Discussion/comments of “Parameterization of nearshore wave breaker index” by Chi Zhang, Yuan Li, Yu Cai, Jian Shi, Jinhai Zheng, Feng Cai and Hong Shuai

Dag Myrhaug[1*], Hong Wang[1], Lars Erik Holmedal[1]

[1]Department of Marine Technology,

Norwegian University of Science and Technology (NTNU), NO-7491 Trondheim, Norway

[*]Corresponding author: Dag Myrhaug, Department of Marine Technology, NTNU,

Otto Nielsens vei 10, NO-7491 Trondheim, Norway

E-mail address: dag.myrhaug@ntnu.no

Abstract

The purpose of these comments and discussion has been to point out that wave statistics can be incorporated in future applications of the nearshore wave breaker index formula proposed by Zhang et al. (2021). This is demonstrated by using a joint distribution of significant wave height and spectral peak period provided by Li et al. (2015).

Keywords: Breaker index; Wave statistics; Joint statistical models; Nearshore

1. Discussion

First, the discussers wish to compliment the authors, Zhang et al. (2021) (hereafter referred to as Z21), on their results developing a new parameterization of nearshore wave breaker index. These comments and discussion have been written to point out that wave statistics can be incorporated in future applications of the authors´ nearshore wave breaker index formula.

Z21 proposed the following formula for the breaker index γ (defined as the breaker wave height-to-breaker depth ratio) (see Eq. (5) in Z21)

$$\gamma = (237 s_0^2 - 34.81 s_0 + 1.46) \cdot \exp\left[1.96 \cdot \ln(38.64 s_0) \times kh\right] \quad (1)$$

where $s_0 = H_0 / L_0$ is the offshore wave steepness, H_0 and L_0 are the offshore wave height and wave length, respectively, *kh* is the normalized local water depth, *k* and *h* are the local wave number and water depth, respectively. Z21 noted that based on the field data used to develop Eq. (1) there might be a limited applicability by the available ranges of data of $s_0(0-0.05)$ and $kh(0.3-1.2)$. By closer inspection of the results in Fig. 3 in Z21 most of the data are for $kh \lesssim 1$, where most of the waves are reasonably represented by shallow water waves for which $kh = (2\pi / T)\sqrt{h/g}$ (Dean and Dalrymple, 1984). Here *T* is the offshore wave period, and *g* is the acceleration due to gravity.

The statistical features of γ will be exemplified based on wave statistics obtained from the joint probability density function (*pdf*) of the significant wave height H_s and the spectral peak period T_p. Then, $s_p = H_s / ((g/2\pi)T_p^2)$ is the spectral wave steepness, $L_p = (g/2\pi)T_p^2$ is the spectral wave length, $k_p = 2\pi / L_p$ is the spectral wave number, and $k_p h = (2\pi / T_p)\sqrt{h/g}$ is the corresponding normalized water depth in shallow water $(k_p h \lesssim 1)$. It should be noted that the results in Z21 are based on using $k = k_p, L_0 = L_p, T = T_p, H_0 = H_{rms}$, i.e. using the rootmean-square (*rms*) wave height where $H_{rms} = H_s / \sqrt{2}$ for a Rayleigh-distributed wave height (Dean and Dalrymple, 1984), and consequently $s_0 = H_{rms} / L_p = s_p / \sqrt{2}$. Thus, substitution of $s_0 = s_p / \sqrt{2}$ (for $s_p(0-0.07)$) and $kh = k_p h(0.3-1)$ in Eq. (1) yields

$$\gamma = (119\, s_p^2 - 24.61 s_p + 1.46) \cdot \exp\left[1.96 \cdot \ln(27.32\, s_p) \times k_p h\right] \qquad (2)$$

with

$$s_p = \frac{H_s}{\frac{g}{2\pi} T_p^2} \quad \text{for} \quad 0 \le s_p \le 0.07 \qquad (3)$$

$$k_p h = \frac{2\pi}{T_p} \sqrt{\frac{h}{g}} \quad \text{for} \quad 0.3 \le k_p h \lesssim 1 \qquad (4)$$

Then, it follows that $\gamma = \gamma(H_s, T_p; h)$ for a given water depth *h*.

Some statistical features of γ will be exemplified based on the joint *pdf* of H_s and T_p provided in Appendix A, which is based on wave data from a location in the North Atlantic 15 km off the French coast. Here the conditional expected value of γ given T_p, $E\left[\gamma \mid T_p\right]$, and the conditional variance of γ given $T_p, Var\left[\gamma \mid T_p\right]$, are considered given by (Bury, 1975)

$$E\left[\gamma \mid T_p, h\right] = \int_0^{0.109 T_p^2} \gamma(H_s, T_p; h)\, p(H_s \mid T_p) dH_s \qquad (5)$$

$$Var\left[\gamma \mid T_p, h\right] = E\left[\gamma^2(H_s, T_p; h)\right] - \left(E\left[\gamma \mid T_p, h\right]\right)^2 \qquad (6)$$

where

$$E\left[\gamma^2(H_s, T_p; h)\right] = \int_0^{0.109 T_p^2} \gamma^2(H_s, T_p; h)\, p(H_s \mid T_p) dH_s \qquad (7)$$

The integration limits 0 and $0.109T_p^2$ for H_s correspond to those obtained from Eq. (3), and $p(H_s \mid T_p)$ is the conditional *pdf* of H_s given T_p obtained from Eq. (A10) in Appendix A.

The conditional coefficient of variation is

$$R\left[\gamma \mid T_p, h\right] = \frac{(Var\left[\gamma \mid T_p, h\right])^{1/2}}{E\left[\gamma \mid T_p, h\right]} \tag{8}$$

Figs. 1 and 2 show the conditional expected value of γ given T_p (and *h*), $E\left[\gamma \mid T_p, h\right]$ (Fig. 1) and the corresponding conditional coefficient of variation, $R\left[\gamma \mid T_p, h\right]$ (Fig. 2) versus T_p in the range 2 s to 10 s and $k_p h = (2\pi / T_p)\sqrt{h/g}$ for the values 0.3, 0.4, 0.5, 0.7 and 1.

From Fig. 1 it appears that $E\left[\gamma \mid T_p, h\right]$ decreases as $k_p h$ increases, i.e. as *h* increases, for given values of T_p, that is, reflecting the features of Eq. (1) for lower values of the wave steepness as depicted in Fig. 3a-h in Z21. Furthermore, $E\left[\gamma \mid T_p, h\right]$decreases as T_p increases for $k_p h = 0.5, 0.7, 1$, while $E\left[\gamma \mid T_p, h\right]$ increases and then decreases as T_p increases for $k_p h = 0.3, 0.4$, with values in the range 0.092 to 0.55 depending on $k_p h$. Overall, this also reflects the features depicted in Fig. 3a-h in Z21. Details of the physical interpretation of Eq. (1) are given in Section 6.2 of Z21 and are hence omitted here.

From Fig. 2 it appears that $R\left[\gamma \mid T_p, h\right]$ increases as $k_p h$ increases, i.e. as *h* increases, for given values of T_p. Moreover, $R\left[\gamma \mid T_p, h\right]$ decreases and then increases slightly as T_p increases for $k_p h = 0.3, 0.4, 0.5, 0.7$, while $R\left[\gamma \mid T_p, h\right]$ increases and then is nearly constant as T_p increases for $k_p h = 1$, with values in the range 0.34 to 1.19 depending on $k_p h$. Overall, these values of $R\left[\gamma \mid T_p, h\right]$ reflect mainly the inherent statistical features of the joint *pdf* of H_s and T_p in Li et al. (2015).

In future applications of the parameterization of the nearshore wave breaker index proposed by Zhang et al. (2021) it should be considered to implement the statistical properties of the waves as demonstrated in this discussion.

Appendix A. Joint *pdf* of H_s and T_p

Here the joint *pdf* of H_s and T_p provided by Li et al. (2015) (hereafter referred to as LGM15) is chosen as an example. This (H_s, T_p) distribution was deduced from a wave hindcast data base from 2001 to 2010 obtained as a best fit to the hindcast data. The data represent swell, wind waves, and combined swell and wind waves conditions at the Sem Rev location 15 km off the French coast at 40 m water depth. The joint *pdf* of H_s and T_p is given as

$$p(H_s, T_p) = p(T_p \mid H_s)\, p(H_s) \tag{A1}$$

where $p(H_s)$ is the marginal *pdf* of H_s given by the combined lognormal and Weibull distribution

$$p(H_s) = \begin{cases} \dfrac{1}{\sqrt{2\pi}\sigma_h H_s} \exp\left[-\dfrac{(\ln H_s - \mu_h)^2}{2\sigma_h^2}\right]; H_s \le 3.5\,\mathrm{m} \\ \dfrac{\alpha}{\beta}\left(\dfrac{H_s}{\beta}\right)^{\alpha-1} \exp\left[-\left(\dfrac{H_s}{\beta}\right)^{\alpha}\right]; H_s > 3.5\,\mathrm{m} \end{cases} \tag{A2}$$

Here μ_h and σ_h are the mean value and the standard deviation, respectively, of $\ln H_s$ given as

$$\mu_h = 0.256 \quad , \quad \sigma_h = 0.583 \tag{A3}$$

and α, β are the Weibull parameters given by

$$\alpha = 1.160 \quad , \quad \beta = 1.309 \tag{A4}$$

Furthermore, $p(T_p \mid H_s)$ is the conditional *pdf* of T_p given H_s given by the lognormal *pdf*

$$p(T_p \mid H_s) = \frac{1}{\sqrt{2\pi}\sigma_t T_p} \exp\left[-\frac{(\ln T_p - \mu_t)^2}{2\sigma_t^2} \right] \tag{A5}$$

where μ_t and σ_t are the mean value and the standard deviation, respectively, of $\ln T_p$ given as

$$\mu_t = c_1 + c_2 H_s^{c_3} \tag{A6}$$

$$\sigma_t^2 = d_1 + d_2 e^{d_3 H_s} \tag{A7}$$

with

$$(c_1, c_2, c_3) = (1.900, 0.429, 0.272) \tag{A8}$$

$$(d_1, d_2, d_3) = (0.001, 0.205, -0.487) \tag{A9}$$

Here H_s is in metres in Eqs. (A6) and (A7) (see LGM15 for more details).

The conditional *pdf* of H_s given T_p is obtained as

$$p(H_s \mid T_p) = \frac{p(H_s, T_p)}{p(T_p)} = \frac{p(T_p \mid H_s)\, p(H_s)}{p(T_p)} \tag{A10}$$

where $p(T_p)$ is the marginal *pdf* of T_p obtained from

$$p(T_p) = \int_0^\infty p(H_s, T_p) dH_s = \int_0^\infty p(T_p \mid H_s)\, p(H_s)\, dH_s \tag{A11}$$

References

Bury, K.V., 1975. Statistical Models in Applied Science. *John Wiley & Sons, New York.*

Dean, R.G., Dalrymple, R.A., 1984. Wave Mechanics for Engineers and Scientists. *Prentice-Hall, Inc., New Jersey, USA.*

Li, L., Gao, Z., Moan, T., 2015. Joint distribution of environmental condition at five European offshore sites for design of combined wind and wave energy devices. *Journal of Offshore Mechanics and Arctic Engineering* 137(3), 031901.

Zhang, C., Li, Y., Cai, Y., Shi, J., Zheng, J., Cai, F., Qi, H., 2021. Parameterization of nearshore wave breaker index. *Coast Eng.* 168, 103914.

Figure caption

Fig. 1 $E\left[\gamma \,|\, T_p, h\right]$ versus T_p and $k_p h = (2\pi / T_p)\sqrt{h/g}$.

Fig. 2 $R\left[\gamma \,|\, T_p, h\right]$ versus T_p and $k_p h = (2\pi / T_p)\sqrt{h/g}$.

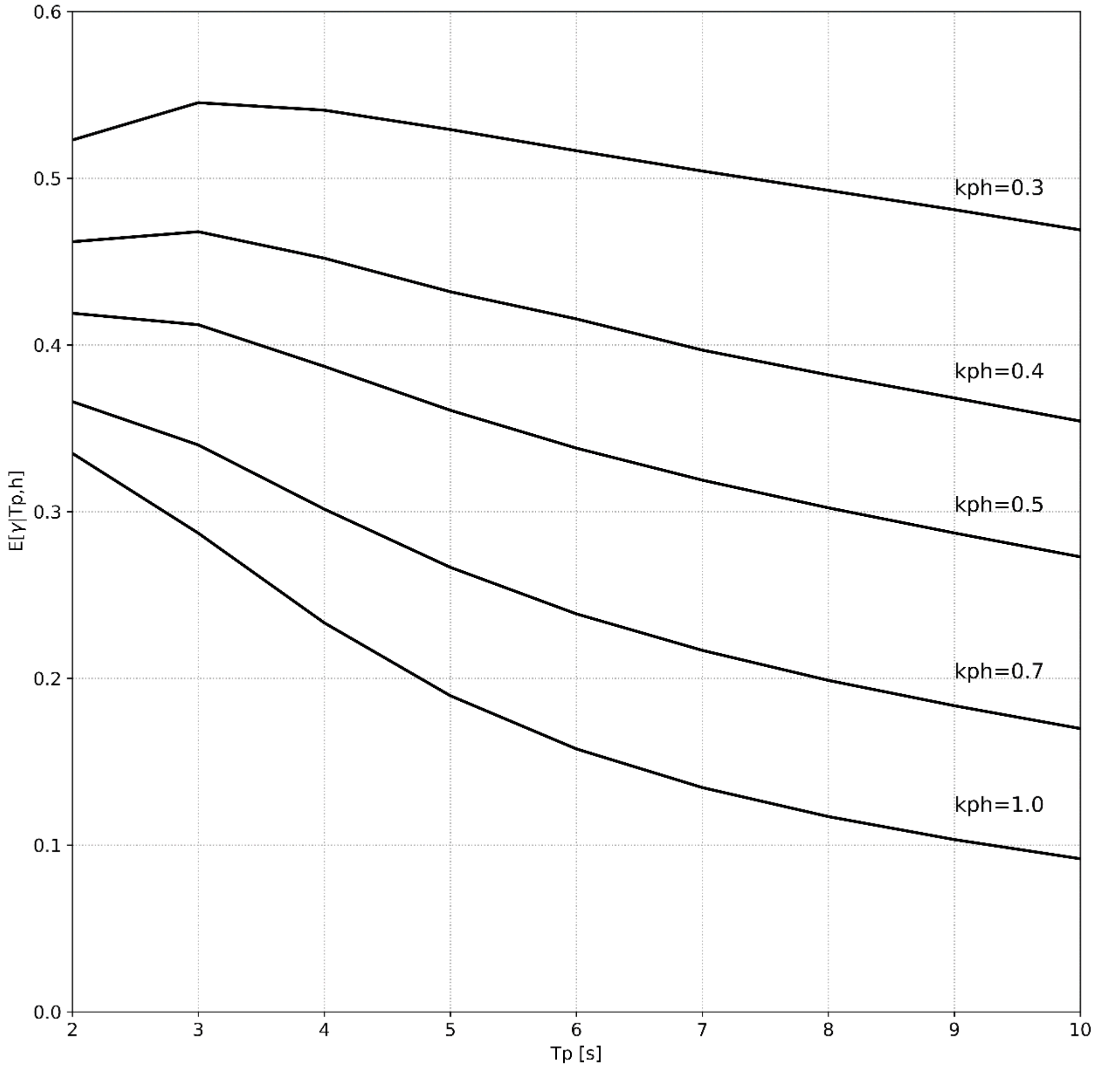

Fig. 1 $E\left[\gamma \mid T_p, h\right]$ versus T_p and $k_p h = (2\pi / T_p)\sqrt{h/g}$.

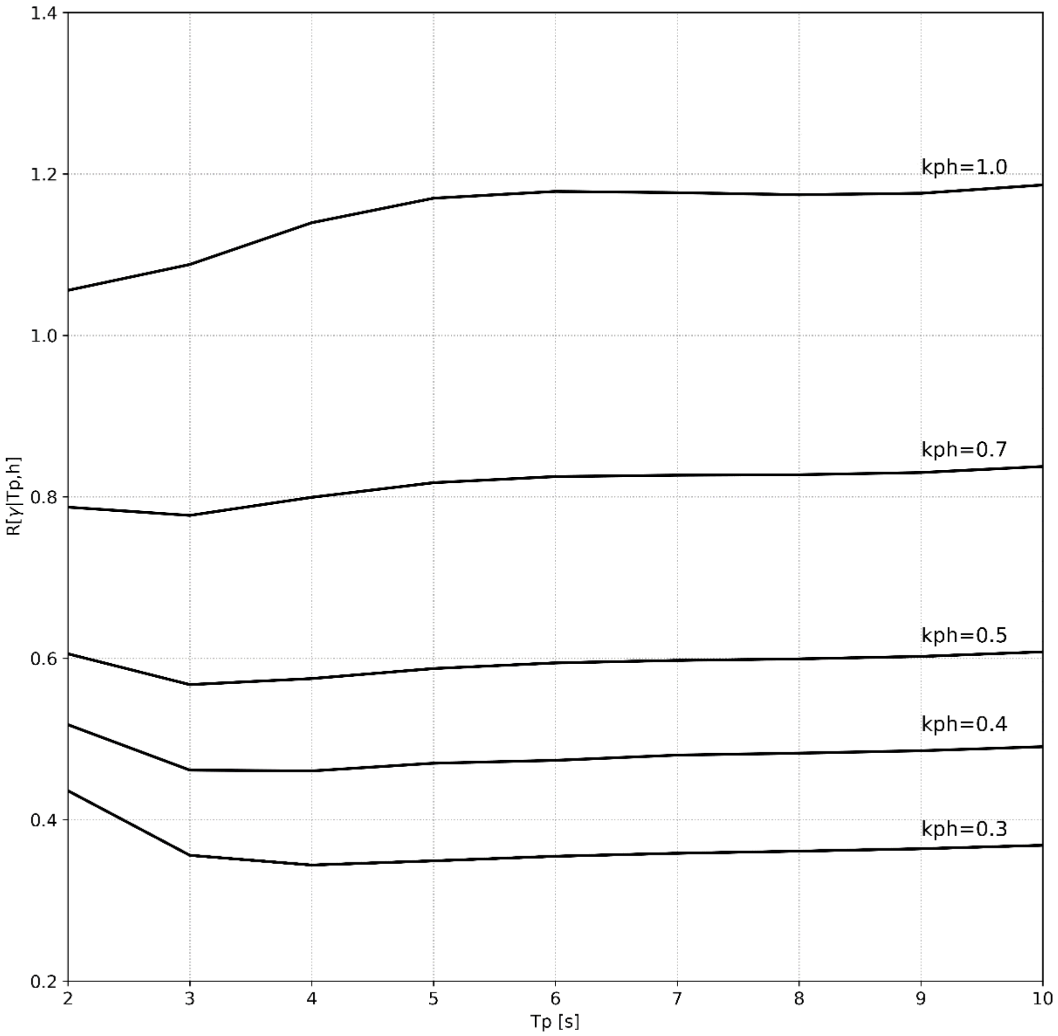

Fig. 2 $R\left[\gamma \mid T_p, h\right]$ versus T_p and $k_p h = (2\pi / T_p)\sqrt{h/g}$.