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6. The ICE Carbon (EUC) and Brent Oil Contracts: Volatility (Co-)Movements and Forecasts

Per Bjarte Solibakke

Abstract This research looks at the conditional mean and volatility densities for the nearest maturities of renewable Carbon and fossil Brent Oil Futures contracts. The primary goal is to characterize the features of volatility across commodity financial markets. Serial and cross-correlation are reported via a Kalman filter and the explicit volatility projection. The enhanced cross-lags should supplement available derivative trading strategies with step-ahead volatility information.

Keywords Stochastic Volatility I Bayesian Estimators I Metropolis-Hastings algorithm I Markov Chain Monte Carlo (MCMC) Simulations I Projection-Reprojection

6.1 INTRODUCTION

The chapter applies a semi-parametric nonlinear model to investigate characteristics of the conditional mean and volatility densities for the ICE Carbon front December and the ICE Brent Oil front month future contracts for the period 2011 to 2021. The chapter uses multifactor stochastic volatility models to obtain step-ahead volatility forecasts for the two contracts. Stochastic volatility (SV) models have an intuitive and simple structure and can explain the major stylized facts of asset, currency and commodity price movements (Solibakke, 2020). Time-varying volatility is endemic in financial markets, and SV models are the main way this time-varying volatility is modelled (Shephard and Andersen, 2009). The motivation for the use of SV models is therefore mainly threefold: Firstly, the number of events is unpredictable on day *t* (Taylor, 1982). The SV methodology is proportional to the number of day *t* events. Secondly, the trading clock runs at different intensities on different days (time deformation) where the clock is often represented by trading volume (Clark, 1973). Finally, Hull and White (1987) show that SV models are a good approximation to diffusion processes for continuous volatility variables (closely related to realized variance). Volatility is a measure of dispersion around the mean return of an asset. When the price returns are tightly bunched together, the volatility is small; conversely, when they are spread apart, the volatility is large. The use of all volatility models entails prediction characteristics for future returns. A special feature of asset volatility is that it is not directly observable. The unobservability of volatility makes it difficult to evaluate the forecasting performance of volatility models. However, market participants who understand the dynamic behaviour of volatility are more likely to have realistic expectations about future prices and the risks to which they are exposed. The step-ahead volatility forecasts are useful to traders of variance swaps[1]. For example, when forecasts are used for trading variance swaps, signals to buy or sell can be obtained by comparing the volatility forecast with the implied volatility (Andersen et al., 2003). Moreover, both the ICE Carbon and Brent Oil have a market for listed derivatives. Bearing in mind that volatility is for most instruments non-traded, which suggests imperfect estimates, the volatility can be interpreted as a latent variable that can be modelled and predicted through its direct influence on the magnitude of returns. The chapter uses the *Bayesian Markov Chain Monte Carlo* (MCMC) modelling strategy used by Gallant and McCulloch (2020) and Gallant and Tauchen (2010a, 2010b)[2]. The method is a systematic approach to generate moment conditions for the *generalized method of moments* (GMM) estimator (Hansen, 1982) of the parameters of a structural model. Moreover, the implemented Chernozhukov and Hong (2003) estimator keeps model parameters in the region where predicted shares are positive for every observed price/expenditure vector. The computationally intensive method enables efficient estimates of parametric SV models. Moreover, the methodology supports restrictions, inequality restrictions, and informative prior information (on model parameters and functionals of the model).

1 A variance swap is a swap between a floating rate and a fixed rate (the variance swap rate). The swap is a pure volatility trade. There are numerous trading applications of variance swaps, including spread trades on forward volatility and on the spread between volatilities on different underlyings. They also provide a natural diversification for long equity investors, since there is strong negative correlation between their pay-offs and returns on equities.

2 The methodology is designed for estimation and inference for models where (1) the likelihood is not available, (2) some variables are latent (unobservable), (3) the variables can be simulated and (4) there exists a well-specified and adequate statistical model for the simulations. The methodologies – *General Scientific Models* (GSM) and *Efficient Method of Moments* (EMM) – are general-purpose implementation of the Chernozhukov and Hong (2003) estimator. That is, the applications for methodologies are not restricted to simulation estimators.

In the SV methodology, the distribution of returns is modelled indirectly (via the structure of the model), and as indicated by the methodology above, the likelihood function is not directly observable. Asset pricing theory implies that higher rewards are required as an asset is exposed to more systematic risk. Knowing that risks change through time in complicated ways, it seems natural to build stochastic models. These models bring financial economics closer to the empirical reality, allowing better decision making, inspiring new theories, and improving model building[3]. Among other features, the SV methodology consistently measures correlation between factors enabling explicit potential return-volatility correlations inducing mean and volatility skewness. That is, negative returns show the lower tail of the log returns distribution that is long and thin, while positive returns show the upper tail of the log return distributions that will be light. Similarly, negative correlation between volatility factors suggests a negative co-movement in volatility, while the opposite is true for positive correlation. Moreover, the number of stochastic factors capability of SV models makes the specification flexible and extendable. For example, a specification with two stochastic volatility factors with consistent correlation structures enables both persistent and strongly mean-reverting volatility factors explicitly detailing the volatility densities. This volatility information allows for data dependence analysis suggesting any form of predictability. In comparison, *general autoregressive conditionally heteroscedasticity* (GARCH) processes, often described as SV, do not follow this nomenclature[4]. These models explicitly model the conditional variance, given past returns observed by the econometrician. The rest of the chapter is organized as follows: Section 6.2 describes the methodology. Section 6.3 presents correlation results, and section 6.4 concretizes these facts from stochastic volatility models. Section 6.5 summarizes and concludes.

6.2 THEORY AND METHODOLOGY

6.2.1 Stochastic volatility models

The SV approach specifies the predictive distribution of price returns indirectly via the structure of the model, rather than directly. The SV model has its own

3 The close connection between SV and realized volatility has allowed financial econometricians to harness the enriched information set available through high-frequency data to improve, by order of magnitude, the accuracy of their volatility forecasts over that traditionally offered by GARCH models based on daily observations. The applications of SV have therefore broadened into the important arena of risk assessment and asset allocation.

4 See Joshua and Grant (2015) and Byon and Cho (2013) and references therein.

stochastic process without considering the implied one-step-ahead distribution of returns recorded over an arbitrary time interval convenient for the econometrician. The starting point is the application of Andersen, Benzoni, and Lund (2002) considering the familiar stochastic volatility diffusion for an observed stock price S_t given by $\frac{dS_t}{S_t} = \left(\mu + c\left(V_{1,t} + V_{2,t}\right)\right)dt + \sqrt{V_{1,t}}dW_{1,t} + \sqrt{V_{2,t}}dW_{2,t}$, where the unobserved volatility processes $V_{i,t}$, i = 1,2, is either log linear or square root (affine). The $W_{1,t}$ and $W_{2,t}$ are standard Brownian motions that are possibly correlated with $corr(dW_{1,t}, dW_{2,t}) = \rho$. Andersen et al. (2002) estimate both versions of the stochastic volatility model with daily S&P 500 stock index data, from 1953 through 31 December 1996. Both SV model versions are sharply rejected. However, adding a jump component to a basic SV model greatly improves the fit, reflecting two familiar characteristics: thick non-Gaussian tails and persistent time-varying volatility. An SV model with two stochastic volatility factors shows encouraging results in Chernov et al. (2003). The authors consider two broad classes of setups for the volatility index functions and factor dynamics: an affine setup and a logarithmic setup. The models are estimated using daily data on the DOW Index, from 2 January 1953 to 16 July 1999. They find that models with two volatility factors do much better than models with only a single volatility factor. They also find that the logarithmic two-volatility factor models outperform affine jump diffusion models and provide an acceptable fit to the data. One of the volatility factors is extremely persistent and the other strongly mean-reverting. The chapter's SV model applies the logarithmic model with two stochastic volatility factors (Chernov et al., 2003). The model is extended to facilitate correlation between the mean (W_{1t}) and the two stochastic volatility factors (W_{2t}, W_{3t}). The main argument for the correlation modelling is to introduce asymmetry effects (correlation between return innovations and the two volatility innovations)[5].

6.2.2 The unobserved state vector using the nonlinear Kalman filter technique

A Kalman filter is an algorithm for sequentially updating a projection for the dynamic system. The algorithm provides a way to calculate exact finite-sample forecasts. From the prior SV model estimation, one by-product is a long-simulated realization of the volatility state vector $\left\{\hat{V}_{i,t}\right\}_{t=1}^{N}, i = 1,2$ and the corresponding

5 See Solibakke (2020), for a detailed specification of a two-factor stochastic volatility model.

returns $\{\hat{y}_t\}_{t=1}^{N}$ for the optimal estimated parameters $\phi = \hat{\phi}$. Hence, by calibrating the functional form of the conditional distribution of volatility functions $\hat{V}_{i,t}, i = 1,2$ given the simulated returns $\{\hat{y}_\tau\}_{\tau=1}^{t}$; evaluating the result on observed returns $\{\tilde{y}_t\}_{t=1}^{n}$; and generating predictions for volatilities $\hat{V}_{i,t}, i = 1,2$ through Kalman filtering returns y_t, very general functions of $\{y_\tau\}_{\tau=1}^{t}$ can be used and a huge data set is available. An SNP model is re-estimated on the simulated returns $\hat{y}_t$, remembering that the model provides a convenient representation of the one-step-ahead conditional variance $\hat{\sigma}_t^2$ of simulated returns $\hat{y}_{t+1}$ given the long simulated returns $\{\hat{y}_\tau\}_{\tau=1}^{t}$. Regressions are run off $\hat{V}_{i,t}, i = 1,2$ on $\hat{\sigma}_t^2$, $\hat{y}_t$ and $|\hat{y}_t|$ and lags (generously long) of these series. These functions are evaluated on the observed return series $\{\tilde{y}_\tau\}_{\tau=1}^{t}$, which give volatility values $\tilde{V}_{i,t}, i = 1,2$ for the two volatility factors at the original data points (Solibakke, 2020). That is, the available data set now consists of $\{\tilde{y}_\tau\}_{\tau=1}^{t}$, $\{\tilde{V}_{1,\tau}\}_{\tau=1}^{t}$ and $\{\tilde{V}_{2,\tau}\}_{\tau=1}^{t}$, where t is the length of the original data set.

6.3 THE ICE CARBON AND THE BRENT OIL CONTRACT SERIES AND SNP DENSITIES

6.3.1 The ICE Carbon and the Brent Oil contract series and stationarity

We impose weak stationarity, and the means, variance and covariances are independent of times (rather than the entire distribution). That is, a process $\{y_t\}$ is weakly stationary if for all t, it holds that $E\{y_t\} = \mu \leq \infty, V\{y_t\} = E\{(y_t - \mu)^2\} = \gamma_0 < \infty$ and $\text{cov}\{y_t, y_{t-k}\} = E\{(y_t - \mu)(y_{t-k} - \mu)\} = \gamma_k$, $k = 1,2,3,\ldots$. A shock to a stationary autoregressive process of order 1 (*AR(1)*) affects all future observations with a decreasing effect. Table 6.1 reports the characteristics of the price movements for the two series. The mean is positive Carbon and negative Brent Oil contracts. The highest extreme values are found for Carbon followed by the highest standard deviation (3.35) as expected. The Brent Oil reports highest kurtosis (23.3) followed by a negative skew of -1.15 (large dumps). The Cramer-von-Mises test statistic reports significant non-normality for both Carbon (2.89) and Brent Oil (2.89) contracts. The Q(12) and the $Q^2(12)$ correlogram statistics (serial correlation) show dependencies for both the mean and volatility for both contracts. Similarly, the 12^{th} lag ARCH test statistic (Engle, 1982) suggests highly significant conditional heteroscedasticity. The RESET test (Ramsey, 1969) reports instability. Finally, for both series, the adjusted series ADF (Dickey & Fuller, 1979) and the KPSS (Kwiatowski et al., 1992) statistics confirm stationarity and the BDS test statistic (Brock and Dechert, 1988; Brock et al., 1996) reports general nonlinear data dependence.

Figure 6.1 reports the levels and the movements series for the Carbon and Brent Oil contracts. The general appearance of the two series is typical for equity market data. We also experimented with breaking trends in the movement equations, but our results suggested little evidence for trend breaks. The *Value at Risk* (VaR) is a well-known concept of measures of risk, and Table 6.1 includes the 2.5% and 1% VaR numbers for market participants.

6.3.2 The SNP density projection

Since the conditional density completely characterizes the price movement process, the density is naturally viewed as the fundamental statistical object of interest. The semi-nonparametric (SNP) model is fitted using conventional maximum likelihood together with a model selection strategy that determines the appropriate order of expansion (BIC). The Schwarz Bayes information criterion (Schwarz, 1978) is computed as $BIC = s_n\left(\hat{\theta}\right) + \left(\frac{1}{2}\right)\left(\frac{p_p}{n}\right)\log\left(n\right)$ with small values of the criterion preferred. Table 6.2 reports the maximum likelihood (ML) estimates[6] of the parameters for the *BIC*-optimal SNP density models[7]. Firstly, for the mean, the intercept is insignificant and the serial correlations (*B[1,x]*) are not significant for Carbon but significant for Brent Oil, implying dependence (η_6). The negative correlation for Brent Oil (η_6) suggests mean reversion for these contracts. Secondly, the conditional variance coefficients (η_7 – η_9) are all strongly significant. Conditional heteroscedasticity is therefore present (η_7 – η_9). Furthermore, asymmetry (η_{10}) and level effects (η_{11}) are only present for Brent Oil (not for Carbon). For the Brent Oil contracts, the reaction from negative price movements is therefore higher than from positive movements (not reported). The largest eigenvalue of the conditional variance function P & Q companion matrix is 1.055 and 0.967 for the Carbon and Brent Oil contracts, respectively. Due to the use of an additional transformation[8] (trigonometric spline) the dictum that the sum of the squared coefficients (squares) must be less than one[9] no longer holds. Finally, the hermite functions coefficients (η_1-η_4), which capture parametric model departures, are *BIC* preferred up to the sixth polynomial lag expansions. Hence, the hermite results

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6 Based on likelihood ratio test statistics (LRT), the student-t log-likelihood function is strongly preferred to a normal likelihood function.

7 The BIC optimal SNP model is the $L_u=14, L_g=1, L_r=1, L_v=1, L_\omega=1, L_p=1, K_z=12, K_x=0$ specification.

clearly suggest departures from the classical normally distributed and parametric conditional model. The SNP projection gives access to one-step-ahead densities $f_K\left(\tilde{y}_t \mid x_{t-1}, \theta\right)$, conditional on the values for $x_{t-1} = (\tilde{y}_{t-1}, \tilde{y}_{t-2}, \ldots, \tilde{y}_{t-L})$, the densities for the conditional mean and volatility together with the conditional one-step-ahead mean densities. Moreover, simulation paths are easily obtainable using the seed for stochastics and bootstrapping. Figure 6.2 reports densities for some of these features. The mean distribution for Carbon contract seems to give a positive mean followed by a negative skewness. The Brent Oil contracts seem to give a negative mean also followed by a negative skewness. The conditional volatility distribution shows a larger right tail for the Carbon contracts than for Brent Oil. Moreover, the Brent Oil contracts seem to show a lower overall volatility.

Table 6.1: Characteristics for the ICE Carbon and Brent Oil contracts for the period 2011–2021

	Mean (all)/ M (-drop)	Median Std.dev.	Maximum / Minimum	Moment Kurt/Skew	Quantile Kurt/Skew	Quantile Normal	Cramer-von-Mises	Serial dependence Q(12)	Q2(12)	VaR (1%; 2,5%)
The ICE Carbon Futures	0,03691	0,00000	23,8234	15,14649	0,17008	3,8532	4,5778	41,3690	130,320	-9,126%
	0,03645	3,35208	-43,2077	-0,88497	0,04732	{0,1456}	{0,0000}	{0,1110}	{0,0000}	-13,509%
	BDS-Z-statistic (e = 1)					Phillips -	Augmented	ARCH	RESET	CVaR
	m=2	m=3	m=4	m=5	KPSS	Perron test	DF-test	(12)	(12;6)	(1% ; 2.5%)
	10,9510	14,5479	17,4774	19,8379	0,04469	-49,5438	-37,1216	85,0100	46,5735	-13,509%
	{0,0000}	{0,0000}	{0,0000}	{0,0000}	{0,6421}	{0,0000}	{0,0000}	{0,0000}	{0,0000}	-9,930%
The ICE Brent Oil Furures	Mean (all)/ M (-drop)	Median Std.dev.	Maximum / Minimum	Moment Kurt/Skew	Quantile Kurt/Skew	Quantile Normal	Cramer-von-Mises	Serial dependence Q(12)	Q2(12)	VaR (1%; 2,5%)
	-0,03419	0,06378	19,0774	23,29382	0,31680	12,4591	9,0716	17,963	663,42	-6,315%
	-0,03828	2,28495	-27,9761	-1,15275	-0,07439	{0,0020}	{0,0000}	{0,1170}	{0,0000}	-9,588%
	BDS-Z-statistic (e = 1)					Phillips -	Augmented	ARCH	RESET	CVaR
	m=2	m=3	m=4	m=5	KPSS	Perron test	DF-test	(12)	(12;6)	(1% ; 2.5%)
	13,8657	16,1690	18,3445	20,3371	0,07196	-50,83143	-50,6842	372,232	104,9609	-9,588%
	{0,0000}	{0,0000}	{0,0000}	{0,0000}	{0,2721}	{0,0000}	{0,0000}	{0,0000}	{0,0000}	-6,988%

8 $$\hat{x}_i = \begin{cases} \tfrac{1}{2}\left\{x_i + \tfrac{4}{\pi}\arctan\left[\tfrac{\pi}{4}(x_i + \sigma_{tr})\right] - \sigma_{tr}\right\} & -\infty < x_i < -\sigma_{tr} \\ x_i & -\sigma_{tr} < x_i < \sigma_{tr} \\ \tfrac{1}{2}\left\{x_i + \tfrac{4}{\pi}\arctan\left[\tfrac{\pi}{4}(x_i + \sigma_{tr})\right] - \sigma_{tr}\right\} & \sigma_{tr} < x_i < \infty \end{cases}$$ where x_i denotes an element of x_{t-1}.

9 Under the spline transformation, it suffices that the sum of squares of the coefficients be less than 2.

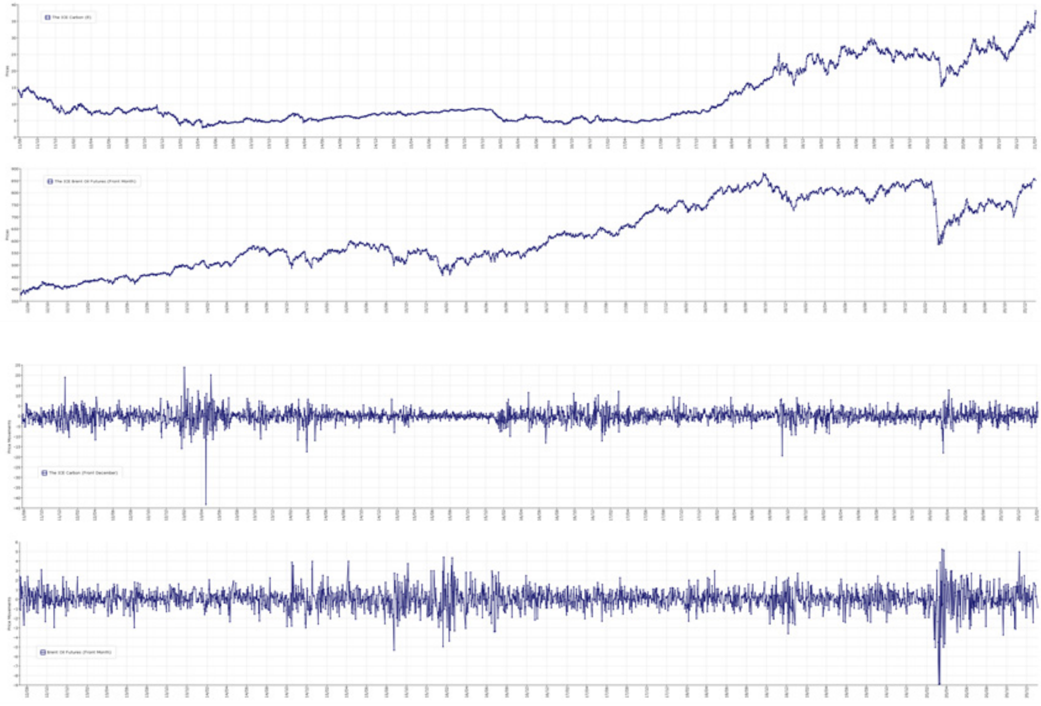

Figure 6.1: The ICE Carbon and Brent Oi contract level (top) and movements (bottom) series for the period 2011–2021.

Table 6.2: SNP-Model Projection Specification Carbon and Brent Oil contracts[10]
Statistical Model SNP (111140000) opt. BIC-fit; semi-parametric-GARCH model

		Mode and {Standard Error}			
Var	SNP Coeff.	The ICE Carbon		The ICE Brent Oil	
Hermite Polynoms					
η_1	$a_0[1]$	-0,02432	{0,0266}	-0,04087	{0,0239}
η_2	$a_0[2]$	-0,18025	{0,0196}	-0,15918	{0,0196}
η_3	$a_0[3]$	0,01124	{0,0155}	-0,02248	{0,0165}
η_4	$a_0[4]$	0,13065	{0,0114}	0,10216	{0,0162}
η_5	$a_0[5]$	-0,00105	{0,0162}	0,00464	{0,0193}
η_6	$a_0[6]$	-0,06886	{0,0172}	-0,07695	{0,0139}
Mean Equation (Correlation)					
η_5	b0[1]	0,05704	{0,0418}	0,05108	{0,0320}
η_6	B(1,1)	0,00789	{0,0213}	-0,04853	{0,0215}
Variance Equation (Correlation)					
η_7	R0[1]	0,12625	{0,0214}	0,07765	{0,0159}
η_8	P[1,1]	0,39379	{0,0357}	0,18454	{0,0524}
η_9	Q[1,1]	0,94843	{0,0057}	0,96608	{0,0047}
η_{10}	V[1,1]	0	{0,0968}	-0,34431	{0,0385}
η_{11}	W[1,1]	0	{0,0}	0,30542	{0,0842}
Model	s_n	1,23241		1,1302302	
selection	*aic*	1,23733		1,1302302	
critera:	*bic*	1,25158		1,1302302	
Largest eigenvalue for mean			0,02641		0,04853
Largest eigenvalue variance			1,05459		0,96737

10 The residual test battery is all insignificant except for the Cramer-von-Mises test for normality, which is strongly reduced in significance (not reported).

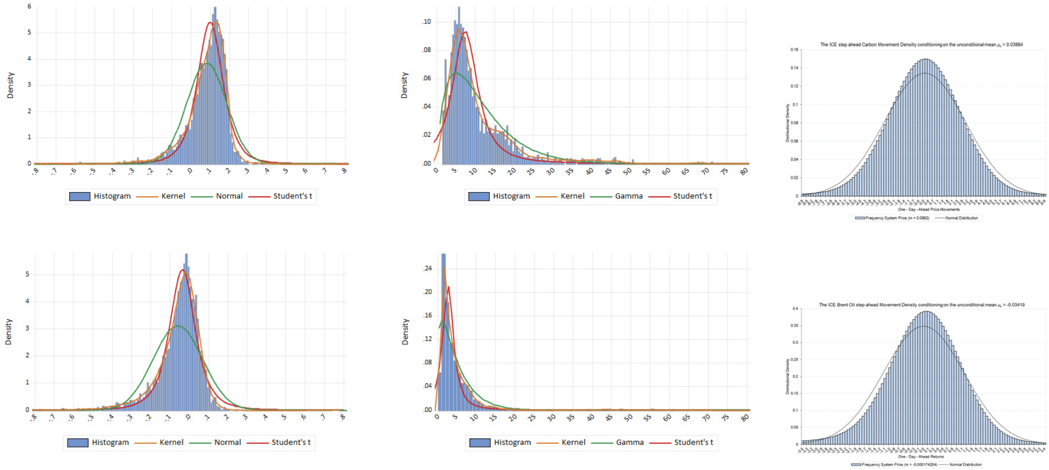

Figure 6.2: Conditional mean and volatility, and one-step-ahead conditional mean density.

The risk seems therefore lower in Brent Oil contracts relative to Carbon contracts. Furthermore, conditioning on the unconditional mean, the Carbon one-step-ahead mean distribution is much wider than for Brent Oil. However, note from Figure 6.2 (right column), the step-ahead mean conditional on the unconditional mean from Table 6.1 is about 0.088 for Carbon and -0.001 for Brent Oil. Normally, these plots therefore suggest a higher positive drift for Carbon than for the Brent Oil contracts.

6.4 STOCHASTIC VOLATILITY

The SNP methodology obtains a convenient representation of one-step-ahead conditional variance $\hat{\sigma}_t^2$ *of* $\hat{y}_{t+1}$ *given* $\{\hat{y}_\tau\}_{\tau=1}^{t}$. From the stochastic volatility model optimization in Table 6.3, we use the by-product of a long simulated realization of the state vector $\{\hat{V}_{i,t}\}_{t=1}^{N}, i=1,2$ together with the corresponding $\{\hat{y}_t\}_{t=1}^{N}$ for the optimally estimated parameter vector $\theta=\hat{\theta}$. Running regressions for V_{it} on $\hat{\sigma}_t^2$, $\hat{y}_t$ *and* $|\hat{y}_\tau|$ and a generous number of lags of theses series, we obtain calibrated functions that give step-ahead predicted values of $V_{it}\,|\{y_\tau\}_{\tau=1}^{t}$, $t=1,2$ at the original data points (see Section 6.2.1). The re-projected volatility and the two volatility factors are reported in Table 6.4 and Figure 6.3.

Table 6.3: Optimal stochastic volatility parameter values for the ICE Carbon and Brent Oil

The ICE Carbon Scientific Model				The ICE Brent Oil Scentific Model			
Parameter values Scientific Model				Parameter values Scientific Model			
θ	Mode	Mean	Standard error	θ	Mode	Mean	Standard error
a0	0,088745	0,078207	0,005025	*a0*	0,039551	0,035373	0,068131
a1	-0,015991	-0,015823	0,009387	*a1*	-0,047363	-0,051652	0,020053
b0	0,763430	0,763470	0,010869	*b0*	0,145630	0,158620	0,168230
b1	0,971250	0,970390	0,002016	*b1*	0,828000	0,824020	0,095037
c1	0	0	0	*c1*	0	0	0
s1	0,088409	0,087666	0,001208	*s1*	0,169340	0,169330	0,000983
s2	0,190860	0,190400	0,001833	*s2*	0,157070	0,157050	0,001290
r1	-0,101200	-0,099473	0,005040	*r1*	-0,331670	-0,330850	0,004434
r2	0,075989	0,076666	0,004590	*r2*	0,117550	0,113550	0,005410
Distributed Chi-square (no. of freedoms)			$\chi 2(5)$	Distributed Chi-square (no. of freedoms)			$\chi 2(5)$
Posterior at the mode			-4,1074	Posterior at the mode			-6,2121
Chi-square test statistic			{0,1420}	Chi-square test statistic			{0,0922}

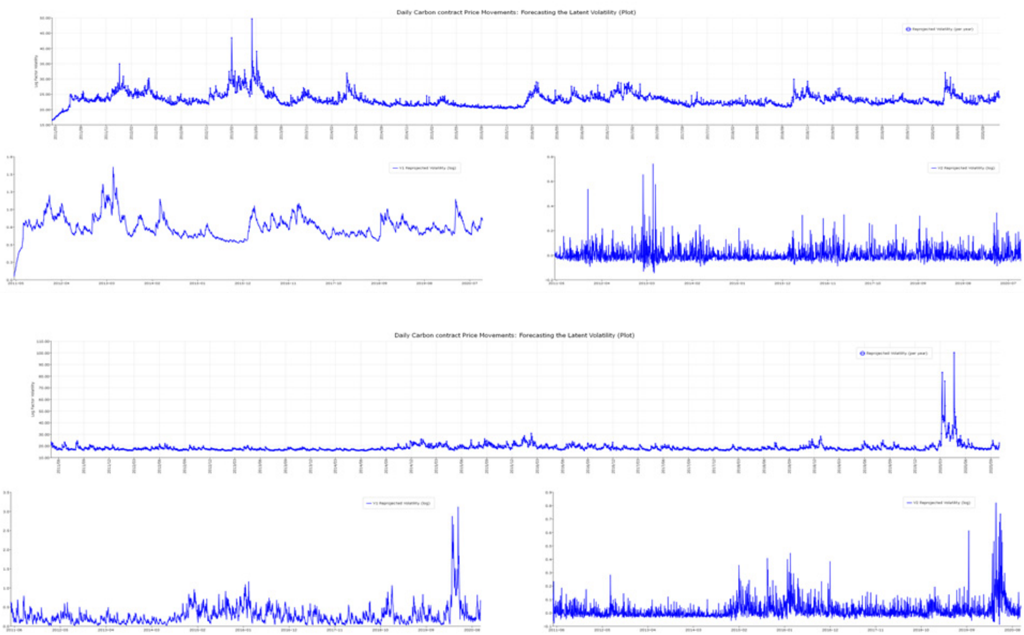

Figure 6:3: Stochastic volatility factors for the ICE Carbon and Brent Oil future contracts.

Table 6.4: Re-projected Volatility Characteristics

	Mean (all)/ Mode	Median Std.dev.	Maximum / Minimum	Moment Kurt/Skew	Quantile Kurt/Skew	Quantile Normal	Cramer-von-Mises	RESET (12;6)
The ICE Carbon Futures	23,93825	23,55800	49,5430	16,00165	0,07210	11,9477	9,3184	43,2384
	#I/T	2,03432	18,2650	2,35653	0,16862	{0,0025}	{0,0000}	{0,0000}
	BDS-Z-statistic (e = 1)				Serial dependence		Phillips -	Augmented
	m=2	m=3	m=4	m=5	Q(12)	KPSS	Perron test	DF-test
	73,2906	84,7795	97,5670	113,7141	15670,0	0,19224	-26,10131	-6,0127
	0,00000	{0,0000}	{0,0000}	{0,0000}	{0,0000}	{0,0251}	{0,0000}	{0,0000}
The ICE Brent Oil Futures	**Mean (all)/ M (-drop)**	**Median Std.dev.**	**Maximum / Minimum**	**Moment Kurt/Skew**	**Quantile Kurt/Skew**	**Quantile Normal**	**Cramer-von-Mises**	**RESET (12;6)**
	18,48380	17,54374	108,5367	162,35814	0,21062	40,0577	53,3745	135,712
	16,24495	4,10456	15,6741	10,04162	0,29835	{0,0000}	{0,0000}	{0,0000}
	BDS-Z-statistic (e = 1)				Serial dependence		Phillips -	Augmented
	m=2	m=3	m=4	m=5	Q(12)	KPSS	Perron test	DF-test
	44,1549	44,8823	44,5400	44,3185	8857,8	0,10739	-29,64429	-4,5407
	{0,0000}	{0,0000}	{0,0000}	{0,0000}	{0,0000}	{0,1135}	{0,0000}	{0,0013}

6.4.1 Volatility characteristics

The volatility factors in Figure 6.3 seem to model two different flows of information to the Carbon and Brent Oil markets and their participants. The slowly mean-reverting factor provides volatility persistence and the rapidly mean-reverting factor provides for the tails (Chernov et al., 2003). For the period 2011 to 2021, the V_1 factor for the Carbon market is clearly moving slower than for the Brent Oil contracts, possibly showing higher serial correlation. For Brent Oil contracts, both V_1 and V_2 report large realization in the start of 2020 (Covid-19). The re-projected volatility is therefore high volatility in both 2020 and 2021. However, for Carbon contracts, volatility for the Covid-19 period from March 2020 seems almost unaffected. The highest volatility period for the Carbon contracts seems to be back in 2012/13 (low prices). For both the contracts, the volatility seems to increase more from negative price changes than from positive price changes. Volatility densities for the front year and the front quarter contract series suggest lognormal densities. Furthermore, the power law $\left(\text{Prob}(\upsilon > x) = Kx^{-\alpha}\right)$ providing an alternative to the normal distributions seems approximately true for the volatility (not reported). Table 6.4 reports statistical details for Carbon and Brent Oil contracts. The statistics indicate substantial data dependence suggesting both clustering (serial correlation) and persistence. However, the statistics (unit root) still suggest mean reversion. The data dependence makes volatility predictions clearly more relevant and probably more informative to market participants.

6.4.2 Volatility co-movements

Table 6.5 reports the cross-correlations, Granger causality and Wald coefficient restriction tests (asymptotically equal to a likelihood ratio test). The Granger (1969) approach to the question of whether x causes y is to see how much of the current y can be explained by past values of y and then to see whether adding lagged values of x can improve the explanation. y is said to be Granger-caused by x if x helps in the prediction of y, or equivalently if the coefficients on the lagged x-es are statistically significant. It is important to note that the statement "*x Granger cause y*" does not imply that y is the effect or the result of x. Granger causality measures precedence and information content, but does not by itself indicate causality in the more common use of the term.

Granger causalities indicate a bivariate regression of the form:

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + ... + \alpha_l y_{t-l} + \beta_1 x_{t-1} + ... + \beta_l x_{t-l} + \varepsilon_t$$
$$x_t = \alpha_0 + \alpha_1 x_{t-1} + ... + \alpha_l x_{t-l} + \beta_1 y_{t-1} + ... + \beta_l y_{t-l} + u_t$$

for all possible pairs of (x, y) series in the group. We report the Wald statistics for the joint hypothesis $\beta_1 = \beta_2 = ... = \beta_l = 0$ for each equation. The daily Carbon front December futures and Brent Oil front month futures reject the hypothesis that Brent oil re-projected volatility *does not* Granger cause Carbon ICE futures (p = 0.004) (and not the other way ($p = 0.5632$)). The BIC optimal bivariate VAR for the re-projected Carbon volatility shows that serial correlation (VAR) is significant up to lag 23 and, suggested by Granger causality, several significant lags of Brent Oil ($\beta \neq 0$). Hence, carbon stochastic volatility reports long-run serial correlation together with significant Brent Oil causality.

The Wald test computes a test statistic based on unrestricted regression. The Wald statistic measures how close the unrestricted estimates come to satisfying the restrictions under the null hypothesis. The test statistics show results that give additional support for volatility influence from Brent Oil to Carbon contracts (and not the other way). The Brent Oil contracts report a significant Wald coefficient of 126.9 ($p = 0$) while Carbon contracts report an insignificant Wald coefficient of 21 ($p = 0.10$). Moreover, Table 6.5 (bottom) reports a Wald Brent Oil factor 1 coefficient of 52.7 ($p = 0$) on Carbon re-projected volatility. The Carbon re-projected volatility does not report significant Wald coefficients to Brent oil volatility.

Table 6.5: Causality statistics for Carbon Futures and Brent Oil Futures contracts

Causality statistics		**Carbon**		**Carbon**		**Carbon**	
		Factor 1 (V_{1t}); prob		**Factor 2 (V_{2t}); prob**		**Reprojected; prob**	
Cross-Correlation	Oil (V1t)	-0,01658	{0,4158}	0,009546	{0,6394}	-0,00803	{0,6936}
	Oil (V2t)	0,012456	{0,5410}	0,052689	{0,0097}	0,027924	{0,1705}
	Oil (Repro)	0,025722	{0,2067}	0,044921	{0,0274}	0,039238	{0,0341}
Causality from Brent Oil to Carbon							
	To:	**Carbon**		**Carbon**		**Carbon**	
	From:	**Factor 1 (V_{1t}); prob**		**Factor 2 (V_{2t}); prob**		**Reprojected; prob**	
Granger	Oil (V1t)	2,43058	{0,0022}	1,24173	{0,2373}	1,85668	{0,0265}
	Oil (V2t)	1,44935	{0,1223}	1,01604	{0,4336}	1,07715	{0,3734}
	Oil (Repro)	3,49463	{0,0000}	1,50165	{0,1021}	2,27503	{0,0044}
Wald coeff.	Oil (V1t)	109,042	{0,0000}	24,19942	{0,0483}	52,69607	{0,0000}
restrictions	Oil (V2t)	23,32456	{0,0552}	19,60125	{0,1432}	23,63356	{0,0507}
	Oil (Repro)	302,2734	{0,0000}	48,07937	{0,0000}	126,8786	{0,0000}
Causality from Carbon to Brent oil							
	To:	**Brent Oil**		**Brent Oil**		**Brent Oil**	
	From:	**Factor 1 (V_{1t}); prob**		**Factor 2 (V_{2t}); prob**		**Reprojected; prob**	
Granger	Oil (V1t)	1,12042	{0,3337}	1,03214	{0,4173}	1,01879	{0,4308}
	Oil (V2t)	2,75905	{0,0004}	1,2104	{0,2600}	1,62793	{0,0646}
	Oil (Repro)	1,43862	{0,1268}	1,39629	{0,1461}	1,42009	{0,1350}
Wald coeff.	Oil (V1t)	8,083335	{0,8849}	10,68981	{0,7102}	9,6049	{0,7905}
restrictions	Oil (V2t)	21,93805	{0,0919}	19,11454	{0,1606}	20,74074	{0,1325}
	Oil (Repro)	6,371669	{0,9562}	19,52757	{0,1458}	21,00248	{0,1016}

6.4.3 Volatility predictions

It is difficult to forecast because the realization of a stochastic process will be influenced by random events that happen in the future. In case of a large market movement at any time before the risk horizon, the forecast needs to take this into account. However, static one-step-ahead forecasts for the single assets Carbon and the Brent Oil movements are presented in Figure 6.4. The estimation period is from 2011 to 1 January 2020 and the static forecasting period from 1 January 2020 to 5 February 2021. Static forecasting performs a series of one-step-ahead forecasts of the dependent variable (Pindyck & Daniel, 1998). For each observation the forecast computes $\hat{y}_{S+k} = \hat{c}(1) + c(j)\, y_{S+k-j}$, where j is the lag number of the forecasting variable y (always using the actual value of the lagged endogenous variable), data for any lagged endogenous variables must be observed. The static daily forecasts do not contain any exogenous variables except for lagged Brent Oil variables for Carbon forecasts. For a "good" measure of fit, using the Theil inequality coefficient (bias, variance and covariance portions), the bias and variance should be small so that most of the bias is concentrated on the covariance proportion. The Granger and Wald test results in Table 6.5 illustrate Brent Oil influence on Carbon contracts that are included in the static predictions in Table 6.6. The covariance proportion for re-projected volatility is 95.5% for the Carbon contracts and 87.6% for the Brent Oil contracts. For the ICE Carbon contracts, the inclusion of Brent Oil correlation for the step-ahead predictions significantly increases the covariance proportion from 90.9% to 95.5% (and not the other way). Only March (Covid-19 outbreak) and possibly April 2020 for the ICE Brent Oil contracts report actual volatility outside of the predicted 95% confidence intervals. Running static forecasts for sub-samples for the period 2018/19 and comparing the Theil covariance measures does not significantly change Theil's covariance portion. In fact, for both Carbon and Brent Oil, the three sub-periods report covariance measures all around (94–95% and 87–88%, respectively). Table 6.6 reports also fit measures of 2020/21 for the sub-factors V_{1t} and V_{2t}. Note especially the high covariance portion for the ICE Carbon V_{1t} factor (99.987%). The cross-correlation from the ICE Brent Oil V_{1t} factor increases this measure to almost 100% (99.994%). The covariance portion for the ICE Carbon V_{2t} factor is considerably lower (32.58%). However, the cross-correlation from the Brent Oil increases the covariance portion to 56.84%. Hence, the major contribution from the ICE Brent Oil contracts is useful tail information for the ICE Carbon contracts.

Table 6.6: Fit measure for the ICE Carbon and the ICE Brent Oil contracts
Estimated Stochastic Volatility Forecast Fit Measures for 2020/21

		Factor		Factor		Reprojected	
Contracts	**Error Measures**	V_{1t}		V_{2t}		**Volatility**	
Pure Carbon Prediction	Root mean square error (RMSE)	0,01647		0,05780		0,93581	
	Mean absolute error (MAE)	0,01036		0,04174		0,64949	
	Mean absolute percent error (MAPE)	1,23304		340,881		2,63468	
	Teil inequality coefficient (U1)	0,01004		0,82302		0,01947	
	Bias proportion		0,00004		0,00031		0,00011
	Variance proportion		0,00010		0,67391		0,09096
	Covariance proportion		0,99987		0,32578		0,90893
	Theil U2 coefficient	0,98673		1,42456		0,82698	
	Symmetric MAPE	1,24308		162,772		2,65605	
		V_{1t}		V_{2t}		**Volatility**	
Pure Brent Oil Prediction	Root mean square error (RMSE)	0,21125		0,12421		6,31330	
	Mean absolute error (MAE)	0,11307		0,07466		2,30957	
	Mean absolute percent error (MAPE)	60,18694		312,606		7,77173	
	Teil inequality coefficient (U1)	0,15982		0,41984		0,13348	
	Bias proportion		0,00236		0,00339		0,00171
	Variance proportion		0,06540		0,30417		0,12267
	Covariance proportion		0,93224		0,69244		0,87562
	Theil U2 coefficient	0,93900		0,69071		0,94765	
	Symmetric MAPE	30,75530		120,438		7,88567	
		V_{1t}		V_{2t}		**Volatility**	
Carbon Prediction incl. Brent Oil	Root mean square error (RMSE)	0,01695		0,06114		0,85214	
	Mean absolute error (MAE)	0,01088		0,04441		0,62682	
	Mean absolute percent error (MAPE)	1,29326		299,072		2,55499	
	Teil inequality coefficient (U1)	0,01033		0,80129		0,01772	
	Bias proportion		0,00002		0,00439		0,00044
	Variance proportion		0,00004		0,42720		0,04499
	Covariance proportion		0,99994		0,56841		0,95457
	Theil U2 coefficient	1,00486		1,30103		0,75906	
	Symmetric MAPE	1,30279		160,959		2,56514	

Figure 6.4: VAR-optimal Carbon and Brent Oil Futures volatility forecasts 2020/21.

6.5 SUMMARY AND CONCLUSIONS

The main objective of this chapter has been to characterize a good volatility model by its ability to forecast and capture the commonly held stylized facts about financial market volatility. The characteristics indicate substantial data dependence in volatility enabling volatility predictions.

The chapter has used the Bayesian M-H estimator and a stochastic volatility representation. The methodology is based on a simple rule: compute the conditional distribution of unobserved variables given observed data. The observables are the asset prices and the un-observables are a parameter vector and latent variables. The inference problem is solved by the posterior distribution. Based on the Clifford-Hammersley theorem (Hammersley & Clifford, 1970), $p(\theta,x|y)$ is completely characterized by $p(\theta|x,y)$ and $p(x|\theta,y)$. The distribution $p(\theta|x,y)$ is the posterior distribution of the parameters, conditional on the observed data and the latent variables. Similarly, the distribution $p(x|\theta,y)$ is the smoothing distribution of the latent variables given the parameters. The MCMC approach therefore extends model findings relative to nonlinear optimizers by breaking the "curse of dimensionality" by transforming a higher dimensional problem, sampling from $p(\theta_1,\theta_2)$, into easier problems, sampling from $p(\theta_1|\theta_2)$ and $p(\theta_2|\theta_1)$ – using the Besag (1974) formula.

This chapter applies stochastic models relating volatility to risks that change through time in complicated ways. The departure from Black-Scholes-Merton option prices and occasional dramatic moves in markets is possible to explain (factors, correlation, and data dependence). In particular, this chapter shows that the stochastic volatility model separates into two distinct factors: a very persistent factor, V_{1t}, showing low mean reversion and a strong mean-reverting factor, V_{2t}. The persistent factor, V_{1t}, provides for the main distribution, and the rapidly mean-reverting factor, while V_{2t}, provides for the tails. The two-factor stochastic volatility model also reflects on the shortcomings of single-factor stochastic volatility models. Moreover, a closer look at the two Brent Oil stochastic factors shows that both the persistent and the strongly mean-reverting factor reacted quite strongly to new information in March 2020 (e.g., Covid-19, low and negative oil prices). An interpretation of these results suggests that the persistent factor signalled a longer period of high volatility while the second factor signalled more short-term mean reversion (i.e., more noise).

The volatility factors report causality from Brent Oil to Carbon Futures contracts (and not the other way around). The influence direction from Brent Oil to Carbon Futures is shown in classical sample analysis (i.e., correlation, Granger, Wald) in Table 6.5, and out-of-sample static forecasts in Table 6.5 and Figure 6.4 (a significant increase in covariance portion) lend support to the influence direction from Brent Oil to Carbon Futures. Furthermore, using an MCMC implementation of a stochastic volatility model with an associated Kalman filter procedure for projection reveals a Theil covariance volatility portion close from 87% to 96% for individual assets. Trading volatility swaps may become less risky for market participants. Although Carbon and Brent Oil price processes are hardly predictable, the variance of the forecast error is time dependent and can be estimated by means of observed past variations. These results suggest that Carbon and Brent Oil contract volatility can be forecast. Furthermore, the observed volatility clustering induces an unconditional distribution of returns at odds with the hypothesis of normally distributed price changes. The stochastic volatility models are therefore an area in empirical financial data modelling that is fruitful as a practical descriptive and forecasting device for all contract series enlightening market participants/managers using, among others, volatility swaps and the associated derivative markets[11]. Irrespective

11 Trading volatility as an asset class provides the market participant with, among other things, excellent diversification. For example, equity volatility is strongly negatively correlated with the equity price (insurance against market crashes).

of markets and contracts, Monte Carlo Simulations should lead us to more insights into the nature of the price processes describable from stochastic volatility models.

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