is no point in incorporating subjective measures that do not perform considerably better than noninformative, objective measures.

REFERENCES

Altman, E. I. & Kishore, V. M. (1996). "Almost Everything You Wanted to Know about Recoveries on Defaulted Bonds." *Financial Analysts Journal,* 52(6).

Arora, N. R., Bohn, J. & Zhu, F. (2005). "Reduced form vs. Structural Models of Credit Riks: A Case Study of Three Models." *Moodys KMV*.

Bharath, S. T. & Shumway, T. (2008). "Forecasting Default with the Merton Distance to Default Model." *Review of Financial Studies,* 21(3).

Black, F. & Cox, C. J. (1976). "Valuing Corporate Securities: Some Effects of Bond Indenture Provisions." *Moodys KMV,* 31(2).

Black, F. & Scholes, M. (1973). "The Pricing of Options and Corporate Liabilities." *Journal of Political Economy,* (2), 637–654.

Bollerslev, T. (1996). "Generalized Autoregressive Conditionial Heteroskedasticity." *Journal of Economics,* 31, 307–327.

Breccia, A. (2012). "Default Risk in Mertons Model." http://www.bbk.ac.uk/ems/forstudents/mscfinEng/pricingemms014p/ab8.pdf

Chan-Lau, J. A. & Sy, A. N. R. (2006). "Distance-to-Default in Banking: A Bridge Too Far?" *International Monetary Fund*.

Chen, T. & Guestrin, C. (2016). "XGBOOST: A Scalable Tree Boosting System." https://dl.acm.org/doi/pdf/10.1145/2939672.2939785

Cheung, Y. W. & Lilian, K. N. (1992). "Stock Price Dynamics and Firms Size: An Empirical Investigation." *Journal of Finance,* 47(5), 1985–1997.

Christoffersen, P. F. (2012). *Elements of Financial Risk Management*. Elsevier.

Darsinos, T. & Satchell, S. (2007). "Bayesian Analysis of the Black-Scholes Option Price." *Forecasting Expected Returns in the Financial Markets.* Academic Press, NY.

de Lange, P. E., Stiberg, K. & Aamo, P. E. (2019). "Estimating Contingent Convertible Credit Spread in the Norwegian Bond Market Using an Option Pricing Approach." *Scandinavian Journal of Business Research,* (2).

Feldhutter, P. & Schaefer, S. (2018). "The Credit Spread Puzzle and the Merton Model – Myth or Reality?" https://ssrn.com/abstract=2363081 or http://dx.doi.org/10.2139/ssrn.2363081

Fruhwirth, M. & Sogner, L. (2006). "The Jarrow/Turnbull Default Risk Model: Evidence from the German Market." *European Journal of Finance,* 12(2), 107–135.

20 The overall belief at the end of 2013, composed by a number of market analysts.

Ho, S. W., Lee A. & Marsden, A. (2011). "Use of Bayesian Estimates to Determine the Volatility Parameter Input in the Black-Scholes and Binomial Option Pricing Models." *Journal of Risk and Financial Management,* 3, 74–96.

Karolyi, G. A. (1993). "A Bayesian Approach to Modelling Stock Return Volatility for Option Valuation." *Journal of Financial and Quantitative Analysis,* 28, 579–594.

Kovacova, M. & Kollar, B. (2018). "Modeling Default Probability via Structural Models of Credit Risk in Context of Emerging Markets." *Intech*.

Koziol, C. (2013). "A Simple Correction of the WACC Discount Rate for Default Risk and Bankruptcy Costs." *Springer* .

Kruskal, W. H. & Wallis, W. A. (1952). "Use of Ranks in One-Criterion Variance Analysis." *Journalal of the American Statisical Association,* 47, 583–621.

Law, D. & Roache, S. K. (2015). "Assessing Default Risks for Chinese Firms: A Lost Cause?" IMF Working Paper. Monetary and Capital Markets Department. https://www.imf.org/external/pubs/ft/wp/2015/wp15140.pdf

Libert, B. & Beck, M. (2016). "Investors Today Prefer Companies with Fewer Physical Assets." *Harvard Business Review Digital Articles*.

Merton, R. C. (1973). "On the Pricing of Corporate Debit: The Risk Structure of Interest Rates." *Journal of Finance,* 29(2).

Mora, N. (2012). "What Determines Creditor Recovery Rates?" *Federal Reserve Bank of Kansas City.*

Oglend, A. & Sikveland, M. (2018). "The Behaviour of Salmon Price Volatility." *Marine Resource Economics,* 23, 507–526.

Ray, B. K. & Tsay, R. S. (2000). "Long-Range Dependence in Daily Stock Volatilities." *Journal of Business Economic Statistics,* 2(18), 254–262.

RiskMetrics. (1996). "J.P. Morgan / Reuters – Risk Metrics – Technical Document." 4.

Shapiro, S. & Wilk, S. (1965). "An Analysis of Variance Test for Normality." *Biometrika,* 52, 591–611.

Statistics-Norway. (2018). "Nyetablerte foretaks overlevelse og vekst." Accessed 2018-10-31. https://www.ssb.no/fordem