Master's thesis

NTNU
Norwegian University of Science and Technology
Faculty of Information Technology and Electrical Engineering

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Inductor application for teaching purposes

Master's thesis in Energy and Environmental Engineering
Supervisor: Robert Nilssen
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***Abstract*—To enable students to learn about inductors through a trial and error method, a simplified user interface for COMSOL Multiphysics has been made called Inductor Analysis Application (IAA). Without deep knowledge about finite elements method simulation tools, the student can simulate inductors and make changes to them, and see how different parameters affects the behavior of the inductor.**

In this thesis, the IAA results have been compared with paper calculations and laboratory testing. IAA provides accurate inductance calculations and can give students good insight to how voltage, resistance and losses changes when different input parameters are used. All the values from IAA are not exact and further work is needed to correct the loss and resistance calculations. For the resistance, this includes adjusting the calculations in the 2D model to capture the shape of the coil. To correct the losses, Steinmetz coefficients for the inductor must be found.

***Index Terms*—Inductors, COMSOL Multiphysics, teaching, losses, inductance, resistance, voltage and current response.**

I. INTRODUCTION

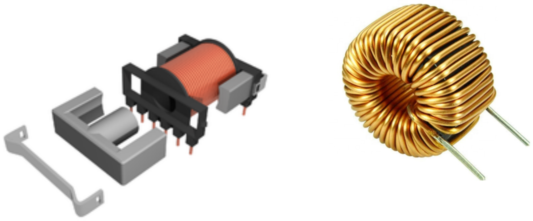
At the university, students learn about inductors and other electrical components. They are taught about the inductor area of use and how to analyse them. Some have tested them in laboratory work, but the setups are often prepared in advance and fixed to one setup. To get an alternative to traditional learning, learning by being able to play around in a laboratory is desired. Here the students could change number of turns, air gap length, wire diameter etc. to see how this affects the inductor parameters. Since it is time and money demanding to let the students build several different inductors, and in addition difficult to make changes to existing inductors, there is need for a virtual laboratory. A COMSOL application model called Inductor Analysis Application (IAA) is therefore made to be used by students and for teaching purposes. This thesis concentrates on IAA and verification of the results it gives compared to paper calculations and laboratory test inductors.

II. THEORY

Inductors are passive linear circuit elements which are widely used for low pass filtering or for impedance matching of capacitive loads. They are used in circuits with frequency ranging from a few Hertz to many megahertz.

A. Inductor types

There are many types of inductors varying from air core inductors, laminated core inductors, toroid inductors and more. For use with power electronics, iron core inductors are typically chosen due to their compactness. Two standard designs are E-core inductors and toroid inductors.



(a) (b)

Fig. 1. Examples of (a) E-core and (b) Toroid inductors

Both types can provide a large variety of impedances. By grinding down one leg of an E-core or inserting a spacer between two E-cores, an air gap can be introduced to change the permeability and avoid saturation in the core. The permeability of the toroid may also be controlled through using different materials in the core. This however, cannot be changed after production.

There are multiple choices of materials in E-cores. Two main ones are iron lamination and ferrite. With ferrite as the core, there are lower copper losses compared with a core having an air gap and lower core loss compared to laminated iron core [1]. Ferrites have lower eddy currents, which means that they can be used with higher frequencies. The two main disadvantages with ferrites is that they typically have lower flux density inductors with iron laminations, and that they are more expensive.

The coil in an inductor is typically a round wire conductor wrapped around the core. To reduce the losses in the coil, Litz wire or foil coils may be used. See Section II-C for details of coil losses.

B. Inductance

The inductance, *L*, of a coil gives the relationship between the current in the coil, *i*, and the flux, φ, in the core of the inductance.

$$L = \frac{N\phi}{i} \tag{1}$$

As can be seen from (1), the flux for a given current is also dependent on the number of turns, *N*, in the coil of the inductor. Though the inductance, *L*, gives the relationship between the current and flux, it is independent of the input current and is calculated:

$$L = \frac{N^2}{\mathscr{R}} \tag{2}$$

$\mathscr{R}$ is the reluctance in the magnetic circuit and is dependent on the permeability, μ, and the length, *l*, and cross-section area, *A*. For any given section of a magnetic circuit, *x*, the reluctance can be calculated using this formula:

$$\mathscr{R}_x = \frac{l_x}{\mu_x A_x} \tag{3}$$

A magnetic circuit can be drawn in the same manner as an electric circuit, where the reluctance can be split into multiple reluctances to represent the different parts of the magnetic circuit. A simplified magnetic circuit model can be represented with two reluctances, one for the reluctance in the magnetic core, $\mathscr{R}_m$ and one for the reluctance in the air gap, $\mathscr{R}_a$ as in Figure 2. As the air gap reluctance is many decades higher than the reluctance for magnetic core materials, the magnetic core reluctance is often neglected.

From (2), it can be seen that the inductance can be changed either by changing the number of turns in the coil, or by changing the reluctance of the magnetic circuit. The number of turns are often limited by the available space for the coil and the current density that is acceptable in the conductor. The reluctance in the core, $\mathscr{R}_m$, can be changed by changing the material (using a material having a different permeability) or the dimensions of the core, while the air gap reluctance, $\mathscr{R}_a$, is changed by the air gap length.

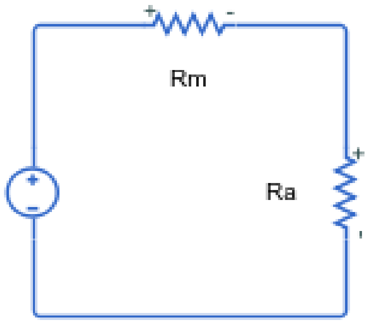


Fig. 2. Equivalent magnetic circuit

Another way of evaluating an inductor is to calculate from an energy perspective. Using (4) the relationship between inductance *L*, coil current *i*, magnetic flux density *B* and volume *V*, the energy *W*, can be found. For an inductor having a core with an air gap, the volume of the air gap is used, as the permeability of air, μ_0, makes the energy in the magnetic core negligible.

$$W = \frac{1}{2}\frac{B^2 V}{\mu} = \frac{1}{2}Li^2 \tag{4}$$

The magnetic flux density in the air gap can be expressed through *N* number of turns and length of air gap, *l*:

$$B = \mu_0 \frac{NI}{l} \tag{5}$$

C. Losses

In an inductor there are mainly two types of losses, copper (or coil) losses and iron (or core) losses. These two types of losses are due to multiple physical phenomena.

Copper losses consist of both current dependent and frequency dependent losses. Copper losses are calculated based on the current, *I*, in the copper and the electrical resistance, *R*, in the copper and can be calculated by the following equation:

$$P_{cu} = I^2 R \tag{6}$$

This equation only takes ohmic losses into account and assumes a evenly distributed current within the conductor. These losses are purely dependent on the amount of current flowing in the copper wire and the resistance in the wire. By reducing the current density by for instance increasing the copper cross section area, the ohmic losses are reduced.

The frequency dependent losses in the copper are created due to the skin effect in the copper wire. This is an effect of having an alternating current in the copper wire which creates eddy currents in the copper. The eddy currents can be explained through Faraday's law, where the induced voltages cause swirls of currents to flow within the conductor [2]. These current flows in the opposite direction of the main current in the center of the wire, and in the same direction along the outer edges of the wire. The result is a higher current density along the surface of the wire, also called skin effect, and causes higher copper losses than if the current density was evenly distributed over the cross section of the wire [3].

To reduce the skin effect, the cross-section area of each conductor can be reduced. To do this without increasing the current density and thereby get increased ohmic losses, each wire can consist of multiple strands. This type of multi-strand conductor is called Litz wire.

Another way to reduce the losses is by using conductor cross sections with large surface area. Using copper foil instead of wire is one solution that can be found. Foil coils consist of thin sheets of conductive material and similarly reduces the losses due to very thin cross sections. The main advantages with foil coil is better fill factor, impact on losses and fault responses [4]. Foil coil makes the inductor lighter, which can be important in some applications. The thermal, mechanical and electrical properties are much better than for a round wire inductor [5]. Foil coils are generally more expensive and requires higher precision during manufacturing. For this reason, round wire is commonly used for most inductors.

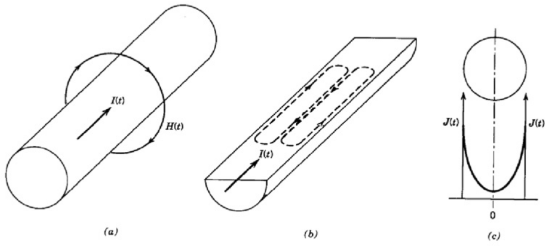


Fig. 3. Isolated copper conductor carrying (a) a current I(t), (b) eddy currents generated by the resulting magnetic field, and (c) the consequences of the skin effect on the current distribution. [3]

Iron losses consist of hysteresis losses and eddy current losses. Hysteresis losses in an iron core is the energy required to accomplish the reorientation of domains during each cycle of the alternating current applied to the core. [2] The hysteresis losses can be difficult to calculate and at present COMSOL does not provide a good solution for this. In many cases, hysteresis losses are therefore neglected.

Eddy currents in the inductor core is induced by the alternating current in the coil, and are explained through Faraday´s law. [2] In the same manner as in the copper wire, currents are created to oppose the changing magnetic field. These currents create losses as in (6).

The use of ferrite cores minimizes eddy currents and are therefore a good choice of magnetic material.

Calculations of iron losses can be complicated and there are multiple ways of doing this. One way is to use Steinmetz equation:

$$P_v = kf^{\alpha}\hat{B}^{\beta} \tag{7}$$

The expression is the product of the coefficient, *k*, (Steinmetz hysteresis coefficient), the remagnetization frequency, *f*, powered with a numerical coefficient, α, and the peak flux density, $\hat{B}$, powered by a numerical coefficient, β. Both the exponents are non-integer numbers, where $1<\alpha<3$ and $2<\beta<3$. [6] The result, P_v, from (7) gives the iron loss density in the core in W/m^3. The equation is only valid for sinusoidal waveforms [7], which is a limitation in power electronics, but it can still be a relevant way of evaluating the efficiency of an inductor.

D. Voltage

When applying a current to an inductor coil, a voltage is induced at the inductor terminals. Depending on the inductor, the voltage may have a large or small real and imaginary part. Each of these parts can be calculated using the resistance and the inductance of the coil with (8). The RI-part gives the real part of the voltage and $2\pi fLI$ gives the imaginary part.

$$V = RI + j2\pi fLI \tag{8}$$

The above formula assumes a real current, I. If the inserted current is complex, this needs to be taken into account by distinguishing between the real and imaginary part of the current applied.

III. METHODOLOGY

COMSOL Multiphysics (called COMSOL in this article) is a finite element method (FEM) tool that enables simulations combining physics within many different disciplines. COMSOL has a functionality called application builder which allows the creator to make applications from COMSOL model so that the model can be utilized by others. The user of the application will only have access to given inputs and outputs defined by the creator.

For the virtual laboratory, the IAA was made using application builder on a COMSOL model of an inductor. The IAA creates a user interface, making it easier to use and present results. With IAA, even users with limited understanding or experience with COMSOL can simulate the model and get useful results. IAA limits the access the user has to COMSOL, reducing the chance of making mistakes. The user only provides input parameters, and can not change the model in COMSOL.

IAA is build to analyse inductors. The analysis is done on a modelled E-core inductor where the geometrical dimensions and input variables are parameterized. All inputs and results in IAA comes from a COMSOL model which is run in the background.

A. *COMSOL Model*

The model behind IAA, includes both a 2D and a 3D model of an inductor. The goal is for students to see how the inductor values changes when the input parameters are changed. The inductor modeled is an E-core type inductor, using two E-cores and one coil. Inductors can also be made by using toroid cores. As IAA is made for teaching purposes, an E-core was chosen to enable variation of the air gap.

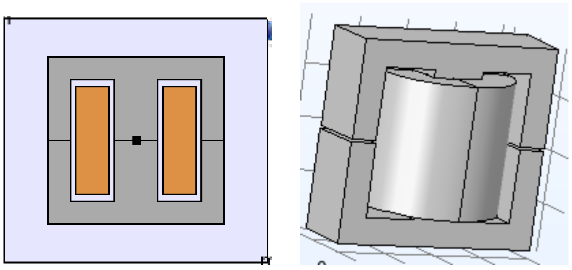
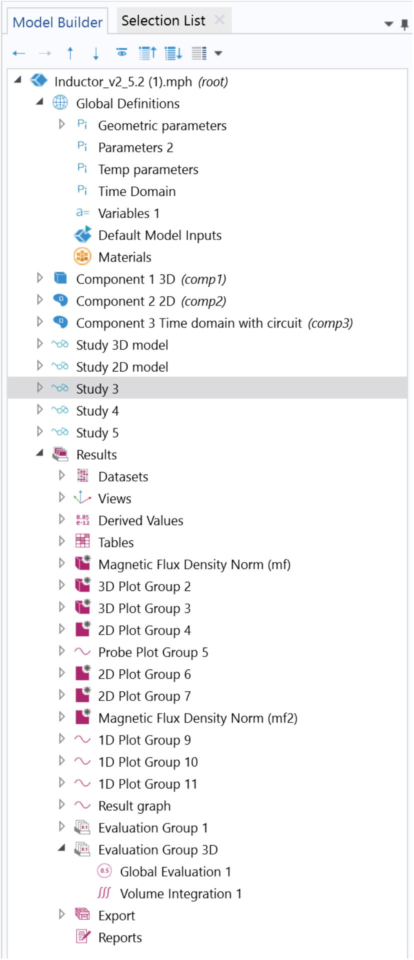

(a) (b)

Fig. 4. Examples of (a) 2D model (b) 3D model

In COMSOL, all the models are based on the same parameter list for dimensions and inputs. The 2D model is a cross section of the inductor through the center. The 3D model is an extruded version of the 2D model where the core is extruded linearly while the coil is rotated around the center. The center leg of the 3D model core is shaved to have a circular form fitting inside the coil. There is also a 2D model which includes an electrical circuit to run time domain simulations. This will be described later. All the models are within the same file and are accessible through the IAA window. Any changes made by the user will therefore affect all the models. Figure 5 shows all the items in the COMSOL model.


Model Builder | Selection List

- Inductor_v2_5.2 (1).mph *(root)*
  - Global Definitions
    - Geometric parameters
    - Parameters 2
    - Temp parameters
    - Time Domain
    - Variables 1
    - Default Model Inputs
    - Materials
  - Component 1 3D *(comp1)*
  - Component 2 2D *(comp2)*
  - Component 3 Time domain with circuit *(comp3)*
  - Study 3D model
  - Study 2D model
  - Study 3
  - Study 4
  - Study 5
  - Results
    - Datasets
    - Views
    - Derived Values
    - Tables
    - Magnetic Flux Density Norm (mf)
    - 3D Plot Group 2
    - 3D Plot Group 3
    - 2D Plot Group 4
    - Probe Plot Group 5
    - 2D Plot Group 6
    - 2D Plot Group 7
    - Magnetic Flux Density Norm (mf2)
    - 1D Plot Group 9
    - 1D Plot Group 10
    - 1D Plot Group 11
    - Result graph
    - Evaluation Group 1
    - Evaluation Group 3D
      - Global Evaluation 1
      - Volume Integration 1
    - Export
    - Reports

Fig. 5. COMSOL model menu including all components, studies, result tables and plots.

The coil in the model is a standard coil based on a round copper wire. A design option for inductors is to use foil coil instead. A test model in COMSOL with a foil coil was created, but not implemented into the final version of IAA.

In COMSOL it is possible to take advantage of symmetry and only model half or a section of the geometry. This can improve simulation time and model size. As IAA is intended for teaching purposes, it was decided that the whole inductor was modelled. It is easier to get a deeper understanding by presenting a complete model and not only a slice of it.

For the meshing of the models a finer mesh was used for the 2D model; with more accurate and finer mesh in the air gap (see Figure 6). In the 3D model a normal mesh was used and free tetrahedral was chosen.

The meshing could always be finer and more accurate, but this gives longer computation time. In this case where IAA is meant for teaching, it is important that the computation time is not too long to avoid loosing the student interest and patience. The meshing only needs to be fine enough to get fairly accurate results.

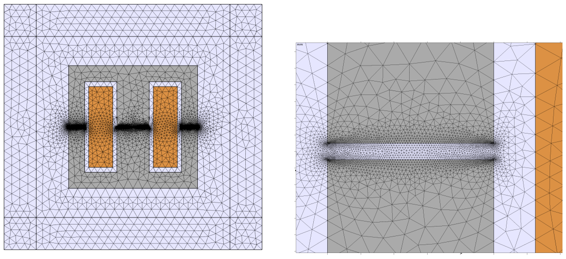

(a) (b)

Fig. 6. (a) Mesh of 2D model. (b) Details of mesh in air gap domain

1) Inductor core: E-cores come in standard sizes and the model is built using parameter cases where one can easily switch between the cases. Each case includes the geometrical parameters describing the E-cores. For the COMSOL model and IAA, three commercially available E-core sizes were chosen to be available for the user. The sizes are E65, E80, E110 ([8], [9], [10]). E80 is the most commonly used and therefore set as default in the model.

The most common inductor cores materials are soft iron laminations or ferrite. Both materials were tried in the model. The magnetic properties of the inductor core are inserted into COMSOL through a B-H relationship. A B-H curve is plotted based on the values that can be found in material datasheets. COMSOL has build in functionalities to convert B-H curves into effective B-H curves, which is needed to emulate the behaviour of the core material. For the inductor model with soft iron, the default parameters were used. For ferrite core, the B-H relationship was taken from datasheet [11].

The air gap in E-core type inductor cores can either be between the two E-core sections in all three legs, or only in the center leg. Since the center leg has twice the cross section area as the other legs, the effect of an air gap in all three legs is doubled. In practice the E-core either has a non-magnetic space between the E-core sections or has grinded the center leg down to create an air gap.

2) Loss calculation: The core losses and coil losses are calculated separately in the model. Coil losses are made available in COMSOL through *Coil Power*, which is based on the multiplication of current and voltage in the coil.

There are multiple ways of calculating the core losses in the inductor, as described in Section II-C Losses. COMSOL provides calculations using resistive heating, Steinmetz method and Bertotti method. Both Steinmetz and Bertotti method require input of coefficients. The availability of reliable coefficients is limited and they should be verified through laboratory testing. For the inductor model, both resistive heating and Steinmetz method were tested and compared with real life results from test coil (see Section V-D Core losses). As the scope of this thesis did not include finding the Steinmetz coefficients, two different sets of coefficients were used. One was from a COMSOL paper by Havez [12], and the other used the COMSOL default values.

In the COMSOL paper by Havez, units were missing and k coefficient had a very low value compared to other examples found. It is assumed that Havez calculated losses in μW. The k was multiplied by a factor of 10^6 to get losses in Watts.

TABLE I
COEFFICIENTS USED FOR STEINMETZ CALCULATIONS

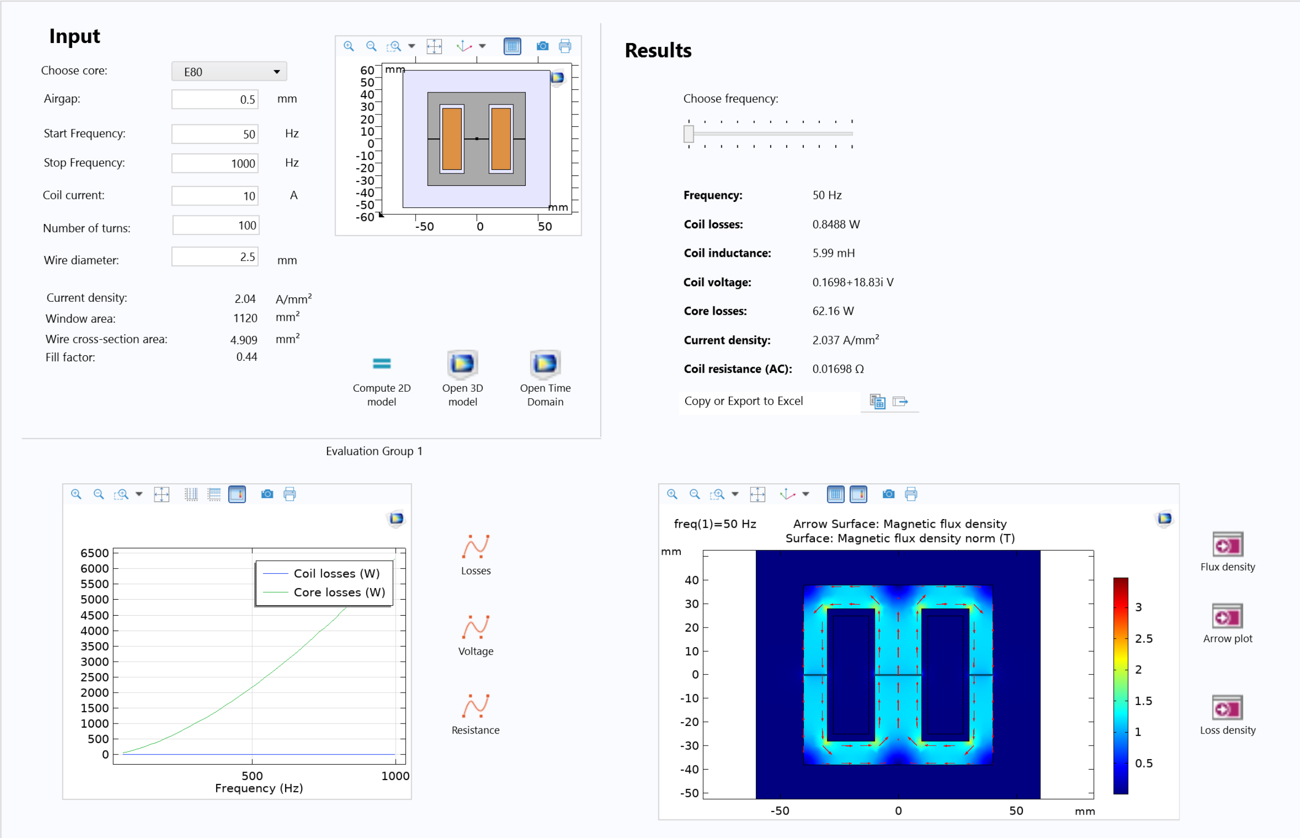
Coefficient	Havez	COMSOL default
k	1055	1000
α	1.544	1
β	2.704	1.5

B. Inductor Analysis Application

The main window of IAA consist of the 2D simulation of the inductor. It gives the user the option to choose from different inductor sizes and specify other inputs. Through running IAA, the user gets different results presented in the same window. For more detailed analysis, the 3D simulation can be run. The advantages with 3D is that the simulation is more accurate since it is not only a cross section. The results are also presented in 3D, this gives the user the opportunity to see the effect of a squared core and circular coil.

For better understanding of how the inductance affects the current and voltage response, a time domain simulation is also available.

1) Inputs: The COMSOL model requires geometrical dimension as well as coil parameters as inputs. Available geometrical dimensions are stored in the model, where the user must choose one from these and is not able to specify the dimension freely. The only geometrical parameter the user can change is the air gap length which is a separate input field. Number of turns and wire diameter of the inductor coil are inserted by the user through the IAA window. For the 2D simulation,



Input

Choose core: E80

Airgap: 0.5 mm

Start Frequency: 50 Hz

Stop Frequency: 1000 Hz

Coil current: 10 A

Number of turns: 100

Wire diameter: 2.5 mm

Current density: 2.04 A/mm²

Window area: 1120 mm²

Wire cross-section area: 4.909 mm²

Fill factor: 0.44

Results

Choose frequency:

Frequency: 50 Hz

Coil losses: 0.8488 W

Coil inductance: 5.99 mH

Coil voltage: 0.1698+18.83i V

Core losses: 62.16 W

Current density: 2.037 A/mm²

Coil resistance (AC): 0.01698 Ω

freq(1)=50 Hz

Arrow Surface: Magnetic flux density

Surface: Magnetic flux density norm (T)

Fig. 7. Inductor Analysis Application window

a sinusoidal current input is used where the user only specifies current amplitude and a frequency range to study.

Changes to the geometry are updated simultaneously and a cross section figure of the E-core is displayed where changes to either E-core size or air gap length can be seen as the user changes the inputs.

The inputs provided for the 2D simulation are also used for the 3D simulation. The user does not need to provide any more inputs to run the 3D version of the model, except specifying at which input frequency the 3D simulation should run at.

For the time domain model, in addition to choosing E-core size, air gap length and coil parameters, the user must also specify the voltage waveform. The required inputs for the voltage are listed in Table II.

2) IAA Results: For the 2D and 3D simulations, which are done in frequency domain, the coil inductance and losses in the copper coil and E-core are calculated. Copper losses, core losses and inductance are found in COMSOL as coil power, heat in the core and coil inductance. These results are displayed as outputs in IAA.

TABLE II
LIST OF INPUTS REQUIRED TO DEFINE THE VOLTAGE WAVEFORM FOR TIME DOMAIN SIMULATION.

Name	Tag	Unit
Voltage amplitude	V_{src}	V
Voltage offset	V_{off}	V
Time delay	t_d	ms
Rise time	t_r	ms
Fall time	t_f	ms
Pulse width	p_w	ms
Period	T_{per}	ms

For the 2D simulation, a graph plotting the induced voltage, losses and resistance is shown. This gives the student the insight of the frequency dependency for these parameters.

There is also a graphical plot giving the user the opportunity to analyse and understand both how the flux flows through the inductor and where losses are generated. The user can choose between surface plot showing flux density, arrow plot displaying the flux and surface plot giving loss density (see examples in Figure

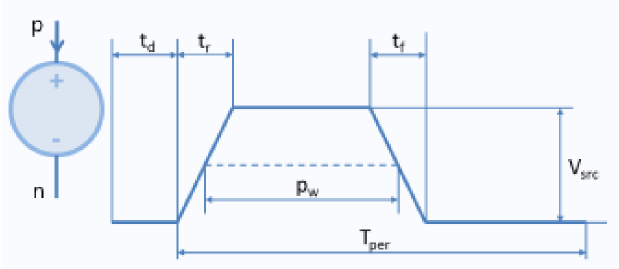


Fig. 8. Voltage source waveform

9). The student is free to zoom in and out, and pan the plot to focus on details if needed.

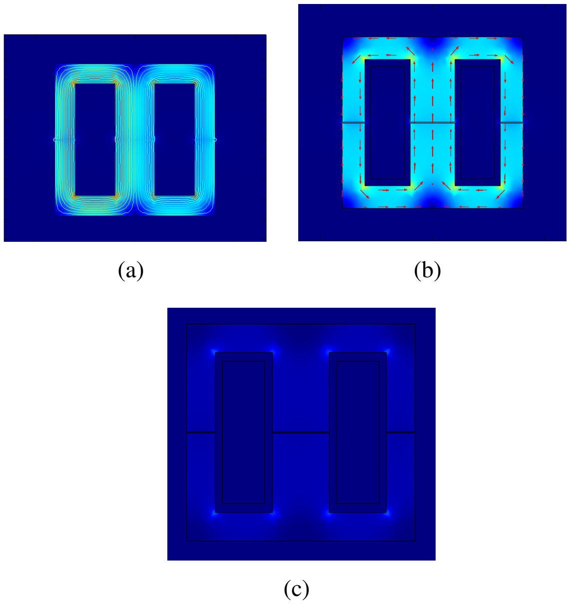
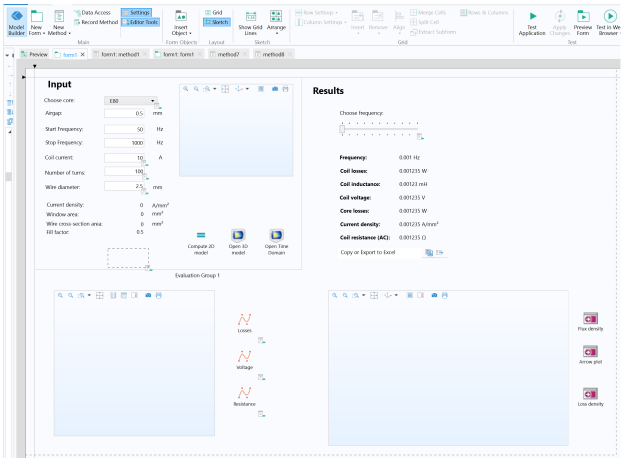

(a)
(b)
(c)

Fig. 9. (a) Surface plot showing flux density. (b) Arrow plot of flux. (c) Surface plot of loss density

All relevant inputs and results are also displayed in a table and the user has the option to export all data for the entire frequency range to Excel. This enables the user to post-process the results and compare them with each other.

3) Build up of IAA: The main window has three different buttons in the input section. The first one is the compute button, which runs the 2D simulation of the inductor over the defined frequency range that is provided by the user. The frequency range is always dived in 51 steps. After the simulation is complete, IAA updates the result tables in the COMSOL model and refreshes the 2D plot. The second and third button each opens a new window, 3D simulation and time domain.

The input section (as seen in Figure 7) contains a drop down list and several text input fields. In the drop down list the user can choose from three different E-cores. Once one is chosen, the parameters belonging to that specific core is inserted into the model and the cross section picture is updated. If additional cores is wanted, one must insert the dimensional data into the model, not in the application, and make that option available in the drop down list by using the application builder.



Input

Choose core: E80
Airgap: 0.5 mm
Start Frequency: 50 Hz
Stop Frequency: 1000 Hz
Coil current: 10 A
Number of turns: 100
Wire diameter: 2.5 mm
Current density: 0 A/mm²
Window area: 0 mm²
Wire cross-section area: 0 mm²
Fill factor: 0.5

Results

Choose frequency:

Frequency: 0.001 Hz
Coil losses: 0.001235 W
Coil inductance: 0.00123 mH
Coil voltage: 0.001235 V
Core losses: 0.001235 W
Current density: 0.001235 A/mm²
Coil resistance (AC): 0.001235 Ω
Copy or Export to Excel

Fig. 10. COMSOL application build window where the application is built.

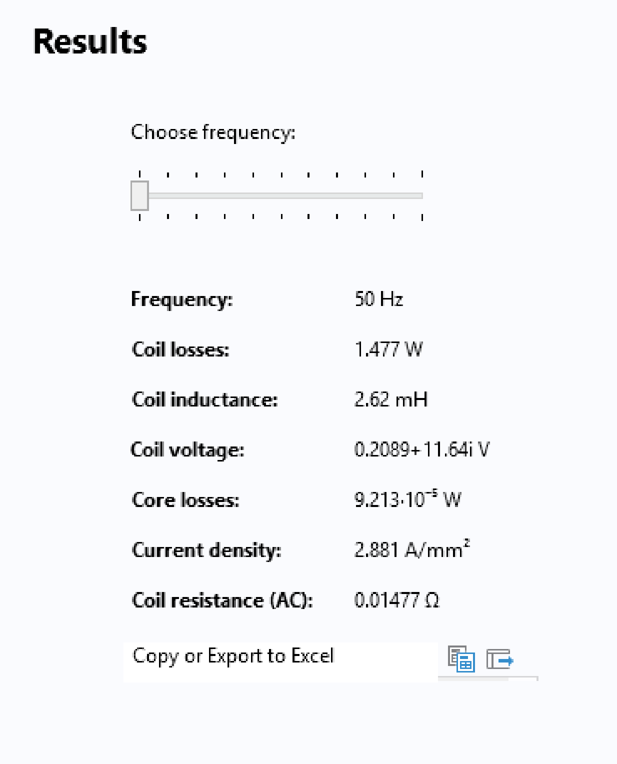
Among the input fields, the user must specify the coil current, number of turns and the wire diameter. All these values affects the current density and fill factor. For the user to see these values, they are calculated directly on data change.

The outputs shown to the user are spilt into to sections. One section gives information on how the parameters are changed over the frequency range. With three buttons the user can choose between losses, voltage and resistance. The second section provides detailed outputs for specific frequencies, see Figure 11. The frequency is chosen by a slider button which updates all the displayed values when moved. For the specified frequency, a 2D plot of the inductor is displayed. By clicking on different buttons the user can choose between flux density, arrow plot and loss density. All these plots are separate 2D plots in the COMSOL model and the buttons only specifies which of them should be displayed in the application window.

In the COMSOL model a result table is made that includes all relevant inputs and outputs for the entire frequency range. In IAA, there are buttons to either copy the entire table or save the table into an Excel sheet.

C. Test inductors

To verify the COMSOL model, physical test coils were produced and tested in a laboratory. All the test



Results

Choose frequency:

Frequency: 50 Hz

Coil losses: 1.477 W

Coil inductance: 2.62 mH

Coil voltage: 0.2089+11.64i V

Core losses: $9.213 \cdot 10^{-5}$ W

Current density: 2.881 A/mm^2

Coil resistance (AC): 0.01477 Ω

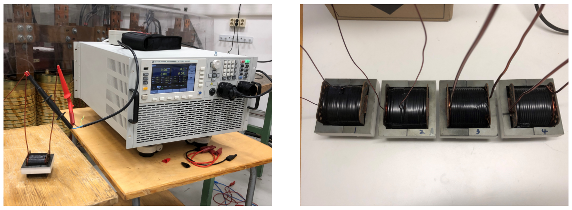
Fig. 11. Output section in IAA

inductors used a E80 core. The coil parameters chosen varied conductor thickness, number of turns and air gap length.

TABLE III
COIL SAMPLES FROM TESTING IN LABORATORY.

Sample number	1	2	3	4
Conductor diameter [mm]	1.0	1.0	2.5	2.5
Number of turns	650	650	90	87
Air gap length [mm]	0.5	1.0	0.5	1.0

Each coil sample was tested with a range of frequencies and currents. The same testes were performed in IAA so that comparison of the results could be made.



(a) (b)

Fig. 12. (a) Test setup. (b) Coil samples

IV. TEACHING PURPOSE

IAA is, as mentioned, designed for teaching purposes, where the student easily can change dimensions and parameters without knowledge of COMSOL. Through calculation of inductors IAA can be used as a virtual lab where the student in an entertaining way. By changing input parameters, the number of turns, the size of the inductor core or the air gap can be altered without needing to modify physical inductors as one would have to do in a laboratory. The student can try and fail without the danger of ruining a laboratory setup and is therefore more free to explore how changes in parameters affects different inductor values. The time needed to change the inductor is close to zero compared to how a change in a physical laboratory would be.

The intended usage is that the student performs paper calculations in advance, and use IAA to verify the calculations. The paper calculation or pre-calculations would build on the knowledge taught in class, using known equations to find inductor losses and inductance. In many cases, the student will also need to evaluate why there are differences between the calculated and the simulated results. This gives the student better understanding of the actual physical phenomenons and how equations may try to explain the world in a simplified manner.

Through simulation of many different inductors, the student can get an increased understanding of the dependencies and trends, and this can be evaluated up against the equations for inductance and losses.

A. *Pre-calculation*

A student assignment may ask the student to create a magnetic equivalent circuit for an inductor. The student may be free in deciding how much the circuit is simplified, for instance if magnetic reluctance should be included or not.

Combining (2) and (3) from Section II-B, (9) can be found; which shows how the inductance of a coil can be altered by changing the number of turns, N, the cross-section area of the flux path in the air gap, A, and the air gap length, l.

$$L = \mu_0 \frac{N^2 * A}{l} \tag{9}$$

Depending on the amount of simplification, the equation may include multiple parts to represent the different reluctances. (9) is reduced to its simplest form where the magnetic core reluctance is omitted.

From (9) it is important for the student to note that the inductance is not dependent on the current. This is true as long as one avoids saturation in the core. When inductors have high flux density, saturation in parts of the inductor core will lead to higher leakage flux, which will give a different inductance.

The student should also use the equations for energy (4) and (5) to see the relationship between inductance, number of turns, cross section area and length of the air gap.

Furthermore, the student should calculate in advance, the expected coil losses in the inductor. This can be done through (6) which gives the DC losses in the copper. The coil resistance *R* is found by (10) where ρ is the electrical resistivity of copper, l is the length of the wire and A the conductor cross section area.

$$R = \frac{\rho * l}{A} \tag{10}$$

Core losses are more difficult to calculate and the student is not expected to calculate these.

B. Verification through IAA

By choosing the correct core dimension and inserting chosen input parameters, the student can verify the pre-calculations through the use of IAA. It performs a 2D simulation of the inductor and provides losses, inductance, coil resistance and more as outputs. There is also a graphical plot of the inductor cross section where magnetic flux density, loss density and flux directions are shown and can be studied.

C. Dependencies

As IAA can simulate different inductors quickly, and the effort to change input parameters is low, IAA is ideal as a playing ground for student to test how different parameters affect the inductor performance. The idea is to get the student interested in playing and understanding dependencies. A typical assignment could be to ask the student to double the inductance. Another could be to ask how one could accommodate for higher currents without the current density increasing in the copper wire.

With the quick computation time, it is possible to optimize the inductor design by altering the input parameters. IAA is not an optimization tool yet, but can be extended as part of further development.

D. Time domain

IAA includes the opportunity to simulate the inductor with a custom voltage input and see how the current respond for the inductor is. The simulation is done in time domain and uses the same model as was used to find the inductance and losses when inserting a sinusoidal current in the coil. With this the student can directly see the effect of different inductances based on different inductor designs.

V. RESULTS

To verify the outputs from IAA, testing in a laboratory and pre-calculations by hand were performed. Each of the outputs parameters were compared, looking at the pre-calculations, 2D results from IAA, and laboratory test results.

A. Inductance

Using (9), the inductance was found through the pre-calculations. Together with measured data from the test inductor and IAA, the results are compared in Figure 13.

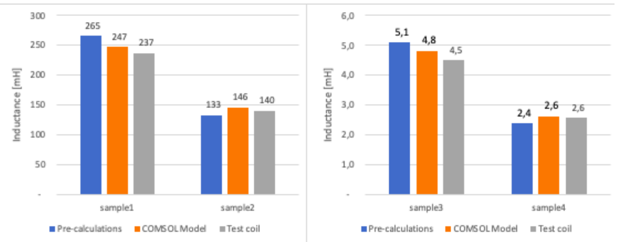


Fig. 13. Comparison between calculated and COMSOL inductance for each of the inductor samples.

All the three calculations methods give similar results. The pre-calculated inductance for *Sample 1* and *Sample 3* are higher than both the IAA results and the measured values of the laboratory test inductor. For *Sample 2* and *Sample 4* the values were lower. Both *Sample 1* and *Sample 3* have 0.5 mm air gap length, while *Sample 2* and *Sample 4* have 1 mm. The difference between calculated and the other results can be explained by the pre-calculations being a simplification, which does not include neither leakage inductance nor the inductance of the inductor core.

TABLE IV
INDUCTANCE RESULTS AND COMPARISON.

	Sample 1	Sample 2	Sample 3	Sample 4
Pre-calculations	265	133	5.1	2.4
IAA	247	146	4.8	2.6
Lab Test inductor	237	140	4.5	2.6
Calc vs. IAA	7%	-9%	6%	-9%
Calc vs. Lab	12%	-5%	13%	-7%
IAA vs. Lab	4%	4%	7%	2%

It can also be noted that the measured values of the test inductor all are a bit lower than the values from

IAA. This may be due to inaccuracies of the air gap. In addition, the permeability of the spacer in the air gap may be different from air which is assumed in IAA.

For teaching purposes, the results provided by IAA will be satisfactory. An essential part of the student´s learning, is to become able to explain the differences between calculated, measured and modelled values.

B. *Coil resistance*

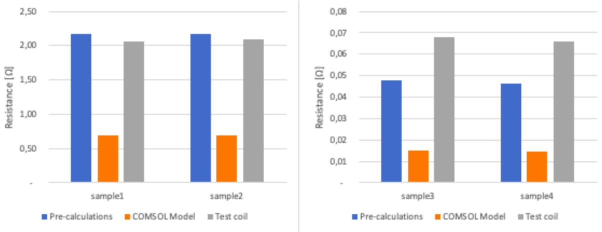


Fig. 14. Resistance

The coil resistance was calculated using (10), with an assumption of average coil radius of 25 mm. From the measurements of the laboratory testing it can be seen that this is relatively accurate for *Sample 1* and *Sample 2*. There is a larger deviation in results for *Sample 3* and *Sample 4*. Since the values are so low, it is assumed that the deviation is due to measurements inaccuracies.

The resistance calculated by IAA are much lower than both the pre-calculated and measured results. Since the values from IAA are calculated using the the 2D cross section model, a hypothesis that IAA only calculates a linearly extruded copper bar was made. To verify the hypothesis, an extra calculation was done using the same formulas for resistance as done in the pre-calculations, but using two linear conductor segments as coil instead of the doughnut shape. The conductor segments were assumed to be 20 mm depth (which is the depth of the inductor core) and gave a resistance of 0.552 Ω for *Sample 1* and *Sample 2*, 0.012 Ω for *Sample 3* and 0.018 Ω for *Sample 4*. This is close to the IAA calculated values and it is therefore assumed that IAA calculates the coil losses by extruding the copper linearly instead of using a coil with doughnut shaped.

Based on the results shown in Table V, a different calculation method should be implemented in IAA. The 3D model made for this thesis was meant to calculate the resistance more accurately, but by now, the model is not working accordingly. This can be done in future studies.

TABLE V
RESISTANCE RESULTS WITH COMPARISON. LINEAR CALCULATIONS TO CHECK HOW IAA HAS CALCULATED RESISTANCE

	Sample 1	Sample 2	Sample 3	Sample 4
Pre-calculation	2.17	2.17	0.05	0.05
IAA	0.69	0.69	0.02	0.01
Lab test inductor	2.06	2.09	0.07	0.07
Calc vs lab	5%	4%	-29%	-30%
Linear calc.	0.55	0.55	0.01	0.01
Lin calc vs IAA	-20%	-20%	-20%	-20%

C. *Coil losses*

The losses in the coil were calculated using (6) and the resistances from Table V. The laboratory test inductor only measured total loss in the inductor. By using (6) and the measured current and resistance in the laboratory inductor, the coil losses were calculated. As can be seen in Figure 15, there are large differences between the coil losses calculated in IAA and the other coil losses. The relative deviations are similar to what was found for the coil resistance shown in Figure 14. With the improved calculation method of the resistance, as suggested in Section V-B, it is expected that the IAA coil losses will be closer to the results from the pre-calculations and laboratory testing.

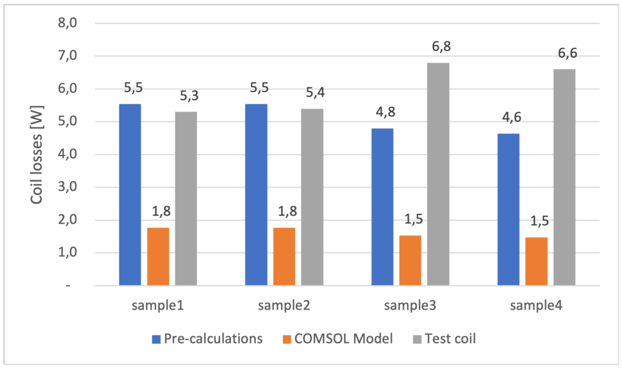


Fig. 15. Comparison of coil losses calculated through pre-calcualtion and Inductor Analysis Application.

Based on the results for coil losses, it is suggested that a different calculation method for coil losses in the 2D model is introduced in future versions of IAA. This has not been implemented in the current version.

In the laboratory testing, the losses are not divided into coil and core losses, only total losses were measured. The formula used in the pre-calculations, (6) does not take frequency dependencies into account. It is only in IAA we can see this dependency. Even though the coil losses in IAA gives wrong results, they were analysed and are shown in Figure 16.

The coil loss for each sample is relatively constant, with a slight increase for higher frequencies. It is assumed that the dependency of frequency is due to eddy currents, see Section II-C for more details.

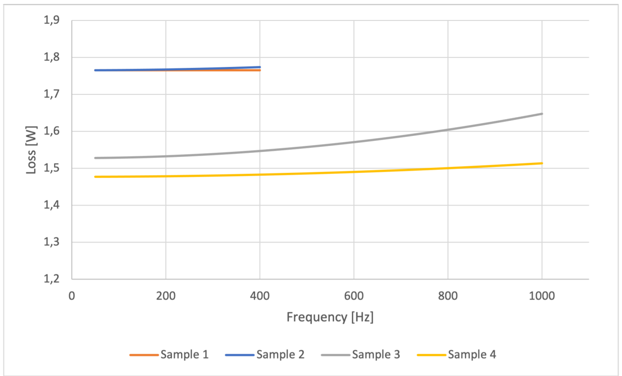


Fig. 16. Coil losses for each sample

D. *Core losses*

There is no easy way of calculating core losses, so only the results from IAA and laboratory testing are compared. The results from the laboratory testing only provide total losses, not coil and core losses separated. Most of the losses are from the core, so it is still interesting to compare the total losses with the simulated core losses.

From Figure 17 and Figure 18 it is clear that non of the two different sets, see Table I, with Steinmetz coefficients fits with the measured values.

The measured values from the laboratory testing were in the 100s of Watts for *Sample 1* and *Sample 2*. Using Steinmetz method of calculation with Havez coefficients, the core losses were estimated to be in the 1000s. Using the COMSOL standard coefficients IAA gave losses in the 10s. This shows that the coefficients are not correct, but it is possible to get some more accurate values in laboratory testing. The last method tested, resistive heating, gave losses in the milliwatt range. This is not very realistic and the results are therefore not presented here.

The results from the resistive heating calculations shows that there is more work to be done to get more correct results. In the COMSOL model a soft iron is used as material. Different ferrite cores were tested in IAA. BH-curves and effective BH-curves were implemented based on datasheets [11], but the model was not able to complete the calculations and soft iron was used instead. It was not expected that soft iron and ferrite have such different values of losses. Further improvements of the modelling in COMSOL is needed.

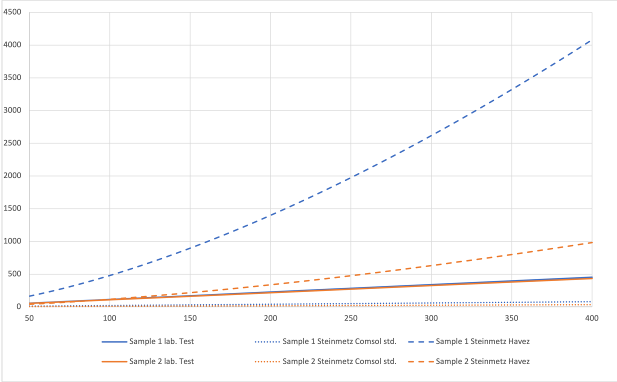


Fig. 17. Core losses with different frequencies, Sample 1 and 2

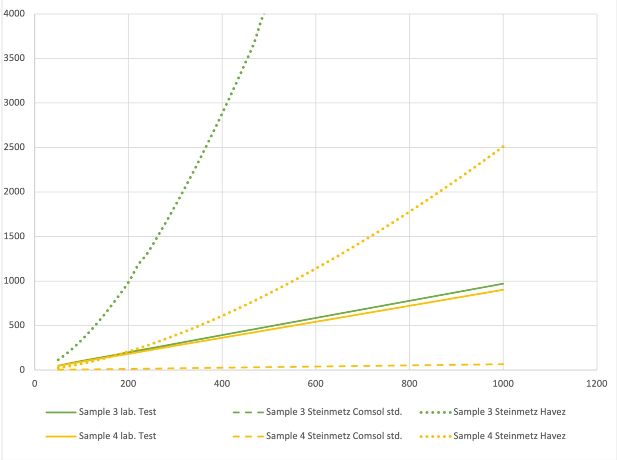


Fig. 18. Core losses with different frequencies, Sample 3 and 4

It was observed that during testing in both the laboratory and IAA, the inductor core went into saturation. This can be observed in Figure 19. It seemed difficult for COMSOL to calculate accurate values during saturation. Further testing with lower current should be preformed.

E. *Induced voltage*

The results for voltages from pre-calculations, IAA and the laboratory tests were compared. In the pre-calculations formula (8) was used to find the voltage, using the calculated inductance and resistance as described in Section V-A and V-B. Since the resistance is wrong in IAA, the voltage will also be wrong, but nevertheless, one may see trends.

The real value of the voltage is constant in pre-calculation and COMSOL simulation, as the resistance and current does not change with frequency. As seen

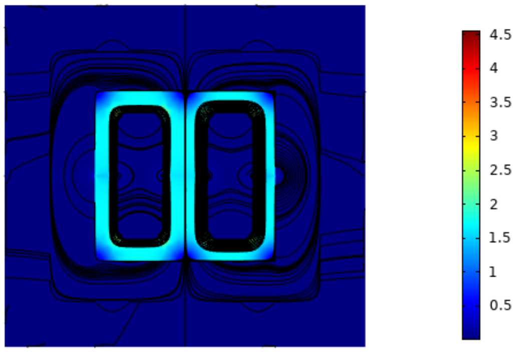


Fig. 19. Flux density plot of sample 1, 120 Hz

in the laboratory results, Figure 20 and 21, the values fluctuated a bit. This is assumed to be measurement inaccuracies. It should also be noted that the results for *Sample 1* and *Sample 2*, both from pre-calculations and IAA, are identical, as the there is no difference between the coil in the two designs, only the air gap length of the core. Similarly *Sample 3* and *4* are close to identical as the coils are almost identical, except number of turns, which is 90 for *Sample 3* and 87 for *Sample 4*.

As expected after the evaluation of the results on coil resistance from IAA, the voltage level is also wrong as the coil is not simulated as a doughnut shape. In Figure 21 it is registered that the voltage difference between pre-calculation and laboratory test is significant. It is assumed that the difference comes from measurement inaccuracies as the value are low. New and more accurate measurements should be done to verify this assumption.

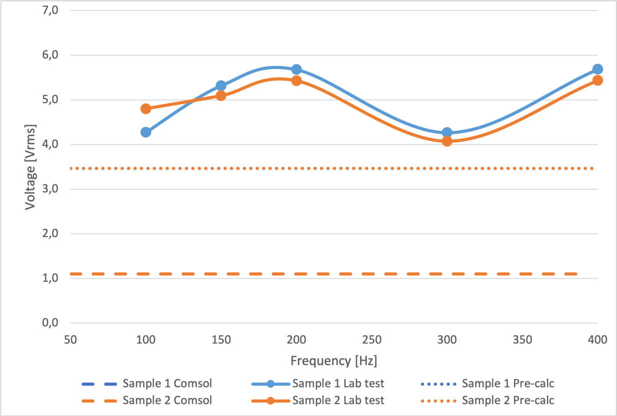


Fig. 20. Induced real voltage of Sample 1 and Sample 2

In Figure 22 and 23 the imaginary part of the voltage is shown. It can be seen that the voltage is increased with increased frequency, as expected. There are differences between pre-calculations, IAA and the laboratory testing. Pre-calculations are simplified and assumes ideal conditions without eddy currents and saturation. Both in the laboratory testing and with COMSOL modelling the inductor was saturated. During saturation the inductance will change with frequency and it is unsure if COMSOL is able to calculate this correctly.

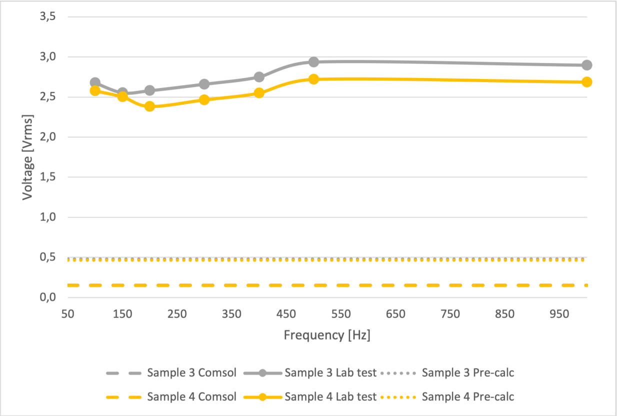


Fig. 21. Induced real voltage of Sample 3 and Sample 4

It is noted that the laboratory test results does not have large differences between *Sample 1* and *Sample 2*, and between *Sample 3* and *Sample 4*. As the test results from the laboratory coil only registered real and imaginary power, calculating a power factor ($\cos \phi$), and measuring the total voltage, there possible errors in the measurement and in the splitting the voltage into real and imaginary parts.

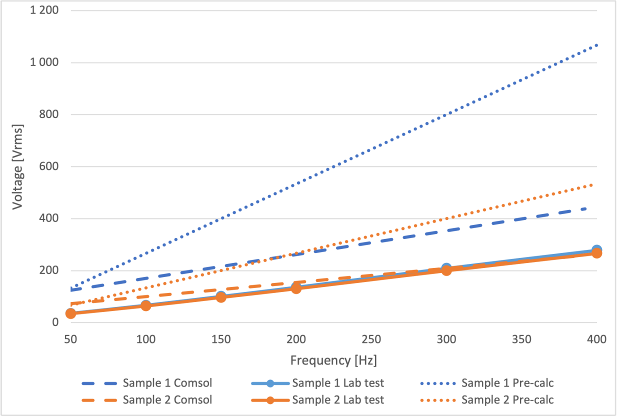


Fig. 22. Induced imaginary voltage of Sample 1 and Sample 2

F. Time domain

Using the time domain feature in IAA the voltage and current waveforms are found. This feature allows the

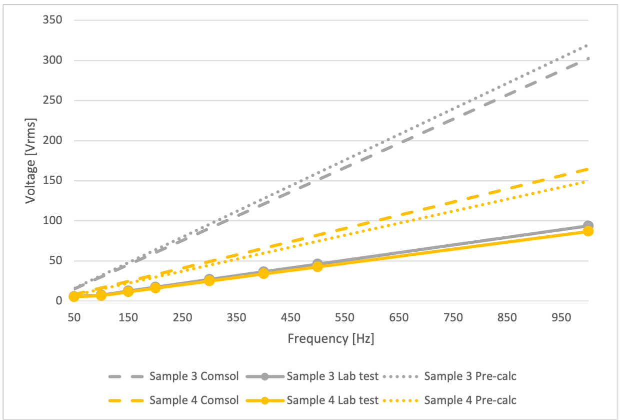


Fig. 23. Induced imaginary voltage of Sample 3 and Sample 4

student to see how the current response is when different voltage waveforms are applied. An example is shown in Figure 24 using inductor *Sample 4*.

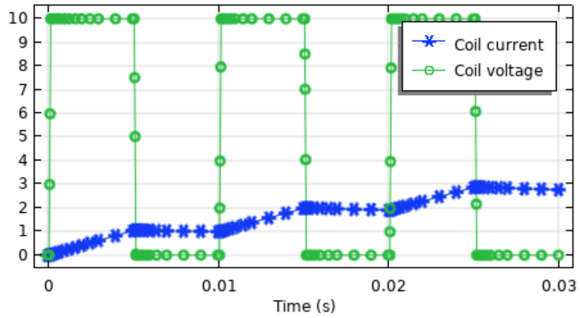


Fig. 24. Voltage and current waveform for Sample 4

The parameters used in this example are shown in Table VI. These values can be changed by the student to alter the voltage input waveform.

The student can also learn how the inductance affects the current output when changing the inductor design.

TABLE VI
INPUT PARAMETERS FOR TIME DOMAIN

Parameter	Value	Units
Voltage	10	V
Offset voltage	0	V
Delay	0.1	ms
Rise time	0.1	ms
Fall time	0.1	ms
Pulse width	5	ms
Period	10	ms

VI. CONCLUSION

The Inductor Analysis Applcication (IAA), works as intended regarding calculating inductance as a function of number of turns and air gap length. The results from IAA for the inductance are close to both the pre-calculations and the laboratory test results.

IAA does not provide the same values for resistance, voltage or losses as the pre-calcualtions and the laboratory tests did. Different reasons have been identified and suggestions for further work is given that could correct this. The IAA results does give similar output trends as the other calculation methods. When altering the frequency, air gap length or number of turns, the outputed values from IAA changes in the same manner as the results from the pre-calculations and laboratory testing.

To correct the outputed values from IAA, further testing in the laboratory is suggested to be able to insert correct Steinmetz coefficients. Improved comparisons of test results is also expected if saturation of the inductor is avoided. The coil resistance and voltage is not calculated correctly by COMSOL for the 2D model, and further work should include improving this to take into account that the coil is circular.

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