https://doi.org/10.2352/issn.2169-2629.2021.29.282

Optimising a Euclidean Colour Space Transform for Colour Order and Perceptual Uniformity

Luvin Munish Ragoo and Ivar Farup; Norwegian University of Science and Technology; Gjøvik, Norway.

Abstract

In this paper, we attempt to optimise a colour space transform for colour order and perceptual uniformity to verify if a trade-off could be achieved between the two. The IPT colour space is used as basis for the optimisation. An optimisation model consisting of a modified XYZ-to-LMS matrix, a non-linearity factor, and two geometric transformation matrices is proposed. Two objective functions are constructed based on the optimisation model, where one would improve perceptual uniformity primarily and the other would improve colour order instead. Finally, the two objective functions are combined, in an attempt to optimise both simultaneously and see if a trade-off between the seemingly incompatible features can be achieved. The performance of the optimised IPT transform is then compared to the original IPT transform, in terms of relative improvements in perceptual uniformity and colour order. Finally, the results show that there is indeed an inverse relationship between the two objectives. However, by adjusting the bias of the optimisation, a balance could be achieved between the two, where both colour order and perceptual uniformity was improved with respect to the original IPT transform.

Introduction

Numerous colour transforms have been proposed for either perceptual uniformity or colour order, but not both [1]. In fact, attempts at creating a perceptually uniform space, often results in spaces that are not hue linear. While other spaces, such as the IPT colour space, are designed to be hue linear, they suffer from lack of perceptual uniformity [2]. There are many indicators that a perceptually uniform Euclidean colour space does not exist [3, 4].

In this paper, we propose an optimisation pipeline for improving both perceptual uniformity and colour order, for a Euclidean colour space. The goal is to test if a balance can be achieved between these two desirable yet seemingly incompatible features. The IPT colour space is used as the baseline for optimisation. While there are better perceptually uniform colour spaces that have been proposed, such as CAM16 [5], we have chosen to use the IPT colour space due to its inherent simplicity.

First, a brief background to the IPT colour space and the datasets used in the optimisation is given. Then, the optimisation model describing the main modifications to the original IPT transform is introduced. Next, the steps involved in combining the two optimisation objectives into a single optimisation cost function are listed. Finally, the performance of optimised IPT transform is compared to the original, with respect to the two optimisation objectives.

Background

IPT Colour Space

Ebner and Fairchild derived a simple uniform colour space that aimed to accurately model constant perceived hues [6]. The IPT colour space is named such that its coordinates have some degree of relationship to the meaning of the dimensions. The lightness dimension is denoted as I, which can be loosely related to the word intensity. The P vector represents the red-green dimension, which is also the dimension lost by protanopes. Similarly, the T vector represents the yellow-blue dimension or the dimension lost by tritanopes. The model consists of a 3×3 matrix, followed by a non-linearity adjustment and followed by another 3×3 matrix. The steps for converting from XYZ to IPT are listed below.

1. Calculate LMS

$$\begin{bmatrix} L \\ M \\ S \end{bmatrix} = \begin{bmatrix} 0.4002 & 0.7075 & -0.0807 \\ -0.2280 & 1.1500 & 0.0612 \\ 0.0 & 0.0 & 0.9184 \end{bmatrix} \begin{bmatrix} X_{D65} \\ Y_{D65} \\ Z_{D65} \end{bmatrix} \quad (1)$$

2. Adjust gamma for each of the L, M, S components

$$X' = \text{sgn}(X) \cdot |X|^{0.43} \quad (2)$$

where $X = L, M$ or S and $\text{sgn}(X)$ is the signum function of X.

3. Gamma-adjusted LMS to IPT

$$\begin{bmatrix} I \\ P \\ T \end{bmatrix} = \begin{bmatrix} 0.4000 & 0.4000 & 0.2000 \\ 4.4550 & -4.8510 & 0.3960 \\ 0.8056 & 0.3572 & -1.1628 \end{bmatrix} \begin{bmatrix} L' \\ M' \\ S' \end{bmatrix} \quad (3)$$

Datasets

Perceptual uniformity is based on how well geometric differences in a colour space relates to perceived colour differences, while colour order specifies perceptual attributes of colours [7]. Given how the two optimisation objectives are fundamentally different, finding a single dataset that includes testable aspects for both objectives is difficult. Thus, two datasets were used instead, namely the Munsell dataset for colour order and the RIT-Dupont dataset for perceptual uniformity.

1. **Munsell Dataset**
The Munsell colour system is arranged as three dimensional solid with three dimensions of Value, Chroma and Hue. The hue circle is arranged so that there is an equal perceptual distance between each major hue category [8]. The main hue categories were red, yellow, green, blue and purple. These are further divided into 40 total hue categories. The Value axis goes from 0-black to 10-white. The Chroma scale goes from 0-neutral to an open ended high chroma number. The Munsell notation for a particular colour is "Hue" "Value"/"Chroma". For example, a green colour may have the Munsell notation of 7.5G 5/8.
Along with the Hue,Value,Chroma notation, each patch in the dataset is also defined in xyY coordinates, making transformation to IPT space relatively easy. In this project, we use the Munsell dataset mainly for colour order optimisation.

2. **RIT-DuPont Dataset**
In the 1980s, a large-scale colour experiment was performed to determine CIELAB's lack of effectiveness in predicting supra-threshold visual colour tolerances. DuPont prepared over 1000 samples that were designed to sample CIELAB in specific univariate directions [9]. These sample pairs were compared to a single, near-neutral anchor pair with a constant colour difference of 1.02 ΔE^*_{ab} and judged by two sets of 50 observers. The observers' task was to judge whether a sample pair's total colour different ΔE^*_{ab} was perceived as "greater than" or "less than" the anchor pair's total colour difference. These judgements of the 50 observers were used to calculate a frequency of rejection for each colour-difference pair. The frequency data were later transformed to equal-interval visual data, ΔV, through a psychometric function. The RIT-DuPont experiment was designed so that probit analysis could be used to transform the frequency data to the T_{50} for each direction [10].
Using the RIT-DuPont dataset, the performance of a colour metric in estimating perceptual colour differences can be quantified. In this project, we use the dataset to optimise for perceptual uniformity.

Methods

In this section, the optimisation model and the rationale behind it are described. The steps involved in the design of the cost functions for the two different optimisation objectives are also explained.

Optimisation Model

The original IPT transform as listed in Eq. (1), (2) and (3) is modified as follows to generate the optimised IPT transform which we denote as, $\overline{IPT}$:

$$\begin{bmatrix} L \\ M \\ S \end{bmatrix} = M_1 \begin{bmatrix} X_{D65} \\ Y_{D65} \\ Z_{D65} \end{bmatrix} \tag{4}$$

$$X' = \mathrm{sgn}(X) \cdot |X|^{\gamma} \; ; \;\; \text{where } X \rightarrow L, M \text{ and } S \tag{5}$$

$$\begin{bmatrix} \bar{I} \\ \bar{P} \\ \bar{T} \end{bmatrix} = M_2 M_3 \begin{bmatrix} 0.4000 & 0.4000 & 0.2000 \\ 4.4550 & -4.8510 & 0.3960 \\ 0.8056 & 0.3572 & -1.1628 \end{bmatrix} \begin{bmatrix} L' \\ M' \\ S' \end{bmatrix} \tag{6}$$

M_2 and M_3 in Eq. (6) are defined as follows :

$$M_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \alpha \\ 0 & 0 & 1 \end{bmatrix} \; ; M_3 = \begin{bmatrix} 1 & \varepsilon_1 & \varepsilon_2 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{bmatrix} \tag{7}$$

The parameter α in M_2 act as a skewing factor in the chromatic plane. In M_3, there are three parameters to optimise, namely ε_1 and ε_2 which would apply a tilt in the *I-T* or *I-P* plane, and θ, which is the angle of a rotation transformation in the *P-T* plane. The γ is also optimised to best suit the optimisation objectives.

The M_1 matrix from Eq. (4) is generated in the following steps :

1. **Optimising primaries**
The original transformation matrix from XYZ to LMS can be optimised by adjusting the primaries. We start by first inverting the original XYZ to LMS matrix in Eq. (1), to obtain the LMS to XYZ matrix in Eq. (8).

$$\begin{bmatrix} 1.8502 & -1.1383 & 0.2384 \\ 0.3668 & 0.6439 & -0.0107 \\ 0 & 0 & 1.0889 \end{bmatrix} \tag{8}$$

The original LMS primaries in the original IPT transform are defined as LMS_{p1}, LMS_{p2} and LMS_{p3} as shown in Eq. (9).

$$LMS_{p1} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \; ; LMS_{p2} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \; ; LMS_{p3} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{9}$$

Using the LMS to XYZ transform in Eq. (8), we can obtain the XYZ tri-stimulus values of the primaries. The *xy*-chromaticities of the primaries are then derived using Eq. (10) , which can then be modified as shown in Eq. (11).

$$xy_i = \begin{bmatrix} \frac{X_i}{X_i+Y_i+Z_i} & \frac{Y_i}{X_i+Y_i+Z_i} \end{bmatrix} \tag{10}$$

where $i \rightarrow p1, p2$ or $p3$

$$\begin{bmatrix} \bar{x}_{p1} & \bar{y}_{p1} \\ \bar{x}_{p2} & \bar{y}_{p2} \\ \bar{x}_{p3} & \bar{y}_{p3} \end{bmatrix} = \begin{bmatrix} x_{p1}+\alpha_1 & y_{p1}+\alpha_2 \\ x_{p2}+\alpha_3 & y_{p2}+\alpha_4 \\ x_{p3}+\alpha_5 & y_{p3}+\alpha_6 \end{bmatrix} \tag{11}$$

The parameters α_1 to α_6, are additive factors that can be optimised to slightly adjust the chromaticities of the primaries towards the relevant optimisation objective.

2. **Reconstructing the optimised transformation matrix**
The optimised *xy*-chromaticities of the primaries, $\bar{xy}_{p1}$, $\bar{xy}_{p2}$ and $\bar{xy}_{p3}$ are converted back to their XYZ tri-stimulus values using Eq. (12).

$$\begin{bmatrix} \bar{X}_i \\ \bar{Y}_i \\ \bar{Z}_i \end{bmatrix} = \begin{bmatrix} \bar{x}_i \times (X_i+Y_i+Z_i) \\ \bar{y}_i \times (X_i+Y_i+Z_i) \\ \bar{z}_i \times (X_i+Y_i+Z_i) \end{bmatrix} \tag{12}$$

where $i \rightarrow p1, p2$ or $p3$; and $\bar{z}_i = 1 - \bar{x}_i - \bar{y}_i$.

$$\begin{bmatrix} S_{P1} \\ S_{P2} \\ S_{P3} \end{bmatrix} = \begin{bmatrix} \bar{X}_{p1} & \bar{X}_{p2} & \bar{X}_{p3} \\ \bar{Y}_{p1} & \bar{Y}_{p2} & \bar{Y}_{p3} \\ \bar{Z}_{p1} & \bar{Z}_{p2} & \bar{Z}_{p3} \end{bmatrix}^{-1} \begin{bmatrix} X_{D65} \\ Y_{D65} \\ Z_{D65} \end{bmatrix} \tag{13}$$

Given that $p1$,$p2$ and $p3$ are LMS primaries, M_1 from Eq. (14) will be the optimised transformation matrix from XYZ to LMS, replacing the original XYZ to LMS matrix in Eq. (1).

$$M_1 = \begin{bmatrix} S_{p1}\bar{X}_{p1} & S_{p2}\bar{X}_{p2} & S_{p3}\bar{X}_{p3} \\ S_{p1}\bar{Y}_{p1} & S_{p2}\bar{Y}_{p2} & S_{p3}\bar{Y}_{p3} \\ S_{p1}\bar{Z}_{p1} & S_{p2}\bar{Z}_{p2} & S_{p3}\bar{Z}_{p3} \end{bmatrix}^{-1} \tag{14}$$

Colour Order Optimisation

For this optimisation objective, the main goal is to modify the colour space transform such that colours are mapped to coordinates that make intuitive sense. As mentioned in previous sections, the Munsell colour ordering system is used as reference. The steps in colour order optimisation are as follows:

1. **Creating a reference position dataset**
Since the Munsell Colour system is defined in a cylindrical coordinate system, a translation from the latter to a similar space as IPT is needed. In this new space, which we denote as IPT^{ref}, the colour order of the Munsell patches is preserved.
The Value dimension in the Munsell system can readily be mapped to the I^{ref} dimension using Eq. (15), where s is a scaling factor. The P^{ref} and T^{ref} coordinates can be obtained using Eq. 16, where h is the hue angle. Each hue category is assigned a hue angle from 0 to 351, starting at '5RP' and moving clockwise (refer to Figure 1), where each step corresponds to a 9° increment. This particular hue was arbitrarily chosen to have a Hue angle of 0, since a preliminary conversion of Munsell's xyY values to IPT showed that the maximum P value occurred at Hue notation of '5RP'.

$$I_i^{ref} = \frac{Value_i}{Value_{max}} \times s \tag{15}$$

$$P_i^{ref} = \frac{Chroma_i}{Chroma_{max}} \times cos(h_i) \qquad T_i^{ref} = \frac{Chroma_i}{Chroma_{max}} \times sin(h_i) \tag{16}$$

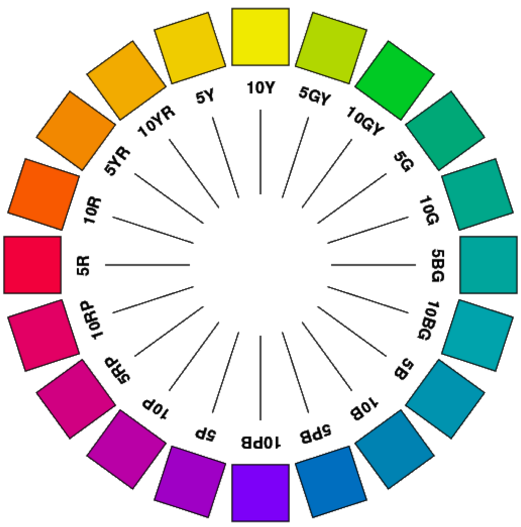

Figure 1. The Munsell system in the Hue-Chroma Plane. (Figure reproduced under the CC-BY 3.0 licence from https://commons.wikimedia.org/wiki/File:MunsellColorWheel.svg)

The results of this step is shown in Figure 2. As comparison, Figure 3 shows the position of the Munsell patches if the original IPT transform is applied on the available xyY data. Looking normal to the I and T axes in Figure 3, a tilt can be observed in the planes of constant value. Ideally, they should be parallel to the P-T plane and thus appear as horizontal lines in the I-T plane as is the case in the IPT^{ref} in Figure 2. Additionally, the planes of constant hue, which appear as radial lines in the P-T plane, are not equidistant from each other. The colour order optimisation seeks to minimise these irregularities in colour position.

2. **Designing a Colour Order cost function**
Now that the coordinates of the Munsell dataset in IPT^{ref} have been derived, they are used as ground truth for the

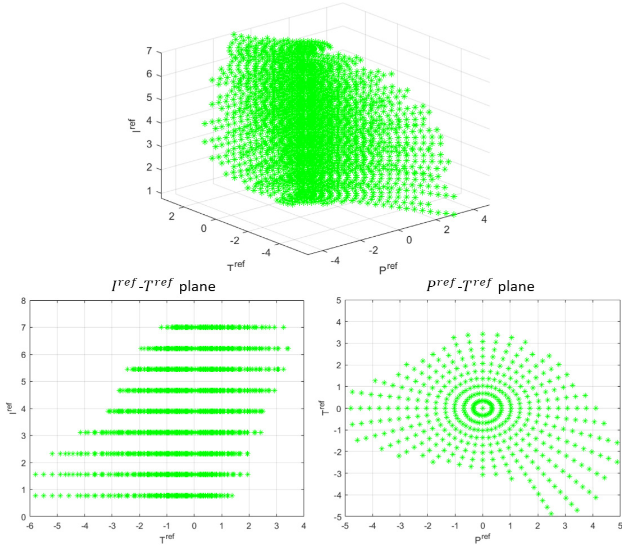

Figure 2. The Munsell patches in IPT^{ref} space with colour order preserved.

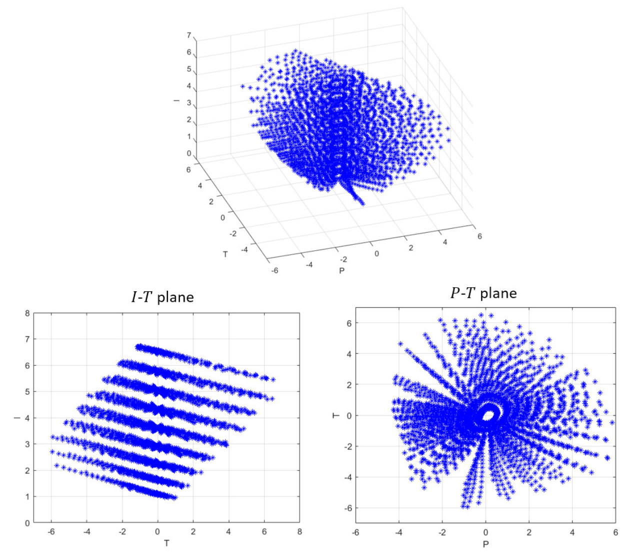

Figure 3. The Munsell patches in the original IPT space.

optimisation algorithm. Each iteration will compute a position error metric between the $\overline{IPT}$ coordinates of the Munsell dataset and their reference ones, IPT^{ref}. The sum of Euclidean distances is used as position error metric in this case and it is computed for the following two cases:

(a) The positions of Munsell Patches in the original IPT space with respect to IPT^{ref}, which we denote by E_p as shown in Eq. (17).

$$E_p = \sum_{i=1}^{N} \sqrt{(I_i^{ref} - I_i)^2 + (P_i^{ref} - P_i)^2 + (T_i^{ref} - T_i)^2} \tag{17}$$

where $i = 1 \ldots N$ and $N \rightarrow$ Number of samples in the dataset.

(b) The new positions of the Munsell Patches in the $\overline{IPT}$ which we denote by $\bar{E}_p$ in Eq. (18).

$$\bar{E}_p = \sum_{i=1}^{N} \sqrt{(I_i^{ref} - \bar{I}_i)^2 + (P_i^{ref} - \bar{P}_i)^2 + (T_i^{ref} - \bar{T}_i)^2}$$

(18)

The relative error in Colour Order can thus be calculated using Eq. (19)

$$\Gamma_p = \frac{\bar{E}_p}{E_p} \quad (19)$$

Γ_p would have a value of less than 1, if $\overline{IPT}$, generates a better colour order than the original IPT transform with respect to the reference Munsell positions in IPT^{ref}. Γ_p can thus be minimized to optimise for colour order.

Perceptual Uniformity Optimisation

For this optimisation objective, the main goal is to modify the colour space transform such that the general colour space solid is perceptually uniform. For example, if two colour patches, A and B are compared, the Euclidean distance between the two should be equal to that of another two colour patches that may be different from each other along a different vector than A and B, but otherwise have the same perceptual colour difference as A and B. As mentioned earlier, we use the RIT-DuPont dataset here. The dataset contains XYZ coordinates of 312 colour pairs which are said to have a constant perceptual colour difference, ΔV of 1.02. By computing the Euclidean distance between the two patches in a colour pair, we can obtain the *actual* colour difference, ΔE for each colour pair. The steps in perceptual uniformity optimisation are as follows :

1. **Converting to IPT Space**
   The colour coordinates of each pair is converted to the IPT and $\overline{IPT}$ spaces, using the original IPT transform and the optimised IPT transform respectively.
2. **Computing Euclidean Distances**
   The Euclidean Distance between the two colours (A and B) in each colour pair is computed using Eq. (20).

$$\Delta E = \sqrt{(I_A - I_B)^2 + (P_A - P_B)^2 + (T_A - T_B)^2} \quad (20)$$

3. **Designing a Perceptual Uniformity cost function**
   In order to create a cost function that optimises perceptual uniformity, the relationship between perceived colour differences, ΔV, and measured colour differences, ΔE, should be established. To this end, Garcia et al. showed that the performance of a colour difference metric in predicting perceptual difference can be quantified using $STRESS$ (Standardised Residual Sum Of Squares) [11].
   Given the ΔV and ΔE values for each colour pair, the $STRESS$ can be computed using Eq. (21).

$$STRESS = \left(\frac{\sum(\Delta E_i - F\Delta V_i)^2}{\sum F^2 \Delta V_i^2} \right)^{1/2} \quad (21)$$

$$\text{with } F = \frac{\sum \Delta E_i^2}{\sum \Delta E_i \Delta V_i}$$

   The lower the $STRESS$, the better the colour metric performs in estimating perceptual differences.
   $STRESS$ is computed for both cases in step (1) and denoted by E_u for the original IPT transform and $\bar{E}_u$ for the optimised IPT transform. The relative improvement in $STRESS$ can then be calculated using Eq. (22).

$$\Gamma_u = \frac{\bar{E}_u}{E_u} \quad (22)$$

   Similar to Γ_p, Γ_u would have a value of less than 1, if the optimised IPT transform results in a better perceptually uniform colour space.

Combining the two Objective Functions

In both cost functions, we try to minimise a relative term instead of absolute terms like the sum of Euclidean distances or $STRESS$. This is done so that the cost functions have the same importance when they are ultimately combined. Γ_p and Γ_u are both in the same scale where:

1. a value of 1 would mean no improvement of the objective function.
2. a value of less than 1 would mean that the optimised IPT transform performs better than the base IPT transform for a particular objective.
3. a value of more than 1 would mean that the optimised IPT transform performs worse than the base IPT transform for a particular objective.

The final objective function $F(M)$ can now be defined as shown in Eq. (23).

$$F(M) = f \times \Gamma_p(M) + (1 - f) \times \Gamma_u(M) \quad (23)$$

M is the array containing the 11 optimisation parameters introduced in the Optimisation Model, as shown in Eq. (24).

$$M = [\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \gamma, \theta, \alpha, \varepsilon_1, \varepsilon_2] \quad (24)$$

Since both optimisation objectives, Γ_p for colour order and Γ_u for perceptual uniformity, have the optimised IPT transform in their pipeline, they can both be represented as functions of M. f is a weighting factor for biasing the optimisation towards either colour order or perceptual uniformity. An f factor of 1 will result in the focus of $F(M)$ to be entirely on colour order, while an f factor of 0 will shift the focus to perceptual uniformity exclusively. The MATLAB routine *fminsearch* was used to solve this unconstrained multivariable optimisation function.

Results and Discussion

The optimisation was run for f factors ranging from 0 to 1. Figure 4 shows the results from these optimisation runs.

It can be observed that improvements in colour order(Γ_p) are quite large, with the best-case scenario having the optimised IPT transform performing more than 3 times better than the base IPT transform. The improvements in perceptual uniformity(Γ_u), on the other hand, are relatively modest. Only around 20% improvement from the base IPT transform was achieved in the best-case scenario. However, it is important to note that if optimising only for perceptual uniformity, as is the case with an f factor of 0, colour order is severely impaired. The reverse is also true, where an f factor of 1 results in the best performance for colour order, while also giving the worst performance for perceptual uniformity. The two objective functions seem diametrically opposed.
For f factors above 0.2, further improvements in Γ_p become marginal, while noticeable improvements in Γ_u, are only obtained for f factors below 0.2. An f factor of 0.1 is thus chosen as a good middle ground between the two optimisation objectives.

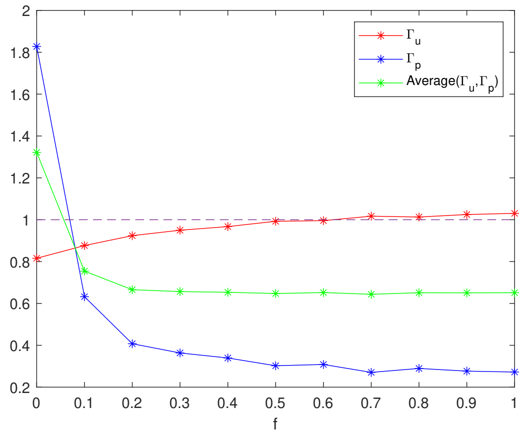

Figure 4. Relationship between f and the two optimisation objectives, Γ_p and Γ_u

It is worth noting that the same datasets are used to both train and test the model in this case. Ideally, the new optimised IPT transform should be tested on new datasets to test its robustness to new data.

Optimised IPT transform

With an f factor of 0.1, the following M_1, M_2, M_3 and γ, as introduced in the Optimisation Model, are obtained:

$$M_1 = \begin{bmatrix} 0.4321 & 0.6906 & -0.0930 \\ -0.1793 & 1.1458 & 0.0226 \\ 0.0631 & 0.1532 & 0.7226 \end{bmatrix} ;\ \gamma = 0.4071 \quad (25)$$

$$M_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -0.1964 \\ 0 & 0 & 1 \end{bmatrix} ;\ M_3 = \begin{bmatrix} 1 & -0.0456 & 0.1327 \\ 0 & 0.9837 & -0.1797 \\ 0 & 0.1797 & 0.9837 \end{bmatrix} \quad (26)$$

The optimised IPT transform, $\overline{IPT}$, can thus be summarised as follows :

1. Calculate LMS

$$\begin{bmatrix} L \\ M \\ S \end{bmatrix} = \begin{bmatrix} 0.4321 & 0.6906 & -0.0930 \\ -0.1793 & 1.1458 & 0.0226 \\ 0.0631 & 0.1532 & 0.7226 \end{bmatrix} \begin{bmatrix} X_{D65} \\ Y_{D65} \\ Z_{D65} \end{bmatrix} \quad (27)$$

2. Adjust gamma for each of the L, M, S components

$$X^{'} = \mathrm{sgn}(X) \cdot |X|^{0.4071} \ ;\ \text{where } X \rightarrow L, M \text{ and } S \quad (28)$$

3. Gamma-adjusted LMS to $\overline{IPT}$

$$\begin{bmatrix} \overline{I} \\ \overline{P} \\ \overline{T} \end{bmatrix} = \begin{bmatrix} 0.3037 & 0.6688 & 0.0276 \\ 3.9247 & -4.7339 & 0.8093 \\ 1.5932 & -0.5205 & -1.0727 \end{bmatrix} \begin{bmatrix} L^{'} \\ M^{'} \\ S^{'} \end{bmatrix} \quad (29)$$

The performance of the above optimised IPT transform, $\overline{IPT}$, compared to the original transform is highlighted in Table 1. A 14% improvement in $STRESS$ is achieved. While, the colour order metric features an improvement of 37%.

Performance of Optimised IPT w.r.t Original IPT

IPT Space	STRESS(E_u)	Sum of Euclidean distances(E_p)
Original	0.2944	2995
Optimised	0.2581	1893

Conclusion and Future Work

In this article, we described in detail the pipeline for a colour space transform optimisation, namely the IPT system. The modifications proposed includes adjusting the gamma, optimising the primaries and several geometric transformations. From the results obtained, we can conclude there is a clear trade-off between colour order and perceptual uniformity. Neither of optimisation objectives could not be optimised fully at the same time. Yet, by adjusting the weighting factor f, a balance between perceptual uniformity and colour order could be achieved, where both features are somewhat improved with respect to the original IPT transform. However, further testing with other data sets than the ones used in training, is required for more conclusive results. It is well established in the literature [12] that a colour space cannot be Euclidean if it is satisfying both perceptual uniformity and colour order. Perhaps exploring non-Euclidean spaces could effectively satisfy both of these objectives as the work done by Farup suggests [1].

Acknowledgments

This research was funded by the Research Council of Norway over the project ‘Individualised Colour Vision-based Image Optimisation’, grant number 287209.

References

[1] Farup, Ivar. "Hyperbolic geometry for colour metrics." Optics Express 22.10 (2014): 12369-12378.

[2] Lissner, Ingmar, and Philipp Urban. "How perceptually uniform can a hue linear color space be?." Color and Imaging Conference. Vol. 2010. No. 1. Society for Imaging Science and Technology, 2010.

[3] Judd, D. B. "Ideal color space: the super-importance of hue differences and its bearing on the geometry of color space." Palette 30 (1968): 21-28.

[4] Judd, D. B. "Ideal color space: ideal color space redefined." Palette 31 (1969): 23-29.

[5] Li, Changjun, et al. "Comprehensive color solutions: CAM16, CAT16, and CAM16-UCS." Color Research & Application 42.6 (2017): 703-718.

[6] Ebner, Fritz, and Mark D. Fairchild. "Development and testing of a color space (IPT) with improved hue uniformity." color and Imaging Conference. Vol. 1998. No. 1. Society for Imaging Science and Technology, 1998.

[7] Ikeda, Koichi, and Kiyoshige Obara. "Linear uniform colour space composed of simple transformations of tristimulus values X, Y and Z." Journal of Light & Visual Environment 25.1 (2001): 1_49-1_59.

[8] Ebner, Fritz. "Derivation and modelling hue uniformity and development of the IPT color space." (1998).

[9] Berns, Roy S., et al. "Visual determination of suprathreshold color-difference tolerances using probit analysis." Color Research & Application 16.5 (1991): 297-316.

[10] Hou, Bingxin. "Extending the RIT-DuPont suprathreshold data set: Weighted individual discrimination pair data and new chroma dependency visual data." (2010).

[11] Garcia, Pedro A., et al. "Measurement of the relationship between perceived and computed color differences." JOSA A 24.7 (2007): 1823-1829.

[12] Newhall, Sidney M., Dorothy Nickerson, and Deane B. Judd. "Final report of the OSA subcommittee on the spacing of the Munsell colors." josa 33.7 (1943): 385-418.

Author Biography

Luvin Munish Ragoo received his Bachelor in Electrical Engineering and Computer systems from Monash University (2016) and his MSc in Colour Science from University of Eastern Finland (2019). He is currently employed as a PhD candidate at the Norwegian University of Science and Technology. His PhD topic is titled 'Individualised Colour-vision based Image Optimisation', where the goal is to propose an image optimisation pipeline based on individualised cone fundamentals and uniform colour spaces.

Ivar Farup received his MSc in technical physics from the Norwegian Institute of Technology in 1994 and his PhD in applied mathematics from the University of Oslo, Norway, in 2000. He currently serves as a professor of computer science at the Norwegian University of Science and Technology, mainly focusing on colour science and computational colour image processing.