Geophysical electromagnetic modeling and evaluation: a review

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Abstract

Electromagnetic forward modeling is the cornerstone of geophysical electromagnetic inversion. During the last 50 years, numerical simulation methods have been rapidly developed and widely used in geophysical area as the computational capacity continued to increase, such as from single-core to the most modern multi-core processing cards. This paper reviews the literature of electromagnetic fields simulation, particularly focusing on the forward modeling methods include finite difference method, finite element method, integral equation method, and several hybrid methods. We also discuss the possibility of deep learning methods for EM modeling. By sorting out the work done by the predecessors, this review briefly introduces the basic principles and traces back the development of these methods. We propose a Qualitative Evaluation Model named STAMP Model and some criteria of qualitative evaluation on these methods will be discussed in this model.

Keywords: electromagnetic modeling; finite difference method; finite element method; integral equation method

1 Introduction

Geophysical electromagnetic (EM) methods are effectively and widely applied in geophysical researches, applied geophysics and engineering. They mainly reflect the contrast of the electrical conductivity and the magnetic permeability between the targets and surrounding rocks. Accurate simulation of the EM fields distribution has become the primary goal for the EM exploration. Since it is impossible to obtain the analytical solutions of multi-dimensional Maxwell's equations in real complex medium underground, geophysicists devote to find out the numerically approximate solutions for EM fields modeling. EM inversion estimates the realistic subsurface electromagnetic fields distribution, which depends on the precise solution of EM forward modeling problem.

During the past 50 years, the development of the modeling has experienced from low to high dimension (generally from one-dimensional (1D) to two-dimensional (2D), three-dimensional (3D) and two-and-half-dimensional (2.5D) problems), from simple to complex geometry, and from isotropic to anisotropic structure. Application of EM fields modeling has been used in the EM exploration with onshore, offshore, airborne, and borehole environments. With the rapid development of computers and numerical methods, the complexity of the problems to be solved has gradually increased, which has caused a enormous calculational burden. In order to improve computing efficiency, modern computer distributed platforms provide good technical supports for parallel computing.

Geophysical EM forward modeling is sometimes regarded as an engine for EM inversion and commonly used to obtain the verification of conductivity models or conduct various related feasibility studies (Avdeev, 2005). Virieux et al. (2011) mentioned that the key indicators for choosing the forward modeling method mainly included the accuracy, the efficiency, the practicality of the method and the gradient of the misfit function in an inversion algorithm. Researchers committed to advance the forward modeling to higher accuracy and faster computational speed (Kosloff and Baysal, 1982; Zhang et al., 1995; Heagy et al., 2019).

There are some review articles on the EM forward modeling methods. Some articles are focusing on some specific problems, such as EM applications in hydrocarbon exploration and monitoring, partly mentioned the modeling problems (Kaikkonen, 1986; Sheard et al., 2005; Siemon et al., 2009; Strack, 2014; Streich, 2016). Avdeev (2005) mentioned numerical methods from theory to application focusing on the 3D problem. Zhdanov (2010) discussed the EM methods exhaustively, including the developments of data acquisition, modeling, inversion and interpretation, as well as a new approach to EM-field characterization. Börner (2010) considered the numerical solution of the 3D time-domain and frequency-domain EM induction problems, restricted to finite difference method (FD) and finite element method (FEM) and consciously ignoring integral equation method (IE). From the perspective of numerical calculations, Miensopust et al. (2013) compared different algorithms from the two aspects of forward and inversion methods and discussed the applicability of these different algorithms to the two Dublin models. Newman (2014) reviewed high performance computational (HPC) strategies for large-scale 3D EM modeling and imaging and discussed the future of HPC applied to EM modeling.

Differently from the above reviews, in this paper we mainly review the three most widely applied methods for EM modeling, including FD, FEM and IE, and several hybrid methods derived from them. We will intentionally not elaborate on some details of numerical calculation and parallel computing. Our aim is to catch the recent development of the EM forward modeling methods. In order to give readers a clearer understanding of the EM forward modeling, we stand at the point of the development history and the improvement of these methods.

In Section 2, we first review FD, FEM, IE respectively and divide some key technologies of them more carefully according to their respective characteristics. Then we also mention several hybrid methods derived from the above three methods. The hybrid methods generally combine advantages of at least two of these conventional methods to make up for the shortcomings of single methods. Additionally, we discuss the possibility of the application of the deep learning (DL) method in EM modeling.

In Section 3, we discuss on the basis of the traditional EM modeling methods reviewed in the previous part and build up an evaluation model, called STAMP Model, to qualitatively describe the advantages and disadvantages of the forward modeling methods. In the last, we make a conclusion for this review paper.

The statistics for the years and quantities of major references cited are given in Figure 1. What we can find interesting is that after 2000, researchers began to study FEM gradually, and in the past five years, deep learning began to become the focus of researchers' attention.

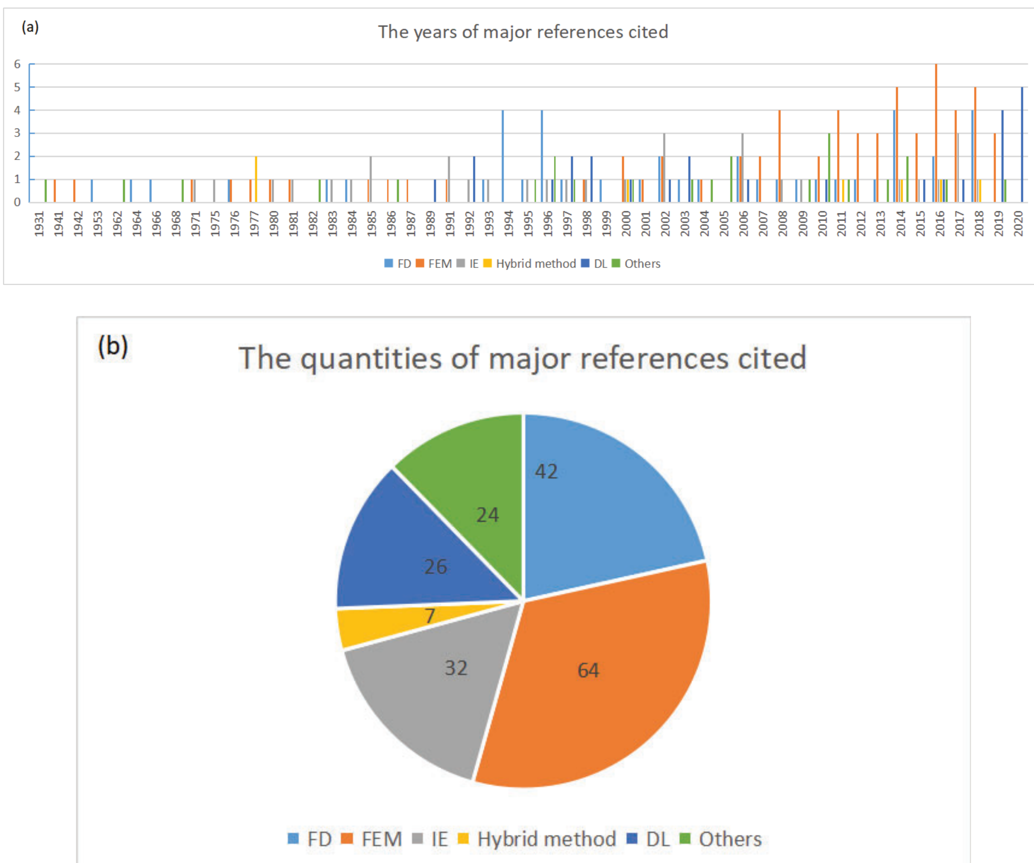

Figure 1. The statistics for the years (a) and quantities (b) of major references cited.

2. Commonly use numerical approaches

2.1 Finite difference method

Finite difference (FD) method is an approximate numerical solution for differential equations (DE), which discretizes the derivative in the governing equations mainly by Taylor series expansion. With it, the algebraic equation with unknown variables on the grid can be established and then the differential equation system is directly turned into an algebraic problem.

Finite difference method is one of the earliest methods used in computer numerical simulation. In the 1960s, Yee (1966) first adopted FD to solve the initial boundary problem of time-domain Maxwell's equations in the isotropic medium, and proposed the staggered-grid finite-difference method (SFD). According to the different solution domain, it can be divided into finite-difference time-domain method (FDTD) and finite-difference frequency-domain method (FDFD). SFD as the most commonly used difference format can solve for the EM fields in both the time and frequency domain.

2.1.1 SFD in EM modeling

The staggered-grid finite-difference method (SFD) is a method meshing in space. In the conventional staggered grids (SG), the scalar is stored and calculated on the normal grid node, and the components of the vector are stored and calculated on the dislocated grid. The center of the dislocated grid is located on the interface of the original control volume. And the purpose of using SG is to solve the discontinuity problem caused by the discrete governing equations on ordinary grids. In view of this advantage, Yee (1966) proposed SFD suitable for EM modeling.

Yee grids

Yee's SFD defines the discrete electric field components at the midpoint of the edges of the discrete elements, while the discrete magnetic field components are defined at the centers of each side facet of the discrete elements. The defined position of the EM fields can be exchanged. On the one hand, Yee's SFD naturally expresses Faraday's right-handed spiral law of electromagnetic induction. On the other hand, it solves the problem of the discontinuity of the tangential components of the electric field caused by the electric field definition at the nodes of the elements in the conventional grids. Figure 2 shows the staggered grid (Yee, 1966) and the conventional grid for the FD method on the EM modeling. Unless otherwise specified, the "SFD" used below is Yee's SFD.

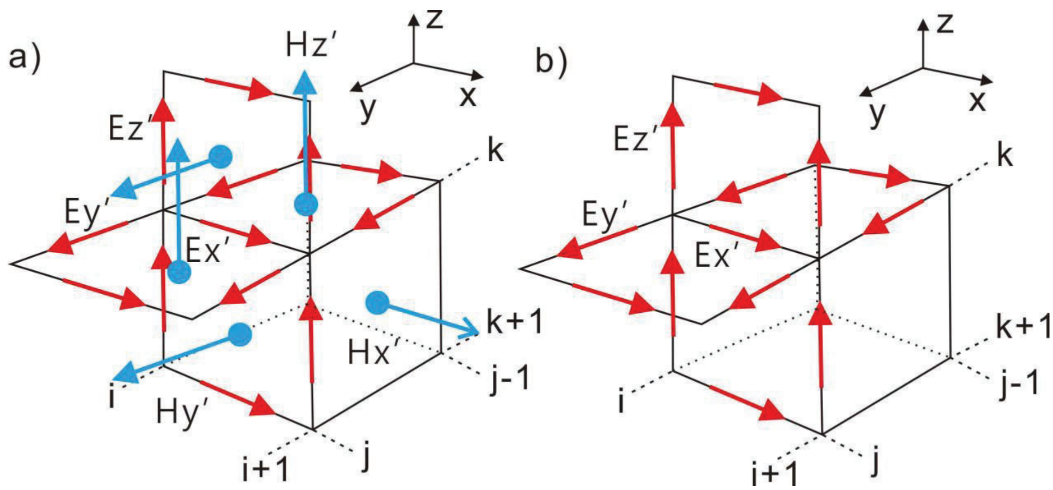

Figure 2. The staggered grid (a) and the conventional grid (b) for the FD method. The red arrows represent the electric field components E' and the blue arrows represent the magnetic field components H' (the electric field components E' and the magnetic field components H' are coincident in b). (Modified from Weiss and Newman, 2002)

Due to the limitations of computer development, the SFD algorithm was not widely used until the 1990s. Newman and Alumbaugh (1995) employed SFD method to the 3D frequency-domain AEM response and Alumbaugh et al. (1996) applied it for solving the 3D earth wideband EM response. Smith (1996a) developed the SFD for 3D electromagnetic induction directly on the inhomogeneous rectangular grid and discussed the derivation process of SFD equation. The solution was compared with the 2D quasi-analytical solution and the accuracy of the method was proved. Smith also pointed out that the various differential relationships between different field components must be completely maintained in the SFD form, which was the most important feature of the SFD. In another article, Smith (1996b) applied the Schur complement introduced by Haynsworth (1968) to divide the computational domain into some smaller subdomains.

In the 1990s, the use of FD for EM researches on anisotropy of EM field still stayed in 2D. Pek and Verner (1997) proposed a FD algorithm of MT fields in 2D generally anisotropic block structures. At that time, the solution to the 2D anisotropy problem was still limited by computer performance. At the beginning of the 21st century, the study of the anisotropy problem of FD began to consider 3D models. Wang and Fang (2001) developed the SFD to simulate the multicomponent EM response in 3D inhomogeneous medium with arbitrary anisotropy. By using the coupled Maxwell's equation, the computation time of anisotropy was approximately equal to the isotropic calculation time. From another perspective, for simulation of EM induction in 3D anisotropic medium, Weiss and Newman (2002) proposed a new SFD algorithm which accurately simulated the effect of the geological structures on induction tool response. The new work extended the previous isotropic work by Newman and Alumbaugh (1995) to anisotropy and also effectively controlled the calculation cost of anisotropy to be similar to that of isotropy.

Gradually, the SFD has been widely applied in induction logging, borehole, airbone and marine modeling. Followed the work by Newman and Alumbaugh (1995) and Alumbaugh et al. (1996), Newman and Alumbaugh (2002) developed the SFD approach for simulating the 3D induction logging response with quasi-static limit and transverse anisotropy. The approach used the decoupled vector potential and dc scalar potential functions. The new developed preconditioner significantly sped up the solution of low induction numbers (LINs) and low frequency. Differently from Newman and Alumbaugh (2002), Hou et al. (2006) proposed a new SFD algorithm using coupled scalar-vector potential formulas. Averaged conductivities and potential components not defined on the same points were calculated by the similar methods used by Wang and Fang (2001) and Weiss and Newman (2002). The proposed algorithm could efficiently and accurately simulate various types of the frequency and arbitrary electrical anisotropy in the complex 3D borehole EM modeling. Applying the SFD algorithm to the AEM system, Liu and Yin (2014) obtained the solution of the coupled the partial differential equations (PDEs) for the scattered electrical fields. They adopted the specifically designed divergence correction technique for the 3D anisotropic model to accelerate the process of the simulation. The technique greatly improved the calculation efficiency and the convergence speed of the solution. By using the SFD method, Li et al. (2018) applied the complex frequency-shifted perfectly matched layer (CFS-PML) boundary (Kuzuoglu & Mittra, 1996) to 3D marine CSEM modeling. Compared to using the Dirichlet boundary condition, the algorithm using the CFS-PML saved more computing time and memory and could be more efficient.

Lebedev grids

The above mentioned SFDs are mostly based on Yee's SFD variants. Actually before Yee (1966), Lebedev (1964) presented a different SG scheme. Davydycheva and Druskin (1999) introduced Lebedev grids to solve the Maxwell's equation. Differently from Yee grids, Lebedev grids places all components of the electric fields at one set of nodes, and all components of the magnetic fields at another set of nodes. This maintains the current conservation property in the grid cells and addressed the modeling with anisotropy in the physical properties. Figure 3 shows the difference between Lebedev grids and Yee grids. Lebedev grids can be split in four uncoupled standard Yee grids but four times the computational cost is needed compared to the similar isotropic problem in the standard Yee grids (Wang and Fang, 2001; Weiss and Newman, 2002). In order to reduce the cost of computation in the Lebedev grids, Davydycheva et al. (2003) employed a proper averaging of the sources, solutions, and error elimination and a spectrally optimal grid refinement scheme to calculate the electromagnetic field of the 3D anisotropic inhomogeneous media in the EM induction logging. The grid size was significantly reduced and the 3D calculation was greatly accelerated without sacrificing accuracy. Jaysaval et al. (2016) also presented an algorithm based on the Lebedev grid with a multigrid preconditioner for the numerical simulation of 3D CSEM general electrical anisotropic conductive medium. They accurately simulated layered and 3D tilted transverse isotropic (TTI) typical marine CSEM model and proved the importance of considering the fully anisotropy of the conductivity tensor for the inversion. And a rule was observed that the solution time of a linear system increases linearly with the increase of unknowns.

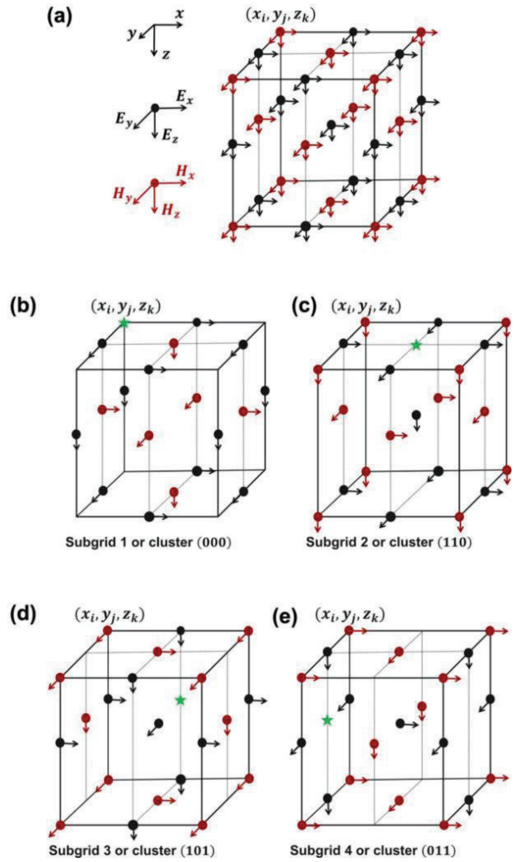

Figure 3. (a) Lebedev grid cell used to define the collocated electric and magnetic field components. (b) Standard Yee grid, and the complementary Yee grids constructed by shifting the components of E and H by half a cell in the (c) x-, (d) y- and (e) z-directions. (Jaysaval et al., 2016)

2.1.2 FDTD

Finite-difference time-domain method (FDTD) has been widely applied into the simulation of the time-domain EM fields. It directly solves the Maxwell's equation in the time domain without requiring any form of derived equation. The prominent advantage of FDTD is that it can simulate various complex structures only by assigning the components of the EM fields contained by the difference format to each grid. Due to adopting the stepping method for calculation, FDTD can easily realize the simulation of various complex time-domain wideband signals, and it is very convenient to obtain the time-domain signal waveform at a certain point in space.

Goldman and Stoyer (1983) used the FDTD of a coaxial loop or wire loop transient electromagnetic (TEM) detection system to simulate the EM fields generated by a 2D buried cylindrical conductor. Based on the Du Fort-Frankel FD scheme (Du Fort and Frankel, 1953), Oristaglio and Hohmann (1984) applied FDTD to solve the time-stepping Maxwell's equations in a 2D conductive earth. Wang and Hohmann (1993) applied this method to the 3D TEM model. And in order to solve the boundary conditions, Berenger (1994) first proposed the concept of FDTD perfectly matched layer (PML) absorbing boundary conditions to calculate the boundary condition problems in 2D time domain. Subsequently Katz et al. (1994) and Chew and Weedon (1994) extended the FDTD PML to 3D time-domain calculations, and Debroux (1996) applied the FDTD code to the 3D modeling of the EM response.

For reducing the computational time of 3D modeling, Commer and Newman (2004) presented a parallel FDTD approach for 3D TEM modeling. By combining a modified Du Fort-Frankel method with the FD scheme presented by Wang and Hohmann (1993), Maxwell's equations were stepped in time. For simulating a real large-scale earth model economically, the approach was parallelized to save the large consumption of computational time. Maaø (2007) improved the FDTD based on mathematical transformation rather than physical approximation (Oristaglio and Hohmann, 1984) for marine-subsurface EM problem. The improvement significantly reduced the frequency dependence of the propagation velocities and cur down the computational time. Continuation of this mathematical transformation improvement, Mittet (2010) presented a numerically cost-efficient high-order FDTD scheme, which used a correspondence principle of wave and diffusion fields, for efficiently simulating marine CSEM data. And de la Kethulle de Ryhove and Mittet (2014) developed it to solve Maxwell's equations for marine MT 3D modeling. They pointed out that the FDTD method completely avoided solving the linear system of equations. It allowed the calculation of the unknowns of EM fields at all frequencies in only one simulation, with very low computation complexity and low memory.

2.1.3 FDFD

Frequency-domain finite-difference method (FDFD) based on Maxwell's equation is simple and intuitive in both principle and formulas, and can be used to deal with various EM problems. However, the classic FDFD needs to discretize the entire calculation area, and a difference equation must be established at each grid node. In spite of the final matrix equation is sparse, the scale of the matrix will increase rapidly as the computational domain increases, resulting in a huge burden of calculation and storage. Therefore, FDFD is usually combined with some other techniques to reduce the computational cost.

The most common combination of FDFD is the use of Yee grid to discretize the EM fields. Mackie et al. (1994) combined the SFD algorithm with the minimum residual relaxation method to calculate the MT response of general 3D models in the frequency domain. Frequency-domain SFD has been used successfully to solve EM fields in isotropic medium (Newman and Alumbaugh, 1995; Alumbaugh et al., 1996; Smith, 1996a) and then has been developed into anisotropy (Wang and Fang, 2001; Weiss and Newman, 2002; Hou et al., 2006). Even this method has been applied to the forward modeling of the EM inversion problem (Egbert and Kelbert, 2012; Grayver et al., 2013). It is worth noting that Egbert and Kelbert (2012) developed a module system of computer codes, ModEM, for EM inversion. ModEM has already been widely applied into 3D MT and CSEM problems (Kelbert et al., 2014).

FDFD is relatively simple and suitable for CSEM surveys which extracting only a few discrete frequencies from data (Streich, 2009). Streich (2009) discussed the iterative and direct solvers for solving the system of equations and used a massively parallel sparse direct solver (MUMPS) (Amestoy et al., 2000) to solve the system of equations from FDFD. The staggered scheme with electric-field components located on the cell faces and the magnetic-field components on the edges was better for the CSEM surveys than the more commonly used SG. Similarly, using the same scheme described by Streich (2009), Grayver et al. (2013) applied FDFD and MUMPS in the forward algorithm for 3D CSEM data inversion. Modern distributed-memory platforms solved the high memory demand of the direct solver. In order to obtain a stable system at low frequencies using a direct solver, a static divergence correction (Smith, 1996b) was enforced for the static limit in the low-conductivity air layer. However, it is not necessary for typical CSEM frequencies (~0.1-10Hz) (Streich, 2009; Jaysaval et al., 2014). And Jaysaval et al. (2014) applied a Schur complement scheme (Haynsworth, 1968; Smith, 1996b) to FDFD with the commonly used staggered scheme for fast multi-model 3D CSEM modeling. The Schur complement system was solved by using MUMPS and the scheme overcame the shortcoming of standard FDFD method, which required repeated forward modeling of the whole earth model at each iteration, so that reducing the computation complexity greatly. The efficiency of the FDFD code was validated against the FDTD code developed by Maaø (2007) and Mittet (2010).

In addition, there are some other optimization schemes. Yavich and Zhdanov (2016) developed a new efficient frequency-domain SFD method for calculating discrete 1D layered background conductivity, based on introducing a contraction operator (CO) to construct an effective FD EM-modeling preconditioner. The contraction preconditioner can significantly accelerate the convergence of the FD iterative solver and save the memory storage of the computation. Considering that there is no need for fine grids in deep underground, Cherevatova et al. (2018) presented a multi-resolution (MR) FD approach for frequency-domain 3D MT forward modeling. The MR staggered-grid (SG) scheme was implemented to decrease the horizontal resolution with depth. Three ways of handling the interface layers were considered and the best one retained the symmetry of the coefficient matrix with a similar accuracy result as the SG solution. Compared with the basic SG, MR scheme improved the computation efficiency without the loss of the solution accuracy. Varilsuha and Candansayar (2018) studied different EM formulation approaches, including direct EM formulation, ungauged and gauged (Lorenz, Coulomb, and axial) vector and scalar potential formulations, to solve the problems of 3D MT modeling. Comparing the accuracy and the speed of the FD solution for each method, the ungauged method provided faster simulation with the same precision. Furthermore, the axial specification system forward modeling also had a faster speed of CSEM field simulation than other methods.

2.1.4 2.5D problem

Although the use of high-performance computing can achieve 3D forward modeling, the cost of discretizing the computation domain of fully 3D models is extremely expensive. To avoid direct 3D solution, there is a reasonably assumption that the geological structure with topography within a certain range is a 2D model with uniform electromagnetic characteristics in the strike direction. It should be noted that such an assumption is not suitable for discussing fully anisotropy. The coordinate in the strike axis is transformed into the wavenumbers by Fourier transforms. For each of a number of wavenumbers, only a 2D problem need to be solved (Stoyer and Greenfield, 1976) and the response is still that of a 3D model with the properties invariant along one of the axes. Such 2D problem is described as 2.5D problem and it simplifies the 3D solution and significantly reduces the number of unknowns and computational cost. The 2.5D problem also applies to FEM and IE that will be reviewed in the corresponding section later.

The 2.5D FD method was first proposed by Stoyer and Greenfield (1976). Abubakar et al. (2006a) put forward a 2.5D SFD forward algorithm for marine CSEM. The algorithm solved all source-receiver configurations simultaneously, which greatly improved the calculation efficiency and helped the realization of fast inversion algorithms. Based on Abubakar et al. (2006a), Abubakar et al. (2008) presented efficient 2.5D forward and inversion algorithms for the interpretation of low-frequency EM measurement. And the forward algorithm used a multifrontal LU decomposition (Davis and Duff, 1997) to invert the stiffness matrix. Chen et al. (2011) developed a 2.5D SFD code for simulating the responses of logging-while-drilling (LWD) deep directional EM tools and wireline tensor induction tools in high angle and horizontal (HA/HZ) wells. The code was applied for 2D formation conductivity distributions and 3D well trajectories. Zeng et al. (2018) proposed a 2.5D FD method based on Yee's grid using the weighted average method (Weiss and Newman, 2002) to simulate the anisotropy of 2D resistivity logging. The Fourier transform was performed by adopting the Gauss-Legendre quadrature rule and the inverse Fourier transform was accelerated by employing the Gaussian quadrature method (Quarteroni et al., 2010), which greatly reduced the calculation time and improved the computation efficiency. This made the 2.5D FD has a better applicability than 3D scheme.

Above all, FD is a simple and practical numerical simulation method for solving PDEs by approximating the derivative with a difference. Because of the approximate solution obtained by differential approximation and interpolation, FD has the advantage for the general model. Due to the own characteristics of the SG, the SFD method can largely solve the problems of EM fields discontinuity caused by the difference in electromagnetic properties of the medium. FDTD and FDFD have different applicability and whether in the time or frequency domain, the whole domain needs to be discretized so that FD is not suitable for solving more complex domains. In addition, since the interpolation calculation is in the whole domain, different discretized grid sizes will get different solution results.

2.2 Finite element method

Finite element method (FEM) is based on the variational method and the weighted equivalent integral method. According to the principle of variation or the principle of orthogonalization between the remainder of the equation and the weight function, an integral expression equivalent to the initial boundary value problem of the differential equation is established. Although both belong to the DE method, FEM is very different from FD. In the process of solving, an interpolation function is used to connect all the discrete units, and the PDEs group becomes a total stiffness matrix to be solved. No matter how complicated the calculation domain is, it can be discretized into finite elements and these elements are connected through interpolation functions to realize the solution of the complex domain.

Originally FEM was applied to solve the elastic and structural analysis problems (Hrennikoff, 1941; Courant and Robbins, 1942). Until the 1970s, Coggon (1971) firstly employed it to calculate EM fields. Rodi (1976) proposed the FEM for a numerical simulation of MT data on 2D conductivity model with a new rectangular grid. Rijo (1977) put forward a single-module FEM algorithm which can deal with 2D symmetry problems in electromagnetic methods. This algorithm greatly improved the accuracy and the speed of 2D EM simulation. Wannamaker et al. (1986) employed this method to simulate the 2D MT response with terrain. These methods can be used to simulate both the MT and the CSEM data. In 3D space, the computational cost of FEM for simulating 3D EM response increased dramatically. The limitation of computer operation speed and physical memories hindered the usage of FEM. Since 1980s, 3D EM modeling methods have been gradually proposed with the development of computational resources (Pridmore et al., 1981; Mur, 1991). Considering the huge computational burden of 3D EM modeling, Zyserman and Santos et al. (2000) applied parallel FEM 3D EM modeling.

2.2.1 Mesh generation technology

Since mesh generation is a very important step in FEM numerical analysis, and it directly affects the accuracy of the subsequent numerical calculation and analysis results, the predecessors have fully studied this technology.

The structure mesh is generated fast with high quality and it can be easily applied to simulate curves or surfaces only by parameterized methods or interpolation (Wannamaker et al., 1987). However, this also limits its scope of application, making it inadequate for calculations in complex domains. In order to overcome limitations of computational complexity and the inherent constraints of EM fields, structured-grid FEM were proposed in some reviews (Sugeng, 1998; Yoshimura and Oshiman, 2002).

Compared to structured grid, unstructured gird can accurately segment curved boundaries of complex geological structures such as terrain or seafloor topography because of the flexibility of meshing and reduce the size of the system of linear equations arising from the forward problem (Börner et al., 2008; Schwarzbach and Haber, 2013).

Unstructured grid generation technology solves the discretization of complex calculation domains while the speed and quality of mesh generation will decrease and the difficulties in boundary recovery will also be introduced. In order to prevent excessive meshing and find the optimal meshes, adaptive mesh refinement technology is an effective method to solve this problem.

2.2.2 Adaptive FEM.

The adaptive FEM is a numerical method that can automatically adjust the algorithm through adaptive analysis to improve the solution process. It is based on the conventional FEM, with a posteriori error estimation and adaptive mesh improvement technology. The method can successfully save physical memories and significantly improve the computational efficiency and accuracy.

Key and Weiss (2006) applied the adaptive FEM for 2D MT modeling. They replaced the rectangular grids with irregular triangular grids since it was easier to simulate complex structural boundaries. The adaptive refinement method based on the dual-error weighting approach (DEW) (Ovall 2006) refined the mesh with insufficient precision through iteration in order to ensure the accuracy. Li and Key (2007) applied the DEW approach into 2.5D marine CSEM, enabling the unstructured grids to adjust themself automatically to calculate EM fields effectively. Since the analytical solution is used to calculate the primary field, it was the most accurate solution during that time. Li and Pek (2008) presented a similar goal-oriented self-adaptive FEM with DEW as a guide. The algorithm improved the quality of numerical solutions in a general 2D MT anisotropic conductivity media. For 3D case, Ren and Tang (2010) presented an adaptive FEM for direct current (DC) resistivity modeling. It started with the initial coarse mesh and then based on a gradient recovery scheme. The mesh refined adaptively according to the indication of a recursive error estimator. The whole process of adaptation is shown in Figure 4. Where CFEM is the abbreviation of conventional finite element method, $\overline{\eta_e}$ is the average element error percentage, and η^* is the given error criterion. Schwarzbach et al. (2011) proposed a 3D adaptive higher order FEM for modeling a realistic marine CSEM scenario. The adaptive mesh refinement strategy and the higher-order polynomial (HOP) FEM improved the accuracy of the solution.

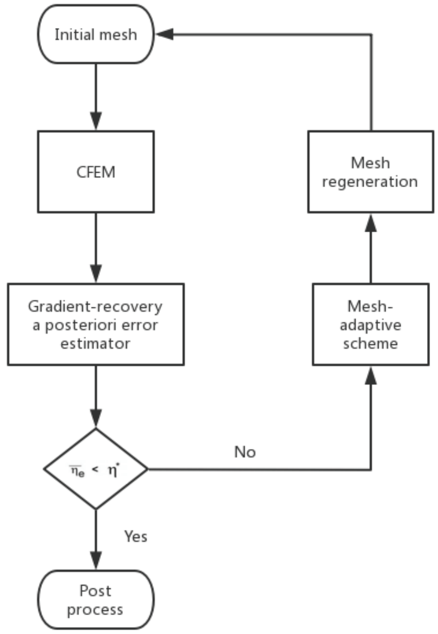

Figure 4. The whole process of the adaptive FEM scheme. (Modified from Ren and Tang, 2010)

Actually, according to the posterior error estimation method, the adaptive mesh refinement technology can be divided into two types. One is based on the super-convergence characteristic of the variant of the EM field (Key and Weiss, 2006; Ren and Tang, 2010; Schwarzbach et al., 2011). The other is based on the continuity of the EM field or the current density (Yin et al., 2016). Ren et al. (2013) applied the continuity-condition-based adaptive FEM for plane wave 3D EM modeling based on electric field differential equations. And then, Yin et al. (2016) presented a goal-oriented, continuity-condition-based adaptive FEM for 3D scattered AEM modeling in the frequency domain. In addition, Zhang et al. (2018) employed the method with the backward Euler scheme to perform time-domain 3D airborne full-wave EM field simulation. The random grid-selection technique improved the stability of the forward modeling and controlled the number of meshes in the adaptive process, thereby the efficiency of EM simulation was improved.

In fact, Ovall (2006) also proposed another called the dual weight residual (DWR) method. Compared to the DEW method, the DWR method calculated the weight of residual instead of the gradient recovery. In other words, DWR replaced the absolute error of DEW with the relative error. Such a goal-oriented refinement strategy could dramatically reduce the density of adaptive refinement grids to a greater extent and save more a large number of computational resources while keep high numerical accuracy. Therefore, Key and Ovall (2011) developed a parallel goal-oriented adaptive meshing technique based on the DWR method and implemented this technique into a parallel Fortran code named Modeling with Adaptively Refined Elements for 2D EM (MARE2DEM). Figure 5 shows a typical synthetic marine CSEM model for hydrocarbon exploration on the continental shelves. The technique was tested and proved on this model. The optimal distribution of mesh density was discovered and the accuracy of 2.5D EM numerical simulation was improved. The MARE2DEM software, Key (2016) published, also applied the method to automatically generate and refine an unstructured triangular element mesh for forward modeling and inversion. It ensured the accurate model response with various conductivity parameters. Liu et al. (2018b) put forward a goal-oriented adaptive FEM algorithm for 3D MT modeling in generally anisotropic conductivity media. A global residual based posterior error estimator was employed to guide the refinement of unstructured tetrahedral meshes. The algorithm realized the modeling of arbitrary bathymetries and structural boundaries.

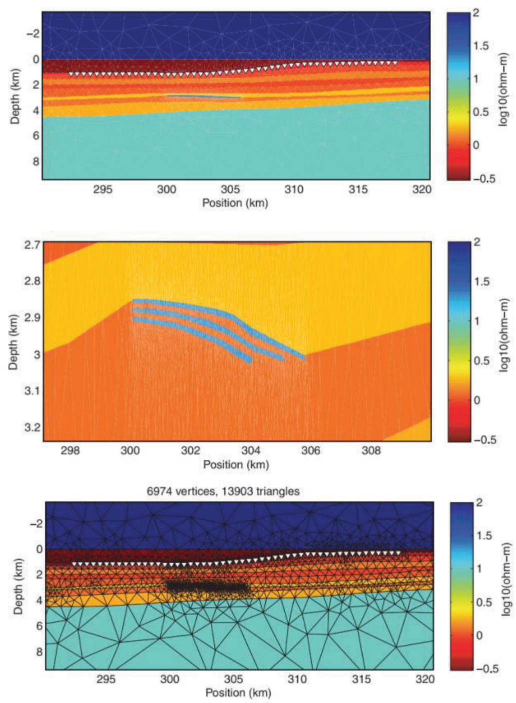

Figure 5. Complex marine conductivity model including typical features that are difficult to discretize on a rectangular grid: a large bathymetric slope, tilted regional strata and closely spaced thin and dipping reservoir intervals. The three panels show the model (top panel), a close-up of the stacked reservoir layers (middle panel, note the vertical exaggeration) and the unstructured grid used as the starting mesh (bottom panel). Inverted white triangles show the location of the seafloor EM receivers. Only the central portion of the model is shown (Key and Ovall, 2011).

The unstructured grid solves the complexity of the solution area, and the adaptive scheme solves the problem of grid division, so that they have improved the calculation accuracy and calculation efficiency of the finite element solution process to a certain extent. However, due to the diffusion of the EM field itself, the convergence rate of the solution has not been resolved.

2.2.3 More optimization solutions

Divergence correction

Due to the characteristics of electric field diffusion, the speed of convergence, especially at low frequencies, was very slow with the low convergence rate. Farquharson et al. (2011) proposed a divergence correction method applied in the FEM based on Smith (1996b) on divergence correction. This correction method promoted the process that the discontinuous conductivity in the approximate electric field generated the discontinuous normal component. The convergence speed of the equations was accelerated and the computation efficiency was improved. Kordy et al. (2016) described the divergence correction as the typical procedure for removing spurious curl-free fields caused by current divergences over the discretized model domain during the iterative solution process.

Edge-based or Vector FEM

The conventional FEM has an obstacle that the node-based FEM cannot handle the discontinuity of the normal electric field component. One solution is to use the electromagnetic potential formulation (Badea et al., 2001). Because the electric field can be decomposed into vector and scalar potential in Helmholtz equation and the charge conservation equation, Mitsuhata and Uchida (2004) proposed a FEM method for 3D MT conductivity response based on the T-Ω Helmholtz decomposition. In particular, the vector field T is approximated by the twelve components assigned at the centers of edges of each element and the scalar field Ω is approximated by the eight components at the vertices of each element. Mukherjee and Everett (2011) put forward an edge-based tetrahedral mesh FEM algorithm based on an ungauged potential formulation to simulate near-surface 3D CSEM induction response, which addressed local inhomogeneities in the electrical conductivity and magnetic permeability distribution near the surface.

However, potential formulation may introduce more numerical instability (Puzyrev et al., 2013). Ansari and Farquharson (2014) proposed an unstructured tetrahedral mesh vector FEM solution for frequency domain 3D EM modeling. The discretization of the edge element and node element were used to approximate the vector potential and the scalar potential, respectively. The scheme adopted the Galerkin method (Jin, 2002) variant of the weighted residual method to discretize the equations of the sparse linear system, and applied the generalized minimum residual solver with incomplete LU preprocessor (Saad, 2003) to solve the system iteratively. Based on this scheme, Ansari et al. (2017) proposed a new gauged finite-element potential formulation for 3D EM modeling. The block diagonal preprocessing scheme based on the Schur complement of the potential system stabilized the iterative solution of the estimation system. Both the iterative solution and the direct solver had the same response to the potential, which proved the uniqueness of the potential solution. And then Dunham et al. (2018) employed the 3D finite element code provided by Ansari and Farquharson (2014) to the real exploration prospects of the Flemish Pass basin offshore Newfoundland, Canada, and extended the application of FEM for the 3D marine CSEM. Models were discretized with unstructured tetrahedral meshes. The edge length constraints as an optimization reduced the total number of tetrahedral elements and refined specific areas of the mesh. It accurately simulated the complex structures in the model, simultaneously minimizing the numbers of model unknowns.

Another solution is using edge-based FEM (Nédélec, 1980; Jin, 2002). The use of edge elements can ensure the continuity across different medium of the vector basis function, while ensuring zero curl and non-zero divergence, and can ideally express electromagnetic physical characteristics such as current density. Edge-based FEM is also known as vector FEM. Since the vector FEM could satisfy the discontinuity of the normal component of the electric field, the advantage of retaining the calculation accuracy and high computation efficiency is to avoid the divergence correction. Nam et al. (2007) used vector FEM to realize 3D MT forward modeling. Liu et al. (2008) applied the unstructured grid to improve the method which used the vector FEM to efficiently simulate the 3D MT response and it further improved the forward modeling accuracy.

In order to increase modeling efficiency while ensuring accuracy of 3D CSEM modeling, da Silva et al. (2012) introduced MUMPS into the edge-based FEM for solving the linear system of equations. The scheme of non-uniform Cartesian conforming hexahedra made the grid generation more convenient. And it was demonstrated that the presented approach was robust for indefinite and ill-conditioned linear systems. Chung et al. (2014) used edge-based FEM based on a hexahedral mesh with a direct solver PARDISO (Schenk and Gärtner, 2004) for 3D CSEM modeling. The results of a series of comparative experiments verified the effectiveness of the edge-based FEM and the advantages of the direct over iterative solver. However, both da Silva et al. (2012) and Chung et al. (2014) indicated that there was a limitation in the hexahedral mesh when the model complexity increased and maybe a tetrahedral mesh could be adopted. Li et al. (2016) combined the total-field algorithm, local refinement of unstructured tetrahedral mesh and vector FEM for EM modeling. The MUMPS was used to solve the linear equations. And differently from the unnecessary large distances to the truncation boundaries as Chung et al. (2014) set, appropriate truncation boundaries for the computational domain was determined by numerical experiments which reduced the waste of calculations to a certain extent.

Utilizing the computation power of modern distributed-memory platforms, Ren et al. (2014a) developed a new parallel vector FEM code combined with unstructured meshes for plane wave 3D EM modeling. Based on a domain-decomposition approach, the code recombined the unknowns. Parallel implementation used the robust direct solver PARDISO. Compared with the traditional FEM, the code could solve more complicated large-scale models. Grayver and Bürg (2014) also studied a robust and scalable approach for large-scale 3D EM modeling in the frequency domain. They applied the flexible generalized minimal residual (FGMRES) iterative Krylov subspace method (Saad, 2003), which significantly reduced the computational time and memory. However, both Ren et al. (2014a) and Grayver and Bürg (2014) only employed the lowest order Nédélec elements (Nédélec, 1980). Grayver and Kolev (2015) extended the Grayver and Bürg (2014) approach to the arbitrary order. Combining the high-order FEM with the relationship of target local mesh, the computation time was saved and at the same time, the multi-degree of freedom calculation was also reduced. Based on these works, Grayver (2015) employed adaptive FEM on parallel 3D MT modelling and inversion. The adaptive mesh refinement technology avoided over-parameterization and accurately calculated the EM response based on goal-oriented error estimator. The computation effort was significantly saved by a locally refined decoupling grid and the calculation of the electromagnetic field at each frequency using an independent grid further improved the computation efficiency. Then, Grayver et al. (2019) applied it combined with high-order meshes to calculate the high-resolution solution of 3D MT modelling in spherical earth.

The above papers are all discussions on isotropic media. Some differently, Cai et al. (2014) presented a linear edge-based FEM for the numerical simulation of the 3D CSEM data in anisotropic conductive medium. The scheme used a non-uniform rectangular mesh to capture the rapid changes of the diffused electromagnetic field in the abnormal conductivity region and around the source, which also can be transformed to hexahedral mesh to simulate the effect of sounding. Later, based on the previous work, Cai et al. (2017a) employed total field formulation and unstructured tetrahedral mesh. A new hybrid boundary condition was used to reduce the computation domain while improve the accuracy of forward modeling. The MUMPS was used to speed up the solution of the system of equations.

Castillo et al. (2016) developed an edge-based FEM parallel code for the isotropy of 3D marine CSEM forward modeling. The scalability testing and the evaluation of the error norm of the different size meshes verified that the method still maintained high accuracy with good parallel efficiency. And then, Castillo et al. (2018) developed a Parallel Edge-based Tool for Geophysical Electromagnetic modeling (PETGEM), which is the first open-source modeling toolbox for 3D marine CSEM problems, to study the 3D CSEM problem of an infinitesimal dipole arbitrary isotropic medium with low frequency approximation. They provided an adaptive scheme for frequency and specific source locations, and developed a scalable study of HPC architecture based on basic metrics. However, PETGEM only support first-order polynomials, isotropy and cannot process the multiple horizontal electric dipoles without surface topography. Therefore, Rochlitz et al. (2018) developed an open-source toolbox custEM (customizable electromagnetic modeling) for complex 3D CSEM modeling. The custEM is similar to the PETGEM but support HOP, anisotropy and multiprocessing.

Time-domain finite-element method

For simulating transient EM fields in 3D diffusive earth media, Um et al. (2010, 2012) put forward a time-domain finite-element method (FETD). An unstructured grid and adaptive time-stepping doubling (ATSD) scheme was used to simulate the diffusion of 3D electromagnetic waves. Compared with the analytical method and the 3D FDTD, the algorithm was demonstrated. Although the FETD with unstructured grid and ATSD have a potential to reduce the number of unknowns and time steps, the FETD method was often difficult to extend with available parallel computing resources, due to each step required solving a large-scale unstructured sparse matrix.

Fu et al. (2015) designed a parallel FETD method for improving modelling speed. Multi-threading sped up the key steps of solving large sparse matrices and the convergence, and greatly reduced computation time while ensuring accuracy. Cai et al. (2017b) implemented FETD with a hybrid boundary condition to simulate CSEM data. They employed the unstructured tetrahedral mesh to discretize the model domain and adopted the ATSD to keep the same step size. Then, the semi-adaptive method was also adopted to discrete the model domain. The new hybrid boundary condition used the primary field corresponding to the layered background model to approximate the total field on the boundary. Additionally, based on the unstructured tetrahedral mesh and the ATSD scheme, Cai et al. (2017c) also developed an adaptive Padé series method (Baker and Graves-Morris, 1996) to approximate the Cole-Cole model. The method improved the accuracy of the Padé approximation over a wide time range to enhance the simulation accuracy. Applying the edge-based FEM for the spatial discretization and the second-order of backward Euler scheme for the time discretization (Um et al., 2010), Liu et al. (2019) adopted the direct solver MUMPS to factorize the large sparse matrices obtained by FEM and utilized the iteration scheme from an initial field for all time channels to solve the TEM forward modeling with topography using unstructured tetrahedral grids efficiently.

2.2.4 2.5D problem

As early as 1985, Lee and Morrison (1985) have already used 2.5D FEM for the electromagnetic scattering by a 2D inhomogeneity. Everett and Edwards (1992), Unsworth et al. (1993), and Mitsuhata (2000) applied 2.5D FEM to obtain the EM induction over a 2D earth. The previously mentioned papers, such as Li and Key (2007), Key and Ovall (2011) and Key (2016) applied the 2.5D FEM into the simulation of the marine EM. Kong et al. (2008) also presented a 2.5D FEM difference method for marine CSEM in stratified anisotropic media. Kang et al. (2012) used a 2.5D FEM to calculate the secondary field caused by a subsurface anomalous for marine CSEM response.

In general, FEM is an effective forward modeling method with high precision of simulation. More degrees of freedom for meshing enables FEM to solve the problem that the FD method cannot cope with complex solution domains. However, the property of the mesh will sacrifice some simulation speed to some extent. With the development of unstructured grids, adaptive schemes and parallel computing, the simulation speed of FEM has been solved partially, which makes the precision advantage of FEM more obvious than other forward methods. This is why researchers are gradually interested in the study of FEM after 2000, what we have mentioned in Section 1. At present, geophysicists have tried to combine the FEM with other methods for more efficient forward modeling with high precision and high speed.

2.3 Integral equation method

Integral equation method (IE) is a method for solving the unknowns of the model using integral equations. Usually, the Maxwell's equations in the form of differential equations are converted into integral equations, and then applied the Green's function (Wait, 1962) to obtained the scattering equations (SE). The linear system is generated by the discretization of the SE and the solution to the forward modelling is the solution to the linear system. However, differently from the discretizations of the DE method (FD and FEM), IE only meshes the scattering area, that is the anomalous bodies, instead of the whole computational domain. The efficiency advantage of IE is critical to save the computation time of EM modeling, especially for simple 3D models.

Integral equation method (IE) was first proposed by Hohmann (1971) for solving 2D inhomogenious EM response. And then, Hohmann (1975) developed a volume integral equation method (VIE) based on a hexahedral mesh in order to calculate the 3D induced polarization and EM responses.

2.3.1 VIE

Volume integral equations method (VIE) is a very useful method for simulating 3D EM models. On the basis of the works done by Hohmann (1975), a series of studies about the VIE had been conducted. Ting and Hohmann (1981) used a structured grid to perform 3D MT forward modeling. And then, Hohmann (1983) improved the general 3D IE solution by using the vector-scalar potential method and introducing symmetry through a series of theories. Wannamaker et al. (1984) used IE to simulate 3D EM response in a layered structure. SanFilipo and Hohmann (1985) established a time-domain integral equation for TEM response in a restricted region of half-space electrical conductors with anomalous conductivity.

Because of the limitation of the computation condition at that time, some improved methods were developed. Wannamaker (1991) abandoned the original charge estimation formula (Wannamaker et al., 1986) and utilized the real surface charge with the potential difference. The improvement maintained the internal consistency of the pulse basis function, meanwhile obtained a good approximation result. So that the IE forward modeling of the 3D MT response could further deal with the complex model structure. Differently from SanFilipo and Hohmann (1985), Walker and West (1991) proposed an IE solution that could stably simulate the EM scattering of thin plates. It was suitable for scattering models in fully resistive or conductive medium. The uncertainty of the solution was eliminated by the robustness of the IE solution, so this method had a strong applicability to simulate broadband EM response.

In order to overcome the limitation of discretization cells number caused by the restriction of the computer memory, Xiong (1992) developed a new IE for the simulation of 3D earth conductivity structure. The scattering matrix was divided into multiple sub-matrices, and the block iteration method was used to solve the whole system, which greatly reduced the calculation time of solving matrix equations. Because each sub-matrix was independent of each other, the method also had a potential for parallelization. And then, continued the works of Xiong (1992), Xiong and Tripp (1993) used the spatial homogeneity and the symmetry relationship of the

Green's tensor (Wannamaker et al., 1984) to greatly reduce the computing time.

Actually, due to computer memory level restrictions in the 1980s, Singer & Fainberg (1985) have already proposed an iterative dissipative method (IDM) applied into integral equation based on a contraction operator, which was introduced by Fainberg & Zinger (1980). The IDM was mostly adopted at that time; however, the convergence of the method was slow. Then, Singer (1995) improved it and put forward a modified iterative dissipative method (MIDM). On this basis, Singer (2008) developed a new complex 3D EM fields modeling code, and employed the iterative perturbation approach to generate a series of convergence solutions. The solution optimization in the Krylov subspace significantly reduced the number of iterations, meanwhile weakened the dependence on the lateral contrast of the model, so that the accuracy, robustness and efficiency of the code was ensured.

In addition, combining the MIDM proposed by Singer (1995) and the Krylov subspace iterative solution scheme (Krylov, 1931), Avdeev et al. (1997, 1998, 2002) applied a 3D frequency-domain solution based on the VIE to simulate the response of MT, CSEM, AEM and induction logging. Following these works, Avdeev and Knizhnik (2009) improved the solution for modeling 3D EM fields by using its inherent 3×3 dyadic Green's tensor (Avdeev et al., 1997) separability. The linear dependence on all three dimensions overcame the quadratic dependence of the traditional IE on the size of the model, not requiring the calculation or storage of the entire Green's matrix. Thereby, the improvement significantly reduced the computation load and improved the computation efficiency.

From another perspective of improvement, Farquharson and Oldenburg (2002) studied the application of edge element basis vectors in the IE solution of 3D electromagnetic simulation, and realized the edge-element basis function in the numerical solution of the electric field integral equation. The system of equations was solved by using the Galerkin approach. Later, Farquharson et al. (2006) employed the electric-field VIE to calculate the numerical results of EM response, and implemented the agreement of the numerical modeling and physical scale modeling results.

2.3.2 IE based on a contraction operator

The contraction operator mentioned earlier is from Banach theorem. The theorem allows bounded linear operators defined on a certain vector space to expand to the entire space, and states that there are "sufficient" continuous linear functions. That means if operator is a contraction operator, successive iterations converge. Therefore, the contraction operator will accelerate the convergence of the solution of the integral equation.

By using IE based on a contraction operator, Zhdanov and Fang (1996) presented a new approach called quasi-linear (QL) approximation to solve the EM induction problem. The QL approximation was able to accurately estimate the broadband EM response and had the potential to be applied into the fast 3D EM inversion. Similarly, Zhdanov et al. (2000) proposed a quasi-analytical (QA) approximation method for electromagnetic forward modeling based on the IE of scattering current. The approximate solution was proposed by constructing the quasi-analytical expressions of the anomalous EM fields for 2D and 3D models. The new method adopted iterative methods to extend the quasi-analytic method and developed the approximation into high order, which improved the accuracy. As a result, quasi-analytic series were obtained. The stability and efficiency of the method were guaranteed by the improved accuracy of the simulation and the greatly accelerated convergence speed of the calculation. Hursan and Zhdanov (2002) presented a contraction integral equation (CIE) technique, which replaced the original IE with the modified Green's operator equation. The CIE technique significantly improved the convergence of the iterative method. Later, it was developed into parallel by Čuma et al. (2017), and the fast forward modeling has been implemented to a large extent. In order to improve the validity of IE method for complex model calculation, Zhdanov et al. (2006) developed a new IE method for 3D EM modeling in the complex structures with inhomogeneous background conductivity. The new method overcame the limitation of traditional IE which only used to simulate horizontal layered backgrounds, and improved the calculation accuracy by iterative methods.

2.3.3 Effective 3D numerical solvers

Three-dimensional interpretation of EM data from different sources and scales was increasingly becoming the key to 3D EM data analysis. However, in terms of the computation complexity, accuracy and actual level of spatial detail, 3D EM numerical simulation still existed challenge.

In view of this, using an effective 3D numerical solver was a good solution. Sun and Kuvshinov (2014) proposed a method of Green's function matrix compression based on singular value decomposition (SVD), which was used to accelerate the solution of global geomagnetic induction by the electromagnetic IE forward solver. The method significantly reduced the memory usage and Central Processing Unit (CPU) time of the Krylov subspace iterative solution scheme under the premise of less precision sacrifice. Similarly, Kruglyakov had done some work committing to the optimization of IE forward solver for 3D modeling. Kruglyakov et al. (2016) developed a new open-source 3D MT forward solver based on the CIE method. The solver could accurately calculate the Green's function (Ting and Hohmann, 1981) and its integral, at the same time it could solve high-contrast complex models and support massive parallelization. Furthermore, Kruglyakov and Bloshanskaya (2017) developed a new parallel VIE solver. The Galerkin method was used to ensure the convergence of numerical solutions with high precision, stability and high parallelization. Memory usage was eight times lower than other VIE solvers (Avdeev et al., 1997; Hursan and Zhdanov, 2002). The solver had no additional restrictions on the background media, so that it could achieve non-uniform discretization in any layered background and vertical direction. On this basis, Kruglyakov and Kuvshinov (2018) cooperatively proposed a new 3D numerical solver, which used HOP to improve computation efficiency and greatly reduced the number of unknowns under the premise of ensuring accuracy. The solver sped up the calculation and saved the computing memory significantly.

2.3.4 2.5D problem

Several 2.5D EM forward modeling studies have been mentioned (Li and Key, 2007; Key and Ovall, 2011; Zeng et al., 2018). Something different for IE is that after Fourier transformation in the invariant direction, for each Fourier parameter, the problem is reduced to the problem of solving 2D integral equations (Abubakar et al., 2006b).

Abubakar et al. (2006b) proposed an IE forward algorithm for the solution of 2.5D low-frequency electromagnetic response over the scattering domain. The algorithm employed a standard conjugate gradient normal residual method (CGNR) to solve the linear system of equations and simplified the 3D problem by Fourier transform into solving multiple 2D integral equations, which greatly reduced the computation complexity. Dyatlov et al. (2015) developed and successfully validated a boundary integral equation algorithm based on the four tangential components of the electric field and magnetic field for simulating the response of the LWD EM tool in complex 2D and 3D structures. The Fourier transform simplified the high dimension problems into a series of 1D frequency-independent integral equation, and simultaneously calculated the whole set of measurement points with the same matrix, which greatly shortened the calculation time. The boundary integral equation method avoided the so-called near-offset problem of 2.5D FD simulation with singular sources and had a potential for parallelization. Then, Dyatlov et al. (2017) improved this method in a 2D model with plane boundaries, and calculated the solution of the two-layer model (TLM) corresponding to the nearest boundary by explicit formula. The solutions of the TLM improved the computation accuracy while maintaining the original computation efficiency when the transmitter and the boundary were close. However, when the transmitter was close to the boundary endpoints, the efficiency was very poor. And the anisotropy couldn't be solved. These two points had yet to be further studied.

On the whole, IE is very useful for 3D EM fields simulation. Although its applicability is not as extensive as FEM and FD, it only meshes the scattering anomaly region, which greatly reduces the number of meshes compared with FEM and FD meshing in the whole half space. It reduces the calculation of the unknowns, so that there is a clear advantage in the calculation speed. In addition, since converting the Maxwell's differential equations into the form of the integral equations, in principle, IE has a characteristic of semi-analytical solution, and the solution accuracy is not affected too much by the meshing. However, IE is difficult to handle the complex anisotropy and non-horizontal layered background media but FEM and FD are more suitable for such issues. Moreover, due to the heavy limitation to the accuracy of solving linear equations, most EM software developers avoid using the IE method (Avdeev, 2005). Therefore, taking advantages and drawbacks of these, coupling IE and FEM or FD to form a new hybrid scheme, it can better achieve a relatively balance between the accuracy and the efficiency of forward modeling.

2.4 Hybrid methods

With the further development of FEM, FD and IE, for more complex models and a larger amount of data, a single forward modeling method is not enough for more accurate and efficient simulation. Some hybrid schemes which combined the characteristics and advantages of several methods has been developed.

2.4.1 The solutions to calculate the field boundary

It is well-known that edge-based FEM (or vector FEM) is widely used to solve the Maxwell’s differential equations about the secondary electric fields, however, the approximation on the field boundaries is limited to computational domain. In order to resolve such boundary restriction, Ren et al. (2014b) proposed a hybrid boundary-element finite-element method (BEM-FEM) for goal-oriented adaptive multi-level fast algorithm to simulate the 3D EM induction response of plane waves. This method, which coupled the Galerkin vector FEM (Jin, 2002) method with the point collocation boundary-element method, had the ability to simulate the problems of the large-scale complex earth EM induction, and it was better than the conventional FEM method at high frequency. Differently from the method combined FEM and BEM, Liu et al. (2018a) applied IE method to calculate the boundary values by solving the Green’s functions. They developed a hybrid solver based on IE and boundary-based vector FEM to simulate 3D CSEM model. They applied the vector FEM to solve Maxwell’s differential equation, and calculated the secondary electric field at the receivers by IE, as Figure 6 showed. A more accurate and efficient solution for high conductivity contrast medium was obtained compared to the traditional method.

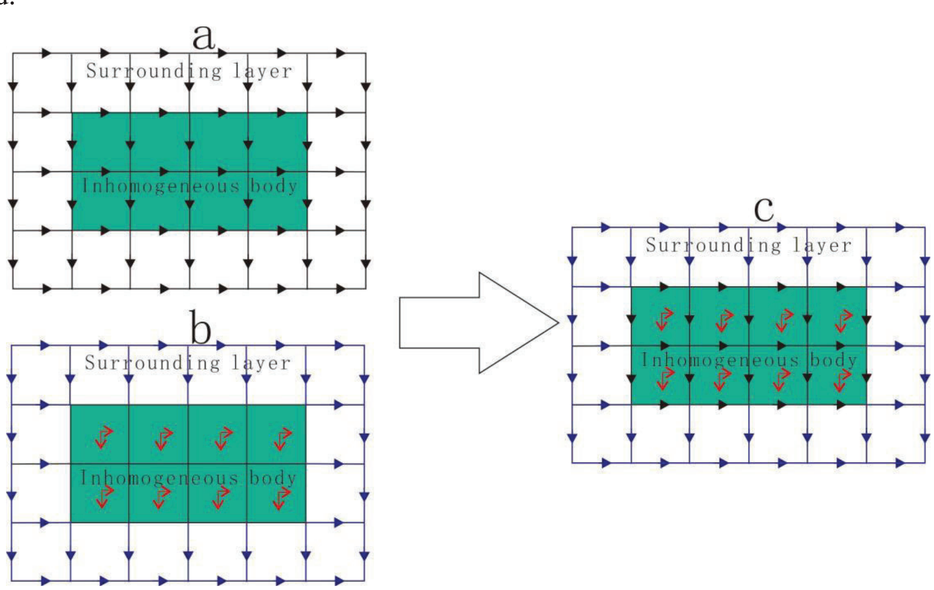

Figure 6. Plane view of the evolution process of the hybrid grid, from the vector FEM and IE grids. (a) The vector FEM grid with electric fields (black arrow lines) defined on grid edges; (b) IE grid with scattering currents (red arrow lines) defined at the

center of inner cells (green color), and boundary electric fields (blue arrow lines) located at the edges of boundary cells (white color); and (c) in the hybrid scheme, the boundary electric fields are given in terms of scattering currents by IE; the scattering current within each cell in turn can be represented by electric fields (black arrow lines) within the cell using the edge-shape function of vector FEM (Liu et al. 2018a).

Nowadays, a coupling method of FEM and infinite element method (IFEM) is still in a research stage in the field of geophysical EM method. IFEM was first proposed by Bettess (1977a) mainly applied in the research of acoustics, electromagnetism, geotechnical mechanics engineering, etc. In the same year, Bettess (1977b) proposed the coupling method of FEM and IFEM. Fu and Wu (2000) introduced IFEM into the geophysical field to deal with the boundary conditions of absorbing elastic waves. IFEM overcame some of the difficulties encountered in conventional absorption techniques, so that it took up less memory space and reduced more computation time. The coupling between FEM and IFEM was achieved by adding an infinite element outside the boundary of the finite element splitting unit and then mapping it to infinity through coordinate mapping to achieve integration of infinity and neglect the boundary condition. Although this hybrid scheme is not very mature in the study of geophysical EM method, it has a good application prospect.

2.4.2 The improvements of the computing accuracy and efficiency

According to the foregoing, the linear system of IE is independent from grid meshing but difficult to settle down the anisotropy. However, FD has the advantage of high-discretization to make up for this deficiency. Based on this thinking, Zaslavsky et al. (2011) proposed a hybrid finite-difference integral equation method (FDIE) for CSEM, single-well and crosswell EM modeling, along with the complex structure and anisotropy. The FDIE overcame the large condition number of the system of the traditional FD and decreased size of the computation domain. The optimization formed from combining with FD homogenization and optimal meshing algorithms was suitable for discretization. In the same idea, Yoon et al. (2016) developed a new hybrid 3D marine CSEM modeling method combining the advantages of FD and IE. Something different from Zaslavsky et al. (2011), the precondition operator for FD solver was replaced with the MUMPS direct solver. And SFD was used to solve the Maxwell's equations in the electric field. The Green's tensor of the corresponding background conductivity model was calculated by IE. The hybrid solution overcame the problem of marine CSEM consuming a lot of time and memory in the case of multiple transmitters and receivers.

As a conclusion, the hybrid scheme is more efficient and accurate than the traditional single method. Compared with the traditional method, the hybrid scheme can save more computation time and memory, and get a faster and more accurate solution. However, there remains many difficulties in the technology of matching hybrid scheme. Although the hybrid scheme is still in the research stage, considering the wide applicability, the high efficiency and the value of these schemes, it will become a main development trend of future EM forward modeling.

2.5 Deep learning

Artificial neural network (ANN) is a research hot spot that has emerged in the field of artificial intelligence since the 1980s. ANN refers to a complex network structure formed by a large number of processing units (neurons) connected to each other. It is a certain abstraction, simplification and simulation of the human brain tissue structure and operating mechanism and it has a strong ability to approximate nonlinear functions. ANN has been applied in geophysical EM problems (Poulton et al., 1992a, 1992b; Poulton & Birken 1998), such as well-log (Huang et al., 1996; Zhang et al., 2002; Maiti and Tiwari, 2010), MT (Zhang and Paulson, 1997; Spichak and Popova, 2000; Manoj and Nagarajan, 2003) and AEM (Seiberl, 1998; Ahl, 2003; Andersen et al., 2016).

Deep learning (DL) can be simply understood as the development of ANN. The concept of DL comes from the research of ANN. Through multi-layer processing, after the initial low-level feature representation is gradually transformed into the high-level feature representation with the high-dimensional data transforming into low-dimensional, the simple models can be used to complete complex classification and other learning tasks (Hinton and Salakhutdinov, 2006). The "deep" of deep learning not only represents the depth of the multilayer neural network structure, but also represents the deep extraction of feature information (LeCun et al., 2015).

The most typical deep learning model is convolutional neural network (CNN) (LeCun et al., 1989) and CNN has been applied in geophysics EM imaging or inversion due to the rapid development nowadays. Puzyrev (2019) used the DL method for EM inversion based on fully CNN for 2.5D inversion and this is the first application of deep CNN to EM inverse problems as we know. Inspired by this, Moghadas (2020) proposed a new method of DL inversion based on CNN, which can estimate the subsurface electrical conductivity layering from electromagnetic induction data. Oh et al. (2019) successfully identified salt bodies from towed streamer EM data with a CNN, and the prediction results demonstrated the applicability of CNN for imaging resistivity from EM data. Haber et al. (2019) trained a VNet CNN architecture to interpret 3D AEM inversions. For imaging subsurface resistivity inversion from 1D AEM data in the frequency domain, Noh et al. (2020) applied the deep neural network (DNN) method and the potential of DNNs for AEM inversion interpretation was validated by comparison with the conventional Gauss-Newton inversion algorithm. Similarly, Li et al. (2020) developed a new fast imaging method of 1D AEM data in the time domain using a long short-term memory (LSTM) DNN (Hochreiter and Schmidhuber, 1997). DL can also eliminate multi-source noise of AEM data (Wu et al., 2020).

In addition, the training synthetic data generation process of the above deep learning is still inseparable from the traditional forward modeling method to solve the PDEs. Puzyrev (2019) used the parallel 3D SFD code based on the curl-curl electric field formulation and Oh et al. (2019) adopted the 2.5D FEM method in the frequency domain proposed by Kang et al. (2012). An efficient and accurate forward algorithm will provide DL with more reliable training data to get more credible prediction results.

Above all, since the method of establishing a nonlinear relationship and solving the gradient is adopted, in a sense, the DL method is similar to inversion in a broad sense. Compared with the iteration of traditional inversion, deep learning uses a multi-layer feature extraction method to maximize the extraction of useful information in the original data, without causing too much accuracy loss under the interference of random noise. Unlike traditional inversion, which requires re-iterative calculations for each inversion, a neural network model trained based on certain research data has a certain degree of commonality for similar survey.

In fact, DL technology also has the potential to simulate the EM field not going through the complex solution of Maxwell's equations. Tang et al. (2017) made an investigation on it. They applied CNN into the simulation of 2D electrostatic problem and the results of the study demonstrated the possibility and by building up a fast FD solver the computing complexity was exactly reduced. In addition, Khan et al. (2019) efficiently estimated the distribution of the magnetic field by using the DL field estimator model learning from the finite-element analysis. Although there is still room for improvement in the structure of the network, the study has reduced calculation time cost and has the advantage of parallelization. Shahriari et al. (2020) examined the potential of DNN that can replace the traditional PDE solution method for forward simulation of borehole resistivity measurements. However, the paper also pointed out that it requires a sufficiently large data set to produce a reasonably accurate forward function and the application still faces many difficulties and challenges.

At present, there are very few related researches, exclusively in the exploratory research stage. The existing data-driven DL forward modeling requires the use of traditional forward solvers, such as FD (Tang et al., 2017; Shahriari et al., 2020) or FE (Khan et al., 2019), to make a training data set. The difference from DL "inversion" is that DL "forward modeling" uses the EM models as the input of the neural network, and the output is the responses. Therefore, for DL, there is no distinction between forward and inverse problems, only training, validation and testing, and the intermediate processes are similar. For neural networks, there is only the difference between input and output, or known and unknown. In other words, it depends on what kind of task we want to accomplish in order to achieve what kind of goal.

However, there are still two problems to be solved. One is that the essential feature of DL is data-driven. The realization of DL is based on training with a large amount of data. To a certain extent, the more training data, the more adequate the training of the neural network, and then the more accurate prediction results will be. How to obtain adequate training data or how to deal with the problem of insufficient training due to inadequate sample size? The other is the applicability of network model to EM modeling. What kind of network structure is more conducive to learning and predicting the realistic EM field distribution? And for EM modeling, the aforementioned DL methods for PDEs are all solutions to 1D or 2D problems, how to achieve 3D modeling? Although DL methods seem to have become a hot spot for geophysical applications in the past five years mentioned in Section 1, these issues need to be further studied in the future.

3. Discussions

Based on the collation and summary of all corresponding references involved, we put forward a Qualitative Evaluation Model named STAMP Model, which is shown in Figure 7. Storage, time, accuracy, model complexity and parallelization these five criteria are used to evaluate the advantages and disadvantages of the forward modeling methods. We divided the five key criteria into five levels from 1 to 5. When the value of level is higher, they respectively represent less computing memory, shorter computing time, higher calculation accuracy, higher model complexity and higher degree of parallelization.

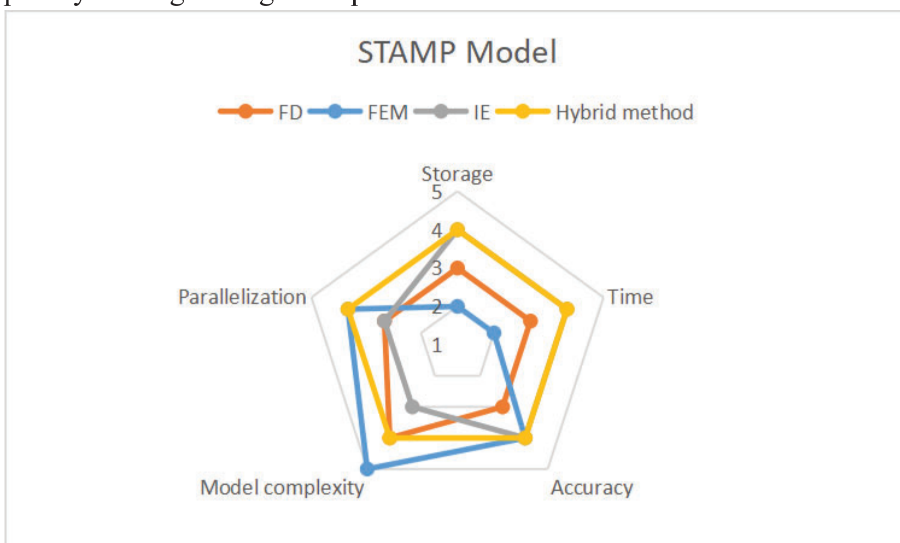

Figure 7. STAMP Model for modeling evaluation.

FD is an efficient tool to solve the EM modeling with simple implementation (Streich, 2009; Yavich and Zhdanov, 2016). It can handle the discontinuity of the magnetic field and electric field caused by the electromagnetic difference in the internal medium very well, because of the characteristics of the staggered grid (Yee, 1966; Smith, 1996a). However, the regular structure mesh of the model restricts the application of FD in complex geophysical models, which also affects its calculation accuracy (Key and Weiss, 2006; Key and Ovall, 2011).

FEM is the most flexible for simulating complex and large-scale geometry models with high computation accuracy (Avdeev, 2005; Börner, 2010). However, The number of unknowns is often on the order of millions and it performs with higher computer memory and computational cost (Puzyrev et al., 2013; Ren et al., 2014a). The flexibility of unstructured grids, such as tetrahedral or hexahedral grid, improves the calculation accuracy of complex geoelectric structures to a certain extent. The accuracy of the finite element depends on the size of the element and the order of the shape function (Jin, 2002). Unfortunately, it still lacks the analytic solutions to 3D problems (Smith, 1996a).

IE is suitable for simple 3D models in a layered earth, which only needs to discretize the computational area within the range of the scattering anomaly resulting in small system metrices, so that it takes up less computing memory and has higher efficiency (Hohmann, 1971; Avdeev et al., 2002). Due to the numerical results of IE have the accuracy of a semi-analytical solution, IE is often used to test the accuracy of newly developed algorithms (Ren and Tang, 2010). However, as the size of the model becomes larger and the complexity increases, the computational efficiency of IE will be greatly reduced (Mackie et al., 1994). And the accuracy of the solution is heavily dependent on the accuracy of the complicated and time-consuming system matrix which is an extremely tedious and nontrivial problem itself (Avdeev, 2005). These drawbacks limit the solution of IE to solve complex EM models, especially the complex, high-contrast inhomogeneous anisotropic medium (Zaslavsky et al., 2011).

Hybrid method has the advantage to solve some special problems, because it combines the advantages from different modeling methods. Hybrid methods are indeed effective strategies for improvements (Zaslavsky et al., 2011; Ren et al., 2014b; Yoon et al., 2016; Liu et al., 2018a). Given that, in our STAMP Model, the hybrid method maybe has a balance between accuracy and efficiency.

It is important to point out that for the criteria of parallelization, the specific situation requires specific analysis, such as utilizing a direct solver or iterative solver and computing by single-core or multi-core processing cards. It depends on the models of parallelization such as shared memory or distributed memory and multithreading or multiprocessing. So, it is very difficult to quantitatively analyze the degree of parallelization of FD, FEM and IE. However, from the perspective of the algorithm itself, FEM is suitable for complex and large-scale models and has the characteristics of high accuracy but large memory usage and time-consuming calculations. In this case, the parallelization scheme is more conducive to the balance of high precision and high efficiency. In addition, according to the literature citations in this review, researchers indeed have more research on parallel FEM than FD or IE. Therefore, a qualitative comparison was given that the degree of the parallelization of FEM is higher than FD and IE.

DL is not included in the STAMP model for comparison with other methods, because the current researches are not enough to explain its advantages and disadvantages with traditional forward modeling methods in geophysical EM. As far as we know, in modern deep learning, the number of parameters is increasing, and the data sets are getting larger, so that it is difficult to load all the data sets into the memory. To train a complex deep learning model on a larger data set, machine learning on a single node takes too long, and multi-node parallel computing has to be used. Furthermore, according to the current research situation, the introduced DL method may provide high speed to compute the model. The complexity of the training model and the accuracy of the prediction model need to be improved.

4. Conclusions

In this paper, we review the mainly simulation method in EM field modeling. Three most widely employed methods in EM modeling include FEM, FD and IE method. Based on the published 195 papers, we summarized the advantages and disadvantages of these modeling methods. It is complex to judge which is the best modeling method, due to different applications. So, we proposed the STAMP Model for qualitative evaluations of FD, FEM, IE, and hybrid methods. We also reviewed and discussed the application of DL in geophysical EM forward and inversion problems.

Above all, the EM field simulation methods have been developed to solve the different problems in high-dimensional, complex geometry model, high accuracy, high computational speed. And in the future, the EM modeling research will focus on the high accuracy and low computational cost solutions in large-scale, high-dimensional and anisotropic medium combining HPC and artificial intelligence.

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Data availability Statement: 'Not Applicable'.
This review includes the number of the EM publication could be found by Google.

Appendix A

Finite difference solution of Maxwell equations: taking the MT for the isotropic media as an example

First, we discrete the research area. A series of parallel planes are used to divide the research area into several small rectangular cells at different distances along the X, Y, and Z axis directions. Assuming that they are divided into N_x, N_y, and N_z segments along the X, Y, and Z axis directions, respectively.

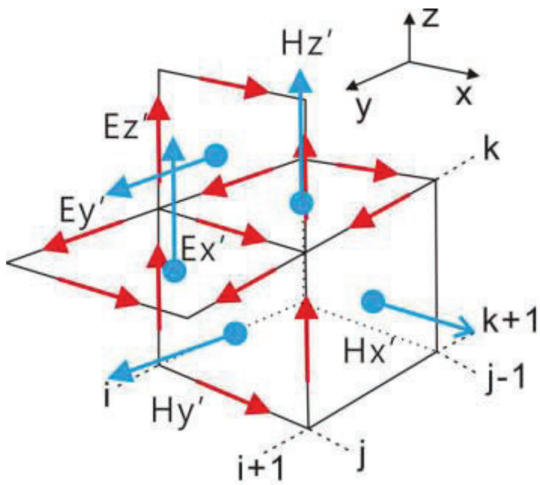

Figure 8. The staggered grid.

We use the staggered grid in Figure 2 (a). Figure 8 shows the grid element numbered (i, j, k). The value of resistivity is ρ (i, j, k). Its length, width and height are respectively Δx_i ($i = 1, 2, ..., N_x$), Δy_j ($j = 1, 2, ..., N_y$) and Δz ($k = 1, 2, ..., N_z$). And we define $E_x^{'}$, $E_y^{'}$, $E_z^{'}$ as the average value of the electric field at the central point of the edge of the grid cell; $H_x^{'}$, $H_y^{'}$, $H_z^{'}$ respectively represent the average value of the magnetic field at the central point of the plane of the grid cell.

According to the theory of the MT method, under the quasi-static condition, the integral form of Maxwell's equations in the frequency domain is as follows.

$$\begin{aligned}
&\oint_l \mathbf{E} dl = i\omega\mu \iint_S \mathbf{H} dS, \\
&\oint_l \mathbf{H} dl = \iint_S \sigma \mathbf{E} dS, \\
&\oint_l \mathbf{H} dS = 0, \\
&\oint_l \mathbf{E} dS = 0,
\end{aligned} \qquad (1)$$

Where **E** is the electric field strength, **H** is the magnetic field strength, σ is the conductivity, i is the imaginary unit, ω is the circular frequency, μ is the permeability of underground media, *dl* is the enclosure line of the enclosed integration, and *dS* is the area contained in the enclosure line.

Discretization of equation (1) can be obtained as follows. μ is approximated to μ_0 ($= 4\pi \times 10^{-7}$ H/m), which is the magnetic permeability for free space.

$$\begin{cases} \left[E_z(i,j,k)-E_z(i,j-1,k)\right]\bullet\frac{\Delta z_{k-1}+\Delta z_k}{2}-\left[E_y(i,j,k)-E_y(i,j,k-1)\right]\bullet\frac{\Delta y_{j-1}+\Delta y_j}{2} \\ =i\mu_0\omega H_x(i,j,k)\bullet\frac{\Delta z_{k-1}+\Delta z_k}{2}\bullet\frac{\Delta y_{j-1}+\Delta y_j}{2}, \\ \left[E_x(i,j,k)-E_x(i,j,k-1)\right]\bullet\frac{\Delta x_{i-1}+\Delta x_i}{2}-\left[E_z(i,j,k)-E_z(i-1,j,k)\right]\bullet\frac{\Delta z_{k-1}+\Delta z_k}{2} \\ =i\mu_0\omega H_y(i,j,k)\bullet\frac{\Delta x_{i-1}+\Delta x_i}{2}\bullet\frac{\Delta z_{k-1}+\Delta z_k}{2}, \\ \left[E_y(i,j,k)-E_y(i-1,j,k)\right]\bullet\frac{\Delta y_{j-1}+\Delta y_j}{2}-\left[E_x(i,j,k)-E_x(i,j-1,k)\right]\bullet\frac{\Delta x_{i-1}+\Delta x_i}{2} \\ =i\mu_0\omega H_z(i,j,k)\bullet\frac{\Delta x_{i-1}+\Delta x_i}{2}\bullet\frac{\Delta y_{j-1}+\Delta y_j}{2}, \end{cases} \quad (2)$$

$$\begin{cases} \left[H_z(i,j+1,k)-H_z(i,j,k)\right]\bullet\Delta z_k-\left[H_y(i,j,k+1)-H_y(i,j,k)\right]\bullet\Delta y_j=\sigma_x(i,j,k)E_x(i,j,k)\bullet\Delta y_j\bullet\Delta z_k, \\ \left[H_x(i,j,k+1)-H_x(i,j,k)\right]\bullet\Delta x_i-\left[H_z(i+1,j,k)-H_z(i,j,k)\right]\bullet\Delta z_k=\sigma_y(i,j,k)E_y(i,j,k)\bullet\Delta x_i\bullet\Delta z_k, \\ \left[H_y(i+1,j,k)-H_y(i,j,k)\right]\bullet\Delta y_j-\left[H_x(i,j+1,k)-H_x(i,j,k)\right]\bullet\Delta x_i=\sigma_z(i,j,k)E_z(i,j,k)\bullet\Delta x_i\bullet\Delta y_j, \end{cases} \quad (3)$$

Where σ_x, σ_y, σ_z are respectively the conductivities in X, Y and Z directions. The expressions are as follows.

$$\begin{cases} \sigma_x(i,j,k)=\frac{1}{\rho_x(i,j,k)}=\frac{\Delta x_i+\Delta x_{i-1}}{\rho(i,j,k)\Delta x_i+\rho(i-1,j,k)\Delta x_{i-1}}, \\ \sigma_y(i,j,k)=\frac{1}{\rho_y(i,j,k)}=\frac{\Delta y_j+\Delta y_{j-1}}{\rho(i,j,k)\Delta y_j+\rho(i,j-1,k)\Delta y_{j-1}}, \\ \sigma_z(i,j,k)=\frac{1}{\rho_z(i,j,k)}=\frac{\Delta z_k+\Delta z_{k-1}}{\rho(i,j,k)\Delta z_k+\rho(i,j,k-1)\Delta z_{k-1}}, \end{cases} \quad (4)$$

After simultaneous polynomials and elimination, the system of linear equations for the electric field component or the magnetic field component can be obtained as follows.

$$\mathbf{Ax}=\mathbf{b}$$

Then, we can solve the system of linear equations by using various methods, including direct solver, LU decomposition, the Krylov subspace iterative solution and so on.

Appendix B

Finite element solution of Maxwell equations: taking the the electric field of MT for the isotropic media as an example

According to the theory of the MT method, under the quasi-static condition, the differential form of Maxwell's equations in the frequency domain is as follows.

$$\begin{cases} \nabla \times \mathbf{E} = -i\omega\mu\mathbf{H}, \\ \nabla \times \mathbf{H} = \sigma\mathbf{E}, \\ \nabla \bullet \mathbf{H} = 0, \\ \nabla \bullet \mathbf{E} = 0, \end{cases} \quad (5)$$

Where **E** is the electric field strength, **H** is the magnetic field strength, σ is the conductivity, i is the imaginary unit, ω is the circular frequency, and μ is the permeability of underground media.

We simultaneously calculate the curl of both sides of the first formula in the system of equation (5), and combining the second formula, the solution can be obtained as follows.

$$\nabla \times \nabla \times \mathbf{E} + i\omega\mu_0\sigma\mathbf{E} = 0 \; (6)$$

We use Galerkin method (Jin, 2002) to derive the system of equation (6).

And we use the simple Dirichlet boundary conditions (Nam et al., 2007).

$$\mathbf{E} \times \mathbf{n} = \mathbf{E}_o \times \mathbf{n} \; (7)$$

Where **n** is the outer unit normal vector of the outer boundary, $\mathbf{E_o}$ is the given known electric field strength on the outer boundary

Equations (6) and (7) are the boundary value problems for MT forward modeling. Using the vector formula $\mathbf{B} \bullet (\nabla \times \mathbf{A}) = \mathbf{A} \bullet (\nabla \times \mathbf{B}) + \nabla \bullet (\mathbf{A} \times \mathbf{B})$ and Green's integral equation (Nam et al., 2007), we obtain the corresponding variation expression by combing equation (6) and (7).

$$b(\mathbf{E}, \mathbf{V}) = f(\mathbf{V}), \mathbf{V} \in H(curl) \; (8)$$

Where $H(curl) = \{\mathbf{V} \,\|\, \mathbf{V} \in L_2(\Omega), \mathbf{V} \times \mathbf{n} = \mathbf{n} \times \mathbf{V}_o\}$, $L_2(\Omega)$ is the second-order derivative continuous function space, **n** is the unit normal vector on outer boundary, b and f are expressed as:

$$b = \int_\Omega (\nabla \times \mathbf{E} \cdot \nabla \times \mathbf{V} - i\omega\mu\sigma\mathbf{E} \cdot \mathbf{V}) d\Omega \; (9)$$

$$f = \int_{\partial\Omega} V \cdot E_o d\Gamma \; (10)$$

We use the vector FEM to solve the electric field distribution represented by equation (6) and use unstructured tetrahedral element grid for spatial discretization. The numbering rules of the edges of the tetrahedral elements are shown in the Figure 9.

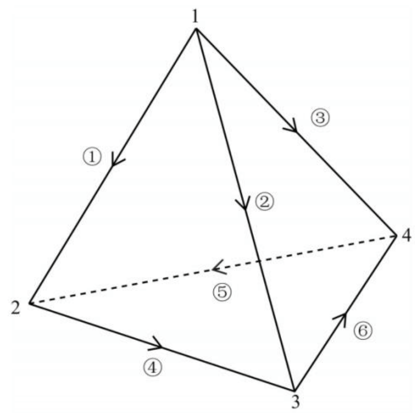

Figure 9. Edge defination of tetrahedal element.

For any tetrahedral element e, approximate expression of the electric field in this element is as follows.

$$\mathbf{E}^e = \sum_{i=1}^{n} \mathbf{N}_i^e E_i^e \quad (11)$$

Where n=6, which is the numbers of edge in each element, E_i is the tangential electric field on the *i*th edge of the *i*th element, and $\mathbf{N_i}$ is the vector shape function on the *i*th edge of the *i*th element (Jin, 2002). We designate the *i*th edge vector shape function as:

$$N_i^e = l_i^e \left(L_{i1}^e \nabla L_{i2}^e - L_{i2}^e \nabla L_{i1}^e \right) (12)$$

where l_i^e is the length of *i*th edge. L_{i1}^e and L_{i2}^e are the origin node shape functions defined at each node of the *i*th edge.

Then, substituting the equation (11) and (12) into the equation (9), we can get a large complex linear matrix equation

$$\mathbf{KU} = \mathbf{F}$$

Where **K** is a stiffness matrix, which reflects the topological relationship between grid nodes, **U** is a matrix of electric field vectors to be calculate at all nodes and **F** is a mass matrix and only the edge on the outer Dirichlet boundary is not zero. K and F can be:

$$\begin{aligned} K_{ij}^e &= \int_{\Omega^e} \nabla \times N_i \cdot \nabla \times N_j - i\omega\mu\sigma N_i N_j d\Omega \\ F_{ij}^e &= \int_{\partial L^e} N_i \cdot E_o d\Gamma \end{aligned} \quad (13)$$

Appendix C

Integral equation solution of Maxwell equations: taking the the electric field of MT for the isotropic media as an example

According to the EM theory, the EM field at any point in the model can be expressed as the superposition of the incident fields $\mathbf{E_i}$ ($\mathbf{H_i}$) and scattered fields $\mathbf{E_s}$ ($\mathbf{H_s}$)

$$\begin{cases}\mathbf{E}=\mathbf{E}_i+\mathbf{E}_s\\ \mathbf{H}=\mathbf{H}_i+\mathbf{H}_s\end{cases} \quad (14)$$

The incident field is generated by the incident source in the layered ground, while the scattered field is caused by the difference in conductivity $\Delta\sigma=\sigma_b-\sigma_s$ between the anomalous body and the layered ground.

According to the Green's Function Theory (Ting and Hohmann, 1981; Wannamaker et al., 1984), the scattering field $\mathbf{E_s}$ can be written as

$$\mathbf{E_s}(r)=\oiiint_{\Omega}\mathbf{G^{JE}}(\mathbf{r},\mathbf{r'})\bullet\Delta\sigma(\mathbf{r'})\bullet\mathbf{E}(\mathbf{r'})dV \quad (10)$$

Where $\mathbf{G^{JE}}$ is the Green's Function of the electric field, Ω is the area where the anomaly is located. Substituting the equation (10) into the the first formula in the system of equation (9), we can obtain

$$\mathbf{E}(\mathbf{r})=\mathbf{E}_i(\mathbf{r})+\oiiint_{\Omega}\mathbf{G^{JE}}\bullet\Delta\sigma(\mathbf{r'})\bullet\mathbf{E}(\mathbf{r'})dV$$

We discretized the anomalous body into several small volume units Ω_i (i = 1, 2, ..., N). Assuming that the electric field and conductivity in each cell are constant and equal to the value of the cell center r_i, we can get a discretized matrix equation

$$\mathbf{A}\bullet\mathbf{E}=\mathbf{E}_i \quad (11)$$

Where $\mathbf{E}=[\mathbf{E}(\mathbf{r_1}),\mathbf{E}(\mathbf{r_2}),...,\mathbf{E}(\mathbf{r_M})]^T$ and $\mathbf{E}_i=[\mathbf{E}_i(\mathbf{r_1}),\mathbf{E}_i(\mathbf{r_2}),...,\mathbf{E}_i(\mathbf{r_M})]^T$ are respectively the 3M order vectors consisting of the unknown electric field and the incident electric field at the midpoint of each discrete unit. **A** is a 3M×3M square matrix, whose elements are composed of the following series of 3×3 sub-matrices

$$A_{\alpha\beta}=I\delta_{\alpha\beta}-\Delta\sigma(r_\beta)\bullet\oiiint_{\Omega_\beta}G^{JE}(r_\alpha.r')dv$$

Where α, β = 1, 2, ..., M and $\delta_{\alpha\beta}=\begin{cases}1, & \alpha=\beta\\ 0, & \alpha\neq\beta\end{cases}$. Solving the equation (11) can get the electric field on each discrete element of the anomaly and then substituting the solution into equation (10), the scattered field distribution at any position in space can be obtained.

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