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Non-linear cross-sectional analysis of concrete shells

Trondheim, June 2021

Master's thesis

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Non-linear cross-sectional analysis of concrete shells

Ikke-lineær tverrsnittsberegning av betongskall

BY:

Micael Mebrahtu Hailemicael

SUMMARY:

This thesis deals with the development of a computer program that implements the iteration method. The iteration method is a non-linear numerical method used to calculate the capacity of reinforced concrete shells. A user manual is developed to make the program more accessible to users.

The theory behind the iteration method and its derivation are presented. Moreover, a detailed study of the materials used in a reinforced concrete shell (reinforcement steel and concrete) and corresponding material models is conducted. The choice of material models has a considerable impact on the results of the computer program. The iteration method procedure is then further developed to expand its application to calculate beams and columns.

The primary purpose of the thesis is to develop a user-friendly computer program that uses the iteration method correctly in the calculation of reinforced concrete shells, beams, and columns.

To ensure that the program gives correct results, results obtained by the program are compared to results from hand calculations and an approved computer program. There are, in some cases, relatively small differences, but they can be explained by the fact that the iteration method is an approximation and not 100% accurate. The comparisons show that the results from the program are consistent with the hand calculations and the approved computer program.

RESPONSIBLE TEACHER: Professor Jan Arve Øverli

SUPERVISOR: Professor Jan Arve Øverli

CARRIED OUT AT: The Department of Structural Engineering, NTNU

Abstract

This thesis deals with the development of a computer program that implements the iteration method. The iteration method is a non-linear numerical method used to calculate the capacity of reinforced concrete shells. A user manual is prepared to make the program more accessible to users.

The theory behind the iteration method and its derivation are presented. Moreover, a detailed study of the materials used in a reinforced concrete shell (reinforcement steel and concrete) and corresponding material models is conducted. The choice of material models has a considerable impact on the results of the computer program. The iteration method procedure is then further developed to expand its application to calculate beams and columns.

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Sammendrag

Denne oppgaven omhandler å utvikle et dataprogram som iverksetter iterasjonsmetoden. Iterasjonsmetoden er en ikke-linear numerisk beregningsmetode som beregner kapasiteten i armerte betongskall. For at det skal være enkelt å bruke programmet, en brukermanual er laget.

Teorien og derivasjon av iterasjonsmetoden er først presentert. Dessuten, er det tatt en gjennomgang av materialer brukt in armert betongskall (armering og betong) og tilsvarende materialmodellene er utført. Valget av materialmodeller har en stor innvirkning på resultatet av dataprogrammet. Iterasjonsmetoden er dermed utviklet videre for å utvidet den til beregning av bjelker og søyler.

Hovedhensikten med oppgaven er å lage og utvikle et brukervennlig dataprogram som regner riktig armerte betongskall, -bjelke og -søyle, i henhold til iterasjonsmetoden.

For å forsikre at dataprogrammet regner riktig, resultater hentet fra dataprogrammet er sammenlignet med resultater fra håndberegninger og et godkjent dataprogram. Det finnes, i noen tilfeller, relativt lite avvik, men disse kan forklares med at iterasjonsmetoden er en tilnærming og ikke 100% nøyaktig. Sammenligningene viser at resultatene fra dataprogrammet er i samsvar med resultatene fra håndberegninger og det godkjente dataprogrammet.

Preface

This master thesis is written at the Department of Structural Engineering, Norwegian University of Science and Technology (NTNU). The thesis accounts for 30 credit points and is conducted during the spring semester of 2021.

Working on this thesis has been an interesting and instructive process. It has given me a better understanding of the capacity control of reinforced concrete shells and the development of computer programs.

On this occasion, I would like to thank my supervisor Professor Jan Arve Øverli, for his guidance throughout the entire process.

Trondheim, 10. June 2021

Micael Mebrahtu Hailemicael

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List of Symbols

Capital Latin letters

Thesis	**Computer Program**	**Description**
A_i	Ai	Matrix that transforms middle plane strains and curvature to concrete layer strains
A_{sxj}	Asxj	Matrix that transforms middle plane strains and curvatures to reinforcement layer strain in x-direction
A_{syj}	Asyj	Matrix that transforms middle plane strains and curvatures to reinforcement layer strain in y-direction
A_{sx1}	Asx1	Reinforcement in x-direction, bottom layer
A_{sx2}	Asx2	Reinforcement in x-direction, top layer
A_{sy1}	Asy1	Reinforcement in y-direction, bottom layer
A_{sy2}	Asy2	Reinforcement in y-direction, top layer

Thesis	Computer Program	Description
C_0	Co	Initial material matrix of concrete layers
C_i	Ci	Material matrix of concrete layers
C_{sx0}	Csx0	Initial material matrix of reinforcement layers is x-direction
C_{sxj}	Csyj	Material matrix of reinforcement layers in x-direction
C_{sy0}	Csy0	Initial material matrix of reinforcement layers is y-direction
C_{syj}	Csyj	Material matrix of reinforcement layers in y-direction
E_{11}	E11	Secant modulus in principal direction 1
E_{12}	E12	Secant modulus in direction 1-2
E_{22}	E22	Secant modulus in principal direction 2
E_{cd}	Ecd	Design elasticity modulus of concrete
E_{cm}	Ecm	Initial secant modulus for concrete
E_{sx1}	Esx1	Elasticity modulus for reinforcement in x-direction, bottom layer
E_{sx2}	Esx2	Elasticity modulus for reinforcement in x-direction, top layer
E_{sy1}	Esy1	Elasticity modulus for reinforcement in y-direction, bottom layer
E_{sy2}	Esy2	Elasticity modulus for reinforcement in y-direction, top layer
K	K	Stiffness matrix of the section
K_c	Kc	Stiffness matrix for concrete
K_{c0}	Kc0	Initial stiffness matrix for concrete
K_{s0}	Ks0	Initial stiffness matrix for reinforcement
K_s	Ks	Stiffness matrix for reinforcement
M_x	mx	Bending moment in x-direction
M_{xy}	mxy	Bending moment in xy-direction
M_y	my	Bending moment in y-direction
N_x	mx	Membrane force in x-direction
N_{xy}	mxy	Membrane force in xy-direction
N_y	ny	Membrane force in y-direction
R	R	External force vector
S	S	Internal force vector
S_c	Sc	Concrete internal force vector
S_s	Ss	Reinforcement internal force vector
T(θ)	Tepsci	Transformation matrix for concrete layers
V_x		Shear force in x-direction
V_y		Shear force in y-direction
W_e		External virtual work
W_i		Internal virtual work

Small Latin letters

Thesis	Computer Program	Description
a		Dimension of the shell element
b	b	Width of the section
c_1	c1	Cover, distance from the bottom edge to between A_{sx1} and A_{sy1}
c_2	c2	Cover, distance from the top edge to between A_{sx2} and A_{sy2}

Thesis	Computer Program	Description
c_{x1}	cx1	Cover, distance between the bottom edge and the center of bottom reinforcement in x-direction
c_{x2}	cx2	Cover, distance between the top edge and the center of top reinforcement in x-direction
c_{y1}	cy1	Cover, distance between the bottom edge and the center of bottom reinforcement in y-direction
c_{y2}	cy2	Cover, distance between the top edge and the center of top reinforcement in y-direction
f_{cd}	fcd	Concrete design compressive strength
f_{ck}	fck	Concrete characteristic cylinder compressive strength
f_{cm}	fcm	Concrete mean compressive strength at 28 days
f_{yd}	fyd	Reinforcement design yield strength
f_{yk}	fyk	Reinforcement characteristic yield strength
h	h	Height of the section
n	n	Number of concrete layers in the section
r_x		Generalized displacement in x-direction
r_y		Generalized displacement in y-direction
r_{xy}		Generalized displacement in xy-direction
z_i	zc	Distance from the shell section mid-plane to mid-plane of concrete layer
z_j	zs	Distance from the shell section mid-plane to the reinforcement layer

Small Greek letters

Thesis	Computer Program	Description
α_1	alfa1	angle between x-axis and neutral axis in column
α_2	alfa2	Angle between y-axis and neutral axis in column
α_3	alfa3	Angle between diagonal and x-axis in column
β	beta	Convergence criterium for the iteration method
ε_{c1}		Compressive strain at peak stress (non-linear concrete model)
ε_{c2}	epsc2	Compressive strain at reaching the maximum strength (parabola-rectangle concrete model)
ε_{c3}	epsc3	Compressive strain at reaching the maximum strength (bilinear concrete model)
ε_{cu1}		Compressive nominal ultimate strain(non-linear concrete model)
ε_{cu2}	epscu2	Compressive ultimate strain(parabola-rectangle concrete model)
ε_{cu3}	epscu3	Compressive ultimate strain(bilinear concrete model)
$\boldsymbol{\varepsilon_p}$	epspci	Strain in principal directions matrix
$\boldsymbol{\varepsilon_t}$	epst	Vector with strains and curvatures of the middle plane of the shell element

Thesis	Computer Program	Description
ε_{ud}	epsud	Reinforcement design yield strain
ε_{x}	epscxi	Strain in x-direction
ε_{xm}		Strain in the middle plane of the shell element in x-direction
ε_{ym}		Strain in the middle plane of the shell element in y-direction
ε_{xym}		Strain in the middle plane of the shell element in xy-direction
ε_{y}	epscyi	Strain in y-direction
γ_{xy}	gammacxyi	Shear strain
κ_{x}		Curvature of the middle plane of the shell element in x-direction
κ_{y}		Curvature of the middle plane of the shell element in y-direction
κ_{xy}		Curvature of the middle plane of the shell element in xy-direction
ν	vc	Concrete Poisson's ratio
σ_{ci}		Concrete stress in layer i
$\boldsymbol{\sigma_{p}}$	sigpci	Stress in principal directions matrix
σ_{sxj}		x-direction reinforcement stress in layer j
σ_{syj}		x-direction reinforcement stress in layer j
θ	thetaci	Angle between concrete layer local direction and the global direction

1 Introduction

Concrete shells are structural constructions that can be structurally and economically effective as well as architecturally attractive. Since a shell element is subjected to both normal forces and moments in two directions, it is difficult and unpractical to calculate its capacity by hand. Therefore capacity control methods and algorithms are implemented to calculate it.

The thesis aims to develop a user-friendly computer program to calculate the capacity of a shell section subjected to membrane forces and bending moments. The capacity control is implemented by the iteration method, a non-linear numerical method that analyses a shell section's capacity. The iteration method is further expanded to calculating the capacity control of beams and columns.

The primary workload in the thesis preparation is to understand the iteration method in the calculation of reinforced concrete shells, beams, and columns and then implement it in a computer program by using the programming language Python. The program is then tested, and at last, a user manual is prepared.

The thesis consists of five chapters:

1. Introduction: The background, objective, and structure of the thesis are presented.

2. Theory: The technical description of shells, material models of concrete and reinforcement, methods for designing and calculating reinforced concrete shells, and extension of the iteration method to beams and columns are presented.

3. Computer Program: The computer program is described in detail, and the user manual for the program is presented.

4. Verification: The computer program is run, and its results are compared to examples with known results.

5. Conclusion: The results obtained in the previous chapter are summarized, and a list of proposals for further development of the calculation program is presented.

In the Appendix, derivation of the formulas used in calculations, hand calculation of the examples used in testing are presented.

2 Theory

Shells are defined as elements subjected to both membrane and bending forces and can be plane or curved with respect to either one or two directions.

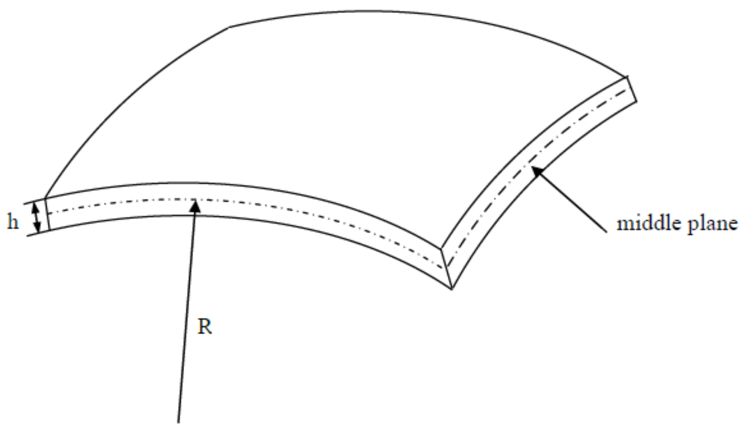

Figure 2.1: Middle plane, curvature radius and thickness of a thin shell [1]

The classical thin shell theory, Love-Kirchoff theory, is based on the following assumptions[1]:

- The shell thickness is considerably smaller compared to its other dimensions and its radius of curvature.
- Plane sections normal to the shell mid-surface prior to deformation remain plane and perpendicular and perpendicular to the deformed mid-surface.
- Stresses normal to the shell mid-surface are negligible.
- Strains and stresses are small.

2.1 Material Models

Reinforced concrete shells consist of concrete and reinforcement steel. Both concrete and reinforcement steel have non-linear strain-stress relations. However, Eurocode 2 (EC2) allows the use of simplified material models, which can be found in EC2-3[2].

2.1.1 Concrete

In the standard EC2, three strain-stress relation models for concrete are presented. These are:

- Non-linear model EC2-3.1.5
- Idealized parabola-rectangle model EC2-3.1.7(1)
- Bilinear model EC2-3.1.7(2).

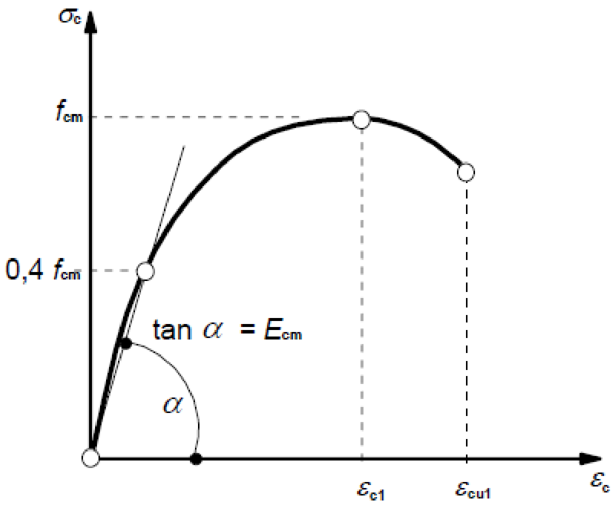

Figure 2.2: Non-linear concrete model [2]

The non-linear model is shown in Figure 2.2, and the following formulas represent the strain-stress relation:

$$\frac{\sigma_c}{f_{cm}} = \frac{k\eta - \eta^2}{1 + (k-2)\eta} \qquad \text{for} \qquad 0 < |\varepsilon_c| < |\varepsilon_{cu1}| \qquad (2.1.1)$$

$$\eta = \varepsilon_c / \varepsilon_{c1} \qquad (2.1.2)$$

$$k = 1.05 E_{cm} \cdot |\varepsilon_{c1}| / f_{cm} \qquad (2.1.3)$$

Where:

f_{cm} : mean compressive strength at 28 days

E_{cm} : modulus of elasticity of concrete

ε_{c1} : strain at peak stress

ε_{cu1} : nominal ultimate strain

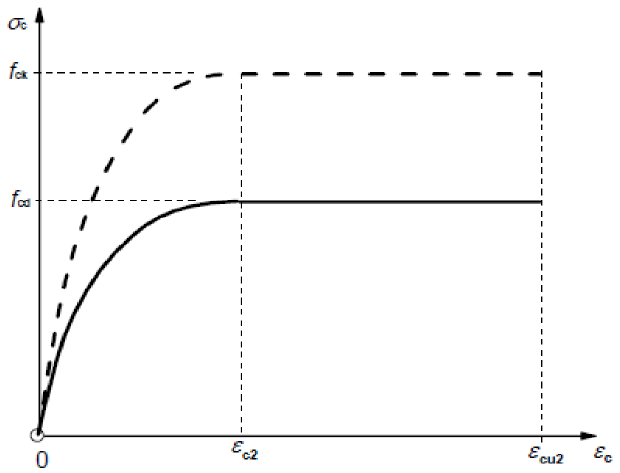

Figure 2.3: Parabola-rectangle diagram for concrete under compression [2]

The idealized parabola-rectangle model is shown in Figure 2.3, and the following formulas represent the strain-stress relation:

$$\sigma_c = f_{cd}\left[1-\left(1-\frac{\varepsilon_c}{\varepsilon_{c2}}\right)^n\right] \quad \text{for} \quad 0 \le \varepsilon_c \le \varepsilon_{c2} \tag{2.1.4}$$

$$\sigma_c = f_{cd} \quad \text{for} \quad \varepsilon_{c2} \le \varepsilon_c \le \varepsilon_{cu2} \tag{2.1.5}$$

Where:

f_{cd} : design compressive strength

ε_{c2} : strain at reaching the maximum strength

ε_{cu2} : ultimate strain

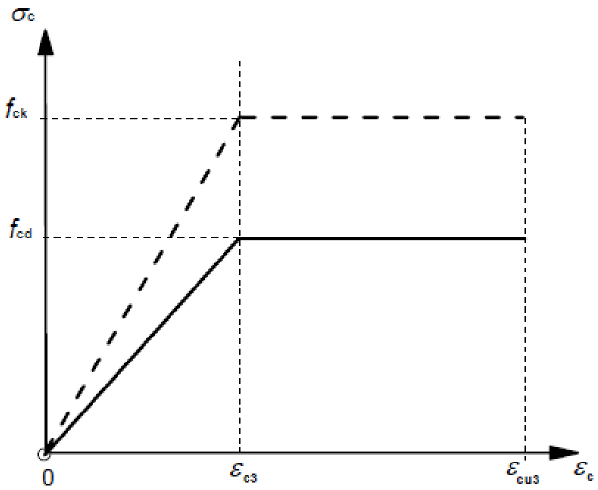

Figure 2.4: Bilinear diagram for concrete under compression [2]

The bilinear model is shown in Figure 2.4, and the following formulas represent the strain-stress relation:

$$\sigma_c = f_{cd} \bullet \frac{\varepsilon_c}{\varepsilon_{c3}} \quad \text{for} \quad 0 \le \varepsilon_c \le \varepsilon_{c3} \tag{2.1.6}$$

$$\sigma_c = f_{cd} \quad \text{for} \quad \varepsilon_{c3} \le \varepsilon_c \le \varepsilon_{cu3} \tag{2.1.7}$$

Where:

f_{cd} : design compressive strength

ε_{c3} : strain at reaching the maximum strength

ε_{cu3} : ultimate strain

2.1.2 Reinforcement Steel

As previously mentioned, reinforcement steel has a non-linear strain-stress relationship, as shown in Figure 2.5.

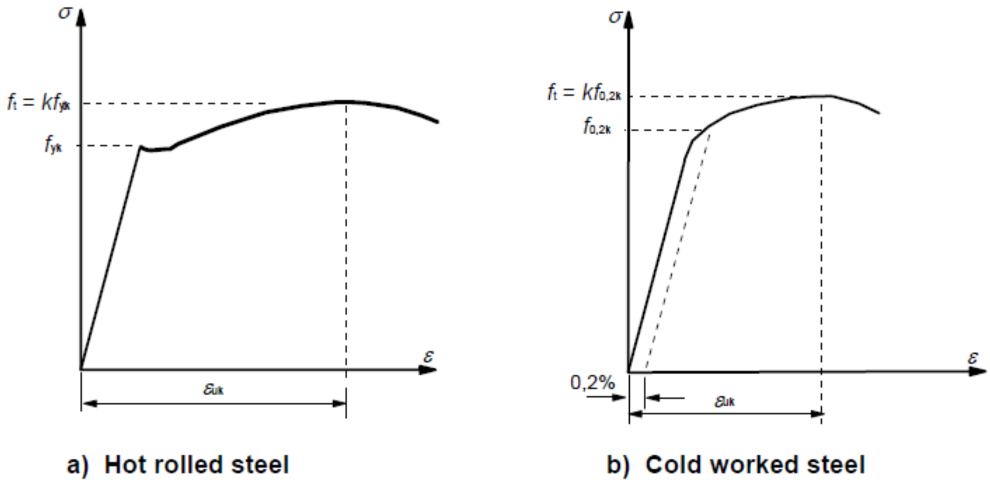

Figure 2.5: Stress-strain diagrams for typical reinforcing steel [2]

However, EC2 allows the use of two simplified design models. These are two bilinear models, a model with an inclined top branch and a model with a horizontal top branch, as shown in Figure 2.6.

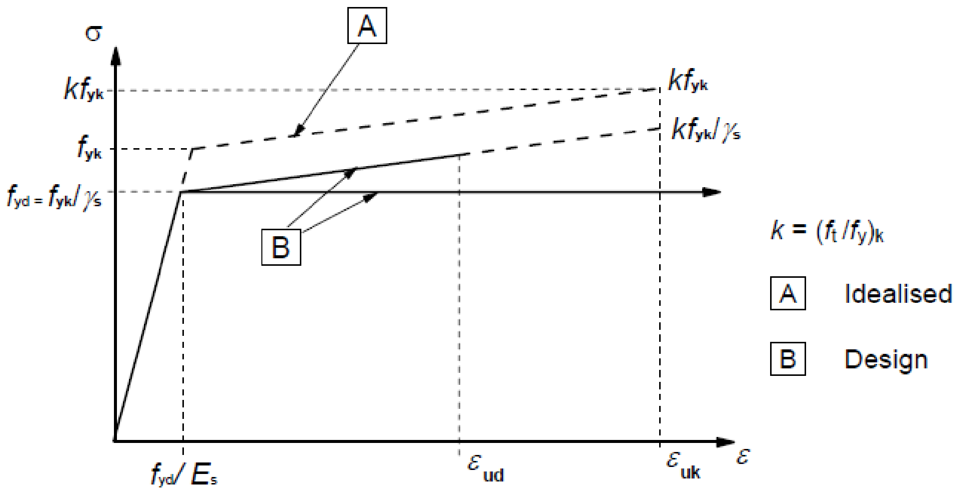

Figure 2.6: Idealized and design stress-strain diagrams for reinforcing steel [2]

The strain-stress relationship for the model with a horizontal top branch represented by the following formulas:

$$\sigma_s = \varepsilon_s E_s \quad \text{for} \quad 0 \leq \varepsilon_s \leq \frac{f_{yd}}{E_s} \tag{2.1.8}$$

$$\sigma_s = f_{yd} \quad \text{for} \quad \frac{f_{yd}}{E_s} \leq \varepsilon_s \leq \varepsilon_{uk} \qquad (2.1.9)$$

Where:

f_{yd} : design yield stress

E_s : modulus of elasticity of reinforcement

ε_{uk} : elongation at maximum force

The computer program implements the design stress-strain relationship with a horizontal top branch. According to EC2-3.2.7(2), when using this model, there is no need to check the strain limit [2].

2.2 Design of shells

The design of reinforced concrete shells consists of finding the necessary concrete dimensions and steel reinforcement amounts such that there is equilibrium between internal sectional forces and external forces.

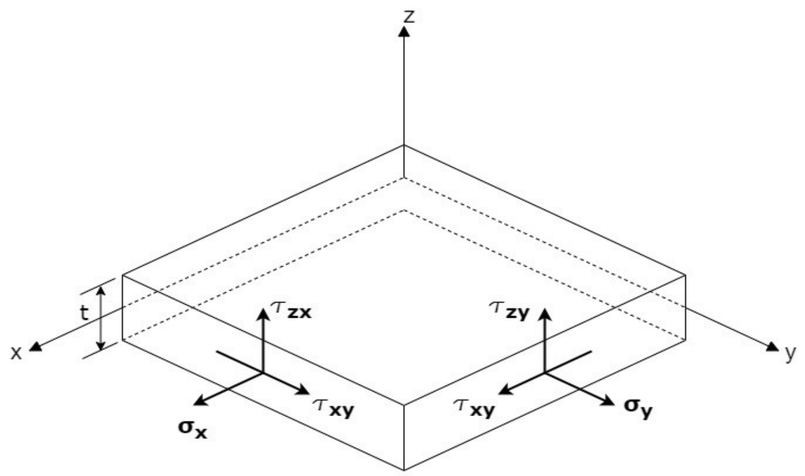

Figure 2.7: Stresses in a shell element

The stresses along the shell thickness, based on the Love-Kirchoff theory, are shown in Figure 2.7. The resulting forces and moments are shown in Figure 2.8 and consist of two bending moments (M_x and M_y), one torsional moment (M_{xy}), two transverse shear forces (V_x, V_y), three membrane forces (N_x, N_y, N_{xy}).

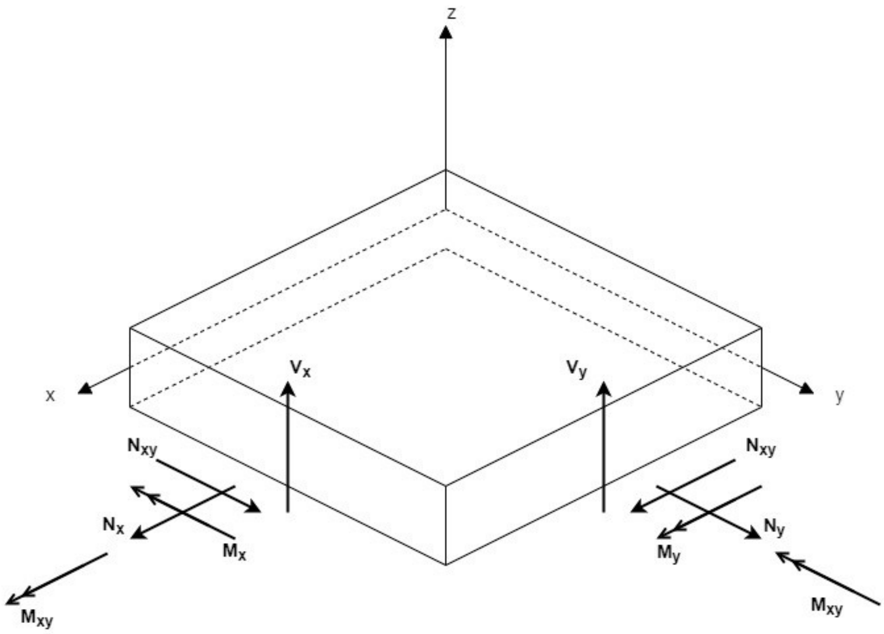

Figure 2.8: Stress resultants in a plane shell element

The stress resultants shown in Figure 2.8 are obtained by integrating the stresses on Figure 2.7 along the shell thickness t.

$$N_x = \int_{-t/2}^{t/2} \sigma_x dz \qquad N_y = \int_{-t/2}^{t/2} \sigma_y dz \qquad N_{xy} = \int_{-t/2}^{t/2} \sigma_{xy} dz$$

$$M_x = \int_{-t/2}^{t/2} \sigma_x z dz \qquad M_y = \int_{-t/2}^{t/2} \sigma_y z dz \qquad M_{xy} = \int_{-t/2}^{t/2} \sigma_{xy} z dz$$

$$V_x = \int_{-t/2}^{t/2} \tau_{zx} z dz \qquad V_y = \int_{-t/2}^{t/2} \tau_{zy} z dz$$

The stress resultants calculated above are then subdivided into longitudinal reinforcement stresses, concrete stresses, and shear. Generally, these calculations present some difficulties due to varying stresses along the shell thickness. Therefore, in order to approach such a complex problem, the introduction of simplifying assumptions is necessary. Two methods that assume the use of orthogonal reinforcement are the Membrane Method and the Sandwich Method.

2.2.1 Membrane Method

In the membrane method, the shell section is subdivided into two layers(one top and one bottom) which resist the moments and in-plane forces, while the transverse shear forces are neglected.

Equilibrium equations in the x and y direction are used to calculate n_{x1}, n_{x2}, n_{y1}, n_{y12}, n_{xy1}, n_{xy2}, as shown in Figure 2.9. Once these forces are calculated, the two membranes are designed using the compression field theory[1].

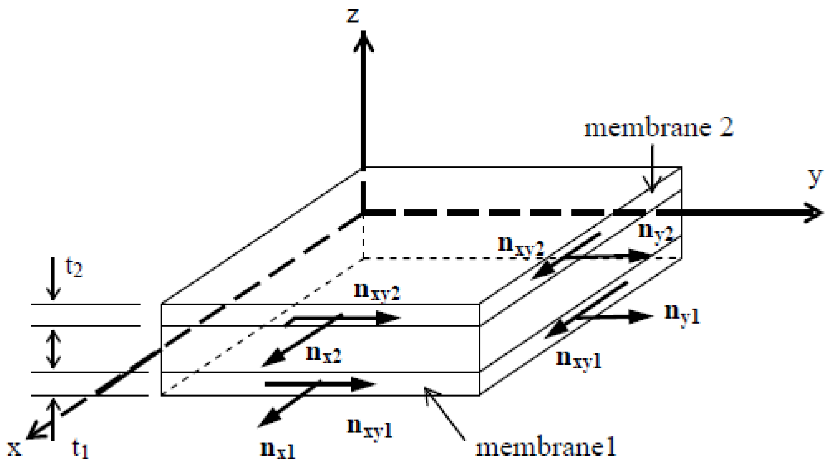

Figure 2.9: Equivalent membrane forces [1]

The membrane method is a simplified approach to shell design and is based on many assumptions. The cracking of concrete is only checked in the middle plane of the membranes, transverse shear is neglected, and strain compatibility is ignored. Notwithstanding the shortcomings mentioned above, it can be used for preliminary design, and its results can subsequently be checked and improved by more accurate methods.

2.2.2 Sandwich Method

In the sandwich method, the shell section is subdivided into three layers. The two outer layers support the inner layer and resist the moments and in-plane forces, while the inner layer carries the transverse shear forces as a beam in the principal shear direction[1].

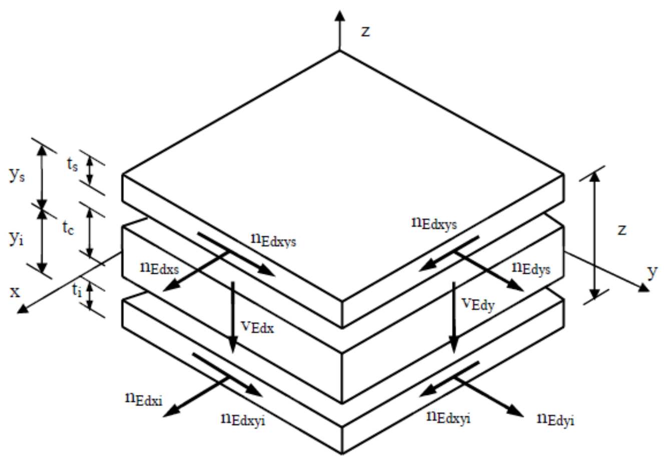

Figure 2.10: Definition of forces in different layers [1]

2.3 Iteration Method

The iteration method is a general method for the capacity control of a reinforced concrete shell, where the geometry and reinforcement amount is given. The method is based on Kirchoff's hypothesis about linear strain distribution over the thickness of a shell. Therefore, out-of-plan normal stresses are assumed to be zero and excluded from the analysis.

External forces and moments acting on the shell are obtained using FEM or other design methods. Based on these results, the method finds the strain distribution for both concrete and reinforcement in an iterative manner, which ensures equilibrium between external and internal sectional forces.

2.3.1 Derivation of the iteration method

As previously mentioned, the iteration method aims to find a state where internal and external sectional forces are in equilibrium. It means finding a strain distribution that ensures equilibrium, where the internal forces are functions of strain[1]:

$$\mathbf{R} = \mathbf{S}\left(\boldsymbol{\varepsilon}_{t,r}\right) \qquad (2.3.1)$$

Where:

$\mathbf{R}$: external load vector

$$\mathbf{R} = \begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} \qquad (2.3.2)$$

$\mathbf{S}$: internal load vector

$\boldsymbol{\varepsilon}_{t,r}$: generalized strain vector

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \boldsymbol{\varepsilon}_\mathbf{m} \\ \boldsymbol{\kappa} \end{bmatrix} = \begin{bmatrix} \varepsilon_{xm} \\ \varepsilon_{ym} \\ \varepsilon_{xym} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} \qquad (2.3.3)$$

$\boldsymbol{\varepsilon}_\mathbf{m}$: strain of the middle plane of the shell element

$\boldsymbol{\kappa}$: curvature of the middle plane of the shell element

The distribution of strain over the shell thickness can be represented as follows:

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \boldsymbol{\varepsilon}_{\mathbf{m}} - z \cdot \boldsymbol{\kappa} = \mathbf{A} \cdot \boldsymbol{\varepsilon}_t = \begin{bmatrix} 1 & 0 & 0 & -z & 0 & 0 \\ 0 & 1 & 0 & 0 & -z & 0 \\ 0 & 0 & 1 & 0 & 0 & -z \end{bmatrix} \cdot \begin{bmatrix} \varepsilon_{xm} \\ \varepsilon_{ym} \\ \varepsilon_{xym} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} \quad (2.3.4)$$

The relationship between strain and stress represented by eq. (2.3.1) is non-linear and is illustrated in Figure 2.11.

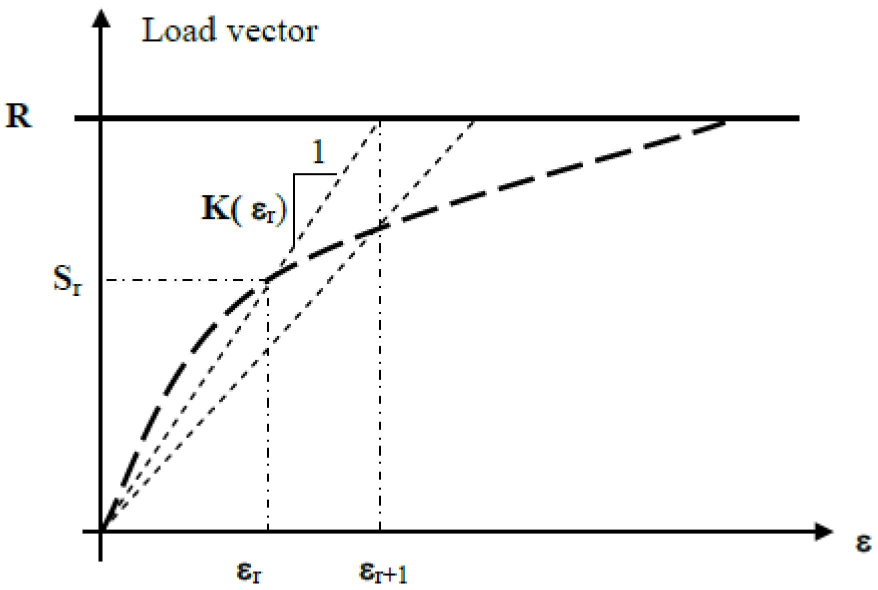

Figure 2.11: Non-linear stiffness relationship[1]

The strain-stress relationship in Figure 2.11 is defined as:

$$\mathbf{R} = \mathbf{K}\left(\boldsymbol{\varepsilon}_{t,r}\right) \cdot \boldsymbol{\varepsilon}_{t,r+1} \quad (2.3.5)$$

Where $\mathbf{K}\left(\boldsymbol{\varepsilon}_{t,r}\right)$ is the secant stiffness matrix for concrete and reinforcement combined at iteration number r.

The material stiffness matrix $\mathbf{K}$ is obtained by using the principle of virtual work. The generalized displacement and rotation are represented by the vector $\mathbf{r}$:

$$\mathbf{r} = a\begin{bmatrix} \boldsymbol{\varepsilon}_{\mathbf{m}} \\ \boldsymbol{\kappa} \end{bmatrix}\begin{bmatrix} r_x \\ r_y \\ r_{xy} \\ \theta_x \\ \theta_y \\ \theta_{xy} \end{bmatrix} \tag{2.3.6}$$

Where a is the dimension of the shell element.

The principle of virtual work can be represented as follows:

Virtual displacement vector: $\delta\mathbf{r} = a\,\delta\,\boldsymbol{\varepsilon}_t$ (2.3.7)

External virtual work: $W_e = \delta\mathbf{r}^T \cdot a \cdot \mathbf{R}$ (2.3.8)

Internal virtual work: $W_i = \int_V \delta\boldsymbol{\varepsilon}^T \cdot \boldsymbol{\sigma} \cdot dV$ (2.3.9)

Since the material model is defined in a general form, the in-plane stress can be written as:

$$\boldsymbol{\sigma} = \mathbf{C}(\boldsymbol{\varepsilon}) \cdot \boldsymbol{\varepsilon} \tag{2.3.10}$$

Where:

- **C** : material matrix, which includes both concrete and reinforcement
- $\boldsymbol{\sigma} = \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}$; $\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}$

$$W_e = a^2\,\delta\boldsymbol{\varepsilon}_t^T\,\mathbf{R} \tag{2.3.11}$$

$$W_i = \int_V \delta\boldsymbol{\varepsilon}^T \boldsymbol{\sigma}\, dV = \int_V \delta\boldsymbol{\varepsilon}^T \mathbf{C}\boldsymbol{\varepsilon}\boldsymbol{\sigma}\, dV = \int_V \delta\boldsymbol{\varepsilon}_t^T \mathbf{A}^T \mathbf{C}\mathbf{A}\boldsymbol{\varepsilon}_t \boldsymbol{\sigma}\, dV \tag{2.3.12}$$

According to the principle of virtual work:

$$W_e = W_i \rightarrow \quad a^2\,\delta\boldsymbol{\varepsilon}_t^T\,\mathbf{R} = a^2\,\delta\boldsymbol{\varepsilon}_t^T \int_{-h/2}^{h/2} \mathbf{A}^T\mathbf{C}\mathbf{A}\, dz\, \boldsymbol{\varepsilon}_t \tag{2.3.13}$$

Consequently, the equilibrium equation for a shell element is:

$$\mathbf{R} = \int_{-h/2}^{h/2} \mathbf{A}^T \mathbf{C} \mathbf{A} \, dz \, \boldsymbol{\varepsilon}_t = \mathbf{K} \boldsymbol{\varepsilon}_t \tag{2.3.14}$$

where the stiffness matrix of the shell is:

$$\mathbf{K} = \int_{-h/2}^{h/2} \mathbf{A}^T \mathbf{C} \mathbf{A} \, dz \tag{2.3.15}$$

and by a congruence multiplication of the integrand, the stiffness matrix can be represented as:

$$\mathbf{K} = \int_{-h/2}^{h/2} \begin{bmatrix} \mathbf{C} & -z\mathbf{C} \\ -z\mathbf{C} & z^2\mathbf{C} \end{bmatrix} dz \tag{2.3.16}$$

The strains and curvatures at the middle plane of the shell can therefore be calculated by applying the following equilibrium equation:

$$\boldsymbol{\varepsilon}_t = \mathbf{K}^{-1} \cdot \mathbf{R} \tag{2.3.17}$$

The integrand in the formula for stiffness matrix $\mathbf{K}$ is solved by dividing the shell cross-section into layers. The concrete is divided into n layers; each layer has a thickness of $\Delta h = h / n$, where h is the thickness of the shell. The reinforcement is subdivided into layers, where each layer has a distance z from the middle plane. The stiffness matrices for concrete and reinforcement are:

Concrete:

$$\mathbf{K}_c = \sum_{i=1}^{n} \Delta h \cdot \mathbf{A}_i^T \cdot \mathbf{C}_i \cdot \mathbf{A}_i = \Delta h \sum_{i=1}^{n} \begin{bmatrix} \mathbf{C}_i & -z_i\mathbf{C}_i \\ -z_i\mathbf{C}_i & z_i^2\mathbf{C}_i \end{bmatrix} \tag{2.3.18}$$

Reinforcement:

$$\mathbf{K}_s = \sum_{j=1}^{m} \left(A_{sxj} \cdot \begin{bmatrix} \mathbf{C}_{sxj} & -z_j\mathbf{C}_{sxj} \\ -z_j\mathbf{C}_{sxj} & z_j^2\mathbf{C}_{sxj} \end{bmatrix} + A_{syj} \cdot \begin{bmatrix} \mathbf{C}_{syj} & -z_j\mathbf{C}_{syj} \\ -z_j\mathbf{C}_{syj} & z_j^2\mathbf{C}_{syj} \end{bmatrix} \right) \tag{2.3.19}$$

$$\mathbf{K} = \mathbf{K}_c + \mathbf{K}_s \tag{2.3.20}$$

The internal vector $\mathbf{S}$ can be represented as:

$$\mathbf{S} = \begin{bmatrix} \mathbf{S}_N \\ \mathbf{S}_M \end{bmatrix} = \begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} \tag{2.3.21}$$

The stress resultants $\mathbf{S}_N$ and $\mathbf{S}_M$ can be expressed as:

$$\mathbf{S}_N = \int_{-h/2}^{h/2} \boldsymbol{\sigma}\, dz \tag{2.3.22}$$

$$\mathbf{S}_M = \int_{-h/2}^{h/2} -z\boldsymbol{\sigma}\, dz \tag{2.3.23}$$

which can be solved numerically as the summation of concrete and reinforcement contributions:

$$\mathbf{S}_N = \sum_{i=1}^{n} \Delta h \cdot \boldsymbol{\sigma}_{ci} + \sum_{j=1}^{m} \begin{bmatrix} A_{sxj} \cdot \sigma_{sxj} \\ A_{syj} \cdot \sigma_{syj} \\ 0 \end{bmatrix} \tag{2.3.24}$$

$$\mathbf{S}_M = \sum_{i=1}^{n} \Delta h \cdot (-z) \cdot \boldsymbol{\sigma}_{ci} + \sum_{j=1}^{m} \begin{bmatrix} -z \cdot A_{sxj} \cdot \sigma_{sxj} \\ -z \cdot A_{syj} \cdot \sigma_{syj} \\ 0 \end{bmatrix} \tag{2.3.25}$$

Where:

$\boldsymbol{\sigma}_{ci}$: concrete stress in layer *i*

σ_{sxj} : x-direction reinforcement stress in layer *j*

σ_{syj} : y-direction reinforcement stress in layer *j*

In the iteration method, concrete and reinforcement are considered non-linear. To take into account the cracking of concrete in tension and non-linear behavior in compression, an orthotropic material model in the directions of the principal stress is used.

$$\boldsymbol{\sigma}_p = \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \mathbf{C}_p \boldsymbol{\varepsilon}_p = \frac{1}{1-\nu^2} \begin{bmatrix} E_{11} & \nu E_{12} & 0 \\ \nu E_{12} & E_{22} & 0 \\ 0 & 0 & \dfrac{(1-\nu)E_{12}}{2} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} \qquad (2.3.26)$$

Where:

- $\boldsymbol{\sigma}_p$: stresses in principal directions
- $\boldsymbol{\varepsilon}_p$: strains in principal directions
- E_{11}, E_{22} : secant modulus in the principal directions
- $E_{ii} = \dfrac{\sigma_i}{\varepsilon_i}$ for i=1,2 ; $E_{12} = \dfrac{E_{11} + E_{22}}{2}$ (2.3.27)
- ν : Poisson's ratio

To obtain the stresses and strains in principal directions, they must be transformed from the stresses and strains in global directions *x* and *y* by the following formula:

$$\boldsymbol{\varepsilon}_p = \mathbf{T}(\theta) \cdot \boldsymbol{\varepsilon} \qquad (2.3.28)$$

where:

- θ : angle for the principal direction; $\theta = \dfrac{1}{2} \cdot \arctan\left(\dfrac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}\right)$ (2.3.29)
- $\mathbf{T}(\theta)$: Transformation matrix

$$\mathbf{T}(\theta) = \begin{bmatrix} \cos^2\theta & \sin^2\theta & \sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & -\sin\theta\cos\theta \\ -2\sin\theta\cos\theta & 2\sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix} \qquad (2.3.30)$$

Assuming that principal strains and principal stresses have the same axis, it is possible to transform both the principal stresses and principal stiffness matrix to the corresponding global stresses and global stiffness matrix.

$$\boldsymbol{\sigma}_c = \mathbf{T}^T(\theta) \cdot \boldsymbol{\sigma}_p = \mathbf{T}^T(\theta) \cdot \mathbf{C}_p \cdot \boldsymbol{\varepsilon}_p = \mathbf{T}^T(\theta) \cdot \mathbf{C}_p \cdot \mathbf{T}(\theta) \cdot \boldsymbol{\varepsilon} \qquad (2.3.31)$$

$$\mathbf{C}_p = \mathbf{T}^T(\theta) \cdot \mathbf{C}_p \cdot \mathbf{T}(\theta) \qquad (2.3.32)$$

A similar approach is used for the reinforcement layers.

If the longitudinal reinforcement directions are assumed in the global x-y directions, the stress-strain relationship for a layer is:

$$\boldsymbol{\sigma}_s = \mathbf{C}_s \cdot \boldsymbol{\varepsilon} \tag{2.3.33}$$

$$\boldsymbol{\sigma}_s = \begin{bmatrix} \sigma_{sx} \\ \sigma_{sy} \\ \tau_{sxy} \end{bmatrix} = \begin{bmatrix} E_{sx} & 0 & 0 \\ 0 & E_{sy} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \tag{2.3.34}$$

Where:

E_{sx} , E_{sy} : secant modulus for the reinforcement in x- and y-direction, respectively

Suppose the longitudinal reinforcement directions don't correspond with the global x-y directions. In that case, the material matrix must be transformed by using a transformation matrix similar to that used in the concrete layers:

$$\mathbf{C}_s^{xy} = \mathbf{T}^T(\alpha) \cdot \mathbf{C}_s \cdot \mathbf{T}(\alpha) \tag{2.3.35}$$

Where α is the angle of the reinforcement relative to the global directions.

To decide whether equilibrium between internal and external forces is reached, a convergence criterium must be defined. One method is the use of the relative difference between each of the internal and external stress resultants. The iteration stops on two conditions:

1. The relative differences are under the convergence criterium β, which typically is in order of magnitude 0.01.
2. The number of iterations is higher than the allowed maximum iteration number.

The convergence criterium is defined as:

$$\left| \frac{\mathbf{R}_k - \mathbf{S}_{i,k}}{\mathbf{R}_k} \right| < \beta \tag{2.3.36}$$

- *k*= 1,2,...,6
- *i*: iteration number

2.3.2 Iteration method procedure

To have an overview of how the iteration method is implemented, a step-by-step procedure is presented[1].

1. Calculate the external load vector ***R*** and the reinforcement amount.

2. Assume linear elastic isotropic behavior for concrete and linear elastic behavior for reinforcement, and calculate the initial stiffness matrix $\mathbf{K}_0$.

Concrete: $$\mathbf{K}_{c0} = \sum_{i=1}^{n} \Delta h \cdot \mathbf{A}_i^T \cdot \mathbf{C}_{0i} \cdot \mathbf{A}_i = \Delta h \sum_{i=1}^{n} \begin{bmatrix} \mathbf{C}_{0i} & -z_i \mathbf{C}_{0i} \\ -z_i \mathbf{C}_{0i} & z_i^2 \mathbf{C}_{0i} \end{bmatrix}$$

Reinforcement:

$$\mathbf{K}_{s0} = \sum_{j=1}^{m} \left(A_{sxj} \cdot \begin{bmatrix} \mathbf{C}_{0sxj} & -z_j \mathbf{C}_{0sxj} \\ -z_j \mathbf{C}_{0sxj} & z_j^2 \mathbf{C}_{0sxj} \end{bmatrix} + A_{syj} \cdot \begin{bmatrix} \mathbf{C}_{0syj} & -z_j \mathbf{C}_{0syj} \\ -z_j \mathbf{C}_{0syj} & z_j^2 \mathbf{C}_{0syj} \end{bmatrix} \right)$$

$$\mathbf{K}_0 = \mathbf{K}_{c0} + \mathbf{K}_{s0}$$

3. Calculate strains and curvatures at the middle-plane of the shell
$$\boldsymbol{\varepsilon}_{t0} = \mathbf{K}_0^{-1} \cdot \mathbf{R}$$

4. Calculate in-plane strains for each concrete and reinforcement layer
$$\boldsymbol{\varepsilon}_{0i} = \mathbf{A}_i \cdot \boldsymbol{\varepsilon}_{t0}$$

5. Calculate the principal directions and principal strains in each concrete layer
$$\boldsymbol{\varepsilon}_{p0i} = \mathbf{T}_{\varepsilon i}(\theta_i) \cdot \boldsymbol{\varepsilon}_{0i}$$
$$\theta_i = \frac{1}{2} \cdot \arctan\left(\frac{\gamma_{xy}^{i}}{\varepsilon_x^{i} - \varepsilon_y^{i}} \right)$$

6. Calculate concrete stress in local principal directions for each concrete layer. The principal stresses are calculated based on the stress-strain relationship model used for concrete.

7. Transform principal stresses in each concrete layer to stresses in global directions
$$\boldsymbol{\sigma}_{c0i} = \mathbf{T}_{\varepsilon i}^T(\theta_i) \cdot \boldsymbol{\sigma}_{p0i}$$

8. Calculate reinforcement stresses in each reinforcement layer
$$\boldsymbol{\sigma}_{s0j} = \mathbf{C}_{s0j} \cdot \boldsymbol{\varepsilon}_{0j}$$

9. Calculate the internal stress resultants
$$\mathbf{S}_0 = \mathbf{S}_{c0} + \mathbf{S}_{s0}$$

$$\mathbf{S}_0 = \Delta h \cdot \sum_{i=1}^{n} \begin{bmatrix} \boldsymbol{\sigma}_{c0i} \\ -z_i \cdot \boldsymbol{\sigma}_{c0i} \end{bmatrix} + \sum_{j=1}^{m} \begin{bmatrix} A_{sxj} \cdot \sigma_{sx0} \\ A_{syj} \cdot \sigma_{sy0} \\ 0 \\ -z_j \cdot A_{sxj} \cdot \sigma_{sx0} \\ -z_j \cdot A_{syj} \cdot \sigma_{sy0} \\ 0 \end{bmatrix}$$

10. Calculate the maximum relative difference between external and internal forces.

    Maximum relative difference = $\max\left(\left|\frac{\mathbf{R}_k - \mathbf{S}_{0,k}}{\mathbf{R}_k}\right|\right)$

11. Check for convergence based on the chosen convergence criterium β.

    If $\max\left(\left|\frac{\mathbf{R}_k - \mathbf{S}_{0,k}}{\mathbf{R}_k}\right|\right) \le \beta$ equilibrium is achieved and the iteration stops.

    If $\max\left(\left|\frac{\mathbf{R}_k - \mathbf{S}_{0,k}}{\mathbf{R}_k}\right|\right) > \beta$ equilibrium is not achieved and the iteration continues.

12. Calculate a new secant modulus for every concrete and reinforcement layer.

13. Calculate a new material matrix for every concrete and reinforcement layer using the secant modulus obtained in step 12.

    $\mathbf{C}_{pli}$; p: principal, l: iteration number, i: layer number

14. Transform the principal material matrices obtained in step 13 to global material matrices.

    $\mathbf{C}_{li} = \mathbf{T}_{\varepsilon i}^{T} \cdot \mathbf{C}_{pli} \cdot \mathbf{T}_{\varepsilon i}$

    Repeat steps 2 to 12 with the newly obtained material matrix for both concrete and reinforcement until the convergence criterium is satisfied.

2.3.3 Utilization ratio

The utilization ratio is used to evaluate the degree of utilization of an element compared to its maximum capacity. When using the iteration method and there is convergence, the maximum strain values in concrete and reinforcement layers are obtained. These are then compared to their respective strain limit values [1].

The utilization ratio for concrete is:

$$UR_c = \frac{\varepsilon_c}{\varepsilon_{cu}} \qquad (2.3.37)$$

Where:

ε_c : maximum compressive principal strain in concrete

ε_{cu} : ultimate strain

The utilization ratio for reinforcement is:

$$UR_s = \frac{\varepsilon_s}{\varepsilon_{ud}} \tag{2.3.38}$$

Where:

ε_s : maximum strain in reinforcement

ε_{ud} : strain limit for the reinforcement

As described in chapter 2.1.2, the stress-strain model adopted in the computer program does not need to check the strain limit. Based on these premises, the user can decide the value ε_{ud} but needs to consider that it directly affects the utilization ratio. In the following calculations, the value ε_{ud} is set to 1%, which is relatively high compared to the reinforcement strain at reaching the maximum strength ε_{yd} of 2.17‰. In the verification of the computer program, it is preferable to use a high ε_{ud} to test the program in extreme load cases and high strain values.

2.3.4 Application of the iteration method

As previously mentioned, the main objective of the iteration method is to control the capacity of concrete shells. The forces considered in the analysis are shown in Figure 2.12**.**

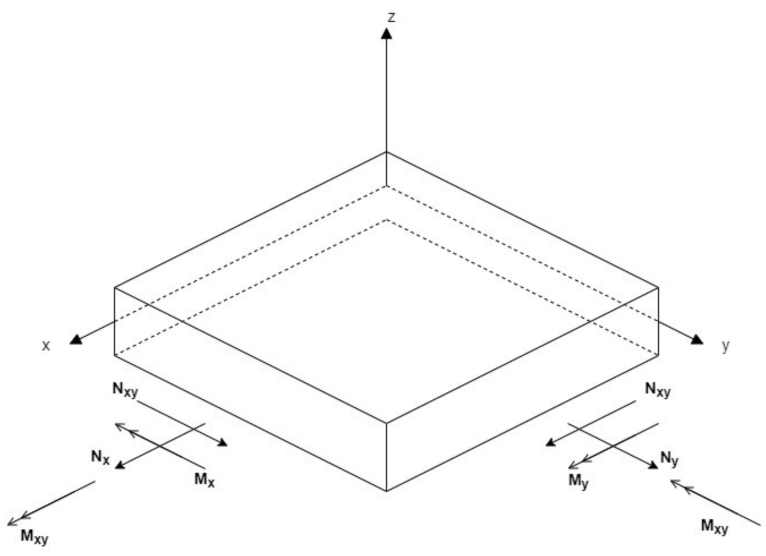

Figure 2.12: Shell, forces and moments in the iteration method

A beam can be considered as a shell subjected to an axial force and moment in one direction. Consequently, the iteration method can be easily applied to a beam. As the iteration method calculates a shell size of *1m x 1m*, the force and moment values, reinforcement amount, and geometry need to be transformed accordingly.

A column is subjected to a uniaxial force and two bending moments with respect to x- and y-direction, respectively, as shown in Figure 2.13

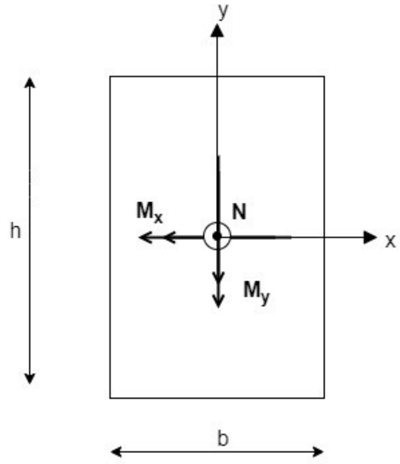

Figure 2.13: Column, forces and moments

In order to implement the iteration method in a column, the moments are combined by the following formula:

$$M_s = \sqrt{M_x^2 + M_y^2}$$

$$\alpha_1 = \arctan\left(\frac{M_y}{M_x}\right)$$

The resulting moment M_s acts about the *s*-axis, which is at an angle α_1 with the x-axis. Consequently, the section can be considered subjected to uniaxial force and a moment in one direction, with the *s* -axis as the middle plane of the section. The reinforcement layers are generally not parallel to the s-axis. Therefore the layer subdivision is applied to both concrete and reinforcement. There are four different cases to be considered based on the value and direction of the moments.

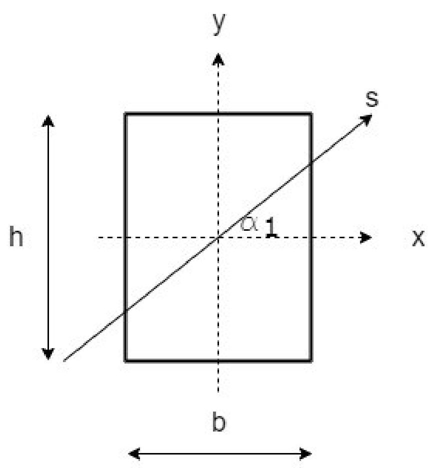

Figure 2.14: Column, *s*-axis

A detailed description of the calculations that allow the use of the iteration method for the capacity control of columns is presented in Appendix E.

3 Computer Program

The computer program is written in Python programming language. Python is an open-source and cross-platform programming language that was first released in 1991 and has become increasingly popular over the last ten years. It is an object-oriented programming language that can be used for multiple purposes such as scientific computing, web development, etc., by downloading and installing the appropriate packages. Python packages for science and numerical computations used in this program are NumPy(fundamental package for scientific computing) and Matplotlib (Python 2D plotting library)[3].

The editor used in the development of the computer program is Spyder. It is an open-source, cross-platform integrated development environment (IDE) for scientific computing in Python[4].

During the preparation of the computer program, the main aim was to make a robust algorithm able to take every possibility into account. To make the script easily accessible to others and ensure a direct connection between the theory and the script, the symbols and variables used in the script are taken directly from the Theory chapter 2.

The calculation program is subdivided into three main parts: beam, column, and shell. All three parts follow the main algorithm described in chapter 2.3. As the version of the algorithm used in the capacity calculation of a shell is the complete one, it will be used in the detailed description of the program in chapter 3.1.

3.1 Description of the Program

The computer program follows all the steps of the iteration method algorithm described in chapter 2.3.2. In this section, important syntaxes and the implementation of some important steps in the algorithm are presented and explained.

3.1.1 Step 1: External load vector *R* and the reinforcement amount

The external load vector ***R*** contains three forces N_x, N_y, N_{xy}, and three moments M_x, M_y, M_{xy}. The units accepted by the program are *kN* for forces and *kNm* for moments, while all subsequent steps are implemented in N and mm. The input data is converted into *N* and *mm* to ensure compatibility between units.

```python
# Input

self.__nx = nx                #kN/m  = N/mm
self.__ny = ny                #kN/m  = N/mm
self.__nxy = nxy              #kN/m  = N/mm

self.__mx = mx * 1000          #kNm * 1000  =  Nmm/mm
self.__my = my * 1000          #kNm * 1000  =  Nmm/mm
self.__mxy = mxy * 1000        #kNm * 1000  =  Nmm/mm
```

Figure 3.1: Python code, external load vector

The same approach is used for reinforcement amount data, which is inserted into the program as mm^2/m.

```python
#Reinforcement: 1: lower rf, 2: upper rf
self.__Asx1 = Asx1 / 1000       #mm^2/mm
self.__Asx2 = Asx2 / 1000       #mm^2/mm
self.__Asy1 = Asy1 / 1000       #mm^2/mm
self.__Asy2 = Asy2 / 1000       #mm^2/mm
```

Figure 3.2: Python code, reinforcement amount

3.1.2 Step 3: Middle-plane strains and curvatures

In step 3, the strains and curvatures at the midplane of the shell are calculated by the following formula, where the stiffness matrix is inverted:

$$\boldsymbol{\varepsilon}_{t0} = \mathbf{K}_0^{-1} \cdot \mathbf{R}$$

A matrix can be correctly inverted if it is regular (non-singular) and well-conditioned (not ill-conditioned). A singular matrix has a determinant equal to zero, while an ill-conditioned matrix has a high condition number. In order to take such possibilities into account, when the matrix is either singular or ill-conditioned, the program implements an alternative method known as the Moore-Penrose pseudo-inverse of a matrix[5].

```python
#Strains and curvatures in the middle plane of the shell

cond = np.linalg.cond(K, p=1)

try:
    epst = np.linalg.solve(K,R)
except np.linalg.LinAlgError as err:
    if 'Singular matrix' in str(err):
        epst = np.matmul(np.linalg.pinv(K,  hermitian= True), R)


if cond > 10:
    epst = np.matmul(np.linalg.pinv(K, hermitian= True), R)

```

Figure 3.3: Python code, middle-plane strains and curvatures

3.1.3 Step 6: Concrete stress in principal directions

In this step, the concrete stress for each concrete layer in local principal directions is calculated by using the stress-strain relationship model of concrete. The computer program allows the user to choose between two concrete models: the parabola-rectangle model (concModel == 1) and the bilinear model (concModel == 2). Subsequently, two sets of formulas are used to obtain the principal stresses of each layer.
It should be noted that the script shown in Figure 3.4 is within a for-loop, and the calculation is implemented for each concrete layer. As previously mentioned, the tensile strength of concrete is assumed to be zero. Therefore, if the strain is positive, the concrete stress value is set to zero. If the compressive strain is higher than the ultimate strain (ε_{cu2}), the concrete stress value is zero.

```
#Principal stresses ( In principal directions)
sigpci = np.zeros((3,1))
if concModel == 1:
    for j in range(2):
        if (epspci[j][0])<0:
            if epspci[j][0]>-epsc2:
                sigpci[j][0] = -fcd*(1-(1-(-epspci[j][0]/epsc2))**nc)
            elif epspci[j][0]<=-epsc2 and epspci[j][0]>=-epscu2:
                sigpci[j][0] = -fcd
            else:
                sigpci[j][0] = 0
        else:
            sigpci[j][0] = 0
elif concModel == 2:
    for j in range(2):
        if (epspci[j][0])<0:
            if epspci[j][0]>-epsc3:
                sigpci[j][0] = fcd*epspci[j][0]/epsc3
            elif epspci[j][0]<=-epsc3 and epspci[j][0]>=-epscu3:
                sigpci[j][0] = -fcd
            else:
                sigpci[j][0] = 0
        else:
            sigpci[j][0] = 0
```

Figure 3.4: Python code, concrete stress in principal directions

3.1.4 Step 8: Reinforcement stress

The stresses in each reinforcement layer are obtained by using the following formula:

$$\boldsymbol{\sigma}_{s0j} = \mathbf{C}_{s0j} \cdot \boldsymbol{\varepsilon}_{0j}$$

In a shell, the reinforcement is categorized by direction and position. Consequently, there are four layers, namely: x-direction bottom, x-direction top, y-direction bottom, y-direction top.
The program implements a for-loop in relation to the reinforcement position. At the same time, instead of using matrix multiplication, the elasticity modulus values of each layer are used to calculate the reinforcement stress.

These calculations are an implementation of the design stress-strain model with the horizontal top branch of the reinforcement.

```
        #Stresses in principal directions

            #x-direction
        if abs(epspsj[0][0]) <= fyd/Esx[j]:
            sigpsj[0][0] =  (epspsj[0][0]) * Esx[j]
        elif abs(epspsj[0][0]) <= epsud:
            sigpsj[0][0] = np.sign(epspsj[0][0]) * fyd
        else:
            sigpsj[0][0] = 0

            #y-direction
        if abs(epspsj[1][0]) <= fyd/Esy[j]:
            sigpsj[1][0] =  (epspsj[1][0]) * Esy[j]
        elif abs(epspsj[1][0]) <= epsud:
            sigpsj[1][0] = np.sign(epspsj[1][0]) * fyd
        else:
              sigpsj[1][0] = 0

        # no need to specify sigpsj[2][0] since it is already 0
```

Figure 3.5: Python code, reinforcement stress

The resulting stress vector is a 3x1 vector, where the first two values are stresses in x- and y-direction, respectively. The third value represents shear, and it is set to zero as it is not considered in the reinforcement stress calculations.

3.1.5 Step 10: Maximum relative difference

The maximum relative difference between external and internal forces is calculated by using the following formula:

$$\max\left(\left|\frac{\mathbf{R}_k - \mathbf{S}_{0,k}}{\mathbf{R}_k}\right|\right)$$

However, some exceptions should be taken into account. As previously discussed, the external forces vector is composed of 6 elements, and some could be zero. In such a case, the formula will be a division by zero and the result will be infinite. Whenever a value in the external forces vector is zero, an alternative method using the difference between external and internal forces is used. The resulting algorithm is as follows:

```
        #Calculation of the maximum difference between S and R

        Diff = R - S
        RelDiff = np.zeros((6,1))       #Matrix containing the relative differences
        diff = np.zeros((6,1))          #Matrix containing the differences
        for d in range (6):
            if R[d][0] == 0:
                RelDiff[d][0] = 0
                diff[d][0] = abs(Diff[d][0])
            else:
                RelDiff[d][0] = abs(Diff[d][0]/R[d][0])

        devMax = np.amax(RelDiff)           #Maximum realtive deviation
        diffNMax = np.amax(diff[0:3,0:1])   #Maximum difference for the values where values of N in R are 0
        diffMMax = np.amax(diff[3:6,0:1])   #Maximum difference for the values where values of M in R are 0
```

Figure 3.6: Python code, maximum relative difference

The resulting values are three: maximum relative difference (devMax), maximum difference for forces (diffNMax), and maximum difference for moments (diffMMax).

Therefore, the convergence criterium is subdivided into three parts:

- β : relative difference
- β_N : difference for Forces
- β_M : difference for Moments

The use of different convergence criterium for forces and moments is because they are defined in terms of N and mm they have different orders of magnitude. These convergence criteria are determined by taking into account all value possibilities of the external load vector.

```
        #Convergence values (differences are taken in consideration)
        RN = R[0:3, 0:1]
        RM = R[3:6, 0:1]

        if np.any((RN != 0)):
            RNAbs = abs(RN)
            RNMin = min([i for i in RNAbs if i > 0])

            betaN = RNMin * beta

            if np.any((RM != 0)):
                RMAbs = abs(RM)
                RMMin = min([i for i in RMAbs if i > 0])
                betaM = RMMin/1000 * beta

            else:       #RM==zeros
                betaM = betaN

        else:           #RN==zeros
            RMAbs = abs(RM)
            RMMin = min([i for i in RMAbs if i > 0])
            betaM = RMMin/1000 * beta
            betaN = betaM
```

Figure 3.7: Python code, convergence criterium

3.1.6 Step 12: Updating concrete secant modulus

The new secant modulus for every concrete and reinforcement layers are calculated by using the following formulas:

$$E_{ii} = \frac{\sigma_i}{\varepsilon_i} \text{ for } i=1,2 \text{ ;} \qquad E_{12} = \frac{E_{11} + E_{22}}{2}$$

An exception that needs to be taken into account is when the strain is equal to zero. According to the formula above: if the strain is equal to zero, the secant modulus will be without a solution, as the expression becomes a division by zero. To prevent that, when the strain is zero, the secant modulus is set to zero. This is shown in Figure 3.8 for concrete layers.

```
        # New Young's Modulus and Material Matrix for Concrete

        CcgiMat = np.zeros((3,3*n)) #Matrix containing all Material Matrices
        for i in range(n):
            if epspciMat[0][i] == 0:
                E11 = 0
            else:
                E11 = (sigpciMat[0][i])/(epspciMat[0][i])

            if epspciMat[1][i] == 0:
                E22 = 0
            else:
                E22 = (sigpciMat[1][i])/(epspciMat[1][i])

            E12 = (E11+E22)/2
```

Figure 3.8: Python code, new concrete secant modulus

3.2 User Manual

The program is designed to be as user-friendly as possible. To run the program, the user opens the folder where the program is downloaded and runs the application *startMain.exe*. The user manual section is composed of three sections: input, output, and exceptions.

3.2.1 Input

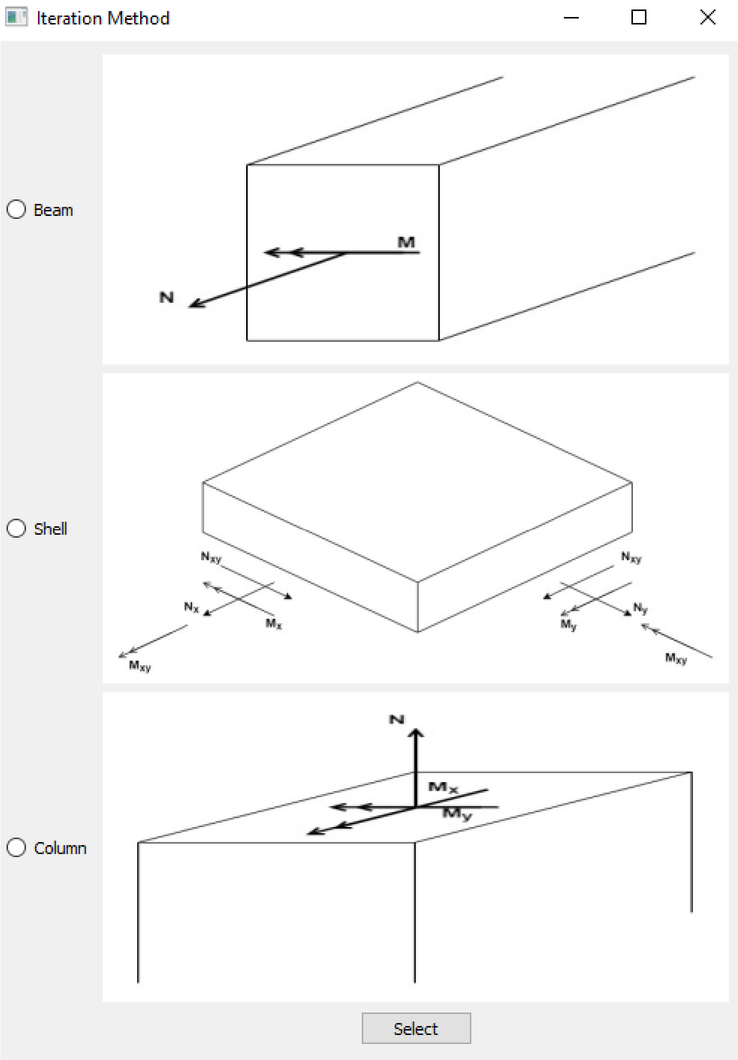

Figure 3.9: Screenshot of the structure selection window

The first window shown in Figure 3.9 allows the user to select the structure type.

Once the structure is selected, a new window appears depending on the selected structure type. The input windows for beam, shell, and column are presented below.

As shown in Figure 3.10, Figure 3.11, and Figure 3.12, the input values are categorized into Forces, Geometry, Reinforcement, Concrete, and Iteration. Common inputs for all structure types are described here, while those specific to the structure types are described in their respective sections.

The sign of forces and moments follow the directions shown in the section figure. In the case of axial force, compression has a negative value and tension has a positive value. The moment is positive when the bottom part is under tension and the top part is under compression.

In the reinforcement part, f_{yk} is the reinforcement yield strength, γ_s is the partial safety factor for reinforcement, and ε_{ud} is the reinforcement strain limit.

In the concrete section, the concrete model is selected from a drop-down list where the user can choose between two concrete models: parabola- rectangle and bilinear. f_{ck} is the concrete compressive yield strength, γ_c is the partial safety factor for concrete.

The iteration part controls the number of concrete layers n, convergence criterium β, and the maximum number of iterations *maxIt*.

1. Beam

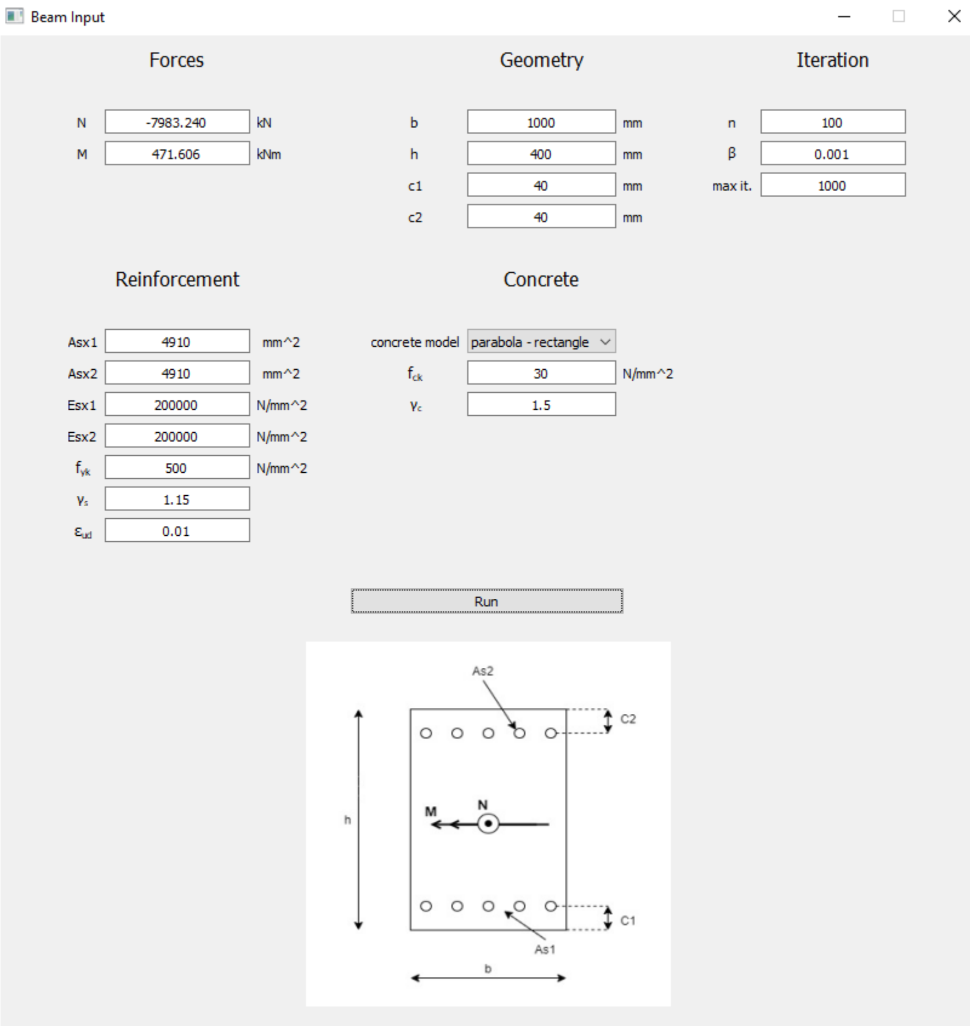

Figure 3.10: Screenshot of the beam input window

In the geometry part, *b* and *h* are the width and height of the section, *c1* and *c2* represent the distance between the bottom and top reinforcement and their corresponding concrete section edges.

In the reinforcement part, *Asx1* and *Asx2* are the bottom and top reinforcement area, while *Esx1* and *Esx2* are their respective modulus of elasticity.

2. Shell

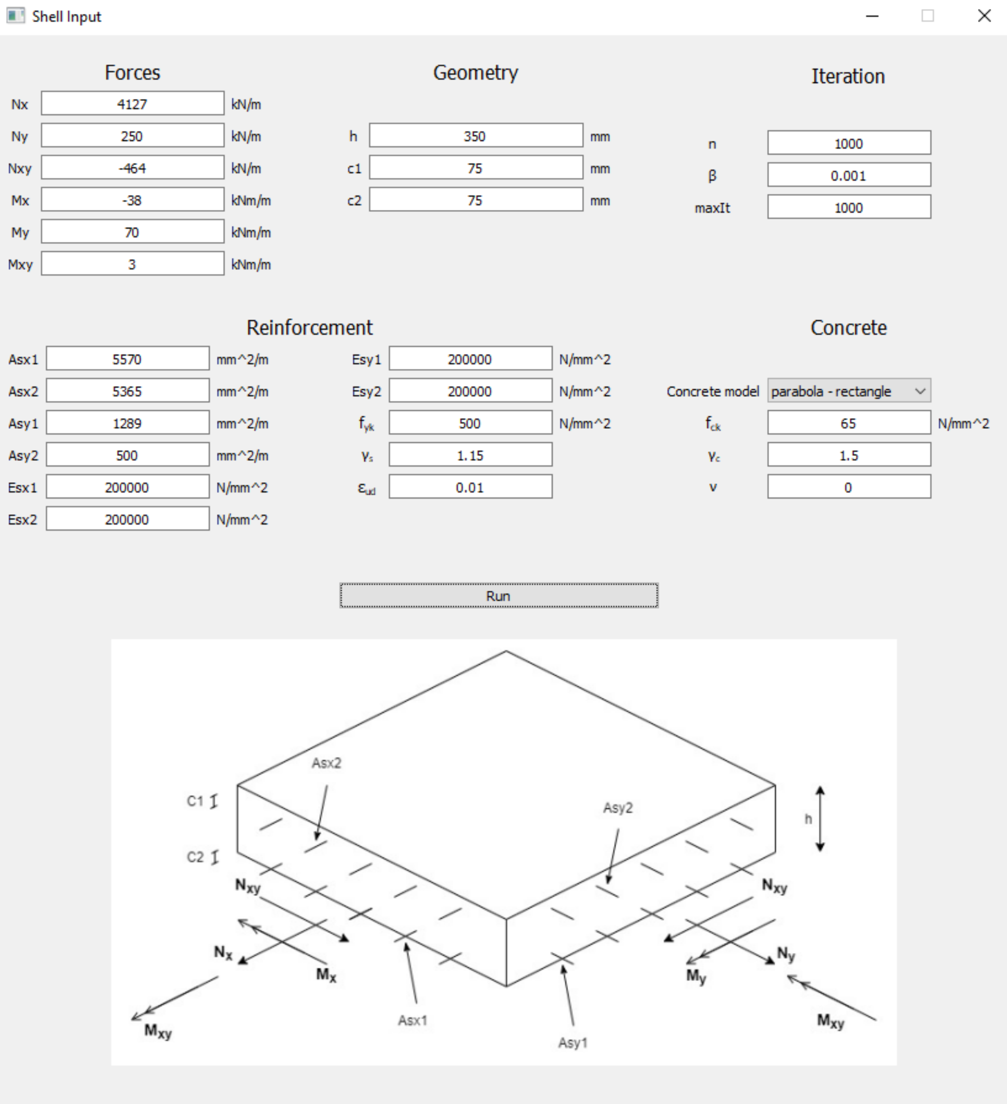

Figure 3.11: Screenshot of the shell input window

In the geometry part, *h* is the height of the section, *c1* and *c2* represent the distance between the bottom and top reinforcement cover and their corresponding edges as shown in the section figure.

In the reinforcement part, *Asx1* and *Asx2* are the bottom and top reinforcement area in the x-direction, while *Asy1* and *Asy2* are the bottom and top reinforcement area in the y-direction. *Esx1*, *Esx2*, *Esy1,* and *Esy2* are their respective modulus of elasticity.

3. Column

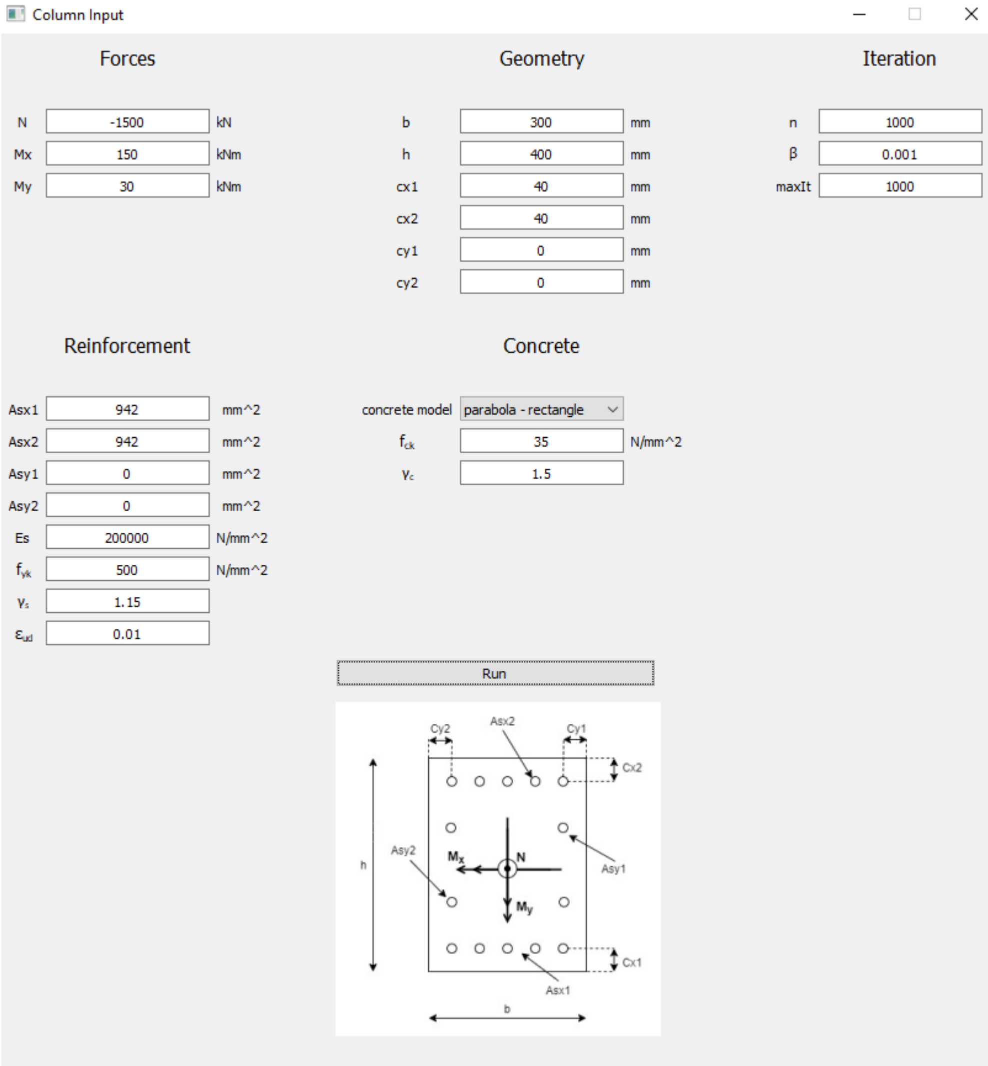

Figure 3.12: Screenshot of the column input window

In the forces part, the value of the three section forces is inserted. The sign of forces and moments follows the directions shown in the section figure. In the case of axial force, compression has a negative value and tension has a positive value. The x-direction moment (Mx) is positive when the bottom of the section is under tension and the top of the section is under compression. The y-direction moment (M_y) is positive when the right part of the section is under tension and the left part of the section is under compression.

In the geometry part, *b* and *h* are the width and height of the section. *cx1* and *cx2* represent the distance between the bottom and top reinforcement and their corresponding edges. In contrast, *cy1* and *cy2* represent the distance between the right and left reinforcement and their corresponding edge, as shown in the section figure.

In the reinforcement part, *Asx1* and *Asx2* are the bottom and top reinforcement areas, while *Asy1* and *Asy2* are the right and left reinforcement areas. *Esx1*, *Esx2*, *Esy1,* and *Esy2* are their respective modulus of elasticity.

3.2.2 Output

When the user clicks the *run* button, the program calculates according to the inserted values, and a new output window displays the results. The result windows for beams, shells, and columns are presented below.

The window is divided into parts showing concrete, reinforcement, internal forces, and iteration number results. A graphic representation of the results above is also displayed.

1. **Beam**

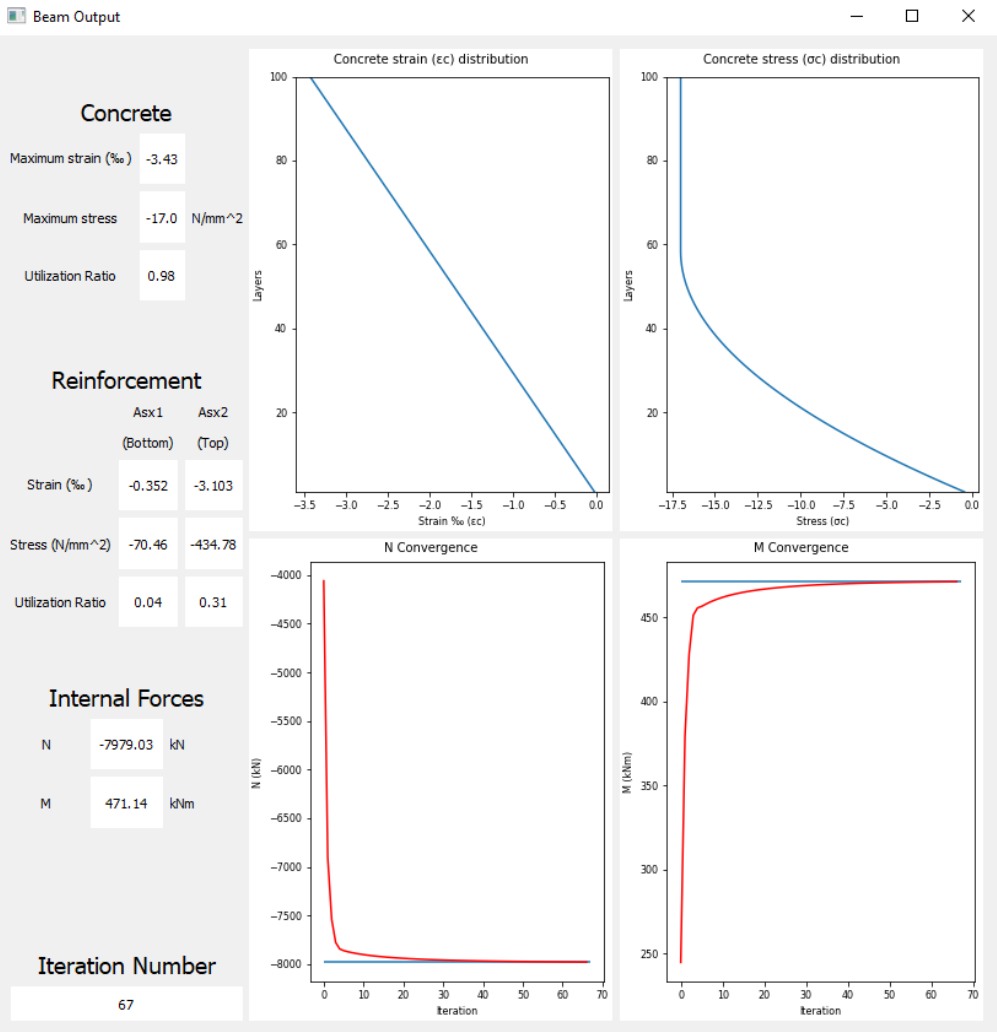

Figure 3.13: Screenshot of the beam output window

In the concrete part, the maximum strain and stress for concrete are displayed. The utilization ratio is a strain ratio between the maximum concrete strain and the ultimate strain ($\varepsilon_{c,\max}$ / ε_{cu}).

In the reinforcement part, the strain and stress values for both bottom and top reinforcements are shown. The utilization ratio is a strain ratio between the reinforcement strain values and reinforcement strain limit (ε_s / ε_{ud}).

The internal forces part shows the value of the internal forces reached after the iteration. The number of iterations used to achieve convergence is displayed in the iteration number part.

The graphs in the first row show the strain and stress distribution in concrete in the graphic representation. The graphs in the second row present the convergence process of the force and the moment during the iteration.

2. Shell

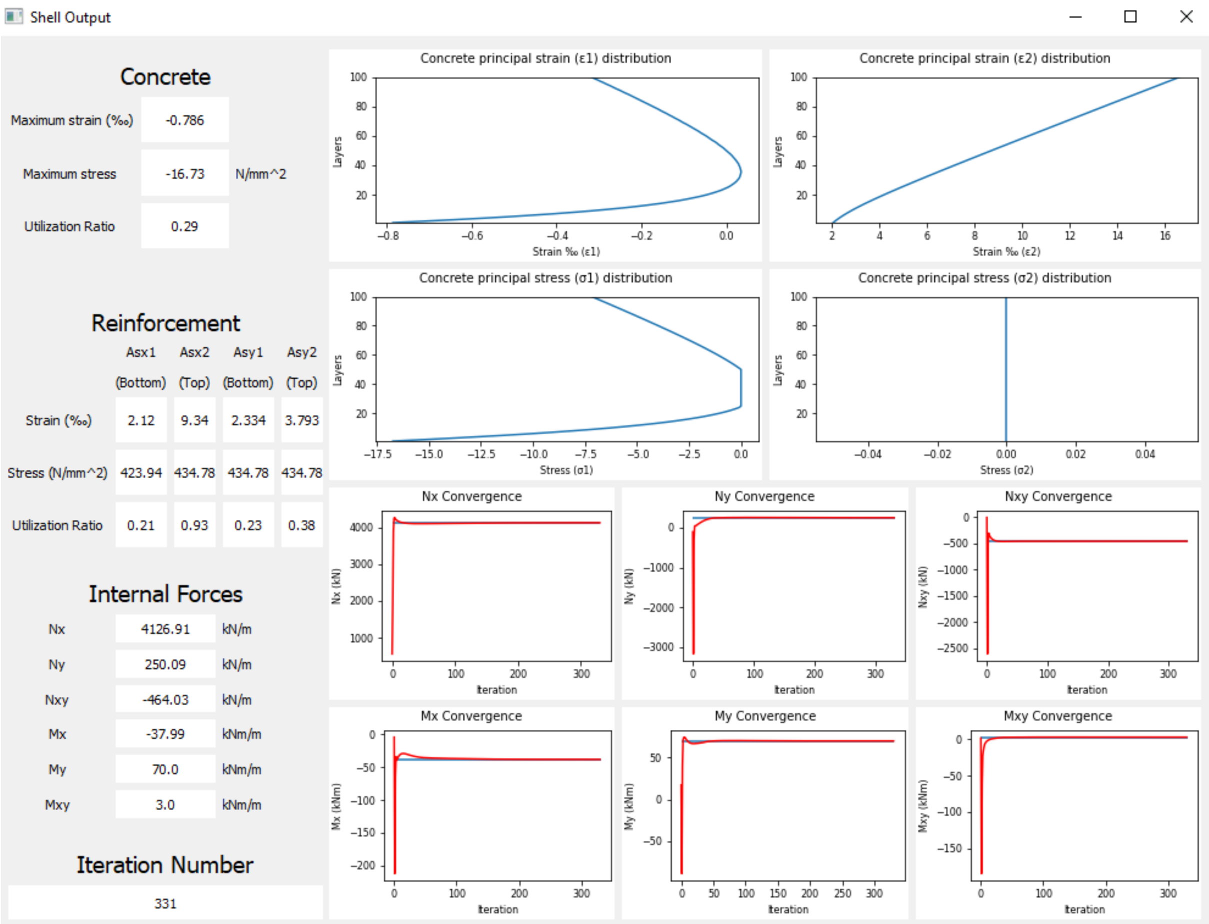

Figure 3.14: Screenshot of the shell output window

In the concrete part, the maximum strain and stress for concrete are presented. The utilization ratio is the strain ratio between the maximum concrete strain and the ultimate strain ($\varepsilon_{c,\max}$ / ε_{cu}).

In the reinforcement part, the strain and stress values for the bottom and top reinforcements in both x- and y- directions are presented. The utilization ratio is the strain ratio between the reinforcement strain values and reinforcement strain limit (ε_s / ε_{ud}).

The internal forces part shows the value of the internal forces reached after the iteration. The number of iterations is shown in the iteration number part.

The graphic representation is composed of four rows. The graphs in the first row display the concrete strain in the principal directions. The graphs in the second row show the concrete stress distribution in the principal directions. The third- and fourth-row graphs display the convergence process of the forces and moments, respectively, during the iteration process.

3. Column

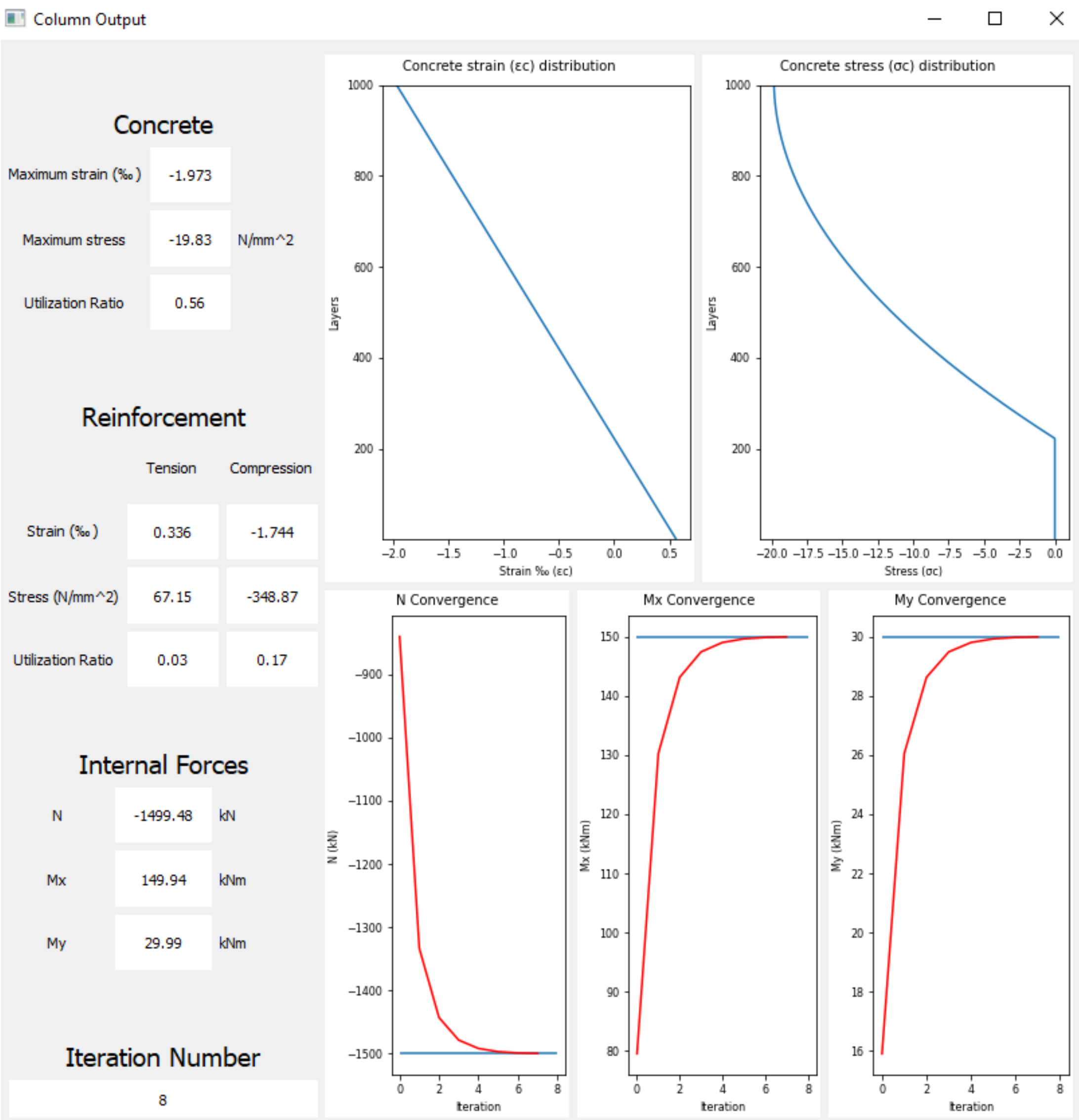

Figure 3.15: Screenshot of the column output window

In the concrete part, the maximum strain and stress for concrete are presented. The utilization ratio is a strain ratio between the maximum concrete strain (tension and compression) and the ultimate strain ($\varepsilon_{c,max}$ / ε_{cu}).

In the reinforcement part, the maximum reinforcement strain values for compression and tension are displayed. The utilization ratio is a strain ratio between the maximum reinforcement strain values and reinforcement strain limit ($\varepsilon_{s,max}$ / ε_{ud}).

The internal forces part shows the value of the internal forces reached after the iteration. The number of iterations used to achieve convergence is shown in the iteration number part.

The graphs in the first row show the strain and stress distribution in concrete. This distribution does not necessarily follow the height of the column. As detailed in Appendix E, the layer subdivision direction is at an angle (α_1) from the x-direction: $\alpha_1 = \arctan(M_y / M_x)$

The graphs in the second row display the convergence process of the force and moments during the iteration.

3.2.3 Exceptions

This section covers situations when the iteration program doesn't converge and when a non-numerical value is inserted.

If the program doesn't converge, the iteration stops, and a dialog box, as shown in Figure 3.16, pops up.

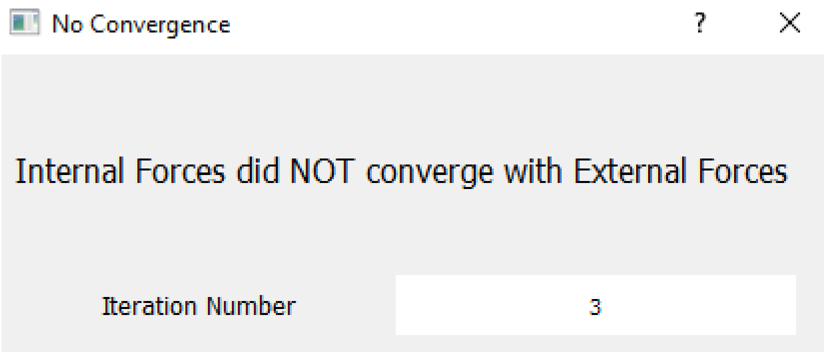

Figure 3.16: Screenshot of the no-convergence dialog box

If a non-numerical value is inserted, the program doesn't run and a dialog box, as shown in Figure 3.17, pops up.

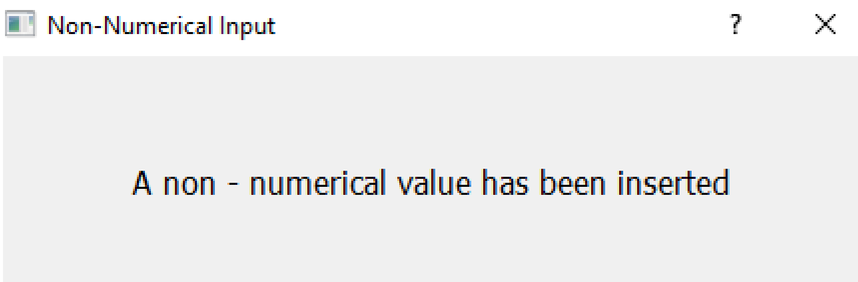

Figure 3.17: Screenshot of the dialog box when a non-numerical value is inserted

4 Verification

Software verification is defined as a process of exercising a software system by using various inputs to validate its behavior, discover bugs or defects, and improve the software's quality. Defects in a program can have many causes, such as mistakes in writing code, wrong requirements, ambiguous instructions, etc.[6]

Software testing can be subdivided into four levels: unit testing, integration testing, system testing, and acceptance testing [6]. However, the testing of this computer program is implemented using a simplified approach subdivided into three parts:

1. Example with known results: an example is calculated using basic hand calculations or an approved computer program.
2. Use of the program: the computer program is used to calculate the same example.
3. Comparing the results: the results from hand calculations and the program are compared.

This method can have two possible outcomes:

1. Results from both methods are equal, which means the program is functioning as expected. This outcome is a green light for the further development of the program.
2. Results from both methods are different, which means the program is not functioning correctly. Therefore, the program needs to be rectified, and the verification is rerun.

The computer program is designed to calculate shell sections for capacity control and lower loads. With the appropriate modifications detailed in chapter 2.3.4, beams and columns can also are calculated. The following verification examples are set up in order of complexity.

The formulas for the design of concrete beams in EC-2.6 apply to concrete sections at ultimate limit state (ULS) [2]. The computer program uses the stress-strain relationship formulas presented in chapter 2.1 to calculate the internal forces and moments in a section. These concrete and reinforcement stress-strain relationships are used to derive formulas for calculating the internal forces and moments in a section. The computer program results can thus be compared to exact hand calculation results. The derivation of the hand calculation formulas is detailed in Appendix A.

4.1 Shells and beams at load capacity

The examples used in this section are first calculated by hand by using formulas for obtaining the maximum capacity of the section. The results from the program are then compared to the hand-calculated results, which are referred to as control results.

The hand calculations and control results of the following examples are shown in Appendix B and Appendix C.

4.1.1 Compression

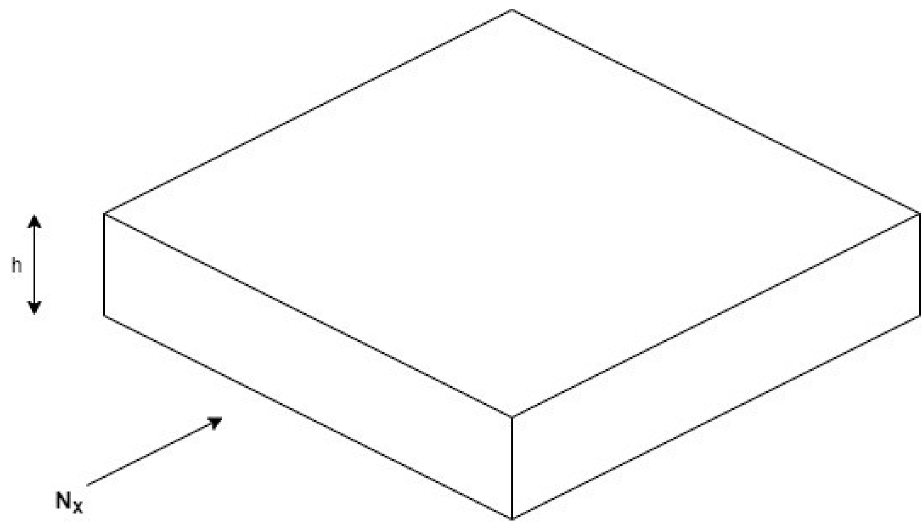

Figure 4.1: Shell, compression in one direction

Input

Symbol	Value	Unit		Symbol	Value	Unit
N_x	-1700	kN/m		A_{sx1}	0	mm²/m
N_y	0	kN/m		A_{sx2}	0	mm²/m
N_{xy}	0	kN/m		A_{sy1}	0	mm²/m
M_x	0	kNm/m		A_{sy2}	0	mm²/m
M_y	0	kNm/m		E_{sx1}	200000	N/mm²
M_{xy}	0	kNm/m		E_{sx2}	200000	N/mm²
				E_{sy1}	200000	N/mm²
h	100	mm		E_{sy2}	200000	N/mm²
c_1	0	mm		f_{yk}	500	N/mm²
c_2	0	mm		γ_s	1.15	
				ε_{ud}	0.01	
n	variable					
β	variable			concrete model	parabola-rectangle	
max it.	1000			f_{ck}	30	N/mm²
				γ_c	1.5	
				ν	0	

Table 4.1: Shell input, compression in one direction

Results

Concrete

	β=0.001		**β=0.0001**	
Concrete Layers (n)	**Stress** (N/mm²)	**Iteration number**	**Stress** (N/mm²)	**Iteration number**
10	-16.98	32	-17.00	100
30	-16.98	32	-17.00	100
100	-16.98	32	-17.00	100
1000	-16.98	32	-17.00	100
Control	-17.00		-17.00	
Concrete Layers (n)	**Strain** (‰)	**Iteration number**	**Strain** (‰)	**Iteration number**
10	-1.938	32	-1.980	100
30	-1.938	32	-1.980	100
100	-1.938	32	-1.980	100
1000	-1.938	32	-1.980	100
Control	-2.000		-2.000	

Table 4.2: Shell concrete results, compression in one direction

Comments

- Convergence criterium (β): Both stress and strain values increase in accuracy as the value of β decreases; however, lower values of β lead to an increase in the number of iterations.

- Concrete layers (n): The number of concrete layers does not affect the results. Since the only force acting on the section is a compressive force N_x, the stress and strain values are the same for any number of subdivisions of concrete layers.

4.1.2 Tension

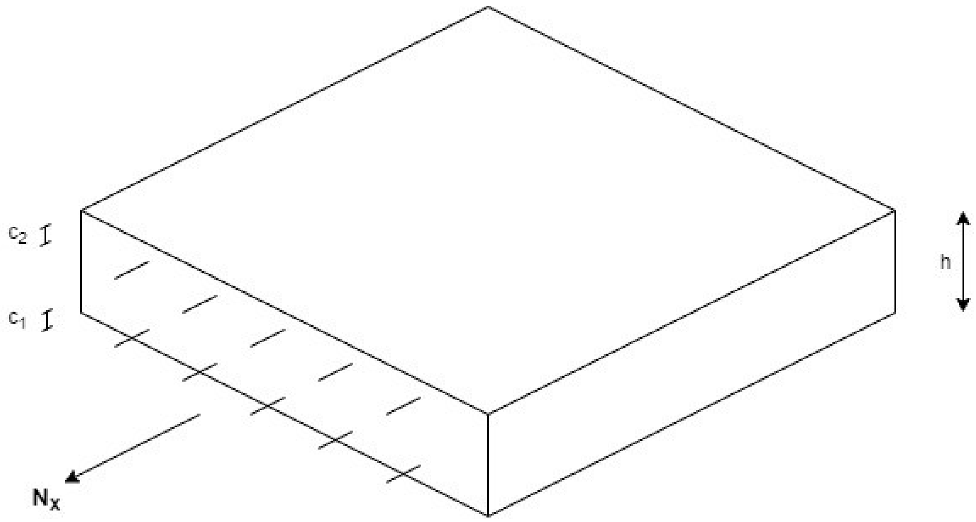

Figure 4.2: Shell, tension in one direction

Input

Symbol	Value	Unit		Symbol	Value	Unit
N_x	500	kN/m		A_{sx1}	580	mm^2/m
N_y	0	kN/m		A_{sx2}	580	mm^2/m
N_{xy}	0	kN/m		A_{sy1}	0	mm^2/m
M_x	0	kNm/m		A_{sy2}	0	mm^2/m
M_y	0	kNm/m		E_{sx1}	200000	N/mm^2
M_{xy}	0	kNm/m		E_{sx2}	200000	N/mm^2
				E_{sy1}	200000	N/mm^2
h	100	mm		E_{sy2}	200000	N/mm^2
c_1	35	mm		f_{yk}	500	N/mm^2
c_2	35	mm		γ_s	1.15	
				ε_{ud}	0.01	
n	variable					
β	variable			concrete model	parabola-rectangle	
max it.	1000			f_{ck}	30	N/mm^2
				γ_c	1.5	
				v	0	

Table 4.3: Shell input, tension in one direction

Results

Reinforcement

	β=0.001			β=0.0001		
Concrete Layers (n)	**Stress**(N/mm^2)		**Iteration number**	**Stress**(N/mm^2)		**Iteration number**
	Sx1	**Sx2**		**Sx1**	**Sx2**	
10	431.03	431.03	2	431.03	431.03	2
30	431.03	431.03	2	431.03	431.03	2
100	431.03	431.03	2	431.03	431.03	2
1000	431.03	431.03	2	431.03	431.03	2
Control	431.03	431.03		431.03	431.03	
Concrete Layers (n)	**Strain** (‰)		**Iteration number**	**Strain**(‰)		**Iteration number**
	Sx1	**Sx2**		**Sx1**	**Sx2**	
10	2.155	2.155	2	2.155	2.155	2
30	2.155	2.155	2	2.155	2.155	2
100	2.155	2.155	2	2.155	2.155	2
1000	2.155	2.155	2	2.155	2.155	2
Control	2.155	2.155		2.155	2.155	

Table 4.4: Shell reinforcement results, tension in one direction

Comments

- Convergence criterium (*β*): Both stress and strain values remain unchanged for different values of *β*.

- Concrete layers (*n*): The number of concrete layers does not affect the results. Since the only force acting on the section is a tension force N_x, and the tensile strength of concrete is assumed zero, the stress and strain values are the same for any number of subdivisions of concrete layers.

4.1.3 Moment in one direction

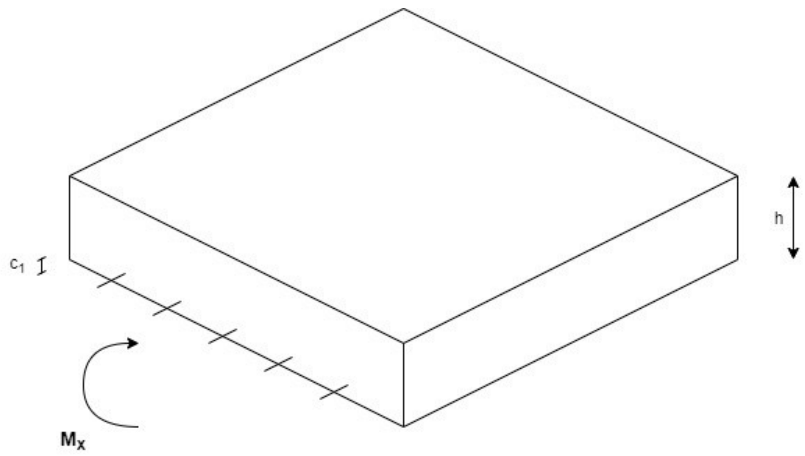

Figure 4.3: Shell, moment in one direction

Input

Symbol	Value	Unit		Symbol	Value	Unit
N_x	0	kN/m		A_{sx1}	3768	mm²/m
N_y	0	kN/m		A_{sx2}	0	mm²/m
N_{xy}	0	kN/m		A_{sy1}	0	mm²/m
M_x	516.780	kNm/m		A_{sy2}	0	mm²/m
M_y	0	kNm/m		E_{sx1}	200000	N/mm²
M_{xy}	0	kNm/m		E_{sx2}	200000	N/mm²
				E_{sy1}	200000	N/mm²
h	400	mm		E_{sy2}	200000	N/mm²
c_1	35	mm		f_{yk}	500	N/mm²
c_2	0	mm		γ_s	1.15	
				ε_{ud}	0.01	
n	variable					
β	variable			concrete model	parabola-rectangle	
max it.	1000			f_{ck}	30	N/mm²
				γ_c	1.5	
				ν	0	

Table 4.5: Shell input, moment in one direction

Results

Concrete

	β=0.001		**β=0.0001**	
Concrete Layers (n)	**Stress** (N/mm^2)	**Iteration number**	**Stress** (N/mm^2)	**Iteration number**
10	-	255	-	255
30	-17.00	283	-17.00	511
100	-17.00	260	-17.00	518
1000	-17.00	256	-17.00	515
Control	-17.00		-17.00	
Concrete Layers (n)	**Strain** (‰)	**Iteration number**	**Strain** (‰)	**Iteration number**
10	-	255	-	255
30	-3.226	283	-3.373	511
100	-3.259	260	-3.396	518
1000	-3.299	256	-3.438	421
Control	-3.500		-3.500	

Table 4.6: Shell concrete results, moment in one direction

Reinforcement

	β=0.001		**β=0.0001**	
Concrete Layers (n)	**Stress** (N/mm^2)	**Iteration number**	**Stress** (N/mm^2)	**Iteration number**
10	-	255	-	255
30	434.78	283	434.78	511
100	434.78	260	434.78	518
1000	434.78	256	434.78	515
Control	434.78		434.78	
Concrete Layers (n)	**Strain**	**Iteration number**	**Strain**	**Iteration number**
10	-	255	-	255
30	7.017	283	7.460	511
100	6.711	260	7.103	518
1000	6.684	256	7.073	515
Control	7.232		7.232	

Table 4.7: Shell reinforcement results, moment in one direction

Comments

- Convergence criterium (β): The stress values in concrete and reinforcement are unchanged for both values of β and equal to the control value. In contrast, the strain values in both materials increase in accuracy as β decreases. The iteration number increases as β decreases.

- Concrete layers (*n*): For *n* equal to 10, the iteration doesn't converge, which means the concrete layer subdivision is not enough. In the other three concrete layer numbers, the iteration converges, and the stress values for concrete and reinforcement are unchanged and equal to the control value. The strain values for concrete increase in accuracy as n increases. However, strain values for reinforcement don't have a uniform response to increase in *n*. However, it should be noted that the maximum strain difference, which occurs for n=1000 and β=0.001, the relative difference compared to the control value is 7.48%.

4.1.4 Moment and axial force in one direction

Moment and axial force can be combined in various ways. In the following cases, the choice of combinations is based on examples similar to those presented in the book 'Betongkonstruksjoner – Beregning og dimensjonering etter Eurocode2' [7]. The following examples represent various capacity extremes for a reinforced concrete section due to fracture in concrete and high reinforcement strains. The hand calculations for this section are presented in Appendix B.4.

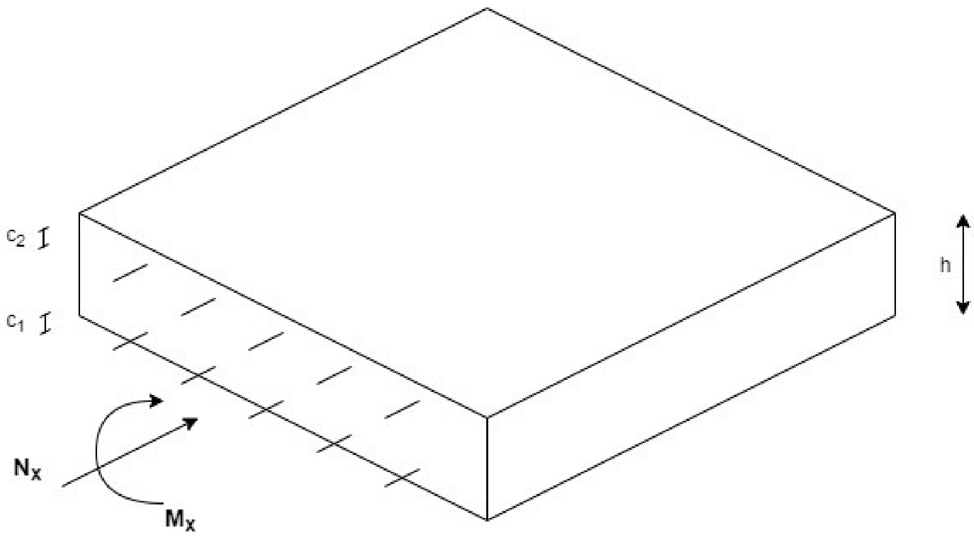

Figure 4.4: Shell, moment and axial force in one direction

Input

Symbol	Value	Unit		Symbol	Value	Unit
N_x	variable	kN/m		A_{sx1}	4910	mm^2/m
N_y	0	kN/m		A_{sx2}	4910	mm^2/m
N_{xy}	0	kN/m		A_{sy1}	0	mm^2/m
M_x	variable	kNm/m		A_{sy2}	0	mm^2/m
M_y	0	kNm/m		E_{sx1}	200000	N/mm^2
M_{xy}	0	kNm/m		E_{sx2}	200000	N/mm^2
				E_{sy1}	200000	N/mm^2
h	400	mm		E_{sy2}	200000	N/mm^2
c_1	40	mm		f_{yk}	500	N/mm^2
c_2	40	mm		γ_s	1.15	
				ε_{ud}	0.03	
n	variable					
β	variable			concrete model	parabola-rectangle	
max it.	2000			f_{ck}	30	N/mm^2
				γ_c	1.5	
				v	0	

Table 4.8: Shell input, moment and axial force in one direction

1. **Compression fracture in concrete**

N_x = -7983.240 kN
M_x = 471.606 kNm

Results

Concrete

	β=0.001		**β=0.0001**	
Concrete Layers (n)	**Stress** (N/mm^2)	**Iteration number**	**Stress** (N/mm^2)	**Iteration number**
10	-17.00	70	-17.00	125
30	-17.00	68	-17.00	118
100	-17.00	67	-17.00	118
1000	-17.00	67	-17.00	118
Control	-17.00		-17.00	
Concrete Layers (n)	**Strain** (‰)	**Iteration number**	**Strain** (‰)	**Iteration number**
10	-3.353	70	-3.404	125
30	-3.399	68	-3.446	118
100	-3.430	67	-3.478	118
1000	-3.444	67	-3.493	118
Control	-3.500		-3.500	

Table 4.9: Shell concrete results, case 1

Reinforcement

	β=0.001			β=0.0001		
Concrete Layers (n)	**Stress**(N/mm^2)		**Iteration number**	**Stress**(N/mm^2)		**Iteration number**
	Sx1	**Sx2**		**Sx1**	**Sx2**	
10	-68.49	-434.78	70	-68.02	-434.78	125
30	-70.25	-434.78	68	-69.83	-434.78	118
100	-70.46	-434.78	67	-70.03	-434.78	118
1000	-70.48	-434.78	67	-70.05	-434.78	118
Control	-70.00	-434.78		-70.00	-434.78	
Concrete Layers (n)	**Strain** (‰)		**Iteration number**	**Strain** (‰)		**Iteration number**
	Sx1	**Sx2**		**Sx1**	**Sx2**	
10	-0.342	-3.176	70	-0.340	-3.223	125
30	-0.351	-3.112	68	-0.349	-3.154	118
100	-0.352	-3.103	67	-0.350	-3.146	118
1000	-0.352	-3.102	67	-0.350	-3.145	118
Control	-0.350	-3.150		-0.350	-3.150	

Table 4.10: Shell reinforcement results, case 1

Comments

According to the hand calculations detailed in Appendix B.4.1, the whole section is under compression, and the failure is due to compression fracture in concrete. As for reinforcement, the top reinforcement yields while the bottom reinforcement does not.

- Convergence criterium (β): The stress values for concrete and top reinforcement are unchanged for both values of β and equal to the control value. The stress values for the bottom reinforcement increase in accuracy as the value of β decreases, except for n=10. The strain values for concrete and top reinforcement increase in accuracy as β decreases. The strain values for bottom reinforcement have a similar trend except for when n=10. The iteration number increases with lower β.

- Concrete layers (n): The stress values for concrete and top reinforcement are unchanged and equal to the control value for all values of n. The strain values for concrete increase in accuracy as n increases. In contrast, strain values for reinforcement don't have a uniform response to the increase in n. However, it should be noted that the maximum strain difference, which occurs in the bottom reinforcement for n=10 and β=0.0001, the relative difference compared to the control value is 2.86%.

2. Compression fracture in concrete and yield strain in reinforcement

N_x = -3056.574 kN

M_x = 1012.053 kNm

Results

Concrete

	β=0.001		**β=0.0001**	
Concrete Layers (n)	**Stress** (N/mm²)	**Iteration number**	**Stress** (N/mm²)	**Iteration number**
10	-17.00	27	-17.00	76
30	-17.00	27	-17.00	38
100	-17.00	28	-17.00	40
1000	-17.00	27	-17.00	40
Control	-17.00		-17.00	
Concrete Layers(n)	**Strain** (‰)	**Iteration number**	**Strain** (‰)	**Iteration number**
10	-3.215	27	-3.242	76
30	-3.388	27	-3.397	38
100	-3.461	28	-3.469	40
1000	-3.487	27	-3.497	40
Control	-3.500		-3.500	

Table 4.11: Shell concrete results, case 2

Reinforcement

	β=0.001			**β=0.0001**		
Concrete Layers(n)	**Stress**(N/mm²)		**Iteration number**	**Stress**(N/mm²)		**Iteration number**
	Sx1	**Sx2**		**Sx1**	**Sx2**	
10	434.68	-434.78	27	434.78	-434.78	76
30	434.52	-434.78	27	434.78	-434.78	38
100	434.46	-434.78	28	434.74	-434.78	40
1000	434.41	-434.78	27	434.74	-434.78	40
Control	434.78	-434.78		434.78	-434.78	
Concrete Layers(n)	**Strain** (‰)		**Iteration number**	**Strain** (‰)		**Iteration number**
	Sx1	**Sx2**		**Sx1**	**Sx2**	
10	2.173	-2.898	27	2.196	-2.923	76
30	2.173	-2.864	27	2.174	-2.872	38
100	2.172	-2.863	28	2.174	-2.870	40
1000	2.172	-2.861	27	2.174	-2.870	40
Control	2.173	-2.870		2.173	-2.870	

Table 4.12: Shell reinforcement results, case 2

Comments

According to the hand calculations detailed in Appendix B.4.2, the failure is due to compression fracture in concrete. The top reinforcement yields due to compression, while the bottom reinforcement yields due to tension with a strain value $2.173 \cdot 10^{-3}$.

- Convergence criterium (β): The stress values for concrete and reinforcement are unchanged for both values of β and equal to the control value. The strain values for concrete increase in accuracy as β decreases. In the case of strain in reinforcement, bottom reinforcement values don't have a uniform response to changes in β, while top reinforcement values increase in accuracy as β decreases. The iteration number increases as β decreases.

- Concrete layers (n): The stress values for concrete and reinforcement are unchanged and equal to the control value for all values of n. The strain values for concrete increase in accuracy as n increases. Strain values for the bottom reinforcement don't have a uniform response to increase in n, while the strain values for the top reinforcement increase in value as n increases. However, it should be noted that in the maximum strain difference, which occurs in the bottom reinforcement for n=10 and β=0.0001, the relative difference compared to the control value is 1.85%.

3. **Compression fracture in concrete and double yield strain in reinforcement**

N_x = -2039.995 kN

M_x = 965.340 kNm

Results

Concrete

	β=0.001		**β=0.0001**	
Concrete Layers(n)	**Stress** (N/mm^2)	**Iteration number**	**Stress** (N/mm^2)	**Iteration number**
10	-17.00	165	-	280
30	-17.00	95	-17.00	393
100	-17.00	98	-17.00	349
1000	-17.00	96	-17.00	342
Control	-17.00		-17.00	
Concrete Layers(n)	**Strain** (‰)	**Iteration number**	**Strain** (‰)	**Iteration number**
10	-3.287	165	-	280
30	-3.033	95	-3.381	393
100	-3.133	98	-3.421	349
1000	-3.163	96	-3.456	342
Control	-3.500		-3.500	

Table 4.13: Shell concrete results, case 3

Reinforcement

Concrete Layers(n)	β=0.001			β=0.0001		
	Stress(N/mm^2)		Iteration number	Stress(N/mm^2)		Iteration number
	Sx1	**Sx2**		**Sx1**	**Sx2**	
10	434.78	-434.78	165	-	-	280
30	434.78	-434.78	95	434.78	-434.78	393
100	434.78	-434.78	98	434.78	-434.78	349
1000	434.78	-434.78	96	434.78	-434.78	342
Control	434.78	-434.78		434.78	-434.78	
Concrete Layers(n)	**Strain** (‰)		**Iteration number**	**Strain** (‰)		**Iteration number**
	Sx1	**Sx2**		**Sx1**	**Sx2**	
10	5.563	-2.767	165	-	-	280
30	4.334	-2.338	95	5.094	-2.582	393
100	4.350	-2.338	98	4.935	-2.534	349
1000	4.334	-2.333	96	4.922	-2.529	342
Control	5.000	-2.556		5.000	-2.556	

Table 4.14: Shell reinforcement results, case 3

Comments

According to the hand calculations detailed in Appendix B.4.3, the failure is due to compression fracture in concrete. The top reinforcement yields due to compression, while the bottom reinforcement yields due to tension with a strain value $5.00 \cdot 10^{-3}$.

The iteration doesn't converge when *n*=10 and *β*=0.0001.

- Convergence criterium (*β*): The stress values for concrete and reinforcement are unchanged for both values of *β* and equal to the control value. The strain values for concrete and reinforcement increase in accuracy as *β* decreases. The iteration number increases as *β* decreases.

- Concrete layers (*n*): The stress values for concrete and reinforcement are unchanged and equal to the control value for all values of *n*. The strain values for concrete increase in accuracy as *n* increases. Strain values for both bottom and top reinforcement don't have a uniform response to increase in *n*. The maximum strain difference occurs in the bottom reinforcement for *n*=30, *n*=1000, and *β*=0.001; the relative difference compared to the control value is 13.32%. This is a high relative difference. However, when *β*=0.0001, the results' accuracy improves considerably.

4. Compression fracture in concrete and high strain level in reinforcement

N_x = -220.956 kN
M_x = 729.419 kNm

Results

Concrete

	β=0.001		**β=0.0001**	
Concrete Layers(n)	**Stress** (N/mm²)	**Iteration number**	**Stress** (N/mm²)	**Iteration number**
10	-17.00	457	-17.00	619
30	-17.00	769	-17.00	1244
100	-17.00	643	-17.00	1045
1000	-17.00	621	-17.00	1010
Control	-17.00		-17.00	
Concrete Layers(n)	**Strain** (‰)	**Iterations**	**Strain** (‰)	**Iterations**
10	-2.920	457	-2.933	619
30	-3.239	769	-3.279	1244
100	-3.368	643	-3.404	1045
1000	-3.449	621	-3.485	1010
Control	-3.500		-3.500	

Table 4.15: Shell concrete results, case 4

Reinforcement

	β=0.001			**β=0.0001**		
Concrete Layers(n)	**Stress**(N/mm²)		**Iteration number**	**Stress**(N/mm²)		**Iteration number**
	Sx1	**Sx2**		**Sx1**	**Sx2**	
10	434.78	-313.32	457	434.78	-314.07	619
30	434.78	-289.56	769	434.78	-290.25	1244
100	434.78	-288.32	643	434.78	-288.94	1045
1000	434.78	-288.19	621	434.78	-288.80	1010
Control	434.78	-288.89		434.78	-288.89	
Concrete Layers(n)	**Strain** (‰)		**Iteration number**	**Strain** (‰)		**Iteration number**
	Sx1	**Sx2**		**Sx1**	**Sx2**	
10	20.080	-1.567	457	20.233	-1.570	619
30	15.750	-1.448	769	16.097	-1.451	1244
100	14.783	-1.442	643	15.058	-1.445	1045
1000	14.700	-1.441	621	14.962	-1.444	1010
Control	15.000	-1.444		15.000	-1.444	

Table 4.16: Shell reinforcement results, case 4

Comments

According to the hand calculations detailed in Appendix B.4.4, the failure is due to compression fracture in concrete. The top reinforcement is under compression below yield value, while the bottom reinforcement yields due to tension with a high strain value $1.50 \cdot 10^{-2}$.

- Convergence criterium (β): The stress values for concrete and bottom reinforcement are unchanged for both values of β and equal to the control value. In contrast, the stress values for top reinforcement increase in accuracy as β decreases only when n has a high value of 100 or 1000. The strain value for concrete increases in accuracy as β decreases. On the other hand, the strain values for reinforcement increase in accuracy as β decreases only when the value on n is either 100 or 1000. The iteration number increases as β decreases.

- Concrete layers (n): The stress values for concrete and bottom reinforcement are unchanged and equal to the control value for all n. In contrast, the stress values for top reinforcement don't have a uniform response to increase in n. The strain values for concrete increase in accuracy as n increases. Strain values for both bottom and top reinforcement for β=0.0001 increase in accuracy as n increases, while for β=0.001, the response is not uniform.

4.1.5 Moment and axial force in two directions

The following is an example of calculating a shell element where all six sectional forces are present. The shell is a part of a box girder bridge in reinforced concrete. The material properties and sectional forces are taken from a FEM analysis [1].

The control calculations for this example are calculated by an iteration-method computer program developed and approved by NTNU.

The example is subdivided into two parts:

- In the first part, the input data is obtained from a hand calculation design and run. The result shows that the top reinforcement in the y-direction (A_{sy1} = 1241 mm^2/m) is over-dimensioned.
- In the second part, the top reinforcement in the y-direction is reduced (A_{sy1} = 500 mm^2/m), and the program is rerun.

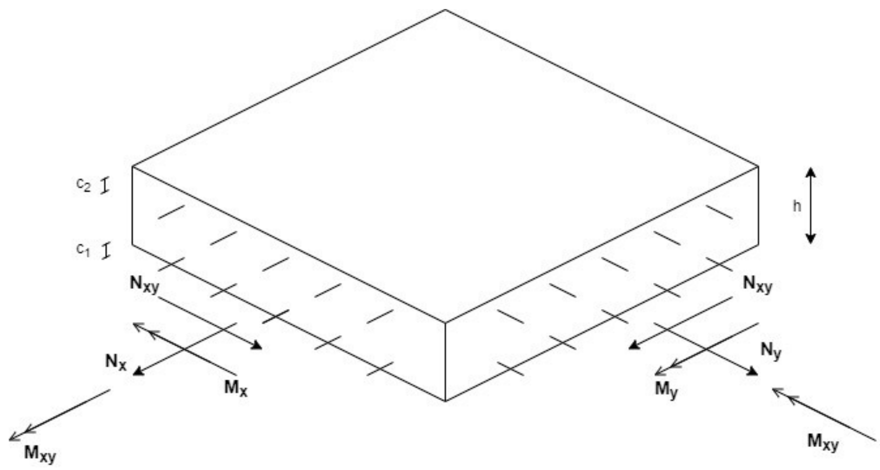

Figure 4.5: Shell, moment, and axial force in two directions

Input

Symbol	Value	Unit		Symbol	Value	Unit
N_x	4127	kN/m		A_{sx1}	5570	mm^2/m
N_y	250	kN/m		A_{sx2}	5365	mm^2/m
N_{xy}	-464	kN/m		A_{sy1}	1289	mm^2/m
M_x	-38	kNm/m		A_{sy2}	variable	mm^2/m
M_y	70	kNm/m		E_{sx1}	200000	N/mm^2
M_{xy}	3	kNm/m		E_{sx2}	200000	N/mm^2
				E_{sy1}	200000	N/mm^2
h	350	mm		E_{sy2}	200000	N/mm^2
c_1	75	mm		f_{yk}	500	N/mm^2
c_2	75	mm		γ_s	1.15	
				ε_{ud}	0.01	
n	variable					
β	0.001			concrete model	parabola-rectangle	
max it.	2000			f_{ck}	65	N/mm^2
				γ_c	1.5	
				v	0	

Table 4.17: Shell input, moment, and axial force in two directions

1. A_{sy2} = 1241 mm²/m

Results

Concrete

Concrete Layers (n)	**Stress** (N/mm²)	**Iteration number**
10	-9.50	367
30	-10.63	355
100	-11.07	355
1000	-11.25	354
Control	-12.00	365
Concrete Layers (n)	**Strain** (‰)	**Iteration number**
10	-0.427	367
30	-0.481	355
100	-0.502	355
1000	-0.511	354
Control	-0.4	365

Table 4.18: Shell concrete result, case 1

Reinforcement

Concrete Layers (n)	**Stress** (N/mm²)				**Iteration number**
	Sx1	**Sx2**	**Sy1**	**Sy2**	
10	401.89	434.78	434.78	264.01	367
30	401.71	434.78	434.78	262.61	355
100	401.69	434.78	434.78	262.45	355
1000	401.69	434.78	434.78	262.44	354
Control	401	435	435	262	365
Concrete Layers (n)	**Strain** (‰)				**Iteration number**
	Sx1	**Sx2**	**Sy1**	**Sy2**	
10	2.009	4.328	3.232	1.320	367
30	2.009	4.233	3.206	1.313	355
100	2.008	4.222	3.203	1.312	355
1000	2.008	4.221	3.203	1.312	354
Control	2.0	4.1	3.1	1.3	365

Table 4.19: Shell reinforcement results, case 1

Comments

- Concrete: The stress values for concrete increase in accuracy as the number of concrete layers increases. In contrast, concrete strain values decrease in accuracy as *n* increases. It should, however, be noted that the obtained strain values are relatively accurate.

- Reinforcement: The stress values of *Sx2* and *Sy1* are unchanged and equal to the control value at yield stress. The stress values of *Sx1* and *Sy2* increase in accuracy as *n* increases. The strain values of both bottom and top reinforcements increase in accuracy as *n* increases.

2. A_{sy2} = 500 mm²/m

Results

Concrete

Concrete Layers	**Stress** (N/mm²)	**Iteration number**
10	-11.75	314
30	-15.20	332
100	-16.73	331
1000	-17.38	331
Control	-18	349
Concrete Layers	**Strain** (‰)	**Iteration number**
10	-0.353	314
30	-0.707	332
100	-0.786	331
1000	-0.821	331
Control	-0.7	349

Table 4.20: Shell concrete results, case 2

Reinforcement

Concrete Layers	**Stress** (N/mm²)		**Stress** (N/mm²)		**Iteration number**
	Sx1	**Sx2**	**Sy1**	**Sy2**	
10	426.51	434.78	434.78	434.78	314
30	424.15	434.78	434.78	434.78	332
100	423.94	434.78	434.78	434.78	331
1000	423.93	434.78	434.78	434.78	331
Control	423	435	435	435	349
Concrete Layers	**Strain** (‰)				**Iteration number**
	Sx1	**Sx2**	**Sy1**	**Sy2**	
10	2.132	10.126	2.168	4.063	314
30	2.121	9.410	2.319	3.815	332
100	2.120	9.340	2.334	3.793	331
1000	2.120	9.333	2.336	3.790	331
Control	2.1	9.2	2.2	3.7	349

Table 4.21: Shell reinforcement result, case 2

Comments

- Concrete: The stress values of concrete increase in accuracy as the number of concrete layers increases. In contrast, the concrete strain does not have a uniform response to increase in *n*. However, it should be noted that the strain results for concrete when *n* is higher than 10 are relatively accurate.

- Reinforcement: The stress values of *Sx2*, *Sy1,* and *Sy2* are unchanged and equal to the control value at yield stress. The stress value of *Sx1* increases in accuracy as *n* increases. The strain values of both bottom and top reinforcements increase in accuracy as *n* increases.

4.2 Shells and beams below load capacity

The computer program is designed to give accurate results regarding the maximum capacity of the section and when forces and moments below the maximum capacity are applied to a section. To verify that, moments and forces lower than the capacity of the section are inserted into the program, and the resulting strain values are used to calculate the corresponding moments and forces by hand. These are then compared to the original forces and moments. Examples of the hand calculations are presented in Appendix C. The following verifications are executed for sections subjected only to moment in one direction. This simplified method is implemented to verify the accuracy of the algorithm in the program. The verification is executed for both concrete models used in the computer program, the parabola-rectangle and bilinear models.
The results are presented in table form, where the strain values obtained from the program are inserted into the second and the third column. The resulting moments (M) and forces (N), as well as the difference to the original values (δ_N, δ_M) and the relative difference to the actual values (dev_N, dev_M), are presented.

This verification will also compare the effect of the number of concrete layer subdivisions (n) and the convergence criterium (β) on the result accuracy.

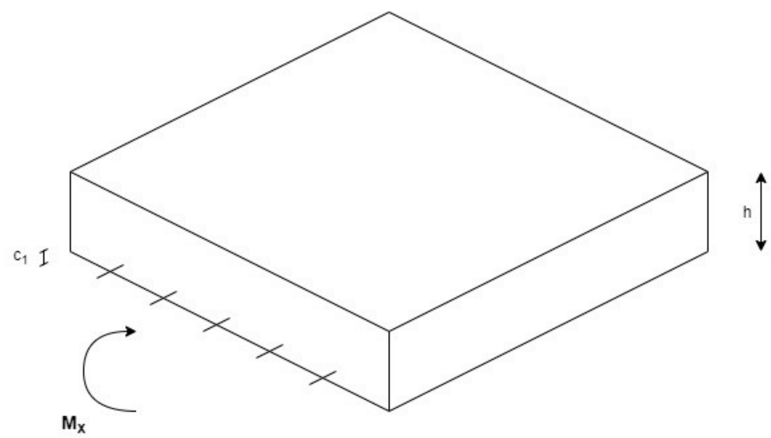

Figure 4.6: Shell, moment in one direction

Input

Symbol	Value	Unit		Symbol	Value	Unit
N_x	0	kN/m		A_{sx1}	3768	mm²/m
N_y	0	kN/m		A_{sx2}	0	mm²/m
N_{xy}	0	kN/m		A_{sy1}	0	mm²/m
M_x	variable	kNm/m		A_{sy2}	0	mm²/m
M_y	0	kNm/m		E_{sx1}	200000	N/mm²
M_{xy}	0	kNm/m		E_{sx2}	200000	N/mm²
				E_{sy1}	200000	N/mm²
h	400	mm		E_{sy2}	200000	N/mm²
c_1	35	mm		f_{yk}	500	N/mm²
c_2	0	mm		γ_s	1.15	
				ε_{ud}	0.01	
n	variable					
β	variable			concrete model	variable	
max it.	1000			f_{ck}	30	N/mm²
				γ_c	1.5	
				v	0	

Table 4.22: Shell input, load below capacity

Results

1.

M_x = 200 kNm
concrete model: parabola-rectangle

β=0.001									
n	ε_c (compression)	ε_s (tension)	N (kN)	δ_N (kN)	dev_N	M (kNm)	δ_M (kNm)	dev_M	Iteration number
10	$4.909157*10^{-4}$	$8.456347*10^{-4}$	123.62	123.62	-	183.75	16.25	$8.10*10^{-2}$	3
10^2	$5.560051*10^{-4}$	$8.426155*10^{-4}$	12.79	12.79	-	198.26	1.74	$9.00*10^{-3}$	4
10^3	$5.628869*10^{-4}$	$8.426000*10^{-4}$	1.20	1.20	-	199.84	0.16	$8.03*10^{-4}$	4
10^4	$5.635795*10^{-4}$	$8.425993*10^{-4}$	0.03	0.03	-	200.00	0.001	$6.93*10^{-6}$	4
10^5	$5.636490*10^{-4}$	$8.425993*10^{-4}$	0.09	0.09	-	200.02	0.02	$7.30*10^{-5}$	4
β=0.0001									
n	ε_c (compression)	ε_s (tension)	N (kN)	δ_N (kN)	dev_N	M (kNm)	δ_M (kNm)	dev_M	Iteration number
10	$4.910125*10^{-4}$	$8.457463*10^{-4}$	123.59	123.59	-	183.78	16.22	$8.10*10^{-2}$	4
10^2	$5.559494*10^{-4}$	$8.425070*10^{-4}$	12.87	12.87	-	198.25	1.75	$9.00*10^{-3}$	5
10^3	$5.628247*10^{-4}$	$8.425908*10^{-4}$	1.29	1.29	-	199.82	0.18	$8.78*10^{-4}$	5
10^4	$5.635177*10^{-4}$	$8.425905*10^{-4}$	0.121	0.121	-	199.99	0.02	$8.11*10^{-5}$	5
10^5	$5.635870*10^{-4}$	$8.425905*10^{-4}$	0.004	0.004	-	200.00	0.00	$1.40*10^{-6}$	5

Table 4.23: Shell results, load below capacity, case 1

2.

M_x = 350 kNm
concrete model: parabola-rectangle

β=0.001									
n	**ε_c** (compression)	**ε_s** (tension)	**N** (kN)	**δ_N** (kN)	**dev_N**	**M** (kNm)	**δ_M** (kNm)	**dev_M**	**Iteration number**
10	$9.230866*10^{-4}$	$1.496565*10^{-3}$	203.34	203.34	-	324.75	25.25	$7.21*10^{-2}$	6
10^2	$1.043691*10^{-3}$	$1.489687*10^{-3}$	20.68	20.68	-	347.30	2.70	$7.72*10^{-3}$	5
10^3	$1.056221*10^{-3}$	$1.489616*10^{-3}$	2.37	2.37	-	349.68	0.32	$9.15*10^{-4}$	5
10^4	$1.057479*10^{-3}$	$1.489615*10^{-3}$	0.54	0.54	-	349.92	0.08	$2.32*10^{-4}$	5
10^5	$1.057605*10^{-3}$	$1.489615*10^{-3}$	0.35	0.35	-	349.95	0.06	$1.63*10^{-4}$	5
β=0.0001									
n	**ε_c** (compression)	**ε_s** (tension)	**N** (kN)	**δ_N** (kN)	**dev_N**	**M** (kNm)	**δ_M** (kNm)	**dev_M**	**Iteration number**
10	$9.231257*10^{-4}$	$1.496574*10^{-3}$	203.30	203.30	-	324.76	25.24	$7.21*10^{-2}$	7
10^2	$1.043957*10^{-3}$	$1.489734*10^{-3}$	20.35	20.35	-	347.35	2.65	$7.56*10^{-3}$	7
10^3	$1.056482*10^{-3}$	$1.489662*10^{-3}$	2.04	2.04	-	349.73	0.27	$7.64*10^{-4}$	7
10^4	$1.057738*10^{-3}$	$1.489661*10^{-3}$	0.21	0.21	-	349.97	0.028	$8.13*10^{-5}$	7
10^5	$1.057863*10^{-3}$	$1.489661*10^{-3}$	0.03	0.03	-	350.00	0.00	$1.33*10^{-5}$	7

Table 4.24: Shell results, load below capacity, case 2

3.

M_x = 200 kNm
concrete model: bilinear

β=0.001									
n	**ε_c** (compression)	**ε_s** (tension)	**N** (kN)	**δ_N** (kN)	**dev_N**	**M** (kNm)	**δ_M** (kNm)	**dev_M**	**Iteration number**
10	$6.905206*10^{-4}$	$8.659070*10^{-4}$	73.25	73.25	-	189.35	10.65	$5.32*10^{-2}$	3
10^2	$7.665291*10^{-4}$	$8.633115*10^{-4}$	7.53	7.53	-	198.85	1.15	$5.74*10^{-3}$	4
10^3	$7.746059*10^{-4}$	$8.632344*10^{-4}$	0.69	0.69	-	199.89	0.12	$5.77*10^{-4}$	3
10^4	$7.754212*10^{-4}$	$8.632261*10^{-4}$	0.02	0.02	-	199.99	0.011	$5.57*10^{-5}$	3
10^5	$7.755025*10^{-4}$	$8.632256*10^{-4}$	0.07	0.07	-	200.00	$7*10^{-4}$	$3.60*10^{-6}$	3
β=0.0001									
n	**ε_c** (compression)	**ε_s** (tension)	**N** (kN)	**δ_N** (kN)	**dev_N**	**M** (kNm)	**δ_M** (kNm)	**dev_M**	**Iteration number**
10	$6.905206*10^{-4}$	$8.659070*10^{-4}$	123.59	123.59	-	183.78	16.22	$8.10*10^{-2}$	3
10^2	$7.665291*10^{-4}$	$8.633115*10^{-4}$	12.87	12.87	-	198.25	1.75	$9.00*10^{-3}$	4
10^3	$7.745680*10^{-4}$	$8.632733*10^{-4}$	0.75	0.75	-	199.88	0.12	$5.79*10^{-4}$	4
10^4	$7.753757*10^{-4}$	$8.632730*10^{-4}$	0.08	0.08	-	199.99	0.01	$5.79*10^{-5}$	4
10^5	$7.754565*10^{-4}$	$8.632730*10^{-4}$	0.01	0.01	-	200.00	0.001	$5.80*10^{-6}$	4

Table 4.25: Shell results, load below capacity, case 3

4.

M_x = 350 kNm
concrete model: bilinear

β=0.001									
n	**ε_c** (compression)	**ε_s** (tension)	**N** (kN)	**δ_N** (kN)	**dev_N**	**M** (kNm)	**δ_M** (kNm)	**dev_M**	**Iteration number**
10	$1.208411*10^{-3}$	$1.515337*10^{-3}$	128.16	128.16	-	331.37	18.63	$5.32*10^{-2}$	3
10^2	$1.341426*10^{-3}$	$1.510795*10^{-3}$	13.17	13.17	-	347.99	2.01	$5.74*10^{-3}$	4
10^3	$1.355560*10^{-3}$	$1.510660*10^{-3}$	1.21	1.21	-	349.80	0.20	$5.77*10^{-4}$	3
10^4	$1.356987*10^{-3}$	$1.510646*10^{-3}$	0.04	0.04	-	349.98	0.02	$5.56*10^{-5}$	3
10^5	$1.357129*10^{-3}$	$1.510645*10^{-3}$	0.12	0.12	-	350.00	0.00	$3.6*10^{-6}$	3
β=0.0001									
n	**ε_c** (compression)	**ε_s** (tension)	**N** (kN)	**δ_N** (kN)	**dev_N**	**M** (kNm)	**δ_M** (kNm)	**dev_M**	**Iteration number**
10	$1.208411 *10^{-3}$	$1.515337*10^{-3}$	128.16	128.16	-	331.37	18.63	$5.32*10^{-2}$	3
10^2	$1.341426*10^{-3}$	$1.510795*10^{-3}$	13.17	13.17	-	347.99	2.01	$5.74*10^{-3}$	4
10^3	$1.355494*10^{-3}$	$1.510728*10^{-3}$	1.32	1.32	-	349.80	0.20	$5.79*10^{-4}$	4
10^4	$1.356907*10^{-3}$	$1.510727*10^{-3}$	0.13	0.13	-	349.98	0.02	$5.83*10^{-5}$	4
10^5	$1.357049*10^{-3}$	$1.510728*10^{-3}$	0.01	0.01	-	350.00	0.00	$5.60*10^{-6}$	4

Table 4.26: Shell results, load below capacity, case 4

Comments

- Convergence criterium (β): the value of β has a negligible effect on the accuracy of the result. However, it should be noted that the values of β used are 10^{-3} and 10^{-4}, which are both relatively accurate convergence criteria. The iteration number increases as the value of β decreases.
- Concrete layers (n): the accuracy of both forces and moments increases as the value of n increases. It should be noted that for n=10 and 100, the obtained values of the moments and especially the forces are very different from the original values.

4.3 Columns at load capacity

The examples used in this section are first calculated by hand by using formulas for obtaining the maximum capacity of the section. The results from the program are compared to the hand-calculated results, which are referred to as control results. Due to few available examples for calculating columns, one of the previously used cases with moment and axial force in one direction is used.

4.3.1 Biaxial moment and axial force

The following example is taken from the book 'Betongkonstruksjoner – Beregning og dimensjonering etter Eurocode2' [7]. The corresponding hand calculation is detailed in Appendix E.

The result of the hand calculation is the value of the section's moment capacity in x- and y-direction.

M_{rdx} = 210 kNm

M_{rdy} = 132 kNm

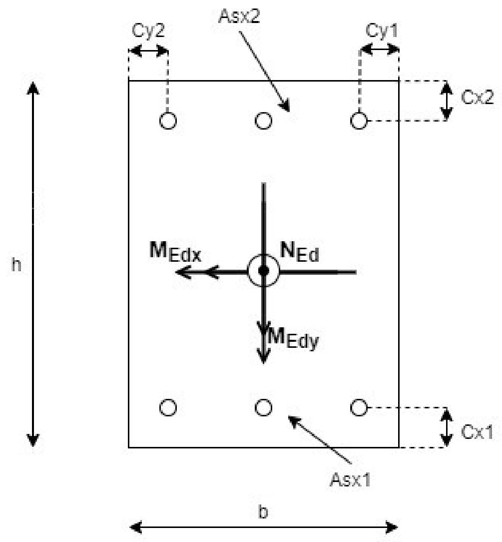

Figure 4.7: Column, axial force and biaxial moment

Input

Symbol	Value	Unit		Symbol	Value	Unit
N	-1500	kN		A_{sx1}	942	mm^2
M_x	variable	kNm		A_{sx2}	942	mm^2
M_y	variable	kNm		A_{sy1}	0	mm^2
				A_{sy2}	0	mm^2
b	300	mm		E_s	200000	N/mm^2
h	400	mm		f_{yk}	500	N/mm^2
c_{x1}	40	mm		γ_s	1.15	
c_{x2}	40	mm		ε_{ud}	0.03	
c_{y1}	0	mm				
c_{y2}	0	mm		concrete model	parabola-rectangle	
				f_{ck}	35	N/mm^2
n	variable			γ_c	1.5	
β	variable					
max it.	1000					

Table 4.27: Column input, biaxial moment and axial force

Results

1. M_x = 210 kNm
   M_y = 0 kNm

	Convergence	
Concrete Layers (n)	**β=0.001**	**β=0.0001**
10	Yes	Yes
30	Yes	Yes
100	Yes	Yes
1000	Yes	Yes

Table 4.28: Column results, case 1

2. M_x = 0 kNm
   M_y = 132 kNm

	Convergence	
Concrete Layers (n)	**β=0.001**	**β=0.0001**
10	Yes	Yes
30	Yes	Yes
100	Yes	Yes
1000	Yes	Yes

Table 4.29: Column results, case 2

Comments

The program shows that the section has the moment and axial force capacity calculated by hand. The solution converges for all values of concrete layer *n* and convergence criterium *β.*

4.3.2 Uniaxial moment and axial force

The following example is the same as the one presented in section 4.1.4, example 1. The only difference is that the calculation is implemented by the algorithm version used to calculate columns.

Input

Symbol	Value	Unit		Symbol	Value	Unit
N	-7983.240	kN		A_{sx1}	4910	mm^2
M_x	471.606	kNm		A_{sx2}	4910	mm^2
M_y	0	kNm		A_{sy1}	0	mm^2
				A_{sy2}	0	mm^2
b	1000	mm		E_s	200000	N/mm^2
h	400	mm		f_{yk}	500	N/mm^2
c_{x1}	40	mm		γ_s	1.15	
c_{x2}	40	mm		ε_{ud}	0.03	
c_{y1}	0	mm				
c_{y2}	0	mm		concrete model	parabola-rectangle	
				f_{ck}	30	N/mm^2
n	variable			γ_c	1.5	
β	variable					
max it.	1000					

Table 4.30: Column input, uniaxial moment and axial force

Results

Concrete

	β=0.001		**β=0.0001**	
Concrete Layers	**Stress** (N/mm^2)	**Iteration number**	**Stress** (N/mm^2)	**Iteration number**
10	-17.00	9	-17.00	11
30	-17.00	46	-17.00	81
100	-17.00	60	-17.00	105
1000	-17.00	66	-17.00	116
Control	-17.00		-17.00	
Concrete Layers	**Strain** (‰)	**Iteration number**	**Strain** (‰)	**Iteration number**
10	-2.158	9	-2.160	11
30	-2.798	46	-2.824	81
100	-3.218	60	-3.257	105
1000	-3.421	66	-3.469	116
Control	-3.500		-3.500	

Table 4.31: Column concrete results, uniaxial moment and axial force

Reinforcement

	β=0.001		**β=0.0001**	
Concrete Layers	**Max Stress** (N/mm^2)	**Iteration number**	**Max Stress** (N/mm^2)	**Iteration number**
10	-431.69	9	-431.91	11
30	-434.78	46	-434.78	81
100	-434.78	60	-434.78	105
1000	-434.78	66	-434.78	116
Control	-434.78		-434.78	
Concrete Layers	**Max Strain** (‰)	**Iteration number**	**Max Strain** (‰)	**Iteration number**
10	-2.158	9	-2.160	11
30	-2.616	46	-2.640	81
100	-2.931	60	-2.966	105
1000	-3.083	66	-3.126	116
Control	3.150		-3.150	

Table 4.32: Column reinforcement results, uniaxial moment and axial force

Comments

According to the hand calculations detailed in Appendix B.4.1, the whole section is under compression, and the failure is due to compression fracture in concrete. As for reinforcement, the top reinforcement yields while the bottom reinforcement does not. The column algorithm displays only the maximum compressive strain and stress for concrete and the maximum compressive and tensile strain and stress for reinforcement. The obtained values are not as accurate as those calculated by the shell and beam version of the program.

- Convergence criterium (β): The stress and strain values for both reinforcement and concrete increase in accuracy as the value of β decreases. The iteration number increases as β decreases.
- Concrete layers (n): The stress and strain values for both concrete and reinforcement increase in accuracy as n increases. The iteration number increases with n.

5 Conclusion

The computer program developed in this thesis fulfills the expected results. It can perform the capacity control for beams, shells, and columns.

The literature studies and theories represent the basis for the development of the program. The comparison between known results and the results from the program shows that the method used and its code implementation are correct. However, there is room for improvement and optimization of the code.

The known results used in the verification of the program are hand calculations and results from an approved iteration method program. The comparison with these known results aims to test the program in general and the effect of the number of concrete layers n, convergence criterium β on its accuracy.

For beams and shells at load capacity, the value of n should be at least 100. A more accurate convergence criterium (β=0.0001) is needed to obtain satisfactory results in the case of very high reinforcement strains. When calculating columns, the reinforcement is also subdivided into n layers. Therefore, a high ($n=10^3$) subdivision number is needed to obtain accurate results. When calculating with loads below capacity, layer subdivision has a significant impact on the result accuracy. It is preferable to use a value of n of at least 10^3 to obtain relatively precise results.

Based on the testing, it can be stated that the accuracy of the results increases with n and decreases as β increases. In the following discussion on the accuracy of the program, the most accurate results obtained with $n=10^3$ and $\beta=0.0001$ are considered.

When comparing the results between hand calculations and the computer program, it can be shown that they are very similar with some minor differences. The differences could be caused by the fact that the program uses more decimal numbers than the corresponding hand calculations.

When comparing the results with those from the approved iteration computer program, the reinforcement stress and strain are nearly equal to the control results. In contrast, the concrete strain and stress values show some minor differences. Lack of information about the material models and general criteria used in the approved computer program makes it difficult to comment on the cause of the differences. However, given the complexity of the example, with six sectional forces and reinforcement in both directions, the comparison is satisfactory.

The verification of the column results consists of two examples. The comparison with these known results is satisfactory. However, more testing and comparisons need to be carried out to ensure the accuracy of the method.

Based on the examples, the programs can accurately calculate beams, shells, and columns. However, as a new program, it needs to be improved, tested with complicated examples, and updated.

A list of proposals for further development of the program follows:

- Verification of the program with other programs that don’t implement the iteration method and improving it accordingly.
- Develop the program with an option to implement the effect of multiaxial effects of the uniaxial stress-strain relationship of concrete. This would mean a reduced compressive strength for cracked concrete.
- Update the program such that it displays the utilization ratio in case of no-convergence, giving the user a better understanding of the no-convergence causes.

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[6] S. Cha, R. N. Taylor, and K. Kang, Eds. *Handbook of Software Engineering*. Cham, Switzerland: Springer, 2019.

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Appendices

A. Internal forces and moments in a reinforced concrete section

Based on the concrete and reinforcement models described in chapter 2, the derivation of the formulas for calculating internal forces and moments when the strain distribution in a reinforced concrete section is known is presented here.

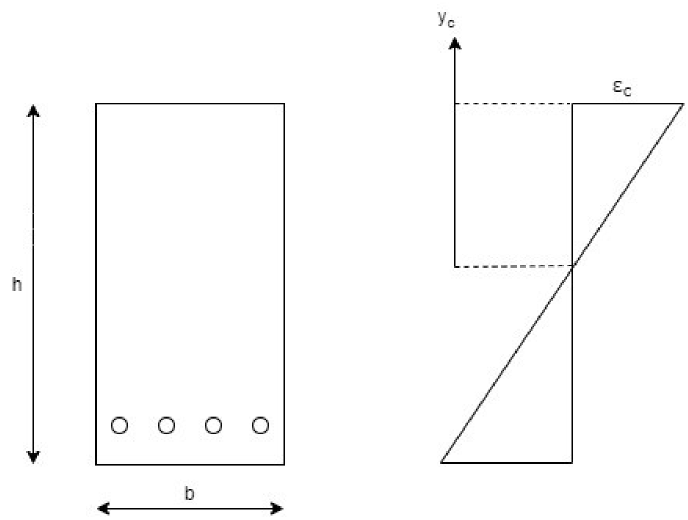


Figure A.1: Strain distribution

A.1 Parabola – rectangle Concrete model

As described in chapter 2, the parabola-rectangle concrete model is an idealized model, and the following formulas calculate the strain-stress relation.

A.1.1 Force and moment when $0 \le \varepsilon_c \le \varepsilon_{c2}$

$$\varepsilon_c = y_c \cdot \kappa$$

$$\sigma_c = f_{cd}\left[1-\left(1-\frac{\varepsilon_c}{\varepsilon_{c2}}\right)^n\right] \text{ for } 0 \le \varepsilon_c \le \varepsilon_{c2}$$

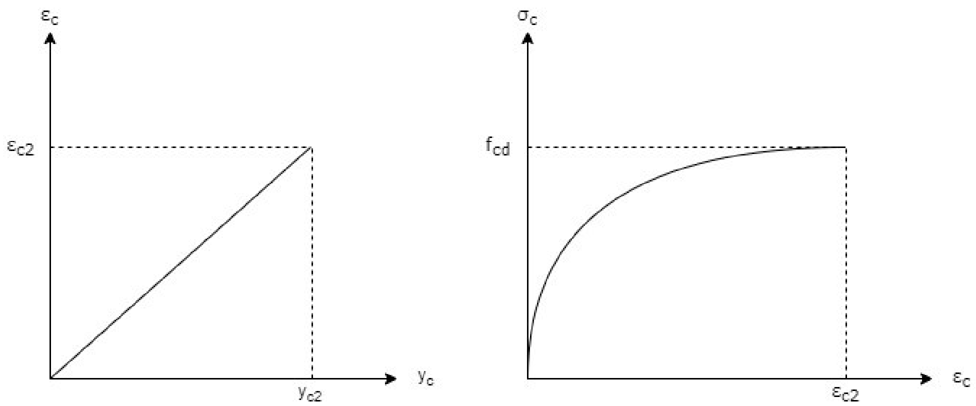


Figure A.2: Strain distribution, Stress-strain relationship

The strain distribution across the compressed part of the concrete section and the stress-strain relationship when $0 \le \varepsilon_c \le \varepsilon_{c2}$ is shown in Figure A.2.

Force F_c when $0 \le \varepsilon_c \le \varepsilon_{c2}$

Based on the previously described stress-strain relations, the resulting force Fc can be calculated by integrating the stress values across the compressed concrete section.

$$\sigma_c(y_c) = f_{cd} \cdot \left[1 - \left(1 - \frac{y_c \cdot \kappa}{\varepsilon_{c2}}\right)^n\right]$$

$$F_c = b \cdot \int_0^{y_c} \sigma_c(y_c)\, dy_c = b \cdot \int_0^{y_c} f_{cd} \cdot \left[1 - \left(1 - \frac{y_c \cdot \kappa}{\varepsilon_{c2}}\right)^n\right] dy_c$$

$$F_c = bf_{cd} \cdot \left[y_c - \left(-\frac{\varepsilon_{c2}}{\kappa(n+1)} \left(1 - \frac{y_c \cdot \kappa}{\varepsilon_{c2}}\right)^{n+1} \right) \right]_0^{y_c}$$

$$F_c = bf_{cd} \cdot \left\{ y_c + \frac{\varepsilon_{c2}}{\kappa(n+1)} \left[\left(1 - \frac{y_c \cdot \kappa}{\varepsilon_{c2}}\right)^{n+1} - 1 \right] \right\}$$

This is a general formula for strains below or equal to the strain at reaching the maximum strength (ε_{c2}). If the strain is equal to ε_{c2}, the formula can be simplified as:

If $y_c = y_{c2} \longrightarrow \varepsilon_c = \varepsilon_{c2}$: $\varepsilon_{c2} = y_{c2} \cdot \kappa$

$$F_c = bf_{cd} \cdot \left\{ y_{c2} + \frac{\varepsilon_{c2}}{\kappa(n+1)} \left[\left(1 - \frac{y_{c2} \cdot \kappa}{\varepsilon_{c2}} \right)^{n+1} \right] \right\}$$

$$F_c = bf_{cd} \cdot \left\{ y_{c2} - \frac{\varepsilon_{c2}}{\kappa(n+1)} \right\}$$

$$F_c = bf_{cd} \cdot \left(y_{c2} - \frac{y_{c2}}{(n+1)} \right)$$

$$F_c = b \cdot f_{cd} \cdot y_{c2} \cdot \left(1 - \frac{1}{n+1} \right)$$

The coefficient n has a value of 2 for concrete strength class 50 and below. In that case, the formula for the force F_c is further simplified:

For n=2: $F_c = \frac{2}{3} b \cdot f_{cd} \cdot y_{c2}$

Moment M_c when $0 \le \varepsilon_c \le \varepsilon_{c2}$

In order to calculate the resulting moment, first, the neutral axis of the parabolic shape has to be found, as the formula for the moment is:

$$M_c = F_c \cdot \overline{y}$$

The neutral axis of the compressed concrete section is calculated by:

$$\overline{y} = \frac{\int_0^{y_c} \sigma_c(y_c) \cdot y_c \, dy_c}{\int_0^{y_c} \sigma_c(y_c) \, dy_c} \quad \rightarrow \quad \frac{(1)}{(2)}$$

(2) $$\int_0^{y_c} \sigma_c(y_c) \, dy_c = f_{cd} \cdot \left\{ y_c + \frac{\varepsilon_{c2}}{\kappa(n+1)} \left[\left(1 - \frac{y_c \cdot \kappa}{\varepsilon_{c2}} \right)^{n+1} - 1 \right] \right\}$$

$$\int_0^{y_c} \sigma_c(y_c)\cdot y_c dy_c = \int_0^{y_c} f_{cd}\left[1-\left(1-\frac{y_c\cdot\kappa}{\varepsilon_{c2}}\right)^n\right]\cdot y dy$$

(1)
$$= f_{cd}\left[\int_0^{y_c} y_c dy_c - \int_0^{y_c} y_c\left(1-\frac{y_c\cdot\kappa}{\varepsilon_{c2}}\right)^n dy_c\right]$$

$$= f_{cd}\left\{\frac{y_c^2}{2}-\left(\frac{\varepsilon_{c2}}{\kappa}\right)^2\cdot\left[\left(1-\frac{y_c\cdot\kappa}{\varepsilon_{c2}}\right)^{n+1}\left(\frac{1-\frac{y_c\cdot\kappa}{\varepsilon_{c2}}}{n+2}-\frac{1}{n+1}\right)-\left(\frac{1}{n+2}-\frac{1}{n+1}\right)\right]\right\}$$

The obtained results are difficult to simplify. However, when the strain value is ε_{c2}, the expression can be simplified as:

If $y_c = y_{c2} \longrightarrow \varepsilon_c = \varepsilon_{c2}$: $\quad \varepsilon_{c2} = y_{c2}\cdot\kappa$

(2): $f_{cd}\cdot y_{c2}\cdot\left(1-\frac{1}{n+1}\right)$

(1): $f_{cd}\cdot y_{c2}^2\cdot\left(\frac{1}{2}-\frac{1}{n+1}+\frac{1}{n+2}\right)$

Furthermore, if the coefficient *n=2*, the expression can be simplified as:

(2): $\frac{2}{3}\cdot f_{cd}\cdot y_{c2}$

(1): $\frac{5}{12}\cdot f_{cd}\cdot y_{c2}^2$

The neutral axis of the compressed concrete section when the strain value is ε_{c2} and the coefficient *n=2,* can be written as:

$$\bar{y} = \frac{(1)}{(2)} = \frac{\frac{5}{12}\cdot f_{cd}\cdot y_{c2}^2}{\frac{2}{3}\cdot f_{cd}\cdot y_{c2}} = \frac{5}{8}y_{c2}$$

A.1.2 Force and moment when $\varepsilon_{c2} \le \varepsilon_c \le \varepsilon_{cu2}$

The following calculations represent the case when the strain is higher than ε_{c2} and lower than the ultimate strain (ε_{cu2})

$$\varepsilon_c = y_c\cdot\kappa \qquad\qquad \sigma_c = f_{cd} \text{ for } \varepsilon_{c2}\le\varepsilon_c\le\varepsilon_{cu2}$$

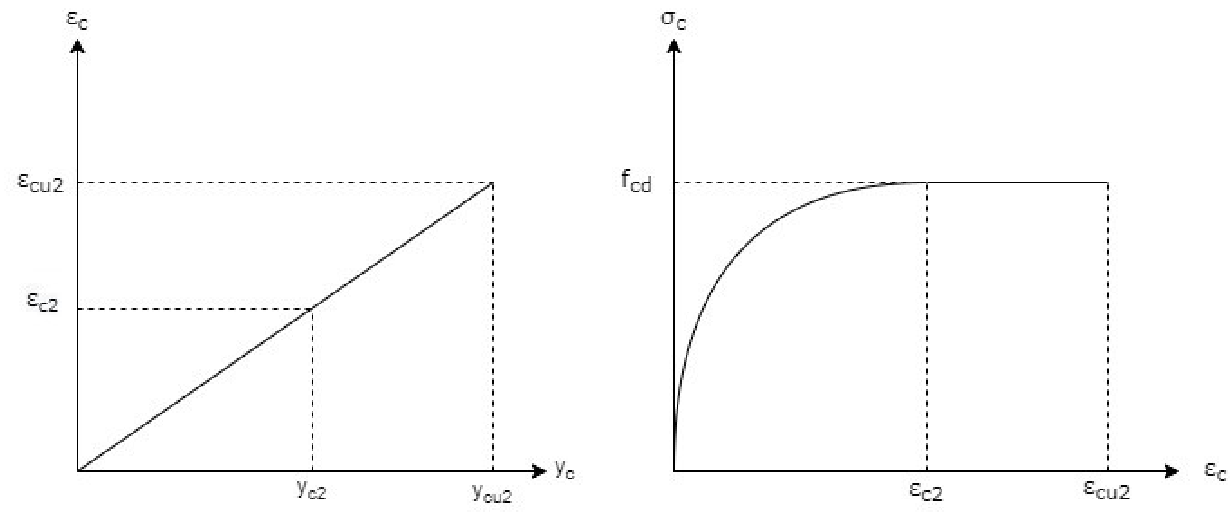


Figure A.3: Stress distribution, Stress-strain relationship

In the following calculations, concrete strain values will be assumed equal to the values for

concrete classes C12 to C50 where ε_{c2} is 2.00‰ and ε_{cu2} is 3.5‰.

Force *F_c* when $\varepsilon_{c2} \le \varepsilon_c \le \varepsilon_{cu2}$

By using the formula for similar triangles, $y_{c2} = \frac{4}{7} y_{cu2}$.

The resulting force is the sum of the force when $0 \le \varepsilon_c \le \varepsilon_{c2}$ (calculated previously) and the force when $\varepsilon_{c2} \le \varepsilon_c \le \varepsilon_{cu2}$. The latter can be easily calculated as the stress is constant and equal to f_{cd}.

The resulting force is:

$$F_c = \frac{2}{3} b \cdot f_{cd} \cdot \frac{4}{7} y_{cu2} + b \cdot f_{cd} \cdot \frac{3}{7} y_{cu2} = \frac{17}{21} b f_{cd} y_{cu2}$$

Moment *M_c* when $\varepsilon_{c2} \le \varepsilon_c \le \varepsilon_{cu2}$

The position of the neutral axis, where the force F_c acts, is calculated by the static formula: $\bar{y} = \frac{\sum A_i \cdot y_i}{\sum A_i}$, which is equal to:

$$\bar{y} = \frac{139}{238} y_{cu2}$$

The resulting moment is:

$$M_c = F_c \cdot \bar{y} = \frac{139}{294} b f_{cd} y_{cu2}^2$$

A.2 Bilinear Concrete model

As described in chapter 2, the bilinear concrete model is an idealized model, and the following formulas calculate the strain-stress relation.

A.2.1 Force and moment when $0 \le \varepsilon_c \le \varepsilon_{c3}$

$$\varepsilon_c = y_c \cdot \kappa \qquad\qquad \sigma_c = f_{cd} \cdot \frac{\varepsilon_c}{\varepsilon_{c3}} \quad \text{for} \quad 0 \le \varepsilon_c \le \varepsilon_{c3}$$

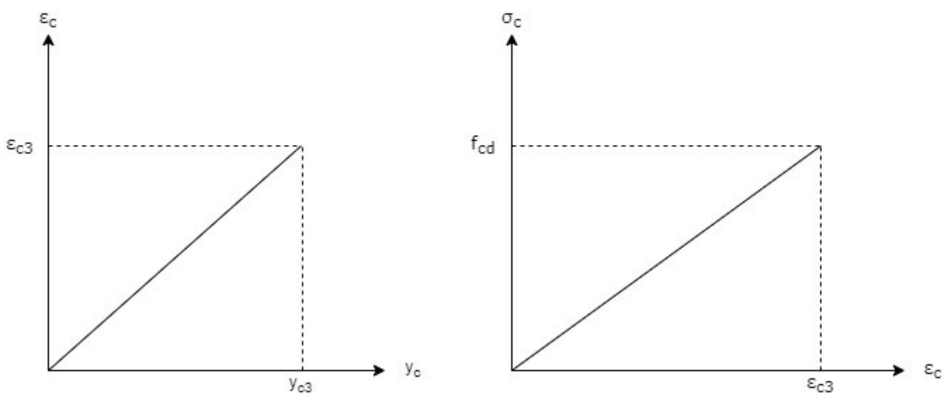


Figure A.4: Strain distribution, Stress-strain relationship

Force F_c when $0 \le \varepsilon_c \le \varepsilon_{c3}$

Based on the previously described stress-strain relations, the resulting force Fc can be calculated by integrating the stress values across the compressed concrete section.

$$\sigma_c(y_c) = f_{cd} \cdot \frac{\varepsilon_c}{\varepsilon_{c3}} = f_{cd} \cdot \frac{y_c \cdot \kappa}{\varepsilon_{c3}}$$

$$F_c = b \cdot \int_0^{y_c} \sigma_c(y_c)\, dy_c = b \cdot \int_0^{y_c} f_{cd} \cdot \frac{y_c \cdot \kappa}{\varepsilon_{c3}} dy_c$$

$$F_c = b f_{cd} \cdot \left[\frac{y_c^2 \cdot \kappa}{2\varepsilon_{c3}} \right]_0^{y_c}$$

$$F_c = \frac{b \cdot f_{cd} \cdot y_c^2 \cdot \kappa}{2\varepsilon_{c3}}$$

This is a general formula for strains below or equal to the strain at reaching the maximum strength (ε_{c3}). If the strain is equal to ε_{c3}, the formula can be simplified as:

If $y_c = y_{c3} \longrightarrow \varepsilon_c = \varepsilon_{c3}$: $\qquad \varepsilon_{c3} = y_{c3} \cdot \kappa$

The resulting force is:

$$F_c = \frac{1}{2} b \cdot f_{cd} \cdot y_{c3}$$

Moment *Mc* when $0 \le \varepsilon_c \le \varepsilon_{c3}$

In order to calculate the resulting moment, first, the neutral axis position of the compressed section has to be found.

$$\bar{y} = \frac{2}{3} y_c$$

The moment M_c is thus calculated as:

$$M_c = F_c \cdot \bar{y} = \frac{1}{3} \frac{b f_{cd} y_c^3 \kappa}{\varepsilon_{c3}}$$

This is a general moment formula when $0 \le \varepsilon_c \le \varepsilon_{c3}$. However, in the specific case when the strain has value ε_{c3}, the expression is simplified, and the following expression is obtained.

If $y_c = y_{c3}$: $\bar{y} = \frac{2}{3} y_{c3}$ $\qquad$ $M_c = F_c \cdot \bar{y} = \frac{1}{3} b f_{cd} y_{c3}^2$

A.2.2 Force and moment when $\varepsilon_{c3} \le \varepsilon_c \le \varepsilon_{cu3}$

The following calculations represent the case when the strain is higher than ε_{c3} and lower than the ultimate strain (ε_{cu3}), as shown in Figure A.5.

$$\varepsilon_c = y_c \cdot \kappa \qquad \sigma_c = f_{cd} \text{ for } \varepsilon_{c3} \le \varepsilon_c \le \varepsilon_{cu3}$$

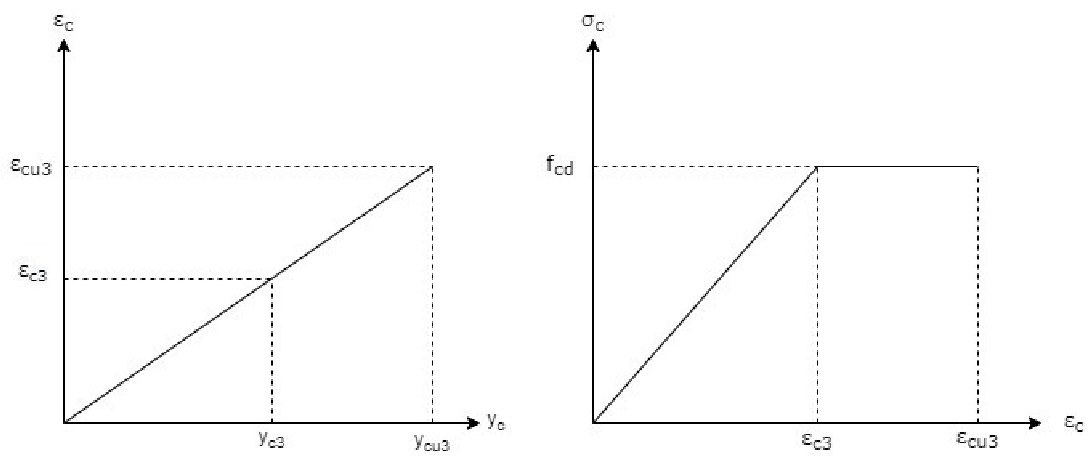


Figure A.5: Strain distribution, Stress-strain relationship

In the following calculations, concrete strain values will be assumed equal to the values for concrete classes C12 to C50, where ε_{c3} is 1.75‰ and ε_{cu3} is 3.0‰.

Force *Fc* when $\varepsilon_{c3} \leq \varepsilon_c \leq \varepsilon_{cu3}$

By using the formula for similar triangles, $y_{c3} = \frac{1}{2} y_{cu3}$.

The resulting force is the sum of the force when $0 \leq \varepsilon_c \leq \varepsilon_{c3}$ (calculated previously) and the force when $\varepsilon_{c3} \leq \varepsilon_c \leq \varepsilon_{cu3}$. The latter can be easily calculated as the stress is constant and equal to f_{cd}.

$$F_c = \frac{1}{2} b \cdot f_{cd} \cdot y_{c3} + b f_{cd} \cdot (y_{cu3} - y_{c3}) = \frac{3}{4} b f_{cd} y_{cu3}$$

Moment *Mc* when $\varepsilon_{c3} \leq \varepsilon_c \leq \varepsilon_{cu3}$

The neutral axis position, where the force F_c acts is calculated by the static formula:

$\bar{y} = \frac{\sum A_i \cdot y_i}{\sum A_i}$ which is equal to:

$$\bar{y} = \frac{11}{18} y_{cu3}$$

The resulting moment is:

$$M_c = \frac{11}{24} b f_{cd} y_{cu3}^2$$

B. Hand calculations for capacity control of beams

Examples 1 to 3 represent single forces or moments acting in one direction

Examples 4 to 7 represent combinations of forces and moments acting in one direction

B.1 Compression

$$\sigma_c = \frac{N_x}{A} = \frac{1.7 \cdot 10^6 N}{10^5 mm^2} = 17 N/mm^2$$

$$\sigma_c = f_{cd} \rightarrow \varepsilon_c = \varepsilon_{c2} = 0.002$$

B.2 Tension

$$A_s = \frac{N_x}{f_{yd}} = \frac{500 N/mm}{434.78 N/mm^2} = 1.15 mm^2/mm$$

The reinforcement area is set to $A_s = 1.16 mm^2/mm = 1160 mm^2/m$

Since concrete is assumed to have zero strength in tension, all the force is taken by the reinforcement:

$$\sigma_s = \frac{N_x}{A} = \frac{5.0 \cdot 10^5 N}{1160 mm^2} = 431.03 N/mm^2$$

$$\varepsilon_s = \frac{\sigma_s}{E_s} = \frac{431.03 N/mm^2}{200000 N/mm^2} = 0.002155$$

B.3 Moment in one direction

Yield strain: $\varepsilon_{yd} = \frac{f_{yd}}{E_s} = \frac{434.782 N/mm^2}{200000 N/mm^2} = 2.173 \cdot 10^{-3}$

Balanced reinforcement ratio:

$$\alpha_b = \frac{3.5 \cdot 10^{-3}}{3.5 \cdot 10^{-3} + 2.173 \cdot 10^{-3}} = 0.617$$

$$A_{s,b} = \frac{\frac{17}{21} f_{cd} \cdot \alpha_b \cdot d \cdot b}{f_{yd}} = \frac{\frac{17}{21} 17 \cdot 0.617 \cdot 365 \cdot 1000}{434.782} = 7128.2 mm^2$$

$A_s = 3768 mm^2 < A_{s,b} = 7128.2 mm^2$, the section is under-reinforced

The reinforcement ratio of the section is:

$$\alpha = \frac{f_{yd} \cdot A_s}{\frac{17}{21} f_{cd} \cdot b \cdot d} = \frac{434.782 \cdot 3768}{\frac{17}{21} \cdot 17 \cdot 1000 \cdot 365} = 0.3261$$

Reinforcement strain:

$$\varepsilon_s = \frac{1-\alpha}{\alpha} \cdot \varepsilon_{cu} = \frac{1-0.3261}{0.3261} \cdot 0.0035 = 7.232 \cdot 10^{-3} < \varepsilon_{ud} = 3.0 \cdot 10^{-2}$$

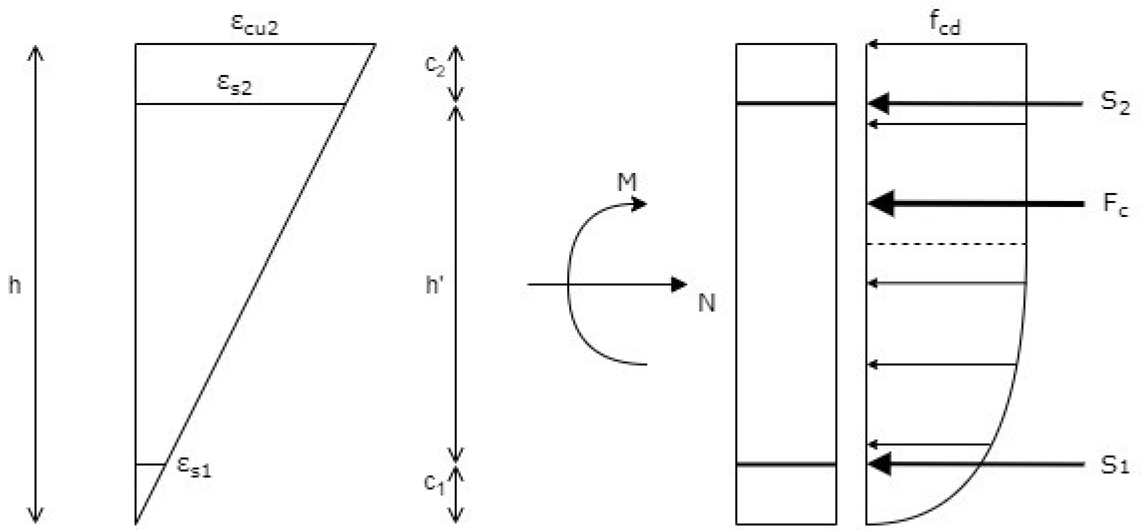
Moment capacity of the section:

$$M_{rd} = \frac{17}{21} \cdot f_{cd} \cdot \alpha \cdot \left(1 - \frac{99}{238}\alpha\right) b \cdot d^2$$

$$M_{rd} = \frac{17}{21} \cdot 17 \cdot 0.3261 \cdot \left(1 - \frac{99}{238} \cdot 0.3261\right) \cdot 1000 \cdot 365^2 = 516.780 kNm$$

B.4 Moment and axial force in one direction

B.4.1 Compression fracture in concrete



$$\varepsilon_{s1} = \frac{c_1}{h} \cdot \varepsilon_{cu2} = \frac{40}{400} \cdot 0.0035 = 3.5 \cdot 10^{-4} < \varepsilon_{yd} = 2.17 \cdot 10^{-3}$$

$$\sigma_{sd1} = E_s \cdot \varepsilon_{s1} = 200000 \cdot 3.5 \cdot 10^{-4} = 70 N/mm^2$$

$$\varepsilon_{s2} = \frac{h - c_1}{h} \cdot \varepsilon_{cu2} = \frac{360}{400} \cdot 0.0035 = 3.15 \cdot 10^{-3} > \varepsilon_{yd} = 2.17 \cdot 10^{-3}$$

$$\sigma_{sd2} = f_{yd} = 434.782 N/mm^2$$

$$F_c = \frac{17}{21} \cdot f_{cd} \cdot h \cdot b = \frac{17}{21} \cdot 17 \cdot 400 \cdot 1000 = 5504.761 kN$$

$$S_1 = \sigma_{sd1} \cdot A_{s1} = 70 \cdot 4910 = 343.700 kN$$

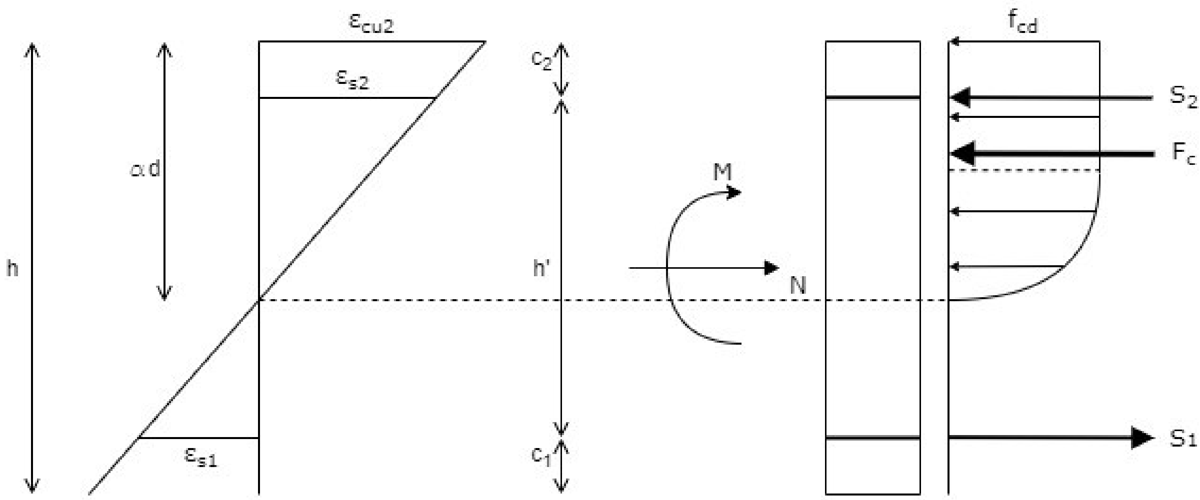
$$S_2 = \sigma_{sd2} \cdot A_{s2} = 434.782 \cdot 4910 = 2134.779 kN$$

$$N = F_c + S_1 + S_2 = 5504.761 + 343.700 + 2134.779 = 7983.240 kN$$

$$M = F_c\left(\frac{h}{2} - \frac{99}{238}h\right) + S_2\left(\frac{h}{2} - c_2\right) - S_1\left(\frac{h}{2} - c_1\right)$$

$$M = 5504.761 \cdot \frac{10}{119} \cdot 0.4 + 2134.779(0.2 - 0.04) - 343.7(0.2 - 0.04)$$
$$M = 471.606 kNm$$

B.4.2 Compression fracture in concrete and yield in reinforcement



$$\varepsilon_{s1} = \varepsilon_{yd} = 2.173 \cdot 10^{-3}$$

$$\sigma_{sd1} = f_{yd} = 434.782 N/mm^2$$

$$\alpha d = \frac{3.5 \cdot 10^{-3}}{3.5 \cdot 10^{-3} + 2.173 \cdot 10^{-3}} \cdot 360 = 222.104mm$$

$$\varepsilon_{s2} = \frac{\alpha d - c_2}{\alpha d} \cdot \varepsilon_{cu2} = \frac{222.104 - 40}{222.104} \cdot 0.0035 = 2.870 \cdot 10^{-3} > \varepsilon_{yd} = 2.17 \cdot 10^{-3}$$

$$\sigma_{sd2} = f_{yd} = 434.782 N/mm^2$$

$$F_c = \frac{17}{21} \cdot f_{cd} \cdot \alpha d \cdot b = \frac{17}{21} \cdot 17 \cdot 222.104 \cdot 1000 = 3056.574kN$$

$$S_1 = \sigma_{sd1} \cdot A_{s1} = 434.782 \cdot 4910 = 2134.779kN$$

$$S_2 = \sigma_{sd2} \cdot A_{s2} = 434.782 \cdot 4910 = 2134.779kN$$

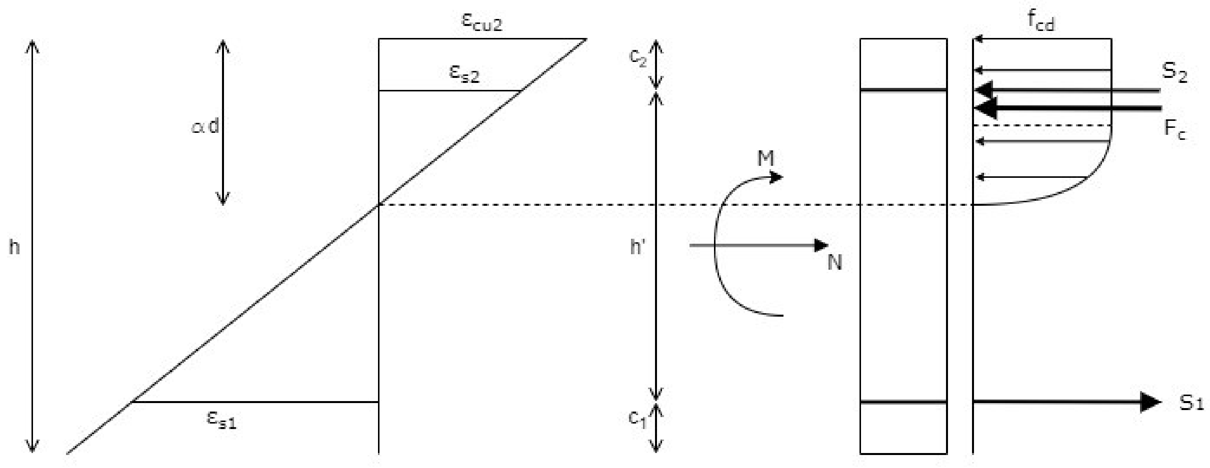
$$N = F_c - S_1 + S_2 = 3056.574 - 2134.779 + 2134.779 = 3056.574kN$$

$$M = F_c\left(\frac{h}{2} - \frac{99}{238}\alpha d\right) + S_1 \cdot h'$$

$$M = 3056.574\left(200 - \frac{99}{238} \cdot 222.104\right)10^{-3} + 2134.779 \cdot 320 \cdot 10^{-3}$$

$$M = 1012.053kNm$$

B.4.3 Compression fracture in concrete and double yield strain in reinforcement



$$\varepsilon_{s1} = 2 * \varepsilon_{yk} = 5.0 \cdot 10^{-3}$$

$$\sigma_{sd1} = f_{yd} = 434.782 N/mm^2$$

$$\alpha d = \frac{3.5 \cdot 10^{-3}}{3.5 \cdot 10^{-3} + 5.0 \cdot 10^{-3}} \cdot 360 = 148.235mm$$

$$\varepsilon_{s2} = \frac{\alpha d - c_2}{\alpha d} \cdot \varepsilon_{cu2} = \frac{148.235 - 40}{148.235} \cdot 0.0035 = 2.556 \cdot 10^{-3} > \varepsilon_{yd} = 2.17 \cdot 10^{-3}$$

$$\sigma_{sd2} = f_{yd} = 434.782N/mm^2$$

$$F_c = \frac{17}{21} \cdot f_{cd} \cdot \alpha d \cdot b = \frac{17}{21} \cdot 17 \cdot 148.235 \cdot 1000 = 2039.995kN$$

$$S_1 = \sigma_{sd1} \cdot A_{s1} = 434.782 \cdot 4910 = 2134.779kN$$

$$S_2 = \sigma_{sd2} \cdot A_{s2} = 434.782 \cdot 4910 = 2134.779kN$$

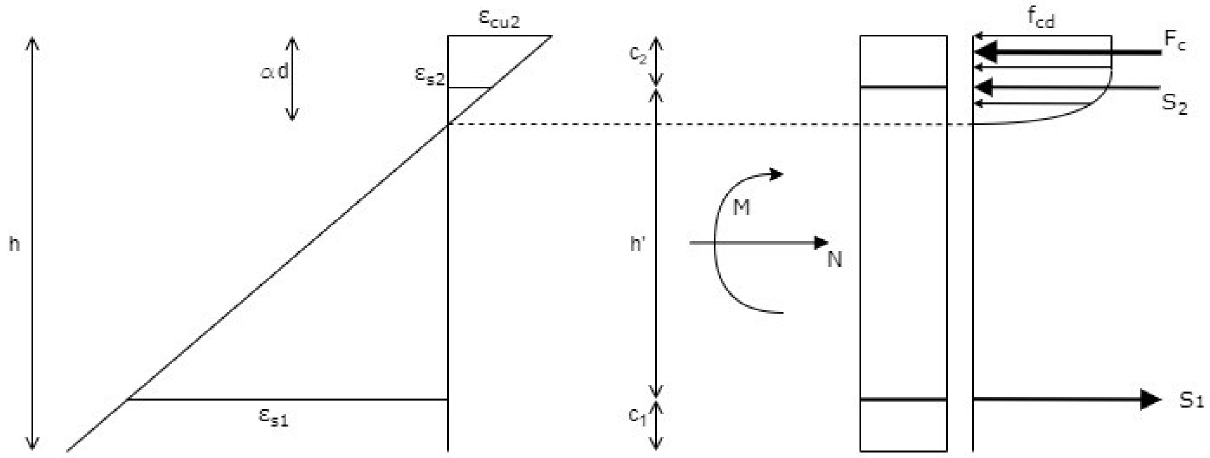
$$N = F_c - S_1 + S_2 = 2039.995 - 2134.779 + 2134.779 = 2039.995kN$$

$$M = F_c\left(\frac{h}{2} - \frac{99}{238}\alpha d\right) + S_1 \cdot h'$$

$$M = 2039.995\left(200 - \frac{99}{238} \cdot 148.235\right)10^{-3} + 2134.779 \cdot 320 \cdot 10^{-3}$$

$$M = 965.340kNm$$

B.4.4 Compression fracture in concrete and high strain level in reinforcement



$$\varepsilon_{s1} = 1.5 \cdot 10^{-2}$$

$$\sigma_{sd1} = f_{yd} = 434.782N/mm^2$$

$$\alpha d = \frac{3.5 \cdot 10^{-3}}{3.5 \cdot 10^{-3} + 1.5 \cdot 10^{-2}} \cdot 360 = 68.108mm$$

$$\varepsilon_{s2} = \frac{\alpha d - c_2}{\alpha d} \cdot \varepsilon_{cu2} = \frac{68.108 - 40}{68.108} \cdot 0.0035 = 1.444 \cdot 10^{-3} < \varepsilon_{yd} = 2.17 \cdot 10^{-3}$$

$$\sigma_{sd2} = E_s \cdot \varepsilon_{s2} = 200000 \cdot 1.444 \cdot 10^{-3} = 288.888N/mm^2$$

$$F_c = \frac{17}{21} \cdot f_{cd} \cdot \alpha d \cdot b = \frac{17}{21} \cdot 17 \cdot 68.108 \cdot 1000 = 937.295kN$$

$$S_1 = \sigma_{sd1} \cdot A_{s1} = 434.782 \cdot 4910 = 2134.779kN$$

$$S_2 = \sigma_{sd2} \cdot A_{s2} = 288.888 \cdot 4910 = 1418.440kN$$

$$N = F_c - S_1 + S_2 = 937.295 - 2134.779 + 1418.44 = 220.956kN$$

$$M = F_c\left(\frac{h}{2} - \frac{99}{238}\alpha d\right) + S_1 \cdot \frac{h'}{2} + S_2 \cdot \frac{h'}{2}$$

$$M = 937.295\left(200 - \frac{99}{238} \cdot 68.108\right)10^{-3} + 2134.779 \cdot 160 \cdot 10^{-3} + 1418.440 \cdot 160 \cdot 10^{-3}$$

$$M = 729.419kNm$$

C. Hand calculations for loads below the capacity of beams

The formulas for calculating the resulting forces and moments from strain values in concrete and reinforcements are obtained in Appendix A. The calculation process is implemented in two Mathcad templates, one for each concrete model. An example of each template is presented here.

C.1 Parabola-rectangle concrete model

Input

$M_x := 350\ kN \cdot m$

$A_s := 3768\ mm^2$

$b := 1000\ mm$

$d := 365\ mm$

$\varepsilon_c := 1.057863 \cdot 10^{-3}$

$\varepsilon_{c2} := 0.002$

$f_{cd} := 17\ \frac{N}{mm^2}$

$\varepsilon_s := 1.489661 \cdot 10^{-3}$

$E_s := 2 \cdot 10^5\ \frac{N}{mm^2}$

Calculations

Force

$$k := \frac{\varepsilon_c + \varepsilon_s}{d} = \left(6.98 \cdot 10^{-6}\right) \frac{1}{mm}$$

$$F_c := f_{cd} \cdot b \cdot \left(\frac{\varepsilon_c}{k} + \frac{\varepsilon_{c2}}{3 \cdot k} \cdot \left(\left(1 - \frac{\varepsilon_c}{\varepsilon_{c2}} \right)^3 - 1 \right) \right) = \left(1.123 \cdot 10^3\right) kN$$

$$S := \varepsilon_s \cdot E_s \cdot A_s = \left(1.123 \cdot 10^3\right) kN$$

$$\delta_N := \left| F_c - S \right| = 0.032\ kN$$

Moment

$$eq1 := f_{cd} \cdot \left(\frac{1}{2} \cdot \left(\frac{\varepsilon_c}{k} \right)^2 - \left(\frac{\varepsilon_{c2}}{k} \right)^2 \cdot \left(\left(1 - \frac{\varepsilon_c}{\varepsilon_{c2}} \right)^3 \cdot \left(\frac{1 - \frac{\varepsilon_c}{\varepsilon_{c2}}}{4} - \frac{1}{3} \right) - \left(\frac{1}{4} - \frac{1}{3} \right) \right) \right) = \left(1.104 \cdot 10^5\right) \, \mathbf{N}$$

$$eq2 := f_{cd} \cdot \left(\frac{\varepsilon_c}{k} + \frac{\varepsilon_{c2}}{3\ k} \cdot \left(\left(1 - \frac{\varepsilon_c}{\varepsilon_{c2}} \right)^3 - 1 \right) \right) = \left(1.123 \cdot 10^3\right) \, \frac{\mathbf{N}}{\mathbf{mm}}$$

$$y_{cN.A.} := \frac{eq1}{eq2} = 98.341 \ \mathbf{mm}$$

$$z := d - \left(\frac{\varepsilon_c}{k} - y_{cN.A.} \right) = 311.774 \ \mathbf{mm}$$

$$M_c := F_c \cdot z = 349.99 \ \mathbf{kN \cdot m}$$

$$M_s := S \cdot z = 350 \ \mathbf{kN \cdot m}$$

$$M_{av} := \frac{M_c + M_s}{2} = 349.995 \ \mathbf{kN \cdot m}$$

$$\delta_M := \left| M_{av} - M_x \right| = 0.005 \ \mathbf{kN \cdot m}$$

$$dev_M := \frac{\delta_M}{M_x} = 0.0000133$$

C.2 Bilinear concrete model

Input

$M_x := 350\ kN \cdot m$

$A_s := 3768\ mm^2$

$b := 1000\ mm$

$d := 365\ mm$

$\varepsilon_c := 1.357049 \cdot 10^{-3}$

$\varepsilon_{c3} := 0.00175$

$f_{cd} := 17\ \frac{N}{mm^2}$

$\varepsilon_s := 1.510728 \cdot 10^{-3}$

$E_s := 2 \cdot 10^5\ \frac{N}{mm^2}$

$E_c := \frac{f_{cd}}{\varepsilon_{c3}} = \left(9.714 \cdot 10^3\right)\ \frac{N}{mm^2}$

Calculations

Force

$\eta := \frac{E_s}{E_c} = 20.588$

$\rho := \frac{A_s}{b \cdot d} = 0.01$

$\sigma_c := \varepsilon_c \cdot E_c = 13.183\ \frac{N}{mm^2}$

$\sigma_s := \varepsilon_s \cdot E_s = 302.146\ \frac{N}{mm^2}$

$\alpha := \sqrt{(\eta \cdot \rho)^2 + 2 \cdot (\eta \cdot \rho)} - (\eta \cdot \rho) = 0.473$

$\alpha d := \alpha \cdot d = 172.721\ mm$

$F_c := \frac{\sigma_c \cdot \alpha d}{2} \cdot b = \left(1.138 \cdot 10^3\right)\ kN$

$S := \sigma_s \cdot A_s = \left(1.138 \cdot 10^3\right)\ kN$

$\delta_N := \left|F_c - S\right| = 0.013\ kN$

Moment

$$z := d \cdot \left(1 - \frac{1}{3} \cdot a\right) = 307.426\ \mathbf{mm}$$

$$M_c := F_c \cdot z = 349.996\ \mathbf{kN \cdot m}$$

$$M_s := S \cdot z = 350\ \mathbf{kN \cdot m}$$

$$M_{av} := \frac{M_c + M_s}{2} = 349.998\ \mathbf{kN \cdot m}$$

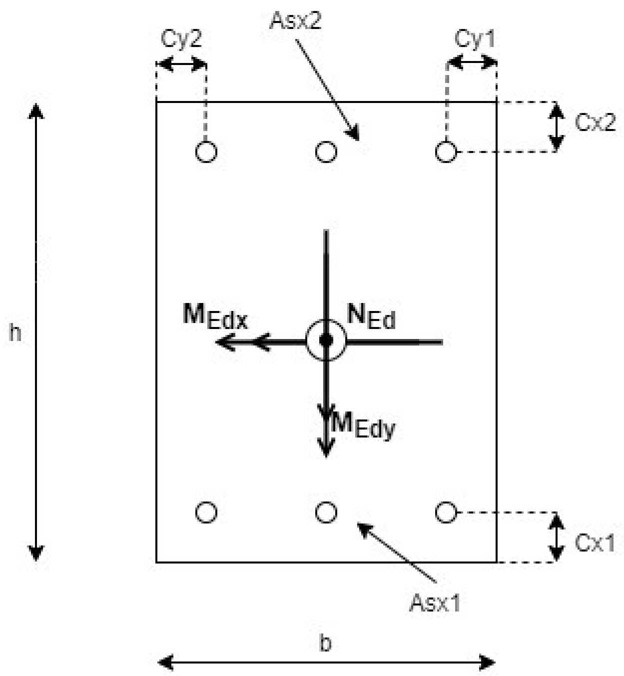
$$\delta_M := \left|M_{av} - M_x\right| = 0.002\ \mathbf{kN \cdot m}$$

$$dev_M := \frac{\delta_M}{M_x} = 0.0000056$$

D. Hand calculation for capacity control of columns

The following example is taken from the book 'Betongkonstruksjoner – Beregning og dimensjonering etter Eurocode2'. It is the capacity control of a section subjected to biaxial moments and an axial force.

The section geometry and material data are presented below.



Concrete B35:

$f_{cd} = 19.8\,N\,/\,mm^2$

Reinforcement B500C:

$f_{yd} = 434\,N\,/\,mm^2$

$b = 300\,mm$

$h = 400\,mm$

$c_{x1} = c_{x2} = c_{y1} = c_{y2} = 40mm$

Design force and moments:

$N_{Ed} = 1500\,kN$ $M_{Edx} = 150\,kNm$ $M_{Edy} = 30\,kNm$

Reinforcement:

$A_{sx1} = A_{sx2} = 3 \cdot 314 = 942\,mm^2$

$A_{sy1} = A_{sy2} = 2 \cdot 314 = 628\,mm^2$

The mechanical reinforcement ratios are calculated:

$$w_x = \frac{f_{yk} A_{sx}}{f_{ck} A_c} = \frac{500 \cdot 2 \cdot 942}{35 \cdot 300 \cdot 400} = 0.224$$

$$w_y = \frac{f_{yk} A_{sy}}{f_{ck} A_c} = \frac{500 \cdot 2 \cdot 628}{35 \cdot 300 \cdot 400} = 0.150$$

Dimensionless axial force:

$$n = \frac{N_{Ed}}{f_{ck} bh} = \frac{1500 \cdot 10^3}{35 \cdot 300 \cdot 400} = 0.36$$

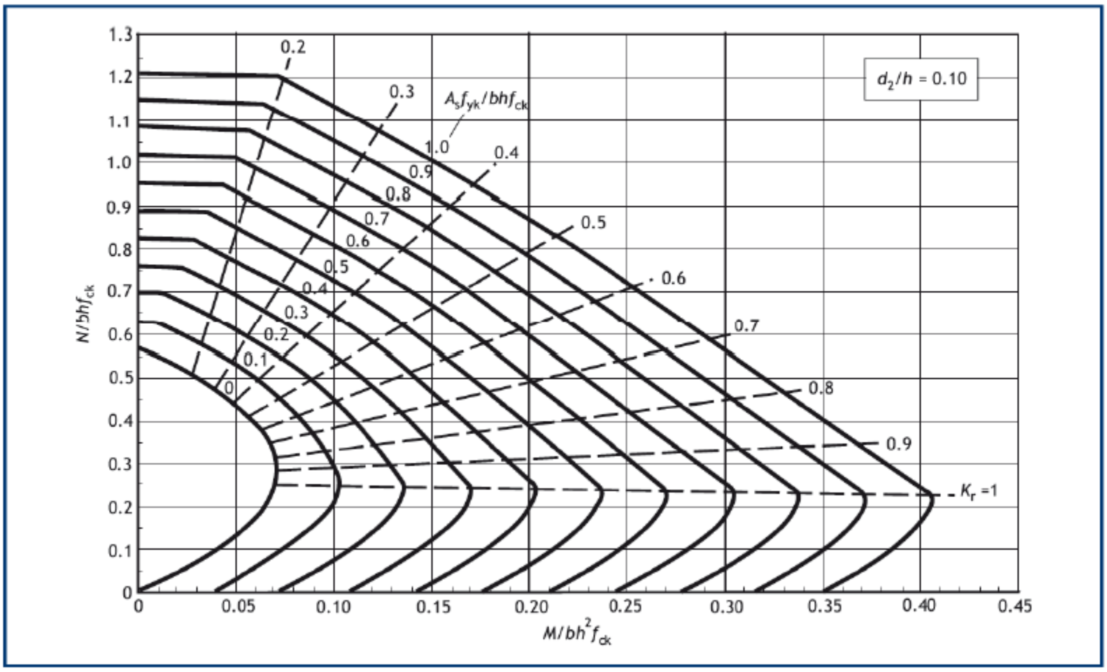


Figure D.1: m-n diagram

The dimensionless M-N diagram for $d_2 = 0.10$ in Figure D.1 gives:

$$m_{Rdx} = 0.125 \quad \rightarrow M_{Rdx} = 0.125 \cdot 35 \cdot 300 \cdot 400^2 \cdot 10^{-6} = 210\, kNm$$

$$m_{Rdy} = 0.105 \quad \rightarrow M_{Rdy} = 0.105 \cdot 35 \cdot 400 \cdot 300^2 \cdot 10^{-6} = 132\, kNm$$

The capacity of the section is controlled by using the following formula from EC2-5.8.9(4):

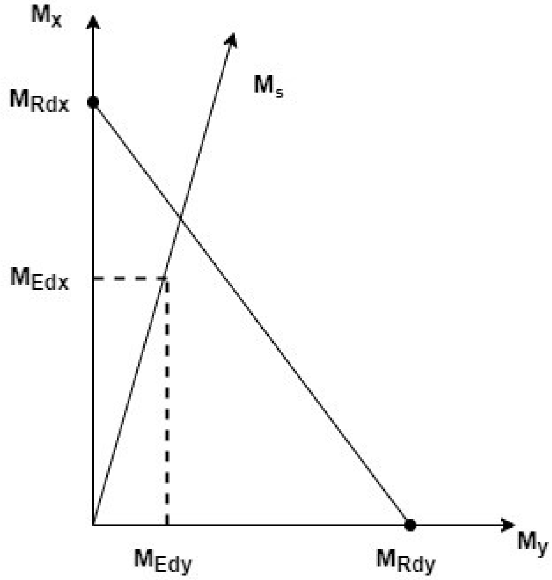
$$\left(\frac{M_{Edx}}{M_{Rdx}}\right)^{a} + \left(\frac{M_{Edy}}{M_{Rdy}}\right)^{a} \leq 1$$

For *a=1*:

$$\frac{150}{210} + \frac{30}{132} = 0.71 + 0.23 = 0.94 < 1$$

The column section has the capacity to support the design forces.

A graphic representation of the capacity curve is presented below:



E. Iteration method implementation for columns

The column is a structure subjected to compression and uniaxial or biaxial bending. In the case of uniaxial bending, it can be calculated by using beam calculation methods. However, in the case of biaxial bending, beam calculation methods cannot be used.

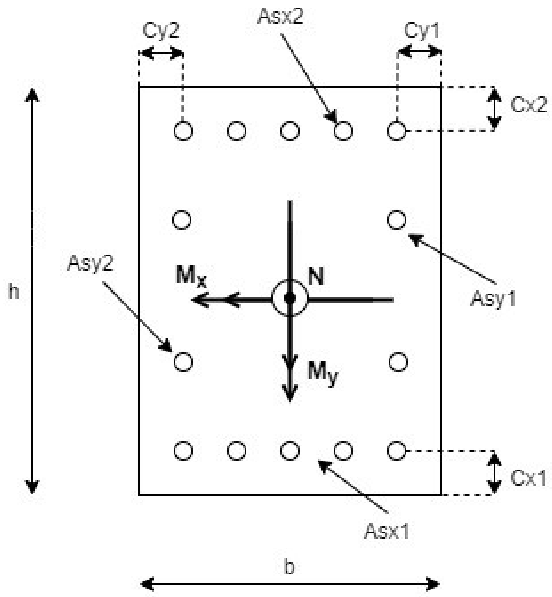


Figure E.1: Column section

In order to apply the iteration method to columns subjected to biaxial bending, the two moments need to be combined.

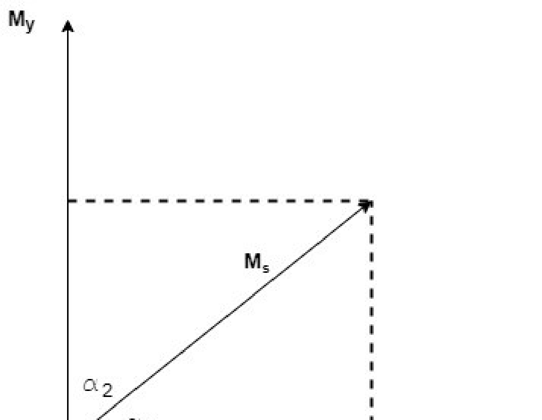


Figure E.2: Moment addition

$$M_s = \sqrt{M_x^2 + M_y^2}$$

$$\alpha_1 = \arctan\left(\frac{M_y}{M_x}\right)$$

The moment *Ms* acts about the *s*-axis. The *s*-axis is considered the new middle plane of the section; its direction is at an angle α_1 with respect to the x-axis direction. Based on the direction of the *s*-axis with respect to the diagonal of the section, the task can be subdivided into four cases.

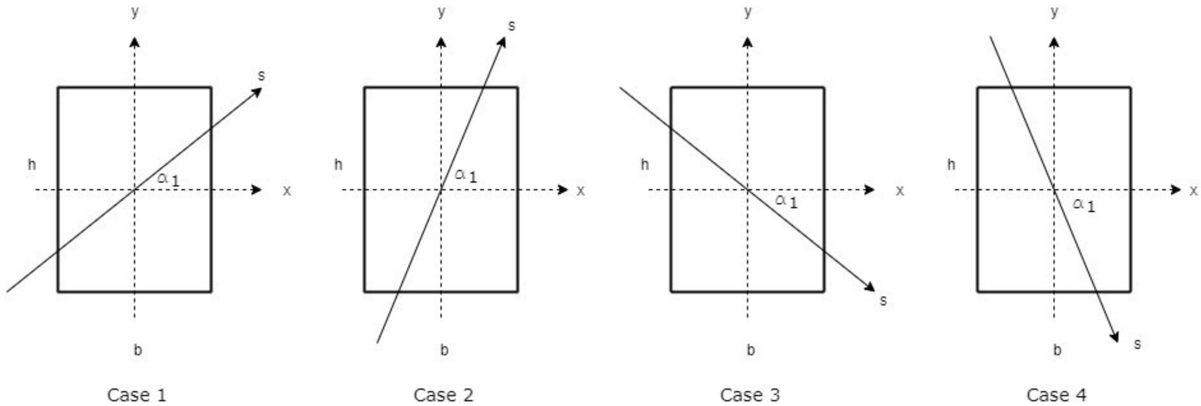


Figure E.3: Axis-s orieintation cases

Based on the four cases, the geometric details of the section, the concrete layer directions and dimensions, and the reinforcement layers' positions compared to the concrete layer dispositions are obtained.

As a result, the task at hand can be summarized as a reinforced concrete section subjected to one moment M_s and an axial force N, where all geometric data is known. Such a task can be calculated by using the iteration method.

The calculations for the four cases are presented in detail in the following sections. In each case, the concrete and reinforcement subdivisions with respect to the layer distributions are approached separately.

E.1 Case 1

$\alpha_1 \geq 0$

$\alpha_1 \leq \theta$

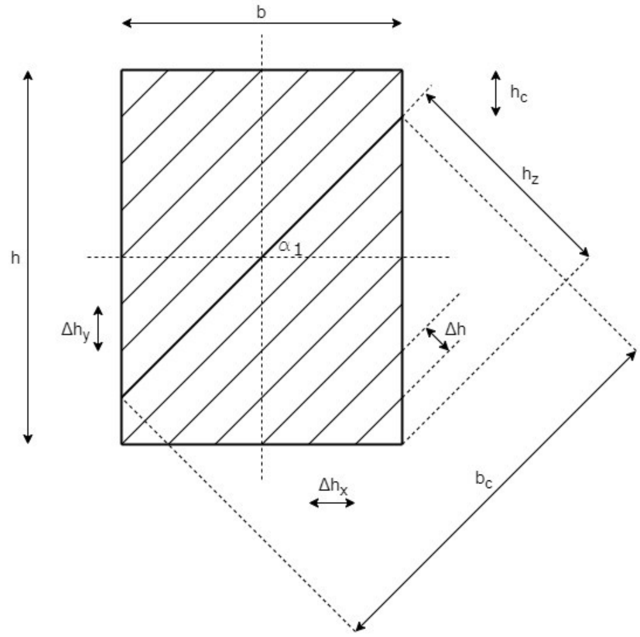


Figure E.4: column section, case 1

E.1.1 Concrete

Once the angle α_1 is obtained, the height of the new section (perpendicular to the *s*-axis) is defined as $2h_z$. Where :

$$h_z = (h - h_c) \cdot \cos\alpha_1$$

As shown in Figure E.4, h_c is the distance from the intersection of the *s*-axis and the right/left edge to the top/bottom edge of the section:

$$h_c = \frac{h}{2} - \frac{b}{2} \cdot \tan\alpha_1$$

The concrete section is divided into *n* layers, and each layer has a thickness of Δh :

$$\Delta h = \frac{2 \cdot h_z}{n}$$

The distance of each concrete layer from the axis *s* is obtained by:

$$z_{ci} = \frac{2h_z}{n} \cdot i + \frac{2h_z}{n} \cdot \frac{1}{2} - h_z = \frac{2h_z}{n} \cdot i + \frac{h_z}{n} - h_z$$

Where:

i : denomination of concrete layers, and can have values from *0* to *n-1*.

The width of the concrete layer varies with the distance from axis *s*.

For concrete layers within a distance $h_c \cdot \cos\alpha_1$ from the axis, the concrete layer width is constant and equal to:

$$b_c = \frac{b/2}{\cos\alpha_1} \cdot 2 = \frac{b}{\cos\alpha_1}$$

When the concrete layers have a distance higher than $h_c \cdot \cos\alpha_1$ from the axis, the concrete layer width is obtained by:

$$b_{ci} = \left(h_z - |z_{ci}|\right) \cdot \left(\tan\alpha_1 + \frac{1}{\tan\alpha_1}\right)$$

E.1.2 Reinforcement

The reinforcement is subdivided into the same layers used for concrete. Therefore, the primary task is to find the position of reinforcement with respect to the layers. It should be noted that the strain value within a layer is considered constant.

Based on the height of the concrete layers, their length in x- and y-direction are obtained.

$$\Delta h_x = \frac{\Delta h}{\sin\alpha_1}$$: horizontal length of the concrete layer.

$$\Delta h_y = \frac{\Delta h}{\cos\alpha_1}$$: vertical length of the concrete layer.

Subsequently, the original reinforcement layers A_{sx1}, A_{sx2}, A_{sy1}, and A_{sy1} are subdivided and matched with the corresponding concrete layers.

The first and last layer number where the reinforcements A_{sx1}, A_{sx2}, A_{sy1}, and A_{sy1} are calculated is presented in the table below:

	i_{First}	i_{Last}
A_{sx1}	$i_{Asx1F} = \dfrac{c_{x1}}{\Delta h_y}$	$i_{Asx1L} = i_{Asx1F} + \dfrac{b}{\Delta h_x}$
A_{sx2}	$i_{Asx2F} = \dfrac{h - c_{x2}}{\Delta h_y}$	$i_{Asx1L} = i_{Asx2F} + \dfrac{b}{\Delta h_x}$
A_{sy1}	$i_{Asy1F} = \dfrac{c_{y1}}{\Delta h_x}$	$i_{Asy1L} = i_{Asy1F} + \dfrac{h}{\Delta h_y}$
A_{sy2}	$i_{Asy2F} = \dfrac{b - c_{y2}}{\Delta h_x}$	$i_{Asy2L} = i_{Asy2F} + \dfrac{h}{\Delta h_y}$

i_{First} : concrete layer number where a reinforcement layer begins.

i_{Last} **:** concrete layer number where a reinforcement layer finishes.

For example i_{Asx1F} is the concrete layer number where the bottom reinforcement in x-direction starts, while i_{Asx1L} is the last concrete layer number for that reinforcement layer. In that way, it is possible to map the reinforcement layers, which are vertical or horizontal, to an inclined concrete layer distribution.

The results of the calculations are decimals, while the layer numbers are integers. Therefore, they are converted to integers. The choice of converting to integers as opposed to rounding is to consider the fact that the number of the first layer is zero and the last layer is *n-1*.

A similar approach is used for the other three cases, which are presented below.

E.2 Case 2

$\alpha_1 \geq 0$

$\alpha_1 > \theta$

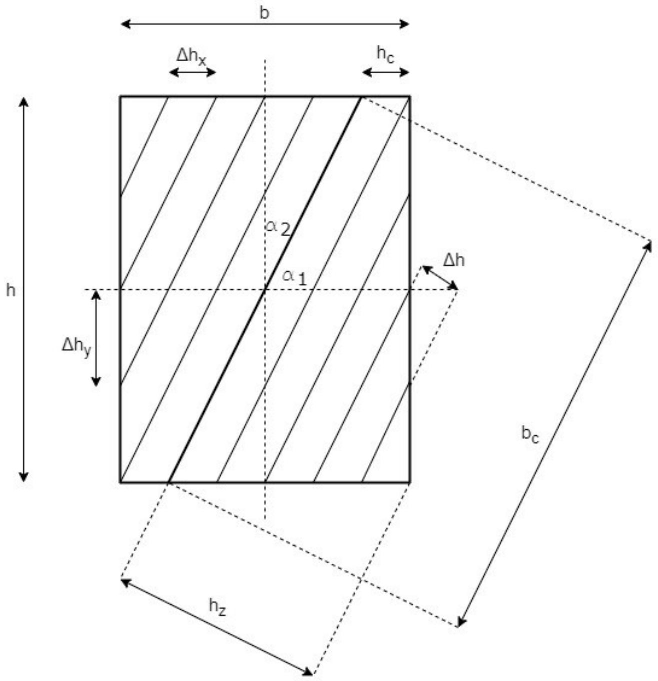


Figure E.5: Column section, case 2

E.2.1 Concrete

The angle between the axis *s* and y-axis (α_2) is calculated as: $90^\circ - \alpha_1$.

The height of the new section (perpendicular to the axis *s*) is defined as $2h_z$. Where :

$$h_z = (b - b_c) \cdot \cos\alpha_2$$

As shown in Figure E.5, h_c is the distance from the intersection of the *s*-axis and the top/bottom edge to the right/left edge of the section:

$$h_c = \frac{b}{2} - \frac{h}{2} \cdot \tan\alpha_2$$

The concrete section is divided into *n* layers, and each layer has a thickness of Δh :

$$\Delta h = \frac{2 \cdot h_z}{n}$$

The distance of each concrete layer from the axis *s* is obtained by:

$$z_{ci} = \frac{2h_z}{n} \cdot i + \frac{2h_z}{n} \cdot \frac{1}{2} - h_z = \frac{2h_z}{n} \cdot i + \frac{h_z}{n} - h_z$$

Where:

i : denomination of concrete layers, and can have values from *0* to *n-1.*

The width of the concrete layer varies with the distance from axis *s*.

For concrete layers within a distance of $h_c \cdot \cos\alpha_2$ from the axis, the concrete layer width is constant and equal to:

$$b_c = \frac{h/2}{\cos\alpha_2} \cdot 2 = \frac{h}{\cos\alpha_2}$$

When the concrete layers have a distance higher than $h_c \cdot \cos\alpha_2$ from the axis, the concrete layer width is obtained by:

$$b_{ci} = \left(h_z - |z_{ci}|\right) \cdot \left(\tan\alpha_2 + \frac{1}{\tan\alpha_2}\right)$$

E.2.2 Reinforcement

Based on the height of the concrete layers, their length in the x- and y-direction are calculated.

$$\Delta h_x = \frac{\Delta h}{\cos\alpha_2}$$

$$\Delta h_y = \frac{\Delta h}{\sin\alpha_2}$$

Subsequently, the original reinforcement layers A_{sx1}, A_{sx2}, A_{sy1}, and A_{sy1} are subdivided and matched with the corresponding concrete layers.

The first and last layer number for the reinforcement A_{sx1}, A_{sx2}, A_{sy1}, and A_{sy1} is presented in the table below.

	i_{First}	i_{Last}
A_{sx1}	$i_{Asx1F} = \frac{c_{x1}}{\Delta h_y}$	$i_{Asx1L} = i_{Asx1F} + \frac{b}{\Delta h_x}$
A_{sx2}	$i_{Asx2F} = \frac{h - c_{x2}}{\Delta h_y}$	$i_{Asx1L} = i_{Asx2F} + \frac{b}{\Delta h_x}$
A_{sy1}	$i_{Asy1F} = \frac{c_{y1}}{\Delta h_x}$	$i_{Asy1L} = i_{Asy1F} + \frac{h}{\Delta h_y}$
A_{sy2}	$i_{Asy2F} = \frac{b - c_{y2}}{\Delta h_x}$	$i_{Asy2L} = i_{Asy2F} + \frac{h}{\Delta h_y}$

E.3 Case 3

$\alpha_1 < 0$

$|\alpha_1| \leq \theta$

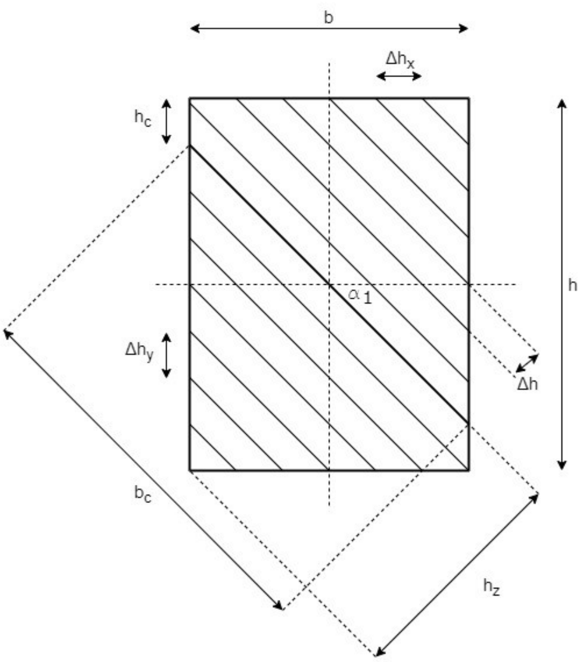


Figure E.6: Column section, case 3

E.3.1 Concrete

Once the angle α_1 is obtained, the height of the new section (perpendicular to the *s*-axis) is defined as $2h_z$. Since α_1 is negative, its absolute value is used in the following calculations.

$$h_z = (h - h_c) \cdot \cos|\alpha_1|$$

As shown in Figure E.6, h_c is the distance from the intersection of the axis *s* and the right/left edge to the top/bottom edge of the section:

$$h_c = \frac{h}{2} - \frac{b}{2} \cdot \tan|\alpha_1|$$

The concrete is divided in *n* layers and each layer has a thickness of Δh:

$$\Delta h = \frac{2 \cdot h_z}{n}$$

The distance of each concrete layer from the axis *s* is obtained by:

$$z_{ci} = \frac{2h_z}{n} \cdot i + \frac{2h_z}{n} \cdot \frac{1}{2} - h_z = \frac{2h_z}{n} \cdot i + \frac{h_z}{n} - h_z$$

Where:

i : denomination of concrete layers, and can have values from *0* to *n-1*

The width of the concrete layer varies with the distance from axis *s*.

For concrete layers within a distance of $h_c \cdot \cos|\alpha_1|$ from the axis, the concrete layer width is constant and equal to:

$$b_c = \frac{b/2}{\cos|\alpha_1|} \cdot 2 = \frac{b}{\cos|\alpha_1|}$$

When the concrete layers have a distance higher than $h_c \cdot \cos|\alpha_1|$ from the axis, the concrete layer width is obtained by:

$$b_{ci} = \left(h_z - |z_{ci}|\right) \cdot \left(\tan|\alpha_1| + \frac{1}{\tan|\alpha_1|}\right)$$

E.3.2 Reinforcement

Based on the height of the concrete layers, their length in the x- and y-direction are obtained.

$$\Delta h_x = \frac{\Delta h}{\sin|\alpha_1|}$$

$$\Delta h_y = \frac{\Delta h}{\cos|\alpha_1|}$$

The original reinforcement layers A_{sx1}, A_{sx2}, $A_{sy1,}$ and $A_{sy1,}$ are subdivided and matched with the corresponding concrete layers.

The first and last layer number for the reinforcement A_{sx1}, A_{sx2}, $A_{sy1,}$ and A_{sy1} is presented below.

	i_{First}	i_{Last}
A_{sx1}	$i_{Asx1F} = \frac{c_{x1}}{\Delta h_y}$	$i_{Asx1L} = i_{Asx1F} + \frac{b}{\Delta h_x}$
A_{sx2}	$i_{Asx2F} = \frac{h - c_{x2}}{\Delta h_y}$	$i_{Asx1L} = i_{Asx2F} + \frac{b}{\Delta h_x}$
A_{sy1}	$i_{Asy1F} = \frac{b - c_{y1}}{\Delta h_x}$	$i_{Asy1L} = i_{Asy1F} + \frac{h}{\Delta h_y}$
A_{sy2}	$i_{Asy2F} = \frac{c_{y2}}{\Delta h_x}$	$i_{Asy2L} = i_{Asy2F} + \frac{h}{\Delta h_y}$

E.4 Case 4

$\alpha_1 < 0$

$|\alpha_1| > \theta$

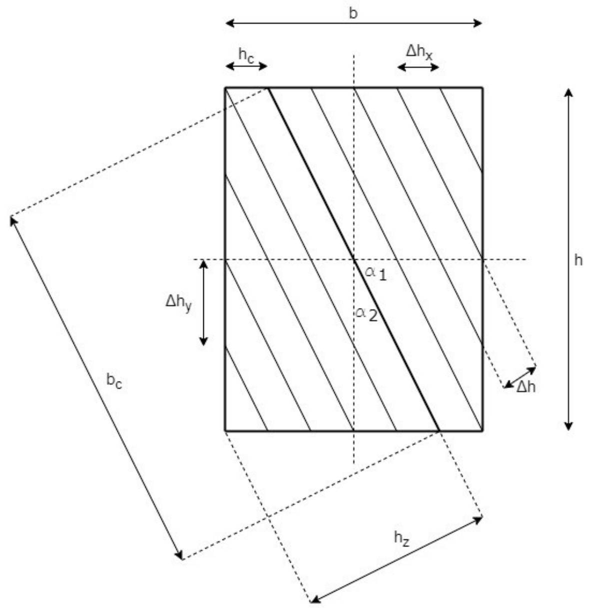


Figure E.7: Column section, case 4

E.4.1 Concrete

The angle between the axis *s* and y-axis (α_2) is calculated as: $90° - |\alpha_1|$.

The height of the new section (perpendicular to the axis *s*) is defined as $2h_z$. Where :

$$h_z = (b - b_c) \cdot \cos|\alpha_2|$$

As shown in Figure E.7, h_c is the distance from the intersection of the axis *s* and the top/bottom edge to the right/left edge of the section:

$$h_c = \frac{b}{2} - \frac{h}{2} \cdot \tan|\alpha_2|$$

The concrete section is divided into *n* layers, and each layer has a thickness of Δh :

$$\Delta h = \frac{2 \cdot h_z}{n}$$

The distance of each concrete layer from the axis *s* is obtained by:

$$z_{ci} = \frac{2h_z}{n} \cdot i + \frac{2h_z}{n} \cdot \frac{1}{2} - h_z = \frac{2h_z}{n} \cdot i + \frac{h_z}{n} - h_z$$

Where:

i : denomination of concrete layers, and can have values from *0* to *n-1.*

The width of the concrete layer varies with the distance from axis *s*.

For concrete layers within a distance of $h_c \cdot \cos|\alpha_2|$ from the axis, the concrete layer width is constant and equal to:

$$b_c = \frac{h/2}{\cos|\alpha_2|} \cdot 2 = \frac{h}{\cos|\alpha_2|}$$

When the concrete layers have a distance higher than $h_c \cdot \cos|\alpha_2|$ from the axis, the concrete layer width is obtained by:

$$b_{ci} = (h_z - |z_{ci}|) \cdot \left(\tan|\alpha_2| + \frac{1}{\tan|\alpha_2|} \right)$$

E.4.2 Reinforcement

Based on the height of the concrete layers, their length in x- and y-direction are obtained.

$$\Delta h_x = \frac{\Delta h}{\cos|\alpha_2|}$$

$$\Delta h_y = \frac{\Delta h}{\sin|\alpha_2|}$$

Subsequently, the original reinforcement layers A_{sx1}, A_{sx2}, A_{sy1}, and A_{sy1} are subdivided and matched with the corresponding concrete layers.

The first and last layer number for the reinforcement A_{sx1}, A_{sx2}, A_{sy1}, and A_{sy1} is presented below.

	i_{First}	i_{Last}
A_{sx1}	$i_{Asx1F} = \frac{c_{x1}}{\Delta h_y}$	$i_{Asx1L} = i_{Asx1F} + \frac{b}{\Delta h_x}$
A_{sx2}	$i_{Asx2F} = \frac{h - c_{x2}}{\Delta h_y}$	$i_{Asx1L} = i_{Asx2F} + \frac{b}{\Delta h_x}$
A_{sy1}	$i_{Asy1F} = \frac{b - c_{y1}}{\Delta h_x}$	$i_{Asy1L} = i_{Asy1F} + \frac{h}{\Delta h_y}$
A_{sy2}	$i_{Asy2F} = \frac{c_{y2}}{\Delta h_x}$	$i_{Asy2L} = i_{Asy2F} + \frac{h}{\Delta h_y}$