

Mika Krogtoft

# Portfolio optimization with cryptocurrencies

Combining gold and bitcoin in modern portfolios

Bachelor's project in Science in Business Administration

Supervisor: Denis Becker

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Norwegian University of Science and Technology  
Faculty of Economics and Management  
NTNU Business School





## Abstract

This paper investigates whether bitcoin and gold can be added to a market portfolio to make more efficient portfolios. Using historical data and the efficient frontier theory, we simulate 2,500 portfolios with different weights for bitcoin, gold, and a market portfolio. Our results suggest that adding bitcoin to the portfolio significantly increases the return for given standard deviations. We also show that adding gold to the portfolio reduces the standard deviations for efficient portfolios. Splitting the weekly returns of SP500 from the lowest to greatest returns into equal-sized groups, we see some evidence of the safe-haven properties of gold, not for bitcoin.

«Innholdet i denne oppgaven står for forfatteren(e)s regning».

Supervisor: Denis Becker

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## 1 Introduction

Many investors have observed a new type of financial technology, namely cryptocurrencies. Cryptocurrencies are digital currencies that are often decentralized using blockchain technology. There have been launched many different cryptocurrencies lately, each with its technicalities and value proposition, the most popular one being bitcoin. With a fixed supply and a decentralized system without any centralized control or monetary policy, some investors expect that bitcoin will see an increased demand in the future. However, as a novel currency with having no tangible value, it is not easy to make accurate predictions for its future price. Many cryptocurrencies promise a new way of doing finance digitally and selling phrases and marketing. In this paper, we want to see how cryptocurrencies work in a portfolio. Having had much attention in the media and academia, we will examine its properties and see how cryptocurrencies can work in a portfolio together with gold.

An essential part of the finance field is diversifying the investments by combining different financial assets to reduce the risk. Several economists and theories have written on the subject. One of the most important ones is Markowitz's Portfolio Theory, focusing on generating as much profit as possible for a given level of risk.

During market crashes, there is a stronger correlation between global assets (Onnela *et al.*, 2003). Here, financial assets that are considered uncorrelated to the indexes in the market downturns can be valuable to include in the portfolio. These assets are so-called safe havens, assets that have a negative correlation to the market during crashes. Many academics regard gold as a safe-haven asset. Since gold is relatively uncorrelated to the market, many investors find it a helpful diversifier and a hedge against inflation.

The emergence of many new cryptocurrencies poses a critical question for investors. Should the new cryptocurrencies be a part of a well-diversified and modern portfolio? If so, how much should they be weighted? With these questions taken into consideration, the main research question of this paper is:

*Does bitcoin and gold offer diversification advantages in a market portfolio?*

We will also examine the following question:



*How can gold affect the risk and return of a market portfolio with bitcoin?*

To answer this, we will use Markowitz's portfolio theory to simulate different market portfolios with different weights of gold and Bitcoin. We will also examine the correlations between bitcoin and gold in different groups of SP500 returns. Using a rank scale for SP500, Bitcoin, and Gold, we get to check if there are any similarities in their variance at different levels of returns.

We will start this paper with a theoretical overview examining the properties of bitcoin and gold and making a comparison. We can better understand the asset's properties by examining the different financial assets qualitatively. After this, we will look at Markowitz's efficient frontiers. The methodology and data collection will be presented before showing the resulting efficient frontiers, descriptive statistics, correlations, and scatterplots, followed by a conclusion.

## 2 Theoretical Overview

### 2.1 What are cryptocurrencies?

Cryptocurrencies are digital currencies, where "crypto" refers to the cryptography behind the technology and "currency" referring to medium of exchange. Relying on a secure distributed ledger data structure in which mining adds records of past transactions to the blockchain, cryptocurrencies allow users to reach secure, robust consensus for each transaction (Mukhopadhyay *et al.*, 2016). Since Bitcoin emerged, more than 1 000 altcoins and crypto-tokens were created, with most of them trading on unregulated or registered exchanges (Chuen, Guo og Wang, 2017). Figure 2-1 list the top 10 cryptocurrencies sorted by market capitalization.

#	Name	Price	24h %	7d %	Market Cap	Volume(24h)	Circulating Supply	Last 7 Days
1	Bitcoin BTC	\$49,657.33	▲ 0.76%	▼ 9.23%	\$929,679,747,260	\$41,545,887,462 835,270 BTC	18,690,981 BTC	
2	Ethereum ETH	\$2,293.55	▲ 3.83%	▲ 8.68%	\$267,157,318,869	\$29,140,188,443 12,610,548 ETH	115,613,532 ETH	
3	Binance Coin BNB	\$503.45	▲ 1.53%	▲ 11.09%	\$77,911,894,826	\$2,736,762,104 5,389,541 BNB	153,432,897 BNB	
4	Tether USDT	\$1.00	▲ 0.03%	▲ 0.01%	\$49,962,550,070	\$82,666,545,063 82,671,019,631 USDT	49,965,254,439 USDT	
5	XRP XRP	\$1.06	▲ 0.08%	▼ 15.87%	\$48,464,263,195	\$6,628,803,612 6,210,233,463 XRP	45,404,028,640 XRP	
6	Cardano ADA	\$1.12	▼ 0.72%	▼ 6.12%	\$35,822,600,500	\$2,356,840,023 2,101,942,720 ADA	31,948,309,441 ADA	
7	Dogecoin DOGE	\$0.272	▲ 3.91%	▼ 11.42%	\$35,365,964,692	\$6,272,896,209 22,940,275,830 DOGE	129,334,992,638 DOGE	
8	Polkadot DOT	\$30.18	▲ 1.08%	▼ 15.89%	\$28,253,695,643	\$1,847,106,506 60,968,332 DOT	932,583,308 DOT	
9	Uniswap UNI	\$33.08	▲ 8.20%	▲ 9.93%	\$17,491,867,122	\$655,336,296 19,608,695 UNI	523,384,244 UNI	
10	Litecoin LTC	\$227.57	▲ 0.70%	▼ 12.50%	\$15,271,042,234	\$3,615,698,325 15,804,854 LTC	66,752,415 LTC	

Figure 2-1: Top 10 cryptocurrencies sorted by market capitalization. From Coinmarketwatch.com (25.04.2021).

The biggest cryptocurrency, which we will discuss in more detail here, is Bitcoin, with a market cap of nearly 930 billion dollars (April 2021). Compared to the market capitalization of SP500 of 33.62 trillion, it is approximately 2.8 percent.

## 2.2 Hedges and safe-havens

This paper will use the definition from (Baur og McDermott, 2010) for hedge and safe-havens.

A strong (weak) hedge is defined as an asset that is negatively correlated (uncorrelated) with another asset or portfolio on average.

A strong (weak) safe haven is defined as an asset that is negatively correlated (uncorrelated) with another asset or portfolio in certain periods only, e.g. in times of falling stock markets.

## 2.3 What is Bitcoin?

Bitcoin has a market capitalization of almost \$930 billion and considered one of the most popular cryptocurrencies. Having a transaction log that is distributed across a network of participating computers, the Bitcoin's rules were designed by engineers with no apparent influence from lawyers or regulators (Böhme *et al.*, 2015). The fact that the currency is decentralized and not controlled by banks is a frequently discussed topic in media and academic literature. As a decentralized currency in which there is no central government, some investors argue that it can diversify their portfolio. However, many of the positive qualities of Bitcoin, such as no governing body and irreversible transactions, also constitute some of its problematic properties.

Looking at Figure 2-2, we see the Google Trends results for the digital asset. The interest peaked in December 2017 and February 2021. However, the interest indicated by Google is still high from its peak in February 2021.

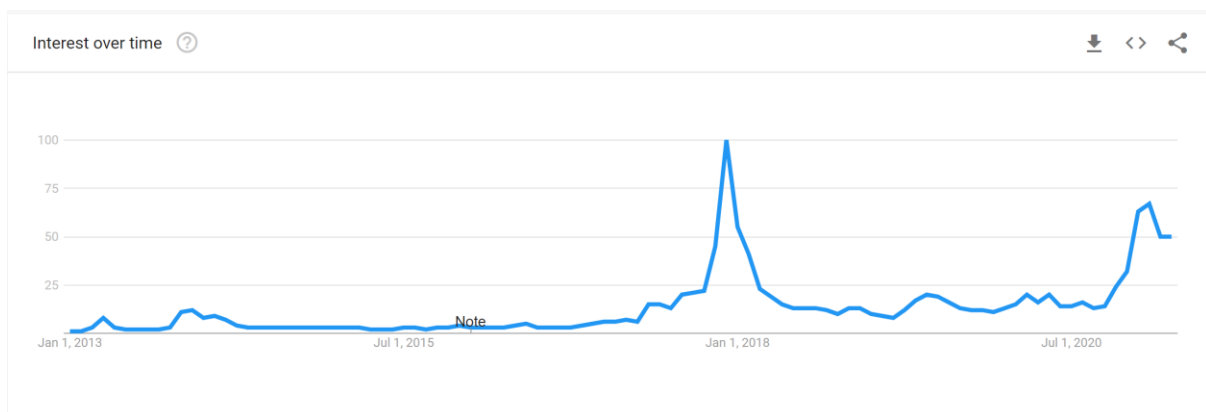


Figure 2-2: Google Trends, search interest of Bitcoin.

## 2.4 What is Gold?

Gold can be an effective hedge against inflation (Ghosh *et al.*, 2004). With a long history as an inflation hedge gold outperformed assets such as bonds, equities and even other commodities between 1974 and 2008 when U.S. inflation was high (Dempster og Artigas, 2010). Many academics and investors regard gold as both an inflation hedge and a safe haven. Being both a hedge and a safe haven for U.S. and European markets, gold may act as a stabilizing force in market downturns (Baur og McDermott, 2010). The price volatility of gold is relatively low. Gold has been used for centuries as a luxury good, and some claim that it is safe because it is time-tested. Gold has inherent value because it is a metal and can be used in various products. There are different applications for gold because of its physical properties and corrosion resistance. Some of the physical properties are high electrical potential (V), electrical resistivity, and thermal conductivity, creating many industrial uses (Corti og Holliday, 2004).

At the start of January 1973, the price per ounce of gold was \$64. At the end of 2020, it was \$1951 per ounce, an increase of 30.5 times. The price development of SP500 was from \$119 to \$3756 in the same period, 31.5 times increase in dollars. However, gold has had less volatility than SP500, especially in economic downturns.

One dollar could purchase 5.86 more goods and services in 1973 than in 2021. Saving dollars between 1973 and 2021 would yield a decrease in investors' wealth by a factor of 5.86. In other words, only 17% of the initial dollar value is stored over that period. If the investor stored the money as gold in that period, the wealth adjusted for inflation would have grown over five times in the same period. Having the savings in SP500 yields a similar return of 5.3 in the period. In figure 2-3 we see the decline of the purchasing power of one dollar between 1899 and 2021.

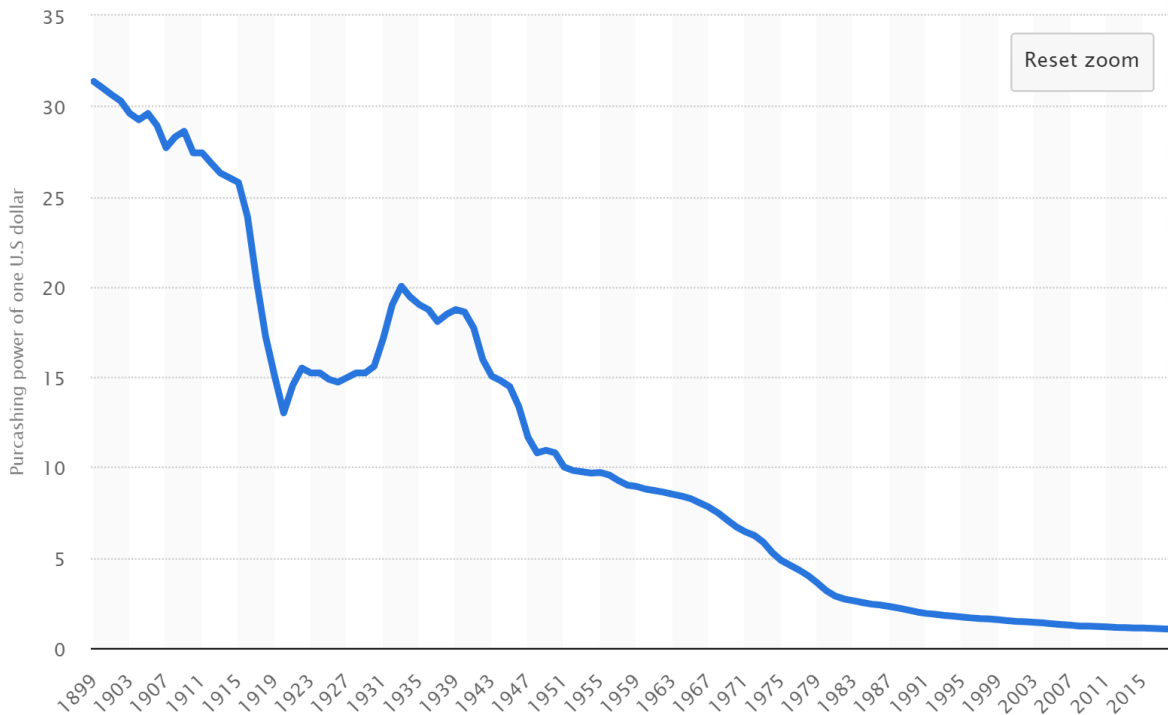


Figure 2-3: The purchasing power of one US dollar from 1800 to 2021

## 2.5 Markowitz and the theory of the efficient frontier

The efficient frontier is a set of optimal portfolios where there is no other portfolio with a lower standard deviation for that expected return. The model was created by the economist Harry Markowitz, which also contributed to Modern Monetary Theory (MMT). Markowitz's emphasized that risk must be considered when evaluating an investment opportunity. The theory of discounted fails to imply diversification and never implies a diversified portfolio that is preferable to all non-diversified (Markowitz, 1952).

By investing in several assets and thus diversifying, one can reduce the firm-specific risk of one company. It is possible to diversify to reduce the firm-specific risk, but the market risk will still exist. The market risk is also called systematic risk, and the company-specific risk is called non-systematic risk. The non-systematic risk and systematic risk are illustrated in figure 2-4.

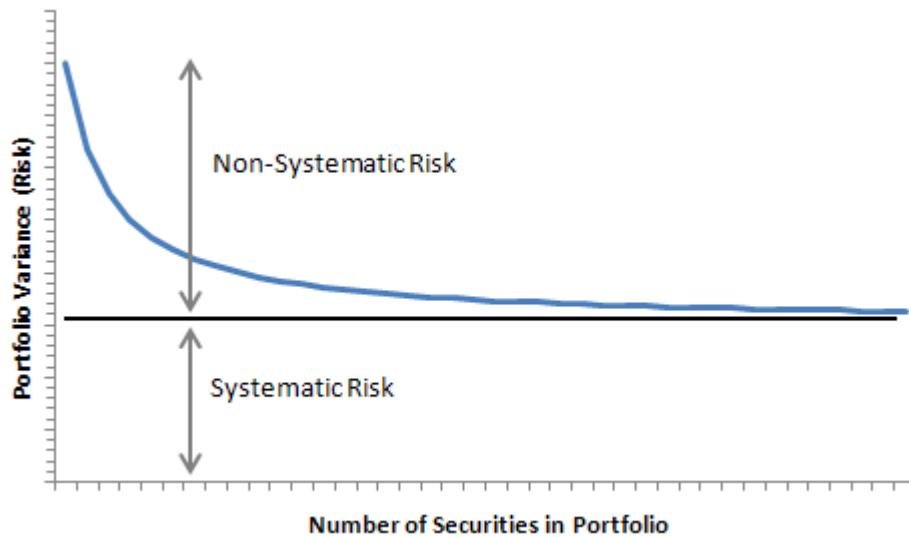


Figure 2-4: Systematic Risk and Non-Systematic Risk on the number of securities in the portfolio

As the illustration shows, by increasing the number of securities in a portfolio, an investor can reduce the risk of the portfolios. The risk goes drastically down with the first 3-5 number of assets, and after more assets, the risk will go towards the systematic risk existent in the markets.

The assets on the efficient frontier are the portfolios with the lowest possible standard deviation for its return. As stated at the beginning of this chapter, the efficient frontier is the datapoints or portfolios where the investor will obtain the highest return for a given standard deviation (risk). Said in other words, when a portfolio is optimal, there is no other portfolio with lower risk for that amount of return. The efficient portfolios represent a concave line, as shown in figure 2-5 below.

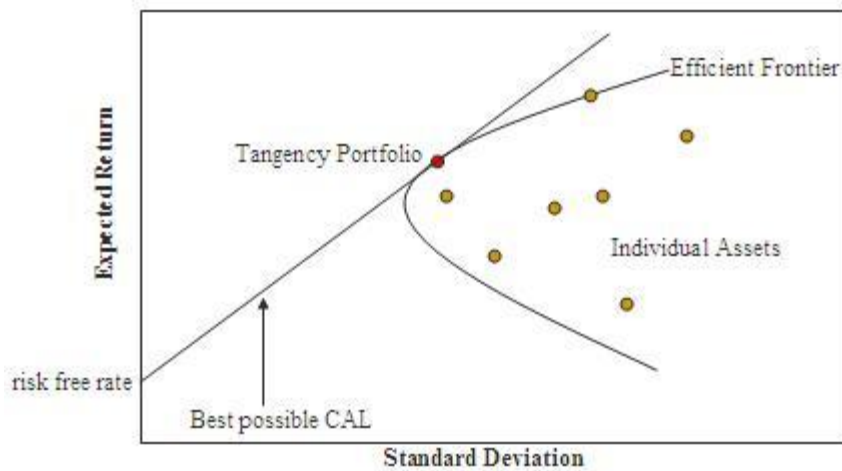


Figure 2-5: The efficient frontier with the Tangency portfolio

Using a tangent from the risk free rate to the efficient frontier, the best portfolio can be selected. We assume that the investor can borrow and lend, thereby leveraging the portfolio. We find the tangency portfolio in the intersection between the tangent and the efficient frontier. In theory, we want to find the optimal portfolio by adding leverage. This is the tangency portfolio, in which we can leverage up to the desired rate of risk and return. When we compare the returns for the transformation line and the efficient frontier, it becomes apparent that one can obtain a higher return for the same standard deviation with leverage.

The illustration shows that the risk-free rate is assumed to have no standard deviation and some return. These are often the rate of government bonds issued by governments. This is important to define because the risk-free rate will affect the steepness of the transformation line and, thereby, the tangency portfolio.

The shape of the efficient frontier is affected by, among other factors, the correlation between the assets. As we will show in figure 2-6, an investor can reduce the portfolio's risk by selecting uncorrelated assets.

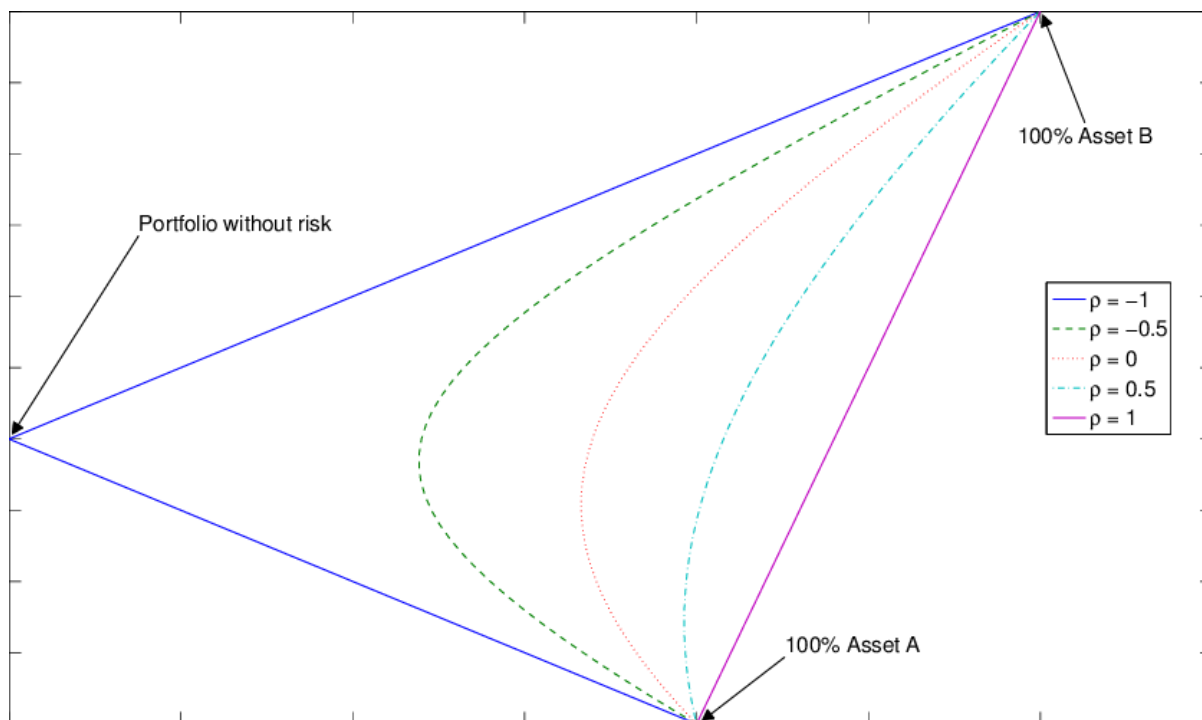


Figure 2-6: Portfolio risk with different correlations.

In the figure, we see that if the assets are entirely negatively correlated, there will be no risk. This suggests that an investor would want to combine uncorrelated assets. However, in practice, it is hard to find completely uncorrelated assets. The green line shows how the efficient frontier will look like when they are negatively correlated with the value of -0.5.

The theory of efficient frontier has many assumptions. One of them is that the returns are normally distributed. This is not realistic, and several of the returns we will analyze are several standard deviations away from the mean. Other assumptions are that the investor can access borrowing and lending at a risk-free rate, and no brokerage fees are included. In addition, Markowitz assumes that the investors are rational and that market prices cannot be influenced by the investors.

## 2.6 Hedge, safe-haven, and diversifiers

As we have seen, when constructing a portfolio, one investor can add assets that are uncorrelated to each other. The investor could also make sure to have assets uncorrelated with the market, especially in the market downturn, to secure the portfolio against economic uncertainty. Especially in today's markets, some analysts see the increasing money supply and



increased government spending after the corona crisis factors driving inflation. In such periods, investors could use assets defined as hedges or safe-havens to protect their wealth.

## 2.7 The properties of Bitcoin

Bitcoin has different properties that set it apart from other cryptocurrencies. Firstly, the currency is entirely decentralized and is not maintained by any bank. This means that it is not subject to monetary policy, which can create inflation in the currency. It uses a decentralized validation network of different contributing nodes that validates the transactions. One of the big arguments from Bitcoin enthusiasts is that Bitcoin has a fixed money supply. After 21 million bitcoin is mined, it will not make any more Bitcoin. The predictable growth of supply is one of the main advantages of bitcoin. Thinking in such simple terms as supply and demand, many investors argue that people will demand more of the currency in the future, and with the fixed demand, the price will increase by a significant amount. In addition, the risk-return tradeoff of cryptocurrencies distinct from stocks, currencies, and metals, cryptocurrency has no exposure to the most common stock market and macroeconomic factors (Liu og Tsyvinski, 2018).

Money has three critical functions, medium of exchange, unit of account, and store of value (Mattke, Maier og Reis). It is not certain that Bitcoin serves all these three functions. Many cryptocurrency exchanges emerged, and bitcoin can be traded on most of these platforms. One of the disadvantages of Bitcoin is its amount of volatility. Therefore, it could be challenging for it to be a unit of account. With loss aversion, many investors would be scared of investing in such an asset. Some crypto enthusiasts argue that the price will stabilize when the currency gets wider acceptance. Cryptocurrencies can be used for cybercrimes like money laundering, cyber extortion, hacking because of their unregulated and pseudo-anonymous nature (Reddy og Minnaar, 2018) that may reduce its future reliability and price.

## 2.8 Comparison of Bitcoin and Golds properties

Following is a table showing some of the differences between the properties of bitcoin and gold. The list of properties is by no means exhaustive but sheds light on some of the most important aspects to compare. The properties of comparison are scarcity, counterfeit resistance, decentralization, portability, and divisibility.

	<b>Bitcoin</b>	<b>Gold</b>
Scarcity	21 million maximum supply.	Limited supply, difficult to estimate.
Counterfeit Resistance	Preventing counterfeiting by validator nodes.	Spectrometer test. Expensive.
Decentralization	Decentralized issuance.	Centralized corporate control of much of the production and supply.
Portability	Not physical and portable	Physical assets, transport is a big cost.
Divisibility	Can be divided into Satoshis (100 000 000 denomination units).	The smallest unit is a gold grain of 0.06 grams. Expensive process.

*Table 1: Comparison of bitcoin and gold*

### 3 Data and Method

#### 3.1 Data gathering

The data used in this paper is historical prices starting with the first available date price for BTC, 4 November 2013 to present-day 12 April 2021. The weekly historical prices for BTC were gathered from coindesk.com. Prices for stock indices were collected from the Eikon database. Daily prices were gathered and aligned with the weekly BTC price each Monday. Inspired by (Klabbers, 2017) major stock indices were gathered, together with one commodity index and a global real estate index.

Base portfolio	
Stock indices	Non-stock indices
S&P 500	SP GSCI commodity total
FTSE 100	FTSE EPRA/NAREIT gobal
Nikkei 225	
Shanghai A-share	
Dax 30	
MSCI World	

Table 2: The base portfolio

To test the research question of whether bitcoin or gold could be used as a diversifier in a portfolio, these instruments were added to the portfolio.

Diversification/safe haven Portfolio	
Bitcoin	Gold

Table 3: Diversification/safe haven Portfolio

#### 3.2 Excel, mean-variance, and Markowitz.

Mean-variance is a type of analysis considering both the risks and the possible return. As discussed, Markowitz introduced the theory behind the efficient frontier.

First, the weekly returns of all the assets are calculated. Using the weekly returns of the assets, the standard deviation, the variance, and the covariance between all the assets are calculated. Random weights for each asset are created by generating a random number and

normalizing the number creating a random weight for each asset summing up to 100%. Using a cross multiplying function, the return and standard deviation can be calculated for each portfolio. Simulation runs 2500 times, generating random weights for the different assets. Each portfolio in turn produces a return and a standard deviation. Then generate the efficient frontier with and without bitcoin. After this, we produce an efficient frontier for the bitcoin portfolio with and without gold to check the effect of gold in the portfolio.

Using the returns and standard deviations produced by the simulation, we get a scatterplot with the different market portfolios. Then we use the solver function in excel to find the portfolios on the efficient frontier. This is done by choosing a return and minimizing the standard deviation for that return. This is done for the market portfolio with bitcoin and gold, market portfolio with only gold, and then only bitcoin.

Rank scales are created by sorting from the most positive return to the most negative. The returns are then ranked, and the number is converted to a normalized number. This is done for Bitcoin, Gold, SP500. The rank scale for FTSE and SP500 is used to check if there are any patterns between two assets we know correlate.

To check the correlation of bitcoin and gold to SP500 in different periods of positive and negative return, the return of the SP500 is split into 12 groups. This is done to make sure there are enough data points in each bucket to get significant results.

## 4 Results

In this part of the paper, we will start to look at the summary statistics of the base portfolio's different assets, including bitcoin and gold. Following the summary statistics, correlations and rank scales will be shown. After this, we will look at the different correlations fo bitcoin and gold to check its correlation to the SP500 in this period. The last part will show the efficient frontiers with the base portfolio with bitcoin and gold, only with bitcoin and base portfolio, and only with gold. Here we can see the different effects of both bitcoin and gold on the portfolio. Lastly, we will analyze the different weights used of bitcoin, gold, and SP500 for different levels of expected return on the efficient frontier.

### 4.1 Summary Statistics

The summary statistics are color-coded to represent their weight in the row. This is done to make it easier to see any outliers in the data. As we can see, bitcoin has a mean weekly return of 2.14%. That is much compared to gold and SP500 with 0.10 and 0.25 mean weekly return, respectively. We see that bitcoin also has much volatility, with its standard deviation of 11.90% compared to gold with 2.02% and SP500 of 2.3%.

	% BTC	% Gold	% SP500	% FTSE	% MSCI	% SSEA	% GDAXI	% N225	% GSCI_C	% MWO.PA
Mean	2.137%	0.098%	0.247%	0.007%	0.179%	0.160%	0.143%	0.226%	-0.160%	0.058%
Standard Error	0.604%	0.103%	0.120%	0.145%	0.116%	0.174%	0.145%	0.138%	0.153%	0.134%
Median	1.043%	0.110%	0.384%	0.194%	0.350%	0.294%	0.420%	0.318%	0.061%	0.251%
Standard Deviation	11.891%	2.024%	2.370%	2.855%	2.277%	3.424%	2.856%	2.722%	3.006%	2.643%
Sample Variance	1.414%	0.041%	0.056%	0.082%	0.052%	0.117%	0.082%	0.074%	0.090%	0.070%
Kurtosis	366.037%	253.602%	1155.911%	1291.178%	1297.173%	438.676%	934.262%	336.216%	398.015%	1944.518%
Skewness	82.206%	8.558%	-19.502%	-25.479%	-28.232%	-32.883%	-46.365%	-54.877%	-60.637%	-63.755%
Range	101.862%	18.710%	30.520%	39.129%	31.408%	33.302%	35.332%	26.700%	30.691%	38.270%
Minimum	-34.440%	-9.283%	-13.123%	-19.343%	-14.657%	-19.744%	-19.632%	-13.690%	-17.904%	-20.304%
Maximum	67.422%	9.427%	17.397%	19.785%	16.751%	13.558%	15.700%	13.011%	12.787%	17.966%
Sum	8.29280733	0.37989235	0.9580639	0.02773537	0.69627156	0.62240582	0.55454755	0.87784383	-0.61942687	0.22677649

Figure 4-1: Descriptive statistics

Looking at the correlation matrix, we see some distinct properties of the variation of gold and bitcoin. Gold has a low correlation for all assets, and in most cases, has a negative sign of the correlation. In the next part of the analysis, we will look at the correlations of BTC and Gold for different groups of SP500 weekly returns.

	% BTC	% Gold	% SP500	% FTSE	% MSCI	% SSEA	% GDAXI	% N225	% GSCI_C	% MWO.PA	% CPI	% PPI	% 10T
% BTC	100.00%												
% Gold	-2.31%	100.00%											
% SP500	10.75%	1.74%	100.00%										
% FTSE	11.44%	-2.91%	81.40%	100.00%									
% MSCI	11.84%	1.30%	97.96%	88.75%	100.00%								
% SSEA	7.41%	-4.33%	35.01%	33.04%	37.60%	100.00%							
% GDAXI	14.39%	0.34%	77.68%	86.18%	85.66%	34.12%	100.00%						
% N225	15.60%	-2.06%	70.90%	64.63%	75.58%	36.91%	67.03%	100.00%					
% GSCI_C	10.07%	0.48%	40.71%	42.16%	43.41%	22.23%	36.58%	30.27%	100.00%				
% MWO.PA	5.62%	-3.84%	70.38%	73.96%	74.51%	23.58%	67.13%	49.71%	25.88%	100.00%			
% CPI	3.19%	-14.60%	4.31%	8.51%	5.40%	3.01%	5.31%	1.70%	8.11%	11.49%	100.00%		
% PPI	7.29%	-16.24%	2.15%	7.99%	3.42%	1.42%	5.57%	4.52%	13.62%	8.84%	74.29%	100.00%	
% 10T	8.10%	3.88%	11.04%	10.03%	12.06%	6.86%	13.86%	29.39%	23.32%	-0.82%	2.08%	17.00%	100.00%

Figure 4-2: Correlation matrix of the different assets in the portfolio.

## 4.2 Correlations and rank scales

To get big enough groups to get reliable results for the correlation analysis, the dataset of SP500 was split into 12 equal-sized groups of 32 data points. The different percentage change intervals for the SP500 in the different groups are shown in the illustration below. The two most extreme values both on the negative and positive side of the returns, were removed. This is because they deviate from the other data points and their effect on the correlation.

	From	To	Corr. BTC	Corr. Gold
G1	-10.0%	-2.6%	12.7%	-7.3%
G2	-2.6%	-1.2%	20.4%	-31.5%
G3	-1.2%	-0.6%	-28.6%	15.6%
G4	-0.6%	-0.2%	-12.3%	14.3%
G5	-0.2%	0.1%	0.8%	37.9%
G6	0.1%	0.4%	-21.4%	-14.1%
G7	0.4%	0.6%	8.2%	-20.2%
G8	0.7%	1.0%	-3.9%	8.6%
G9	1.0%	1.3%	-28.6%	-14.2%
G10	1.3%	1.7%	9.4%	-22.7%
G11	1.7%	2.5%	-22.2%	-8.8%
G12	2.5%	6.6%	-12.8%	0.2%

Figure 4-3: The intervals of weekly SP500 returns for the different groups.

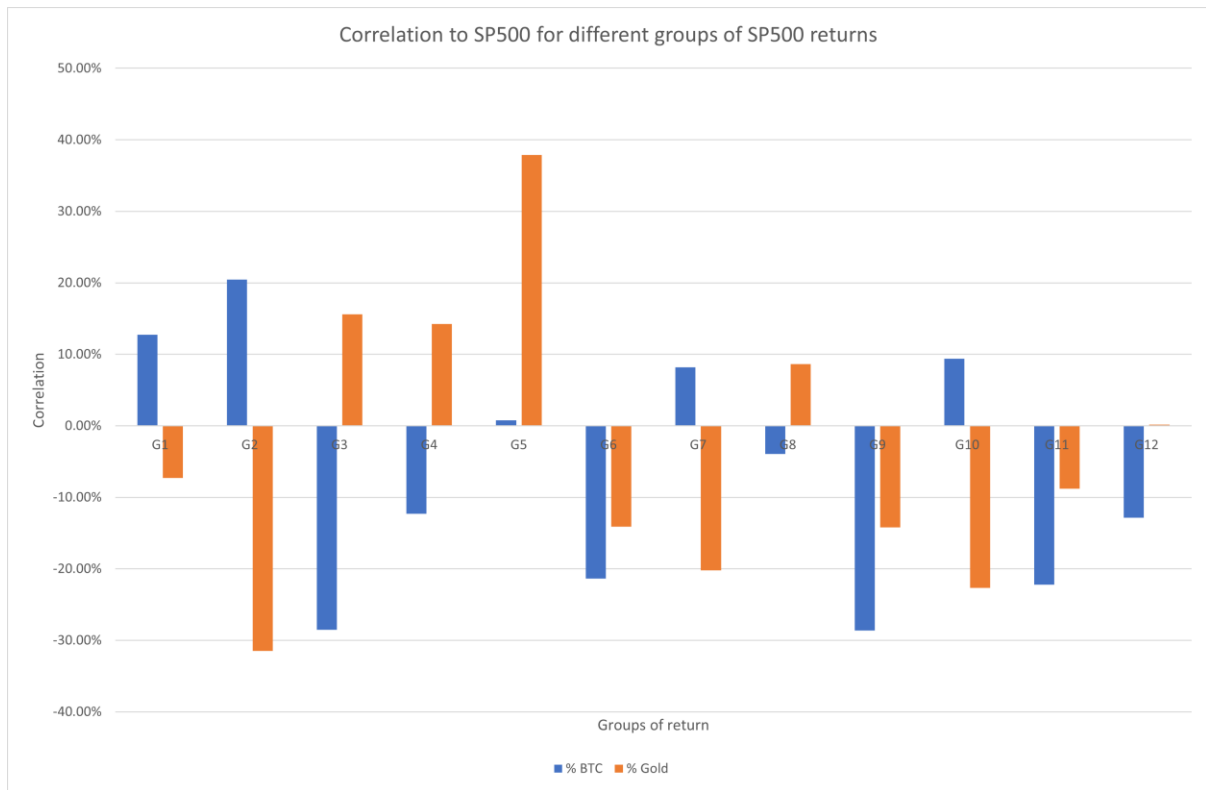


Figure 4-4: Correlation of BTC and Gold sorted by the percentage of weekly change in the SP500.

We see that gold is negatively correlated to SP500 in the two first groups (-10% to -1.2%). The correlation is especially negatively strong in G2 where SP500 is (-2.6% to -1.2%). On the other hand, bitcoin has a low positive correlation to SP500 in the first two groups. Having a low correlation to SP500 in the first two groups, this may be an indication of the safe-haven properties of gold.

The rank scale of SP500 and FTSE produces a predictable result. With its high correlation of 81.40% in the period, we can also see how the scaled values produce a cluster toward the same values.

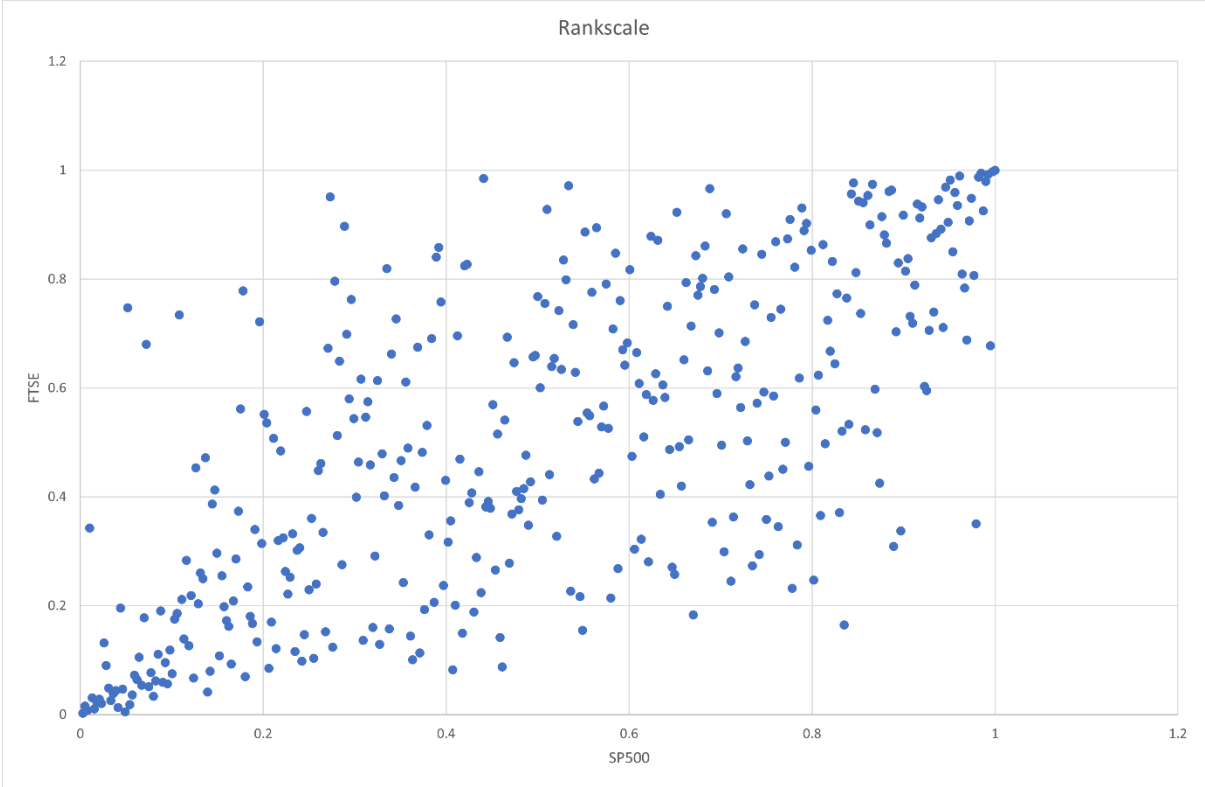


Figure 4-5: Rank scale of SP500 (x-axis) and FTSE (y-axis).

Looking at the rank scale of SP500 and bitcoin we see no distinct pattern. There are no apparent similarities between BTC and Gold either. This poses an interesting opportunity to use the assets together to benefit diversification. The same applies to SP500 and Gold, with no distinct pattern suggesting that their returns are not related.



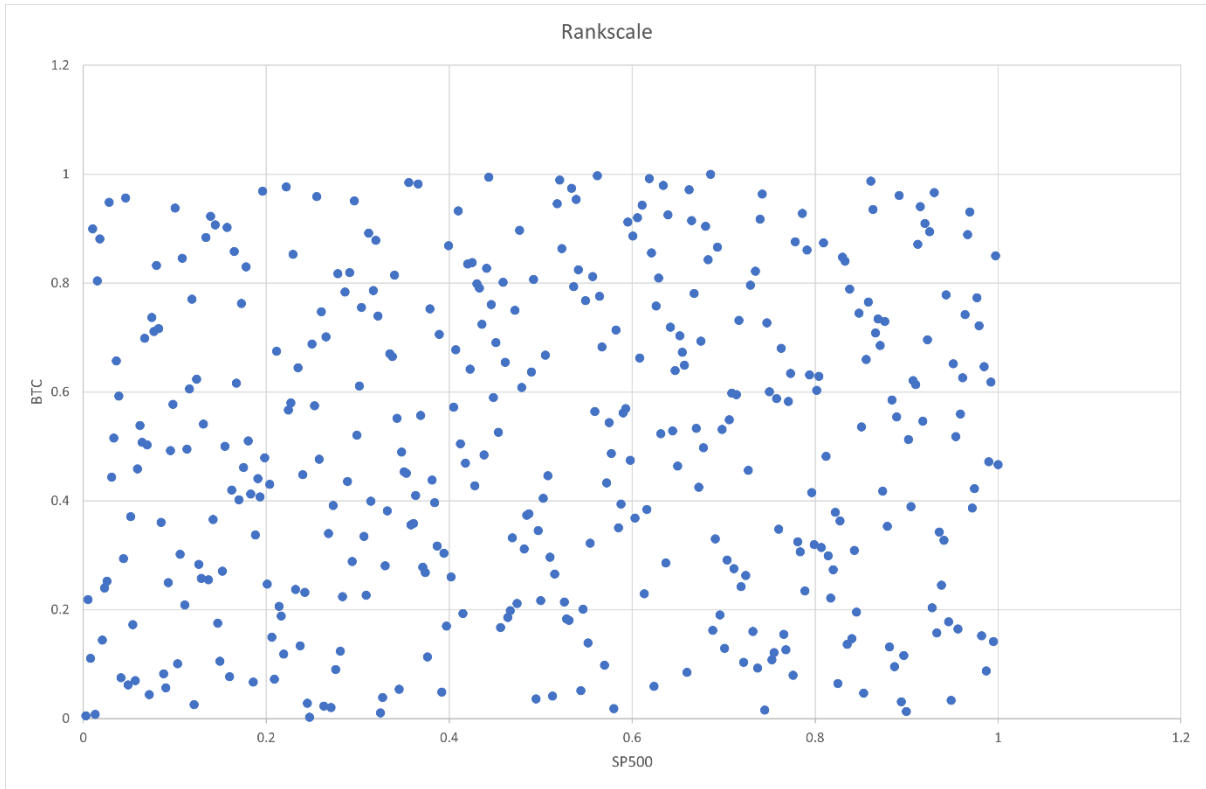


Figure 4-6: Rank scale of SP500 (x-axis) and BTC (y-axis)

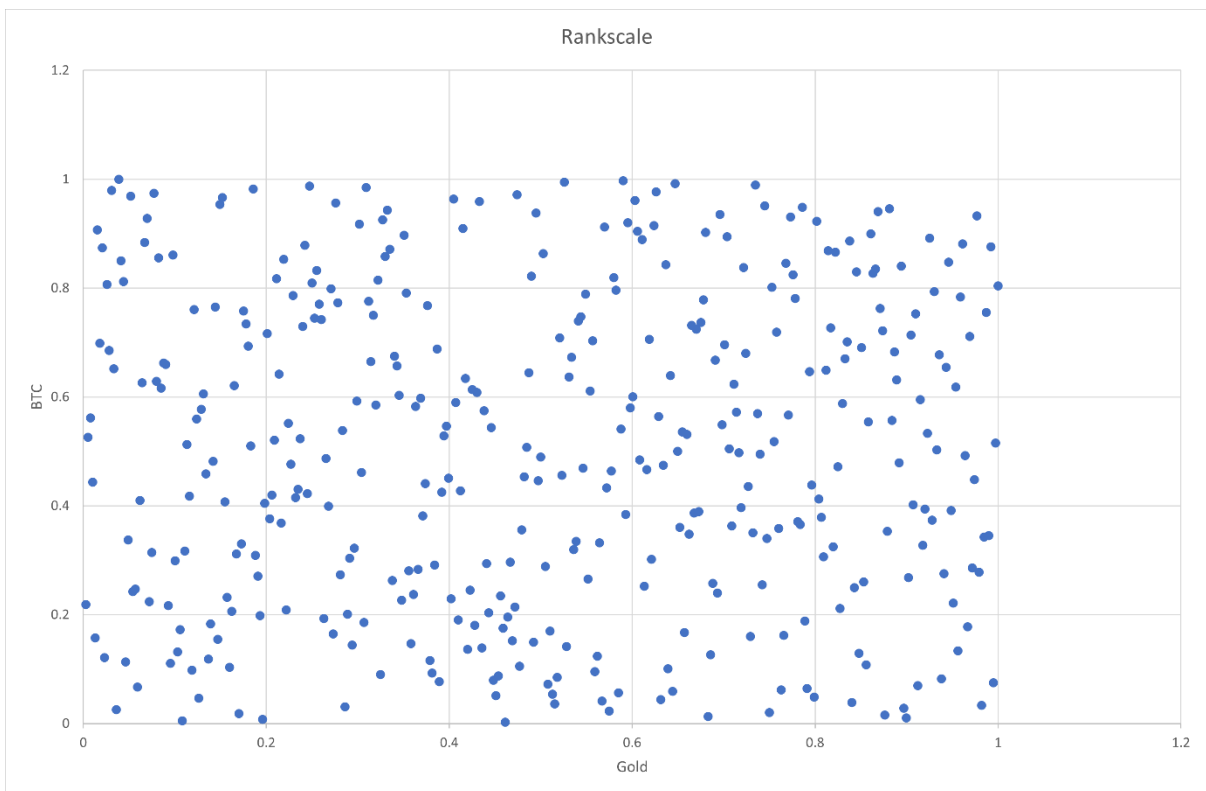


Figure 4-7: Rank scale of gold (x-axis) and BTC (y-axis)

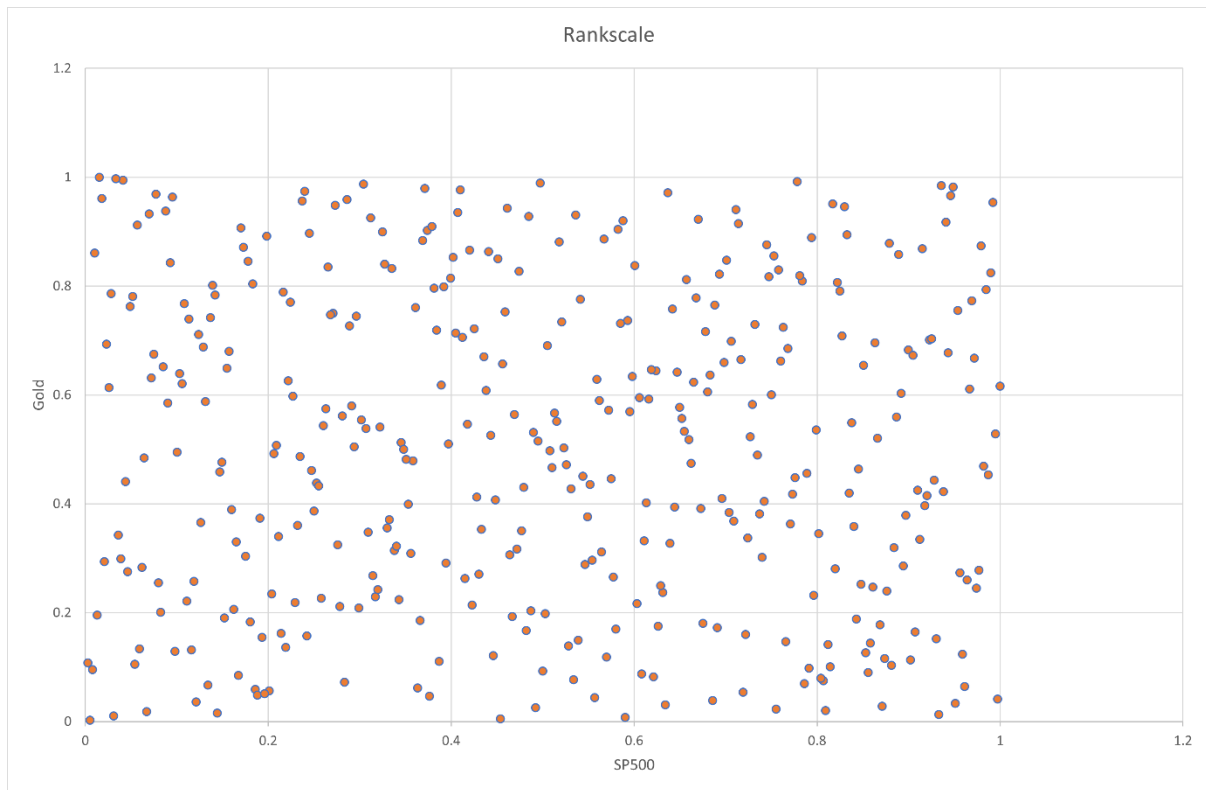


Figure 4-8: Rank scale of SP500 (x-axis) and gold

### 4.3 Efficient frontiers

#### 4.3.1 Portfolio with and without BTC

In figure 4-9, we can see the two efficient frontiers, the orange one is the base portfolio with both bitcoin and gold. The blue efficient frontier is the base portfolio with only gold. The results indicate that with bitcoin in the portfolio, the investors can get a significantly higher return for the same levels of standard deviation. The lowest standard deviation achievable is around 1.40% starting with a return of 0.1%.

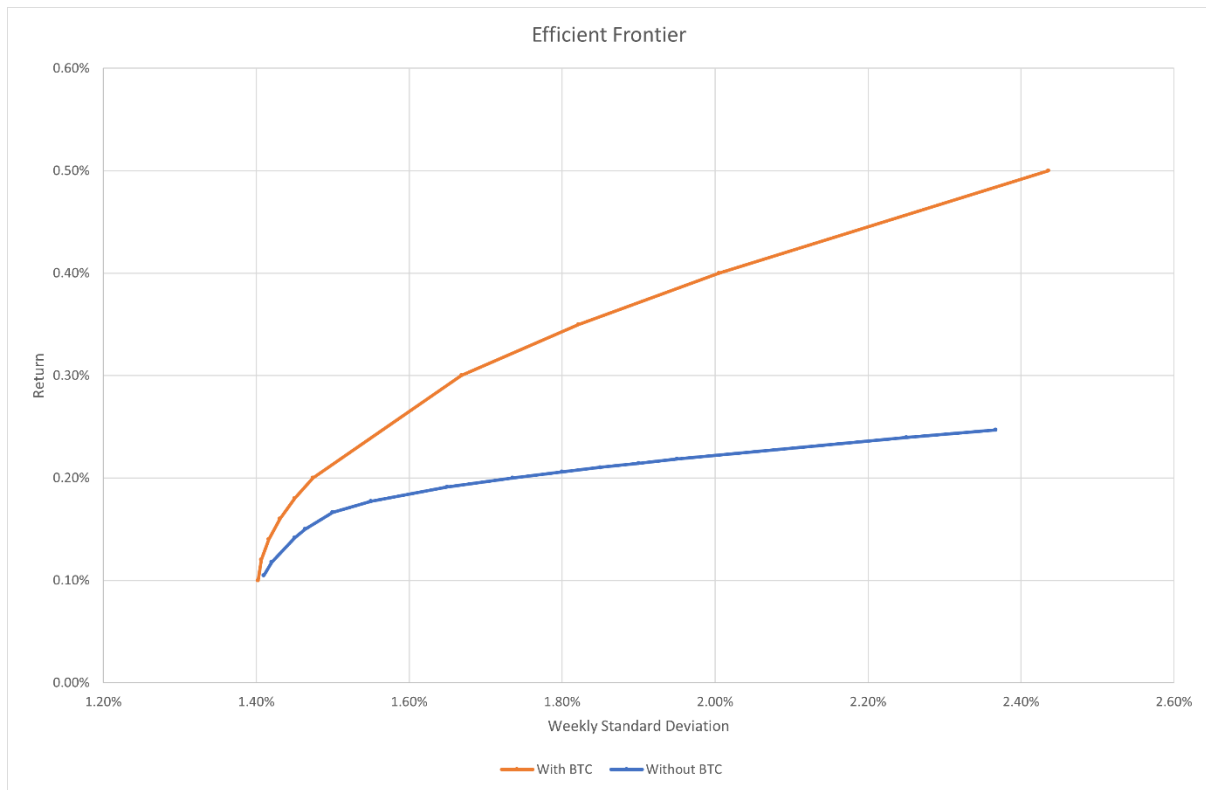


Figure 4-9: Efficient frontier for the portfolio with BTC and without BTC.

#### 4.3.2 The effect of gold on the portfolio

When looking at the effect of gold in the portfolio in figure 4-10, we may see some of the benefits of having gold in the portfolio. The two efficient frontiers in figure 4-9, are both including gold. In the new efficient frontier, we see the baseline portfolio with bitcoin and no gold. This shows us the effect gold has on the portfolio. The three points calculated for the base portfolio without bitcoin started at a standard deviation of almost 2.1% but a return of just a little over 2.2%. By adding gold to the portfolio, the investor can transform the investment from a 2.2% return to a 4.1% return for the same standard deviation of 2.1%.

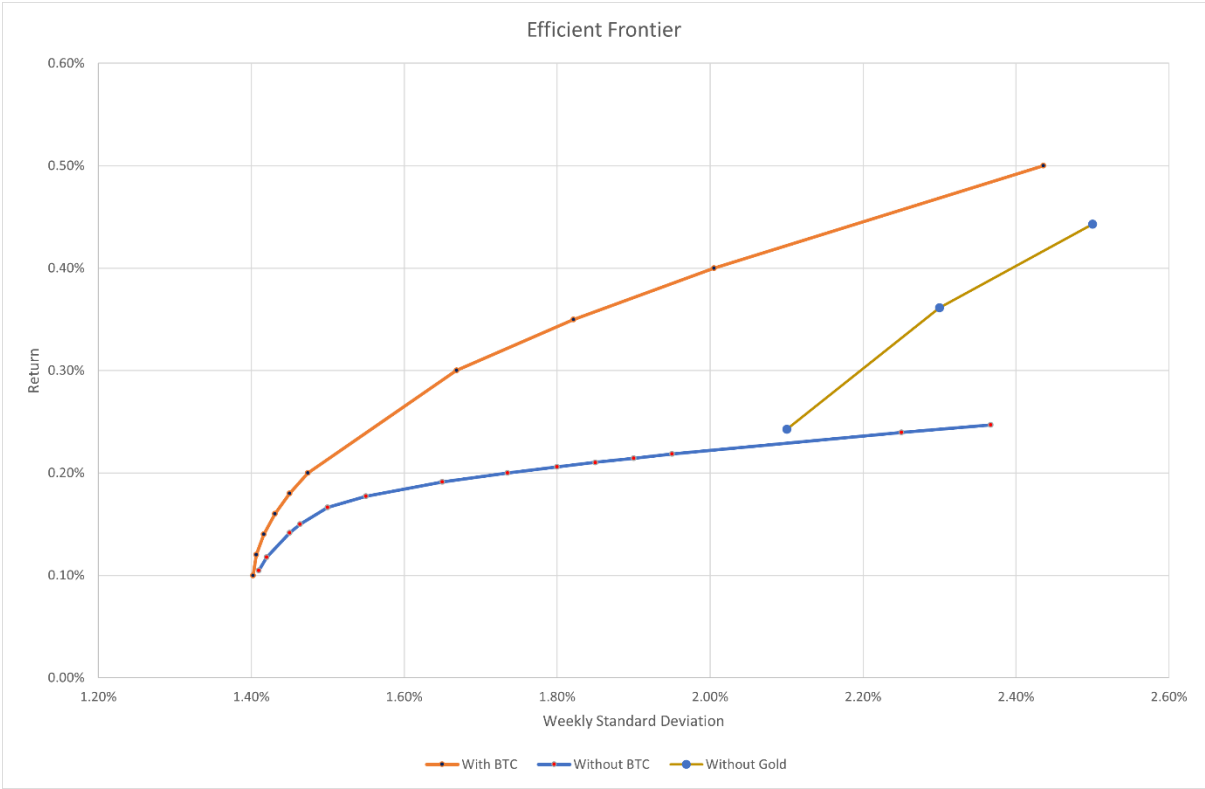


Figure 4-10: Efficient frontier for the portfolio with both BTC and Gold (Orange), without BTC (Blue), and without Gold (Yellow).

### 4.3.3 The changing weights of BTC, Gold, and SP500 for the Efficient Frontier

The more return the investor wants in the portfolio, the more bitcoin and SP500 are weighted. We see that this affects the amount of gold invested in the portfolio. Gold starts with a constant allocation of 50% between the returns of 0.10% and 0.20%. It is interesting how much is allocated in gold. It may be explained by the return offered of 0.098% but a standard deviation of almost as low as 2% on average over the time series.

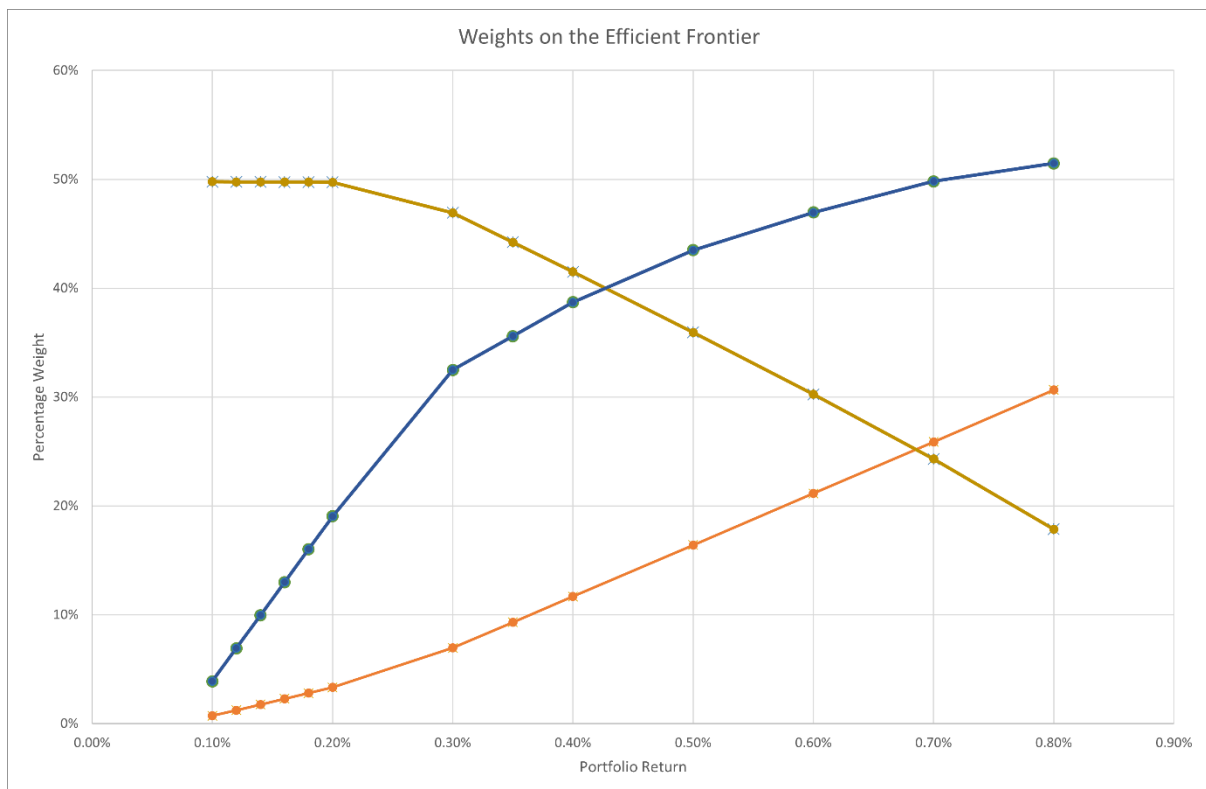


Figure 4-11: The percentage weight Gold (Yellow), BTC (orange), and SP500 (blue) with the portfolio return.

## 5 Conclusion

In the results of this paper, we see that adding bitcoin to the portfolio can result in a higher return for the same levels of standard deviation. This was as expected, seeing the huge increase in bitcoin asset prices over the last eight years. However, one could argue that this price development will persist in the future, with the fixed supply and future demand for the cryptocurrency. On the other hand, there are many fundamental or qualitative factors contributing to bitcoin's price in the future. If regulators decide to ban the currencies or implement further fees on bitcoin, that might negatively impact the asset's price.

An interesting finding in this paper is gold's contribution to reducing risk in the efficient frontier. Comparing the efficient frontier with and without gold showed the effect of adding gold to the portfolio. This could be due to the low correlation of gold to the market, also in the market downturns. The correlation between gold and SP500 was negative in the two groups with the most negative returns for SP500. However, for bitcoin, there was a positive correlation with SP500 in these two groups. Consistent with other literature, we demonstrated some of the safe-haven qualities of gold with its negative correlation to the market when there were substantial losses in SP500.

It is important to emphasize that bitcoin would perhaps not be included as an asset in this paper if it was not for its extreme price development in the last years. This hindsight could have contributed to including the cryptocurrency and, therefore, affecting this paper. In addition, there are many assumptions in Markowitz's portfolio theory. One of them is assuming that one can use statistical analysis of historical data to guide investments in the future.

Future papers could look at how an investor could utilize other cryptocurrencies to construct an optimal portfolio. In addition, one could split the period between 2013 and 2021 up into four different periods and making an efficient frontier for each period. In this way, one could see if the analysis from 2013-2015 would propose a strategy that could be useful from 2015 to 2017. Other work could also focus on incorporating bonds into the portfolio. In this paper, gold prices were used, but one could also have included prices of other metals.

## 6 References

- Baur, D. G. og McDermott, T. K. (2010) Is gold a safe haven? International evidence, *Journal of Banking & Finance*, 34(8), s. 1886-1898.  
<https://doi.org/https://doi.org/10.1016/j.jbankfin.2009.12.008>
- Böhme, R. *et al.* (2015) Bitcoin: Economics, Technology, and Governance, *Journal of economic Perspectives*, 29(2), s. 213-238. <https://doi.org/10.1257/jep.29.2.213>
- Chuen, D. L. K., Guo, L. og Wang, Y. (2017) Cryptocurrency: A new investment opportunity?, *The Journal of Alternative Investments*, 20(3), s. 16-40.
- Corti, C. W. og Holliday, R. J. (2004) Commercial aspects of gold applications: from materials science to chemical science, *Gold Bulletin*, 37(1), s. 20-26.
- Dempster, N. og Artigas, J. C. (2010) Gold: Inflation hedge and long-term strategic asset, *The Journal of Wealth Management*, 13(2), s. 69-75.
- Ghosh, D. *et al.* (2004) GOLD AS AN INFLATION HEDGE?, *Studies in Economics and Finance*, 22(1), s. 1-25. <https://doi.org/10.1108/eb043380>
- Klabbers, S. (2017) Bitcoin as an investment asset: The added value of bitcoin in a global market portfolio, i.
- Liu, Y. og Tsyvinski, A. (2018) *Risks and returns of cryptocurrency*. (0898-2937): National Bureau of Economic Research.
- Markowitz, H. (1952) Portfolio Selection, *The Journal of Finance*, 7(1), s. 77-91.  
<https://doi.org/10.2307/2975974>
- Mattke, J., Maier, C. og Reis, L. Is Cryptocurrency Money?, i, 2020. ACM.
- Mukhopadhyay, U. *et al.* (2016) A brief survey of Cryptocurrency systems, i *2016 14th Annual Conference on Privacy, Security and Trust (PST)*, 12-14 Dec. 2016. s. 745-752.
- Onnela, J. P. *et al.* (2003) Dynamics of market correlations: Taxonomy and portfolio analysis, *Physical Review E*, 68(5), s. 056110. <https://doi.org/10.1103/PhysRevE.68.056110>
- Reddy, E. og Minnaar, A. (2018) CRYPTOCURRENCY: A TOOL AND TARGET FOR CYBERCRIME.

*Figure 6-1: The purchasing power of one US dollar from 1800 to 2021*

<https://www.statista.com/statistics/1032048/value-us-dollar-since-1640>

*Figure 6-2: Systematic Risk and Non-Systematic Risk on the number of securities in the portfolio* <https://appliedfinancejulianshovlin.wordpress.com/2011/12/12/how-can-we-minimize-systematic-risk/>

*Figure 6-3: The efficient frontier with the Tangency portfolio*

[https://en.wikipedia.org/wiki/Efficient\\_frontier#/media/File:Markowitz\\_frontier.jpg](https://en.wikipedia.org/wiki/Efficient_frontier#/media/File:Markowitz_frontier.jpg)

*Figure 6-4: Portfolio risk with different correlations.*

[https://www.researchgate.net/figure/Risk-and-return-of-a-two-assets-portfolio-as-function-of-the-correlation-r-12\\_fig3\\_229005093](https://www.researchgate.net/figure/Risk-and-return-of-a-two-assets-portfolio-as-function-of-the-correlation-r-12_fig3_229005093)

