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An Adaptive Non-Linearity Detection Algorithm for Process Control Loops ⋆

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Abstract: Non-linearities are considered to be a major source of oscillations and poor performance in industrial control systems, as 20-30 % of loops are reported to be oscillating due to valve non-linearities (Srinivasan et al. (2005)). This fact has led to a significant effort aimed at the detection and diagnosis of non-linearities; in particular for valve non-linearities in the control loops. The current paper presents an adaptive algorithm, based on HHT (Hilbert Huang Transform), for non-linearity detection and isolation in process systems. The HHT is an adaptive data analysis technique that is applicable to non-linear and non-stationary time series. An index termed the *Degree of Non-Linearity* (DNL), based on intra-wave frequency modulation, is used to identify the presence of non-linearity in the signal generating system. The proposed method is shown to be more robust in differentiating between linear and non-linear causes of oscillations when compared to existing methods, and can handle non-stationary effects.

Keywords: Control non-linearities, adaptive algorithm, intra-wave frequency modulation, instantaneous frequency

1. INTRODUCTION

Control loop performance assessment (CLPA) is an important concept owing to the fact that only about one-third of all industrial control loops are reported to be giving satisfactory performance (Srinivasan et al. (2005)). These performance issues can have adverse effect on the productivity and profitability of any industrial process. Poor performance may manifest itself as poor set point tracking, excessive control actions and presence of oscillations. Among these manifestations of poor performance, the detection of oscillations and identification of their root cause has attracted significant research recently, as oscillations are the most common indicator of performance degradation. There can be a variety of different sources of oscillations, such as poor controller tuning, presence of non-linearities, disturbances etc. The high complexity of industrial systems necessitates the provision of robust monitoring mechanism that can detect the root cause of the oscillations, hence minimizing the critical maintenance time, energy consumption and ensuring product quality.

Of all sources of oscillations in control loops, valve non-linearities (backlash, stiction, hysteresis) are considered to be one of the major causes of oscillations. Therefore, distinguishing between linear and non-linear causes of oscillations is an important aspect of performance monitoring.

The oscillation characterization process can be divided into sub-categories like oscillation detection, grouping the loops oscillating due to (apparently) the same cause, and oscillation diagnosis. Oscillation detection is mainly concerned with the detection of oscillating loops and related characteristics such as the frequency and the amplitude of oscillations. Different methods to detect oscillation in individual loops as well as plant wide oscillations are reported in literature. Hägglund (1995) proposed a procedure based on monitoring the Integral Absolute Error (IAE) to detect the oscillations, whereas Thornhill et al. (2003) proposed to use the Auto Covariance Function (ACF) for detection and characterization of oscillations. A Modified Empirical Model Decomposition (EMD) method is used by Srinivasan et al. (2007). For grouping loops with similar oscillation patterns (and presumably also a common cause of oscillations), the use of tools like the Power Spectral Correlation Map (PSCMAP) (Tangirala et al. (2005)) and Principal Component Analysis (PCA) (Thornhill et al. (2002)) have been proposed.

The diagnosis part, that deals with the identifying the root cause of oscillations, is rather tricky owing to complex underlying plant dynamics and presence of multiple sources of oscillations. Most of the research revolves around differentiating between linear and non-linear causes of oscilla-

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tions; with non-linearities in process systems often being associated with valves, in particular due to valve stiction. Data driven approaches to detect non-linearities in the control loops are more practical as they do not require an accurate model of the process dynamics, which is seldom available. Some important non-linearity detection methods will be discussed in this paper; while readers are referred to Thornhill and Horch (2007) and Capaci and Scali (2015) for a more complete of oscillation detection and grouping methods.

A simple approach is to use the cross correlation between controller and plant output to differentiate between linear and stiction caused oscillations, for stable and non-integrating plant, is presented in Horch (1999). An odd correlation indicates the presence of stiction, whereas an even correlation indicates other causes for oscillation.

Different shape analysis formalism methods are proposed in Hägglund (2011) and Srinivasan et al. (2005) where the shape of the process output is compared with different patterns like square, triangular and sinusoidal waves to assess the presence (or absence) of stiction. Another similar shape analysis method is given in (Yamashita (2005)) where the phase portrait of the input and output of the controller is used to detect the presence of sticton.

It is well known that non-linearities in valves can generate limit cycles that have waveforms different from ordinary sinusoids, thereby giving rise to harmonics in the power spectrum. These harmonics can be used as a signature of non-linearity (Thornhill and Horch (2007)), but the high frequency peaks in the power spectrum due to fast disturbances may be mistaken for the non-linearity. Chaudhry et al. (2004) proposed a non-linearity detection method based on higher order statistics. The method claims to distinguish between oscillations caused by linear or non-linear sources, with a prior assumption of stationary plant data; the assumption of stationarity may not hold in actual practice. The limitations of cross correlation and bi-spectrum based techniques are highlighted in Rossi and Scali (2012). All the existing methods for detecting non-linearity in control loops thus rely on certain prior assumptions or conditions that may not be fulfilled in actual practice, therefore limiting the practical scope and application. This is not intended to say that these methods cannot be useful in practice, but rather that some care and expertise is required in their application.

In an effort to overcome the shortcomings in the listed approaches, an adaptive non-linearity detection method based on the Hilbert Huang Transform (HHT) Huang et al. (1998) is proposed in this paper. The essence of the concept is that the non-linearity manifests itself as intra-wave frequency modulation in the instantaneous frequency (IF). Babji et al. (2009) used the same concept to detect non-linearity in the control loops, but the procedure required manual inspection of instantaneous frequency plot and is therefore not applicable for automated non-linearity detection.

In this paper we propose the use of a non-linearity index called the degree of non-linearity (DNL), based on intra-wave frequency modulation, to detect the presence of non-linearity in the process control loop. This method is widely applicable and not limited to valve non-linearities. Furthermore, in this work, it is assumed that oscillations are being detected and objective is restricted to identify the linear or non-linear source of these oscillations. The proposed method can handle non-stationary time series, which is an inherent benefit of the HHT. This paper is organized as follows. Section 2 gives a brief overview of HHT and instantaneous frequency concepts. Section 3 presents the proposed non-linearity measurement index, DNL, that can be used to detect the non-linearity. Simulation studies are presented in Section 4.

2. HILBERT-HUANG TRANSFORM (HHT)

2.1 Overview

The Hilbert Huang Transform (HHT) is an adaptive data processing tool, recently developed by Huang et al. (1998). The HHT is finding applications in many different areas due to its adaptive nature and ability to handle non-stationary and non-linear time series. The procedure involves the decomposition of the time series into components called Intrinsic Mode Functions (IMFs). IMFs can be used to calculate the instantaneous frequency (IF) through the application of the Hilbert transform. In order to get a meaningful IF the IMFs must fulfil certain criteria, i.e. they must be symmetric, zero mean and their number of extrema and zero crossings should at-most differ by one. The component IMFs are generated through a decomposition method called Empirical Mode Decomposition (EMD).

The EMD process sifts out high frequency components from the data by iteratively subtracting low frequency components. These low frequency components are local means $m(t)$ of the envelope defined by spline fitting of the extrema.

$$d(t) = x(t) - m(t) \tag{1}$$

where $d(t)$ represents the local high frequency component Rilling et al. (2003). The sifting process is iterated on d until it qualifies as an IMF; and named as $c_1(t)$. Once the IMF is extracted it is subtracted from the original signal and the sifting procedure is started again on the residue. This continues until there are no more IMFs to be extracted. If $c_i(t)$ is the i^{th} IMF and $r(t)$ is residue, the sifting procedure gives

$$x(t) = \sum_{i=1}^{N} c_i(t) + r(t) \tag{2}$$

where $x(t)$ is the input time series and N is the total no of IMFs. The details of the procedure can be seen in Huang et al. (1998) and Rilling et al. (2003).

2.2 Instantaneous Frequency (IF)

An important consequence of the HHT is the Instantaneous frequency (IF) for time-frequency analysis. The Instantaneous frequency is calculated by the application of the Hilbert Transform to the IMF. A brief overview of the IF calculation is provided here. The Hilbert transform $Y(t)$ of a signal $X(t)$, also regarded as convolution of $x(t)$ and $1/\pi t$, is given by

$$Y(t) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{X(\tau)}{t-\tau} d\tau = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{X(t-\tau)}{\tau} d\tau \quad (3)$$

where P indicates Cauchy's principal value of the integral. The analytic signal $Z(t)$ is therefore defined as

$$\begin{aligned} Z(t) &= X(t) + jY(t) = a(t)e^{j\theta t} \\ a(t) &= \sqrt{X^2(t) + Y^2(t)}, \quad \theta(t) = \arctan \frac{Y(t)}{X(t)} \end{aligned} \quad (4)$$

Here the amplitude and phase are functions of time and the instantaneous frequency (IF) is defined as the time derivative of the phase function $\theta(t)$; given by (Wu and Huang (2009))

$$\omega(t) = \frac{d\theta(t)}{dt} = \frac{1}{A^2}[X\dot{Y} - Y\dot{X}] \quad (5)$$

The original signal $x(t)$ can be represented as

$$x(t) = \Re(a(t)e^{j\int \omega t}) \quad (6)$$

where $\Re$ is the real part of the analytic signal. The instantaneous frequency gives insight into the characteristics of the signal under analysis and can be used to detect non-linearity, as explained in the next section.

2.3 Discarding Spurious IMFs

The EMD process may generate spurious IMFs, as large swings at the ends of spline fitting may creep inwards and generate additional IMFs than necessary (Peng et al. (2005)). These spurious IMFs can be regarded as pseudo-components of the input signal and must be eliminated. As the IMFs generated through the EMD process are nearly orthogonal components, the significant IMFs will be more correlated with the input signal than the pseudo-components (Peng et al. (2005)). The correlation coefficient, therefore can be used to extract significant IMFs as reported in (Peng et al. (2005)) and (Sirinavasan and Rengasawamy (2012)). The correlation coefficient ρ_i of normalized i^{th} IMF c_i with normalized signal $x(t)$ is calculated from

$$\rho_i = \frac{Cov(c_i, x)}{\sigma_x \sigma_{c_i}}, \quad i = 1, 2, 3 \ldots n \quad (7)$$

where Cov is the covariance of i^{th} IMF and input signal $x(t)$; σ_x and σ_{c_i} are standard deviations of signal and IMF respectively and n is total number of IMFs. IMFs with normalized coefficient $\lambda > 0.5$ are retained , while the others are eliminated and added to the residue.

$$\lambda_i = \frac{\rho_i}{max(\rho_i)}, \quad i = 1, 2, 3 \ldots n \quad (8)$$

3. NON-LINEARITY DETECTION

3.1 Intra-wave Frequency Modulation

It is an established fact that non-linearity, in HHT, manifests itself as an intra-wave frequency modulation, i.e fluctuation of the IF within one period of oscillation (Huang et al. (1998),Babji et al. (2009),Wang et al. (2012)). The same fact is used in this paper to detect non-linearity in process control loops. The intra-wave frequency modulation in a non-linearly distorted wave form can further be elaborated by a simple example.

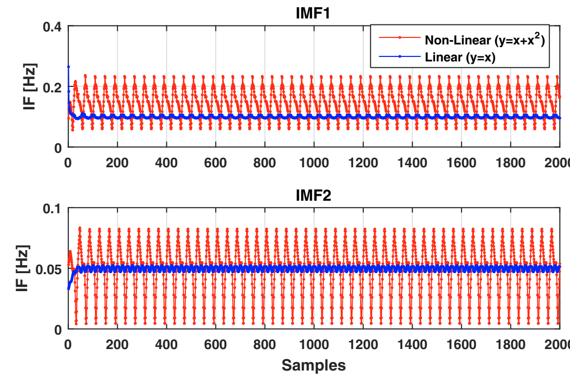

Fig. 1. Non-linearity effect: Intra-wave frequency modulation

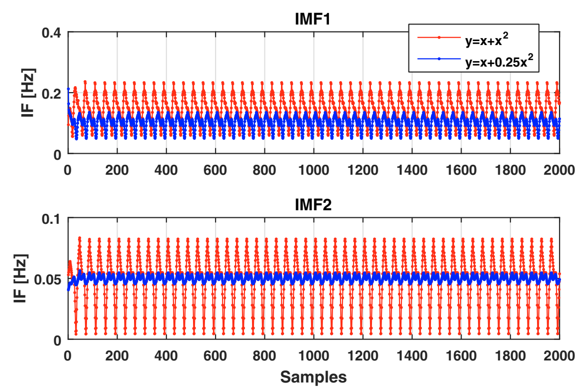

Fig. 2. Intra-wave frequency modulation spread increases with increased non-linearity

Example I: Intra-wave Frequency Modulation A signal is generated by sum of two sinusoids with frequencies $f_1 = 0.05Hz$ and $f_2 = 0.1Hz$. This signal is added to its square to create a non-linearly distorted waveform as given in (9).

$$\begin{aligned} x(k) &= \sin(2\pi f_1 k) + \sin(2\pi f_2 k) \\ y(k) &= x(k) + x^2(k) \end{aligned} \quad (9)$$

Signals $x(k)$ and $y(k)$ are subjected to EMD process to generate 2 IMFs each. The corresponding IF plots, given in figure 1, show significant intra-wave frequency modulation for the non-linearly distorted waveform $y(k)$).

3.2 Non-linearity Measurement Index

To develop the non-linearity index, it is being highlighted that the presence of non-linearity effects can be identified by intra wave frequency modulation. The difficult issue is to have a quantitative measure that can automatically detect the presence of non-linearity. The spread of the intra wave frequency modulation is proportional to the extent of non-linearity in the signal (IMF) as highlighted in figure 2.

As for the non-linearly distorted waveforms the IF changes within one oscillation cycle with variation proportional to the extent of non-linearity; mathematically we can define the degree of non-linearity in an IMF as (Huang et al. (2014))

$$DNL \propto var(IF)$$
$$DNL \propto \left\langle \left\{ \frac{IF - IF_z}{IF_z} \right\}^2 \right\rangle^{1/2} \quad (10)$$

where IF is the instantaneous frequency and IF_z is the full wave zero crossing frequency. However, it would be more appropriate for the measure to also include the effect of the amplitude in the DNL; as the distorted wave (non-linearity) with large amplitude should contribute more towards the degree of non-linearity. Therefore DNL, weighted by the amplitude,for an i^{th} IMF can be defined as (Huang et al. (2014))

$$DNL_i = std\left\langle \left\{ \frac{IF_i - IF_{z_i}}{IF_{z_i}} \right\} . \frac{a_{z_i}}{\bar{a}_{z_i}} \right\rangle \quad (11)$$

where a_z is the zero crossing amplitude; defined as the absolute value of the extremum between successive zero crossings, $\bar{a}_z$ is the mean of a_z and std represents the standard deviation of the resulting vector. Equation (11) gives the measure of Non-Linearity for individual IMF; but the actual signal may consists of number of IMFs. Therefore a **Total non-linearity (TDNL)** for the complete signal consisting of N IMFs can be given as sum of individual DNLs weighted by the energy of each IMF. The TDNL is given by (Huang et al. (2014))

$$TDNL = \sum_{j=1}^{N} \left\langle DNL_j \frac{|c_j|^2}{\sum_{k=1}^{N} |c_k|^2} \right\rangle \quad (12)$$

Where $|c_j|^2$ is 2-norm of j^{th} IMF.

3.3 Non-linearity Measure Threshold

Ideally the linear signal will have no intra-wave frequency modulation and hence zero DNL and TDNL; but in actual practice the linear signal may have slight fluctuations in IF due to spline fitting issues. This will be further elaborated by application of the non-linearity measure to the signal in Example-I. Though the exact threshold value to distinguish between linearity and otherwise is under investigation; for preliminary analysis threshold value of 0.1 is assumed in this work. Therefore detection of linear or non-linear source of oscillations is based on the rule summarized in Table 1.

Table 1. Non-linearity detection threshold

if DNL > 0.1
then non-linear source
else
linear source

3.4 Non-Linearity Measure for Example I

The DNL defined in (11) and (12) is applied to the signals in Example-I; the results are summarised in Table 2. The indices calculated in Table 2 show that the proposed measures can detect the varying non-linearity quite well. The strength of the proposed scheme in differentiating between linear and non-linear causes of oscillations are further established by simulation results of closed loop system with First Order plus Delay Time (FOPDT) plant model and PI controller in Section 4.

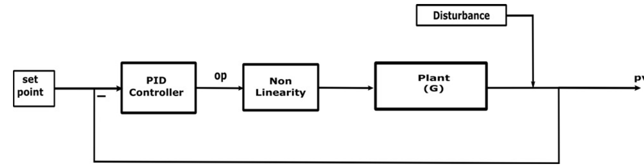

Fig. 3. Closed loop model

4. SIMULATION EXAMPLE

The objective is to distinguish between linear and non-linear causes of oscillation using the non-linearity measure given in section 3. The same scenario is discussed in the bi-spectrum method proposed by Chaudhry et al. (2004); but the scheme proposed in this paper can work even in case of transient effects and non-stationary data. The simulation example is single input single output closed loop system with a FOPDT plant and a PI controller. The plant dynamics are given by

$$G(s) = \frac{2.25}{4.5s + 1} e^{-3s} \quad (13)$$

Nominal PI controller gains are $K_c = 0.1$ and $K_i = 0.05$. The block diagram of the simulation setup is shown in Figure 3. The non-linearity is modelled as stiction in the valve; whereas for linear cases the non-linear block is removed. Simulation data is recorded for 4000 seconds with a sampling rate of 1 sec. The transient effects are also included to prove the effectiveness of the proposed scheme in the case of non-stationary data. In the following, oscillations are induced in the closed loop, with both linear and non-linear causes, in order to illustrate the ability of the HHT to differentiate between these causes.

4.1 Inappropriate controller tuning

The simulation is done with the stiction block removed and with increased integral gain in the controller ($K_i = 0.19$), thereby inducing oscillations in the system. The EMD process generates four IMFs but only the first one is significant and hence retained for non-linearity analysis; the corresponding oscillatory response and IMF is shown in row 1 of Figure 4. The Degree of Non-linearity (DNL), is calculated to be 0.057. The value establishes the absence of any non-linear element in the closed loop system. It is to be noted that the data also contains transient effects that can cause false detection of non-linearity if analysed by other tools such as the bi-spectrum based method.

4.2 External Sinusoidal Disturbance

The closed loop system is subjected to external sinusoidal disturbance with amplitude 5 and frequency $0.25 rad.sec^{-1}$. Here again only the first IMF is significant and is therefore analysed. The plant output and IMF are shown in row 2 of Figure 4. The DNL value of 0.023 again confirms the absence of any non-linearity as a source of oscillations.

Table 2. Non-linearity Example I

Signal	DNL (IMF1)	DNL (IMF2)	TDNL
$x(k)$	0.04	0.04	0.04
$x(k) + 0.25x^2(k)$	0.21	0.05	0.13
$x(k) + x^2(k)$	0.29	0.47	0.37

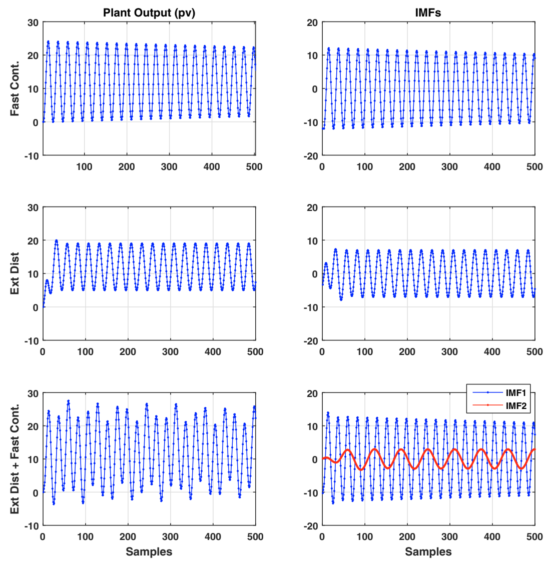

Fig. 4. Plant output and significant IMFs for oscillations from linear sources

4.3 Oscillations due to Multiple Linear Effects

In this case both increased integral action and sinusoidal disturbance of amplitude 10 and frequency $0.1rad.sec^{-1}$ are applied to generate oscillatory response of the system. The two IMFs showing two distinct frequency bands, one because of compromised loop stability and other due to external disturbance, and plant output are shown in row 3 of Figure 4. The non-linearity measures for both the IMFs are 0.036 and 0.022 ; indicating linear cause of oscillation. Again the presence of transient/ non-stationary effects are emphasized here that will hinder the diagnostics in other methods. The instantaneous frequencies (IF), given in row 1 of Figure 6, also shows the absence of any non-linearity as there are no intra-wave frequency modulations in the IF.

4.4 Stiction Non-linearity

In order to assess whether the proposed scheme can identify the presence of non-linearity as source of oscillations, the stiction non-linearity is introduced in the closed loop system as shown in Figure 3. Stiction is simulated using both data driven and physical models to illustrate the robustness of the proposed scheme. The data driven model used is the two parameter model, taken from Chaudhry et al. (2008); with stick-band S=7 and slip jump J=5, while the physical model is the LuGre model presented in Olsson (1996); the details and parameters of LuGre model are given in appendix A. The process output for both the models is shown in Figure 11. The parameters for both models are tuned to get similar response.The non-linearity measures for data driven model and LUGRe model are found to be 1.6 and 1.38 respectively; thereby indicating presence of similar extent of non-linearity in the system. The IF plot for both models, given in row 2 of Figure 6, shows significant intra-wave frequency modulation.

The non-linearitry meaures for different cases are summarized in Table 3.

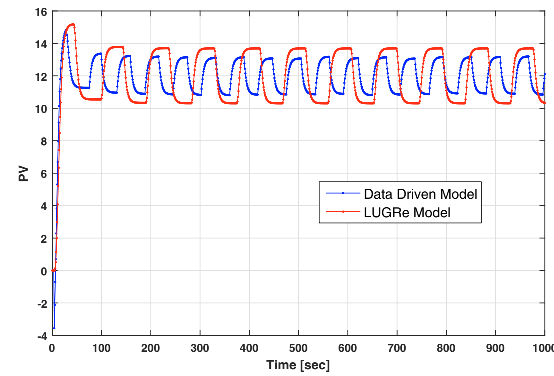

Fig. 5. Plant output: oscillations from non-linear sources

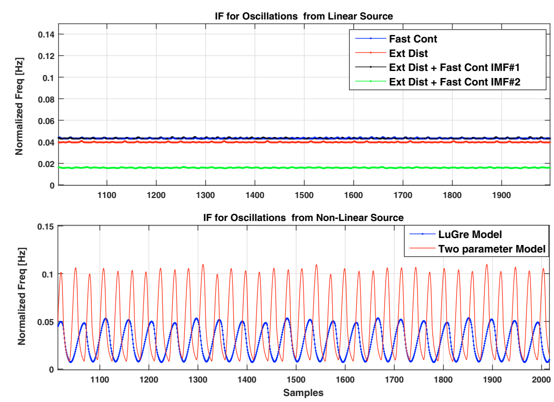

Fig. 6. Instantaneous frequencies (IF) for linear and non-linear sources of oscillations

Table 3. Non-linearity measures (closed loop FOPDT process)

Signal	DNL(IMFs)	TDNL
Fast Cont	0.057	0.057
Ext Dist	0.023	0.023
Ext Dist + Fast Cont	0.036 & 0.022	0.024
stiction (data driven model)	1.6	1.6
stiction (physical model)	1.38	1.38

5. CONCLUSIONS

In this paper an adaptive non-linearity detection algorithm based on the HHT is presented. The method can differentiate between linear and non-linear causes of oscillations solely from the recorded data and doesn't require any prior knowledge about the underlying process dynamics. The proposed algorithm can handle transient and non stationary effects that can limit the effectiveness of existing performance monitoring tools to a great extent. The application of the scheme to actual plant data is in progress and results will be published later.

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Appendix A. LUGRE FRICTION MODEL

The LuGre model used in the simulation is based on the work by Olsson (1996). A brief overview of the model is discussed here. The distance x travelled by the valve stem under the influence of the applied force F_e is governed by Newton's second law as

$$m\ddot{x} = F_e + F_f + F_k \tag{A.1}$$

where F_f and F_k are friction and spring forces respectively. The spring force given by $-kx$, with spring constant k acting as a proportional control action, and the valve stops when the spring force becomes equal to applied force F_e. The frictional force F_f arises due to the friction the stem experiences while travelling through the packaging element, the details can be seen in Chaudhry et al. (2008). The frictional force $F_f$is modelled as

$$\begin{aligned}
\dot{z} \quad &= v - \frac{|v|}{g(v)} z \\
F_f \quad &= \sigma_0 z + \sigma_1 \dot{z} + F_v v \\
g(v) &= \frac{1}{\sigma_0}(F_c + (F_s - F_c)e^{-(v/v_s)})
\end{aligned} \tag{A.2}$$

where F_s is the static friction; F_c is Coulomb friction and F_v is the coefficient of viscous friction, v_s is the Stribeck velocity, $\sigma_0 > 0$ is stiffness and $\sigma_1 > 0$ is velocity dependant damping coefficient. z is an intermediate variable representing relative average deflection of bristles (friction is modelled as a contact among bristles); (Olsson (1996)) The values of these parameters used in simulation are given in table A.1. The simulations are done with unit mass actuator and spring constant $k = 1$.

Table A.1. LuGre model parameters

F_s	F_c	F_v	σ_0	σ_1	v_s
1	0.5	0.3	10^4	$0.2\sqrt{10^5}$	0.01