Bootstrapped nonlinear impulse-response analysis: the FTSE100 (UK) and the NDX100 (US) indices 2012–2021

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Abstract: This paper presents bootstrapped nonlinear impulse response function analyses for general step ahead mean and volatility densities. From strictly (ergodic and) stationary series and BIC optimal nonlinear model coefficients, the paper establishes step-ahead densities for both the conditional mean and volatility. For sampling variances using one thousand samples and conditioning all paths on the daily impulses –5, –3, …, 5% all mean and volatility responses show mean reversion. For the volatility, all increases seem to arise from negative index movements suggesting strong asymmetry. Furthermore, the model coefficients for the volatility exhibit data dependence suggesting ability to predict volatility. The indices report some striking step-ahead differences for both the mean and the volatility. For the mean, only the NDX100 seems to show overreactions. For the volatility, for both positive and negative impulses the NDX100 reports higher volatility responses then FTSE100. However, asymmetry manifested for both indices suggesting that trading volatility as an asset may insure against market crashes and be an excellent diversification instrument. Finally, using a stochastic volatility model to obtain calibrated functions that give step-ahead predicted values for static predictions, enriches participants' derivative trading strategies (i.e., volatility swaps).

Keywords: bootstrapping; conditional heteroscedasticity; equity markets; impulse-response functions; nonlinearity; volatility predictions.

JEL codes: C61, Q4.

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1 Introduction

This paper presents nonlinear impulse-response analyses for two central international equity indices. For statistical inference, the paper uses simulations (bootstrapping) to consider the sampling variation. Impulse-response analysis is often used in contemporary macroeconomic modelling describing for example how the economy reacts over time to exogenous impulses (shocks). This paper treats impulses as exogenous shocks to stock markets; that is positive and negative daily impulses generating changes in the index level. Impulses or shocks can originate from interest rates (central banks/federal reserve), tax rates, and other policy parameters as well as changes in raw material prices or other technological parameters. Response functions for stock markets describe the reaction (responses) from endogenous market variables at the time of shocks and subsequent points in time. The impulse response analysis is therefore a tool for inspecting the inter-relation of the model variables. Moreover, this paper emphasises measures of sampling uncertainty.

The paper focuses on two indices both containing approximately 100 assets [FTSE100 (UK) and NDX100 (US)].[1] The analysis is univariate indicating that non-synchronous trading effects do not exist. However, for the daily settlement, note that European markets close several hours before the US markets. The analysis has three objectives. First, to find general step ahead densities, second, identify data dependence for predictability and third, report systematic market features. The paper starts with a description of the bootstrapped impulse-response function analysis. The empirical index analysis follows three steps. First, the univariate index movements for all index models are expanded sequentially using the BIC criterion (Schwarz, 1978). Second, the BIC optimal univariate models are bootstrapped and conditioned. Third, the impulse-response analysis report step ahead profiles with confidence intervals with associated distributions for the European and US indices for the period 2012–2021.

The methodology is the semi-non-parametric time series analysis (SNP densities) introduced by Gallant and Tauchen (1988, 1992, 2014). The method uses an expansion in hermite functions to approximate the conditional density of the time series processes. The leading term of the model expansion process is therefore an established parametric model already known to give a reasonable approximation to the process; higher order terms (hermite functions) capture departures from the model (Robinson, 1983). The SNP model is fitted using conventional maximum likelihood (ML) together with a model selection strategy [BIC (Schwarz, 1978)] that determines the appropriate order of expansion. The model is well designed for the computation of nonlinear functionals of the densities bringing econometrics closer to empirical reality. Extending the SNP model to bootstrapped impulse-response distribution analysis is challenging but made possible using bash scripting tools in Linux and access to clusters of CPUs/GPUs and optimisation[2] using the OpenMPI[3] software. Firstly, 1.000 simulations of the BIC optimal SNP models $(\hat{\theta})$ are generated (changing the seed) and all extended with impulses from –5% to 5% (iterations). For each simulation and impulses, mean and volatility responses are calculated and reported for several days ahead applying densities and confidence intervals. Hence, the simulations, iterations and density reports are Linux bash script unique while the SNP models calculate the mean and volatility responses using C/ C++. Together the Linux scripts and the SNP models originate the work.

The remainder of this paper is organised as follows. Section 2 introduces the impulse-response functions and describes the bootstrapping techniques to obtain numbers for statistical inference. Section 3 gives a literature review over Monte Carlo impulse-response literature together with an introduction to the SNP-software and methodology. Section 4 for the FTSE100 and NDX100 indices, reports the SNP specification consistent mean and volatility equation specifications. The hermite function expansions extend model approximation for the conditional density[4], which summarises the probability distribution and characterises the index movement processes. Residual characteristics are used to assess model fit. Section 5 performs the impulse-response[5] analysis put forth in Sims (1980) and refined by Doan et al. (1984) and others. The impulse response dynamics from the SNP models are elicited in Section 5.2 by perturbing the vector of conditioning arguments in the conditional density function (Gallant et al., 1993; Gallant and Tauchen, 2010, 2014). Section 6 summarises and concludes.

2 The impulse-response functionals

The paper applies the methodologies outlined by Gallant et al. (1993), Gallant and Tauchen (2014) defining step-ahead forecasts for the mean conditioned on the history as $g\left(y_{t-\infty+1},...,y_t\right) = E\left(y_{t+1} \mid \left(y_{t-k}\right)_{k=0}^{\infty}\right)$ in general and $g\left(y_{t-L+1},...,y_t\right) = E\left(y_{t+1} \mid \left(y_{t-k}\right)_{k=0}^{L\text{-}1}\right)$ for a Markovian process where L is the number of lags. We put $\hat{y}_j(x) = E\left(g\left(y_{t-L+j},...,y_{t+j}\right) \mid x_t = x\right) = E\left(E\left(y_{t+j} \mid y_{t-L+j},...,y_{t+j}\right) \mid x_t = x\right)$ and therefore $\hat{y}_j^i$ for impulse ranges i = –5%, …, 5%, and for five steps-ahead (days) j = 0, …, 3 where $x = (y_{-L+1}, \ldots, y_0)$ and L represents the number of lags in the Markovian process. The conditional mean profiles $\left\{\hat{y}_j^i\right\}_{j=1}^{\infty}$ for i = –5%, …, 5% are the conditional expectations of the trajectories of the one-step conditional mean.[6] Note that $\{\hat{y}_j^{-5\%}\}_{j=1}^{\infty}$ therefore represents the mean response to a negative 5% impulse (error shock). The responses depend upon the initial x, which reflects the nonlinearity. Moreover, the law of iterated expectations implies that $\hat{y}_j(x) = E(y_{t+j} \mid x_t = x)$. The sequences $\{\hat{y}_j^i - \hat{y}_j^0\}_{j=1}^{\infty}$ for i = –5%, …, 5%, represents the effects of the shocks on the trajectories of the process itself. A conditional moment profile can now be defined as $E\left[g\left(y_{t+j-J},...,y_{t+j}\right) \mid \{y_{t-k}\}_{k=0}^{L=1}\right], (j = 0, .., 3),$ where the word moment refers to the time-invariant function $g(y_{-J}, \ldots, y_0)$.

Similarly, the one-step-ahead variance, also called the volatility, is the one-step ahead forecast of the variance conditioned on history becoming

$$Var\left(y_{t+1} \middle| \left(y_{t-k}\right)_{k=0}^{\infty}\right) =$$

$$E\left\{\left[y_{t+1} - E\left(y_{t+1}\{y_{t-k}\}_{k=0}^{\infty}\right)\right] \times \left[y_{t+1} - E\left(y_{t+1} \mid \{y_{t-k}\}_{k=0}^{\infty}\right)\right]' \middle| \{y_{t-k}\}_{k=0}^{\infty}\right\}$$

or $Var(y_{t+1} \mid (y_{t-k})_{k=0}^{L-1})$ for a Markovian process ($L << \infty$). By appropriately defining the function $g(.)$, we can measure the effect of impulses on volatility. Now writing $\widehat{\Psi}_j(x) = E\left(g\left(y_{t-L+j}, ..., y_{t+j}\right) \mid x_t = x\right) = E\left(Var\left(y_{t+j} \mid x_{t+j-1}\right) \mid x_t = x\right)$ for $j = 0, \ldots, 3$ where $x = (y_{-L+1}, \ldots, y_0)$. $\hat{\Psi}_j(x)$ is the conditional expectation of the trajectories of the step-ahead conditional variance matrix j, conditional on $x_t = x$. Therefore, as for the conditional mean, the $\{\widehat{\Psi}_j^{-5\%}\}_{j=1}^{\infty}$ represents the volatility response from a negative 5% impulse (shock). The net effects of perturbations on volatility are assessed by plotting the profiles compared with the baseline ∂y^i for i = –5%, …, +5%. Note that the above defined conditional volatility profile, is different from the path described by the j-step ahead square error process. Analytical evaluation of the integrals in the definition of the conditional moment profiles are intractable. However, evaluation is well suited to Monte Carlo integration.

Let $\{y_j^r\}_{j=1}^{\infty}$, $r = 1, \ldots, R$ be R simulated realisations of the process starting from $x_0 = x$. That is, y_1^r is a random drawing from $f(y \mid x)$ with $x = \left(y'_{-L+1}, ..., y'_{-1}, y'_0\right)'$; y_2^r is a random drawing from $f(y \mid x)$ with $x = \left(y'_{-L+2}, ..., y'_0, y'_1\right)'$, and so forth. Now applying the invariant function of a stretch of $\{y_j\}$ and length j, we get

$$\hat{g}_j(x) = \int ... \int g\left(y_{j-J}, ..., y_j\right)\left[\prod_{i=0}^{j-1} f\left(y_{i+1} \mid y_{y-L+1}, ..., y_i\right)\right] dy_1 ... dy_j$$

$$\doteq (1/R)\sum_{r=1}^{R} g\left(y_{j-J}^r, ..., y_j^r\right)$$

with the approximation error tending to zero almost surely as $R \to \infty$, under mild regulatory conditions on f and g. For statistical inference, sup-norm bands are constructed by bootstrapping[7], using simulations to consider the sampling variation in the estimation of $\hat{f}(y \mid x)$. That is, changing the seed that generates densities and the basis for impulse-response analyses. The paper applies 1,000 samples and a 95% confidence interval. A 95% sup-norm confidence band is an ε-band around the mean profile $\hat{f}(y \mid x)$ that is just wide enough to contain 95% of the simulated profiles. Moreover, distributions for multiple-step ahead mean and volatility can be plotted for the mean and volatility for days $j = 0, \ldots, 3$. Day 0 is the impulse day and day 1 to 3 are the distributional response forecasts. The one-step ahead response distribution is reported for mean and volatility for all impulses i = –5%, …, 5%.

3 Literature review

3.1 Impulse-response functions literature

Early work (e.g., Campbell and Mankiw, 1987) used univariate linear models and concluded that, at least at business cycle frequencies (e.g., eight to 12 quarters), shocks were persistent. The more recent work by Beaudry and Koop (1993) (BK hereafter), Potter (1995) and Pesaran and Potter (1994) (PP hereafter) has focused on nonlinear models. They argue that linear models are too restrictive. Linear models cannot adequately capture asymmetries that may exist in business cycle fluctuations. Other authors (e.g., Pesaran et al., 1993; Lee and Pesaran, 1993; Blanchard and Quah, 1989) have extended the basic linear univariate literature to a consideration of linear multivariate models. A richer understanding of the persistence of shocks can be achieved by considering information from more than one macroeconomic time series (Blanchard and Quah, 1989) or from more than one sector of the economy (Lee and Pesaran, 1993). Gallant et al. (1993) put greater emphasis on providing measures of sampling uncertainty for impulse response functions produced from non-parametric estimates. That is, a nonlinear impulse response function is estimated by Monte Carlo integration based on estimates of the structural model. The approach tends to be computationally demanding. However, using the SNP software package (Linux), the approach easily implemented with access to all interesting extensions (and programmable C/C++). Hence, when local projections and vector auto regression fails from exogenous serial correlated or endogenous impulses, the nonlinear semi-parametric SNP model applying bootstrapping, is applicable without significant speed drawbacks. A considerable number of structural models has been proposed in the literature. A class of models includes stochastic models, regime switching models, cointegration analysis, mean-reverting models, and other empirical models. These models fail to capture the full volatility dynamics of indices as well as, the price and volatility interrelationships. Another class of models introduces univariate generalised autoregressive conditional heteroscedasticity (GARCH) conditional volatility models, as well as other variations of GARCH modelling, such as EGARCH and TGARCH. These models capture the price and volatility dynamics of financial market prices, as well as price shock transmissions. For this paper, we follow the impulse response methodology of Gallant and Tauchen (1998, 2010, 2014).

3.2 The SNP model

Nonlinear stochastic models will in our study imply conditional models. Autoregressive and moving average (ARMA) is a term applied to the structure of the conditional mean, while GARCH is a term applied to the structure of the conditional volatility. ARMA models can be studied in detail in, for example, Mills (1990), while ARCH specifications were first studied by Engle (1982) and moved furthered by Bollerslev (1986) who specified the generalised ARCH or GARCH. The development to GARCH from ARCH was initially done to the number of lags in the ARCH specification.[8] ARCH/GARCH specifies the volatility as a function of historic price movements and volatility. In the international finance literature, quite several studies have shown how the results from this work has been used. See for example, Bollerslev (1987), Bollerslev et al. (1992), Engle and Bollerslev (1986), Engle and Ng (1993), Nelson (1991) and de Lima (1995a, 1995b). For a comprehensive introduction to ARCH models and applications in finance see Gouriéroux (1997). Ding et al. (1993) extends the symmetric GARCH model into asymmetric GARCH and the truncated GARCH (GJR) is described by Glosten et al (1993).

SNP[9] by Gallant and Nychka (1987), Gallant et al. (1992), Gallant and Tauchen (2010, 2014) stands for SNP, suggesting that it lies halfway between parametric and non-parametric procedures. The leading term of the series expansion is an established parametric model known to give a reasonable approximation to the process; higher order terms capture departures from that model. With this structure, the SNP approach does not suffer from the curse of dimensionality to the same extent as kernels and splines. In regions where data are sparse, the leading term helps to fill in smoothly between data points. Where data are plentiful, the higher order terms accommodate deviations from the leading term and fits are comparable to the kernel estimates proposed by Robinson (1983). The theoretical foundation of the method is the hermite series expansion, which for time series data is particularly attractive based on both modelling and computational considerations. In terms of modelling, the Gaussian component of the hermite expansion makes it easy to subsume into the leading term familiar time series models, including VAR, ARCH, and GARCH models (Engle, 1982; Bollerslev, 1986). These models are generally considered to give excellent first approximations in a wide variety of applications. In terms of computation, a hermite density is easy to evaluate and differentiate. Also, its moments are easy to evaluate because they correspond to higher moments of the normal, which can be computed using standard recursions. Finally, it is practicable to sample from a hermite density, which facilitates simulation.

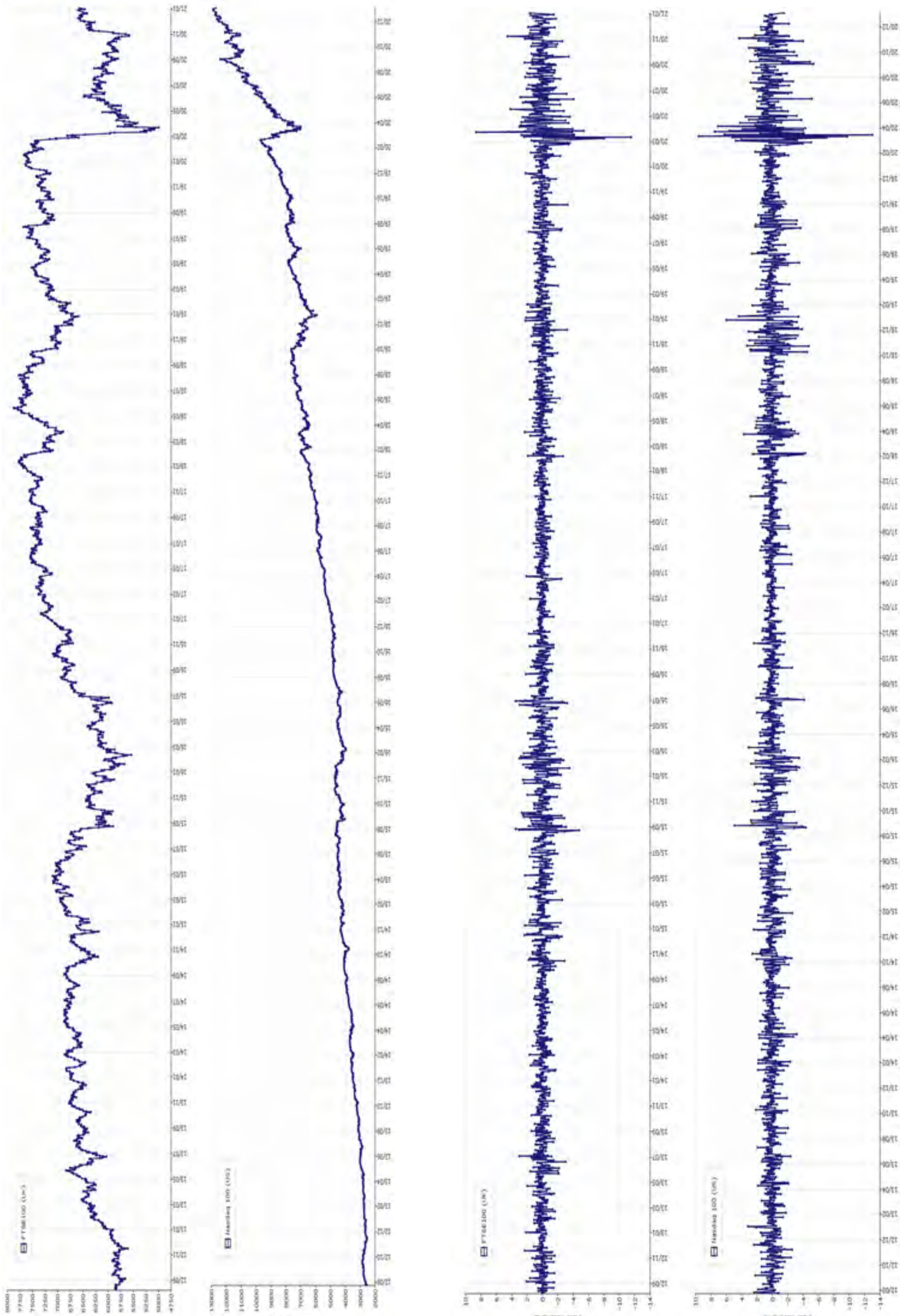
4 The indices, impulse-response functions and empirical findings

4.1 Index data and stationarity

We impose weak stationarity, and the means, variance and covariances are independent of times (rather than the entire distribution). That is, a process $\{y_t\}$ is weakly stationary if for all t, it holds that $E\{y_t\} = \mu \leq \infty$, $V\{y_t\} = E\{(y_t - \mu)^2\} = \gamma_0 < \infty$ and $\text{cov}\{y_t, y_{t-k}\} = E\{(y_t - \mu)(y_{t-k} - \mu)\} = \gamma_k$, $k = 1, 2, 3, \ldots$. A shock to a stationary autoregressive process of order 1 (AR(1)) affects all future observations with a decreasing effect. Table 1 reports the characteristics of the index movement series. The mean is positive. Lowest mean is found for the FTSE100 index (0.006) and is followed by the lowest standard deviation (1.01) as expected. The NDX100 has an expectation of 0.075 with an associated standard deviation of 1.24. A maximum (minimum) mean of 8.7 and 9.6 (–11.5 and –13.0) is found for the FTSE100 and NDX100 indices, respectively. The FTSE100 index (NDX100) reports highest (lowest) kurtosis of 14.3 (12.9) and a negative skew of –0.91 (–0.75). Cramer-von-Mises test statistic reports significant non-normality for both FTSE100 (5.6) and NDX100 (8.1). The Q(12) and the $Q^2(12)$ correlogram statistics (serial correlation) show dependencies for both the mean and volatility for both the FTSE100 and NDX100 indices.

Similarly, the 12th lag ARCH test statistic (Engle, 1982) suggests highly significant conditional heteroscedasticity. The RESET test (Ramsey, 1969) report instability. Finally, for both series, the adjusted series the ADF (Dickey and Fuller, 1979) and the KPSS (Kwiatkowski et al., 1992) statistics confirm stationarity. The BDS test statistic (Brock et al., 1996) reports general nonlinear data dependence. Figure 1 reports the level (top) the movement series (bottom) for FTSE100 and NDX100 indices. The general movement appearances of the two series are typical for equity market data. We also experimented with breaking trends in the movement equations, but our results suggested little evidence for trend breaks. The value at risk (VaR) is a well-known concept for measures of risk and Table 1 includes the 2.5% and 1% VaR numbers for market participants.

Figure 1 (a) FTSE100 and NDX100 index series level and (b) movements for the period for 2012–2021 (see online version for colours)



(a) (b)

Table 1 Characteristics for FTSE100 and NDX100 Indices for 2012–2021

Panel A:	*FTSE100 (UK) Index*								
Mean (all)/	Median	Maximum/	Moment	Quantile	Quantile	Cramer-	Serial dependence		VaR
M (-drop)	Std.dev.	Minimum	Kurt/Skew	Kurt/Skew	Normal	von-Mises	Q(12)	Q2(12)	(1;2.5%)
0.00554	0.05126	8.6664	14.29145	0.21290	5.9167	5.5661	43.091	1199.90	–3.136%
0.00601	1.00843	–11.5117	–0.90815	–0.07326	{0.0519}	{0.0000}	{0.0000}	{0.0000}	–2.135%
BDS-Z-statistic		(*e* = 1)		KPSS (Stationary)		Augmented	ARCH	RESET	CVaR
m = 2	m = 3	m = 4	m = 5	Intercept	I&Trend	DF-test	(12)	(12;6)	(1;2.5%)
11.9713	15.0932	17.4684	19.2878	0.0224	–46.5247	–46.5242	575.2514	82.6138	–4.303%
{0.0000}	{0.0000}	{0.0000}	{0.0000}	{0.5837}	{0.0000}	{0.0000}	{0.0000}	{0.0000}	–3.226%
Panel B:	*Nasdaq100 (US) Index*								
Mean (all)/	Median	Maximum/	Moment	Quantile	Quantile	Cramer-	Serial dependence		VaR
M (-drop)	Std.dev.	Minimum	Kurt/Skew	Kurt/Skew	Normal	von-Mises	Q(12)	Q2(12)	(1;2.5%)
0.07477	0.11739	9.5966	12.85078	0.39647	14.5690	8.1385	239.540	2352.80	–3.853%
0.07229	1.23775	–13.0032	–0.75457	0.04267	{0.0007}	{0.0000}	{0.0000}	{0.0000}	–2.608%
BDS-Z-statistic		(*e* = 1)		KPSS (Stationary)		Augmented	ARCH	RESET	CVaR
m=2	m=3	m=4	m=5	Intercept	I&Trend	DF-test	(12)	(12;6)	(1;2.5%)
12.7312	15.8073	18.1541	20.3465	0.0419	–54.1496	–15.6120	757.8798	139.6197	–5.195%
{0.0000}	{0.0000}	{0.0000}	{0.0000}	{0.4832}	{0.0000}	{0.0000}	{0.0000}	{0.0000}	–3.962%

4.2 The SNP density projection

Since the conditional density completely characterises the price movement process, the density is naturally viewed as the fundamental statistical object of interest. The SNP model is fitted using conventional ML together with a model selection strategy that determines the appropriate order of expansion (*BIC*). The Schwarz (1978) Bayes information criterion is computed as $BIC = s_n(\hat{\theta}) + \left(\frac{1}{2}\right)\left(\frac{p_p}{n}\right)\log(n)$ with small values of the criterion preferred. Table 2 reports the ML estimates[10] of the parameters for the *BIC*-optimal SNP density models.[11] Firstly, for the mean, the intercept is insignificant and the serial correlations (η_6) are insignificant for FTSE100 but significant for NDX100 implying negative dependence. This negative dependence for NDX100 index (η_6) may suggest mean reversion. Secondly, the conditional variance coefficients ($\eta_7 - \eta_9$) are all strongly significant except the ARCH coefficient (η_8) that is close to zero for both series. The significance of η_7 and η_9 suggest conditional heteroscedasticity. Furthermore, asymmetry (η_{10}) is present but level effects (η_{11}) are not (zero coefficient). The largest eigenvalue of the conditional variance function P&Q companion matrix is 0.855 and 0.881 for the FTSE100 and NDX100, respectively. These results confirm mean reversion of the conditional variance. Finally, the hermite functions coefficients ($\eta_1 - \eta_4$), which capture parametric model departures, are *BIC* preferred up to the fourth polynomial lag expansions. Hence, the hermite result clearly suggests departures from the classical normally distributed and parametric conditional model.

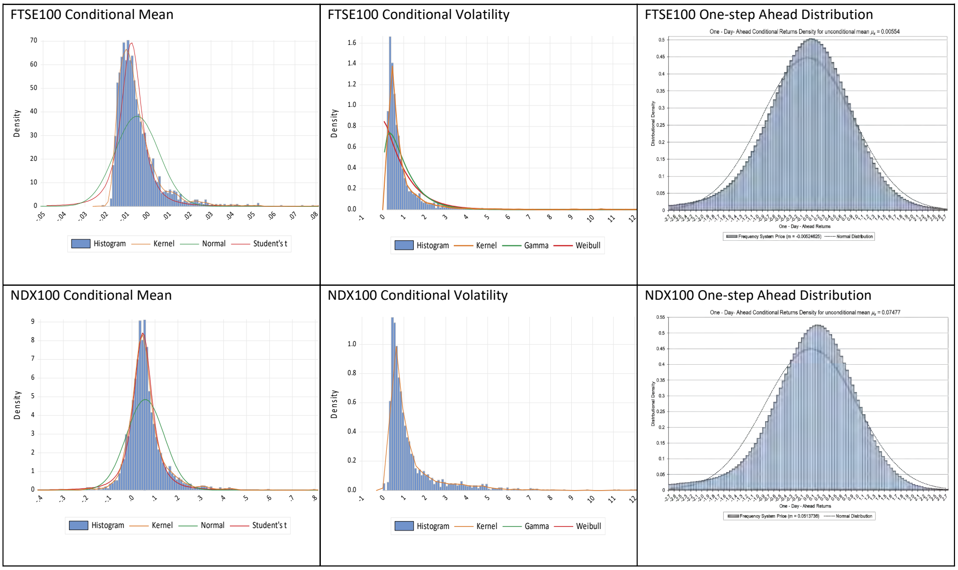
Table 2 SNP-model projection specification European and US indices

Statistical Model SNP (111140000) opt. BIC-fit; semi-parametric-GARCH model					
Var	*SNP Coeff.*	*Mode and {standard error}*			
		FTSE100		*Nasdaq100*	
Hermite Polynoms					
η_1	$a_0[1]$	0.01555	{0.0298}	0.02946	{0.0291}
η_2	$a_0[2]$	–0.03926	{0.0256}	–0.04833	{0.0228}
η_3	$a_0[3]$	–0.05551	{0.0123}	–0.10151	{0.0123}
η_4	$a_0[4]$	0.08170	{0.0115}	0.09363	{0.0125}
Mean equation (correlation)					
η_5	b0[1]	–0.02760	{0.0407}	–0.04317	{0.0365}
η_6	B(1,1)	0.00789	{0.0230}	–0.06278	{0.0217}
Variance equation (correlation)					
η_7	R0[1]	0.18002	{0.0171}	0.15536	{0.0118}
η_8	P[1,1]	0	{0.0}	0	{0.0}
η_9	Q[1,1]	0.92467	{0.0105}	0.93883	{0.0054}
η_{10}	V[1,1]	–0.49898	{0.0401}	–0.44281	{0.0295}
η_{11}	W[1,1]	0	{0.0}	0	{0.0}
Model	s_n	1.18616		1.14026461	
selection	*aic*	1.19086		1.14026461	
criteria:	*bic*	1.20418		1.14026461	
Largest eigenvalue for mean:			0.00789		0.062779
Largest eigenvalue variance:			0.85501		0.881409

Table 3 SNP-model projections residuals for FTSE100 (UK) and Nasdaq100 (US)

Panel A:	*FTSE100 (UK)*							
Mean /	Median /	Maximum /	Moment	Quantile	Quantile	Cramer-	Serial dependence	
Mode	Stand.dev.	Minimum	Kurt/Skew	Kurt/Skew	Normal	von-Mises	Q(12)	$Q^2(12)$
0.00236	0.06857	4.3676	1.5605	0.08516	2.4496	1.0999	7.1255	7.1775
	0.99993	–5.2516	–0.42562	–0.07162	{0.2938}	{0.0000}	{0.8490}	{0.8460}
BDS Z-statistic		(ε = 1)			ARCH	RESET	Breusch-	
m = 2	m = 3	m = 4	m = 5	m = 6	(12)	(12;6)	Godfrey LM	
0.18017	0.06641	0.02421	0.17635	0.17097	7.0632	7.2987	8.1150	
{0.8570}	{0.9471}	{0.9807}	{0.8600}	{0.8642}	{0.8534}	{0.2941}	{0.7761}	
Panel B:	*Nasdaq100 (US)*							
Mean /	Median /	Maximum /	Moment	Quantile	Quantile	Cramer-	Serial dependence	
Mode	Stand.dev.	Minimum	Kurt/Skew	Kurt/Skew	Normal	von-Mises	Q(12)	$Q^2(12)$
0.00597	0.09516	4.1496	3.4111	0.23387	5.2505	2.9399	11.844	6.7110
	0.99977	–7.3320	–0.90911	–0.03474	{0.0724}	{0.0000}	{0.4580}	{0.8760}
BDS Z-statistic		(ε = 1)			ARCH	RESET	Breusch-	
m = 2	m = 3	m = 4	m = 5	m = 6	(12)	(12;6)	Godfrey LM	
1.6787	1.7262	2.1349	2.5169	2.9477	6.7133	8.8303	12.033	
{0.0932}	{0.0843}	{0.0328}	{0.0118}	{0.0032}	{0.8760}	{0.1833}	{0.4431}	

Figure 2 Conditional mean and volatility together with one-step ahead conditional densities (see online version for colours)



The conditional variance function and the quadrature density distributions show that the reaction from negative price movements is clearly higher than from positive index movements (not reported).

The SNP projection densities $f_K\left(\tilde{y}_t \mid x_{t-1}, \hat{\theta}\right)$ give access to the conditional mean and volatility densities. Moreover, conditional on the values for $x_{t-1} = (\tilde{y}_{t-1}, \tilde{y}_{t-2}, \ldots, \tilde{y}_{t-L})$, the one-step-ahead mean densities can be generated. Simulation paths (bootstrapping) are obtainable at any length. For the two series, Figure 2 reports these above-mentioned densities. The mean distribution for FTSE100 is narrower and closer to zero than the NDX100 index. The conditional volatility distribution for NDX100 seem to report a larger right tail than the FTSE100 index. Furthermore, note the skew to the right for the NDX100 index and the one-step ahead mean distribution. These plots suggest that the NDX100 show a higher mean drift than for the FTSE100 index. These results are in full compliance with the statistics from Table 1. Finally, Table 3 reports residual statistics for the two indices. All residual test statistics are insignificant except for the Cramer-von-Mises test for normality. However, the non-normality from the raw data is nearly eliminated for the model residuals.

5 The impulse-response functionals for period 2012–2021

5.1 *The impulse-response analysis*

Section 2 has defined the impulse-response functions and described the bootstrapping techniques to enable distributional reports. Table 4 (top) reports percentiles mean and responses for the FTSE100 and NDX100 indices and impulses i = –5%, –3%, –1%, 1%, 3% and 5%. For all impulses i = –5%, …, 5%, Figures 3 and 5 report mean confidence intervals and one-step ahead mean distributions for the FTSE100 and NDX100 indices. For all plots, the left column contains the conditional mean profiles $\left\{\hat{y}^{i}_{j} - \hat{y}^{0}_{j}\right\}^{3}_{j=0}$ for the impulses i = –5, …, 5% using steps-ahead j = 0, …, 3[12], where day 0 is the impulse day. The impulse response functions for the conditional mean show the well-known characteristics of mean reversion. The baseline mean profile is and negative (positive) response lines are continuous (dotted). For both the FTSE100 and NDX100 indices, the mean for all impulses revert immediately to zero. Moreover, the mean effects seem to be symmetric and totally dissipated within one-step-ahead of the impulse, suggesting very little evidence of nonlinearity in the conditional mean of the movement processes. From the –5% and 5% high price impulses, the step-ahead responses are very close to zero. In fact, all the impulse-response profiles consistently show dissipated responses. However, the mean response differences between positive and negative impulses show higher absolute mean values suggesting that asymmetries may not be neglectable. Implementing bootstrapping, we can report 95% sup-norm bands and step ahead forecast distributions. The bands and distributions use 1,000 re-fittings of the SNP model. The band is computed for all cases i = –5%,…, 5%. The ε-band is located around zero and narrow, suggesting no obvious advantageous positions for market participants. For the FTSE100 (NDX100) index the –5% shock the 95% ε-band is between -0.007 and 0.061 (0.481 and 0.563) with an expectation of 0.029 (0.527). Similarly, for a 5% shock the ε-band is between –0.021 and 0.050 (–0.329 and –0.2389) with an expectation of 0.019 (–0.279). In fact, all mean ε-bands for the FTSE100 index for the index movements ranges between –5% and 5%, include zeroes. This is not so for the NDX100 index. For all negative impulses from –5% to –1%, the 95% response confidence intervals are positive. Moreover, for impulse 5% the 95% response confidence interval is negative. However, all mean impulses show close to zero responses, suggesting immediate market mean reversions.[13] Anyway, the NDX100 index seems to show marginal overreactions (responses) from both negative and positive impulses.

Table 4 (bottom) and Figures 4 and 6 report the impulse-response variance functions (conditional variance profiles) $\{\widehat{\Psi}^{i}_{j} - \widehat{\Psi}^{0}_{j}\}^{3}_{j=0}$ for impulses i = –5%, …, 5%, multi-steps-ahead j = 1, …, 3, where the baseline variance profile is $\widehat{\Psi}^{0}_{j}$. The most conspicuous result is the volatility asymmetry. For both the FTSE100 and NDX100 indices, the asymmetry is visible already from an absolute index movement of 1%. One-step ahead volatility from positive index movement impulses of 1%, 3%, and 5% for the FTSE100 (NDX100) show responses of 0.198, 0.199 and 0.200 (0.269, 0.270 and 0.272) are almost negligible both in size and increase. In contrast, negative index movement impulses of –1%, –3%, and –5% for the FTSE100 (NDX100) index report strongly increasing variance responses of 1.693, 13.468, and 28.351 (2.122, 16.0092, and 34.816), respectively. Hence, volatility seems to follow from negative index movements. The fast-growing and negative asymmetry is manifested. From the bootstrapping implementation (statistical significance), the 95% confidence intervals (sup-norm ε-bands) show ε-bands do not interact (all the differences do not include zeroes from all impulses). For example, for the FTSE100 (NDX100) index, the ε-band responses for the negative –5% relative to positive 5% impulses, one-step ahead is 28.35 (34.82) versus 0.200 (0.272) with a confidence interval of 25.51 – 31.60 (31.54 – 39.14) versus 0.147–0.292 (0.195 – 0.408), respectively. Note also from the volatility figures that the ε-bands for day 0 are naturally wider for negative day 0 impulses. Hence, trading strategies involving volatility changes must depend on negative index movements. Furthermore, Figures 4 and 6 suggest a higher volatility for day t for –5% movements at day t + 1. Therefore, the daily level of volatility may turn out to be a sign for large negative index movements. Hence, volatility may contain information important for the trading position (as an asset class in its own right) of market participants. For example, we have above shown that equity volatility is strongly negatively correlated with the equity price movements. Therefore, adding volatility to an equity portfolio provides both excellent diversification and insurance against market crashes.[14]

Table 4 Mean and volatility characteristics for one-step ahead densities

Mean characteristics one-step ahead from 1,000 impulse-response simulations						
Percentiles:	*–5%*	*–3%*	*–1%*	*1%*	*3%*	*5%*
FTSE100 (UK)						
50%	–0.00713	–0.02517	–0.05482	–0.05391	–0.03604	–0.02060
5%	0.06050	0.04722	0.01973	0.01680	0.03560	0.05008
95%	0.03893	0.02341	–0.00549	–0.00512	0.01155	0.02819
Percentiles:	–0.05000	–0.03000	–0.01000	0.01000	0.03000	0.05000
NDX100 (US)						
50%	0.48090	0.28357	0.07176	–0.08014	–0.20474	–0.32938
5%	0.56257	0.37732	0.16956	0.01126	–0.11336	–0.23803
95%	0.54053	0.34991	0.13821	–0.01777	–0.14237	–0.26701
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
Volatility characteristics one–step ahead from 1,000 impulse-response simulations						
FTSE100 (UK)						
50%	28.35117	13.46767	1.69308	0.19881	0.19853	0.20012
5%	25.51423	12.57237	1.60692	0.14259	0.14581	0.14738
95%	31.59872	13.98139	1.78697	0.29121	0.29037	0.29194
Model	29.07899	13.71144	1.73261	0.21400	0.21479	0.21637
NDX100 (US)						
50%	34.81586	16.09169	2.12190	0.26944	0.27015	0.27160
5%	31.53851	15.46924	2.01144	0.19331	0.19402	0.19546
95%	39.14066	16.53260	2.22514	0.40574	0.40646	0.40792
Model	36.14817	16.44242	2.17904	0.29932	0.30003	0.30147

Figure 3 FTSE100 (UK) impulse-response mean profiles, confidence intervals and one-step ahead distributions (see online version for colours)

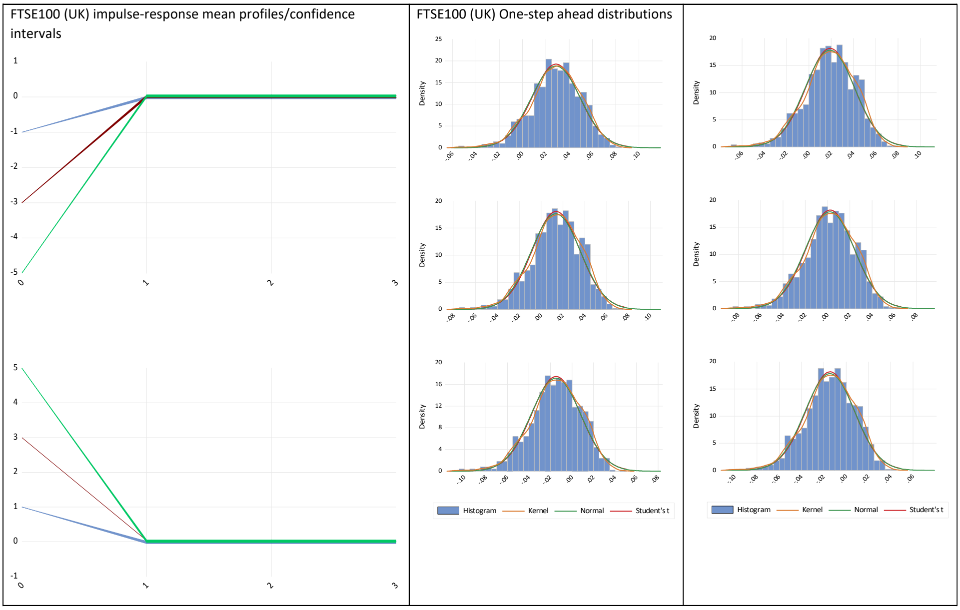


Figure 4 FTSE100 (UK) impulse-response volatility profiles, confidence intervals and one-step ahead distributions (see online version for colours)

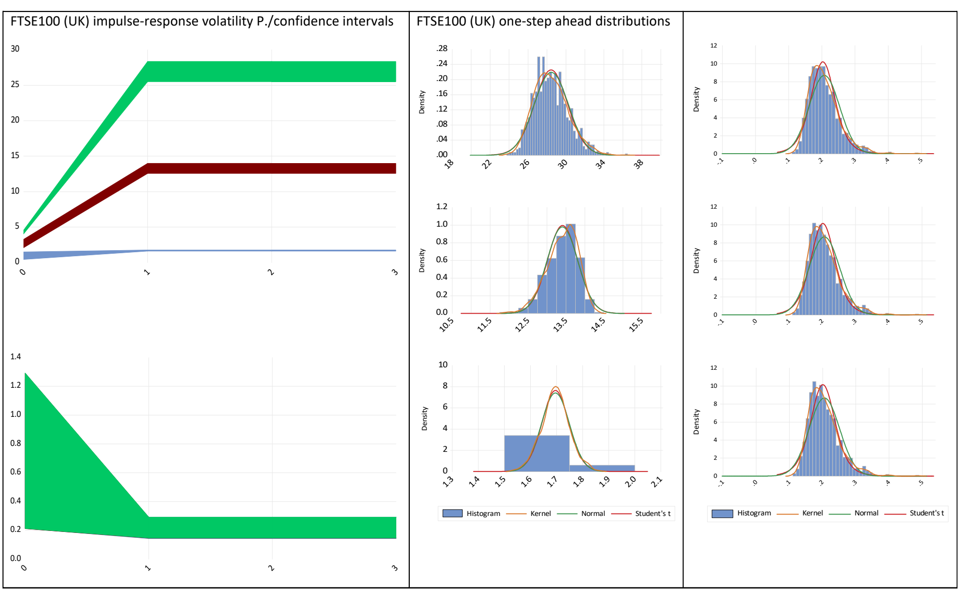


Figure 5 NDX100 (US) impulse-response mean profiles, confidence intervals and one-step ahead distributions (see online version for colours)

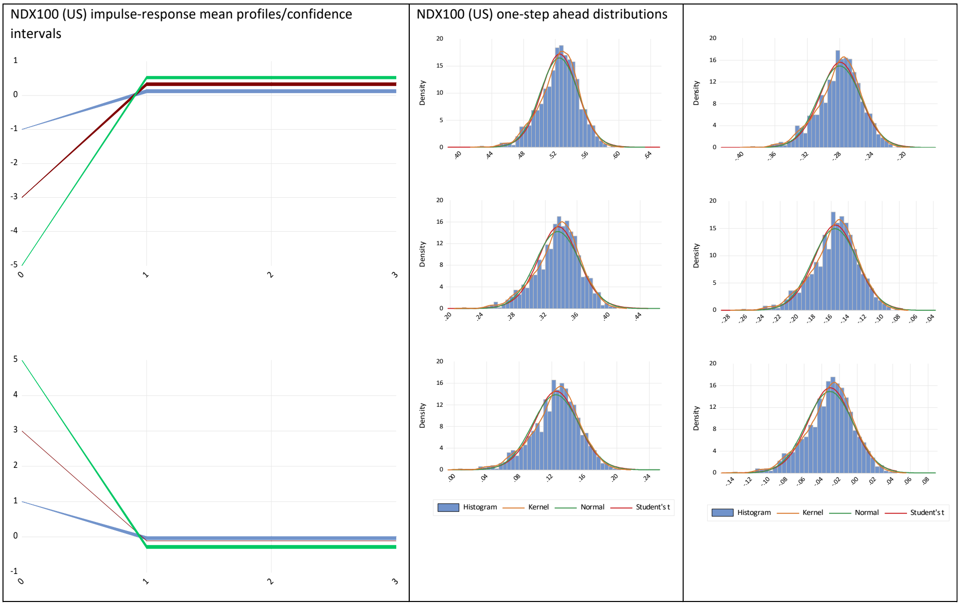
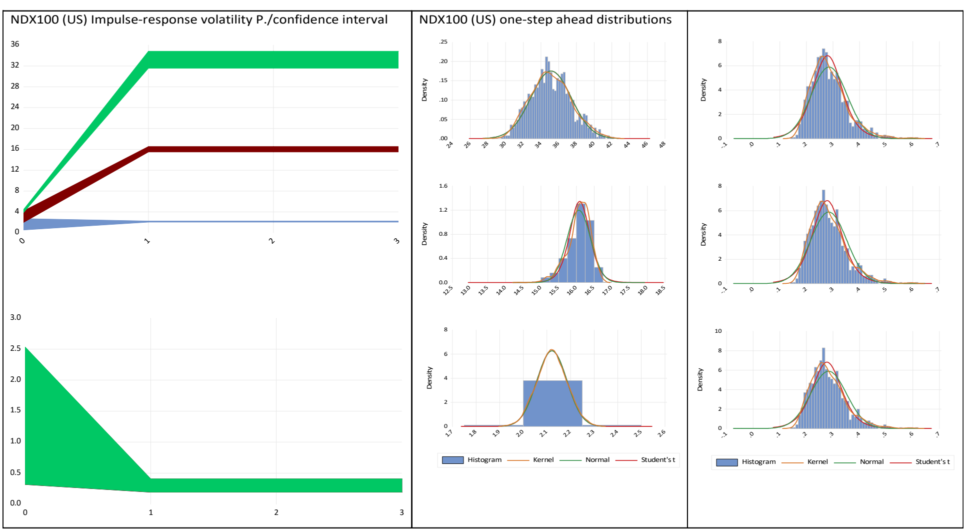


Figure 6 NDX100 (US) impulse-response volatility profiles, confidence intervals and one-step ahead distributions (see online version for colours)



Finally, Figure 7 reports persistence based on the SNP specification $(\hat{f}(y \mid x))$. Each profile uses data up to date $t - 1$. At date t, the profile shows mean reversion typically for GARCH(1,1) processes. The measure of the persistence in a volatility model is the 'half-life' of volatility. This is defined as the time taken for the volatility to move halfway back towards its unconditional mean following a deviation from it. The half-life definition (Engle and Patton, 2001) is given as

$$\tau = k : \left| h_{t+k|t} - \sigma^2 \right| = \frac{1}{2} \left| h_{t+1|t} - \sigma^2 \right|.$$

The volatility from approximately the 2000 latest observations (2012–2021) for the FTSE100 (NDX100) index is defined in the plot to be 8.60 (6.78) days with an associated standard deviation of 2.15 (2.56) days.

5.2 The UK and US Index impulse-response differentials

From Figure 1, the European and US index level plots are clearly more different than the index movements plots. Table 1 confirms these plot differences. The BIC optimal models show coefficient differences for the mean, the volatility and the hermite functions. For the shock analysis, the mean responses for the NDX100 index show overreactions. For example, giving the NDX100 and impulse of –5% (5%), a response one-step ahead of +0.527% (–0.278%) is reported. That is, the index shows overreaction with a 95% confidence interval of 0.481 and 0.563 (–0.238 and 0.393). In contrast, an impulse of –5% (5%) for the FTSE100 index reports one-step ahead of 0.03% (0.02%). The 95% intervals include zero, indicating mean reversion and no overreaction. The mean therefore suggests systematic differences from impulses between the European and US markets. Note that the NDX100 index seems to report an asymmetric mean by showing a 0.527% response to a –5% impulse and only –0.278% response to a 5% impulse. Moreover, the 95% confidence intervals do not overlap signalling statistical significance.

The NDX100 seems also to report asymmetric volatility responses following impulses. For example, giving the NDX100 index a –5% movement impulse show an increased volatility of 34.82% (from 4.54%) with a 95% confidence interval between 31.54% and 39.14%. In contrast, a 5% movement impulse report a calmer response volatility of 0.272% (from 0.599%) with a 95% confidence interval between 0.1957% and 0.408%. Giving the FTSE100 index a similar impulses of –5% (5%) impulse show an increased (calmer) volatility of 28.35% (from 4.44) (0.20% (from 0.377%)) with a confidence interval between 25.51% and 31.6% (0.147% and 0.292%). Note also that both confidence intervals marginally overlap. That is, a distinction between volatility levels are not possible. However, the result suggests that the NDX100 index volatility responses are marginally higher than for the FTSE index (for both negative and positive impulses). Furthermore, for both markets, the asymmetry between positive and negative impulses is clearly manifested through no overlap in the 95% confidence intervals.

5.3 Forecasting FTSE100 and NDX100 Volatility

The SNP methodology obtains a convenient representation of one-step ahead conditional variance $\hat{\sigma}_t^2$ of $\hat{y}_{t+1}$ given $\{\hat{y}_\tau\}_{\tau=1}^t$. From these SNP scores and a stochastic volatility model applying efficient method of moments (Solibakke, 2020), we use the by-product of a long simulated realisation of the state vector $\{\hat{V}_{i,t}\}_{t=1}^N, i = 1, 2$ together with the corresponding $\{\hat{y}_t\}_{t=1}^N$ for the optimally estimated parameter vector $\theta = \hat{\theta}$. Running regressions for V_{it} on $\hat{\sigma}_t^2$, $\hat{y}_t$ and $|\,\hat{y}_\tau\,|$ and a generous number of lags of theses series (data dependence), we obtain calibrated functions that give step ahead predicted values of $V_{it} \mid \{y_\tau\}_{\tau=1}^t$, $t = 1, 2$ at the original data points.

Figure 7 Volatility persistence plots from the 11114000 BIC optimal specifications (see online version for colours)

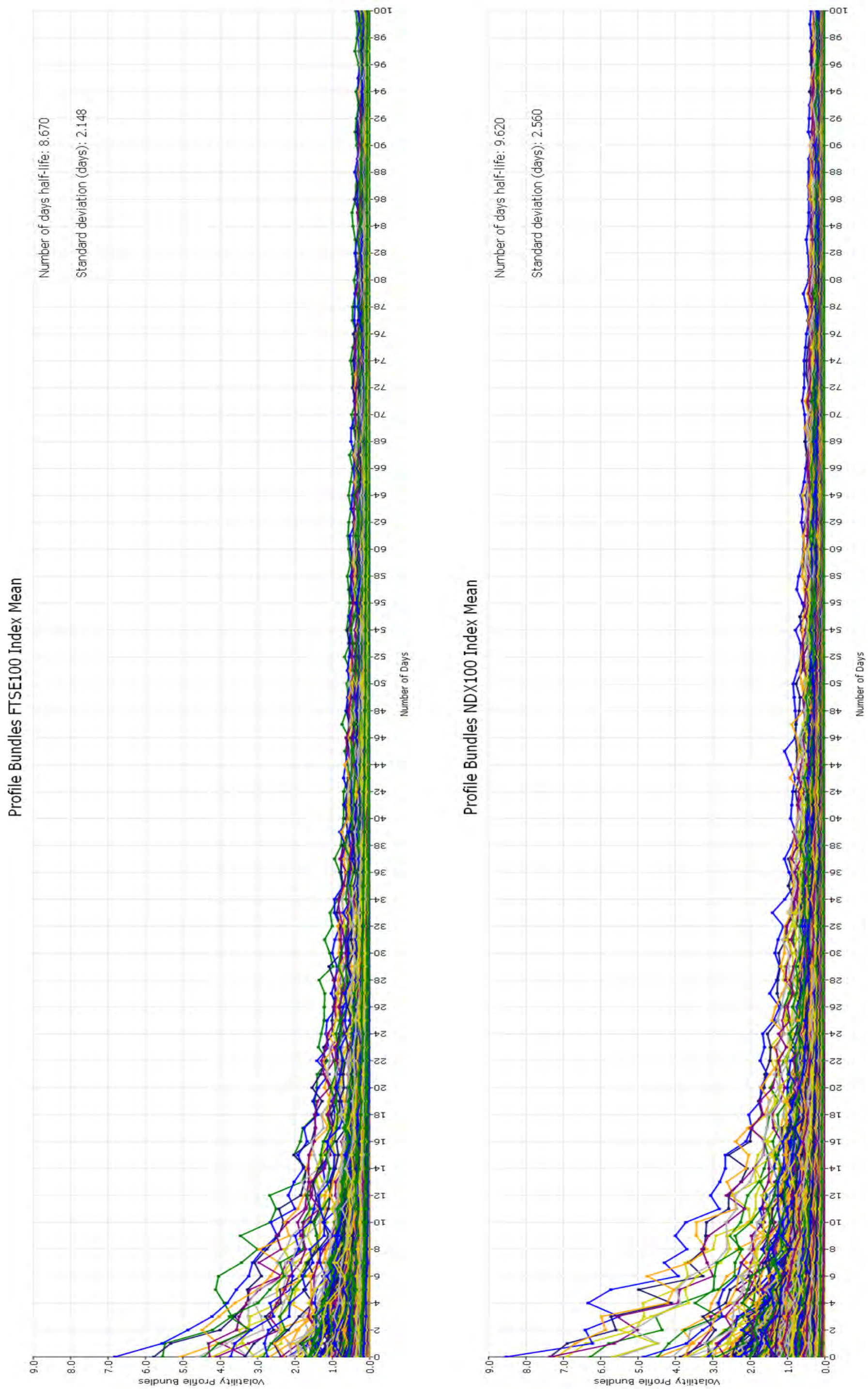
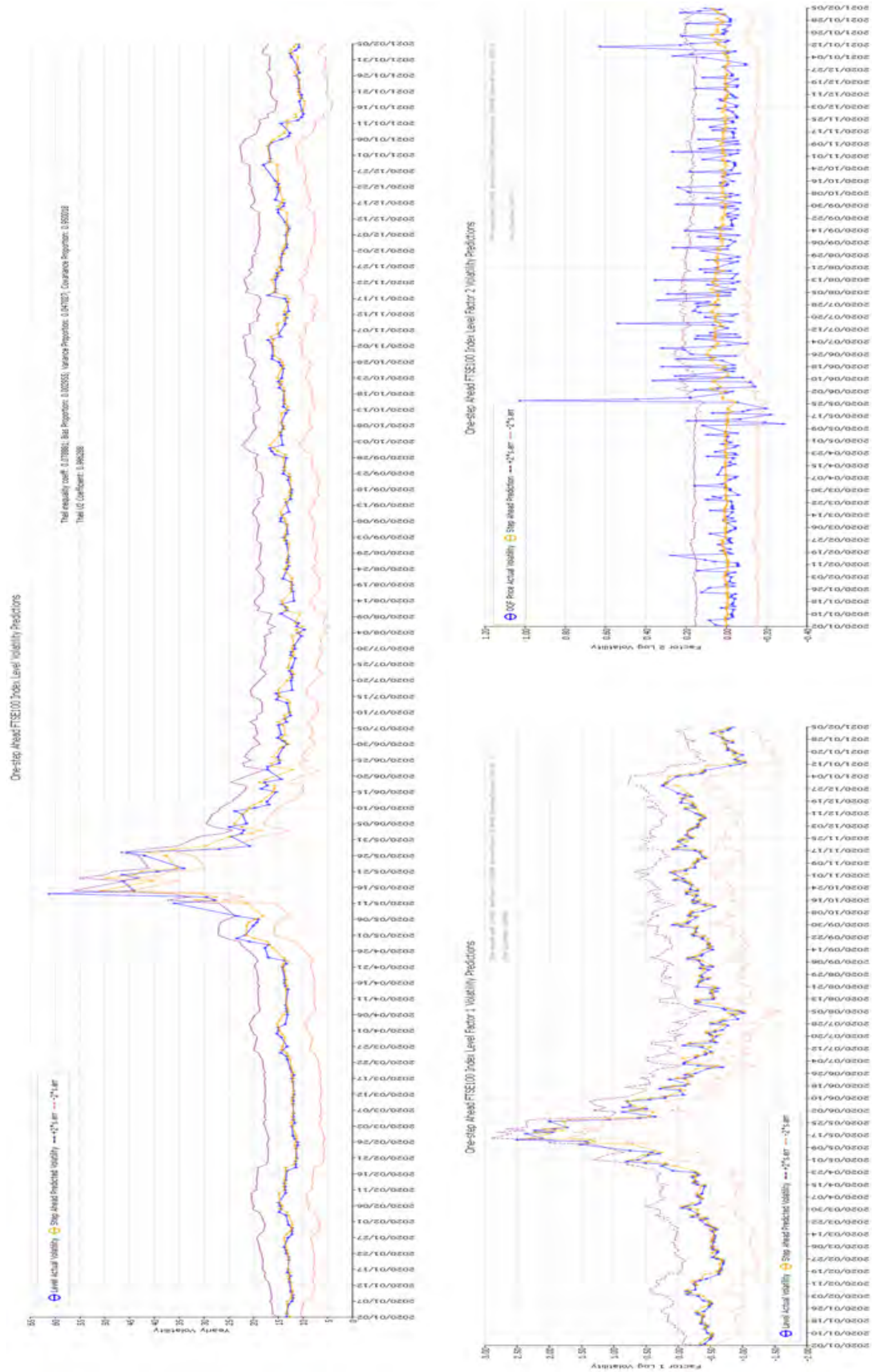
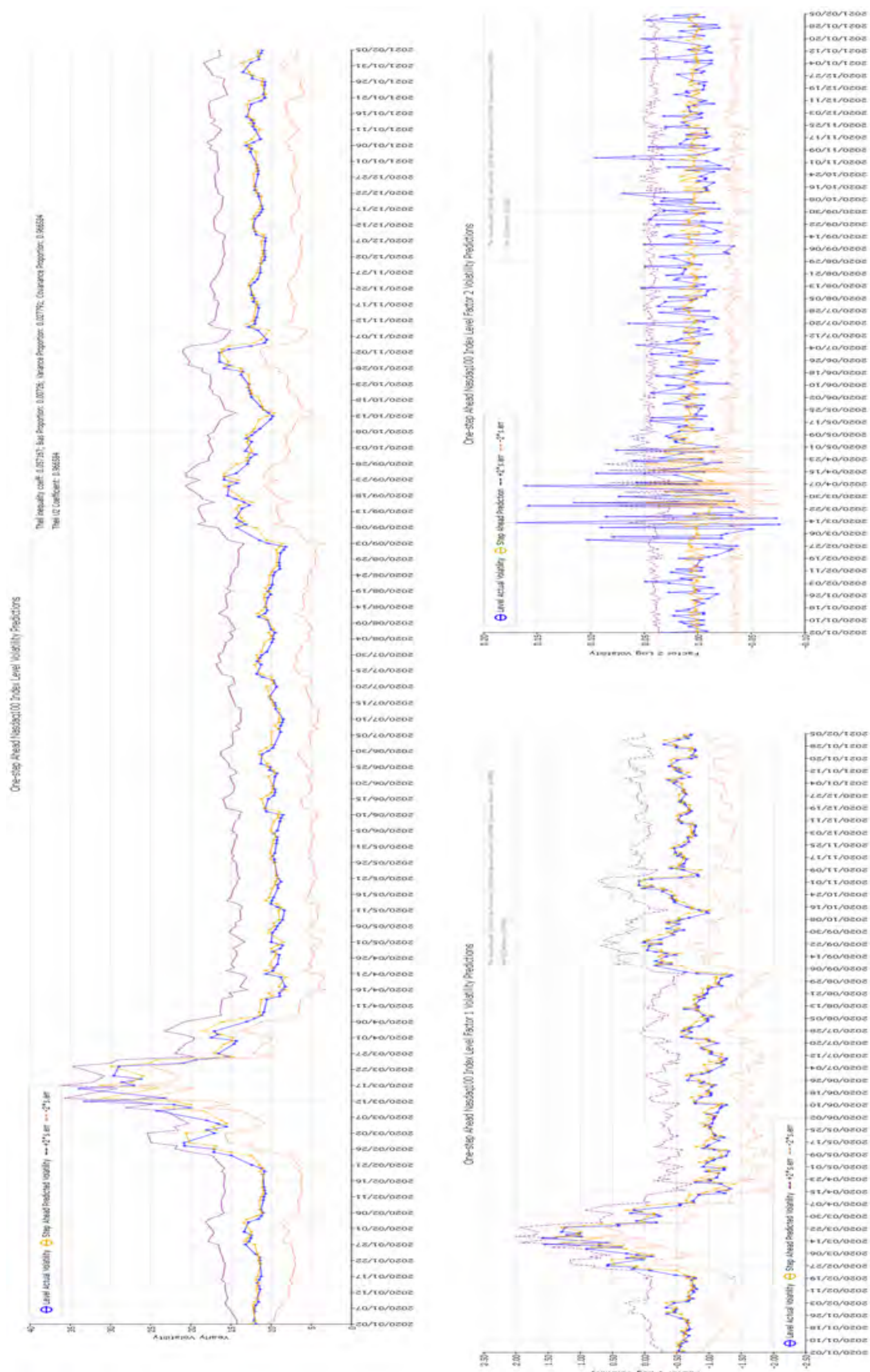


Figure 8 (a) FTSE100 and (b) NDX100 volatility predictions for 2020–2021 (see online version for colours)



(a)

Figure 8 (a) FTSE100 and (b) NDX100 volatility predictions for 2020–2021 (continued) (see online version for colours)



(b)

Table 5 Estimated stochastic volatility forecast fit measures

Contracts	*Error measures*	*Factor 1* V_1t:		*Factor 2* V_2t:		*Reprojected volatility*	
Ftse100 spot index (UK)	Root mean square Error (RMSE)	0.16381		0.12621		2.57933	
	Mean absolute error (MAE)	0.10799		0.09790		1.08914	
	Mean absolute percent error (MAPE)	60.5115		169.235		5.89113	
	Theil inequality coefficient (U_1)	0.14012		0.77683		0.07888	
	Bias proportion		0.002745		0.01092		0.04703
	Variance proportion		0.04146		0.66581		0.04703
	Covariance proportion		0.95580		0.32328		0.95002
	Theil U_2 coefficient	0.99796		0.90377		0.98629	
	Symmetric MAPE	38.6267		162.395		5.95861	
NDX100 spot index (US)	Root mean square error (RMSE)	0.18563		0.02879		1.46596	
	Mean absolute error (MAE)	0.13252		0.02052		0.82305	
	Mean absolute percent error (MAPE)	42.1825		221.626		6.01672	
	Theil inequality coefficient (U_1)	0.12077		0.64417		0.05717	
	Bias proportion		0.028095		0.02678		0.00726
	Variance proportion		0.02810		0.53798		0.02779
	Covariance proportion		0.95936		0.43524		0.96495
	Theil U_2 coefficient	0.79881		1.52722		0.96658	
	Symmetric MAPE	30.1854		147.590		5.98252	

Source: For all measures see Pindyck and Rubinfeld (1998)

It is difficult to forecast because the realisation of a stochastic process will be influenced of random events that happen in the future. In case of a large market movement at any time before the risk horizon the forecast needs to take this into account. However, a static forecast for the FTSE100 index (top) and the NDX100 index (bottom) is in Figure 8 and fit measures are reported in Table 5. The estimation period is from 2012 to January 1st, 2020 and the static forecasting period from January 1st, 2020 to February 5th, 2021. Static forecasting performs a series of one-step ahead forecasts of the dependent variable (Pindyck and Rubinfeld, 1998). For each observation the forecast computes $\hat{y}_{S+k} = \hat{c}(1) + c(j) y_{S+k-j}$, where j is the lag number of the forecasting variable y (always using the actual value of the lagged endogenous variable), requiring that data for any lagged endogenous variables be observed for every observation in the forecast sample. The static daily forecasts do not contain any exogenous variables.

For a 'good' measure of fit, using the Theil inequality coefficient (bias, variance, and covariance portions) the bias and variance should be small so that most of the bias is concentrated on the covariance proportion. The Theil's covariance proportion for re-projected volatility for the FTSE100 index (NDX100 index) is 0.950 (0.965). The two other columns of Table 5 show the fit measures for factor V_1 and V_2. From Figure 8 and the reprojected volatility plots (top), only March 2020 (COVID-19 outbreak) and possibly April 2020 report actual volatility outside of the predicted 95% confidence intervals. These results are also valid for factor V_1 while factor V_2 (tails) show several more breaks of the 95% confidence intervals. However, for the yearly volatility, the influence of V_2 relative to V_1, is considerably lower (see the axes). Running static forecasts for sub-samples for the period 2018, 2019 and compare the Theil covariance measures does not significantly change the Theil's covariance portion.

6 Summary and conclusions

We have modelled and estimated a non-parametric for the conditional mean and variance for the FTSE100 (UK) and NDX100 (US) for the period 2012 to 2021. The time series are estimated using ML and coefficients are optimally selected based on the BIC criterion. Our model captures the serial correlation structure in the return series, the effect of 'thick distribution tails' (leptokurtosis) and residual risk in the conditional mean. The conditional variance equation captures shock, persistence, and asymmetry and the two-equation specification control for conditional heteroscedasticity. The battery of statistical model specification tests cannot reject the BIC-optimal SNP specification (not reported). We summarise our results below.

The drift is close to zero for the FTSE100 index (UK), but positive for the NDX100 index. We find serial correlation structures for the mean as well as mean reversion in the two series. The volatility equation rejects conditional homoscedasticity suggesting some form of data dependence (serial correlation). The empirical impulse-response analysis confirms immediate dissipation (one day) suggesting linearity in the conditional mean equation. The impulse-response analysis reveals asymmetry and long memory. The mean report immediate mean reversion. Moreover, volatility seem to come solely from negative index movements suggesting that to add volatility as an asset class to an equity portfolio provides investors with excellent diversification. Furthermore, by the same token, holding volatility in an equity portfolio provides insurance against market crashes.

The persistence of shock for 2012–2021 is below ten trading days with an associated standard deviation of about 2.5 days. The paper finds significant differentials between European and US indices for the mean (no confidence interval overlap) but not for the volatility (confidence interval overlap). Moreover, the NDX100 negative overreaction response is close to double the size of the positive response. Finally, the index volatilities seem predictable with covariance portions higher than 95% (Theil's covariance portion).

References

Barroga, K. and Tan-Cruz, A. (2018) 'Energy consumption and gross domestic product in the Philippines: an application of maximum entropy bootstrap framework', *International Journal of Computational Economics and Econometrics*, Vol. 8, No. 1, pp.1–17.

Beaudry, P. and Koop, G. (1993) 'Do recessions permanently affect output?', *Journal of Monetary Economics*, Vol. 31, No. 2, pp.149–163.

Blanchard, O.J. and Quah, D. (1989) 'The dynamic effects of aggregate demand and supply disturbances', *The American Economic Review*, Vol. 79, No. 4, pp.655–673.

Bollerslev, T. (1986) 'Generalised autoregressive conditional heteroscedasticity', *Journal of Econometrics*, Vol. 31, No. 3, pp.307–327.

Bollerslev, T. (1987) 'A conditionally heteroscedastic time series model for speculative prices and rates of return', *Review of Economics and Statistics*, Vol. 69, No. 3, pp.542–547.

Bollerslev, T., Chou, R.Y. and Kroner, K.F. (1992) 'ARCH modeling in finance. A review of the theory and empirical evidence', *Journal of Econometrics*, Vol. 52, pp.5–59.

Brock, W.A., Deckert, W.D., Scheinkman, J.A. and LeBaron, B. (1996) 'A test for independence based on the correlation dimension', *Econometric Reviews*, Vol. 15, No. 3, pp.197–235.

Campbell, J. and Mankiw, N.G. (1987) 'Are output fluctuations transitory?', *Quarterly Journal of Economics*, Vol. 102, No. 4, pp.875–880.

Campbell, J.Y., Grossman, S.J. and Wang, J. (1993) 'Trading volume and serial correlation in stock returns', *Quarterly Journal of Economics*, Vol. 108, No. 4, pp.905–939.

de Lima, P.J.F. (1995a) *Nonlinearities and Nonstationarities in Stock Returns*, Working Paper in Economics, Department of Economics, Johns Hopkins University.

de Lima, P.J.F. (1995b) 'Nuisance parameter free properties of correlation integral based statistics', *Econometric Review*, Vol. 15, No. 3, pp.237–259.

Dickey, D.A. and Fuller, W.A. (1979) 'Distribution of the estimators for autoregressive time series with a unit root', *Journal of the American Statistical Association*, Vol. 74, No. 366, pp.427–431.

Ding, Z., Engle, R.F. and Granger, C.W.J. (1993) 'A long memory property of stock market returns and a new model', *Journal of Empirical Finance*, Vol. 1, No. 1, pp.83–106.

Doan, T., Litterman, R. and Sims, C. (1984) 'Forecasting and conditional projection using realistic prior distributions', *Econometric Reviews*, Vol. 3, No. 1, pp.1–100.

Engle, R.F. (1982) 'Autoregressive conditional heteroskedasticity with estimates of the variance of UK inflation', *Econometrica*, Vol. 50, No. 4, pp.987–1008.

Engle, R.F. and Bollerslev, T. (1986) 'Modelling the persistence of conditional variances', *Econometric Reviews*, Vol. 5, No. 1, pp.1–50.

Engle, R.F. and Ng, V.K. (1993) 'Measuring and testing the impact of news on volatility', *Journal of Finance*, Vol. 48, No. 5, pp.1749–1778.

Engle, R.F. and Patton, A.J. (2001) *What Good is a Volatility Model*, Working Paper [online] http://quantlabs.net/academy/download/free_quant_instituitional_books/Volatiliity.pdf; http://archive.nyu.edu/bitstream/2451/26881/2/S-DRP-01-03.pdf (accessed 30 September 2020).

Fan, L. and Mills, T.C. (2009) 'Size and power properties of tests of the martingale difference hypothesis: a Monte Carlo study', *International Journal of Computational Economics and Econometrics*, Vol. 1, No. 1, pp.48–63.

Gallant, A.R. and Nychka, D.W. (1987) 'Semi nonparametric maximum likelihood estimation', *Econometrica*, Vol. 55, No. 2, pp.363–390.

Gallant, A.R. and Tauchen, G. (1992) 'A nonparametric approach nonlinear time series analysis: estimation and simulation', in Parzen, E., Brillinger, D., Rosenblatt, M., Taqqu, M., Geweke, J. and Caines, P. (Eds.): *New Dimensions in Time Series Analysis*, Springer-Verlag, New York, NY.

Gallant, A.R. and Tauchen, G. (2010) 'Simulated score methods and indirect inference for continuous time models', in Aït-Sahalia, Y. and Hansen, L.P. (Eds.): *Handbook of Financial Econometrics*, North Holland, Chapter 8, pp.199–240.

Gallant, A.R. and Tauchen, G. (2014) *SNP: A Program for Non-Parametric Time Series Analysis*, Version 9.1 [online] http://www.aronaldg.org (accessed 2 February 2021).

Gallant, A.R., and Tauchen, G. (1998) 'Reprojecting partially observed systems with application to interest rate diffusions', *Journal of the American Statistical Association*, Vol. 93, No. 441, pp.10–24.

Gallant, A.R., Rossi, P.E. and Tauchen, G. (1992) 'Stock prices and volume', *Review of Financial Studies*, Vol. 5, No. 2, pp.199–242.

Gallant, A.R., Rossi, P.E. and Tauchen, G. (1993) 'Nonlinear dynamic structures', *Econometrica*, Vol. 61, No. 4, pp.871–907.

Glosten, L., Jagannathan, R. and Runkle, D. (1993) 'Relationship between the expected value and the volatility of the nominal excess return on stocks', *Journal of Finance*, Vol. 48, No. 5, pp.1779–1801.

Gouriéroux, C. (1997) *ARCH Models and Financial Applications*, Springer, New York.

Kwiatkowski, D., Phillips, C.B., Schmidt, P. and Shin, Y. (1992) 'Testing the null hypothesis of stationary against the alternative of a unit root', *Journal of Econometrics*, Vol. 54, Nos. 1–3, pp.154–179.

Lee, K. and Pesaran, M.H. (1993) 'Persistence profiles and business cycle fluctuations in a disaggregated model of UK output growth', *Richerche Economiche*, Vol. 47, No. 3, pp.293–322.

Mills, T.C. (1990) *Time Series Techniques for Economists*, Cambridge University Press, UK.

Nelson, D. (1991) 'Conditional heteroscedasticity in asset returns: a new approach', *Econometrica*, Vol. 59, No. 2, pp.347–370.

Pesaran, M.H. and Potter, S. (1994) *A Floor and Ceiling Model of US Output*, Department of Applied Economics Discussion Paper No. 9407, Cambridge University, Cambridge.

Pesaran, M.H., Pierse, R. and Lee, K. (1993) 'Persistence, cointegration and aggregation: a disaggregated analysis of output fluctuations in the US Economy', *Journal of Econometrics*, Vol. 56, Nos. 1–2, pp.57–88.

Pindyck, R. and Rubinfeld, D. (1998) *Econometric Models and Economic Forecasts*, 4th ed., McGraw-Hill, New York.

Potter, S. (1995) 'A nonlinear approach to US GNP', *Journal of Applied Econometrics*, Vol. 10, No. 2, pp.109–125.

Ramsey, J.B. (1969) 'Tests for specification errors in classical least square regression analysis', *Journal of Royal Statistical Society B*, Vol. 31, No. 2, pp.350–371.

Robinson, P.M. (1983) 'Nonparametric estimators for time series', *Journal of Time Series Analysis*, Vol. 4, No. 3, pp.185–207.

Schwarz, G. (1978) 'Estimating the dimension of a model', *Annals of Statistics*, pp.461–464.

Sims, C. (1980) 'Macroeconomics and reality', *Econometrica*, pp.1–48.

Solibakke, P.B. (2020) 'Stochastic volatility models predictive relevance for equity markets', in Valenzuela, O., Rojas, F., Herrera, L.J., Pomares, H. and Rojas, I. (Eds.): *Theory and Applications of Time Series Analysis : Selected Contributions from ITISE 2019*, 1st ed., p.1 online resource (XIV, 460 p. 95 illu, 141 illu in color.), Springer International Publishing, Imprint, Springer.

Notes

1 Ftse100 (UK) is share index of the 100 companies listed on the London Stock Exchange with the highest market capitalisation; NDX100 is a stock market index made up of 102 equity securities issued by 100 of the largest non-financial companies listed on the Nasdaq stock market (New York, USA). The indices are chosen based on the number of assets and both are central indices for the European and US stock markets.

2 The computer cluster at NTNU, Faculty of Economics and Management, Trondheim is used for estimation/implementation. A special thanks to Professor Asgeir Thomasgaard at NTNU, for access to the computer cluster.

3 See web-address: https://www.open-mpi.org

4 The conditional density is a complicated nonlinear function of many arguments.

5 The impulse-response methodology is also recognised under the name error shock methodology.

6 The ± 5% movement interval is chosen for this paper based on an assumption of 99.5% normal index movements.

7 For bootstrapping examples see for example Barroga and Tan-Cruz (2018) and Fan and Mills (2009).

8 Gallant and Tauchen (1998) find 18 (!) ARCH-lags for time series retrieved from the US financial market.

9 The code and user guide are available at http://www.aronaldg.org. The program is free software; you can redistribute it and/or modify it under the terms of the GNU General Public License as published by the Free Software Foundation; either version 2 of the License, or (at your option) any later version.

10 Based on likelihood ratio test statistics (LRT) the student-t log-likelihood function is strongly preferred to a normal likelihood function.

11 The BIC optimal SNP model is the $L_u = 14$, $L_g = 1$, $L_r = 1$, $L_v = 1$, L_ω, $L_p = 1$, $K_z = 12$, $K_x = 0$ specification.

12 The paper uses $j = 1,..,3$ and does not report day 4 to 10. The days 4 to 10 do not change much from day 3 for all impulses.

13 Mean tables are not reported due to manuscript size restrictions. All tables are available from author upon request.

14 Trading forward volatility via calendar spreads provides a vega hedge for forward start and cliquet options. Arbitrage traders and hedge funds may take positions on different volatilities of the same maturities and speculative market participants may simply make a bet on future volatility. These strategies have grown strongly in volume after the financial crisis in 2008.

Supplementary materials

For this paper, datasets for the FTSE100 and the NDX100 spot indices from the international equity markets for the period 2012–2021 are found in the following data files:

- 001_Equity_FTSE100_Spot_index_prices_returns_2012-2021.txt
- 002_Equity_NDX100_Spot_index_prices_returns_2012-2021.txt.