Letter

Temporarily enhanced superconductivity from magnetic fields

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Contrary to the expected detrimental influence on superconductivity when applying a magnetic field, we predict that the abrupt onset of such a field can temporarily strongly enhance the superconducting order parameter. Specifically, we find that the supercurrent in a Josephson junction with a normal metal weak link can increase more than twentyfold in this way. The effect can be understood from the interplay between the energy dependence of Andreev reflection and the abrupt spin-dependent shift in the distribution functions for excitations in the system. The duration of the increase depends on the inelastic scattering rate in the system and is estimated to be in the range of nanoseconds. We demonstrate this by developing a method which solves the Usadel equation for an arbitrary time dependence. This enables the study of ultrafast time-dependent physics in heterostructures combining superconductors with different types of materials.

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Introduction. Time-dependent phenomena in superconductors encompass a variety of both applied and fundamental physics. These phenomena range from the perfect voltage-to-frequency conversion via the AC Josephson effect to excitation of the amplitude mode of the superconducting order parameter, which is the condensed-matter equivalent of the Higgs boson in the standard model.

More recently, interest in time-dependent phenomena in superconductors has been generated by experiments showing optically induced transient states with superconducting properties well above the equilibrium critical temperature [1–3]. In superconducting heterostructures it has also been shown that microwaves can greatly increase the critical current [4,5]. This was given a theoretical explanation based on quasiclassical Green's functions [6]. Another application of quasiclassical Green's functions has been to show that time-dependent exchange fields can produce odd-frequency superconductivity which survives for long distances inside ferromagnets [7,8]. This is a type of superconductivity that is interesting due to its nonlocal temporal symmetry, its direct connection to Majorana states [9], and for its resilient nature, making it practically relevant in, e.g., superconducting spintronics [9].

Discovering new time-dependent physical phenomena in superconducting structures, and explaining existing experimental results, is clearly of substantial interest. Unfortunately, a solution of the quasiclassical Green's function equation is generally not attainable, even numerically, when the system evolves in time. This is because the relevant equations, presented below, are complicated partial differential equations (PDEs) of infinite order. So far, approximate solutions have been found for periodic [6,7,10–12] and slow [13,14] temporal evolutions. Although many situations are either slow or periodic, there is still a multitude of physical systems that are unsolvable with current techniques. For instance, the transient behavior of any sudden change that is not periodic, such as a sudden increase in the applied magnetic field or voltage, would not be possible to study, even numerically, with these methods. Finding a way to solve the Usadel equation that is less restrictive on how it allows the system to evolve in time would therefore open the possibility to study a vast range of new physical phenomena.

Here we accomplish this goal and present a method solving the time-dependent Usadel equation in hybrid nanostructures that places no constraint on the time dependence. We apply this to a superconductor-normal metal-superconductor (SNS) Josephson junction with a time-dependent spin-splitting applied to the N part. Interestingly, we find that the transient behavior can involve a large increase in both the supercurrent and the superconducting order parameter. This is our main result, which stands in stark contrast to the equilibrium effect of an applied magnetic field, which is to exponentially dampen superconductivity [15].

In addition to the curious enhancement of superconductivity, which we suggest can be understood as the interplay between properties of Andreev reflection and the transient behavior of the distribution function, we show how the methodology developed herein can be used to uncover new physics in a wide range of systems. It only requires that the proximity effect is sufficiently weak. In particular, it could be used to study the mostly unexplored territory of explicit time dependence in odd-frequency superconducting condensates, both in the ballistic and diffusive limit.

Equations and notation. The quasiclassical theory is valid when the Fermi wavelength is much shorter than all other length scales. Here we shall focus on the dirty limit, which is valid when the mean free path is short. However, we note that the same derivation can be done with arbitrary impurity concentration, something that is further discussed in the Supplemental Material [16]. The relevant equation for the dirty limit is the Usadel equation [17,18],

$$D\tilde{\nabla} \circ (\check{g} \circ \tilde{\nabla} \circ \check{g}) + i(\check{\sigma} \circ \check{g} - \check{g} \circ \check{\sigma}) = 0. \tag{1}$$

Here, D is the diffusion coefficient, the 8×8 matrix

$$\check{g} = \begin{pmatrix} \hat{g}^R & \hat{g}^K \\ 0 & \hat{g}^A \end{pmatrix} \tag{2}$$

is the isotropic part of the impurity-averaged quasiclassical Green's function, $\check{\sigma}$ is a self-energy that depends on the specific system, and

$$\tilde{\nabla} \circ \check{g} = \nabla \check{g} - ie(\hat{\boldsymbol{a}} \circ \check{g} - \check{g} \circ \hat{\boldsymbol{a}}) \tag{3}$$

is the covariant derivative. The vector $\hat{\boldsymbol{a}}$ includes the effect of the vector potential, but it could also incorporate spin-orbit effects [19,20]. The electron charge is $e = -|e|$. Finally, the circle product is

$$a \circ b = \exp\left(\frac{i}{2}\partial_\varepsilon^a \partial_T^b - \frac{i}{2}\partial_T^a \partial_\varepsilon^b\right) ab, \tag{4}$$

which is what makes Eq. (1) difficult when the constituents depend on the center-of-mass time T. The superscripts in Eq. (4) denote which function the operators acts on and ε is energy. The superscripts R, K, and A are used to denote the upper-left, upper-right, and lower-right 4×4 blocks of 8×8 matrices, respectively.

Equation (1) can be made dimensionless by dividing every term by the Thouless energy, $\varepsilon_{\mathrm{T}} := D/L^2$, where L is the length of the system. With this one can define dimensionless quantities, where lengths are given in multiples of L and energies are given in multiples of ε_{T}. Dimensionless quantities will be used in the rest of this paper. We also use natural units throughout, meaning that $c = \hbar = 1$.

Quasiclassical theory is invalid at interfaces between different materials. Consequently, one needs boundary conditions in order to connect the Green's functions in different materials. Here we use the Kupriyanov-Lukichev boundary condition [21],

$$\boldsymbol{e}_n \cdot (\check{g}_i \circ \tilde{\nabla} \circ \check{g}_i) = \frac{z}{2}(\check{g}_i \circ \check{g}_j - \check{g}_j \circ \check{g}_i), \tag{5}$$

which is valid for low-transparency tunneling interfaces. The subscripts i and j label the two different regions, the unit normal vector $\boldsymbol{e}_n$ points out of region i, and z is the ratio between the bulk resistance of a part of the material that is of length L and the interface resistance. Although we use the Kupriyanov-Lukichev boundary condition here, the same method could also be used with other types of boundaries [22].

The quasiclassical Green's function satisfies the normalization condition $\check{g} \circ \check{g} = 1$ and the relations

$$\hat{g}^A = -\hat{\rho}_3 (\hat{g}^R)^\dagger \hat{\rho}_3, \qquad \hat{g}^K = \hat{g}^R \circ h - h \circ \hat{g}^A, \tag{6}$$

where $\hat{\rho}_3 = \mathrm{diag}(1, 1, -1, -1)$. From Eq. (6) one can see that it is sufficient to solve for the retarded Green's function $\hat{g}^R$ and the distribution function h. Equation (1) does not fully specify h, and we can use this freedom to make h block-diagonal [23].

Finally, we use capital letters to denote Fourier transforms,

$$F(t, T, \boldsymbol{r}) \equiv \mathcal{F}(f)(t, T, \boldsymbol{r}) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\varepsilon f(\varepsilon, T, \boldsymbol{r}) e^{-i\varepsilon t}, \tag{7}$$

and $\bullet$ to denote the circle product between functions of the relative time t, that is, $\bullet$ is the mathematical operation which satisfies $\mathcal{F}(f \circ g) = F \bullet G$.

The aim is to find the Green's function that solves Eq. (1) in a region that is connected through the boundary condition in Eq. (5) to a region with Green's function $\check{g}_s$. This region could, for instance, be a superconducting reservoir. We have developed a method which solves the Usadel equation with an arbitrary time dependence, allowing for the study of quantum quenches and ultrafast dynamics, and present this method below.

The first step is to write the retarded Green's function as $\hat{g}^R = \hat{\rho}_3 + \hat{g} + \hat{f}$, where $\hat{g}$ and $\hat{f}$ are block-diagonal and block-antidiagonal, respectively. Under the assumption that the proximity effect is small, the components of $\hat{g}$ and $\hat{f}$ are all much smaller than 1. One way to formalize this is to Taylor expand $\hat{g}$ and $\hat{f}$ in terms of the interface parameter z. When $\check{\sigma}^R$ is block-diagonal and $z = 0$, we find that $\hat{g}^R = \hat{\rho}_3$ solves the Usadel equation. Hence, assuming $\check{\sigma}^R$ is block-diagonal to lowest order in z, we can write

$$\hat{f} = \sum_{n=1}^{\infty} z^n \hat{f}_n \quad \text{and} \quad \hat{g} = \sum_{n=1}^{\infty} z^n \hat{g}_n. \tag{8}$$

From the normalization condition $\hat{g}^R \circ \hat{g}^R = 1$, we see that $2\hat{\rho}_3\hat{g} + \hat{g} \circ \hat{g} = -\hat{f} \circ \hat{f}$ and $\hat{g} \circ \hat{f} = -\hat{f} \circ \hat{g}$. Hence, $\hat{g}_1 = 0$ and $\hat{g}_2 = -\frac{1}{2}\hat{\rho}_3 \hat{f}_1 \circ \hat{f}_1$.

To first order in z, the retarded part of the Usadel equation reads

$$\hat{\rho}_3 \tilde{\nabla} \circ (\tilde{\nabla} \circ \hat{f}_1) + 2i\varepsilon \hat{\rho}_3 \hat{f}_1 + i(\hat{\sigma}^R \circ \hat{f}_1 - \hat{f}_1 \circ \hat{\sigma}^R) = 0, \tag{9}$$

where $\varepsilon\hat{\rho}_3$ has been extracted from the self-energy and $\hat{\sigma}^R$ is the remaining part. The self-energy $\hat{\sigma}^R$ could also depend on $\hat{g}^R$, for instance, if the system included spin-orbit impurity scattering or spin-flip scattering [24]. In that case Eq. (9) would look slightly different, but the derivation would be similar. To first order in z, the boundary condition (5) reads

$$\boldsymbol{e}_n \cdot \tilde{\nabla} \circ \hat{f}_1 = \hat{f}_s. \tag{10}$$

Despite being linearized, Eqs. (9) and (10) are not much simpler than the original Usadel equation and Kupriyanov-Lukichev boundary condition. They still include the circle product, given in Eq. (4), meaning that they are still PDEs of infinite order. However, one observation can be made which will drastically simplify the equations. This is the fact that all the circle products are between $\hat{f}_1$ and functions that are independent of energy ε. It is this fact, not that the equations are linear, that is crucial for the solvability of Eqs. (9) and (10). As we shall see, this observation allows us to evaluate all the circle products if we first Fourier transform the equations.

When a function $(\varepsilon, T) \mapsto a(T)$ is independent of ε, the Fourier transform, as given by Eq. (7), is simply $A(t, T) = \delta(t)a(T)$, where δ is the Dirac δ distribution. Accordingly, the circle products of a function $(\varepsilon, T) \mapsto f(\varepsilon, T)$ with a function $(\varepsilon, T) \mapsto a(T)$ are, in Fourier space,

$$(A \bullet F)(t, T) = a(T + t/2)F(t, T), \tag{11a}$$

$$(F \bullet A)(t, T) = F(t, T)a(T - t/2). \tag{11b}$$

With this, all the circle products in Eq. (9) turn into normal matrix multiplications when evaluated in Fourier space. This is under the assumption that the self-energy $\hat{\sigma}^R$ does not

depend explicitly on ε. However, it can depend implicitly on energy through its dependence on $\check{g}$, as mentioned above.

Let the subscripts + and − denote $B_\pm(t,T) = b(T \pm t/2)$. Then the equations for the retarded Green's function become

$$2\frac{\partial \hat{F}_1}{\partial t} = \nabla^2 \hat{F}_1 + 2i(\nabla \hat{F}_1 \cdot \hat{\boldsymbol{A}}_- - \hat{\boldsymbol{A}}_+ \cdot \nabla \hat{F}_1) + i(\hat{F}_1 \nabla \cdot \hat{\boldsymbol{A}}_- - \nabla \cdot \hat{\boldsymbol{A}}_+ \hat{F}_1) - \hat{\boldsymbol{A}}_+^2 \hat{F}_1 + \hat{\boldsymbol{A}}_+ \hat{F}_1 \hat{\boldsymbol{A}}_- - \hat{F}_1 \hat{\boldsymbol{A}}_-^2 + i\hat{\rho}_3(\hat{\Sigma}_+^R \hat{F}_1 - \hat{F}_1 \hat{\Sigma}_-^R), \tag{12a}$$

$$\boldsymbol{e}_n \cdot [\nabla \hat{F}_1 - i(\hat{\boldsymbol{A}}_+ \hat{F}_1 - \hat{F}_1 \hat{\boldsymbol{A}}_-)] = \hat{F}_s. \tag{12b}$$

Hence an approximate solution to the full time-dependent Usadel equation can be found by solving a normal PDE of matrices. The approximation is good as long as the proximity effect is weak and, crucially, no assumptions have been made with regard to the time dependence. This approach therefore works for systems that vary both fast and slow in time and regardless of whether or not the system is periodic. The equations for the distribution function h can be obtained in a similar way. This is shown in the Supplemental Material [16].

Application. We now use the above framework to show the counterintuitive result that the abrupt onset of a magnetic field can temporarily strongly increase superconducting order. Consider an SNS junction with no vector potential and a time-dependent, spatially uniform exchange field $m(T)$ that lifts the spin degeneracy of the bands. The geometry is shown in the inset of Fig. 1, where the nanowire geometry allows us to neglect the orbital effect of the magnetic field whereas the thick superconducting regions screen the effect of the magnetic field in the bulk. The self-energy associated with the exchange field is $\hat{\sigma}^R = m\mathrm{diag}(1,-1,1,-1)$. We also include the effect of inelastic scattering through the relaxation time approximation [6], which adds

$$\check{\sigma}_i = \begin{pmatrix} i\delta\hat{\rho}_3 & 2i\delta\hat{\rho}_3 h_{\mathrm{eq}} \\ 0 & -i\delta\hat{\rho}_3 \end{pmatrix} \tag{13}$$

to the self-energy. Here δ is the inelastic scattering rate and $h_{\mathrm{eq}}(\varepsilon) = \tanh(\beta\varepsilon/2)$, where β is the inverse temperature towards which the system relaxes.

If we write the upper-right block of $\hat{F}_1$ as $F_1 = \sigma_1 F_t + \sigma_2 F_s$, where σ_1 and σ_2 are Pauli matrices, the zeroth-order distribution function $H_0 = H_L I_4 + H_{\mathrm{TS}}\mathrm{diag}(1,-1,-1,1)$, and let $m^\pm(t,T) := m(T+t/2) \pm m(T-t/2)$, we find that

$$\left(2\frac{\partial}{\partial t} - \nabla^2 + 2\delta\right)\begin{pmatrix} F_s \\ F_t \end{pmatrix} = \begin{pmatrix} -m^+ F_t \\ m^+ F_s \end{pmatrix}, \tag{14a}$$

$$\boldsymbol{e}_n \cdot \nabla F_s|_{x=0,1} = F_{l,r}^{\mathrm{BSC}}, \quad \boldsymbol{e}_n \cdot \nabla F_t|_{x=0,1} = 0, \tag{14b}$$

$$\left(\frac{\partial}{\partial T} + 2\delta\right)\begin{pmatrix} H_L - H_{\mathrm{eq}} \\ H_{\mathrm{TS}} \end{pmatrix} = \begin{pmatrix} -m^- H_{\mathrm{TS}} \\ m^- H_L \end{pmatrix}, \tag{14c}$$

where $F_l^{\mathrm{BCS}} = \Delta e^{-\delta t} J_0(|\Delta| t)\theta(t)$ and $F_r^{\mathrm{BCS}} = e^{i\phi} F_l^{\mathrm{BCS}}$ are the anomalous Green's functions in the left and right superconductors, respectively. J_0 is the zeroth-order Bessel function of the first kind, Δ is the superconducting gap parameter, and ϕ is the phase difference between the two superconductors. Equation (14) can be solved analytically for arbitrary $m(T)$, and the solution is shown in the Supplemental Material [16].

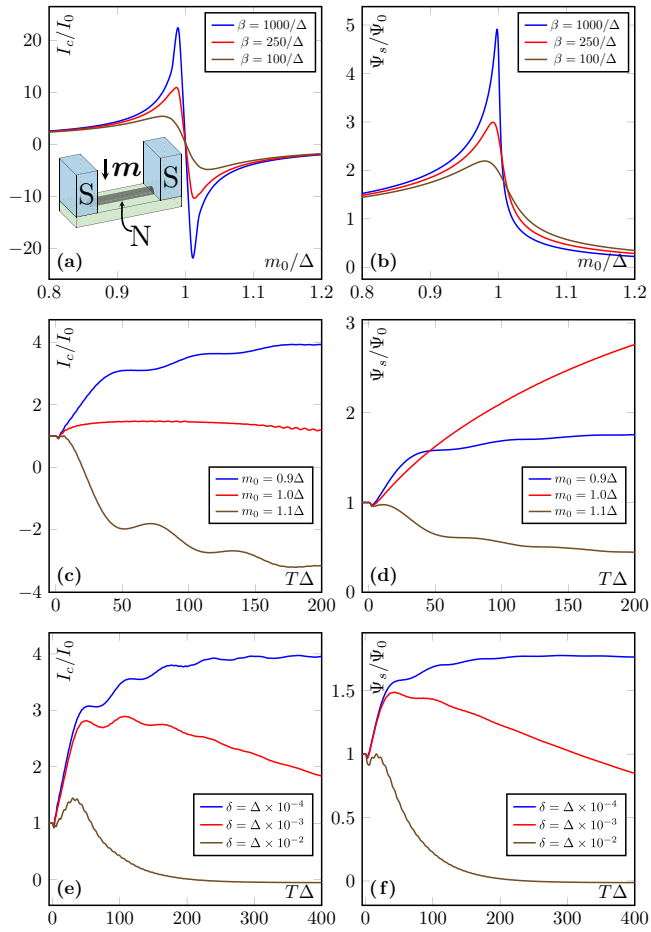

FIG. 1. Critical current I_c and singlet Cooper pair correlation function Ψ_s normalized by the values at zero exchange field, I_0 and Ψ_0. The inset in (a) shows a sketch of the setup. Panels (a) and (b) show the steady-state values obtained with $\delta = 0$ for various values of exchange field m_0. Panels (c) and (d) show the time evolution for different values of m_0 with $\delta/\Delta = 10^{-4}$ and $\beta\Delta = 1000$. Panels (e) and (f) show the time evolution for different values of δ with $m_0/\Delta = 0.9$ and $\beta\Delta = 1000$. In all cases $\Delta/\varepsilon_{\mathrm{T}} = 10$. Ψ_s is computed for $\phi = 0$, which makes the enhancement predicted here applicable also to an SN junction.

The interface parameter z is assumed small enough to fulfill the criterion of a weak proximity effect for all relevant times t and T.

Consider an exchange field that abruptly changes value from 0 to m_0 at time $T = 0$, $m(T) = m_0\theta(T)$. The critical supercurrent

$$I_c = \max_{\phi\in(0,2\pi]} \frac{\pi N_0 e D}{4} \mathrm{Tr}[\hat{\rho}_3(\check{G} \bullet \tilde{\nabla} \bullet \check{G})^K]_{t=0} \tag{15}$$

and the singlet Cooper pair correlation function

$$\Psi_s = -i\pi N_0 (F_s \bullet H_L - F_t \bullet H_{\mathrm{TS}})|_{t=0}, \tag{16}$$

following an abrupt change in the exchange field, are shown in Fig. 1. When the time becomes comparable to the inelastic scattering time, both I_c and Ψ_s are suppressed and the quantities reach their equilibrium values. However, before that, I_c and Ψ_s are significantly enhanced when the exchange field is

close to the superconducting gap Δ. When $m_0 \ll \Delta$ there is only a slight change to the current and Cooper pair correlation function.

We suggest that the behavior of I_c and Ψ_s can be understood from the interplay between the spin dependence of the nonequilibrium distribution function and the energy dependence of both the Andreev reflection probability and the degree of coherence between the participating electrons and holes. To see this, we note that in a time-independent situation both I_c and Ψ_s, jointly denoted A below, can be written as an integral over energy of the form

$$A = \int d\varepsilon (a_\uparrow h_\uparrow + a_\downarrow h_\downarrow), \tag{17}$$

as shown in the Supplemental Material [16]. Here $h_\uparrow$ and $h_\downarrow$ are the distribution functions for electrons with spin $\uparrow$ and $\downarrow$. The explanation can be summarized as three key points.

First, $a_{\uparrow,\downarrow}$ is of largest amplitude at energies close to $\pm\Delta$ and $\pm m$, where $a_\uparrow$ is large close to $\varepsilon = -m$ and $a_\downarrow$ is large close to $\varepsilon = m$. These energies are special in the context of Andreev reflections, which is the process relevant for transferring superconductivity into the normal metal. At $\varepsilon = \pm\Delta$ there is a large peak in the Andreev reflection probability [25] which physically can be understood as resonant scattering produced by subsequent reflections by the interface and the superconducting order parameter [26]. At $\varepsilon = \mp m$ the wave vector of the incoming electrons, $k^e_{\uparrow\downarrow} = \sqrt{2m(E_F + \varepsilon \pm m)}$, match that of the retroreflected holes, $k^h_{\downarrow\uparrow} = \sqrt{2m(E_F - \varepsilon \mp m)}$. Hence, at energies close to $\pm m$ the superconducting correlations penetrate far into the normal metal.

Second, $a_{\uparrow,\downarrow}$ is antisymmetric close to $\mp m$, as long as $m < \Delta$. This is shown in the Supplemental Material [16]. That is, filled states with energy just above $\mp m$ contribute oppositely to filled states with energy just below $\mp m$. Hence, when $m > 0$ and the system is at equilibrium, such that $h_\uparrow(\varepsilon) = h_\downarrow(\varepsilon) = \tanh(\beta\varepsilon/2)$, the contributions to Ψ_s and I_c are suppressed because the coherent states are shifted away from the Fermi surface. However, before inelastic scattering relaxes the system, we find that the distribution functions evolve toward $h_{\uparrow,\downarrow}(\varepsilon) = \tanh[(\varepsilon \pm m)/\beta]$. This is physically reasonable, since an abrupt temporal change induced by the magnetic field not only shifts the energy levels but also preserves the occupation of these states before they have had time to relax. The energy shift in the antisymmetric contribution to $a_{\uparrow,\downarrow}$ coming from the coherent Andreev reflections are thus matched by a similar shift in the distribution function, so Ψ_s and I_c are not suppressed as m is increased.

Third, when $m \approx \Delta$ the enhanced probability of Andreev reflections amplifies the contribution from $\varepsilon \approx m$. In equilibrium, both the positive and negative contributions are amplified, so the overall effect is still a suppression of Ψ_s and I_c when compared to $m = 0$. However, in the transient period with $h_{\uparrow,\downarrow}(\varepsilon) = \tanh[(\varepsilon \pm m)/\beta]$ the consequence is a manifold increase in Ψ_s and I_c. In other words, when $m \approx \Delta$ the Andreev reflections with the longest lifetimes are also the ones with the highest probability of occurring, and the nonequilibrium distribution functions that are present before the system has had time to relax allows this to manifest as a strong enhancement in superconductivity.

We find that the timescale for which the I_c and Ψ_s are able to reach their amplified states is given primarily by Δ. Hence, in order to experimentally detect the enhanced supercurrent it is necessary that δ/Δ is not too large. From Fig. 1 one can see that $\delta < 10^{-2}\Delta$ is sufficient to observe an increase in the supercurrent. Experimental values of the inelastic scattering rate, or Dynes parameter, are often found by parameter fitting, and values as low as $\delta/\Delta = 2.2\times10^{-5}$ have been reported in the millikelvin regime [27]. With $\Delta \approx 1$ meV and $\delta/\Delta = 2.2\times10^{-5}$, the relaxation time is about 10ns. A Zeeman splitting of 1 meV is achieved with a magnetic field strength of around 30 T/g, where g is the Landau factor. This could be either tens of T if $g = 2$ or tens of mT when $g \approx 10^3$. The latter can be found, for instance, in Dirac semimetals [28]. In the former case, an Ising-type superconductor such as $NbSe_2$ can be used to retain superconductivity at high in-plane fields.

The strong enhancement of the proximity-induced singlet order parameter Ψ_s suggests that the order parameter in the superconductor, if solved for self-consistently, could potentially also be enhanced by virtue of the inverse proximity effect. In turn, this would imply an increase in the critical temperature T_c of the superconducting transition. We leave this issue, which requires complicated time-dependent, self-consistent numerical calculations, for a future work.

Conclusion. We have presented a method for solving the time-dependent Usadel equation with arbitrary time dependence. This is made possible by two observations. First, the circle products simplify considerably in Fourier space when one of the arguments is independent of energy; second, by linearizing the equations, only such products remain.

We applied this method to analytically study SNS junctions with time-dependent Zeeman splitting m where a magnetic field is abruptly turned on. We demonstrated a strong enhancement of the supercurrent and Cooper pair correlation function when $m \approx \Delta$, where Δ is the superconducting gap. In particular, if the inelastic scattering rate δ is smaller than $\Delta\times10^{-2}$ and the magnetic field changes value during a time frame shorter than $1/\delta$, our results show an up to twentyfold increase in the magnetic field that potentially lasts for tens of nanoseconds.

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