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Questioning questioning: The "why" and "how" of mathematics

Evaluating the complexity of student-posed questions in high school

Master’s thesis in Science Education (MLREAL)
Supervisor: Associate Professor Yael Fleischmann
June 2021

Master’s thesis

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Norwegian University of Science and Technology
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Abstract

Educators are always looking for new ways to improve their teaching capabilities and methods. By using technology, we have expanded education during the last decades with innovative and unique ways to deliver knowledge. However, we do not have to look into the world of technology to find powerful teaching and learning tools. Some of these are already a vital part of society, questioning being one of them.

This study aimed to find an answer to the research question "*To which extent and how do students following a mathematics 1T course use questions of different levels of complexity in their student-teacher communication?*" I wanted to explore how students used mathematical presuppositions and terminology when posing questions to a teacher in various forms of student-teacher communication. In addition, I wanted to explore the cognitive level of these questions to learn more about the potential of question-posing.

I gathered information about a high-school class's question-posing behavior by conducting a design study using participating observation and digital data collection methods. Specifically, by designing teaching situations revolving around the use of questions seeking questions in return, based on my alteration of Singer's theory of active comprehension, I managed to collect and analyze a large amount of student-posed questions from various situations. My research found that the students participating in this study displayed an ability to pose questions of a higher complexity when communicating through a written format than when communicating orally. The students' oral questions mainly concerned procedural knowledge, or they posed answer- or solution-related questions. None of their oral questions were deemed to be of a higher cognitive level. However, the results suggest that using question-seeking questions in a structured manner could result in a higher amount of high-order cognitive questions. Such question-posing behavior was facilitated using the interactive presentation tool Mentimeter and by asking for questions of reflection in an at-home teaching session. The finding of this study may give a crucial perspective to the future use of inquiry-based education and teaching of question-posing.

Key words: student question-posing, comprehension, Mentimeter, at-home teaching, orality, communication, digital teaching, inquiry-based education

Sammendrag

Utdannere og lærere er alltid på utkikk etter nye måter å forbedre sin undervisningskompetanse og metoder. Gjennom bruk av teknologi har vi utvidet utdanningsfeltet gjennom de siste tiårene med innovative og unike måter å formidle kunnskap. Vi trenger derimot ikke å sette oss inn i teknologiens verden for å finne virkningsfulle og effektive undervisnings- og læringsverktøy. Noen av disse er allerede en viktig del av samfunnet. Å stille spørsmål er en av dem.

Denne studien sikter på å finne et svar på forskningsspørsmålet "*I hvilken grad og hvordan bruker elever som følger et matematikk 1T kurs spørsmål av ulike nivåer av kompleksitet i deres elev-lærer-kommunikasjon?*" Jeg ønsket å utforske hvordan elever brukte matematiske presupposisjoner og terminologi når de stilte spørsmål rettet mot en lærer i ulike former for elev-lærer-kommunikasjon. I tillegg ønsket jeg å undersøke det kognitive nivået til disse spørsmålene for å lære mer om potensialet som ligger i spørsmålsstilling.

Jeg samlet informasjon om en videregåendeklasses spørsmålsstillingsadferd med å gjennomføre en designstudie gjennom deltakende observasjon og digitale datainnsamlingsmetoder. Ved å designe undervisningssituasjoner som omhandler bruk av spørsmål som søker spørsmål i retur, basert på Singers teori om aktiv forståelse (active comprehension), klarte jeg å samle inn og analysere store mengder med elevstilte spørsmål fra varierte situasjoner. Forskningen min fant at elevene som deltok i denne studien viste en evne til å stille spørsmål av en høyere kompleksitet da de kommuniserte gjennom skrevne formater, enn da de kommuniserte muntlig. Elevenes muntlige spørsmål omhandlet hovedsaklig prosedyremessig kunnskap, eller svar-/løsningsrelaterte fokus. Ingen av elevenes muntlige spørsmål ble tolket til å være at et høyere kognitivt nivå. Derimot antyder resultatene at strukturert bruk av spørsmålssøkende spørsmål kan resultere i et større antall høyereordens kognitive spørsmål. Slik spørsmålsstillingsadferd ble fasilitert gjennom bruk av det interaktive presentasjonsverktøyet Mentimeter, og gjennom å stille reflekterende spørsmål under digital hjemmeundervisning. Observasjonene fra denne studien kan gi et avgjørende perspektiv til framtiden av spørsmålsbasert undervisning (inquiry-based teaching) og undervisning av spørsmålsstilling.

Nøkkelord: elevstilte spørsmål, forståelse, Mentimeter, hjemmeundervisning, muntlighet, kommunikasjon, digital undervisning, spørsmålsbasert undervisning

Acknowledgements

To begin with, I would like to address anyone who is in a position where they either are about to or are writing their master's thesis. I have been so lucky as to be able to write over ninety pages of research on a topic in which I am deeply invested. If you are able to decide the matter of your master's, I would greatly advise you to find something you enjoy, find fascinating, and care about. This has helped me in writing this thesis in times where everything seemed impossible. I will not lie and say that performing research at this level is easy; it is not. However, these last six months have been the most giving, intriguing, infuriating, and demanding period of my life, and I would not change that for anything. Look forward to performing your very own research, and enjoy it.

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Lastly, I want to thank my person, Ida Marie Andreassen, for putting up with me even when my brain turned into fog, and everything I talked about was this project. Your words, crisis aversion, and love allowed me to stay above the surface when I felt like everything went wrong. You are my greatest source of inspiration, and I could not have done this without you.

Sincerely,
Thomas Schjem

Contents

Chapter 1

Introduction

1.1 Personal motivation

Through all my years of education, I have developed a deep interest in how learning occurs in an oral setting. Mainly, I find the oral facets of mathematics particularly intriguing. I have discovered that I learn the most from articulating something that I struggle to understand and discussing my thoughts and ideas with others. Further, through my experience as a personal teacher, I have learned that some students who find mathematics challenging may benefit from being taught how to express what they find difficult. In several cases, I have observed that as soon as the student manage to phrase their question, they also find a way to solve it. This experience is what fuels my interest in the topic of question-posing. Moreover, I have discovered that research on student question-posing, more often than not, merely scratches the surface of what I deem an exciting and essential topic to explore. Thus, I want to contribute to a field of research that I believe could positively benefit all classrooms and learning situations.

1.2 The research question

In this thesis, I will examine how students use questions when communicating with a teacher. This communication will be both in lecture-type scenarios and when a teacher provides guidance on exercises. Further, I will design and implement some teaching scenarios aimed to facilitate question-posing in different ways. The questions posed by the students will then be analyzed according to some levels of complexity that I have deemed essential when evaluating a question: the question's presuppositions, the cognitive level of the question, and the use of mathematical terminology. By this, the research question to be examined in this study can be phrased as follows:

> *To which extent and how do students following a mathematics 1T course use questions of different levels of complexity in their student-teacher communication?*

To answer this question, I provide a theoretical framework consisting of several theories, each serving its own purpose in my study. First, I present my adaptation of Harry Singer's theory of active comprehension as the principal didactical theory to be used for further designs and analysis. Following this, I explain how to characterize a question's presuppositions and define the mathematical language. Further, I will examine a question's cognitive level by implementing the Revised Bloom's Taxonomy, a model providing an overview of six levels of cognitive thinking.

1.3 Thesis overview

This thesis is divided into eight sections, each serving its unique purpose to build a complete and structured text. In chapter 2, I present an overview of research on question-posing and introduce the primary theoretical approaches to be used to analyze data and explore the research question. Here I specifically define the levels of complexity from the research question and explain how these can be used to evaluate student questions. Thus, this section is reserved explicitly for the theory used in further analysis and discussion.

Chapter 3 presents the methodological approaches used to provide structure, form, and validity to my study. Here, some theoretical overview of the research methodology didactical engineering and the data analysis method qualitative content analysis are presented alongside how these were used in my research. Additionally, the methods of data collection are presented and discussed in regards to how and why they were chosen as beneficial for my study. Finally, the section is concluded by evaluating several ethical considerations I have taken as a part of this research.

In chapter 4, I present a historical and didactical analysis of triangle trigonometry and differential calculus, as these are the mathematical subjects to be considered in this study. Additionally, an institutional analysis of the setting where the research took place is conducted, focusing on curricula, COVID-19 restrictions, and other frame factors.

Chapter 5 presents how the different didactical choices and designs used in this study were composed. Here I present an analysis of the didactical choices made and how each of these was designed to facilitate the desired results as clearly as possible. Furthermore, as this thesis has been heavily impacted by the ongoing COVID-19 pandemic, I have had to alter and discard several initial ideas. Thus, concluding this chapter, I present the original plan of the study and why it could not be implemented.

In chapter 6, the primary analysis of the collected data is displayed. Here, the theory presented in section 2 is used to evaluate the collected data in regards to the research question. Fascinating cases from the data are displayed and analyzed to find an answer to the research question. Further, the validity of the didactic intention is evaluated, and hypotheses made earlier in the research process are considered.

In chapter 7, the focus is lifted from the data, focusing on the greater picture of question-posing in the current didactical environment. Here I discuss the role of high- and low-order cognitive questions, classroom culture as a possible obstacle to question-posing, the future teaching of question-posing, and provide some critical remarks to the schools' fulfillment of its social mandate.

Lastly, in chapter 8, the study's different points and ideas provided throughout the thesis are collected, and a specific answer to the research question is given.

Chapter 2

Theory

When working with such a vague and vast topic as question-posing, one needs a thorough theoretical background to properly evaluate and analyze different aspects of question-posing situations. Thus, in this chapter, I introduce the main theoretical approaches to be used to strengthen my analysis presented in chapter 6. Firstly, a field overview is presented to give an understanding of the current state of research on question-posing. Following this, I present my adaptation of Harry Singers' theory of active comprehension, which is used as the primary theory for the didactical choices made during my research. It is used in the form of planning and structuring the different phases presented in chapter 5. It is further used in section 6.5 to evaluate the validity of the didactic intention, as is customary when employing Didactical Engineering (see section 3.2) as a research method. Lastly, I present three features of questions, *presuppositions, cognitive level*, and *mathematical terminology* used as the primary theoretical foundation of my analysis. These three features are viewed as the levels of complexity referred to in the research question.

2.1 Field overview

During the last decades, research on student question-posing has become more prominent in the education research community. As a part of the PRIMAS project, a schema visualizing the different dimensions of inquiry-based education (IBE) was produced. Within this schema, student question-posing is presented as a crucial part of their working process (Dorier & García, 2013). While IBE is a widely used term, it misses a commonly shared definition. In short terms, it is related to making students work in the same way scientists and mathematicians explore, inquire, and solve different phenomena (Dorier & García, 2013). Through the last few years, IBE's position in education has been solidified through the European PRIMAS (Promoting Inquiry in Mathematics and Science Education Across Europe) and FIBONACCI projects. IBE started as a learning tool in science education (IBSE), and the migration towards IBE in mathematics (IBME) began to evolve relatively recently (Artigue & Blomhøj, 2013).

Different authors suggest various methods for facilitating student question-posing, for example, the use of active comprehension, that is, helping students become active participants in their own comprehension (Singer, 1978), or the use of microteaching, that is, small teaching segments of a defined focus (Sadker & Cooper, 1974). However, while it is true that different methods have been developed, very few of these have been developed to be implemented in STEM courses. Further, many authors convey the importance of student question-posing, while few articles explore new ways of facilitating it. Thus, as a starting point, I find it essential to investigate how students use questions in their learning process and then use this information to design and investigate methods of promoting question-posing that could be used in most settings without them being too time-consuming or disrupting.

2.2 Teaching (active) comprehension

Comprehension has three dictionary definitions: 1) The act or action of grasping with the intellect, 2) Knowledge gained by comprehending, and 3) The capacity for understanding fully (Merriam-Webster, n.d.-b). By these definitions, comprehension is not limited to understanding something. It also covers the *act* or *action* of understanding, and the *knowledge* gained in this process, including ones *capacity* to absorb a knowledge at any given time. Then, depending on which definition one is to use, comprehension can refer to a process, a product, or a potential (Singer, 1978)[1]. When Singer writes about comprehension, he refers to reading or writing comprehension. In the following, I will refer to *mathematical* comprehension when using the term, which, to some extent, involves both writing and reading comprehension. Based on the dictionary definitions, I define mathematical comprehension to involve the process of understanding a mathematical concept, the knowledge obtained during this process, and one's capacity to understand the concept fully. Thus, I choose to view comprehension as the triple of process,

[1]The following is based on Harry Singer's *Active comprehension: From answering to asking questions* (1978) and will thereby not be cited outside of direct quotes. Here, he presents different views on and possible means to teach comprehension.

product, and potential.

Both reading and writing are essential aspects of learning mathematics. In the Norwegian curriculum of 2020, they are, in addition to the ability to calculate, digital competencies and oral competencies, considered fundamental skills (Utdanningsdirektoratet, 2020a). It should be mentioned that *calculation* (Norwegian: "regne") is not limited to one's ability to perform arithmetic operations. It also involves evaluating the validity of solutions, recognizing mathematical problems, and the ability to formulate questions concerning these. The fundamental skills in mathematics show that reading and writing comprehension, to some extent, overlaps mathematical comprehension. Thus, when Singer presents methods for the teaching of *active comprehension* in a language course (English), it is not far reached to extend his theory to be implemented in mathematics.

There are several means to communicate the process of comprehension. Taba (1965) constructed questioning procedures supporting a sequential approach. She suggested that teachers could ask sequential questions leading to the formulation of a general concept. These can be questions such as "Would you summarize the topic for the class?", followed by "What additional information do we have?" when the student has finished their answer. When this type of discussion has produced a sufficient amount of knowledge, the teacher can then pose a lifting set of questions, causing the students to think on a higher level. These questions can, for example, be of the same manner as the ones suggested for high-order cognitive questions in section 2.3. However, as Singer suggests, although this method emphasizes the sequential manner of the process of thinking, it is the teacher guiding the process, thereby missing what could be argued to be the main objective of teaching comprehension: having students posing their own questions and guiding their own thinking.

As a suggestion to an alternative method, Singer presents what he claims are three necessary steps for teaching comprehension: *Modeling behavior*, *phase-out/phase-in strategy*, and *active comprehension*. There is little doubt that a teacher needs to pose questions in their lessons. However, Singer claims the teacher also needs to educate their students on how to pose similar questions themselves. As mentioned, the ultimate goal of teaching comprehension should be a student group capable of posing self-generated questions. Thus, as a first step, the students should be taught how to model or mimic their teacher's questions as a part of the modeling behavior. Then, in the next step, when the students have insight into how a teacher produces questions, teacher-posed questions should be phased out, and student-posed questions phased in. This can be done by explicitly teaching the students how to produce their own questions. The teacher can go through a topic, task, or problem, showing the students how they would solve, inquire, and phrase their thoughts on different aspects of the process. This way, the teacher demonstrates their process of thinking to go through the problem and comprehending it. This is part of the phase-out strategy. When the students have been taught how to produce questions, they must be stimulated to formulate their own questions. The teacher-posed questions are now phased out, and the student-posed questions are phased in. The students are now developing what Singer calls active comprehension.

Singer's definition of active comprehension is quite heavily tied to reading or writing comprehension. He describes it as "a continuous process of formulating and searching for answers to questions before, during and after reading" (Singer, 1978, p. 904). Thus, the previously mentioned goal of teaching comprehension can be defined as active comprehension. When the students have reached a point where they are actively taking part in their own comprehension, they should be able to formulate self-generated questions and guide their own thinking. Then, by altering Singer's definition, active comprehension in mathematics can be defined as a continuous process of formulating and searching for answers to questions before, during, and after working with a mathematical object. A complete introduction to how the teaching of active comprehension was implemented in this study can be seen in section 5.1. In general, this theory is used to guide my didactical designs to facilitate question-posing, specifically through the use of questions seeking questions for answers as my phase-out/phase-in strategy.

2.3 Student question-posing

> 'Well then,' proposed Socrates, ' if you should ever be charged in actual fact with the upbringing and education of these imaginary children of yours, ... so you will make a law that they must devote themselves especially to the technique of asking and answering questions.' (p. 7, Dillon, 1990, Modified quote from *Republic* VII:534)

The quote from Socrates reveals that education and teaching should produce students capable of not only answering but, equally important, asking questions. However, as Dillon (1990)[2] identifies, children everywhere are schooled to become masters at answering questions while remaining novices at asking them. He further states that the norm is to "induce in the young answers given by others to questions put by others" (Dillon, 1990, p. 7). The norm works against the natural occurrence of student questions. He claims that the vast majority of questions formed in a classroom are generated by a teacher, not the students. Thus, classroom questioning, in reality, refers to teacher questioning. This tendency is not limited to education. Everywhere in the world of questioning, such as courtrooms, interrogation rooms, and medical clinics, we find questions posed by an authoritative figure which is to be answered, not questioned, either by law or by norm.

As a test to Dillon's claim that student questions have a limited place in the classroom as the situation is now, one can examine the quality of student questions based on some preset conditions. Some such conditions may be the validity of the question's presuppositions, the cognitive level of the question, and the use of mathematical terminology. In the following subsections, I will present these three as the three layers of complexity referred to in my research question. Then, a question's complexity could be determined according to the constraints of these three and so be used for further investigation. The choice of precisely these three aspects of questioning is not arbitrarily selected. By referring to my definition of active comprehension in mathematics, the students who have reached active comprehension should continuously formulate and search for answers to questions, and thus, the formulation of high-quality questions should be necessary. Then, by examining a part of the students' mathematical assumptions (presuppositions) based

[2]The following section is based on J. T. Dillon's book *The practice of questioning* from the *International series on communication skills*, and will therefore not be cited outside of direct quotes or when special attention is given to a subject. In the first part of this two-part book, Dillon presents the state of questioning in eight different fields in the 1980s, showing the multidisciplinary nature of questioning. One of these fields is education, where he brings a critical eye to the (lack of) student question-posing in the average classroom. The second part presents different elements of questions, again relying on the multidisciplinary use of questions.

on the formulation of the question (mathematical terminology), an evaluation of the potential knowledge generation (cognitive level) could be analyzed.

2.3.1 Assumptions in questions - Defining a question's presuppositions

There are many elements of questioning that can be considered when analyzing specific question scenarios. What is true in all cases is that in order for a question to be valid and thereby be validly answered, the question's *presuppositions* need to be true. A presupposition can be defined as what is pre-supposed: what information the question conveys as truth. Thus, the question "Is the King of France bald?" presupposes that:

1. there is a king of France;

2. the king is either bald or not-bald.

The question "At what interval is the function f(x) defined?" presupposes that:

1. there exist some function f(x);

2. the function f(x) is defined at some interval.

This last question is a typical question used on a mathematics test or exam. Those familiar with this sort of exercise know that there is no guarantee that the function f(x) is defined at all, thus showing that a question's presuppositions need not be true to pose a question. As Dillon states: "The implication is not that we ask questions that are true, only that we know the truth of the questions we ask" (Dillon, 1990, p. 133). Some, maybe in particular teachers, will ask questions with false or indeterminate presuppositions for the students to show more knowledge. Also, some may unknowingly pose questions with false presuppositions, believing them to be true. For example, a student may ask, "When will the function increase, and when will it decrease?". This presupposes that the function will *both* increase *and* decrease. Unless explicit information is otherwise given, this need not be true. Thus, the presuppositions of a student-posed question can tell us something about the students underlying mathematical knowledge. If some presuppositions imply a fundamental flaw in the student's mathematical understanding, the teacher can use it to strengthen the student's mathematical knowledge.

In the analysis, I will examine student-posed questions, trying to identify whether the question's *mathematical* presuppositions are valid. Thus, if a question does not convey any evident mathematical knowledge as truth, the presuppositions will not be discussed. However, every question has been through such an analysis, even though they are not presented in the analysis chapter.

2.3.2 Cognitive level of questions - Bloom's taxonomy, RBT, and question categories

Another aspect that could be determined when evaluating a question is whether it is of a higher or lower cognitive level. Several different models have been made to classify the levels of cognitive thinking, the most commonly used being Bloom's Taxonomy (or some variation of it). In 2001 a group of psychologists, curriculum theorists and instructional researchers, and testing and assessment specialists published a revision of Bloom's Taxonomy (from now on called RBT, Revised Bloom's Taxonomy), shown in figure 2.1. One of the significant changes in this revision is the neglection of a strict hierarchy, thus allowing the different categories to overlap (Radmehr & Drake, 2019).

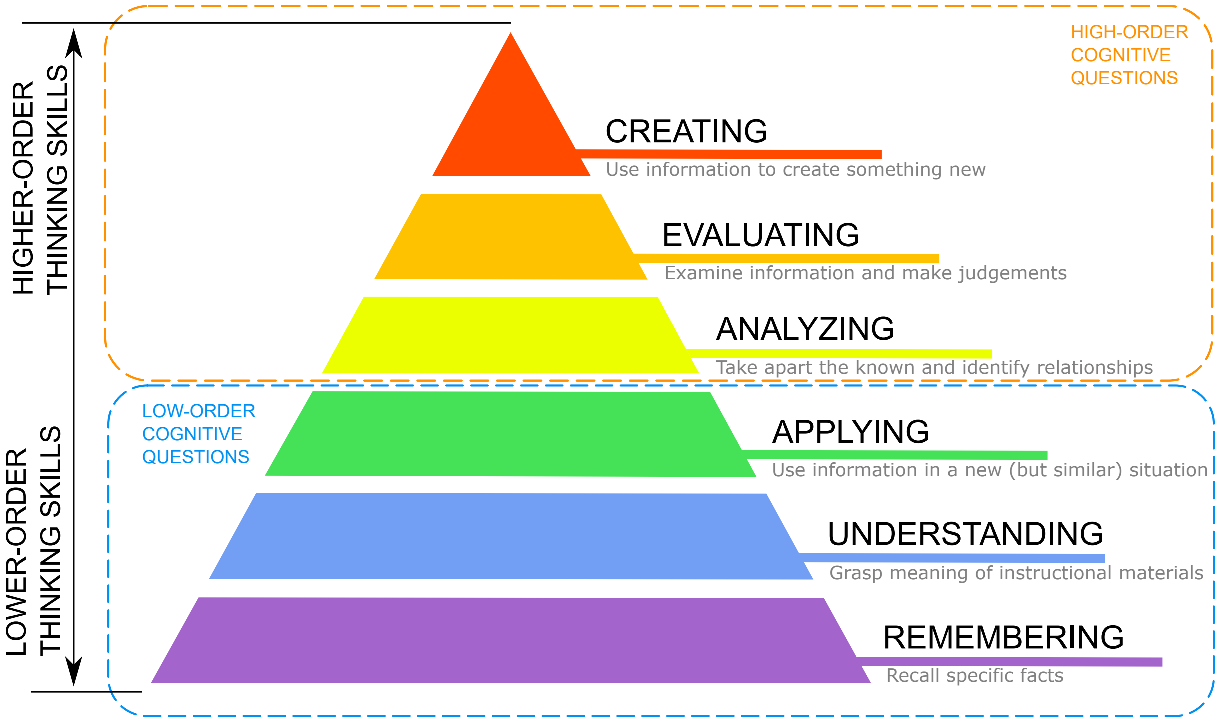


Figure 2.1: A representation of the revised version of Bloom's taxonomy, commonly called "*A Taxonomy for Teaching, Learning, and Assessment*. The pyramid shows the different layers of cognitive thinking skills with the domains of high- (orange stippled area) and low-order (blue stippled area) cognitive questions drawn in. (Self made figure based on figure from Kurt, 2020)

The different levels of cognitive thinking shown in figure 2.1, *remembering, understanding, applying, analyzing, evaluating,* and *creating*, are in total split into nineteen subcategories in this new revision. I will, in the following, go into detail about each category in order to define each level of cognitive thinking properly[3].

Remembering is divided into *recognizing* and *recalling*. *Recognizing* is defined as retrieving relevant knowledge from long-term memory to compare it with presented information. *Recalling* is defined as retrieving relevant knowledge from long-term memory when prompted to do so. Recognizing could then be the retrieval from memory the memorized form of the Pythagorean theorem to compare with a presented formula. Recalling could be the recollection that $7 \cdot 8$ is 56 when facing a problem involving the number 56.

Understanding is split into seven subcategories, *interpreting, exemplifying, classifying, summarising, inferring, comparing*, and *explaining*. *Interpreting* refers to translating from one representational form to another, such as converting from fraction notation to decimal form. Here, I should clarify that this refers to a translation *within* one representation system, not a conversion *between* two separate representation systems, as described by Duval (2006)[4]. *Exemplifying* relates to providing an example or instance of a general principle. *Classifying* refers to identifying that something belongs to a certain category, class, or topic. *Summarising* is related to the development of a statement representing some presented information or abstraction of a general theme. *Inferring*, refers to observing patterns within a series of examples, topics, or situations. *Comparing* involves detecting similarities or dissimilarities between a set of objects, events, ideas, problems, or situations. The final subcategory involving understanding is *explaining* and refers to:

> (...) constructing a cause-and-effect model, including each major part in a system or each major event in the chain, and using the model to determine how a change in one part of the system or one 'link' in the chain affects a change in another part (Radmehr & Drake, 2019, p. 901)

[3]The following definitions are based on "*Revised Bloom's taxonomy and major theories and frameworks that influence the teaching, learning, and assessment of mathematics: a comparison*", by Farzad Radmehr and Michael Drake (2019), which will therefore not be cited outside of direct quotes.

[4]For more information about this topic, one can read *A Cognitive Analysis of Problems of Comprehension in a Learning of Mathematics*, by Raymond Duval, 2006)

In this last subcategory, there is an emphasis on "*major* part," as it otherwise should be considered as analyzing, a high-order cognitive level

Applying is split into two subcategories, *executing*, which refers to using preexisting knowledge in a familiar task, and *implementing*, referring to using preexisting knowledge in a problem, an unfamiliar task.

Analyzing is strongly related to the understanding category. It can be seen as an extension of understanding while also being a prelude to evaluating and creating. It involves breaking a material, topic, problem, or exercise into its constituent parts, determining how each part relates to the overall structure. The subcategories of analyzing are *differentiating, organizing*, and *attributing*. *Differentiating* refers to distinguishing each part of a structure in terms of relevance and importance. *Organizing* relates to the identification of the elements of communication or a situation, determining how each component fits into a joined structure. Finally, *attributing* refers to ascertaining the underlying biases, values, intentions, and points of views in a communication.

Evaluating consists of two subcategories, *checking* which refers to "testing for inconsistencies or fallacies in an operation or act", and *critiquing*, "judging a product or operation based on externally imposed criteria and standards" (Radmehr & Drake, 2019, p. 902). The main aspects of evaluation are making judgments and examining information.

Creating, the highest level of cognitive thinking, is defined as "putting elements together to form a coherent or functional whole" (Radmehr & Drake, 2019, p. 902). This category's main ideas involve creating a new product by mentally manipulating and reorganizing some parts or elements into a new pattern or structure that was not before present. It can be split into three subcategories, *generating, planning* and *producing*. *Generating* involves "representing a problem and arriving at alternatives or hypotheses that meet certain criteria" (Radmehr & Drake, 2019, p. 902). This should not be confused with understanding since generating involves finding various possibilities of solutions, whereas understanding aims to find a single solution through the notions of the seven subcategories. *Planning*, as the name suggests, refers to the development of a plan for solving a problem, while finally, *Producing*, relates to carrying out the said plan that meets certain specifications.

While it is unreasonable to remember each of the nineteen subcategories mentioned above, they are essential in understanding each cognitive level's ideas, aspects, and elements. Now that each cognitive level is adequately defined, an introduction to how one can find these in a question remains unmentioned. The different levels of RBT refer to some action or thought process that should be investigated. However, a question will probably not easily be represented by using any of the categories. The thoughts that the question may promote, however, will more easily coincide with the levels of RBT. Thus, a question's cognitive level is determined through the thoughts and actions of a respondent, whether the respondent is the asker themselves, a teacher, or a student. When evaluating student-teacher questions, the actions of the teacher may not be of interest, and then the question's cognitive level could be determined through the questions *potential* to raise high-order cognitive thinking. This last approach is the one that will be applied in this study.

Sadker and Cooper (1974, p. 503) introduce five types of high-order cognitive questions (HOCQ), based on Bloom's taxonomy:

- **Evaluation:** Questions prompting an evaluation of a subject. Ex. ”In what occasions would we use derivatives in our everyday lives?”
- **Comparison:** Questions asking to determine similarities/dissimilarities between objects. Ex. ”I see that the definition of average and momentary change are quite similar, but I struggle to see in what ways. Could you explain the similarities or dissimilarities between them?”.
- **Problem-solving:** Questions that require the respondent to solve an entirely new problem. Ex. ”We just found that we could express the series with this formula, but is there any way we could draw it?”
- **Cause and Effect:** Questions that require one to perceive relationships. Ex. ”I see that the function $f(x)$ will not be defined at $x = 2$ since the denominator will become zero at this point, but what would happen as we moved closer to this point along the x-axis?”
- **Divergent questions:** Questions that require the respondent to think creatively or offer personal reactions. Ex. ”In how many ways can we represent the series of odd numbers?”

Sadker and Cooper's original text involves teacher-oriented questions. Thus, the examples provided above show my interpretation of the question types from a student perspective. The five question categories can be tied to the three upper levels of RBT by referring to the subcategories described above. Evaluation from Sadker and Cooper are naturally tied to the *evaluation* level of RBT. Comparison, while seemingly belonging to the *comparing* sublevel of the *understanding* level of RBT, is related to the *differentiating* and *organizing* sublevels of the *analyzing* level. Problem-solving may be related to the *analyzing* or the *creating* level, depending on the phrasing and use. Cause and effect relates to the *analyzing* level and divergent question refers to the *creating* or *evaluating* levels of RBT.

Lower-order cognitive questions (LOCQ) do not prompt high-order thinking; that is, LOCQ allows the recipient to rely on memory and recall (Sadker & Cooper, 1974). Thus, by referring to figure 2.1, every question belonging to the three upper levels could be defined as high-order and the bottom three as low-order. However, this does depend on the situation. A question that at one point is deemed to be high-order may at another point be deemed low-order; for example, a high-order question posed during the introductory part of a subject may be considered a low-order question during the conclusion and is thus dependent on preexisting knowledge.

Both RBT and Sadker and Cooper's five categories will be used in the analysis, though with different purposes. RBT will be the primary tool for determining the cognitive level of a question by analyzing its *potential* of generating high-order cognitive thinking. When RBT does not suffice, or some more argumentation is deemed necessary, Sadker and Cooper's five categories will be used to judge if a question should be deemed high-order. As opposed to the question's mathematical presupposition, every question analyzed will to some extent be characterized through its cognitive level. As a question's intention and potential are highly dependent on its phrasing, one needs to define the language of mathematics.

2.3.3 Mathematical language - Defining the vocabulary of mathematics

Bell (1970) published a list of 365 words that were in common use both outside and within mathematics, which he claimed even the "slowest learners" (his term) need to comprehend to deal with mathematics's elementary topics. This list spanned words from simpler terms like "link," "find," and "sort" to more complexly defined "bilateral" and "quadratic". Mulwa (2015) presents three broad categories of words, as first described by Rothery:

1. Words that are wholly specific to mathematics and not usually encountered outside the academic setting. These are words such as "hypotenuse", "parallelogram", "coefficient", and "isosceles". According to Mulwa, many difficulties students face caused by these

words are due to their scarcity in an everyday setting. As the students usually only encounter these words in class where they are often defined only once and never again, they may have trouble remembering or understanding such terms. Further, students often have little experience with or do not have easy access to find such a definition.

2. Words with separate meanings in mathematics and ordinary English (or any language). These words are commonly used in everyday language but have different, and often more complex, definitions and use-cases in mathematics. These words can often be a source of difficulty for students. Such words can, for example, be "product", "volume", "odd", "prime", "power", and "mean". Similar words exist in every language.

3. Words where the everyday and mathematical use and definition aline. These are words such as "between", "similar", "gradient", and "relation". Students' main difficulty with this category of words is knowing that they may, contrary to usual, have the same or similar definition as in everyday language, at least at this level of mathematics. As Mulwa points out, students may think that ordinary words take on some mystical form when put in a mathematical context.

This shows that there is little doubt that phrasing questions with high clarity and the use of mathematical terminology might be difficult for students. Thus, phrasing a question using relevant terminology should be regarded as a desired skill when evaluating student questions. As shown above, this is no easy task. Thus, this can be used to evaluate another layer of complexity in student-posed questions.

Chapter 3

Methodology and data collection

To find an answer to my research question, I had to find some methodology that could aid me in performing the most efficient and thorough study as structurally possible. This section presents the research methodology *didactical engineering* used as the primary method for structuring and organizing my research. Following this, I present the methods used to collect the data necessary to provide an answer to my research question. Further, I present the analysis method *thematic qualitative content analysis*, which purpose is to organize and manage the vast amount of data collected in this study. Though it might be unorthodox to do so in a methodology chapter, I will provide some theoretical background on each of these methods to separate the theory used for further analysis (presented in the last chapter) from the theory solely used to give a sufficient understanding of the methodological approaches.

3.1 The research setting

Before presenting the different methodological approaches used in this project, I will give a short introduction to the setting in which the research was performed. The research was conducted in a mathematics 1T course, which is the more theoretical mathematics course in the first year of high school (*videregående skole*) in the Norwegian school system. The students would then be between sixteen and seventeen years of age, and the class consisted of twenty-four students, with a majority of male students. The students were inducted into the program for general studies (*studiespesialiserende/allmenfag*). The data collection was performed during three weeks; one week of participating observation through field notes, one week of participating observation through audio and video recording, and one week of digital, at-home teaching. A thorough presentation of the data collection is given in section 3.3.

The mathematical content considered in this study was initially to be only differential calculus. However, many parts of the research had to be changed due to the ongoing COVID-19 pandemic. The video and audio collection were originally to be conducted in both lesson-type scenarios and in guidance settings, with some balance between the two. However, due to the class being put in quarantine for one week, I lost valuable time to collect my data and then had to continue my collection during an assessment week. As a result, I felt like I had not collected sufficient data by the time the class moved on from differential calculus. I then planned to collect data for one additional week, this time on an introduction to triangle trigonometry. Unfortunately, the schools were declared to enter a red restriction level, thus forcing all teaching the week I was to collect my additional data to be done at home, again altering the way data could be collected. The data collection method for this at-home teaching session is discussed in section 3.3.2 and a thorough introduction and analysis of the designed lesson is given in chapter 5.

The timeline for the study can then be expressed as followed:

- **Observing classroom culture** when the students worked with differential calculus. One week, five lessons á forty-five minutes,
- **Observing question-posing** during the last stages of an assignment on differential calculus, two days, four lessons á forty-five minutes.
- **Group-based Mentimeter session** to facilitate question-posing concluding differential calculus. One lesson á forty-five minutes.
- **At-home teaching session** on an introduction to triangle trigonometry. The time frame here is somewhat unclear as the students worked with this session individually at home. Initially meant to span two lessons á forty-five minutes.

Some more data were collected, in particular after the at-home teaching session, but this data proved too minuscule to be used effectively and have thus been left out. Each of the different phases mentioned above is described in detail in chapter 5, with a brief presentation of my original plan for the study being presented in section 5.2. By structuring the data collection in this way, I managed to collect data in various settings, thereby aiding me in answering *how* students pose questions of different levels of complexity in different scenarios. This could then give me a broader and more complete answer to my research question. The fact that I collected data from two completely different mathematical topics could indeed help make my results more valid as some tendencies can be analyzed across topics, thus making my general observations more topic-independent. It further allowed me to observe question-posing at both an introductory and concluding level, thus opening for more discussion, though as the situations for data collection were so different, generalizability could be reduced. However, this would again result in a vast assortment of data, and thus, I would need a methodological framework to help me structure my research and manage my data.

3.2 Didactical engineering

To provide such structure to my thesis, I have chosen to implement didactical engineering (DE) as my primary research methodology. The strength of this method lies in the multiple layers of analysis, providing such a thorough understanding of the subjects to be taught and extensive analysis of each part of the research process. DE emerged in France in the 1970s and was founded by the education researcher Guy Brousseau. Initially, it arose alongside the theory of didactical situations (TDS) but has proven to exceed its initial framework (Barquero & Bosch, 2015). According to the renowned education researcher Michèlle Artigue, DE emerged due to a need for a framework considering didactical systems in their concrete functionings, explicitly paying attention to the constraints and forces acting upon them. Further, she claims:

> As a research methodology, DE emerged with this ambition, relying on the conceptual tools provided by the Theory of Didactical Situations (TDS), and conversely contributing to its consolidation and evolution (Brousseau, 1997). It quickly became a well-defined and privileged methodology in the French didactic community, accompanying the development of research from elementary school up to university level [. . .]. From the nineties, DE migrated outside its original habitat, being extended to the design of teacher preparation, and professional development sessions, used by didacticians from other disciplines [. . .] and also by researchers in mathematics education in different countries (Artigue, 2020, p. 203).

Then, DE could prove to be an effective tool when designing and structuring lessons aiming to facilitate question-posing. DE is divided into four stages or phases, each serving its' own purpose. In the following, I will present the four predominant stages of DE, as presented by

Artigue (2020) and Barquero and Bosch (2015). These phases should then be recognized as the remaining chapters of this thesis, one chapter referring to one phase.

3.2.1 Preliminary analyzes

During the preliminary analyzes, we examine the mathematical object in question to gain a historical perspective and map previous research on the topic. We arrange this analysis into three stages: epistemological, didactical, and institutional analysis. The epistemological analysis consists of a historical and mathematical overview of the mathematical object. Here, the mathematical *content* is considered. In many situations, it could be beneficial to have a proper historical overview of a topic to be taught, as this could provide important knowledge of how a piece of knowledge emerged. Further, by examining the mathematical content, one could consider nuances previously unknown or unclear. This could further benefit the effective and well-thought-out design of a lesson. A historical and mathematical evaluation of both trigonometry and calculus, focusing on differential calculus, is provided in section 4.1.

The didactic analysis maps previous insight and research on the topic. The main goal of this stage is to evaluate the necessity to introduce the mathematical knowledge at school. Here, didactical research on the topic is considered, and the pros and cons of introducing it at a specific school level are presented. One of the main benefits of performing such an analysis is gaining crucial insight into potential difficulties and didactic choices that should be considered. This could particularly help in the design process, as one could get ideas and thoughts early in the planning. A didactic analysis on both triangle trigonometry and differential calculus are performed in section 4.2.

The last stage, the institutional analysis, studies the conditions and constraints offered by the institution where the research and teaching are to be conducted. Such an analysis is beneficial as this could help prevent planning something that cannot be executed due to some constraints offered by the institution. As this research is performed in a high school, an evaluation of curricula and other frame factors such as both local and national COVID-19 restrictions are presented in section 4.3.

By analyzing the mathematical content, didactical insight, and institutional conditions relevant to the research, I could more appropriately develop some hypotheses about what to expect when analyzing my data later in my study. In addition, it could further aid me in facilitating the environment necessary to answer my research question by allowing me to predict which possible problems, results, and conditions I could be faced with when collecting my data.

3.2.2 Design and *a priori* analysis

In the design and *a priori* analysis phase, the mathematical content is modeled or considered. The researcher or teacher would then design a lesson according to some desired learning goal, composing it in such a way that the mathematical knowledge would emerge at the end of the lesson. This design is then analyzed before, *a priori*, the lesson is realized. According to Barquero and Bosch (2015), there needs to be a distinction between a mathematical perspective and a didactical perspective. Firstly the mathematical content should be defined or characterized through a mathematical analysis before performing a didactical analysis of how the content at stake may emerge from the designed situation. Here the researcher's hypotheses should be made explicit to be used in a later stage. By performing such an analysis, one can more easily anticipate possible obstacles and take these into account before they emerge. The different designs and corresponding *a priori* analysis trying to facilitate question-posing are presented in chapter 5.

When considered in collection with the preliminary analyzes, this phase could further contribute to the ever-nearing answer to the research question. This would allow me to design and analyze research situations specifically made to promote question-posing in various scenarios, thus giving the *student-teacher communication* more depth. This would again aid me in evaluating to what extent students use different types of questions in different situations.

3.2.3 Realization, observation and data collection

This phase is reserved for the implementation of the designed didactical situation. Here data is collected through the preferred method, e.g., video or sound recordings, answer sheets, or observation notes.

3.2.4 *A posteriori* analysis and validation

A thorough analysis of the collected data is performed during the last phase, *a posteriori* analysis and validation. This analysis is based on the theory presented in chapter 2 and aims to find an answer to the research question. One of the main foci in DE is to perform a comparison between the *a priori* and *a posteriori* analysis. Thus, the hypotheses made in the second phase are considered, and contrasts or key points are presented and evaluated. Further, the validity of the didactical intention is tested, that is, investigating whether the design provided the desired results. The *a posteriori* analysis of this study is presented in chapter 6.

3.3 Methods of data collection

For this project, I have chosen to implement several means of data collection; participating observation through field notes in addition to video and audio collection, anonymously collected data from a Mentimeter session, and student-work collected through Google Forms. These last two data collection methods are closely related to online questionnaires and will thus be discussed in relation to this. In the following, each of the aspects of data collection will be discussed and justified with special care given to the methods' constraints.

3.3.1 Observation

As Robson and McCartan (2016, p. 322) point out, there are, in reality, two outer perimeters of research observation, formal non-participatory and informal participatory observation, the difference between them being the level of structure, rigidity, and participation. Since the primary goal of this study is to investigate how students use and interact with questions in different classroom settings, and this being a highly unpredictable and somewhat chaotic setting, informal participatory observation was deemed to fit the research goal well. This is because I would then be able to talk to the students, gaining first-hand experience with how they reacted to my didactical choices and how they generated questions in various settings. This also served the purpose of inserting myself as a more embedded member of the classroom, thus possibly producing somewhat more natural and normal behavior from the students.

As mentioned, the data from the observation were collected using field notes and audio and video recordings. The field notes served the purpose of quickly writing down impressions and observations of the classroom *culture*, while the recordings were used to collect a broader spectrum of data. Field notes were used for five classroom hours á forty-five minutes, collecting observations of how the class as a whole used questions before any guidance had been given to the teachers on how to implement my research foci. Here, I paid particular attention to the level of orality, which types (if any) questions the students posed, and the classroom dynamic.

As with the field notes, the recordings spanned five classroom hours á forty-five minutes. The video recordings were used to map the classroom dynamic, making the teachers' movements and who was talking easier to manage. The audio recordings provided the majority of data for this study, collecting dialogues between teachers and students and capturing the teacher's voice during a lecture-type teaching scenario.

As Robson and McCartan (2016, p. 334) specify, it is well-nigh impossible to conduct research in a school setting without, to some degree, influencing the participants. They further point out that as soon as the observed (the students in this case) are aware that they are being observed, the observer (the researcher) will become a participant in the situation. As a means to minimize the effect of this, the researcher can usually employ *minimal interaction*, that is, in as many ways possible avoid contact with the research objects, or *habituation*, that is, being repeatedly present in the setting, thus making one's presence less noticeable. As indicated above, I chose to implement this last method as I felt the need to interact with the students to collect the necessary data.

While one can never be sure that one's presence has not influenced the participants, there are several indicators that the effects are somewhat manageable, one of which being that "the pattern of interaction stabilizes over sessions" (Robson & McCartan, 2016, p. 334). When I first began my observation, few of the students sought help or initiated a conversation with me. However, as the week progressed, more and more students started asking me for help, and some carried out non-school-related conversations with me. When I started collecting data through video and audio, I noticed that some of this "trust" was retracted somewhat. This was particularly noticeable when the camera at one point made a sound, and the surrounding students immediately fell silent. However, the students quickly appeared to forget about the cameras and audio recorders, as indicated by them talking about non-school-related topics around the equipment. This indicates that my presence, and the use of recording devices, had a limited effect on the students.

While my presence alone might not have influenced the students too much, my interaction with them could have changed their responses during the different scenarios observed. In particular, the language I employed when interacting with the students may have altered their responses both positively and negatively in terms of using mathematical terminology. It might also have affected their question-posing behavior, in particular in the ordinary teaching setting. As the research focused on question-posing, and this being actively focused on in different ways during data collection, the student responses may have been altered somehow as a reaction to my research. While this can be justified by employing the theory of active comprehension as described in section 2.2, it is still necessary to take into account. An example of how my intervention and use of mathematical terminology might have influenced the data is shown in section 6.2.2.

3.3.2 Online questionnaires

While a proper presentation of the Mentimeter and at-home teaching sessions will be given in chapter 5, a brief introduction will be given here to provide some context to how the data were collected. The Mentimeter session sought to collect data of questions generated by groups of students rather than the (primarily) individually generated questions collected throughout the observation. To do this, groups of three to four students were asked to answer the question "What question should you ask yourselves to answer exercise _?", that is, they were asked to provide questions necessary to solve some tasks they had solved earlier on in the week. I used the interactive presenting tool Mentimeter, where the groups could then send their answers so that they would appear on the screen in the front of the classroom. The answers provided gave no indication of which group they belonged to, and thus, the groups could provide answers anonymously. These questions could later be downloaded both as PDFs and as an Excel sheet. An example of how the presentation looked is shown in figure 3.1.

Figure 3.1: The figure shows a screenshot of the original data set collected in the Mentimeter session. The boxes indicate the students' responses to the question "What question should you ask yourselves to answer exercise 1?". These responses could then be downloaded as a Microsoft Excel file.

The at-home teaching session revolved around the acquisition of *reflective questions*, that is, questions reflecting on some task. The students were to solve an exercise sheet at home, and at two points they were asked to submit some questions generated before and after solving a task to a Google Forms sheet. The students were given no instruction on whether the tasks should be solved individually or in groups, thus possibly resulting in some mix of the two. These questions I then collected in a more structured spreadsheet. Additionally, the students were given a letter so that I, at a later point, could recognize the student according to my preexisting code for that particular student.

While the way Mentimeter and Google Forms were implemented in this study cannot strictly be defined as questionnaires, some closely related advantages and disadvantages should be considered. According to Robson and McCartan (2016, p. 248), there are several advantages to using questionnaires in general, one of these being that "they provide a relatively simple and straightforward approach to the study of attitudes, values, beliefs, and motives". When the students provide answers both in the Mentimeter form and the Google Forms sheet, their meaning might be easier to analyze than if the question were posed orally as they had a longer time to phrase their questions. Further, as the Mentimeter questions were posed anonymously, Robson and McCartan (2016) indicate that the responses may be more sincere. This might also be the case in the Google Forms sheet, as the students never explicitly provide a name, thus maybe giving the impression of anonymity.

Since the Mentimeter session, and possibly also the at-home teaching, involves some group-based elements, it is vital to evaluate the impact of collecting data from groups versus individuals. As pointed to by Zaccaro et al. (2005), it is crucial to consider the influence of the vast social dimension present in all group-based research. As the groups from the Mentimetsr session sent in their questions as one, I had no way of knowing who posed which question. Thus we have no guarantee that all of the students did participate in the generation of questions. Further, some students may not have been comfortable posing questions they genuinely wondered about, possibly removing some sincerity from the responses. On the other hand, some questions may have been more thought-out and polished, as the students could discuss phrasings and the quality of the question. Zaccaro et al. (2005) point to all of these possibilities as integral to all group-based research. Then, the primary difference between group-based and individual research is the difference in the social conditions and constraints offered between these modes of research.

As audio and video collection can be seen as a relatively ineffective data collection method, as it is incredibly time-consuming (at least the transcription process), collecting data through forms could be beneficial when evaluating a project such as this one. Robson and McCartan (2016, p. 248) state that "[questionnaires] can be extremely efficient at providing large amounts of data, at relatively low cost, at a short period of time". As will be indicated by a chart in chapter 6, both the Mentimeter session and the at-home teaching session provided more data (that is, more questions) than the orally communicated questions collected throughout the observation. As the research sought to evaluate *how* students used questions in different student-teacher communications, using Mentimeter and Google Forms allowed me to broaden the horizon of this type of communication. The Mentimeter session allowed me to evaluate the group-based, student-teacher communication, while the at-home teaching generated digital communication, allowing me to evaluate which dimensions benefited or obstructed question-posing.

3.3.3 Pilot-project

Before planning this project in its entirety, I conducted a pilot project to challenge my hypothesis that most students would not pose HOCQ without teacher intervention. In this project, I researched how a small group of students used question-posing when changing semiotic representations of the series of odd numbers by facilitating an inquiry environment. Their process of finding as many representations of the series as possible throughout the ninety-minute lesson was then recorded through video and audio collection. I then monitored and analyzed these student-to-student questions and noticed several fascinating aspects of their interactions. Though it is somewhat unconventional, I will here present some of the results from the pilot study, as these were used to build the hypotheses for this master's project. Firstly, the students posed vastly more questions than anticipated. A total of seventy-seven questions were posed during the ninety-minute lesson. Secondly, out of the thirteen different question categories characterized, the most prominent question type was of a procedural nature. The most striking result was that none of the questions were argued to be of a higher order, though many were interesting nevertheless. A chart representing the distribution of data collected in the pilot project is shown in figure 3.2.

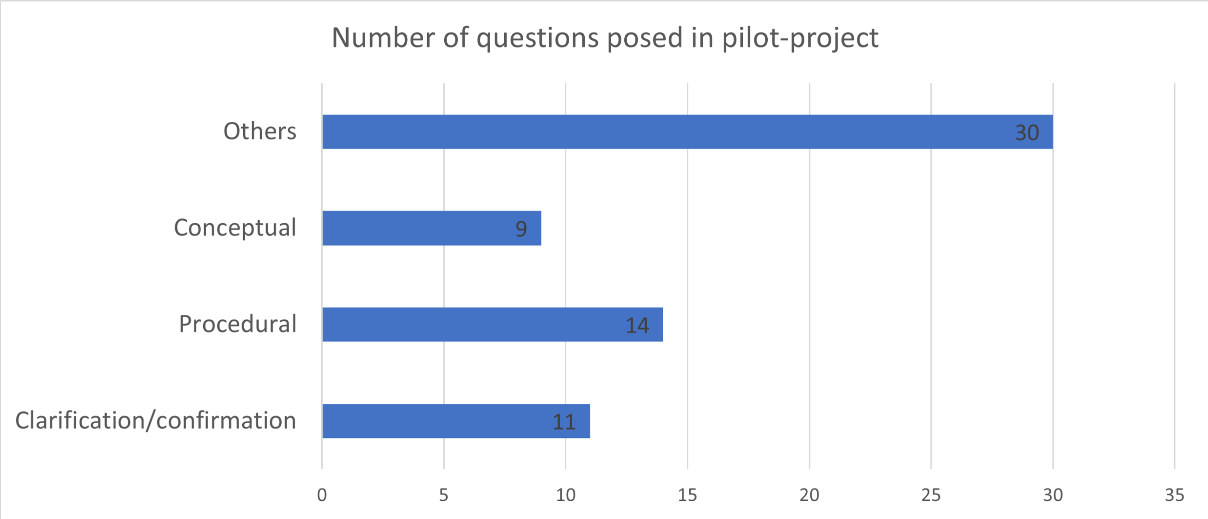


Figure 3.2: Descriptive representation of the distribution of collected data from the pilot project. The different pillars refer to the codes used in this project, and the "other"-category involves a consolidation of ten question-categories too extensive to present here. The data were collected during a ninety-minute inquiry-based lesson, and shows the categories of the seventy-seven questions identified from observing three students throughout this lesson.

As can be seen by the chart, the vast majority of questions emerging in the pilot belonged to the *Clarification/confirmation*, *Procedural*, or *Conceptual* question categories. While most of the categories used in the pilot do not emerge in this study (they have been more thoroughly generated in this master's project and are thus more complexly defined), these three categories, in particular, will have the same definition as the ones to be presented in tables 3.1 and 3.2 in section 3.4.1.5.

3.4 Qualitative content analysis

In this master's project, I have chosen to use qualitative content analysis as my principal method of data analysis. To properly introduce this method, one needs first to present the foundation on which it is built, the history and use of classical content analysis.

Different authors, like Krippendorff and Merten, suggest that the use of content analysis began a long time ago. Merten exemplifies here the exegesis of the Bible or Sigmund Freud's interpretation of dreams (Merten, 1983, p. 35). In other words, the use of content analysis (or some precursor) is in no way new. Then why is it so that the method is not widely known and often excluded from books and articles on qualitative methods? One of the main reasons is that content analysis started, and is mostly known as, a quantitative method. It is used for statistical analysis of preexisting data, often disregarding the qualitative nature of the data set (Kuckartz, 2014). This partly comes from the golden age of content analysis during the second world war. Here it was used to study war reports and propaganda, making it not only scientifically driven but also political. While the political influence paid a vital part in driving content analysis towards a more statistically centered method, the main reason can be argued to stem from the shift towards behaviorism in the social sciences after the second world war. Empirical evidence and testing hypotheses and theories were prioritized over the often vague nature of qualitative research.

According to Kuckartz (2014), we can use *thematic analysis* to see the difference between qualitative and quantitative content analysis particularly well. Quantitative analysis aims to convert verbal data into specific categories represented by numbers, which in turn is used to evaluate a resulting data matrix statistically. On the other hand, the qualitative analysis focuses on the text itself by being based on the text in its entirety. In the following, I will present a general description of thematic content analysis before describing how and why it was chosen to be used in this thesis.

3.4.1 Thematic qualitative content analysis

The most frequently used method of qualitative content analysis is thematic qualitative content analysis (TQCA)[1]. At the center of this method is the construction and use of categories to reduce and focus the collected data. These categories can be generated either inductively, using the data, or deductively, using an underlying theory, or some mix of the two. How TQCA is used is highly dependent on the different types of data collected, thereby making it alterable and adaptable to different research projects. TQCA is at its core a multi-layer (or phase) method. A sketch of the process is shown in figure 3.3.

[1]The following is based on chapter 4 in Kuckartz's *Qualitative Text Analysis: A Guide to Methods, Practice & using Software* (2014), and will thereby not be cited outside of direct quotes. In this chapter, Kuckartz presents a thorough and exemplified description of three basic qualitative content analysis methods, TQCA being one of them.

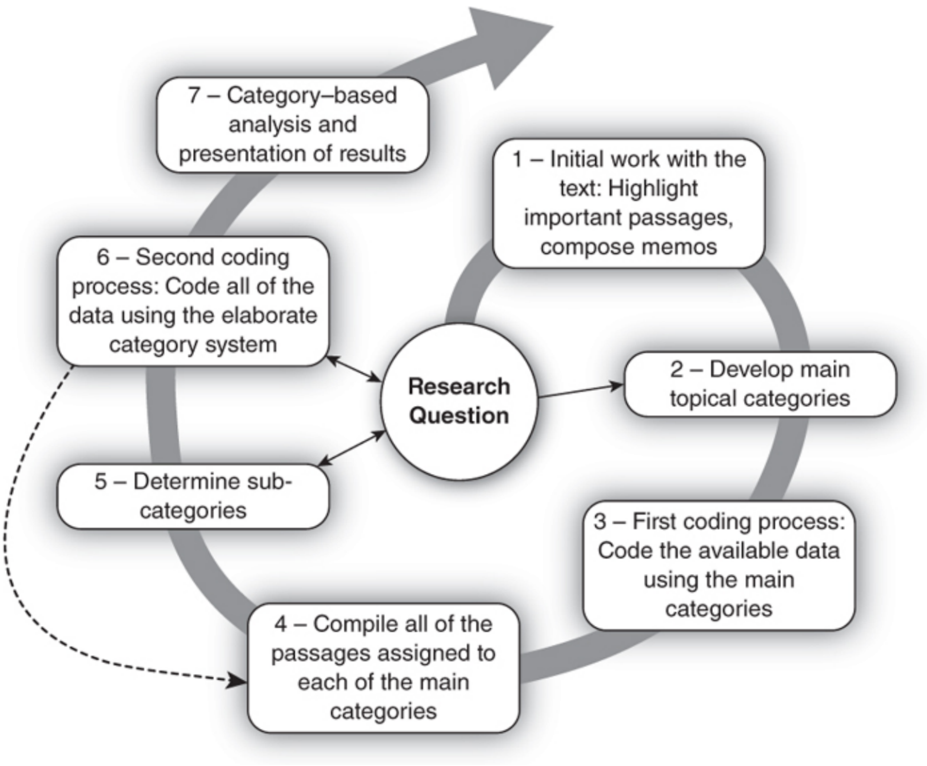


Figure 3.3: A sketch of the process followed in TQCA, highlighting the 7 core phases of the method. (Figure collected from page 2, chapter 4 in Kuckartz, 2014).

It is easy to see from the figure that TQCA in no way can be described as a linear process. Instead, it should be described as a spiral, each new phase building and depending on the previous, ultimately resulting in a thorough, thought-out, and highly structured analysis. I will now give a detailed description of each of the seven phases to give a proper introduction to how it was used in this project.

3.4.1.1 Phase 1: Initial work

The first phase is used to familiarize oneself with the text (transcribed and other textual data) by reading through it multiple times, creating personal memos, and writing comments in the text's margins. This is done to highlight important passages and to make the next steps more comfortable to manage. The goal is to gain a general understanding of the data set based on the research question. By noting initial thoughts and ideas, one can already begin the work by forming an understanding to be used in analysis and structure. In my initial work, verbal data was transcribed in a thorough process. This transcribed data was then read through multiple times while listening to the audio files in order to make sure it was as correct as possible. In other, larger projects, it could have been relevant to have other cowriters or fellow researchers help verify the data. However, to ensure the participants' privacy, I opted to be the only person to evaluate the data. Passages that I initially found particularly interesting were marked and noted for later stages of analysis. The same process was used for the remaining data from the Mentimeter session and the at-home teaching session. The transcribed data, the questions from the Mentimeter session, and questions from the at-home teaching session can be found respectively in Appendix A, B, and C.

3.4.1.2 Phase 2: Developing the main categories

In the second phase, the main thematic categories are generated, often directly from the research question. These categories form the basis for analysis in the future phases. While there is no specific requirement on how these categories are generated, it is often relevant to use open coding; here, one will generate categories based directly on the collected data, structuring and combining similar categories into a fitting collection based on the research question. A rule of thumb is always to record anything that might seem relevant, exciting, or peculiar at first, as one can always discard it later. One is not, however, required by TQCA to use open coding in this phase. It is also relevant to use existing categories from other projects or theories. The only requirement is to in some way generate appropriate categories. Thus, the category system should:

- be established in close connection to the research question(s) and goals of the project;
- neither be too broad or too detailed;
- contain a detailed description of each of the categories, making as little room for interpretation as possible;
- be formulated with thoughts on how they may be presented in the results;
- if possible, be tested on a section of the data material.

The main categories developed for this project divided student questions into high-order cognitive questions and low-order cognitive questions. The primary reason for this choice of main categories was that it would significantly reduce the amount of data to be analyzed in later phases by explicitly using the cognitive level of a question to filter out all data that did not directly involve a question. This further allowed me to perform some surface-level analysis of the cognitive level of all the questions early in the process. An important note here is that the main categories involve a question's *percieved* cognitive level. Thus, a question coded as high-order could later be deemed low-order and *vice versa*.

3.4.1.3 Phase 3: First coding process

The researcher goes through the text section-by-section and line-by-line to assign all relevant passages to appropriate categories during the first coding process. Thus, the researcher needs to interpret which topics a passage is addressing to assign the relevant category. Only passages containing information pertaining to the predetermined topics and sub-topics must be associated with a category (or code), ignoring irrelevant data. One passage can be assigned multiple codes, as there often can be more than one way to interpret the situation. As a result, some passages may overlap and intertwine, possibly making them especially interesting to analyze.

This first coding process divided my data into the two main categories, high- and low-order cognitive questions. The data was then deductively coded using the theory from section 2.3.2 to determine whether a question could (possibly) belong to its respective cognitive level. This resulted in some questions being coded both as low-order and high-order, depending on how the question was interpreted.

3.4.1.4 Phase 4 and 5: Compiling data and creating subcategories

I have chosen to collapse phases 4 and 5 into one step as they naturally follow each other. After the data has been coded, every segment belonging to the same category is collected and compiled into a list or table. This step is called text retrieval. Following this, relevant categories are split into fitting sub-categories to differentiate between similar but yet different data. These sub-categories are then put into a list, ordered, and defined. Lastly, each sub-category is illustrated using prototypical examples.

3.4.1.5 Phase 6: Second coding process

The sixth phase follows the same pattern as the third, now coding the data according to the newly defined sub-categories. The researcher now has to go through the data once again, interpreting, collecting, and coding. A sufficient amount of data must be used to differentiate between the main topics and define new sub-categories. Categories based on a small amount of data will often prove necessary to alter, resulting in significantly more work and effort from the researcher.

After the data had been collected and compiled in a spreadsheet, every question was coded inductively according to what I deemed the purpose of the question was. These subcategories would then relate to the "how" students use questions of different levels of complexity in my research question. The subcategories, with corresponding definitions and prototypical examples, are provided in table 3.1 and 3.2. These tables are split according to the main categories defined earlier. How each subcategory is related to the main categories is shown in figure 3.4.

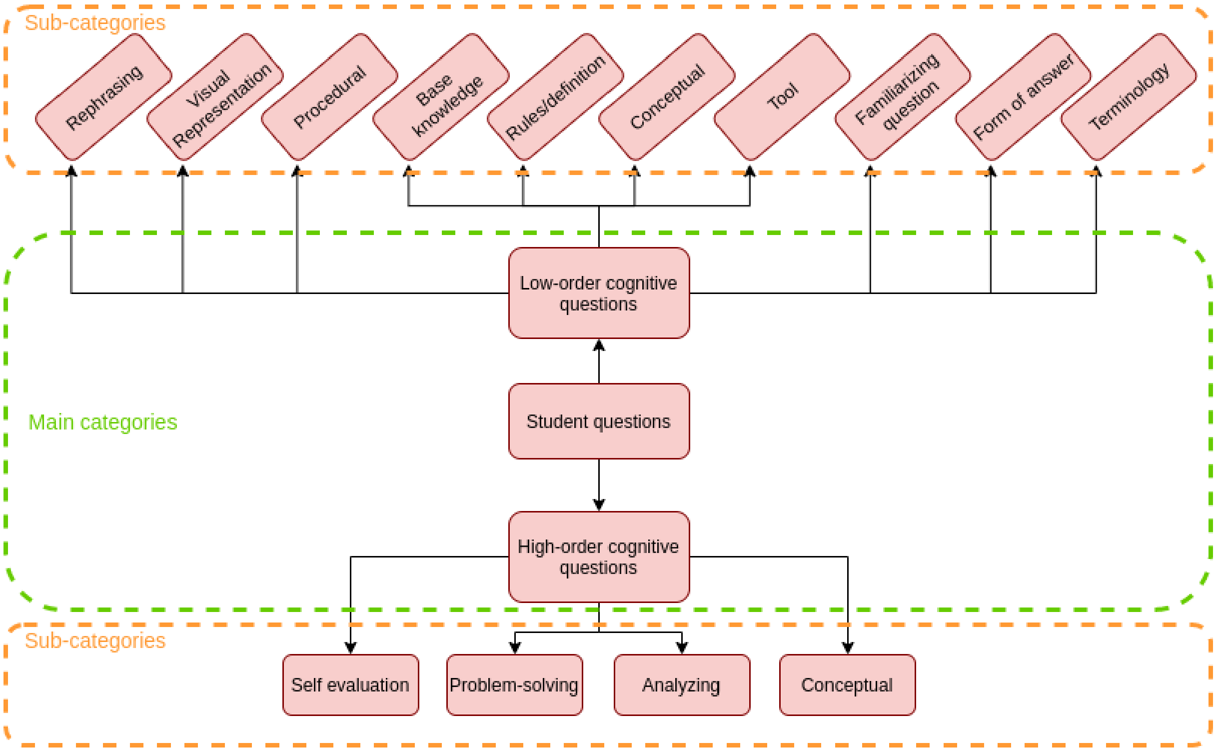


Figure 3.4: Figure depicting how the different categories and subcategories are related. (Self-made)

Some of the strength of TQCA was shown in this phase, as I could go through the data several times, changing and altering subcategories to ensure that they correctly described the different aspects of the questions. By the end of this coding process, the resultant subcategories displayed a rather concrete picture of *how* the students in this study used question-posing. While it may have been more relevant in other projects to put such codes in the appendix, rather than displaying them explicitly as a part of the methodology, I deem it essential to illustrate how each subcategory is defined, as this is crucial to find an answer to my research question. Each definition tells a different story of *how students use questions of different levels of complexity in their student-teacher communication*.

Table 3.1: Subcategories of LOCQ identified in the data analysis, with definitions and examples.

Data code	**Definition**	**Example(s)**
Rephrasing	Questions identified as a rephrasing of an exercise or statement.	What is the equation of the tangent? (when asked about the equation of the tangent)
Visual Representation	Questions concerning a visual representation of a mathematical object, whether it be how something will look or a question concerning a figure.	What can we see in the figure? What does the graph show us?
Procedural	Questions used to gain information about a procedure or method.	How did you get 16? How do I calculate the average rate or change?
Base Knowledge	Questions where the answer can be found without manipulation in the exercise text or figure.	What coordinated do we have? Which points does the graph go through?
Rules/Definitions	Questions seeking information about a rule or a definition.	What is the definition of the derivative? How can we derive long polynomials?
Tool	Questions concerning a mathematical tool, as GeoGebra, a calculator or a computer program.	Which program should I use? What is CAS?
Familiarizing question	Questions concerning how to use the information in the task description.	How can I use the tangents? What does the function tell us?
Form of answer	Questions regarding the correctness of an answer or the phrasing or form of an answer.	Is this the correct answer? How should i phrase my answer?
Terminology	Questions about the meaning of a word. This category should not be confused with asking about a mathematical definition.	What does "increase" mean?

As I only performed a surface-level analysis of each question at this stage in my research, some questions may have been falsely coded and would receive a more suitable code later in my analysis. This shows that the coding process never explicitly stops, the only importance being that the result is as correct and thorough as possible.

Table 3.2: Subcategories of HOCQ identified in the data analysis, with definitions and examples.

Data code	**Definition**	**Example(s)**
Self evaluation	Questions used to analyze and evaluate one's own knowledge or understanding.	What have I learned today? What was it that I did not understand about derivatives?
Problem-solving	Questions promoting the solving of an entirely new problem	We just found that we could express the series with this formula, but is there any way we could draw it?
Analyzing	Questions that promote an analysis or evaluation of some deeper understanding of a topic. This question category can usually be described using the subcategories of *analyzing* or *evaluating* in RBT.	How is this applicable in the real world? Is there a relationship between the Pythagorean theorem and sine, cosine and tangents?
Conceptual	Questions exploring the "why" of mathematics. Questions trying to obtain some higher level understanding of a mathematical topic. This category is often related to the analyzing category above.	Why do we need to find the inverse function?

3.4.1.6 Phase 7: Analysis and presentation of results

The last and final step of the analysis is the Analysis and Presentation of results. As the name suggests, TQCA relies on topics (themes) and sub-topics. There are essentially seven different types of analysis in TQCA, all shown in figure 3.5. Each of the different approaches has different rules and foci, thereby making a presentation of them all of little interest to this project. Hence, I will only present the relevant types.

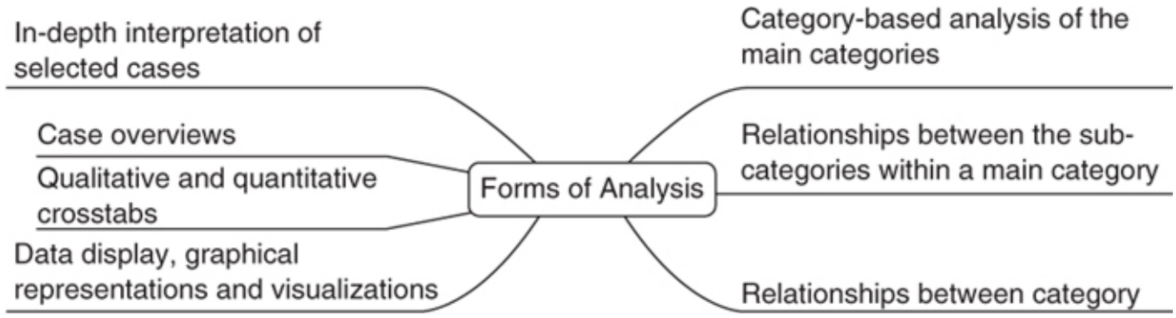


Figure 3.5: Illustration of the seven different types of thematic analyzes. (Figure collected from page 22, chapter 4 in Kuckartz, 2014).

I deemed an in-depth interpretation of selected cases, all defining a prominent theme throughout the data, to be most in line with the purpose of this research project. Here, fascinating situations, cases, or individuals are presented and analyzed according to the underlying theory. In the analysis of chapter 6, the predominant themes identified in this study are used as the headers for each subsection, and particularly compelling cases displaying these themes are presented as the primary data to be analyzed. This analysis method was chosen as it is not as strict as the others presented in figure 3.5. Thus, I was free to display dialogues and other data as I deemed fit, again opening for a more thoroughgoing and in-depth interpretation of only the relevant data.

Further, as the research question explicitly aims to investigate the *extent* of which students use questions of different levels of complexity, some quantitative element of an otherwise qualitative study is required. Thus, I chose to implement some elements of data display and visualizations in the form of diagrams displaying the number and distribution of questions of the different subcategories. These diagrams could then be used to answer this aspect of the research question directly.

3.5 Ethical considerations

As most of the data material was collected through audio and video recordings, many ethical considerations must be taken to preserve the participants' privacy. The participants can be directly identified from the video recordings, and possibly sensitive information may be given unintentionally. Therefore, proper ethical considerations were taken in line with Robson and McCartan (2016, Chapter 10). To ensure the data material could not fall into the wrong hands, an encrypted cloud-based service with two-factor authentification was used to store all raw data. When data had been collected, they were transferred from the equipment to the cloud-based service and instantly deleted from all physical storage media. Further, no data material was used while other people were in the vicinity. Furthermore, all data material presented in this thesis has been anonymized such that no indication of gender is given, all names being pseudonyms and randomly generated. In addition, as recommended by Tjora (2021), no personal information that could refer to specific people was noted in the field notes, thus making them anonymized by default. To make sure the study followed all ethical guidelines, the project was submitted to and approved by NSD - *Norsk Senter for Forskningsdata*, and the students signed an approval form. The approval form can be seen in Appendix G.

Now that each part of this thorough methodology has been presented, I begin my several layers of analysis, starting with the preliminary analysis.

Chapter 4

Preliminary analyses

As described in section 3.2, didactical engineering demands some thorough preliminary analyses of the epistemological, didactical, and institutional levels of the mathematical content and research setting. In this section, such analyses are given, divided into the three respective foci, which again are apportioned into separate analyses of trigonometry and differential calculus. While no specific design concerning differential calculus will be used in this study, most of the data are collected when the students are working on this topic. Thus, to properly evaluate the students' questions, I deem it fit to analyze differential calculus just as thoroughly as trigonometry.

4.1 Epistemological analysis

While the designs to be described in chapter 5 in no way involves a historical perspective, I believe much appreciation should be given to the historical background of all subjects to be taught. The historical aspects can provide a framework for many exploratory tasks and lessons and give the teacher a perspective on which they can base their teaching. Further, such a perspective can help the teacher answer questions of application; in what situations could the subjects be useful?

4.1.1 Trigonometry

The history and place of trigonometry is an old and extensive tale. Through thousands of years and a vast number of mathematicians and philosophers, the practice of trigonometry has evolved into the subject we use today. Trigonometry can be described in a dichotomous manner as either circle or triangle trigonometry. While different traditions and uses come from the two different offshoots, they both have the same origin: the study of the heavens by the ancient Greeks (Bressoud, 2010)[1]. As the lessons concerned in this thesis only rely on triangle trigonometry, I will only present parts of circle trigonometry necessary to present triangle trigonometry properly. The problem that might be the first to be classified as a trigonometry problem was solved by Hipparchus of Rodes around 200 years BC when he tried to explain why the seasons were of unequal length. Using the observed length of each season, he determined the length of the arc traveled by the sun in its orbit each season. With this, he could determine the length of the chords connecting the sun's position at the start of two sequential seasons, allowing him to calculate the distance from the earth to the center of the sun's orbit.

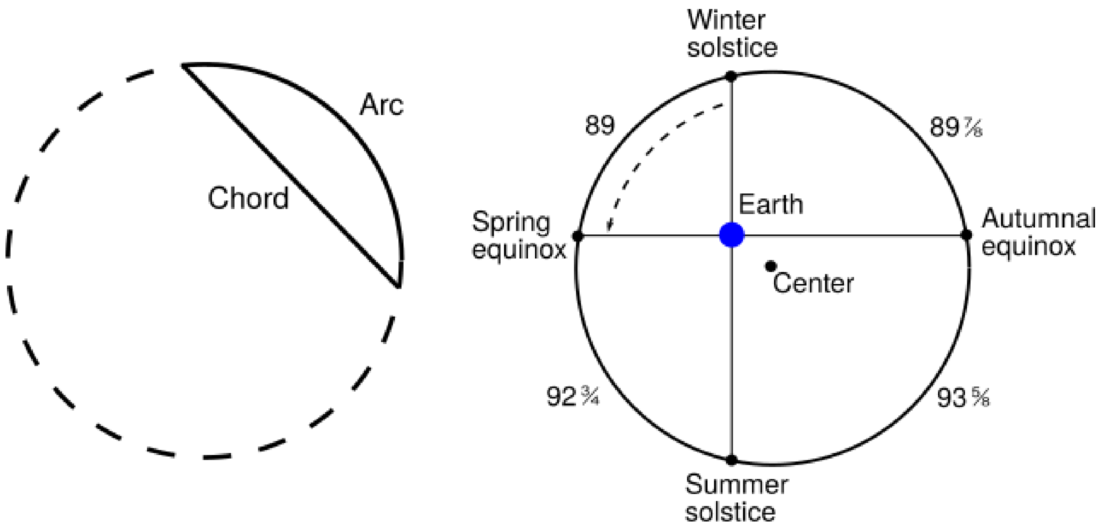


Figure 4.1: Figure to the left shows a classification of chords and arcs in a circle. Figure to the right shows the sun's orbit around the earth and how many days Hipparchus of Rodes counted that each season lasted. Self-made figures based on figure in (Bressoud, 2010).

One of the early problems mathematicians faced was determining the chord length of any arc length. This might be one of the first examples in the history of mathematics of a functional relationship without an explicit formula that can produce an output value for any input value. Arc lengths were usually measured using degrees, with 360° denoting the total circumference of

[1]The following is based on David M. Bressoud's *historical reflections on teaching trigonometry* (2010), which will thereby not be cited outside of direct quotes. In this article, Bressoud describes the emergence of trigonometry from the beginning with the ancient Greeks, through the influence of the 18th-century mathematicians, until the teaching of it in the 19th century.

the circle. By using degrees, some chord lengths could be calculated easily, like

$$crd\,180° = 2R$$
$$crd\,90° = \sqrt{2}R \approx 1,414R$$
$$crd\,60° = R$$

with $crd\,\theta$ referring to the chord length given angle θ and R being the radius of the circle. However, most of the chord lengths were more difficult to calculate. Euclid, using the lengths of the sides of regular inscribed pentagons and decagons, managed to calculate $crd\,72° \approx 1,176R$ and $crd\,36° \approx 0,618R$. More extraordinary, some say that Hipparchus constructed a table consisting of approximate values. Ptolemy of Alexandria built the earliest surviving table in his work on astronomy, *Mathematical Treatise* (or *Almagest*), where he presented the chord lengths for a circle of radius 60 in half-degree increments.

While it may not be readily apparent how chord lengths relate to the trigonometry used today, it can be more easily seen by rotating the circle so that the chord is directed vertically and by inserting some radial lines.

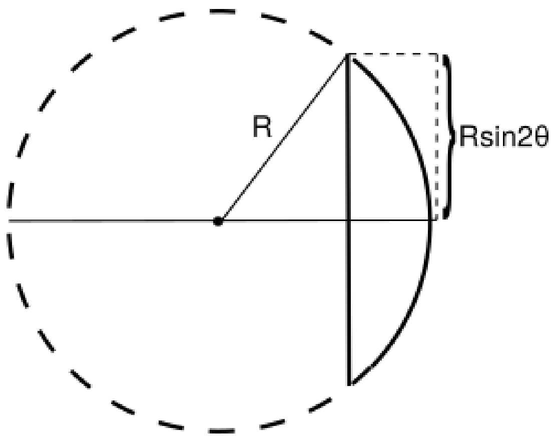


Figure 4.2: The figure shows how a half-chord can be represented using the sine function. Self-made figures based on figure in (Bressoud, 2010).

Then, as seen in figure 4.2, the half-chord may be represented using the usual sine function. Ptolemy's table is then the equivalent of a table of sines in half-degree increments, with seven-digit accuracy. The use of half-chords, or sine, was initially implemented by Indian astronomers somewhere between the third and the fifth century AC, and this is also where the word sine finds its origin in the Sanskrit word *jia* which through some iterations becomes sine in English.

The source problem of triangle trigonometry began by trying to determine the length of a shadow cast by a vertical stick, or *gnomon* (often in the form of a sundial), given the angle of the sun from vertical (figure 4.3a). As this line length would be tangent to the circle, the function came to be known as the *tangent*. The *secant*, from the Latin word *secantem*, meaning "cutting," then was defined as the radial line segment cut off by the tangent. The remaining familiar trigonometric functions, cosine, cotangent, and cosecant, could then be defined for the corresponding line segments for the complementary angle, as seen in figure 4.3b. The six trigonometric functions first make their joint appearance in Abu'l Wafa's commentary of the *Almagest* (Ptolemy's work on astronomy).

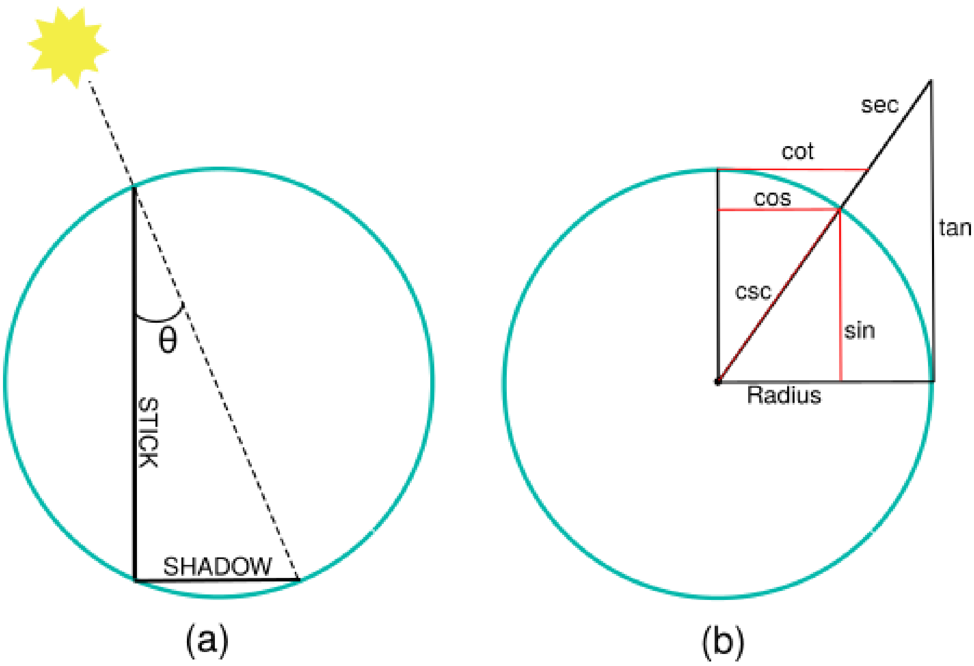


Figure 4.3: (a) Shows how the shadow of a gnomon (vertical stick or sundial) may be represented as a right triangle. (b) Shows how each of the six trigonometric functions can be represented on the unit circle. Self-made figures based on figure in (Bressoud, 2010).

It was not before 1533 that the application of trigonometry to the calculation of sides of right triangles achieved prominence, with Johan Müller's *De Triangulis Omnimodis (On Triangles of Every Kind)*. Bartholomew Pitiscus first used the word *trigonomety* in the form of the book title *Trigonometria*, a Greek-to-Latin transliteration of "triangles measurement". This book marks the beginning of the common use of trigonometry in surveying. According to Victor Katz, many of the trigonometric texts of the sixteenth century illustrated methods of solving plane triangles, but it was not until Pitiscus's work in 1595 that a problem "explicitly involving the solving of a real plan triangle on earth" emerged (Katz, 2009). Mathematicians such as Müller, Rheticus, and Pitiscus used trigonometry and similar triangles to solve for the unknown side of any given right triangle, where at least one of the acute angles and one other side was given. When Euler sometime later fixed the radius of the defining circle at 1 (as in fig 4.2), it became possible to use trigonometric functions as actual ratios of the sides. Then, if the actual purpose of studying trigonometry was to find the unknown sides of a right triangle, thus completely divorcing the purpose from circles, treating the arguments as an arc length no longer made sense. Hence, the definition of a degree as a fraction of a complete revolution arose. A degree would then become 1/360th of a full turn, with 2π corresponding to the circumference of the unit circle (and then a full turn when the defining circle has radius 1). This resulted in the term *radian* as a contraction for "radial angle", first used by either Thomas Muir or James Thomson in the late eighteen hundreds. All that is presented above has then resulted in the familiar triangle trigonometry we use today, with the three basic trigonometric functions:

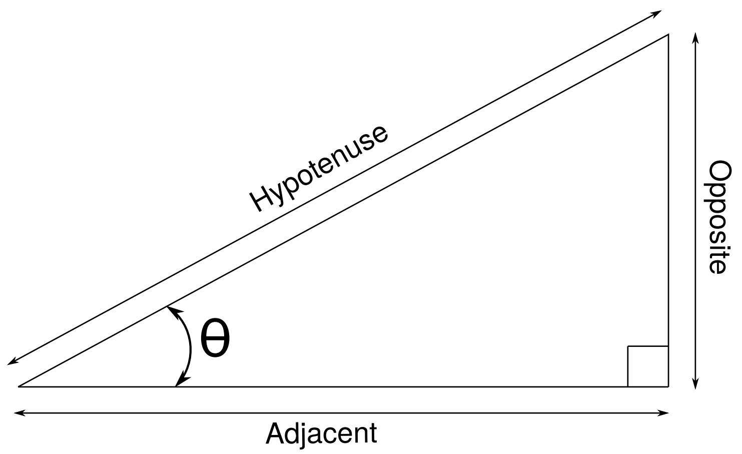


Figure 4.4: Figure depicting the sides on a right triangle in relation to the angle θ. (self-made)

$$sin(\theta) = \frac{Opposite}{Hypotenuse} \qquad cos(\theta) = \frac{Adjacent}{Hypotenuse} \qquad tan(\theta) = \frac{Opposite}{Adjacent}$$

4.1.2 Differential calculus

The invention (or discovery?) of calculus as we know it has been credited to two brilliant minds, Sir Isaac Newton and Gottfried Wilhelm Leibniz, both accusing the other of plagiarism (Encyclopedia of Mathematics, n.d.). While developed from algebra and geometry, calculus builds on two complementary branches; differential and integral calculus. Differential calculus, which is the main focus of this project, studies rates of change, often illustrated by a line's slope. This branch of calculus concerns finding the instantaneous rate of change of one quantity relative to another, i.e., the derivative of one quantity with respect to the other (EDinformatics, n.d.). The other branch, integral calculus, studies the accumulation of quantities, i.e., the areas under a curve, volumes, and distances. Integrals and derivatives are often viewed as opposites, with the integral sometimes called the anti-derivative.

Though Newton and Leibniz have been given credit, historical evidence suggests that many of differential and integral calculus's key concepts were known both to the ancient Greeks and the ancient Egyptians (EDinformatics, n.d.). Some of the more basic ideas, such as calculations of volumes and areas, can be found in the Egyptian Moscow papyrus, stemming from approximately 1850 BC (Spalinger & Clagett, 2001). Archimedes is believed to be the first mathematician to find the tangent of a curve other than a circle, using a method similar to the ones used in differential calculus (Boyer, 1991, pp. 127). Moreover, many mathematicians of great historical and mathematical significance, such as Descartes, De Fermat, and Huygens, should be given credit for the continued development of the field.

Now let us give a formal (algebraic) definition of the derivative. Let a function $y = f(x)$ be defined in some neighborhood of a point x_0. Let $\Delta x \neq 0$ denote the incremental change of the argument and $\Delta y = f(x_0 + \Delta x) - f(x)$ denote the corresponding incremental change of the functions value. The derivative of the function f at the point x_0 is then, if such a limit exits, given by

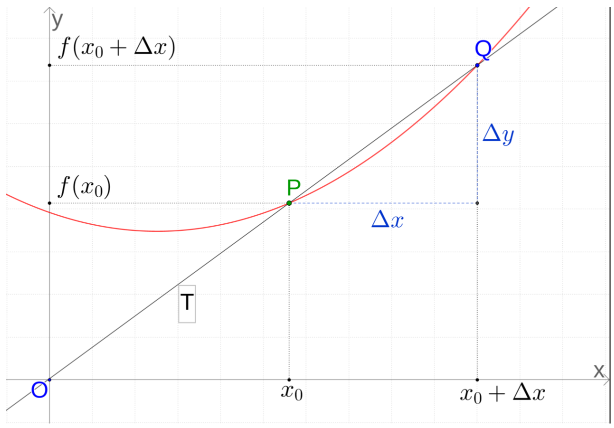
$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

This is usually denoted by $f'(x_0)$, $df(x_0)/dx$, y', y'_x, dy/dx, and similar. Thus, by combining the above denotations

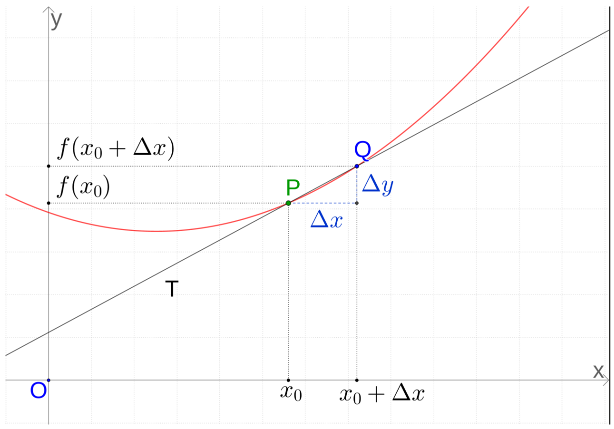
$$f'(x_0) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{\Delta f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

The process of derivation is known as differentiation (Encyclopedia of Mathematics, n.d.).

Using the notation given above, one can also define a geometric (Cartesian) interpretation of the derivative. Draw the graph of the function f and define the points $P = (x_0, f(x_0))$ and $Q = (x_0 + \Delta x, f(x_0 + \Delta x))$. Let T be the line through P and Q. Then as Q approaches P ($\Delta x \to 0$), T tends to the tangent of f for $x = x_0$. We then define the derivative of f at $x = x_0$ to be the slope of the unique tangent of f at the point P (Encyclopedia of Mathematics, n.d.). An illustration is given in figure 4.5.



(a) Large Δx



(b) Small(er) Δx

Figure 4.5: Geometric (Cartesian) representation of the derivative with the red line representing $f(x)$, as Q approaches P. (Self-made)

4.2 Didactical analysis

4.2.1 Trigonometry

As trigonometry is a rather vast subject, I will in this section primarily focus on the teaching and learning of the trigonometric functions sine, cosine, and tangents, as well as more general concerns on trigonometry. According to Hülya Gur (2009), trigonometry is an area of mathematics that many students find particularly difficult and abstract compared to other mathematical subjects. She claims that three generalizable misconceptions in trigonometry, based on Piaget's description of formal operations, are

- misconceptions related to a concept that produces a mathematical object and symbol. Here she points to sine as a concept and sine as the symbol of a trigonometric function.

- misconceptions related to process, that is, the ability to *use* operations. For example, representing the result of calculations of $sin(30°)$ and value of $sin(30°)$.
- misconceptions related to *procept*, that is, the ability to think of mathematical operations and objects. Procept covers both process and concept. An example of this is the sine of x both as a function and a value.

Moreover, a study by Susan Brown (2005) revealed that many students "had an incomplete or fragmented understanding of three major ways to view sine and cosine: as coordinates of a point on the unit circle, as the horizontal and vertical distances that are the graphical entailments of those coordinates, and as ratios of sides of a reference triangle". Further evidence from Brown's study suggests that many students may approach the study of trigonometry as a mystery to be memorized, "rather than an easily-understood logical system of coordinate relationships" (Brown, 2005, p. 2).

As indicated by a study by Delice (2002), "students have misconceptions and learning complexities, which is attributed to the fact that before learning the trigonometry concepts, the students learn some concepts, pre-learning concepts, incorrectly or defectively. These concepts are fundamental for learning the concepts of the trigonometry such as unit circle and factorization" (Gur, 2009, p. 68-69). Further, Blackett and Tall (1991) point to the fact that at an introductory level of trigonometry, students are expected to relate geometric figures to numerical relationships, cope with ratios such as $sin(A) = opposite/hypotenuse$, and manipulate the symbolic representations of such relationships, this being highly complex levels of thinking. Moreover, the students are expected to manage arbitrarily rotated right triangles, making the hypotenuse and opposite sides more challenging to identify (Kamber & Takaci, 2018).

There should, however, be little doubt that trigonometry, when mastered correctly, is helpful in many occupations and subjects. It is highly relevant in many branches of engineering (e.g., mechanical, electrical, informational technology) and subjects such as physics and have many practical implementations in vocational subjects. Performing an analysis of what possible misconceptions students may have in triangle trigonometry have given me insight into potential false presuppositions they may provide when posing questions on this topic. This could help build my hypotheses, which can produce a more thorough answer to my research question. Further, it allows me to design tasks with these misconceptions in mind; thus, I could more easily facilitate a situation that could provide explicit presuppositions and avoid such misunderstandings.

4.2.2 Differential calculus

According to James (1995), many instructors argue that calculus should not be taught in high schools due to the complexity of the subject. The primary concern is that a high school course would have to be "watered down (...) stressing manipulations but slighting subtle processes" (Ferrini-Mundy & Gaudard, 1992, p. 57). As will be shown in the institutional analysis, the different curricula do not advocate the complete teaching of calculus as a whole, only focusing on some of the subject's more basic features. As stated by Jockush and McLoughlin (1990), these can be developed as natural extensions of topics the students already have encountered. On the contrary, James (1995) further emphasizes that by being inducted into the world of calculus too early, students may lack the time to properly develop the necessary background in algebra, functions, and other traditional pre-calculus topics. Furthermore, she claims that students in high school calculus classes usually master the course's procedural aspects while their conceptual understanding is at a minimum. When addressing this problem, Orton (1985, p. 15) states that "we must avoid producing pupils who have learned to 'apply processes mechanically (and) are mystified about the principles.'"

Ferrini-Mundy and Lauten (1994, p. 117) suggest that students can be encouraged to explicitly deal with the conflict between their conceptions and formal concepts by manipulating spreadsheets to explore sequences, series, convergence, and limits in tabular and graphical representations. They further claim that as a means of teaching calculus, we need to emphasize visual learning, pointing to a study by Vinner, which they claim suggests that students may experience a conflict between their geometrical and functional concept images of tangents. Additionally, they suggest that if students solve problems visually, they can gain a deeper understanding than if they solely solve them in an analytic mode (Ferrini-Mundy & Lauten, 1994).

4.3 Institutional analysis

All schools and teaching operate within some external and internal frame factors that both limit and aid their structure. For example, external factors can be national laws, national and local politics, curricula, cultures, and lately, national and local COVID-19 restrictions. Internal factors can, for example, be the specific school's resources, facilities, classroom culture, and the number of students (Braathen, 2007). In the following, I will examine the institutional frames I will be performing my research within.

At the start of the fall semester in 2020, a new curriculum was introduced in the Norwegian school system. As a part of this, many new aspects became a part of teaching mathematics. One of the more intriguing alterations introduced in this curriculum was the withdrawal of many of calculus's key features. In the old curriculum, the students were to know how to

- calculate (...) the average rate of change and find approximate values for the instantaneous rate of change, and provide some practical interpretation of these aspects.
- explain the definition of the derivative, use the definition to obtain a rule for finding the derivative of polynomials, and use this rule to discuss functions. (Utdanningsdirektoratet, 2006, my translation from Norwegian.)

In the new curriculum, however, the students should know how to

- use average and instantaneous rate of change of concrete examples and explain the derivative. (Utdanningsdirektoratet, 2020b, my translation from Norwegian.)

This reveals a noticeable difference in both the phrasing and content of the curricula. Contrary to the old curriculum, the students are no longer expected to use the derivative's definition, only explain it. Neither are they expected (at least explicitly) to provide some practical interpretations of average and instantaneous rate of change. There is, however, some room for interpretation of the words "explain" (*gjøre rede for*) and "use" (*bruke*). Per UDIR's definition, "use" and "explain" are respectively defined:

- To use means to take advantage of or perform an action to obtain a goal. To use is closely related to applying, understood as making use of (...), for example, a method or a tool.

- To explain something is to give a professionally (*faglig*) justified explanation of a case, an issue, or something we should investigate or perform. (Utdanningsdirektoratet, 2020b, my translation from Norwegian.)

Thus, an explanation of the definition of the derivative involves more than simply presenting it. It should be a professionally justified explanation, that is, the students could be expected to atomize the definition, describe the different components and justify the use of the definition. However, the curriculum does not state that the students should explain the derivative's definition, just the derivative. This again leaves room for further examination. It could be argued that the definition is not required to explain the derivative. One could use the slope of the tangent in a point and argue that this is an explanation of the derivative in said point. Personally, I believe that the algebraic definition can yield much exciting discourse. Consequently, I lean towards a more algebraic definition while using geometry as a teaching tool. As a complete examination of the curriculum is too extensive to perform in this thesis, I will end this line of thought here.

When it comes to trigonometry, the new curriculum states that the students should know how to

- explain the definitions of the sine, cosine, and tangent, and use trigonometry to calculate lengths, angles, and areas of arbitrary triangles.
- justify the laws of sine and cosine, and the area sentence (*Arealsetningen*).
- use trigonometry to analyze and solve compound theoretical and practical problems with lengths, angles, and areas. (Utdanningsdirektoratet, 2020b, my translation from Norwegian.)

The second goal is new as of this curriculum, and, contrary to differential calculus, adds some aspects to trigonometry. The learning goals further show that the focus in this course is on triangle trigonometry, not circle trigonometry. The curriculum is relevant to analyze briefly in this study because the students participating in this project are the first ever to be exposed to this new curriculum. Thus, the questions posed by the students may be influenced by how the learning material is presented with respect to the curriculum.

Though research on the area is insufficient, it is reasonable to argue that the COVID-19 pandemic has severely impacted students. Schools were closed on a global scale, abruptly transferring from physical to digital teaching; Norway was no exception. Throughout March 2020, schools across the country began to close. Though some of the schools partially opened in April, most students had to endure digital education. The students in this project were at the end of the 10th grade when the schools closed, thereby "losing" their remaining months in secondary education. A quick examination of different mathematics textbooks for the 10th grade shows that if the teacher chose to follow the textbook's suggested path, the students would most likely have been taught functions and algebra around the time the schools closed. Thus, it can be expected that digital teaching might partially have impacted the students' competence or knowledge in these areas. It is emphasized by Burgess and Sievertsen (2020) that even short periods of missed or lower-quality schooling could have consequences for students' skill growth. It is further indicated by Singh et al. (2020) experience less engagement and participation from students during online teaching. As research on this area is still being performed, and long-term impacts of the pandemic in education are yet to be determined, it is hard to tell how this has impacted my study.

With the mathematical content thoroughly analyzed, didactical perspectives reviewed, and institutional conditions evaluated, I can finally design and analyze specific didactical designs aiming to facilitate question-posing.

Chapter 5

Structuring the research

Many aspects need to be considered when structuring research based on different phases. Firstly, one needs to evaluate the purpose of each individual stage, whether or not it is required, and how to best implement it into the research setting. Secondly, the different stages need to be structured and designed in the desired way, based on the previous analysis and research. This project was divided into four separate phases; observation of the classroom culture, implementation of question-seeking-questions in the classroom, a Mentimeter session, and at-home teaching. Each of the different stages contributes to the research in different ways. In the following, I present the field study's organization as it was ultimately was conducted, excluding the initial observation phase as this was amply described in section 3.3.1. I will here include an *a priori* analysis of each teaching segment. The didactical choices and designs presented in this chapter aim to facilitate a research setting that could generate an answer to my research question; *To which extent and how do students following a mathematics 1T course use questions of different levels of complexity in their student-teacher communication?* Concluding this chapter, I will briefly present the research's initial plan and some consequences of the changes.

5.1 Teaching question-posing

In section 2.2, I presented the three necessary steps for teaching active comprehension: Modeling behavior, phase-out/phase-in strategy, and active comprehension itself. All students have heard their teachers pose different types of questions during their many years of education. Therefore, this experience will naturally fall into modeling behavior, as I hypothesize that students can mimic their teachers' questions, making a specific focus on this step redundant for this research project. However, it is not obvious that they can form their own questions in the same manner. Thereby, it is necessary to implement some changes in the ordinary teaching to ease the students into posing independently generated questions.

I planned to implement two primary changes to urge the students to pose their own questions. These steps would work as the phase-out/phase-in strategy. Firstly, I would host a classroom discussion on how to pose well-formulated questions, why this is necessary, and how the students themselves can use questions to their benefit. I would base my contribution to the discussion on the theory presented in section 2.3 while using the students' responses as my primary tool for progressing the discussion. There are multiple reasons for this approach. Firstly, by directly introducing question-posing as something beneficial and essential to learn, I may spike student interest in the topic. Further, this can contribute to having students phrase questions more elegantly and constructively. Furthermore, as I was warned that the students participated very little in classroom discussion, staying silent when asked questions, it would be beneficial to base my contribution on some theoretical framework to not be surprised if they did not respond. If this were the case, I would then use this time to lecture the students on how to pose well-formulated questions and use them when solving exercises. The goal of this discussion was mainly to introduce question-posing as a theme that would be of interest in the week to come. Further, this allowed me to evaluate how the students used questions in a whole-class setting, thus contributing directly towards answering the research question.

The second addition to be implemented in the class was the purposeful use of what I will define as question-seeking questions (QSQ). I guided the teacher in posing questions, not seeking a right or wrong answer but rather generating a new question. This can be performed in several different settings, e. g., during classroom discourse/lectures or when helping individual students. When a teacher is presenting a new topic, more often than not, they seek a concrete answer when posing questions, though this is dependent on the classroom interaction pattern (Chin, 2007). Hence, by implementing QSQ in common teaching scenarios, it is reasonable to argue that this may conflict with the underlying classroom culture. Both Grouws and Lembke (1996) and Folke Laarsen et al. (2017) explicate the vast time dimension of classroom culture, demonstrating how even small changes may take a relatively long time to implement in an existing culture. Grouws and Lembke further specify that a breach of the classroom's rules and norms may lead to the students rejecting the teaching situation as a cultural shock. Thus, I hypothesize that QSQ in ordinary teaching situations may lead to poor results given the study's timeframe. However, I still deem it essential to attempt implementing them, as this in and of itself would be an exciting result.

From the experience gained in the pilot for this project, I learned that students often struggle to formulate a question to ask for help. They usually ask questions such as "How do we solve this?" or "What does this mean?" or they do not pose a question at all stating, "I do not understand this". To compel the students to pose more concrete and well-structured questions, the teacher, when faced with the previously mentioned situation, may retort with "what questions can we ask ourselves about this task?". Following this, the teacher may continue with "Which of these questions do you know the answer to, and which answers do you seek to know?". This approach may result in a great variety of interesting scenarios. I believe that the first time a student is faced with this type of question, they will not know how to respond appropriately. They will likely doubt themselves and struggle with finding a "good" response. The teacher will probably have to rephrase their question and explain what they expect from the student.

Further, as the students were to work on an assignment in the lessons where these changes would be implemented, I hypothesized that many of the questions posed would be either procedural or concerning the correctness or form of the answer. This hypothesis is based on the results from the pilot project, as seen in figure 3.2 from section 3.3.3. These results suggested that the majority of questions could be procedural if the pattern observed were to repeat itself. As described above, I did not believe that QSQ in ordinary teaching would be very effective in the timeframe given. Thus, I felt the need to design a lesson revolving around QSQ in a way that guaranteed participation from the students.

5.1.1 Designing a Mentimeter lesson

To observe the effectiveness of QSQ as clearly as possible, I designed a lesson revolving around the use of Mentimeter as a teaching tool. Mentimeter is an interactive presenting tool used to make the audience active participants in a lecture or meeting. A depiction of how the presentation slides of such a lesson would look can be seen in figure 3.1 in section 3.3.2. This lesson would be implemented in the last lesson of the week, following the assignment. The lesson's design was quite simple; I chose some of the exercises the students had worked on for their assignment and asked them to pose the questions they needed to solve the task. The students formed groups of three to four when solving the task. The group sizes were chosen to fit the theories of cooperative learning, as presented by Johnson et al. (2006), which states that groups involving between three and four participants facilitate the ideal cooperative environment. As I was still a novice to the classroom dynamic, I asked the teacher to group together students he believed could work effectively.

The tasks to be discussed were chosen to include elements both of different difficulty and visual representation. Thus tasks 1, 3, 4, and 6b from appendix D were deemed to involve the desired complexities. Particularly exercise 6b, which the students had found difficult when working on their assignment, could produce interesting results. Exercise 1 and 3 involve a figure from GeoGebra, though with vastly different use and information. The goal of using these tasks would then be to observe how the students interacted with them and which questions they deemed essential when discussing such exercises. As exercises 1 and 3 are of a visual nature, and by Duval's (2006) theory that students could struggle with the conversion between representation systems, I here hypothesize that the students' questions to a high degree would be of some visual element of the tasks. As exercise 4 involves a very procedural approach, I believe it also would produce procedural questions. Exercise 6b presents information that might not be readily apparent how it should be used. Thus, I hypothesize that many of the questions concerning this task would involve the presented information, that is, a question on base knowledge or of a familiarizing classification as shown in table 3.2. Furthermore, as the exercises were used in an assessment, it could be reasonable to argue that many of the questions could involve the answers or results of the exercises, as the students might be externally motivated by grades.

5.1.2 Designing an at-home teaching lesson

As mentioned in section 3.1, many parts of the research had to be changed due to the ongoing COVID-19 pandemic. One of the more major changes resulted in an at-home teaching lesson concerning an introduction to triangle trigonometry. The teacher had designed an exercise sheet that the students were to work with from home, with the teacher being available to help. Before giving the students the exercises, the teacher gave them to me so that I could change what I needed for them to work with my research. This lead to a collaborative design process between the teacher and me, resulting in the exercise sheet shown in appendix E. The exercises give an iterative introduction to sine, cosine, and tangents by first giving the students some familiar tasks, calculating the ratio between different sides of three different (similar) triangles, and then introducing sine, cosine, and tangents. I mainly did some subtle changes, like giving the different tasks numbers and making sure the document's style looked good. The only significant alteration I did was to include a question perspective to the two reflective exercises, one after the first calculations and one at the end of the exercises. The first such exercise was:

> **What is the relationship between the calculations in 1, 2, and 3?** Write down 2 questions before and after you have written your answer. You should then find 4 questions for this task. Think through how you can phrase the question clearly and comprehensively. You shall hand in these questions in the Google Sheet that you find here: *link*. Do not hand in the questions before every task is done.

At the end of the exercise sheet, the same descriptive text was used, only changing the **bold** text to, "What relationship have you found?". The goal of these exercises was to give the students a moment of reflection both before and after working with the different tasks, not only asking them to find a relationship but also explicitly write down some reflective questions as a part of their process. An added bonus was that I could then collect their written questions and use them in my analysis. I would then have collected oral, written in groups, and individually written questions, thus further expanding the *student-teacher communication* in my research.

I also wanted to give the students some help in posing well-formulated questions and give them some guidance in how they could ask their teacher for help. Thus, I created the informative document in Appendix F, which I based on the theory from section 2.3 and experience gained from the earlier phases of the research, with a specific focus on presuppositions and mathematical language. In particular, I presented some questions that I had observed in the first observation phase, like "How do I find that thing?" and "What do I do here?", giving some guidance on how to make more precise what is being referred to. My examples of proper use of mathematical terminology included, "How was it that I could find an expression for the derivative by using the definition of the derivative?"[1] and "How can I progress to find the slope of the tangent when I only have a figure?"[2]. Careful thought was put into what mathematical terms to use in the questions (i.e., derivatives, slope, definition, expression). I only wanted to use words that I believed the students themselves understood or could manage. Thus, every question included proper mathematical terminology while being written in an oral fashion.

[1] *Norwegian: «Hvordan var det jeg fant et uttrykk for den deriverte med bruk av definisjonen av den deriverte?*

[2] *Norwegian: «Hvordan kan jeg gå fram for å finne stigningstallet til tangenten når jeg kun har et bilde av den?»*

As the students had no time limit for solving the exercises, I expected some more comprehensive and well-written questions. I hoped that they would use more proper mathematical terminology and phrase their questions better. However, this same time condition could lead to less time used on the exercises than planned, as there were no guarantees that they used the entire ninety-minute lesson to solve them. This could ultimately lead to less structured and poorly phrased questions. Further, as this lesson concerned an <u>introduction</u> to triangle trigonometry, the qualifications for a question to be deemed high-order were lower than had been the case for differential calculus. Thus, I expected several more HOCQ to emerge from this lesson, as the students, in general, could not rely on simple recall to solve the exercises. By this, I specifically hypothesized that many questions from the *analyzing* level of RBT would arise, as the reflective exercises explicitly asked for an evaluation of some relationship.

These last two designs, the Mentimeter session and the at-home teaching, reveals a need to define the "*student-teacher communication*" from the research question, as it is not necessarily evident how this "communication" takes place. Communication can be defined as "a process by which information is exchanged between individuals through a common system of symbols, signs, or behavior" (Merriam-Webster, n.d.-a). Thus, some exchange needs to be performed, that is, communication goes both ways, student-to-teacher and teacher-to-student. Then, the student-teacher communication from the Mentimeter session takes the form of group-based questions generated by the students being sent to the teachers through Mentimeter, for then to be discussed and explained by the teachers to the students. The at-home teaching relies on one crucial factor; that the teacher in some way responds to the students either directly through digital messages or by referring to their questions in the following lesson. However, as this study investigates primarily the *student* part of the communication, the fact that the response is not presented and explored should be of no disadvantage; though, it should be mentioned that the teacher did comment on some of the resulting questions in the following digital lesson.

5.2 Original plan

As mentioned, many aspects of this research project had to be changed, sometimes rather quickly, due to the ongoing COVID-19 pandemic. The plan was always to implement QSQ as a critical tool to facilitate question posing. However, I never intended to implement neither the Mentimeter session nor the at-home teaching session. Initially, the research was to be conducted during 1-2 weeks of audio and video collecting, following most of the progress on differential calculus, from introduction to conclusion. I never intended to collect data during an assessment, thus only collecting data from lectures, discussions, and guidance situations.

To conclude my research, I was to design and implement an inquiry-based lesson concerning aspects from differential calculus that the students would not otherwise face at this course level. The plan of the inquiry lesson was to collect student-student questions to compare this master's project more to the pilot project conducted earlier. Thus, the foci of the original project were not on student-teacher communication but student questions in general. Further, Singer's theory of active comprehension was to get a more significant role in the study than it ultimately got. Many aspects of this original project had been designed and analyzed according to the phases of didactical engineering before the resultant changes had to be made. The changes to the original plan were made in a relatively short amount of time, going from a month of planning for the original project to days of planning for the Mentimeter and at-home teaching. I then benefited greatly from the use of questionnaire-type teaching scenarios, as these could relatively quickly be designed and analyzed accordingly.

I ultimately believe that the project that was conducted resulted in fascinating and important results, though entirely different from those I would have collected using the original plan. In the following, these results will be presented along with a meticulous analysis of all aspects relevant to the research question.

Chapter 6

Analysis

In this chapter, I will present a selection of data to be analyzed and interpreted based on the theories presented in chapter 2. This will then work as the *a posteriori* analysis of the collected data. The analysis will be divided into five parts, the first four relating to the different phases of the research; the observation of the classroom culture, the oral classroom communication, the Mentimeter session, and the at-home teaching. Concluding this chapter, I will evaluate the validity of my didactical choices and designs, as demanded by my research method. Different data were observed and collected in the various phases of the research. Thus, the following subsections will entail tendencies and aspects that will sometimes be similar across topics and other times wholly unique to the situation. In figures 6.1 and 6.2, a descriptive representation of the distribution of collected data is presented, separated into each question category as shown in table 3.1 and 3.2. The themes in the following analysis are based on these categories, and only the most prominent or interesting topics will be exhibited. In the coming analysis, we will explore how the students' oral communication presented a scarce use of HOCQ, in contrast to the high amount of HOCQ observed following the at-home teaching session. Quotes and situations obtained through the data collection will be presented as examples of important themes and tendencies throughout the analysis.

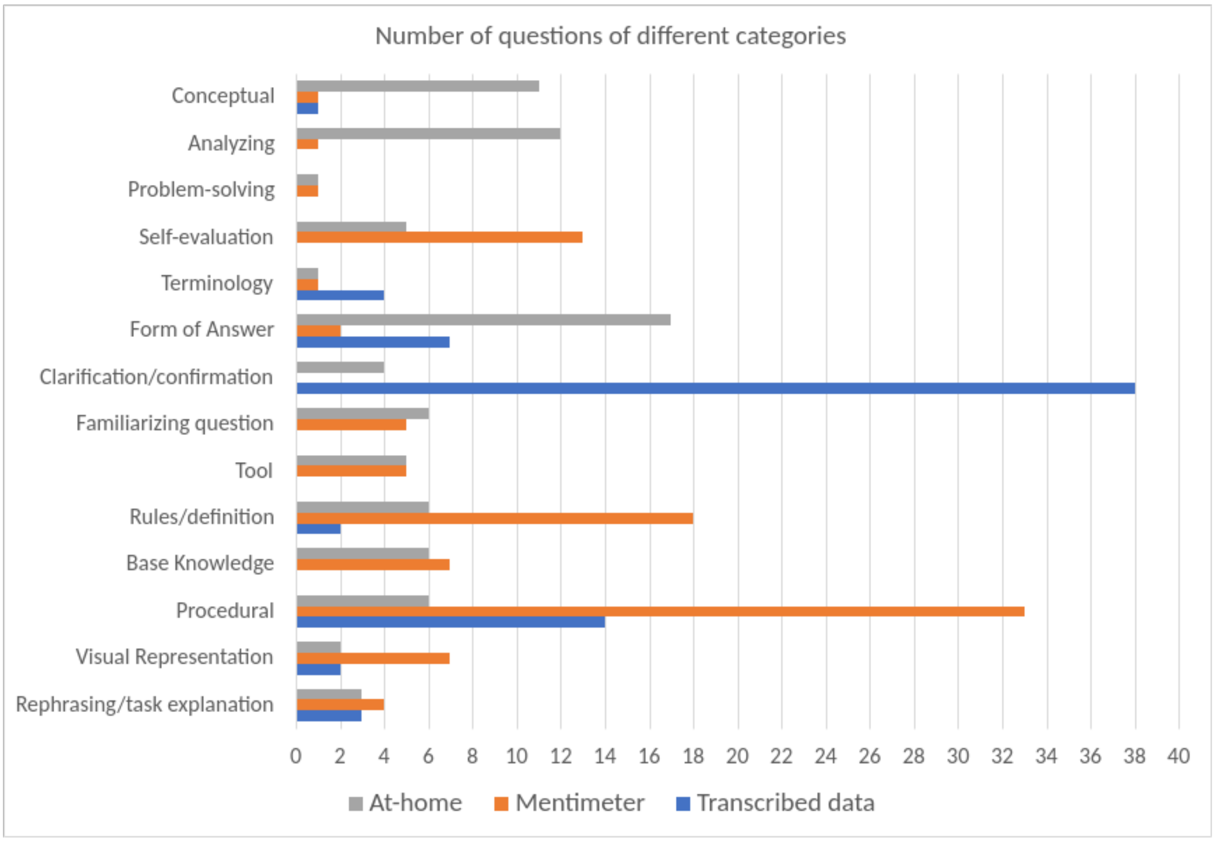


Figure 6.1: The chart shows a descriptive representation of the of collected data, separated into categories as presented in table 3.1 and 3.2. The chart give no indication of the total number of unique questions posed, as some questions belong to multiple categories. The transcribed data spanned 360 minutes and involved 14-24 students (varying from lesson to lesson). 17 students participated in the Mentimeter session spanning 45 minutes. Lastly, 14 students participated in the at-home lesson.

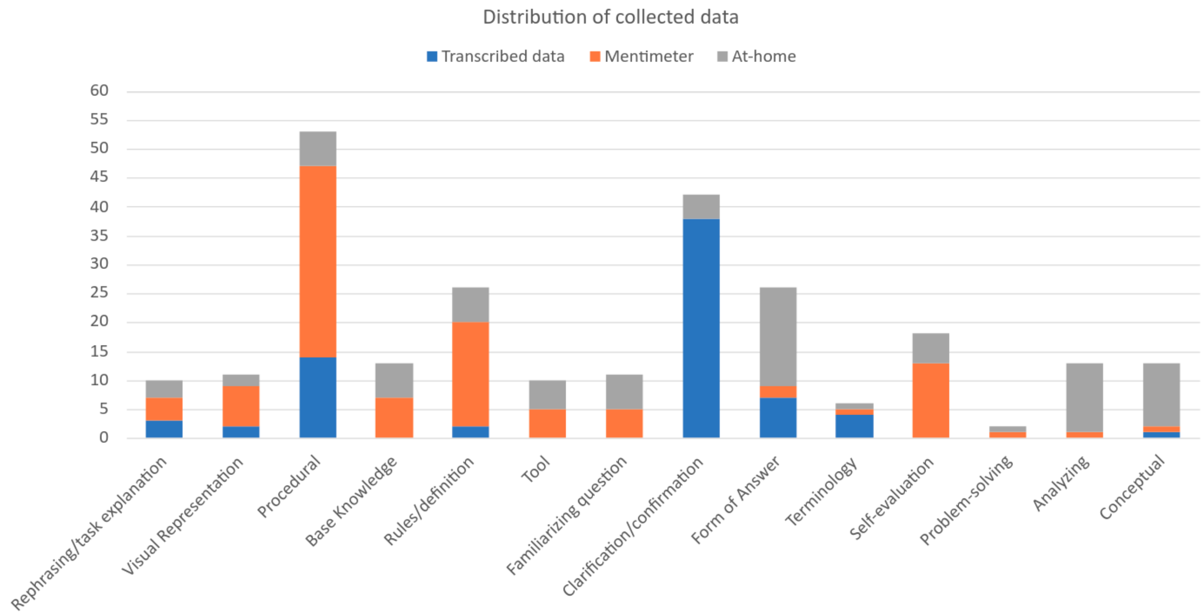


Figure 6.2: The chart shows a descriptive representation of the distribution of collected data, separated into categories as presented in table 3.1 and 3.2. The chart give no indication of the total number of unique questions posed, as some questions belong to multiple categories. The diagram is based on the same data as figure 6.1.

The charts in figures 6.1 and 6.2 could be seen as an answer to the *extent of which the students use question-posing in different forms of student-teacher communication*. While figure 6.1 displays the number of coded questions of each subcategory, figure 6.2 shows a clearer picture of how questions of the same category were distributed across the different data-collection scenarios. In particular, the vast majority of questions posed across the different phases were either of a procedural nature or seeking clarification of confirmation.

All names offered in the dialogues and discussion are fictional, and I will give no physiological markers on any research subjects. If the names suggest a gender, this should be no indication of the students, as the names have been picked at random, not taking gender into account. In addition, if a pronoun suggests a gender, this is purely used to make the text more readable and does not reflect the gender of the student. A list of transcription codes can be seen in Appendix A, the most important of which being <u>underlined text</u> referring to reading from a task description and "..." referring to pauses up to 3 seconds. All data presented have been translated from Norwegian to (sometimes more grammatically correct) English and may have lost some meaning in translation. In cases where the English and Norwegian interpretation does not align, special care will be given to the interpretation of the Norwegian meaning, as this is highly relevant to the use of mathematical terminology and phrasing.

6.1 Evaluating the classroom culture

The initial part of my research involved a participatory observation through field notes. The goal of this phase was to evaluate the classroom culture before any intervention was introduced. In particular, three aspects of the culture were focused upon: the level of orality, the displayed academic confidence, and the displayed mathematical interest. As the last two of these are highly subjective, my observations were discussed with the teacher after each lesson to strengthen future hypotheses and argumentations. In this section, my observations will be presented and analyzed to give a proper understanding of how the class functioned before any significant changes were made to the classroom dynamic.

One of the first observations made was a clear balkanization or grouping of students in the classroom. In particular, two larger groups, one of which dominated the (arguably limited) classroom communication, had formed in separate areas in the classroom. There was no indication of any hostility between the students, as all interactions between groups seemed friendly. With the two prominent groupings, the remaining students looked to form working pairs, with some acting as individuals. Bella and Alice, which will be introduced in section 6.2, is an example of a couple of students who frequently worked together.

Regarding the student groups' oral communication level, a fascinating observation was the amicable and open tone between the teacher and students. Jokes and friendly remarks were a natural part of the classroom communication. However, when it comes to the *academic communication*, a very one-sided communication pattern was observed. During both lecture-type teaching and attempts at classroom discussions, the students remained noticeably silent. This resulted in more extended periods of only the teacher talking while attempting to prompt answers from the students, with little success. Noticeably, the students did not answer any of the teacher's questions posed to the whole class. While the teacher had already warned about the class's lack of oral communication, the shortage of student participation should be viewed in regards to my presence in the classroom. There is little doubt that by me being present as an unknown variable in their ordinary teaching, the students reacted in some way, probably by being more silent than usual.

A particularly fascinating observation regarding the students' question-posing behavior comes from the first day of observation. When the teacher completed an example concerning the largest possible area given some initial constraints, a student asked how $a/2$ became $a/4$ during some algebraic manipulation. When the teacher explained that he had divided everything by two, a noticeable sigh ran through the class, showing that many, if not the majority, of the students struggled with this step in particular. However, none of the students asked about the step when it happened, even though it was apparent that many of them struggled to understand what had occurred. Furthermore, the question did not emerge unprompted, as the teacher had to point out that he noticed that the students struggled to understand something, but he did not know where he lost them. This suggests that the classroom culture, to some degree, did not facilitate whole-class questioning.

In general, only three students participated orally during "whole-class" situations during this week of observation. Two of these students were a part of one of the most prominent groupings described above. The severe lack of oral communication from the students can suggest a lack of mathematical confidence from most of the class. This does, however, not mean that every student who did not speak would struggle in solving a mathematical problem. A student could be perfectly capable of performing well on written mathematics while avoiding oral mathematics. Still, it might suggest that the student lack either the will or the confidence to present their answers or questions, this being either mathematical confidence or social confidence.

6.2 Oral classroom communication

Most of the data for this study were collected during oral student-teacher guidance concerning differential calculus. This is the second data collection phase, directly following the participating observation analyzed in the previous section. This data set spanned four school lessons of forty-five minutes and concerned between fourteen and twenty-four students, depending on the lesson. The dialogues and examples provided in this section come from the transcription of the verbal data presented in Appendix A.

6.2.1 Using questions to clarify or confirm

Surprisingly, the most prominent question type I identified in the student-teacher communication of an ordinary lesson was of a clarifying or confirming character. The students usually initiated a conversation by posing a base level question, a question concerning the phrasing or terminology of the task, or a rephrasing of the task, stating that they needed help or did not understand. When the teacher then started an explanation or guidance, the students mainly posed questions seeking to clarify what the teacher had said or to confirm a belief. This naturally resulted in a pretty one-sided dynamic, where the students could easily avoid answering the teacher's questions by stating that they did not know or did not understand. The following example is taken from the first recorded interaction between Alice and me.

2| **ALICE**: Okay, what does it mean to use the figure to determine when $f(x)$ increases and when $f(x)$ decreases?

3| **STUDENT TEACHER**: Okay, use the figure to determine when... Cause this is... What does the figure show then?

4| **ALICE**: Eh, that one? {Points to the figure} (Student teacher: Mhm) It shows that it crosses at minus four and then at two.

5| **STUDENT TEACHER**: Mhm, and what is it a figure of?... Well, this is a figure of the derivative.

6| **ALICE**: Oh, is it the derivative?

This example shows how most of the student-teacher interactions were initiated. Alice starts by reciting the exercise, asking what it means. When she is then faced with a question she did not know the answer to, "what is it a figure of?", she remained silent rather than attempt an answer. While this may be because she believes she already has answered the question, that is, she might think her answer on transcription line 4 to be sufficient, I think this might stem from a desire not to show any flaws in understanding. This belief comes from the fact that most of my interactions with Alice were of this nature. While she did not answer the question, she did not pose a follow-up or clarifying question to understand my question either. This tendency I observed throughout the study, concerning most of the students. The following examples come from a situation where I provide an extensive explanation of a task to Karl.

127| **STUDENT TEACHER**: The graph of f goes through the point. Find the equation of the tangent. (Karl:yes) Okay, what is it that you need, first of all, what do you know about the tangent?

128| KARL: mmh, is that not the one that goes here?

129| STUDENT TEACHER: mmh, it is not drawn there no.

130| KARL: There {points to another document}

131| STUDENT TEACHER: Yes there, there you have a tangent (Karl:mhm) and what do you know about(...) [segment of explanation removed for readability]

131| STUDENT TEACHER: (...) Then, what information do you have in this task then? What do you know only from the task?

132| KARL: Only from this? {Points to the exercise text}

133| STUDENT TEACHER: Mhm, and then you also have the figure provided in the task as this essentially is the information you have.

134| KARL: That it grows until two and sink after. Like, is that it?

This again shows how another student does not pose any follow-up questions, only relying on clarifying the last question I asked him and confirming that his answer was to my liking. The rest of this interaction with Karl followed the same pattern.

By looking at the questions above, it seems reasonable to argue that they do not promote much thinking of a higher cognitive level. When a question is used to clarify or confirm, it usually involves *explaining, exemplifying, classifying*, or some of the other subcategories of the *understanding* level of RBT. Alice's question "what does it mean to use the figure to determine when $f(x)$ increases and when $f(x)$ decreases?", promotes an explanation, that is, the process of providing an example or instance of a general principle. It might also benefit from some *classification* or *summarizing* to build the base knowledge needed to solve the task. Further, Karl's question "Only from this?" could be argued to belong to the *remembering* level of RBT, as this would only trigger some rephrasing or even a dichotomous, yes or no answer. Therefore, when employing questions of this type, I would argue that they would not promote high-order thinking, thus removing a layer of complexity from the questions.

Such questions also allow the students to partially avoid using mathematical terminology, as the explaining or answering falls on the teacher, not the student. These types of questions can more readily than others be "phrased" using hand gestures and pointing rather than proper mathematical terminology. This again usually results in true (but often, mathematically speaking, uninteresting) presuppositions, as removing the complexity of using mathematical language excludes the students' subject-specific assumptions. Thus, this type of question may be regarded as a question of the very lowest level of complexity, as it generally does not promote high-order thinking, easily can avoid using mathematical terminology, and thus makes the presuppositions unreliable or short of mathematical knowledge.

I began this subsection by stating that I found it surprising that this type of question was the most prominent. My initial hypothesis was that most of the questions would be procedural, as this was the result of the student-student questions I observed in the pilot project. While many of the questions, as a matter of fact, the majority of questions in total, were procedural, the oral questions were overwhelmingly of the confirming or clarifying nature. The procedural level of questions will be extensively discussed in later sections. I will now analyze the procedural elements of the oral student-teacher questions.

6.2.2 Using questions to gain procedural knowledge

As previously mentioned, a significant number of questions posed were procedural. The students usually employed this type of question-posing when the confirmation or clarification questions did not provide a sufficient explanation or response from the teacher. In the following example, Bella is investigating the extrema of a polynomial of the second degree.

35| **BELLA**: Here it says when the ball is at its highest point, does that mean that it is at the maximum point, or on the line?

36| **STUDENT TEACHER**: Eh, when is the ball. That means that it has something to do with time.

37| **BELLA**: Oh, okay like that... so it is after... wait a second... it is the height of the ball after how many seconds (Student teacher: mhm), so when will it be after four.. hours.

38| **STUDENT TEACHER**: That sounds fine, maybe not in hours, though.

39| **BELLA**: No, not hours it is in seconds, and then it.. then the ball is eighty meters high?

40| **STUDENT TEACHER**: That sounds correct.

: [...]

47| **BELLA**: Okay, here I take the derivative of $h(t)$ right? (Student teacher: mhm). And the question is What does the function tell us? How can we see what it tells us?

This dialogue shows a continuation of the same speaking pattern as in Alice's example in the last subsection. Bella starts by posing a task-specific question, shortly followed by a confirming question (transcription line 39), to verify that her answer is reasonable. Then, after some help with how to write functions in Microsoft Word (the removed segment from the dialogue), Bella poses a procedural question regarding how to analyze this function. Bella's questions show a more evolved use of mathematical terminology than we up until now have seen. She does not avoid words such as maximum point (Norwegian: *toppunkt*), derivative or specific function names such as $h(t)$. This may show some higher degree of mathematical confidence than we have seen from Alice or Karl (at least in the dialogues presented). While Alice used the term "derivative," this was in response to my explanation and thus cannot be argued to be naturally generated.

In similar situations like the one presented above, students posed questions such as, "How do I find the average rate of change?", "What do I do with that one?" and "How was it that I could solve this?". The wide variety of phrasings of procedural questions brings into focus the importance of proper use of mathematical terminology. In only the first of the three questions above, the students make evident the question's intent. By the phrasing of this question, we know that the student wants to know something about the average rate of change. The phrasing also makes the mathematical presuppositions more apparent, as one either can or cannot find the average rate of change with the information given. On the contrary, the two other questions avoid using specific language and terminology, leaving more room for interpretation and ambiguity. Henceforth, this can suggest that the students who use proper mathematical terminology also are more likely to offer more evident mathematical assumptions, thus aiding the teacher in guiding them towards their desired learning goal.

This category of questions opens for more discussion regarding their cognitive level, as it often resides in the gray area between *understanding* and *analyzing*, with much depending on the phrasing. Bella's question "How can we see what it tells us?" could belong to the *exemplifying* level of the *understanding* category of RBT. Its potential could lie in providing an example or classifying some crucial aspects of derivatives to help her thinking process for her to get a proper understanding of the topic. On the other hand, it could also be regarded as a question of the *explaining* level of *understanding*, triggering some thought process of cause and effect. This is where the gray area between *understanding* and *analyzing* becomes more evident. Would this type of question begin a surface-level exploration of only the major topics of derivatives, or would it trigger some more extensive investigation of the topic? If this had been a general question, a question posed, for example, introductory during a lecture or discussion, I believe it more easily could have been deemed an analyzing question. In this case, it could be argued to be *organizing*, identifying the components of derivatives, and finding how they fit together. However, as this question is heavily tied to the exercise, actively referring to the phrasing of the task, I would argue that this would not trigger any high-order thinking. It would then belong in the *understanding* or even *applying* level of RBT.

As opposed to procedural knowledge, conceptual knowledge is generally considered to be of a higher cognitive level (Chin & Brown, 2002). However, as we shall see, a conceptual question does not need to be a HOCQ. In the following, I will present in what manner the students produced questions of a conceptual nature in an ordinary student-teacher interaction.

6.2.3 Using questions to gather conceptual knowledge

During the 360 minutes of audio collected for this study, I noticed a singular question that could be argued to be conceptual. This question was generated directly following the situation described in the previous subsection. Here you can see a continuation of this dialogue:

48| STUDENT TEACHER: Okay, then the first thing one should think about is, what does the derivative tell us?

49| BELLA: Is it not that if it is negative, then it drops, and if it is positive, then it grows?

50| STUDENT TEACHER: Yes, amongst other things. So when is the derivative negative, and when is it positive then?

51| BELLA: It is negative... like it sinks from here, does it {Points to where the graph crosses the y-axis} I don't know.

52| STUDENT TEACHER: Ehm, here it crosses the y-axis (Bella: yes), but it is still positive.

53| BELLA: Oh, yes, it sinks here then {Points to where the graph is below the x-axis}

54| STUDENT TEACHER: Mhm, so here it suddenly becomes negative. Then, by looking at that, you can essentially say three things... at least three things...

55| BELLA: But how can you like see that? For like, another question. Here are the function $h(t)$ {shows the function in GeoGebra} (Student teacher: There, yes). And then we have found the derivative.

56| STUDENT TEACHER: yes, and what is the derivative? That is the great question. Do you remember before you heard about the derivative, then you heard about what is called average rate of change? Then you moved on to talk about momentary rate of change? And this momentary rate of change was called the derivative.

57| BELLA: Okay, what are we talking about? When I find the derivative of x, what do I find then?

By evaluating the entire interaction between transcription lines 35 and 57 (the entire dialogue from the one presented in section 6.2.2 including the one shown here), one can get a more thorough understanding of Bella's thought process and argumentation. What becomes apparent is that Bella poses arguably clear, well-formulated questions relative to her classmates. By her using proper mathematical terminology, I could more easily understand what level of explanation she needed. She did not need me to present her with facts or guide her through some calculations as we saw in Alice and Karl's examples. What I instead at the moment evaluated to be an appropriate approach was to use questions to guide her thinking process, grasping on to what she said and helped her by using her words to form questions. This also allowed me to bring focus to the questions I was posing, as I did on transcription line 56. This suggests that Bella might be further in the process towards active comprehension than some of her classmates. Where they still need the teacher to provide ample explanation and pose questions of a lower order, Bella responds well to higher-order questions and explanations. She poses follow-up questions and investigates issues further when faced with topics or questions she does not understand. This further suggests that Bella has a more thorough understanding of the *process* of comprehension, though she still has a way to go in the independent generation of knowledge.

The question on transcription line 57, "When I find the derivative of x, what do I find then?", looks to be investigating some deeper knowledge than the procedural we observed in section 6.2.2. She does not ask about how to find the derivative or what rules apply. It seems like she is trying to get some conceptual knowledge about the topic, and the question could then be evaluated to be of a higher order. According to Sadker and Cooper's categories, a conceptual question could be argued to belong to the divergent questions, as it may trigger creative thinking around a subject. Bella's question could initiate a discussion or creative thinking session concerning what the derivative, in reality, tells us and its usefulness. Thus, the question alone should be considered a HOCQ. However, the students had previously explored this topic, both in lectures and through exercises. Therefore, this would not trigger the highest levels of RBT, *creation, evaluation*, or *analysis*. It would instead rely on remembering the previous discussion or reviewing the textbook, making it a LOCQ. This reinforces the point that a question that in itself is high-order should, given the situation, be regarded as low-order.

6.3 Mentimeter group session

When generating questions in a collaborative environment, the students produced a much greater variety of questions than we have observed in the oral classroom communication. Some of the critical aspects that will be analyzed in the following are the students' more robust use of mathematical terminology and the more prominent place of HOCQs in this form of question-teaching. The exercises referred to in this section can be found in Appendix D, and the complete data in Norwegian in Appendix B. In this Mentimeter session, three teachers, the primary teacher, one secondary teacher, and I participated in the lesson.

6.3.1 Questions of visual representation

Unlike the remaining three tasks, most of the groups produced at least one question concerning some form of visual representation, e.g., the shape of the graph, the look of the sketch, or some detail of the function graph $f(x)$, when working on the first task. Here the students were asked to use a graph representing the derivative of a function to determine when $f(x)$ would increase/decrease, find the tangent in a point on the graph, and draw a sketch of $f(x)$. Three separate groups posed questions such as

What should the sketch look like?
What will $f(x)$ look like?
What can we see in the graph?

This suggests that the students will more naturally generate a question concerning some visual aspect of the exercise or solution due to the task's visual structure. The figure given in this task did not contain more than the graph of the derivative and a coordinate system. This contrasts with the second visual task (exercise 3), where much more information was given in the figure. Here, the students were presented with a function graph, two tangents in two different points with shown coordinates, lengths of segments, and more. Surprisingly, only one question concerning the visual nature of this task was posed. This question was, "What figure do we see in the picture?". Thus, when given more information in a figure, the students posed fewer questions concerning the figure itself, focusing more on definitions and rules, procedures, and solutions. This may also be because they were not explicitly asked to sketch this task, possibly leading to a less visual approach. The questions from the first task are then in line with my initial hypothesis that many of the questions would be of some visual nature. However, the results from exercise 3 display that the students' answers rely upon how the visual tasks are structured rather than the fact that they involve some visual element.

The questions presented above leave some room for interpretation of the cognitive level. Some of these questions can be either of an *understanding* level or *analyzing* level. Let us take "What should the sketch look like?" as an example. This could be seen as rephrasing or altering the exercise text, thus leaving it at the cognitive pyramid's *understanding* level. However, it could also be argued that this is a necessary question to analyze the given material and solve the exercise. This type of question would then fall in the *analyzing* level of RBT, making it a high-order cognitive question. Using Sadker and Cooper's five categories, this could be seen as an evaluation of the task information, again making it high-order. This two-sided view of this question-type illuminates the need for a response to categorize the cognitive level of questioning properly and more validly. Without an answer and corresponding follow-up questions, a question's intent may be vague and unclear. However, as this question is of such importance to gain a proper understanding of the task, I would deem it a HOCQ. This could be further justified by referring to the *differentiation* level of *analyzing*, defined through "distinguishing each part of a structure in terms of relevance and importance". Posing this question could naturally generate an analysis of each segment of the function to evaluate which parts were necessary to draw a sufficiently detailed sketch. Moreover, it could be argued to belong to the highest level of cognitive thinking, *creating*, as this question may facilitate both planning and producing the best approach for drawing the sketch.

6.3.2 Using questions for self-evaluation

One of the first questions posed by one of the student groups was the question "*Do I understand?*". This triggered a "discussion" between the three teachers around the importance of self-evaluation after each exercise, problem, or learning situation. As a part of this discussion, I made a point of telling the students why questions such as "What have I learned?" and "Do I understand this?" were essential to ask themselves. The question "Do I understand" is a question of self-evaluation, thus leaving it somewhere between the second-highest and highest cognitive level, according to RBT. From the *evaluation* category, the question could be said to involve both *checking* for inconsistencies or fallacies in one's understanding and *critiquing* one's knowledge, judging whether or not it should be altered or remain the same. As the *creating* level is defined as "putting elements together to form a coherent or functional whole," where the "functional whole" in this case would be one's mathematical understanding, this type of question also belongs in this category.

This also would be classified as a HOCQ according to the model presented by Sadker and Cooper, where it belongs in the divergent questions, as it offers some personal reaction. However, the phrasing of the question leaves a lot to be desired. In particular, "Do I understand?", which on the surface may seem dichotomous, may be argued to be a very open-ended question, leaving much room for internal interpretation. The question could be separated into several key aspects, for example, "Do I understand how to find the tangent in a point on a graph?", "Do I understand how to draw a correct sketch of a graph?" and "Do I understand how to use the graph of the derivative to evaluate the interval at which a function increases or decreases?". It is, however, not expected that a student should be able to phrase questions quite so extensively. These are meant to be internalized questions, thus leaving room for more personal phrasings and vocabulary.

Following this discussion, this type of question emerged eight times more, usually in the form of "What have I/we learned?". It seems evident that at least most of these questions arose due to me telling them about the importance of using these questions. These types of questions would then come from a desire to meet the teacher's (or my) expectations, not from an inner desire to actually reflect on what the student had learned. However, no matter the intent of this question, I believe it to be plausible that the students, to some degree, would reflect on their learning outcome following the generation of such a question. For this reason alone, I would argue that a question of self-evaluation, independent of intent, phrasing, or mathematical terminology, should be considered a HOCQ. However, this does not mean that the question is used effectively to reflect on one's learning outcome, as I believe to be the case for most students in this scenario.

6.3.3 Using questions to determine or evaluate an answer

On both the second (exercise 3) and the third (exercise 4) task, some groups posed the question,

> *How should I answer this task?*

There are several ways to interpret this question; are they asking a procedural question about the *method* they should use to get an answer? Are they asking about how they should represent the answer? Or are they maybe evaluating which approach would be the most efficient at answering the task? This unspecific phrasing of the question leaves room to judge whether this should be regarded as a high- or low-order cognitive question. Again, the question's intent will be the final decider when evaluating such a question's cognitive level. By the unspecificity of the phrasing, the question alone should be considered a LOCQ, as it seems to be either trying to *classify*, i.e., identify that a solution provides an adequate answer, or *interpret*, i.e., translate the representative form of the answer. Thereby, this question would belong to the *understanding* level of RBT. However, if the question intends to evaluate the best approach for solving the task, then this would be an evaluation of the mathematical process, thus belonging to the *planning* part of a *creating* HOCQ.

The difficulties in deciding whether such a question would be of a higher or lower cognitive level could stem from the lack of mathematical terminology and specificity. This tendency we also observed on the clarifying or confirming questions in section 6.2.1, where the presuppositions of the question easier became true by using unspecific language. By making the question more specific, one group managed to form an HOCQ concerning the third task's answer,

> *What does the answer in CAS tell us?*

Here "CAS" refers to the built-in Computer Algebra System in GeoGebra. This question seeks to evaluate an answer, thus making it an HOCQ both in regards to RBT and Sadker and Cooper's five categories. By belonging somewhere between *analyzing* and *evaluating* in RBT, the question could be said to *differentiating* the information in the answer, *checking* for inconsistencies or fallacies, and *critiquing* the solution based on previously acquired knowledge. It is plausible that this type of questioning could lead to further investigation and analysis of mathematical phenomena or even some level of self-evaluation. The group included some highly relevant information regarding the specific mathematical object in question by directly referring to CAS. If this question had been oral, the teacher could more easily identify the particular problem the students were facing, making the guiding process more efficient. Thus, by phrasing a question so that the mathematical object is prominent, a student may more easily gain the desired knowledge they were seeking.

6.3.4 Questions concerning real-world applications

Most teachers probably recognize questions such as "Why do we need this?", "How is this applicable in the real world?" and "Will we ever use this?". This type of question also emerged on the last task (exercise 6b) in the form of

Will this be useful to us?

This question is fascinating, as it can be viewed in many different ways. It can be referring to a particular piece of information, for example, cost per square centimeter of the sides of the cylinder, asking whether or not it is useful. It would then be considered a HOCQ, as it is analyzing the task information, identifying each element's application. It could also be referring to the larger picture, asking whether the knowledge they have acquired by solving the task will be helpful in a real-world application. In addition, the question's intent may significantly alter how it should be viewed. As familiar to many teachers, this type of question is often posed as a protest against the learning material. If this is the case, an answer to trigger evaluation may neither be desired nor met with an open mind by the student. This should then be considered a LOCQ, as it is not used either for analysis, evaluation, or knowledge creation. However, suppose the student genuinely wonders about the usability of a mathematical object or subject and is thus ready to evaluate or analyze the information. In that case, it should be considered a HOCQ, as it is actively *critiquing* the learning material, judging whether it is useful based on society's standards and criteria for usefulness.

This type of question can get away with not using much mathematical terminology, as it can be very case-specific; it often can only refer to one thing. However, it could also be used in a more general form, referring to, for example, derivatives or sine and cosine in general. In these situations, using sufficient mathematical language may be beneficial to get a more concrete answer. "This" in the phrasing above should then be substituted by the desired mathematical object or subject.

6.4 At-home teaching

When analyzing the data from the at-home teaching session, it quickly became apparent that some of the students had a more remarkable ability to pose high-order cognitive questions than previously expected. For example, the student codenamed Glen provided a high number of questions that were analyzed to be of a high order. Out of eight provided questions, six of Glen's questions were deemed high-order, showing an exceptional capacity to connect mathematical topics, analyze relationships, and question conceptual knowledge. Many other students also showed a higher number of HOCQ than previously displayed, leading to some interesting thoughts on the tasks given to the students. In the following, I will provide an analysis of the most prominent themes and topics identified in the at-home teaching situation while making connections to the previous analysis when relevant. The tasks given to the students can be seen in Appendix E, and all Google Sheets data can be found in Appendix C.

6.4.1 Using questions to make connections

The most prominent question type deemed to be high-order was questions seeking to make connections, either within the task itself or between mathematical subjects. As mentioned, Glen posed several HOCQ, four of these being of this category.

GLEN: Is there a relationship because the triangles are similar?

GLEN: Is there a relationship only because the triangles are similar?

GLEN: Is there such a relationship between all similar triangles?

GLEN: Is there such a relationship between triangles that are not similar?

It is entirely apparent that Glen noticed some connection between sine, cosine and tangents, and similar triangles. What I find incredibly fascinating about these questions is the subtle differences between them. I see similarities between how these questions were phrased and complete mathematical proofs of existence and uniqueness. First, Glen asks about the relationship between the observed results and similar triangles (existence). He then moved on to ask whether this relationship was "only" due to it being similar triangles (uniqueness). Following this, he posed two questions that could help strengthen and generalize his hypothesis, whether this worked for *all* similar triangles, and if it is a comparable relationship when the triangles are not similar. In my opinion, this not only shows an excellent ability to pose relevant, well-phrased HOCQ. It also suggests that Glen has a well-developed mathematical thought process, at least concerning tasks of this level.

Referring to Sadker and Cooper's five categories, Glen's questions check some of the boxes for several HOCQ. If we look at all four questions as one, this can be viewed as a comparison between the observed results and results from similar triangles. Each individual question tries to perceive some relationship between the observed results and similar triangles, whether it is general similarities, if it is *only* for similar triangles, and so on. Thus, each question is analyzing a cause and effect. I would argue that the questions belong somewhere between the *evaluation* and *analysis* level of the hierarchy of RBT because the questions do more than simply taking apart the known and identify relationships. By looking at all four questions, it seems like Glen tries to distinguish several vital parts of knowledge from his questions, thus making the combination of questions belong to the *analyzing* category. Again, each question looks to be testing for inconsistencies or fallacies in his hypothesis, thus making them belong in the *evaluation* category. They also make connections across mathematical subjects, even subjects that would not be fresh in mind, as the topic of similar triangles had not been discussed, at least during the weeks I observed the class.

Another student, Jenna, also observed the relationship between her calculations and similar triangles. She did, however, not use this term. She posed the question:

JENNA: Will this work on all triangles regardless of how long the sides are, as long as they have equal angles?

She observed the relationship between sine, cosine and tangents, and triangles with equal angles. The question then investigates whether or not the effect of having identical results in her calculations is a cause of the triangles having equal angles. Thus, this is a question of cause and effect according to Sadker and Cooper's categories. In contrast to Glen's question, this question does not directly connect the results to similar triangles. It then does not evaluate different mathematical subjects, cementing it firmly in the analyzing dimension of RBT. I would, however, say that she does use a sufficient amount of mathematical terminology. She does not avoid the terms "triangles" (*Norwegian: trekant*), "equal" (*Norwegian: lik*) or "angle" (*Norwegian: vinkel*)[1]. While these terms will not be the most complicated, they are indeed sufficient in the setting. These are words where the everyday and mathematical definition and use align, thus

[1]The Norwegian phrasing of this question was "Vil dette funke med alle trekanter uansett hvor lange sidene er, så lenge de har like vinkler?".

possibly making them easier to implement naturally. While using "similar triangles" should be desired, this does not negate the effect of using more straightforward mathematical terms in a clear, decisive way. As the Norwegian word for "similar" is "*formlik*", it will ultimately not be used in the same setting as one would use "similar" in English. While "*formlik*" does have a relatively intuitive interpretation, it will usually not be used in a setting outside of some geometric context.

The last student presented here who posed questions in this category, John, posed somewhat different questions than Glen and Jenna's in terms of presuppositions. Firstly, he posed the question:

JOHN: Does the length of the hypotenuse matter, or is it only the value of the angles?

While this should be regarded as a HOCQ for the same reasons as Jenna, it presupposes some intriguing points. Two possible presuppositions are

- Either the hypothenuse matter, or it does not.
- Either the hypothenuse matter or only the value of the angles matter

This second presupposition suggests complete independence between the length of the hypothenuse and the value of the angles, which is false. There is indeed a relation between the angles and the length of the sides. This shows how a presupposition need not be valid for a question to be regarded as high-order. Additionally, John asked the question:

JOHN: What connection does Sin, Cos, and Tan have? Are they the different points in the triangle?

This is, in reality, two separate questions, one focusing on the relationship between sine, cosine, and tangents, and the other regarding some structural understanding of these. The first question concerning sine, cosine, and tangents seems only to evaluate some surface-level information about this topic. When he asks about the "connection between them", this looks to be a *comparison* of the information given on the exercise sheet. This question should then belong to the *understanding* level of RBT. However, the second half of the question tries to make some connection between the three trigonometric functions and a geometric figure. It looks like he tries to place the sine, cosine, and tangent within his geometric understanding of the problem by explicitly figuring out where he can see them in the figure. Specifically, he tries to connect the trigonometric functions to the "points", that is, the triangle's vertices. In the didactical analysis of section 4.2, I pointed to a study by Blackett and Tall (1991), displaying students' difficulty in connecting these numerical relationships to geometric figures. John's question could be further indication that their claim is valid. This could then be argued to fit in the *organization* category of the *analyzing* level of RBT by trying to identify how the sine, cosine, and tangent fit into the joined structure of the geometric figure.

6.4.2 Using questions to analyze and gain conceptual knowledge

As seen in figure 6.1, almost all of the conceptual and analyzing questions posed emerged as a result of the at-home teaching. Eleven out of thirteen conceptual and twelve out of thirteen analyzing questions were posed during this part of the research. The conceptual questions posed could be characterized by asking "why" something happened.

GLEN: Why is there a fixed relationship?
CELINE: Why are all the answers for every triangle equal (approximately)?
BELLA: Why are all the answers in exercise 1 the same?

These questions demand an analysis of whether they should be argued to be of the *understanding* or the *analyzing* level of RBT. There should be little doubt that these could trigger some evaluation of *cause and effect*, and thus one should judge to which level this evaluation will occur. If the question's response only involves observing that the ratios between the sides are constant in the triangles provided, this would be a rather surface-level evaluation of the results. Celine's and Bella's questions might promote this line of thinking easier than Glen's, as they directly refer to the answers to the task. This could then lead to an analysis of the numbers provided rather than the mathematical concepts involved in the exercise. This would then refer to the *inferring* or *explaining* parts of the *understanding* level of RBT.

Glen's question, however, never directly refers to the results, focusing on the "fixed relationship" rather than the answers. Thus, this question might "lift" the thought process away from the answers, resulting in an analysis of some greater mathematical concepts. As presented in the last subsection, Glen did evaluate (or at least question) the relationship between his results and similar triangles. It would then be reasonable to argue that he, to some extent, might have followed such a line of thinking, further justifying that this should be considered a HOCQ. As this could involve identifying each element of the task, answer, and related topics (as similar triangles and geometry), this could activate the *organizing* part of the *analyzing* level of RBT, thus making it a HOCQ.

6.4.3 Using questions to evaluate the correctness of an answer

Half of the students posted at least one of these questions, some posed two.

> *Is this correct? it seems so similar.*
> *Is this answer correct?*
> *Did I finish the task?*
> *Did I finish the task correctly?*
> *Did I solve this correctly?*

Questions aiming to determine the correctness or evaluate the form of an answer proved to be the most prominent question category of the at-home teaching session. This should in no regard be surprising, as the students were directly asked to evaluate the relationship between calculations, and thus, the answer should indeed be assessed. However, how this evaluation takes place could result in vastly different reflections. The questions presented above can, in reality, be separated into two purposes, some aiming to determine whether an answer, that is, calculation, is correct, and some are trying to determine whether a solution or procedure would yield a desirable answer. As argued in section 6.3.3 a question aiming solely to identify if a given solution provides an adequate answer, thus fulfilling the requirements for a *classifying* question of the *understanding* level of RBT, would be a LOCQ. However, if the question's purpose is to evaluate the best approach for solving the task, thus evaluating the mathematical process, it could justifiably be argued to be a *planning* question of the *creating* level of RBT.

As in section 6.3.3, the unspecificity of the question leaves much room for interpretation. As such, the questions alone should be considered a LOQC, as, without further explanation or clarification, one cannot say that the question would result in higher-order thinking. As any remaining analysis of these questions would be the same as the one presented in section 6.3.3, I cut this section short.

6.4.4 Using questions of self-evaluation

While questions of self-evaluation were analyzed in section 6.3.2, the observation of this question category in the at-home teaching setting is exceptionally fascinating. This category was deemed high-order in the previous analysis, independent of intent, phrasing, or mathematical terminology. However, one key difference between the two situations, between the Mentimeter session and the at-home teaching session, is that the students had not recently been told to use these types of questions in the at-home session. As argued in section 6.3.2, many of the self-evaluation questions that emerged in that lesson could come from a desire to meet the teacher's expectations, thus limiting the degree of evaluation and reflection. However, as the students had not been explicitly told to use these types of questions in this lesson, one could argue that a plausible outcome could be a higher level of self-evaluation and knowledge creation. Nevertheless, it is still reasonable to say that the students posed these questions because they remembered the previous week's discussion. Still, the fact that they used these questions without being prompted to suggest at least that they recognized their importance.

The way the questions were phrased also suggests some possible improvement in the students' question-posing ability.

CELINE: Did I learn something, and what have I learned?

Here we see Celine not only asking the dichotomous yes or no question of whether she had learned something but also posing a question to reflect on *what* she had learned. We saw this last question emerge as the most prominent self-evaluation question in the Mentimeter session. Three other students asked, "What have I learned?" in this session. What I find interesting about Celine's question is that she does not only reflect on what she learned. She also questions whether she actually learned something new. By posing such a question, both a yes or a no may lead to much essential reflection if the students are to reach active comprehension. As defined in section 2.2, active comprehension can be seen as "a continuous process of formulating and searching for answers to questions before, during, and after working with a mathematical object". Therefore, the questions presented above should be deemed fundamental when searching for answers to questions *after* any given learning process. Then, by this argument, the majority of the students lack some prerequisite question attribute to reach active comprehension. However, not handing in a question does not mean the students never asked themselves such a question. It is perfectly reasonable to argue that some students may have performed some degree of personal reflection without deeming these questions as a desirable "answer" to the given task.

6.5 Evaluating the validity of the didactic intention

As is customary when employing didactical engineering (DE) as a research method, I will in this section evaluate whether my didactical designs proved valid in facilitating question-posing and the respective mathematical objects. I will go through my different didactical choices and designs, the classroom discussion, the use of QSQ, the Mentimeter session, and the at-home teaching session, in the order they were implemented, judging to what extent they managed to facilitate the mathematical knowledge at hand and question-posing. The mathematical knowledge of the first three of these were some conclusions of differential calculus, while the at-home teaching aimed to introduce triangle trigonometry. However, no designs focusing on a specific topic or piece of knowledge of differential calculus were made; thus, no particular focus will be given to this. In addition to evaluating the validity of the didactic intention, this section will also restate and evaluate the remaining hypotheses from chapter 5, as a conclusion to the *a posteriori analysis*.

As suspected, the classroom discussion ended as more of a lecture than a discussion. While being given multiple opportunities to participate or discuss some topic, the students remained silent for the entire duration of this teaching segment. Thus, I believe this portion of my study was more effective in introducing my place in their classroom, what I aimed to research, and some surface-level thoughts on question-posing instead of the goal of teaching the importance of questions. As will be further discussed in section 7.2, implementing such a teaching design in a very non-verbal classroom culture could, and in this case did, end in poor results. Based on the questions received in the remainder of this session being of the exact nature as before the lesson, I deem this teaching session to have been ineffective at promoting question-posing, at least seen as a stand-alone situation. However, it should be mentioned that each of the choices and designs used in this study is meant to work as a whole, each serving its own purpose in a grander scheme to help the students reach active comprehension. Thus, the goal of this session was not to instantly have students posing a vast amount of HOQC. It only sought to introduce the topic. Then, while it is hard to say whether the lesson benefited the students' question-posing ability, it at least worked to place question-posing in the students' short-term memory, thus opening for the possibility of long-term retention of the topic.

Unfortunately, the data collected from the day QSQ were implemented were of such bad quality due to poorly calibrated audio equipment that no concrete examples of how the students reacted to them can be given. However, as hypothesized, the students did not know how to respond to these types of questions, thereby only responding with "I don't know" or silence. Even though several explanations and examples of such questions were given, the students showed no indication that they knew how to generate, and did ultimately not try to phrase, such questions. Thus, following this first lesson, less focus was given to the use of QSQ in ordinary teaching as the responses indicated a timeframe for change that was incompatible with my study. Additionally, QSQ were implemented during an assessment, and thus as an ethical consideration, it was deemed best to focus on teaching the students what types of questions they should ask, instead of using QSQ, so that my research would not negatively impact the students' grades. By this, I mean that the implementation of QSQ demanded using more time than I could without removing the valuable time the students needed to complete their assignments. Then, while directly facilitating modeling behavior from Singer's theory initially was deemed unnecessary, this teaching ended somewhere in the borderland between modeling behavior and phase-in/phase-out strategy. An example of how this was done can be seen on transcription line 48 in the dialogue of section 6.2.3.

The first designed intervention of my study, the Mentimeter session, sought to promote question-posing and, to some extent, teach it. While just as with the classroom discussion, the students remained silent for the duration of this lesson, it still produced essential and exciting results. As Mentimeter was used to allow the students to respond and participate without having to voice their questions, which resulted in a wide variety of responses, the main goal of this lesson was met. The lesson did promote questions. Further, the students who otherwise remained silent in all other whole-class communications gained a voice through a digital interactive tool. As was seen in the analysis, the questions posed in this lesson were significantly different from those posed during student guidance. Notably, the students were taught the importance of posing questions of self-evaluation. While the results from the Mentimeter session alone should not be used to argue that the students *learned* to pose such questions, the data from the at-home teaching suggest that at least some of the students may have. This comes from the fact that some students still posed questions of this character after approximately one week, thus suggesting that some thoughts on question-posing may be retained. Thus, the second goal of this session, teaching question-posing, could be argued to be reached, at least at an introductory level. This does, however, not mean that the students have learned all that is to pose adequate questions. As will be discussed in section 7.2, altering a classroom culture into one of inquiry and questioning could take years to accomplish.

Further, I hypothesized that exercise 4 would produce many procedural questions, as this task was somewhat procedural. The results revealed that this was indeed the case, as all the groups posed at least one procedural question, with twelve in total, concerning. Furthermore, exercise 6b was hypothesized to generate many questions concerning base knowledge and familiarizing information. This task did prove difficult for the students, as fewer questions were posed in total, and, indeed, most of these were of the hypothesized nature.

Lastly, the at-home teaching session aimed to teach an introduction to triangle trigonometry and promote questions of reflection. As shown in the analysis, numerous questions produced from this lesson were of a higher order. Particularly, many questions sought to make connections both between and within mathematical subjects. Further, as mentioned previously, some questions of self-evaluation emerged in this lesson. This suggests that something in this lesson promoted HOCQs in a way that the other designs and didactical choices did not. This could be the phrasing of the task, the topic's nature, or the fact that this was an introductory lesson on a completely unknown topic. As the students had no previous knowledge (at least not taught at school) of trigonometry, the requirements for a question to be deemed high-order were lower than for the conclusion of differential calculus. The students had little opportunity to merely rely on recall and memory, thus, to some higher extent promoting high-order thinking. The tasks they were asked to pose questions to also directly asked the students to perceive some relationship, thus naturally generating questions to make some connections. Thus, this can suggest that using reflection tasks at an introductory level may more easily generate HOCQs than other designs.

Further, the questions from Glen and Jenna suggest that at least some of the students managed to grasp some of the mathematical knowledge sought to promote this lesson. By directly and indirectly referring to similar triangles, these students showed that they had seen the bigger picture of trigonometry, and thus, the mathematical knowledge was conveyed to at least part of the student mass. However, the extent to which the remaining students connected this introductory lesson with previous knowledge is unknown. John's questions could suggest that there still is some confusion regarding the definitions of sine, cosine, and tangents. However, this was only an introduction to the topic, not meant to teach the entire topic of trigonometry, only introduce some basic concepts. This final design should then be deemed a success, as it at least triggered some questioning that could be used further.

Ultimately, the entire collection of designs and choices may, on the surface, tell a story of students that, to some extent, improve their question-posing ability. From the first questions collected in the observation phase to the questions from the at-home teaching, one can see a significant number of HOCQs emerging towards the end. Further, the use of mathematical terminology seems more planned and structured than the ones phrased orally. While there is a fundamental difference in how an oral question and a written question are generated, the use of terminology is still notable. However, this difference in the data cannot confidently suggest that a significant improvement was made due to the completely different ways they were collected. While the oral questions were collected only from students seeking help, and thus greatly restricting the pool of participants, the written questions span a greater variety of students. Then, students who silently manage the tasks would not pose questions orally, and thus many possible high-complexity questions were not posed. While Bellas involvement can be noticed throughout the study, the data does little to suggest a significant improvement in her question-posing, while some progress can be argued. The designs and choices can then be said to have promoted question-posing to a greater extent than traditional teaching would have done, but given the study's timeframe, one cannot assuredly say much about a change of question-posing ability. Though the data might not say much about a significant change, the teacher has indicated that he does find it easier to understand what the students are referring to when posing questions after a few months. This then suggests that at least some retention, either by the teacher or the students, has been generated.

Chapter 7

Discussion

With the data presented and analyzed, this chapter seeks to lift the focus away from the data-specific cases to the more significant tendencies observed in this study. This is done to finally find an answer to the research question:

> *To which extent and how do students following a mathematics 1T course use questions of different levels of complexity in their student-teacher communication?*

7.1 The role of high- and low-order cognitive questions

Throughout this study, several categories of higher- and lower-order cognitive questions have been explored. The results revealed a wide variety of ways to interpret and analyze a question, each interpretation resulting in a different purpose, cognitive level, and meaning. One key observation was that the oral student-teacher communication produced no questions deemed to obviously be of a higher order. So then, what is the role of the low-order cognitive questions that the students ultimately posed?

The tendency observed in the analysis was that most of these questions either sought clarification or confirmation or procedural knowledge. The remaining oral questions relied on task-specific explanations and answer-related phrasings. Thus, these low-order questions looked to attempt to obtain the necessary knowledge to solve the task, determine the correctness of an answer, or understand the exercise description. The role of low-order questions could then be to acquire the knowledge of how to operate on a mathematical object rather than understanding the object in question. This is further reinforced by observing that both the at-home teaching and

Mentimeter session produced several similar low-order questions. In particular, when asked to pose questions *necessary* to solve a task in the Mentimeter session, the vast majority of questions generated sought either procedural knowledge or information about rules and definitions. This tells us that the students saw these categories as significantly more important than questions seeking to *understand* a topic. This could suggest that the students either already feel like they understand the subject or, more extremely, do not feel like understanding a topic is necessary to solve tasks concerning it.

If it is so that understanding a topic is regarded as unnecessary to solve tasks (regardless of how a student decides to define *understanding*), then what would be the role of higher-order cognitive questions? Keeping a similar phrasing, HOCQs are necessary to "solve" a topic or concept. These are the questions that should be posed when exploring an idea and creating a concept. As in Glen's, and to some extent, Jenna's questions, the results from the at-home teaching display questions of reflection, evaluation of solutions and topics, and exploration of ideas. Both the Mentimeter and at-home teaching demonstrated questions of self-evaluation, that is, investigation of the self, our own understanding, and knowledge. While such questions may not be necessary to solve procedural or algorithmic tasks, I believe they should be the foundation of exploratory, inquiry-based, or creativity-driven learning situations. The role of HOCQs are then not to obtain a solution but rather to *evaluate* several possible solutions, *create* new knowledge or *judge* a topic or subject.

In my opinion, there should not be an equal distribution of high- and low-order questions. In a perfect teaching situation, there should be an abundance of lower-order questions, making the connections between the more prominent topics, causing explanations of definitions and rules, and pointing out patterns in topics interspersed by the higher-order questions, exploring the larger picture. While high-order questions should definitely be sought after, I believe it is neither realistic nor beneficial to have an equal distribution between high- and low-order questions. As we have seen, there is a natural majority of low-order questions emerging in a classroom, at least in the one I observed. Further, while HOCQs are of great importance, they require more time to explore fully, thus, making them harder to use effectively if they emerge too frequently. Then, I believe an appropriate distribution between high- and low-order questions would facilitate a reliable learning scenario both for students who excel at a topic and students who could need some additional guidance. Then, if this can be seen as an ideal, what hinders the development of such use of questions?

7.2 The classroom culture as a possible obstacle for the success of question-posing

As a teacher, the knowledge that a lesson may not go as planned comes with the profession. Then, it is essential to evaluate all factors that could contribute to successful teaching. In every classroom and every subject, students and teachers have some shared expectations of how the teaching should occur (Grouws & Lembke, 1996, s. 39-40). This culture, these norms, or these expectations are what Folke Larsen, Hein, and Wedege call the didactical contract (Folke Laarsen et al., 2017, s. 8-9) (Not to be confused with the didactical contract commonly used in the Theory of Didactical Situations, TDS, which comes with a more complex and extensive definition). This can help in explaining many of the results we observed in the analysis of chapter 6. This contract or culture is subconsciously and informally negotiated between the students and the teacher while still being a function of the students' previous experience.

The class in question had produced below-average test results in the previous semester. From discussions with the teacher, I could gather that the class may have lacked some mathematical confidence, again contributing to the non-verbal classroom culture. This lack of confidence can be explained through the notions of ego- and task-oriented students. Ego-oriented students seek to appear high-performing by their peers, while task-oriented students seek to gain intrinsic knowledge and understanding (Skaalvik & Skaalvik, 2013, s. 171-172). From the preexisting classroom culture, the students were used to operating as ego-oriented. Thus, when presented with an attempt to implement a radical change in the didactical contract, from a non-verbal to an oral classroom, many of the students may have felt pushed to reveal information about their level of understanding that they were not comfortable revealing.

The difference in the results from the oral and the written phases of my research suggests that this may be the case. The vast majority of the oral questions were clarifying or confirming a belief; thus, in reality, they did not reveal much about the students' academic performance. This contrasts both the Mentimeter and the at-home questions, as the questions from these sessions were much more exploratory and displayed more terminology and mathematical presuppositions.

Again, trying to implement such changes may have resulted in less participation than usual, as the students may have tried to resist cultural change. Grouws and Lembke (1996) indicate that such a breach of the didactical contract may result in the students actively resisting the change as a cultural shock. As mentioned in section 5.1, they also point to the significant time dimension of the classroom culture, saying that even the smallest of changes may take a great amount of time to implement.

As mentioned in section 2.3, Dillon (1990, p. 7) states that the norm in today's school is to "induce in the young answers given by others to questions put by others". Thus, students are not taught how to *pose* questions, but rather how to *answer* them. This can explain the failure in implementing QSQ in ordinary teaching and many of the typical question categories we observed in the analysis. There is little doubt that the students were unfamiliar or possibly uncomfortable with participating orally in mathematics. Thus, as the classroom culture did not facilitate discussions or oral participation, it would naturally not facilitate the generation of orally communicated questions. According to Nickson (1994), every mathematics classroom assumes a unique culture according to the knowledge, beliefs, and values of each participant and the ways the messages they receive are assimilated. However, the students' knowledge, beliefs, and values are not wholly dependent on what a single teacher, in one year, tries to convey as beneficial or interesting. It would be difficult, if not impossible, for one mathematics teacher to alter some intrinsic values, which may often be unknown or unconscious to the students and that have been fed and nurtured for ten years prior. From Dillon's belief that students are not taught how to pose or generate questions properly, strengthened by what we observed in the oral classroom setting, I would argue that today's school conveys a message that asking questions is inefficient and unnecessary. Thus, with this being the message we communicate to our children and young, it is natural that their ability to generate and pose questions is underdeveloped.

Then, if question-posing is something to be desired, as is my firm belief, the generation of an oral, participatory, and exploring classroom culture should be an ongoing project starting early in the students' education. If mathematics, or the STEM courses in general, are taught to be something rigid, which cannot be questioned or changed, this would naturally result in less exploration and creation in these subjects. Hence, allowing mathematics to be explored, created, investigated, and questioned could positively aid the natural generation of questions, both of a higher and lower order. Luckily, as of the last few years, a more significant focus has been directed towards these ideals, for example, in the form of inquiry-based education and mathematical literacy.

7.3 The future teaching of question-posing

As briefly presented in section 2.1, inquiry-based education (IBE) has emerged as a promising field for facilitating inquiry, exploration, and scientific methods. This didactical approach relies on the generation and exploration of inquiries and is closely related to student question-posing. However, the method mainly focuses on exploring possible solutions to inquiries, not how to generate such inquiries or questions. Thus, while being very close to the field of question-posing, it cannot strictly be argued to be a method that teaches question-posing, at least not directly. There is, however, nothing restricting IBE from doing just that.

As Artigue and Blomhøj (2013) point to, inquiry can develop only if some part of an unknown object can be approached with that which is already known because data and references can suggest hypotheses and inferences. Thus, when faced with something unknown that could, in theory, be explained by something that is known, a student may develop some inquiry or question that should be explored. Glen's questions from section 6.4.1 could be examples of such inquiries that may, at least partially, be explained and explored using already acquired knowledge. A teacher could use questions such as Glen's in an inquiry-based lesson where each of the questions would be explored. In this case, the student's question was generated through QSQ and would be further explored using theories on IBE. Then, by using the theories of active comprehension in conjunction with theories on IBE, the entire question-posing process, from generation, through exploration and creation, to conclusion, may be facilitated. The results from this study suggest that using QSQ in a more structured manner, as in the Mentimeter or at-home teaching, may improve the extent of which students use question-posing. This is shown both in the number of questions and in the amount of HOCQ posed in these lessons. Thus, if using QSQ could positively benefit the generation of higher-order inquiries, this could undoubtedly serve the field of IBE.

The theories of IBE coincide well with the theories of mathematical literacy. While a universal definition of mathematical literacy, or numeracy, is still to be agreed upon, experts on the area generally argue that mathematical literacy is of great importance both in the students' academic and everyday lives (Geiger et al., 2015, p. 2). Geiger et al. show how mathematical literacy goes beyond simple arithmetic operations but extends into one's capability to ”make sense of non-mathematical contexts through a mathematical lens; exercise critical judgment; and explore and bring into resolution real-world problems” (Geiger et al., 2015, p. 1). Different definitions of mathematical literacy also involve ethnomathematics, social or ideological debate, and technology (Geiger et al., 2015; Jablonka, 2003). While the term *literacy* suggests some written competence, I believe that using *numeracy* opens a more significant and substantial pool of possibilities in the teaching of the aspects pointed to by Geiger et al.. This comes from the fact that numeracy does not directly ignore the power of teaching critical judgment, exploration, and creation through oral communication, with this being the natural arena for questioning.

This brings into view the necessity of teaching questioning in a mathematical setting. It could be argued that in order to "exercise critical judgment", a student needs to question either the learning material, the teacher, or some other significant aspects of education. In particular, teaching students to be critical through using questions of the *evaluating* level of RBT could be argued to be essential to reach this goal. If the students' are further to "make sense of non-mathematical contexts through a mathematical lens", teaching them to pose questions of the highest cognitive level, *creating*, relying on the subcategories *generate, plan* and *produce* could be desired. Then in order to teach mathematical numeracy, the teaching of questioning could be considered an effective tool in both creating critical, exploring, and creative students.

Further, suppose one uses a definition of numeracy that includes notions such as ethnomathematics, social or ideological debate, or technology. In that case, questions should get an even more substantial part in education, as this is a natural part of any debate or democratic process. When participating in academic or political debates concerning the field of mathematics, it could be crucial to have a well-developed capacity to use the language of mathematics. As we have seen in this study, the students' mathematical presuppositions are often hidden by a lack of proper use of mathematical terminology. This has often led to questions being ambiguous or possibly misleading as to the question's actual intent. Then, to prepare the students for the debates and quarrels of society, the proper teaching of subject-specific language could be essential. Particularly, if mathematical numeracy is desired, as several political projects suggest, the teaching of question-posing of a higher cognitive level with proper use of terminology should be a fundamental part of any such project (Jablonka, 2003).

7.4 Could the school be failing part of its social mandate?

In the Norwegian school's mission statement (*formålsparagrafen*) it is stated that:

> The education shall provide insight into cultural diversity and show respect for the individual's beliefs. It shall promote democracy, equality, and scientific thought[1].
>
> Pupils and apprentices shall develop knowledge, capability, and attitudes to master their lives and to participate in work and community in society. They shall be allowed to demonstrate creativity, dedication, and need to explore[2]. (Utdanningsdirektoratet, 2016b)

Thus, a large part of the school's social mandate involves the education of democratic citizens so that they can contribute to society. As stated in section 2.3, Dillon (1990) claims that students are not taught to question authority; in fact, he claims they are not taught how to question. The posing of questions is a fundamental part of any democracy, as the people can "interrogate the currently popular official rhetoric of "transparency" by asking critical questions about what is made transparent, at what time, in what forms, through what channels, on whose decision, for what purpose, and in whose interest" (Scholte, 2002, p. 294). If it is indeed the case that students do not learn how to pose questions, how can we claim that we are educating democratic citizens if we actively ignore teaching one of the critical facets of democracy: questioning leaders? This thesis has involved student-teacher question-posing of three levels of complexity in mathematics. I have not explored subjects such as social sciences or religion and can therefore not state that the tendencies observed in this study are universal across subjects. However, the teaching of democratic values is listed as multidisciplinary elements of education in the Norwegian system (Utdanningsdirektoratet, 2016a). This means that the values of democracy shall be taught in all subjects; thus, if they are not met in mathematics, we cannot be said to fulfill the goal of education.

As a part of the educations fundamental values, "critical thinking and ethical consciousness", it is explicitly stated that

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[1] *Opplæringa skal gi innsikt i kulturelt mangfald og vise respekt for den einskilde si overtyding. Ho skal fremje demokrati, likestilling og vitskapleg tenkjemåte.*

[2] *Elevane og lærlingane skal utvikle kunnskap, dugleik og holdningar for å kunne meistre liva sine og for å kunne delta i arbeid og fellesskap i samfunnet. Dei skal få utfalde skaparglede, engasjement og utforskartrong.*

> The school shall contribute to the students becoming curious, *asking questions*, developing scientific and critical thinking, and acting with an ethical consciousness[3] (Utdanningsdirektoratet, 2016c, My italicization).

In connection to Dillon's claims, the results from this study suggest that today's education, in reality, does not contribute to the natural generation of questions. As discussed in section 7.2, the norms in today's school could actively prohibit the generation of student-posed questions based on social standards and cultures. We saw this in the vast difference between the oral and written questions. Thus, the results from this study reveal a possible breach of this fundamental value; the school does not contribute to students asking questions.

As stating that the school may be failing part of its social mandate is quite a bold claim, it is crucial to evaluate the validity and generalizability of my sources. Dillon's claim is based on the American school system and could not be directly transferred to the Norwegian system. However, similar tendencies as the ones Dillon describes are observed in other school systems, such as, for example, the German system, by Bell et al. (2010). Since these tendencies are observed in considerably different educational systems, it is not unreasonable to believe that they may be observed in Norwegian schools. In my research, I have observed a vast amount of low-order questions and questions lacking terminology and displaying unclear presuppositions. While I observed a significant number of questions in total, most of these were not naturally generated; I had to facilitate question-posing to get questions outside of a guidance setting. I would therefore argue that Dillon's claim could be valid, at least to some extent, in the Norwegian school system. However, this relies on one fundamental condition; do the tendencies I have observed in the students' oral questions span across classes, schools, and regions. In my opinion, it does not have to be a majority of classes, only a significant number, as even a small number of cases should be regarded as critical to the values of society. If this is the case, I argue that the Norwegian school fails one of the critical elements of its social mandate and needs to find means to improve these destructive tendencies. Such means may, for example, be some of the aspects discussed in the previous section.

[3] *Skolen skal bidra til at elevene blir nysgjerrige og stiller spørsmål, utvikler vitenskapelig og kritisk tenkning og handler med etisk bevissthet.*

Chapter 8

Summary and concluding thoughts

In this study, I wanted to investigate to which extent and how students use questions of some preset conditions of complexity in their student-teacher communication. The levels of complexity I set out to examine were *mathematical presuppositions, cognitive level*, and *mathematical terminology*. These were deemed to yield a widespread set of variables that could amply describe the quality of a question. In order to investigate these aspects of student-posed questions, I explored and altered Singer's (1978) theory of active comprehension for it to fit into the frame of mathematics. This modified version of Singer's theory was then used to design and analyze several didactical choices and designs intended to facilitate question-posing in various forms of student-teacher communication. Structure and analytical weight were ensured in this study by employing didactical engineering as my primary research methodology. By this, I performed an epistemological, didactical, and institutional analysis to assure that I had the necessary theoretical background to design, implement and analyze didactical choices and designs aiming to facilitate question-posing.

The mathematical presuppositions were defined using Dillon's (1990) multidisciplinary rendition of the state of questioning to provide a general description of a layer of the students' assumptions. Throughout this study, it became clear that a question's mathematical presuppositions were highly dependent on the use of mathematical terminology. While "proper use" of mathematical terminology is relatively subjective, an attempt to define the language of mathematics was made through Mulwa's (2015) three categories of mathematical terminology. In the vast majority of questions, the students' presuppositions had to be deemed valid or undetermined due to highly unspecific phrasings and insufficient use of the language of mathematics. Thus, the students' knowledge and understanding were to a significant extent hidden behind mathematically irrelevant or uninteresting presuppositions. However, when the students exhibited proper use of mathematical terminology, their presuppositions yielded many aspects to consider. For example, John from section 6.4.1 revealed that he possibly understood the sine, cosine, and tangent as the vertices of a triangle by referring directly to the three trigonometric functions and using the words "points" and "triangle". This question, in particular, exposed the power of examining a question's presuppositions, as this revealed an understanding that should be corrected or discussed.

The questions' cognitive level was determined using the Revised Bloom's Taxonomy (RBT), as presented by Radmehr and Drake (2019). When more analysis was deemed necessary or of particular interest, I employed five categories of high-order cognitive questions presented by Sadker and Cooper (1974). This level of complexity was given the most focus in this thesis, as every question, in every situation, reveals some information about a cognitive level. Further, while the use of terminology and, to some extent, the presuppositions of a question could relatively easily be determined, a question's cognitive level proved to require quite extensive analysis to determine appropriately. The data unveiled a fascinating distribution of questions of different cognitive levels.

- Out of seventy-one questions posed in the oral communication, none were deemed high-order.
- Out of ninety-eight questions generated in the Mentimeter session, sixteen were deemed higher-order.
- Out of eighty-five questions posed in the at-home teaching, twenty-nine were deemed high-order.

This displays a crucial contrast between orally communicated questions and written questions. The data suggests that students do not find questions of a higher order necessary to gain the desired information when seeking guidance on a task. This tendency is further extended when the students generate vastly more questions of a lower order when asked to pose questions necessary to solve a task. Thus, the students participating in this study used lower-order questions to gain information to solve a task and obtain a desirable answer. During the Mentimeter session, they were taught to use self-evaluation questions every time they solved an exercise or learned something. This produced the majority of HOCQ in this session. Some students may have learned to use these questions, as they also emerged in the at-home teaching session. The at-home teaching revealed significantly more questions of a higher order, most of which sought to make some connections either within or between mathematical subjects.

Moreover, the written questions illustrated a better use of mathematical terminology and phrasing, which further resulted in clearer presuppositions. Thus, the students participating in this study displayed an ability to produce higher complexity questions when communicating through a written format. When posing questions orally, mathematical terminology was often either neglected or of a lower quality than desired. However, the students showed that they could produce well-phrased and well-structured questions of a higher cognitive level when such questions were facilitated through didactical designs.

By performing this study, I have not only learned a great deal about the topic of question-posing. I have also developed and expanded my definition of what it means to be a teacher. I have learned more about how I fit into the profession I am about to partake in, and I believe that I could have an impact on the world through my research and my knowledge. This project has given me a larger perspective on how to use orality in mathematics and how I can contribute to the further development of education. If I were to do such a project again, I would want to investigate how inquiry-based education can contribute to the teaching of question-posing and which impact this would have on students learning outcomes. I believe that the topic of question-posing is in dire need to be explored further and should be given the attention it warrants.

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A Transcription of verbal data

Transkribsjonskoder:

...	Pause opptil 3 sekunder
_	Avbrytelse
[...]	Uartikulert eller ikke hørbar ytring
Kursiv	Trykk
{Tekst}	Redegjørelse for ikke-verbal handling
(NN: interjeksjon)	Interjeksjon av NN under en annen persons ytring.
Understreket tekst	Resitering av tekst fra oppgave
[	A og B snakker samtidig:
[	A: også er det en [pluss to.
	B: [Ser du her?

Ytring nr.	Navn	Ytring	Tidspunkt
1	L1	Er det noe jeg kan hjelpe deg med?	27:25:00
2	A	Okei, hva vil det si å bruke figuren til å avgjøre når f(x) vokser og når f(x) avtar?	
3	L1	Okei, bruke figuren til å avgjøre når.. For det her er.. Hva viser figuren da?	
4	A	Eh, den der? (L1: mhm). Den viser at den krysser på minus 4 og så med 2	
5	L1	mhm, og hva er det en figur av?... Altså det her er jo en figur av den deriverte	
6	A	Ojja, er den der den deriverte?	
7	L1	Mhm, så den viser grafen til den deriverte	
8	L1	Så hvis oppgaven spør om når f(x) vokser og avtar, når du har grafen til den deriverte, hvilke spørsmål må du stille deg for å på en måte, nå fram da?	
9	A	Jeg vet ikke	
10	L1	nei, men her har vi noe som er den deriverte, så da er kanskje et godt spørsmål, hva er den deriverte? Hva viser den deriverte for noe?	
11	A	okei	
12	L1	Hva forteller den deg om grafen?	
13	L1	Før dere hadde om den deriverte så hadde dere om det som het gjennomsnittlig vekstfart, (A: ja) og den deriverte ble også kalt da momentan vekstfart (A: mhm), husker du det?	
14	A	Eh, ja	
15	L1	Så den deriverte forteller deg noe om hvor fort noe vokser… så her for eksempel så får du vite at I x lik 7 omtrent, litt før 7, så har den deriverte en verdi på 9 (A: ja). Det forteller oss at stigningstallet til funksjonen f I punktet 7 omtrent er [omtrent 9	
16	A	[omtrent 9	

17 L1	Så hvis du bruker den deriverte her da […] hva vet du om den deriverte når noe vokser da?	
18 A	Hvor mye det vokser	
19 L1	Ja for du vet hvor mye det vokser, det vet du, men hvis den deriverte er positiv hva betyr det?	
20 A	At den vokser positivt	
21 L1	At den vokser positivt ja. Og hvis den deriverte er negativ	
22 A	Da betyr det at den vokser negativt	
23 L1	ja, og hvis den vokser negativt er det det samme som at den avtar	
24 A	ja, okei	
25 L1	Så hvor er den deriverte positiv da?	
26 A	[…] jeg vet ikke.. Etter 2, eller?	
27 L1	ja, det er jo en god start! Hvorfor sier du etter 2?	
28 A	For etter 2 er den jo positiv, men […]	
29 L1	Ja, så før 2 er negativ og etter 2 så er den positiv	
30 A	Så er det svaret?	
31 L1	Ja, det kan jo være en start på svaret ihvertfall	
32	[…]	
33 B	Eh, L1?	31:00:00
34 L1	Ja	
35 B	Her så står det når er ballen på sitt høyeste punkt, vil det si at den er I toppunktet, eller på linja?	
36 L1	eh, ja når er ballen ja. Så det vil si at det har noe med tid å gjøre	
37 B	Ojja, okei sånn ja… så det er etter… vent litt da… det er jo høyden på ballen etter hvor mange sekunder (L1: mhm), så når det vil være etter fire.. Timer	
38 L1	det høres veldig greit ut, men det er kanskje ikke I timer	
39 B	nei ikke timer det er I sekunder, og så er det… da er ballen 80 meter høyt?	
40 L1	Ja, det ser rett ut ja	
41 B	Skal jeg skrive det I Word da liksom?	
42 L1	Ja, for du hvis du skriver innleveringen din I Word så må du formulere det I word da.	
43 B	Hvordan bruker man det der… man kunne skrive sånn mattegreie I word	
44 L1	Funksjoner ja. Da trykker du på sett inn, så står d	
45	[…]	
46 L1	Ja	37:20:00
47 B	Okei, her tar jeg den deriverte av h(t) ikke sant? (L1: mhm) og spørsmålet er Hva forteller funksjonen oss? Og hvordan kan vi se liksom hva den forteller	
48 L1	Okei, da er det første man må tenke på, hva forteller den deriverte oss da?	
49 B	Er det ikke sånn at om den er negativ så syker den og positiv så stiger den?	

50 L1	Ja, det gjør den blant annet da, så når er den deriverte negativ, og når er den postiv da?
B	Den er negativ… sånn den synker herfra, gjør den? Jeg vet ikke
52 L1	Eh, her krysser den y-aksen (B: ja), men den er fortsatt positiv
53 B	ojja, ja, her synker den da
54 L1	mhm, så der blir den plutselig negativ ja. Så med å se på den der så er det I all hovedsak 3 ting du kan fortelle… hvertfall 3 ting du kan fortelle om_
55 B	men hvordan er det man ser det liksom? For liksom, ett spørsmål til, her er funksjonen h(t) liksom {viser I GeoGebra}
56 L1	eh, ja, der ja.
57 B	Og så har vi funnet den deriverte (L1: mhm) liksom
58 L1	Ja, hva er den deriverte? Det er jo det store spørsmålet. Husker du før dere hørte om den deriverte så hørte dere om det som heter gjennomsnittlig vekstfart? Og så gikk det videre til å snakke om en momentan vekstfart? (B: ehe) og den momentane vekstfarten ble kalt derivert.
59 B	Okei, hva er det vi snakker om? Når jeg finner den deriverte av x, hva er det jeg finner da?
60 L1	hmm, hvis du finner den deriverte av h(t) (B: ja), da finner du jo en funksjon som forteller deg noe om hvordan h(t) endrer seg. Den deriverte forteller oss alltid om en endring på ett eller annet vis
61 B	okei, ja, så den deriverte forteller oss hvordan den endrer seg liksom, (L1: mhm) hvordan en funksjon endrer seg?
62 B	Så, den her {grafen av den deriverte I GeoGebra} forteller meg at etter 4 så synker den?
63 L1	Ja, etter x lik 4 blir den deriverte negativ (B: ja), og det betyr da at for grafen h at… funksjonen h.. Hvis den deriverte blir negativ, hva skjer med h da?
64 B	Den blir negativ? Jeg vet ikke.
65 L1	mmh, for hvis stigningstallet, den deriverte forteller noe om stigningstallet (B: okei), hvis stigningstallet er negativt [hvis du ser på
66 B	[Da synker den
67 L1	ja, da synker den ja, det er rett ja. Så et eller annet skjer I punktet x=4, der den deriverte går fra positiv til negativ I det punktet… hva skjer I x=4 da? Det burde vi jo nesten spørre oss da.
68 B	Da blir den negativ da
69 L1	mh, hva skjer med grafen til h I x=4, hvis du ser på_
70 B	Den blir negativ, eller den synker.
71 L1	mhm, den begynner å synke når den passerer x=4, men hva skjer akkurat I x=4
72 B	nullpunktet

73	L1	Ja, hvis du ser på den da, hva slags punkt er det på grafen til h?	
74	B	Det er jo… [ja er ikke det toppunktet?	
75	L1	[Det er den grønne grafen	40:35:00
76	L1	Og hvordan kan du vite at det er toppunktet da?	
77	B	Fordi den blir sånn her {viser en bue med hånda}	
78	L1	mhm, så vi kan se det på den grafen der (B: ja), men kan du fortelle det ut fra den deriverte kanskje?	
79	B	eh, nei, eh, jeg vet ikke.	
80	L1	For før x lik 4 da, hva skjer med den deriverte da? Før x=4- Hva kan du si om den deriverte før x=4.	
81	B	Den er positiv	
82	L1	Den er positiv, ja. Så I x=4 hva er den deriverte da?	
83	B	eh, 0	
84	L1	mhm, og etter så er den?	
85	B	den er negativ	
86	L1	Så før så er den postiv, så er den null og så er den negativ	
87	B	Så her er den pluss.. Og så akkurat på toppunktet… ojja fordi toppunktet ikkesant for etter toppunktet så blir den […] (L1: mhm)	
88	B	Så man kan på en måte se, uten å se funksjonen hvordan det her på en måte blir	
89	L1	mhm, så du vet hvordan form grafen vil ha (B: ja) med å se på den deriverte (B: ja). Kan du fortelle noe om hvor høyt opp den går med å se på den deriverte?	
90	B	Nei, jeg tror ikke det.	
91	L1	Nei, hvorfor tror du ikke det?	
92	B	Fordi, den her liksom forteller oss jo bare på 4 er toppunktet, men man kan jo ikke se hva det blir	
93	L1	Nei, alt du vet er at den stiger fram til 4 og synker etterpå (B: ja) så her kunne du ha tegnet en graf som har toppunkt I x=4 og stiger dit og synker dit	
94	B	Okei vent litt, det her har vel en sammenheng med oppgave 1. Her står det tegn en skisse av grafen til f, det betyr at den synker til 2, og så begynner den å stige ikke sant?	
95	L1	Mhm, ja det ser jo sånn ut	
96	B	[…] Etter 2 så er den negativ	
97	L1	ja den deriverte er det ja	
98	B	Blir grafen sånn her eller sånn her? {viser en andregradsfunksjon med toppunkt og en med bunnpunkt med hånda}	
99	L1	Hva tror du da?	
100	`B	Her er toppunktet ikke sant?	
101	L1	Der er ihvertfall den deriverte lik 0	
102	B	Ja lik 0	
103	L1	Er det da et toppunkt eller et bunnpunkt?	

104	B	Ojja bunnpunkt {L1: mhm} så den blir sånn her liksom {viser en andregradsgraf med bunnpunkt}	
105	L1	Det kan virke sånn ja	
106	B		
107	L1	Så er det å kunne forklare hvorfor da.	43:00:00
108		[…]	
109	B	Så den deriverte av h(t) den forteller oss at toppunktet er 4 (L1: mhm), og at før 4 er den positiv (L1: mhm), på 4 er den null (L1: mhm) og etter 4 er den negativ.	
110	L1	Den deriverte er det	
111	B	Den deriverte er det	
112	L1	Så den deriverte forteller bare om akkurat den deriverte, så den forteller oss noe om hvordan h(t) endrer seg. Hva stigningstallet til h(t) er. Så er det, hva betyr det at den deriverte er positiv, hva betyr det at den deriverte er negativ og hva betyr det at den deriverte er 0?	
113	B	Sånn som jeg skjønte det nå, hvis stigningstallet går fra positiv til negativ […]	
114	L1	Hvis den går fra positiv til negativ ja, og hvorfor blir det sånn?	
115	B	Først må den gå oppover, så må den ha et toppunkt, så vil den gå ned. Og hvis den går fra negativ til positiv så må stigningstallet være negativt, så må den ha et bunnpunkt og så må den stige	
116	L1	Ja, kjempebra!	
117	B	Da har jeg det.	
118	A	Hvordan finner jeg den gjennomsnittlige vekstfarten?	
119	L1	Hvordan finner jeg den gjennomsnittlige vekstfarten? (A: ja) okei vet du hva den gjennomsnittlige vekstfarten er da?	
120	A	[...]	
121	L1	Mhm, det er jo det første du må starte med, å finne ut hva den gjennomsnittlige vekstfarten er. Så hvordan kan du finne ut hva den gjennomsnittlige vekstfarten er?	53:37:00

TRANSKRIBSJON L1 24.02

122	K	Den her b-en her, jeg sliter med den	04:20
123	L1	[Skal vi se	
124	K	[Jeg sliter med å forstå den	
125	L1	Det er oppgave 1 det der?	
126	K	Det er oppgave 1 , men det er b, jeg forstår den ikke.	
127	L1	Grafen til f går gjennom punktet. Finn likningen til tangenten. Ja. (K: ja). Okei, hva er det du trenger, for det første hva vet du om tangenten?	
128	K	mmh, er ikke det den som går her?	
129	L1	Mmh, den er ikke tegnet på der nei.	
130	K	Der {viser til et annet dokument}	

131	L1	Jo der ja, der har du jo en tangent (K:mhm) og hva vet du om.. Okei, for nå har du, du har tegnet punktet, også har du laget en tangent I den (K: ja).. Jah ehm… Står det noe om hvordan du skal finne likningen til tangenten?... Mmmh, nei det gjør det ikke, ehm, for det er jo flere måter å finne likningen til tangenten på (K: ja). Jeg kan si at med den løsningen du viste meg der istad med det GeoGebravinuet der (K: ja) så har du i all hovedsak løsningen din skrevet i ehm (K: her?), mhm... Så det er en mulig løsning. En mulig måte å gjøre det på. Men jeg går ut ifra at denne oppgaven her kan ha godt av å forklare litt (K: ja) mulige andre måter (K:mhm).. så da er det, hva slags informasjon har du i oppgaven her da? Hva vet du fra bare oppgaven?
132	K	Bare fra det her? {peker på oppgaven} (L1: mhm)
133	L1	Og da har du jo også den figuren som er oppgitt til oppgaven da for det er jo den informasjonen du har I all hovedsak.
134	K	At den vokser til 2 og så synker den. Er det det liksom?
135	L1	ehm, for, der har du tangenten den er jo grei nok (K: ja) ehm, og så får du vite, okei du har et punkt, det er punktet (3,2) (K: ja) du får vite at figuren viser grafen til den deriverte, og ja, du sa at den den eeh, Hvor vokser og hvor synker den da?
136	K	Den vokser I 2 og så synker den I minus 4.
137	L1	eh, det skjer noe I 2, men den vokser ikke I 2
138	K	Nei, den vokser til 2
139	L1	Hvis du tenker at den går den veien da? {følger grafen med fingren fra høyre mot venstre} (K: ja). Eh, hvis du tenker fra venstre mot høyre da? (K: okei) så vil den jo synke, da den er negativ her da under 2 (K: ja) er den negativ, eller under x-aksen så er den negativ, så før to så vil den vel være
140	K	minus 4
141	L1	Ja for det er jo der den krysser y-aksen (K: ja)… så minus 4 det er, ehm.. Det er, det er jo et viktig punkt for å finne likningen, men så lenge du ser, hvis du bare ser på x=2, hvis du ser på x-aksen (K:ja) så ser du at på alle x-verdier før 2 så vil den linja der være negativ (K: ja), er du enig i det? (K: ja) Så det forteller oss hva for noe? om, om funksjonen f da.
142	K	Altså tenker du på at den er positiv da? Eller
143	L1	em, hvis den deriverte er negativ (K: ja), hva vet vi om selve funksjonen vår da?
144	K	ehm… jeg vet ikke.
145	L1	nei, da ville jeg sagt at det første steget å gå er å gå I boka (K: okei) og se på, å lese om hva den deriverte kan fortelle oss (K:okei) og da spesifikt se på det som kalles monotoniegenskaper. (K: okei) For det kan fortelle [deg litt som faktisk… for da å forstå det litt bedre da (K:ja)

146 K	[Okei, ja det var det ja	
147 L1	Og da kan du forklare det litt bedre i innleveringen din, og når du har gjort det, hvis du trenger mer hjelp da så kan du rekke opp handa på nytt igjen? (K: okei). Ja.	
148 P	{Mens L1 går forbi} Kan du vise meg hvordan? Nei vet du hva? Jeg skal ikke spørre om svaret jeg skal spørre om hvordan man skal gjøre det	08:30
149 N	Ja det er bra	
150 L1	{B rekker opp handa} Yes	08:50
151 B	Hva kaller du en sånn her type funksjon? Er det…	
152 L1	En sånn der type funksjon	
153 B	Eh, graf	
154 L1	Ja en sånn graf ja. Hva den kalles for noe? (B: ja) Det er jo en polynomfunksjon da. (B: ja) eller det kan være en polynomfunksjon burde jeg heller si	
155 B	Fordi, du vet når vi skrev det her inn på GeoGebra (L1: mhm) så så vi at den hadde, at den gikk sånn her {lager en bue, andregradsfunksjon med bunnpunkt, med fingeren} (L1: mhm), eh så jeg kan skrive at, ehm… den.. Hva var det? Du sa at jeg sa det igår. (L1: mhm) Hva var det jeg sa?	
156 L2	Du sa veldig mye så da var det å klare å filtrere bort hva som ikke var viktig. Men når du tegner den inn I GeoGebra, hvordan ser den ut da?	
157 B	Ehm skal vi gjøre det på nytt? (L1: mhm)	
158 L1	Det kan jo være en start {B tegner funksjonen I GeoGebra}	
159 L1	For det viktige er at den funksjonen du har oppgitt der det er den deriverte.	
160 B	Ja	
161 L1	{B har tegnet funksjonen I GeoGebra} mhm, så det er en sånn buet funksjon, en polynomfunksjon. (B: mhm) og_	
162 B	Den har to nullpunkter	
163 L1	Den har to nullpunkter (B: ja)... og hva skjer rundt de nullpunktene da?	
164 B	Rundt dem?	
165 L1	mhm, rundt, før, etter, I dem	
166 B	Ojja før minus to så er jo grafen positiv	
167 L1	mhm, og hva forteller det oss om grafen til f?	
168 B	Det forteller oss at den er positiv først	
169 L1	Det forteller ikke at f er positiv nødvendigvis	
170 B	nei nei, men at den synker først.	
171 L1	At den synker først, eh	
172 B	Eh jeg vet ikke, at det går nedover først.	

173 L1	nei ikke helt, fordi, her ser du på funksjonen til den deriverte. Så hvis du ser på.. For den forteller deg jo I hvert eneste punkt som finnes så kan du finne stigningstallet til funksjonen f I det punktet (B: ja). Så hvis denne grafen her er positiv (B: mhm) hva forteller det om stigningstallet til f?
174 B	At den også er positiv?
175 L1	Da er stigningstallet postitivt ja (B: ja). Og hva betyr det for f da? (B: hmmm) vil den stige eller synke I det punktet?
176 B	Den vil stige
177 L1	Den vil stige ja. Så la oss nå se på denne her {grafen til den deriverte}. Hvor er den positiv og hvor er den negativ den deriverte her?
178 B	Den er jo negativ her {viser på grafen}
179 L1	mhm, så den er negativ etter minus 2 (B: ja), men før 3
180 B	ja, den er jo negativ mellom minus 2 og 3 (L1: mhm).
181 L1	Så da vet du hva om funksjonen f da?
182 B	At den, vent jeg må bare se spørsmålet… eh.. At den, at den avtar (L1: mhm)… hva var det andre ordet?
183 L1	avtar eller øker? Tenker du mellom avtar eller synker
184 B	Nei den der forklaringen på…
185 B	{B logger inn på fagboka} Just wait
186 L1	Det er alltid morsomt å logge inn på de der bøkene, det tar alltid lang tid når det er digitale bøker (B: ja)
187 B	Der, vokser og avtar
188 L1	Ja vokser og avtar ja.
189 B	Okei, så på minus 2 så avtar den (L1: mhm) og på 3 så vokser den.
190 L1	mmh så. Akkurat I punktet minus 2 så verken vokser eller avtar den (B: ja). Der er den deriverte 0 (B: ja), så etter minus 2 (B: ja).
191 B	Så etter minus 2 så avtar den (L1: mhm) og etter 3 så vokser den
192 L1	mhm. Så hvis du vet at den avtar etter minus 2 og den øker etter minus 3, så vet du jo I all hovesak, eller så vet du at den vil avta fra x=-2 til x=3 (B: ojja, ja) Så mellom x=-2 og x=3 vil funksjonen din avta (B: ja).
193 B	Og den vil vokse etter 3
194 L1	Mhm, og så hva skjer_
195 B	Den vil vokse.. Eh. Okei den avtar fra minus 2 (L1: mhm) til 3 (L1: mhm). Sånn etter minus 2 til før 3 (L1: mhm) og så vokser den før minus 2, til etter 3
196 L1	Ja det stemmer
197 B	Så det er hva heter det monogreien
198 L1	Monotoniegenskapene ja (B: ja) ja.
199 B	Ja okei da har jeg det.
200 L1	Kjempebra!

201	K	Ehm, likningen til tangenten (L1: ja) det var det vi skulle finne ut ikke sant (L1: mhm). Så du skulle ta det punktet der den krysser med.. Ja, også skal du I tillegg til stigningstallet. Blir det_	13:50
202	L1	Så.. Hvis du ser på oppgaven da. Jeg har liksom ikke lest oppgavene helt 100% (K: nei okei) så jeg husker ikke all informasjonen dere har.	
203	L1	For da har du et punkt (K: ja).. Eh, så du skal finne likningen til tangenten I det punktet	
204	K	I punktet ja okei	
205	L1	og.. Da.. Kan du bruke, ehm.. Eller det kan du kanskje ikke. Da vet du at I det punktet der (K: ja), hva er x og hva er y I det punktet der da?	
206	K	Er ikke den 3 x og 2 y	
207	L1	Mhm, det stemmer (K: ja). Så da kan du bruke grafen til den deriverte her til å fortelle deg noe om stigningstallet til funksjonen f I punktet x=3 da.	
208	K	Så da skal den gå da fra 3 til 2?	
209	L1	mh, for da ser du, hvis du ser x=3 (K: ja) så kan du, hva er verdien på denne linja her? {Peker på grafen til den deriverte}. For den forteller deg jo noe om stigningstallet er du ikke enig I det?	14:55
210	K	Ja, det er 2	
211	L1	Det er 2, yes så da vet du at stigningstallet er 2. Og tangenten den er jo på formen y=ax+b det så du jo I boka nettopp (K: ja), og hva er det a er for noe her da?	
212	K	eeh, er det der den krysser?	
213	L1	a, a vil være, det kalles jo stigningstallet og b kalles konstantleddet	
214	K	ojja okei, så da er a 2	
215	L1	a vil være 2 ja	
216	K	Hva var det, er det den her? Eller den her (L1: ja) så y, så 2x pluss…	
217	L1	Også vil b være punktet der tangenten vil krysse x-aksen	
218	K	så, burde jeg lage den da?	
219	L1	du kan, du kan lage tangenten.. I GeoGebra (K:ja) det kan du gjøre, og da vet du at den går gjennom punktet (3,2) (K: ja), mhm, så vet du at for hver x-verdi vil stige [med 2 I y-verdi, og da klarer du kanskje å finne noen punkter og trekke noen linjer mellom dem (K: okei) for å finne tangenten	
220	K	med 2	
221	K	Så finne en strek som går der liksom? Og så..	
222	L1	Ja enten den veien eller den veien alt ettersom hva som blir rett da (K: okei) med å bruke det stigningstallet som du har funnet ved å lese av grafen (K: okei).	
223	K	Okei jeg kan prøve	

224 L1	ja, prøv på det. Og så hele tiden når du får den likningen din så er det å spørre seg selv, funker den informasjonen som likningen gir meg nå med den informasjonen jeg har funnet til nå? Er stigningstallet rett? Konstantleddet gir det mening? Og sånne ting	
225 K	okei	
226 S	Har du lyst til å hjelpe meg med oppgave 9?	20:20
227 L1	Jeg kan prøve ja	
228 S	Du skjønner, jeg har skreve alt den der greia inn I excel (L1: mhm), men jeg vet jo ikke hva jeg skal gjøre med det.	
229 L1	Skal vi se, jeg har ikke sett oppgaven før nå så ehm… I denne oppgaven skal vi ta utgangspunkt i funksjonen. Beskriv kort hvordan vi ut fra definisjonen til den deriverte kan finne en tilnærma verdi for f derivert av 5	
230 L1	Har du gjort a eller er du på b?	
231 S	Ehm, beskriv kort nei jeg vet ikke ka, eller vent hva er b? Nei jeg har ikke gjort a	
232 L1	Du har ikke gjort a (S: nei), så da er det jo å ta utgangspunkt I definisjonen av den deriverte (S: ehm) så da er jo det første, hva er definisjonen av den deriverte da?	
233 S	h.. Hva er definisjonen?	
234 L1	Mhm, hvordan kan du finne ut av det?	
235 S	Eehh. Siii, GeoGebra sikkert.	
236 L1	Du kan lete opp I boken din for å finne definisjonen hvertfall. (S: ja) det er det første steget når du skal finne en definisjon på noe, men definisjonen på den deriverte var jo den der store formlen med lim delta x	
237 S	åh ja, det der åh jesus christ… okei.. Så jeg må… {puster tungt ut}	
238 L1	Så her er det.. Det absolutt viktigste å se på denne oppgaven her er hva er det faktisk oppgaven spør om?	
239 S	Ja, det ,det, jeg vet det liksom, men hva spør den om? Men definisjonen det er out of my reach liksom… så Beskriv kort hvordan vi ut fra definisjonen til den deriverte kan finne en tilnærma verdi så vi skal, hvordan finner vi verdien til {L1: mhm) f derivert av 5	
240 L1	Så da er det jo en veldig stor forskjell på denne her og en annen oppgave da I at her skal du beskrive kort hvordan du kan gjøre bruke den, de ber deg ikke om å faktisk bruke den, (S: nei nei) så det er en stor forskjell.	
241 S	og det er out of.. Jeg er clueless på hvor det er I boka {blar I boka}.	
242 L1	Du begynner å nærme deg nå, om du ikke nettopp hoppet rett forbi det.	
243 S	eh, jeg tror det er her borte sikkert… åh Jesus, nei jeg har sikkert hoppet over det.	
244 L1	Jeg tror det stooo.. Huh	
245 S	Deriverte	

246 L1	hmmm, ser ikke noen definisjon der	
247 L1	Du kan også prøve å google definisjonen av den deriverte da (S: ja) og så se hva du finner da	
248 L1	{S googler} mhm gå inn på matematikk for realfag går an da.. Den øverste lenka går an til å se, det er egentlig R1, men det kan hende at selve definisjonen står der uansett (S: ja).	
249 L1	Okei der har du det vet du (S: okei), bare at istedefor delta y over delta x kan du skrive f derivert av x, sånn som den der da {peker}. Så der har du definisjonen av den deriverte. Så skal du[bruke den til å beskrive hvordan du kan bruke definisjonen til å finne en tilnærmet verdi av det	
250 S	[og hva gjør jeg med den?	
251 L1	Så I all hovedsak slik som jeg tolker oppgaven da er å forklare litt hvordan du kan bruke den funksjonen da	
252 S	Ja, det er greit, problemet er jo da at jeg aner ikke. (L1: nei) jeg har null peiling.	
253 L1	Okei, da er jo det en god start da… Den lim greia den ignorerer man fra starten av. (S: ja) så er det da, hva betyr f(x+delta x)? Det er jo, det er kanskje den tingen som er vanligst å gjøre feil når man arbeider med definisjonen av den deriverte. Men hvis du skal finne f(2), hva gjør du da for noe? Hvis du har funksjonen din f hvis du ser på oppgaven igjen	
254 S	Du gjør x-en til 2 da	
255 L1	Ja bare sånn generelt hvis du skal finne verdien av funksjonen I x=2, så da bytter du ut alle x-ene dine med 2 (S: ja) og regner ut. (S: ja) og hvis du skal finne f(8) så bytter du ut alle x-ene dine med 8 (S: ja). Og det er akkurat det samme du gjør når du skal finne f(x+delta x), alle plassene der det står x skal du da bytte ut med x + delta x (S: ja)	
256 L1	Så det kan hende du får noen eksempler på hvordan du kan, deeer kanskje. Her bruker de definisjonen av den deriverte til å derivere en andregradsfunksjon ser det ut som (S: ja). Så da kan du se på det eksemplet der og se hvordan de har brukt den, og se om du kan bruke en liknende forklaring til å løse oppgaven din.	
257 S	ja	
258 L1	Og prøv å henge med på regnestegene. Og det viktigste her er å prøve å forstå det (S: ja) det er det absolutt viktigste. (S: alright) Og om du fortsatt trenger hjelp etter å ha prøvd på dette I fem til ti minutter så kan du rekke opp handa så kan jeg hjelpe deg igjen.	
259 L1	{Går til E} Hei	28:35:00
260 E	Hei, jeg vet ikke hvordan jeg skal gjøre den her oppgaven.	
261 L1	hmm, nei, er du på a eller b?	
262 E	b	

263 L1	B, ja okei I neste del av oppgaven skal vi ta utgangspunkt i tabellen.. Lag tabellen ovenfor i et regneark. Bruk formler der du kan. Okei, så da skal du ehm, har du laget det I regneark?	
264 E	Nei, eller, jeg holder på men jeg kom ikke langt. {Viser det som er gjort I regneark, dette er en kolonne med tall fra 0 til 14}	
265 L1	Mh, Det som går an er at du starter kanskje med å, for den ber deg om å bruke formler der du kan, altså bruke de innebygde funksjonene som er I excel.. (E: ja) Er vel det som blir spurt om her. Så da kan det være start med å kanskje for eksempel bare skrive.. skrive, em... Grunninformasjonen inn i excel. Så da er det, hva er det viktigste, eller hva e den informasjonen som ikke er regnet ut her? for eksempel. (E: ja) Så hvis du ser på den oppgaven igjen (E: okei), så ser du okei, eh, Delta x den er bare et tall så den er 1. f(x) får du vite er minus 10 (E: mhm).. ehm, f(x+delta x) er minus 18. hm skal vi se, ojja for det er i punktet, ja for det er i punktet f.. ehm. det er i punktet x=5 du skal finne den tilnærmede verdien. Så du skal liksom prøve å bruke regnearket her til å regne ut den deriverte, i mer og mer nøyaktig da. er vel det du skal gjøre	
266 E	Ja okei.	
267 L1	Så det går an til å først prøve å bare kopiere akkurat det som står I det regnearket, skrive akkurat det som står I regnearket (E: ja) I oppgaven, du kan prøve å starte med å skrive det. Og så skal du prøve å bruke den informasjonen til å fylle ut resten av cellene der da.	
268 E	{E prøver å skrive av exeltabellen} Men hvordan får jeg det deriv, nei x, (L1: ehm), tegnet	31:40:00
269 L1	Det kan du kanskje finne med å gå inn på symboler I excel.. Sett inn symbol eller noe sånt… Sett inn der oppe eller… ehm… her kanskje, sett inn har de noen symboler der? … ja symboler der ja { peker på symboler I høyre hjørne av exceldokumentet}, så er det på den helt til høyre der da som det står symbol på.. skal vi se på den der, hvis du trykker på den.	
270 E	ojja okei	
271 L1	Så inn på symboler, og den heter jo delta da, går det an å søke på noe vis? Eeh, det er ikke på et valuttasymbol	
272 E	Er ikke det…	
273 L1	Skal vi se, gresk kanskje, for det er jo et gresk symbol. Og så kan du bla til du finner en delta for eksempel. Ikke at det er det viktigste å gjøre da {E blar gjennom symbolene}	
274 L1	Der har du en delta	
275 E	åh der ja.	
276 L1	Så fra nå av alle gangene du må bruke en delta kan du kopiere den og sette den inn der du trenger den, eller noe sånt da, så slipper du å styre med det der da	

277 E	Eh, hvordan gjør jeg det?
278 L1	Den er lagt inn der nå så det er der den skal være
279 E	Jeg må kopiere det her… {E skriver inn kolonneteksten fra tabellen} så delta x (L1: mhm)… nei ikke det.. F […]
280 E	Sånn blir det (L1: mmm) nei ikke det x.
281 L1	x blir det ja. Og da er det, her har du funksjonen din (E: ja). Eh og du er ute etter funksjonen I punktet x=5, eh den deriverte I punktet x=5, så da kan du, hvis du regner ut f(x) først, det er på en måte den enkleste, så alle plassene det står x I den der {peker på funksjonen} skal byttes ut med 5 da. Så du kan enten regne den ut for hånd, siden på alle linjene her vil den være det samme.. ehm, for hva vil f(5) være for noe?
282 E	Eh, minus 5, altså, skal vi se, 10.. 15.
283 L1	Skal vi se, 5 I andre det er 5 ganger 5
284 E	Ja altså minus 5 I andre
285 L1	Så da har du minus 25 pluss 3 ganger 5, så minus 25 pluss 15
286 E	minus 10
287 L1	minus 10 ja… så den blir jo å være det uansett for du ser I punktet x=5.
288 E	Eh, hvordan var det jeg løste den?
289 L1	Hva var det du prøver å gjøre for noe?
290 E	Låse altså sånn at jeg kan kopiere det
291 L1	Du kan sette sånn dollartegn (E: ja […])… Hvis du først bare skriver minus 10 der, og så på neste linje så kan du trykke er lik, så kan du trykke på cellen over og så kan du, må du trykke mellom D og 5 da. Og så kan du sette et dollartegn der. Og da er det Alt Gr og 4. og så kan du ta tak i det hjørnet der og bare trekke den nedover
292 L1	mhm, og så var det den delta x-en, og det som skjer her da, hva er mønstret som den endrer seg med?
293 E	Den minsker..
294 L1	altså du har 1, og så har du
295 E	minus 0.1, og så minus 0.01 (L1: mhm) og så blir det sikkert 0.001
296 L1	Så det fortsetter med samme mønster, du har 1 så har du 0.1, 0.01 og så sånn
297 E	Skal jeg bare skrive det inn?
298 L1	Så, du kan starte med å skrive 1 der. Og så hvis du skal gjøre det her litt enklere for deg selv.. 0.1.. Hvis du tar 1, hva må du delte det på for å få 0.1?
299 E	hva, 0.1?
300 L1	Hvis du har 1, hva må du dele 1 på for å få 0.1?
301 E	10

302 L1	10 ja… så hvis du tar cellen over, altså 1, og deler den på 10 {E regner I cellen der det skal stå 1}, altså, hvis du skriver 1 der først. Hmm, 1 der, også går du ned på cellen under… og så tar du, eh, istede for å skrive 1 der så trykker du på cellen over	
303 E	og så låser jeg det?	
304 L1	Og så løser du bare det ja, bare trykk enter {E begynner å låse cellen} Eh, nei ikke lås denne, bare la den være sånn så skal du få se hvorfor etterpå. Sånn, og så kan du prøve å gjøre akkurat det samme på cellen under. At du tar cellen over, delt på 10, mhm, og så markerer du de to cellene, og så trekker du nedover. {E flytter på cellene} ojj der var det en liten bom.. {E trekker nedover} der ja.	
305 L1	Yes det ser rett ut, og det der betyr jo bare at det er mange desimaltall, så du kan jo, hvis du markerer alle de der igjen så kan du gjøre at det blir flere desimaltall med å trykke på hjem der oppe og så markerer du alle cellene .	
306 E	Alt, nei vent	
307 L1	Derfra og ned ja og så kan du bruke de tastene der, under der som standard er {peker på desimalplasstastene}	
308 E	Hvor hen?	
309 L1	De der to de små.	
310 E	Der?	
311 L1	Ja, hvis du trykker på den eller den andre.. Okei da er det den ved siden av, den på høyre siden der, eh nei den blir det ja, trykk på den der mange ganger, den til venstre mange ganger, den til venstre, den ja trykk på den mange ganger. Sånn så blir det ihvertfall greit sånnsett (E: ja)	
312 L1	Og nå kan du delta x her og det at x er lik, eh, 5 til å regne ut de andre her da. Så prøv å regn ut den delta f-en der, prøv å finn ut hvordan du kan gjøre det først og så sammenlikner du det svaret du får med det som er I regnearket I oppgaven (E: okei) og så ser du om du får det til å gi mening (E: ja). Og da regner du ut den f(x+deltax)-en der på samme måte som du gjorde med x-en, bare at du nå istedefor x bruker x+delta x da. Og da har du delta x-en din her og så er x 5 (E: yes). Okei, prøv det først.	
313 L1	{Går til B} yes	40:30:00
314 B	Eh, jeg forstår ikke oppgave 9	
315 L1	Oppgave 9, nei, er det a-oppgaven du tenker på?	
316 B	Ja	
317 L1	ja, eh, hva er det oppgaven spør om?	
318 B	ehm, vi har fått en funksjon her, den har jeg satt inn her, og så står det Beskriv kort hvordan vi ut fra definisjonen til den deriverte kan finne en tilnærma verdi for f derivert av 5.	

319	L1	Mhm, og da, okei oppgaveteksten sier at du skal beskrive kort hvordan du kan bruke definisjonen av den deriverte til å finne en tilnærmet verdi (B: mhm) og.. Husker du hva definisjonen av den deriverte var?
320	B	av, ehm, den her?
321	L1	Hva definisjonen av den deriverte var?
322	B	Ojja, det er jo hvordan grafen endrer seg.
323	L1	Hva sa du for noe?
324	B	Hvordan grafen endrer seg
325	L1	Ja, det er, det er, det stemmer, den deriverte er hvordan en graf endrer seg (B:ja), men definisjonen av den deriverte er jo den der f'(x)=lim delta x går mot uendelig, den store som han L2 tegnet på tavla, husker du den (B: aaah). Hvis du googler definisjonen av den deriverte
326	B	Okei
327	B	{B googler} den her?
328	L1	Ja det som står på øverste linje der blir jo det da. Så det blir den der som er definisjonen av den deriverte, bortsett fra at de har glemt å skrive den grenseverdien der da, den lim delta x går mot 0 skal være på andre siden av likhetstegnet også. Så det er definisjonen av den deriverte. Og husker du da dere holdt på med de Pythonprogrammene deres, så regnet dere jo ut, eh, tilnærmingsverdier for definisjon.. nei for den deriverte ikke sant?
329	B	Nei, jeg har ikke, ikke peiling på den kode greia
330	L1	Nei, var du her den timen da vi gjennomgikk ganske mange oppgaver med, der dere skulle skrive av koder?
331	B	Ja
332	L1	Ja, har du noen av de kodene?
333	B	Nei
334	L1	nei, for der har du brukt definisjonen av den deriverte til å finne en tilnærmingsverdi… og… da hadde du definisjonen av den deriverte og så valgte du en liten, en veldig liten verdi for delta x, (B: mhm) og så brukte du et pythonprogram for å regne ut, okei hvis jeg sier at delta x er 0,1 og jeg vil ha i x=5, hva vil da, ut ifra definisjonen av den deriverte, hvis jeg bare setter det inn i definisjonen der da hvis jeg sier at delta x er 0.1 og x er 5 og så regner jeg det ut, så får jeg for eksempel 8.2. Okei men hvis jeg velger en mindre delta x da (B: mhm) for eksempel 0,01, ojja men da fikk jeg 8,15 det var jo litt nærmere, så med å velge en mindre og mindre delta x så får du en mer og mer nøyaktig verdi for
335	B	Ojja sånn ja
336	L1	Så du kan se på, dere har en del eksempler på de pythonkodene der I boken deres tror jeg (B: okei) så du kan jo se på dem og se om du finner noen likhet I det
337	B	Takk.

TRANSKRIBSJON L2 24.02

Ytring nr.	Navn	Ytring	Tidspunkt
338	U	Oppgave 5… eh… Jeg forstår det første liksom at jeg skal sette de punktene inn I eh… inn I eh formelene, men lengre enn det kommer jeg ikke	1,35
339	L2	Ehm, ja for du har den {peker på skjermen til eleven}, også har du den, også har du den (U: ja) [så du har de tre likningene du trenger.	
340	U	Skal jeg sette dem inn I CAS, er det det jeg skal gjøre?	
341	L2	Kan du prøve.	
342	U	hmmmhmmhmm, oppgave.. Hvilken oppgave er dette her?	
343	L2	Eh, 5 er det ikke det?	
344	U	Er det 5?... Uh, ja det er oppgave 5. Jeg har jo laget denne her allerede... Det her? Har jeg svart nå?	
345	L2	Ja	
346	U	det, det er så enkelt?	
347	L2	Det er svaret	
348	U	Ok, da trenger jeg ikke hjelp	
349	L2	Hvis du går tilbake og så sjekker du bare kjapt, eh [U: eh, det her?].. Til oppgaven tenkte jeg. Der ja. Prøv å isolere her nå. Hva er det oppgaven ber deg om å gjøre?	
350	U	Ja, jeg skal finne verdien av a,b og c.	
351	L2	Ja, og hvis du switcher til eh CASen din der, så ser du at du har (U: ojja!) bestemt a,b og c. Så det som såklart ville gått an å gjort, og det har du jo gjort ser jeg til og med, er jo å sette det inn I en funksjon og sjekke at det stemmer. (U: mhm) Da har du jo gjort alt som kreves. {L2 røyser seg og snur seg}	
352	Q		04:00
353	M	[…] vis han tabellen vi har laget.	
354	L2	ja, ehm, skal vi se. Vel, det første vi blir spurt om er hva forteller tabellen oss, hva viser tabellen oss.	
355	M	Den viser oss antall, delta x, fx, f(x)+delta x, (Q: ja vi kan jo si det sånn) {alle ler}. Ja det er jo det den viser oss	
356	L2	Ja det er jo en beskrivelse av hva du ser (M og Q: ja), men hva forteller det oss?	
357	M	Det forteller oss at når antallet er 0 {Ler}	
358	Q	Nå leser du bare opp fra tingen.	
359	L2	Ja, vi er litt opptatt av, eller vi er ute etter nå, hva er det som ligger bakom bare akkurat det vi kan se. Hva er betydningen av dette her? Hva de forskjellige kolonnene er for noenting. Det leste du jo opp I stad, men hvilken av dem er det som er mest interessant for oss her kanskje?	
360	M	Det er vel delta f delt på delta x	
361	L2	Ja. Hva er det som er interessant med den?	
362	M	{Ler} Deh, eh, den endrer seg der.	

363 L2	Ja.. Hva mener du med ender seg(?), for for meg ser det ut som den endrer seg hele tiden.
364 M	Ja, den, eh, den blir mer positiv til dit også begynner den å bli mer negativ igjen. (L2: det var vel faktisk til dit da). Jaja, der ja, ja.
365 L2	Ja, men men, der er den jo mer mer positiv igjen (M: mhm). Sånn at, hva ville vi forventet at skulle skje her egentlig?
366 M	At det bare blir mer negativ.
367 L2	Ja, egentlig ikke mer negativ, men eller det som vi sir at den nærmer seg et tall. (M: ja) og da er spørsmålet hvilket tall er det den skal nærme seg.
368 M	7.. Nei minus 7.
369 L2	Ja, og det kan vi jo egentlig regne ut I fra, ja det har du gjort der (Q:mhm), ja. Og det vi ser her er er at når vi øker antall nuller forran delta x her, altså når vi gjør den delta x-en mindre (M:mhm) , så ser vi at vi har det som på veldig fint heter en konvergens her at den størrelsen her den nærmer seg 7 som vi vil forvente fordi at når den delta x-en er 1 så er den delta f delt på delta x 8, så den er ganske unøyaktig, (M: så_)også ser vi at den blir ganske fort mer og mer nøyaktig (M:så. så vi skal_). Det er det vel helt til dit egentlig {L2: peker til punkt i tabellen} er den ikke det? {Q:joo}. Fordi den går ned til 7 komma masse nuller også 11. (M: også går den opp igjen). også går den til 7 også like mange nuller også 22. Det betyr at når vi kommer dit da {peker til samme punkt i tabellen} altså vi lager delta x mindre enn den der {peker} så skjer det et eller annet litt sånn spooky her. Vi får ikke skikkelig fæle verdier her, det fikk jeg på min egen, da fikk jeg den siste her til å bli 0 sånn for sikkerhets skyld.

370 L2 Men det som skjer her.. Helt konkret, nå får dere svaret fra meg da, men det som skjer her helt konkret er at når delta x blir mindre enn den der, altså nærmere null (Q: mhm} enn den der så klarer ikke datamaskinen å håndtere det tallet (Q: ja) som en størrelse (Q: skriv det ned, skriv det ned). Så det betyr at vi kan se det at datamaskiner har en grense for hvor små tall de kan operere med, slik at hvis vi skulle gjøre det her helt nøyaktig så må vi være spesifik på hvilken type maskin vi bruker og vi må programmere den på en spesiell måte slik at vi tar høyde for det her (Q: ja). For det som vil skje er at du vil få.. De fleste verdiene du vil få her vil fortsette å gjøre den delta x-en verre og verre. Så de fleste verdiene vil fortsatt kunne være rundt omkring 7, men du får ikke det der at den nærmer seg 7. Så hvis vi skal bruke denne tabellen til å mene noe om hva den deriverte er når x er 5, så ville vi sett at den nærmer seg til en verdi, og den er jo minus 7 i dette tilfellet (Q: mhm). Så hvis vi ikke har muligheten til å bruke derivasjonsreglene til å finne akkurat den verdien så er det jo sånn der vi er nødt til å gjøre det (Q: ja). Og da er det jo litt kjekt om vi vet hva som, altså hva kan vi stole på og hva kan vi ikke.

371 L2 Og da viser det seg, for å peke litt langt fram da, når vi har forskjellige typer funksjoner som da har grafer som er litt mer sånn, ja, litt rarere enn vi er vant til (Q: mhm), og da kanskje til og med definert på helt andre måter, så er det sånn at, eh, de her deriverte oppfører seg mindre og mindre jevnt og smidig, sånn at vi må være mer og mer nøye på forholdene vi deriverer med (Q: okei). Så dette her er egentlig bare en måte å vise dere at når vi regner for hånd så lar vi denne {peker på delta x} bli i praksis 0 (Q: ja), så nær 0 det går an å komme (Q: mhm), mens hvis du gjør det med en datamaskin så feiler den (Q: okei). ja. Så tabellen sier oss at vi ikke kan ha delta x så liten som vi vil.

372 U L2 (L2: ja). På oppgave 6, mmm, jeg vet ikke helt hva jeg skal starte med. 09:25

373 L2 Nei, det var den vi så på I slutten av mandagstimen.

374 U Ja, jeg skrev jo litt ned, men jeg skjønte ikke

375 L2	Nei, for det som er der, vi kan starte med å se på hva er det vi vet. (U: ja, hm) Og da vet vi jo informasjonen I det første avsnittet. (U:mhm) også punkt a tegn en skisse det kan du jo gjøre, så det som er interessant er oppgave b. Og da kan vi ta utgangspunkt i den.. funksjonen som står der også si hvordan kan jeg knytte inn den infoen jeg har for å lage den k-en? (U: mhm). Så en ting som du såklart også må ta med deg er at hvis du ikke klarer å gjøre oppgave b så klarer du å gjøre oppgave c og d fordi at du har.. du har den {peker på funksjonen k(x)}. men det er på en måte den {oppgave b} som er den mest interessante for, for oss når vi skal se om dere har skjønt det her eller ikke. (U: mhm). Og det første vi legger merke til er at her er det en beskrivelse av arealet, nei unnskyld, volumet (U: ja) og siden vi skal ha ehm.. noe som bruker arealet så må vi vite det også. så hvis vi setter opp uttrykk for eh volumet og for arealet, eller overflaten om du vil, så kan vi se der og se hva er det som ikke passer her? Hvordan kan jeg vri på dette her her for å eh... for å lage en vei inn i det der da... (U:mhm). og det var jo det som vi prøvde å gjøre på tavla sist.
376 U	Ja det var det der vi skulle gjøre ja
377 L2	og fordi at da har vi satt opp volumet og omkretsen.. Nei om_ overflaten heter det. Og det som jeg prøvde å tyne dere på som, hva er det som er I de her to {peker på uttrykk for overflate og volum} som ikke er I den {peker på funksjonen k(x)}.
378 U	Høyde
379 L2	Det er høyden. Så når jeg da puttet vi høyden inn der (U: ja), så kom vi hit. Og så var det jo da, det som du har notert her, at den ene delen her koster 3kr per kvadratcentimeter og det her er antallet kvadratcentimeter (U: mhm), og den her delen koster 5kr per kvadratcentimeter og det her er kvadratsentimeterene. Så hvis du nå ganger inn 3 med det leddet, og 5 med det leddet og forenkler, så kommer du til den (U:mhm). Så det er snakk om algebra fra nå av.
380 L2	Så jeg ser jo at jeg mistet deg et sted her (U: ja).
381 U	Men det her står jo ingen sted I den der.
382 L2	Nei, men det er en sammenknytning mellom de to som er at den der viser seg å være et sted på veien dit. Og det vet ikke du på, på nåværende tidspunkt, men spørsmålet er, hvis vi nå har troa på dette er og fortsetter med å si at vi kan gange det første leddet her med 3 og det siste leddet med 5, og ganger ut og forkorter og forenkler så godt vi kan, kommer vi til den {peker på uttrykket for k(x)} da?
383 U	Jeg vet ikke.
384 L2	nei det er jo det du må prøve.
385 U	Hva var det jeg skulle gjøre nå, si det på nytt igjen.

386 L2	Ja, se her, den ene delen her er sidekanten og den andre delen er toppen og bunnen	
387 U	Kan du markere hva som er hva her?	
388 L2	Ja for du har en pluss der så alt som er forran plussen er ett ledd og alt som er bak plussen er [et annet ledd	
389 U	[ja, og det som er her det er..	
390 L2	Det skulle være sidekanten for det er selve sylinderen (U: eh ja). Så den kom derifra. Så det er 3kr gange det antallet kvadratcentimeter (U: mhm)… mens den bakerste biten her er 2 ganger bunnen, altså det blir bunnen og lokket da (U: ja) så de må vi gange med 5.	
391 U		
392 L2	Eh, bare fortsett å ha pi som et symbol så lenge som mulig	
393 U	Ja okei	
394 L2	Så ser du at det kan jo hende at det er noe som forsvinner.. Det kan hende det er noe som blir stående. Du ser bare så godt du kan.. Til slutt.. Prøv å sett inn sånn og se om du klarer å forenkler og regne videre, fordi at du er ikke langt unna den der nå (U: nei). Det er bare at du må prøve å.. Prøv å slenge noe i det (U: ja) og se om det funker og hvis det ikke funker prøv litt annet og så roper du om hjelp igjen (U: ja okei).	
395 L2	{Beveger seg mot H, peker på E} ser deg.	14:20
396 H	Jeg forsto ikke den.	
397 L2	Ja, da må vi, vi må ha boka… for å peke på noe.	
398 H	Ja hvilken side skal jeg på?	
399 L2	ehm, vi skal I kapittel 5 ihvertfall.	
400 L2	Så er vi… For jvis du nå her så skal vi finne den gjennomsnittlige vekstfarten. Så da ser vi om det kanskje er et kapittel som heter gjennomsnittlig vekstfart.	
401 L2	Hm, der har vi et kapittel (H:mhm). Så det som er at det ikke er voldsomt mange gode eksempler her. Så det beste eksemplet er det som egentlig er I nummer d her. Der har de en funksjon, det har vi også. Her skal de finne den gjennomsnittlige vekstfarten i intervallet fra 30 til 60, og da viser de her hvordan de gjør det (H: ja okei). Som du ser her at de skriver inn de punktene der 30[...]30 og.. skal vi se vi kan ta den her... sånn (H: mhm). Så trekker de en linje gjennom punktene.. sånn som her. Og så finner vi den knappen for stigning ehm... trenger den ikke vi kan lese det ut fra funksjonsuttrykket også.. der er den. ehm, og det stigningstallet til linja det vil være den gjennomsnittlige vekstfarten i det intervallet (H: mhm). Så her har de 30 og 60, mens du har 2 og 4 (H:mhm). Du skal egentlig gjøre akkurat de samme som de gjør der (H: ja), bare med de tallene og den funksjonen istedenfor de tallene og den funksjonen	
402 H	mhm, skal jeg skrive det her.. Skal jeg skrive det her også bak det der liksom? [...]	

403 L2	eh, du må begynne å skrive den inn.. [som en funksjon	
404 H	[ja, jeg har gjort det	
405 L2	Ja, skal vi se… der har du den (H: mhm)… også	
406 H	Skal jeg skrive det ned på CAS eller skal jeg skrive det for hånd	
407 L2	Du kan skrive dem her nede I inntastingsfeltet, og bare at funksjonen din heter g	
408 H	ja, jeg skal bruke g (L2: du skal bruke g der). Stor g?	
409 L2	Liten g (H: okei) fordi at funksjonen din heter liten g (H: ja okei). Og så må du da ha 2 og 2, og 4 og 4	
410 H	Ja okei, ojja sånn ja	
411 L2	Prøv det og se om det funker.	
412 L2	Hva sier du?	17:20
413 E	Nei det er ingen vits jeg trenger ikke noe hjelp	
414 L2	Hva sa du nå?	
415 E	Nei det er ingen vits	
416 L2	Ojja du fant ut av det? (E: ja)	
417 E	Hvis det her er rett da!	
418 L2	Ja, ser veldig rett ut.	
419 B	Eh, L2?	20:30
420 L2	Ja, lite øyeblikk	
421 L2	Sånn, hva lurte dere på	21:10
422 B	På oppgave b… Det står avgjør om de stasjonære punktene på grafen til f er toppunkter eller bunnpunkter. (L2: ja). Det her er jo de punktene ikke sant? (L2: eeh, ja). Og, det… det er på minus 2 og minus 3 ikke sant? (L2: ja). Og.. Så det er enten bunnpunkt eller toppunkt? (L2: ja). begge to?	
423 L2	Eh ja. De kunne ha vært noe annet.. Hvis den deriverte her ikke hadde skiftet fortegn (B: ja). Altså hvis den hadde vært negativ og så blitt null og så fortsatt å være negativ, eller tilsvarende positiv og så null og så positiv. Da ville den selve f-funksjonen da som vi ser nå den ville hatt det vi kaller et terassepunkt (B: ja). Så det vi gjerne ser på her nå er at, som du har påpekt, der er et stasjonært punkt og der er et stasjonært punkt (B: ja), og da kan vi jo spørre oss hva som skjer med fortegnet til den deriverte her?	
424 B	Den blir minus	
425 L2	Ja den går fra positiv til negativ (B: ja). Så hva betyr det for f?	
426 B	At den og går fra positiv til negativ.	
427 L2	Ja, altså [...] det her er jo den deriverte (B: mhm), så selve f da, hva vil den gjøre når den deriverte skifter fra positiv til negativ?	
428 B	Den avtar? Eller den blir negativ.	

429 L2	Altså den vil begynne å synke (B: ja) når den deriverte er negativ, og når den er positiv så.. Så stiger den jo, så selve f-funksjonen den stiger dit og så synker den deretter og hvis den først stiger og så synker så bør det være en topp.	
430 B	Ja	
431 L2	Og så kan vi spørre oss hvordan kan vi bruke det til å avgjøre om.. Hva som skjer her?	
432 B	Okei så den her er.. Der? Så den er jo først negativ (L2: ja) og så blir den positiv (L2: ja) så man kan ha et bunnpunkt? (L2: ja)	
433 L2	Fordi f vil først synke (B: mhm) og så vil den stige (B: ja), så da må den være på en [bunn et sted	
434 B	[Sånn her på en måte? (L2: ja)	
435 L2	Så da har du jo bestemt om det er et toppunkt eller et bunnpunkt (B: ja)	
436 Q	Okei, to ting (L2: mhm)… jeg har ingen anelse på hvordan man.. Hvordan man skal fortsette med, med det her, og hvordan man skal utvide programmet.	23:50
437 L2	Ja, eh, Hva er det du blir bedt om å gjøre nå?	
438 Q	Jeg blir bedt om å angi c og delta x der c er x-verdien der den deriverte ønskes beregnes og delta x angir inkrementet	
439 L2	Ja, inkrementet er altså bare delta x, det er den (Q: ja)… så det du egentlig skal gjøre da med programmet er at du skal få brukeren til å spytte inn c og delta x (Q: okei), og så skal programmet regne ut den deriverte der da (Q: okei). Så hvis du switcher til Spyder så ser du at her har den jo brukt ehm (Q: 5,01), 5,01. Det forteller oss jo sånnsett at x verdien som blir brukt her er 5 (Q: ja), og delta x verdien som blir brukt er 0,01 (Q: mhm). Så det du trenger å gjøre er å få en bruker, altså be brukeren om å bestemme hva som skal være x-verdien (Q: ja) og hva som skal være delta x, og så må du da sette inn det han finner, eller det som da tastes inn der.	
440 L2	Så.. Hvis jeg kan lede deg litt på vei (Q: ja). Hvis vi bruker en verdi her, her er en variabel som heter d, nei c, (Q: mhm) og en som vi kaller for bare dx	
441 Q	Er dx delta x?	
442 L2	Ja det får funke, det betyr egentlig noe litt annerledes men det får funke her. Ehm så det vil jo si at den her.. Det sa vi jo istad var 5+ delta x da (Q og L: ja), så her burde den kanskje være c pluss delta x eller pluss dx (Q: ja). Og så spør vi, hva som skal stå her inni montro?	
443 Q	c	
444 L2	Og hva skal vi dele på?	
445 Q	Delta x	
446 L2	mhm	
447 Q	Og så må du skrive at c er 5 og delta x er 0.01	

448 L2	Ja hvis du gjør det så kjører den, men den ville at du skal spørre brukeren	
449 Q	Ojja så jeg må legge sån der eh.. Nå husker jeg ikke.. Sånn print? Må jeg skrive sånn print?	
450 L2	Eh, vi bruker den som heter input (Q: input), ja for å få eh.. (Q: input, sånn) sånn ja	
451 Q	Og så skriver jeg bare mellomrom	
452 L2	Ikke mellomrom	
453 Q	Ikke mellomrom	
454 L2	Og så får du inni her så skriver du den beskjeden du har lyst til å gi til brukeren (Q: okei)	
455 L2	Men det er litt viktig at vi passer på en ting. Fordi at det som brukeren skriver inn kommer I utgangspunktet til å være en tekst (Q og L: mhm ja), altså det vi kaller en streng. Så for å fikse det så kan vi si at vi vil at det skal, det som kommer inn her, det skal være.. et flyttall, en float	
456 Q	Så et tall liksom	
457 L2	Jeg er ikke 100% sikker på at det her er riktig så vi prøver, vi gjør det som alle gode programmerere gjør, vi prøver oss fram	
458 Q	Da må vi kanskje legge på en ekstra parentes, du har to på slutten og på starten [...]	
459 L2	Hvis vi prøver å gjøre sånn så sier vi at den skal være 5 og den skal være 0.01, så ja da funker det	
460 Q	Så det er riktig det vi har gjort da?	
461 L2	Ja faktisk, så nå har jeg gjort nesten alt for deg.	
462 Q	Jeg må bare bytte, eller ikke skrive [...] et spørsmål	
463 L2	Ja du må justere på den der (Q: ja)	
464 Q	Og det, det er liksom hele oppgaven?	
465 L2	Ja det er ikke mer å gjøre (Q: okei)	
466 U	L2? (L2: ja) Jeg forstår ingenting. Jeg har prøvdt å gjøre den greia […] men det går ikke.	27:40:00
467 L2	Ehm skal vi se, hvor kommer alle tre-tallene fra?	
468 U	Jeg trodde vi skulle gange det med 3?	
469 L2	Ja, men eh, sånn som her (U: ja) her står det 2 gange pi gange h (U: ja). Hvis du skal gange det med 3, ville du ha ganget hver av dem med 3?	
470 U	ja	
471 L2	Så hvis jeg gir deg regnestykket her {skriver regnestykket I boka} Sånn (U: ja)… Klarer du å regne ut hva det er for noen ting?	
472 U	6, eh… 24.. Nei det er litt mye tall her	
473 L2	Ta 2 ganger 5 først det blir 10 (U: ja) og så 3 ganger 4 blir 12	
474 U	Ja, 12 ganger 10 blir 120 (L2: blir 120)	

475 L2	Og så sier jeg, hvis vi skal gange det der med 3 da… (U: ja) Tror du vi helst bør gjøre sånn, eller ikke ta 3 la oss gange med 7. Skal vi gjøre sånn {L2 skriver 2*3*4*5*7} eller skal vi gjøre sånn {L2 skriver 2*7*3*7*4*7*5*7}	
476 U	Hmm I midten?	
477 L2	Du ville gått for den ja (U: ja). Det ville jeg også gjort. For her har vi jo strengt tatt ikke ganget med 7 vi har ganget med 7 ganger 7 ganger 7 ganger 7 (U: hm). Og derfor får du trøbbel her. Den trenger ikke være der, den trenger ikke være der, den trenger ikke være der, den trenger ikke være der og den trenger ikke være der (U: så). og... den trenger ikke være der og den trenger ikke være der, men de skulle uansett ha vært ganget med 5 da fordi at de er den der (U: mhm)	
478 L2	Så hvis du går til angrep med hviskelæret littegran. Og tar bort 3-tallene {U hvisker bort}	29:37:00
479 L2	Sånn ja, så har vi, nå har du den, 6 ganger, ja da er den sånn som den trenger å være {L2 peker på første ledd av det som skal bli k(x)}. Også må du her ha, du hadde en 2-er der I utgangspunktet, den skal ganges med 5, det burde gi deg	
480 U	10	
481 L2	Ja… også bare sett 10 forran der. Så nå har jeg litt lysst til å peke på den igjen. Ser du nå noe felles med den og den {peker mellom siste ledd I k(x) og uttrykket}?	
482 U	mmmmh, ja… (L2: mhm) eller nei egentlig ikke. (L2: ikke?) jeg skjønner jo at.. Nei jeg vet ikke skjønner ikke.	
483 L2	Det der {peker} (U: ja?) finner du ikke det der? (U: ja) for det gjør nå jeg (U: ja). Ja. Så da blir spørsmålet hva med resten her da? Er det noe måte vi kan få det, det der {peker} til å bli det der?	
484 U	eeeh, jeg vet ikke hvordan jeg skal få bort den der x^2.	
485 L2	eeh, nei det kan vi vel se. Her er det 6 ganger pi ganger x ganger 1000 delt på pi gange x^2, eller delt på både pi og x^2 (U: mhm). Og hva er pi delt på pi?	
486 U	0.. [Eller 1	
487 L2	[nnn.. Ikke helt. 1 ja. Så bort med de. (U: ja). Hva er x delt på x^2?	
488 U	x.. Jeg vet ikke. [Er det ikke x?	
489 L2	[mmm.. Sånn {viser til noe på arket} [blir det x?	
490 U	[Jeg vet ikke	
491 L2	Nei tenk at vi kan.. Hvis jeg sir eh, 2 delt på 4 blir jo ikke 2	
492 U	Så det blir en halv x. [en halv x^2	
493 L2	[mmmm.. Jeg skjønner hvordan du tenker, men det blir en x-del. På samme måte som at om vi forkorter den {peker på 2/4} så kan vi dele på 2 oppe og nede (U: ja). Samme kan vi gjøre her, vi kan dele på x oppe og nede (U: ja). Så x delt på x blir bare 1. og x^2 delt på x, det blir x	
494 U	Ja, så jeg hadde riktig	

495 L2	Nei for du sa det ble x	
496 U	ja, var det ikke x?	
497 L2	Ja men det der er jo 1 delt på x	
498 U	Så det blir 1 delt på x	
499 L2	Det er ganske stor forskjell på om du har 2 eller en halv. Så vi kan gjøre sånn (U: mhm). Så når vi da ganger 6 med 1000 så får du den der	
500 U	Ja nå skjønner jeg	
501 L2	Ja nå har du ihvertfall kommet fram til den samme. (U: ja). Så det som er at… eh c her, kan du gjøre selv om du ikke klarte å finne denne her selv, du har jo fått den her, så den kan du jo dytte inn I geogebra og så finner du prisen oppover og radiusen bortover (U: ja). Så det som oppgaven spør om her er kostnaden i oppgave b, radiusen i oppgave c og høyden i boksen, høyden finner du med hjelp av den der {peker}. Så du har alle verktøyene tilgjengelig nå (U: ja) så nå er det bare å dytte den inn i geogebra og finne det du trenger. bare {lager anførselstegn}. Prøv det (U: ja) og se hvordan det går.	
502 N	ja, (L2: sí) kunne du kanskje ha sett over dette her? Om at jeg har gjort noen sånne her småfeil innimellom… for jeg ble bare litt sånn usikker.	
503 L2	Se, jeg som leser ville kanskje hatt en kobling mellom der, og der kan du rett og slett skrive at det gir, eller noe sånt	
504 N	Sånn bare.. Hva er det som skjer med pcen min, ser du det? Den får sånn der (L2: ja) jeg vet ikke hvorfor.	
505 L2	Har du veldig mange programmer oppe, lenge siden du har startet den på nytt?	
506 N	Jeg startet på nytt I dag tidlig så.. Det har vært veldig mye sånn	
507 L2	Skal vi.. Vi kan prøve en ting som, du har lagret dokumentet sant? (N: ja)	
508 N	Kan bare dobbeltsjekke	
509 L2	Hvis du bare gjør sånn her nå så ser vi om… ser vi om vi klarer å lage noe.. Nei jeg klarte å få den til å blinke litt, men det ser ikke ut som det er noe kontaktfeil.	
510 L2	men okei så har du overflaten (N: ja), også har du_	34:35:00
511 N	Det er jo det det koster	
512 L2	Ja… også har du den ja	
513 N	[...]	
514 N	Det jeg var litt usikker på den der geogebragreia der jeg skrev alt det her om det ble rett	
515 L2	Ja jeg tror det blir riktig (N: mhm). Det lignet veldig på det jeg hadde ihvertfall (N: ja)	
516 L2	Ja, jeg tror jo dere skjønte det (N: ja) vesentligste med den oppgaven da	
517 N	For det er det her, sånn her, at jeg må gå inn på det på en måte hver gang… så må jeg trykke der	
518 L2	Hva for noe?	

519 N	Men så var det litt sånn, jeg er jo ferdig med alle oppgavene (L2: ja), men er det slik at du ønsker at jeg skal skrive enda mer (L2: Nei nei nei) på hver besvarelse her. For det var litt sånn, her har jeg prøvd så godt jeg kan å vurdere noen greier, men det kan jeg se litt mer på	
520 L2	Ja, men jeg tenker nå at det næremer seg at du kan levere den. Jeg skal lage en sånn innleveringsmappe nå, ja om jeg skal gjøre de nå eller om jeg skal gjøre det imorgen	
521 N	Men det her ble rett, for jeg lagde en ny sånn	
522 L2	Den ser veldig rett ut ja	
523 N	Hvordan kan jeg få det til å se fint ut, vil du at jeg skal ha oppgave 8 på en side eller?	
524 L2	Nei jeg syns det ser greit ut sånn som det der, det viktigste for meg er at det er lett å skjønne hele tiden hva jeg leser (N: ja) sånn at jeg slipper å lure på leser jeg nå oppgave 8 eller leser jeg oppgave 9 nå? Ehm, så det trenger ikke å være på hver enkelt side (N: nei okei)	
525 N	For jeg får bare litt sånn panikk når det er sånn her.	
526 L2	Nei det er helt lov. {ler} sånn for det første så er det jo, det her, det er jo ikke sånn at dere har kniven på strupen, at hvis ikke det her er bra så går det skikkelig dårlig med dere, men også det her med å føre de her tingene digitalt er dere ikke vant med (N: nei), så jeg tror du kan ta det helt mer ro	
527 N	Men får vi karakter? Eller vil du sette høy, middels	
528 L2	I utgangspunktet var det snakk om sånn godkjent/ikke godkjent (N: ja) og så kommer det mer vurdering på dette her senere (N: mhm)	
529 S	Jeg er faktisk på bærtur. For se, se hva jeg har svart på a… og så så jeg hva de andre har svart og jeg har ikke svart I nærheten av.. Jeg vet ikke hva jeg skal gjøre (L2: på a). Har du lyst til å hjelpe meg her?	36:50:00
530 L2	Skal vi se, hva har du skrevet her… Ja for ehm.. Det som definisjonen av den deriverte sier (S: ja), ehm skal vise det her {L2: går til tavla}	
531 P	Men jeg forstår ikke den der, den definisjonen av den deriverte	
532 L2	Jeg er midt I en forklaring så så det kan hende du forstår det […] Hvis du er veldig […] på at du ikke forstår det så kommer du deg ingen vei så ta det med ro, jeg skal prøve å forklare litt nå.	
533 L2	Hvis vi bruker den der {peker på definisjonen av den deriverte}, altså med en funksjon, så får vi den nøyaktige deriverte. Og det oppgaven ber oss om å gjøre er jo å finne en tilnærmet verdi	
534 S	Hva er tilnæremt verdi?	
535 L2	Circa verdi	
536 S	ja, på hva?	
537 L2	På den deriverte	

538 S	Hva er. Hvordan finner vi.. Det er jo en graf. Nei det er ikke en graf
539 L2	Altså når vi vil finne […] {går til pcen til S og leser opp oppgaven}
540 L2	Så hvis vi ikke skal bruke den her til å finne en nøyaktig derivert, men en tilnærmet derivert, ehm så glemmer vi den {viser til grenseverdien}. Og det vi vet er at vi skal ha x=5. Og her kommer det som kanskje kan være nøkkelen til at han P skal klare å knytte en forståelse opp imot det den definisjonen sier, fordi at nå bestemmer vi oss for en liten delta x. For det som sto forran her sa at den delta x-en skulle bli fryktelig liten. Når vi skal finne en tilnærmet verdi gidder vi ikke det. så sånn kanskje? (S: ja)
541 L2	Så det betyr at_
542 P	Men hva er delta x egentlig?
543 L2	Den lille forskjellen som blir I x retning mellom to punkter. Ehm, jeg kan illustrere det med en tegning straks etterpå for å vise at den blir sånn. Vi går for å si at, hvis vi skal finne den gjennomsnittlige vekstfarten mellom to punkter, ett punkt der for eksempel og ett punkt der for eksempel, så tar vi linja mellom de to og så tar vi stigningstallet og det er den gjennomsnittlige vekstfaten (P: ja). og det som er her, det er forskjellen i y, er høydeforskjellen, og forskjellen i x (P: ja, ja ja) er breddeforskjellen. Men når vi vil finne den deriverte som også er kjent som den momentane vekstfarten som vi også kan beskrive som bratthet eller noe slikt, vil være som om vi tar på samme måte som her bare at vi tar et til punkt som er veldig nært
544 P	aaaaah, ja.
545 L2	Og da sier jeg at hvis jeg lar det der to punktene gå nærmere og nærmere sånn at den biten I mellom dem er delta x (P: ojja). Og når jeg lar den, de punktene, komme nærmere og nærmere, det var det den der lim greia betyr.
546 P	Så den deriverte er på en måte bratthet (L2: ja) eller?
547 S	Bratthet mellom delta x?
548 L2	nei, bratthet akkurat rundt det punktet vi sjekker
549 P	Og det som er imellom dem, det er delta x (S: I know)
550 L2	Det som er mellom I den retningen {peker horisontalt}. Det som er imellom I den retninen {peker vertikalt} er delta y.
551 P	jaaaa, det gir jo mening!
552 L2	Og det som står her {viser til telleren I definisjonen} det er delta y, og det som står her er delta x, sånn nei det blir dumt å bruke de uttrykkene der. Ehm...

553 L2	Okei, prøv å henge med på det som skjer her, for det jeg skal gjøre nå jeg skal ta høyden til det punkte der, det er når jeg har gått bort til x der og så går jeg en delta x til da finner jeg høyden der med å regne ut den der, altså det er den vi kaller f(x+delta x). (S: ja). Og så skal jeg trekke fra hvis jeg går til x der opp til høyden, den finner jeg med f(x). for da har jeg forskjellen i høyde. Også skal jeg ha forskjellen i bredde, vel den ene var jeg gikk til x og så gikk jeg en delta x videre og så skal jeg trekke fra det stykket som går til x. så når jeg lar delta x bli bitte liten, da har jeg definisjonen av den deriverte
554 P	Hva er lim?
555 L2	limit, eller egentlig limes
556 S	Uansett, okei nå skjønte jeg det, nå skjønte jeg hva vi gjør, men hvordan kan jeg skrive oppgaven.
557 L2	Det som vi gjør her nå for å finne den ca. verdien (S: ja) det vil være at nå skal jeg finne f(5+0,1)
558	{elevene pakker sammen}.

B Collection of Mentimeter data

Oppgave 1	Code
Når vokser den og når synker den?	Rephrasing
Hva viser dette oss?	Visual representation
Hvordan finner man likningen til tangenten?	Procedural
Kan vi se på grafen?	Visual representation
Må vi bruke geogebra?	Procedural
Hva viser grafen?	Visual representation
Hvilke koordinater vet vi?	Base knowledge
Kan vi se/finne stigningstallet?	Procedural
Hvordan finner man et uttrykk ut av en graf?	Procedural
Hvorfor er det så mange tall?	
Hvordan løser vi dette her?	
Hvorfor gjør de dette vanskelig?	
Hvordan vet man når en figur vokser og når den avtar?	Rules/definition
Hvordan finner man likningen til en tangent?	Procedural
Må jeg gjøre dette her?	
Hva er en graf?	Rules/definition
Når vokser f(x) og når avtar f(x)?	Rephrasing
Hvordan finner man likningen til tangenten?	Procedural
Hvilket program skal jeg bruke til å tegne en skisse?	Tool
Hvordan vil f(x) se ut?	Visual representation
Hvordan tegne en skisse av grafen på PC?	Procedural
Forstår jeg?	Self evaluation
Hvordan finner vi likningen til tangenten?	Procedural
Hva betyr "avtar"?	Terminology
Hva er en tangent?	Rules/definition
Hva er likningen til tangenten?	Rephrasing
Hvordan skal skissen se ut?	visual representation
Kan man finne den nøyaktige grafen?	Problem-solving
Hvor vil grafen skjære?	VIsual representation

Oppgave 3	
Hva forteller tangentene oss, og hvordan skal vi bruke det til å finne ut hvordan uttrykk f'(x) har?	Familiarizing question, rephrasing, procedural
Hvilken type funksjon er den deriverte?	Analyzing
Hva er stigningstallet til den deriverte?	
Hvilke punkt går grafen gjennom?	base knowledge
Hvorfor bruker vi to tangenter for å finne et uttrykk f'(x)?	conceptual
Hva er en funksjon?	rules/definition
Hva er en tangent?	rules/definition
Hva er den deriverte?	rules/definition
Hvilken figur ser man på bildet?	visual representation
Hvordan regner man det ut?	procedural
Hvor begynner vi?	Procedural

Hvordan finner vi et uttrykk for den deriverte?	procedural
Hva kan vi bruke informasjonen i bildet til?	base knowledge
Hva kan jeg bruke tangentene til?	Familiarizing question
Hva har vi lært?	Self evaluation
Hvordan finner vi et uttrykk for den deriverte?	procedural
Hvordan skal vi regne dette ut?	procedural
Hvordan skal vi besvare oppgaven?	Form of answer
Oppgave 4	
Hva er derivasjonsreglene?	rules/definition
Hvordan skal jeg svare på dette?	Form of answer
Hvordan skal man bruke CAS?	procedural
Hva har jeg lært?	self evaluation
Hvordan deriverer man?	procedural
Hvordan bruker man CAS?	procedural
Hva er regnereglene for derivasjon?	rules/definition
Hvilket program bruker vi?	Tool
Hvilken ny lærdom har vi fått fra denne oppgaven?	self evaluation
Hvordan bruker man CAS?	procedural
Hvordan løser man så lang funksjon?	procedural
Hvordan deriverer man en funksjon?	procedural
Hvordan gjør man dette her, hva er formlene?	procedural, rules/definition
Hvordan bruker man CAS?	procedural
Hvordan starter man?	procedural
Hva starter man med slutten?	
Hvorfor må det være så avansert?	
Hvordan deriverer man med hjelp av regnereglene?	rules/definition
Hva er regnereglene for derivasjon?	rules/definition
Hvordan deriverer man en funksjon?	procedural
Hva har vi lært?	self evaluation
Hvilken informasjon gir oppgaven oss?	base knowledge
Hva er CAS?	Tool
Hvordan skrive inn i CAS?	procedural, tool
Hva forteller svaret i CAS oss?	Answer evaluation, tool
Hva har vi lært?	self evaluation
Hva vet vi om derivasjon?	Self evaluation
Hvordan derivere?	rules/definition
Hvordan derivere med CAS?	procedural
Hvordan derivere lange uttrykk?	rules/definition
Hva har vi lært?	self evaluation
Skjønner vi dette nå?	self evaluation
Oppgave 6b	
Hva vet vi? Hva trenger vi for å løse dette?	base knowledge, analyzing
Hva er formelen for volum?	rules/definition
Hvordan finne høyden?	procedural
Hva har vi lært?	self evaluation
Skjønner vi nå dette?	self evaluation
Hva forteller k(x) oss?	familiarizing question
Hva har jeg lært?	self evaluation
Hvordan regner man arealet til en sirkel?	procedural, rules/definition

Hva er volum?	rules/definition
Får vi bruk for dette?	Application to Real-World
Hva er det likningen forteller oss?	familiarizing question
Hvorfor er PI med i oppgaven, hvis vi ikke har jobbet med det?	
Hvordan gjør man om en formel?	procedural, rules/definition
Nøyaktig hva er det vi skal gjøre med oppgaven?	procedural or Application in Real-World
Hva spør oppgaven om?	base knowledge
Hvordan skal jeg løse dette?	procedural
Hvor skal vi starte?	procedural
Hva er formelen?	base knowledge
Hva har vi lært?	self evaluation
Hvor begynner vi?	procedural
Hvordan informasjon trenger vi?	familiarizing question
Hvordan finner vi volumet?	rules/definition

C Collection of Google sheets data

		Student 1	Student 2	Student 3	Student 4
First task	Before	Hvorfor er det en fast sammenheng?	hva skal jeg dele hva på? lengden eller graden?	kor starta æ?	hva er sinus, cosinus og tangens?
	Before	Er det en sammenheng fordi trekantene er formlike?	er det bare vanlig deling?	hva æ skal løs?	har graderne til trekantene noe å si til oppgaven?
	After	Er det bare fordi de er formlike at de har sammenheng?	er dette riktig? det virket så likt	er ikke dette det samme svaret som i forje oppgave?	hvorfor er alle svarene til hver trekant like (sånn ca.)?
	After	Hva er sinus, Cosinus og tangens og hva har de med denne oppgaven å gjøre?	skal jeg runde opp svaret? eller la det stå og deretter skrive alle desimalene?	e det samme spørsmål bare utdypet på to forskjellige måter?	har jeg lært noe og hva har jeg lært?
Second task	Before	Gjelder dette bare for vinkelrette trekanter?	blir ikke det nesten det samme som i forje oppgave? vanlig deling på lengden?	ble dette riktig?	hva menes det med motstående og hosliggende?
	Before	Hva er sammenhengen mellom svarene?	lure spørsmål?	samme svar igjen?	hvor skal jeg begynne?
	After	Er det en slik sammenheng mellom alle formlike trekanter?	dette virker litt for lett for å være t-matte, skal det virkelig gjøres sånn? har jeg misforstått?	lova æ må ha gjort noe feil, alt har samme svar. eller er det riktig?	fullførte jeg oppgaven?
	After	Blir det en slik sammenheng mellom trekanter som ikke er formlike?	va dette svaret riktig?	ka e dette egt?	fullførte jeg oppgaven riktig?

		Student 5	Student 6	Student 7	Student 8
First task	Before	hva forteller figurene oss?	Hva forteller trekantene oss?	Hvilke verdi har a, b og c i alle trekantene?	Hvor forteller oppgaven?
	Before	hva forteller formelen oss?	Hva forteller formelen oss?	Skal jeg gjøre noe mer enn å bare dele?	Hvordan skal jeg løse oppgaven?
	After	har vi funnet en sammenheng til utregningen	Har vi funnet en sammenheng mellom utregningene?	Hvorfor er svarene i oppgave 1 det samme?	Hvordan kom jeg fram til svaret?
	After	Har jeg løst oppgaven på en rett måte	Har jeg løst dette på en riktig måte?	Hva har jeg lært?	Hva forteller svaret oss?
Second task	Before	hvordan finner vi sin(a)?	Hvordan finner vi SIN A?	x	Hva spør oppgave om?
	Before	hvordan finner vi cos(a)?	Hvordan finner vi COS A?	x	Hvordan skal man skrive inn i CAS?
	After	hva er sin, cos og tan i figurene?	Hva er Sin, Cos og tan i trekantene?	x	Hva forteller svaret?
	After	har jeg løst dette riktig?	Har jeg løst dette riktig?	x	Hva har jeg lært?

		Student 9	Student 10	Student 11	Student 12
First task	Before	hva er poenget med oppgaven?	hva er a, b og c ?	Vil svarene på de tre trekantene bli like, siden vinklene er like store?	Hva er sinus, cosinus og tangens?

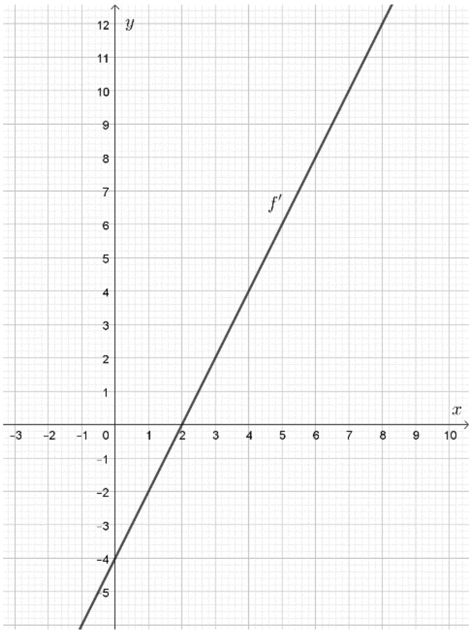
	Before	hvorfor skal vi stille spørsmål?	hva spør oppgaven om?	Hva betyr sin v, cos v og tan v?	har lengden på hypotenusene noe å si eller er det bare vinkelene sine verdi?
	After	hvorfor får alle tre trekantene det samme svaret?	Er det riktig svar ?	Hvorfor ble svarene like?	Hvorfor får jeg kun de to samme verdiene 0,85 og 0,51
	After	hva er poenget med å stille to spørsmål etter oppgaven?	Hva har jeg lært?	Hvordan bruker jeg CAS i denne oppgaven?	Hva betyr tegnene cos, sin og tan
Second task	**Before**	hva skal jeg gjøre?	Hva er sinus, cosinus og tangens?	Har dette noe med hypotenus, katet 1 og katet 2 å gjøre?	Er hypotenusen den motsideliggende kateten til punkt B på trekanten?
	Before	hva skal jeg gjøre?	Hvordan finner jeg svaret	Vil disse svarene bli like som noen av de i den andre oppgaven?	Hvordan løser jeg oppgavene i cas?
	After	hva betyr bruk cas til å finne? hva skal cas finne? det står jo bare sin 31 grader?	Hvilken sammenheng har jeg funnet ?	Vil dette funke med alle trekanter uansett hvor lange sidene er, så lenge de har like vinkler?	Hvilken sammenheng har sin, cos og tan. er det de ulike punktene i trekantene
	After	hva skal cas finne?	Hva har jeg lært?	Er det en sammenheng mellom disse oppgave og de første oppgavene på arket?	

		Student 13	**Student 14**
First task	**Before**	Hva forteller trekantene?	Hva betyr sinus, cosinus og tangens?
	Before	Hvordan skal jeg løse denne oppgaven?	hvordan bruker man disse begrepene for å regne ut a/b?
	After	Har jeg funnet en sammenheng mellom utregningene?	hvorfor blir svarene det samme uansett størrelse?
	After	Har jeg løst oppgaven rett?	hva kan man gjøre med svarene man får?
Second task	**Before**	Hva ønsker oppgaven at jeg skal gjøre?	hvilke type svar er man ute etter?
		Hvordan finner man sin a og hvordan finner man cos a?	hva er det man skal skrive inn i cas?
	After	Hvordan finner man sin, cos og tan i trekantene	hva er det man egentlig har regnet ut?
	After	Har jeg løst oppgaven riktig?	hvorfor må vi bruke Cas?

D Assignment on differentiation

Innleveringen skal i sin helhet føres digitalt, der det er ønskelig at dere bruke formel-editoren som er innebygget i Word til å skrive formler. Grafer henter dere fra GeoGebra eller lignende.

Oppgave 1



Figuren viser grafen til den deriverte $f'(x)$.

a) Bruk figuren til å avgjøre når $f(x)$ vokser og når $f(x)$ avtar.

b) Grafen til f går gjennom punktet $(3,2)$. Finn likningen til tangenten i dette punktet på grafen til f.

c) Tegn en skisse av grafen til f.

Oppgave 2

En funksjon er gitt ved:

$$g(x) = x^2 - 5x + 4$$

Finn den gjennomsnittlige vekstfarten i intervallene:

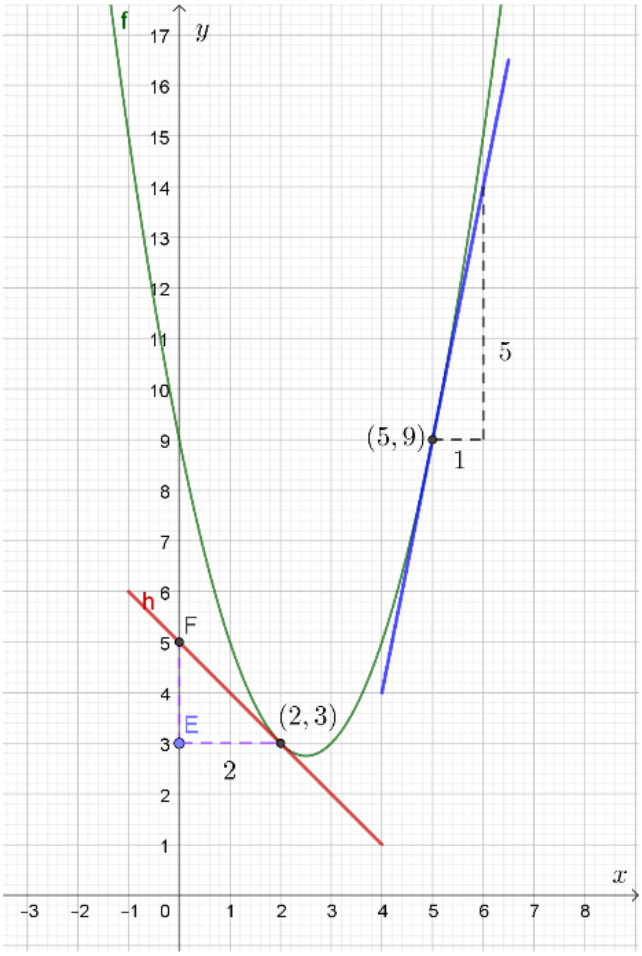
a) $[2,4]$

b) $[0,3]$

Den gjennomsnittlige vekstfarten i intervallet er $[2, a]$ er 3, der $a \epsilon \mathbb{R}$.

c) Bestem verdien til a.

Oppgave 3



Figuren viser grafen til funksjonen $h(x)$ med tangenter i to punkt på grafen. Bruk figuren til å finne et uttrykk for den deriverte funksjonen $f'(x)$.

Oppgave 4

Deriver funksjonene, både ved hjelp av regnereglene for derivasjon og CAS:

a) $f(x) = 3x^2 + 6x - 7$

b) $g(x) = \frac{1}{4}x^4 - 5x^3 + \frac{1}{4}x^2 + 3x + 2.3$

Oppgave 5

Grafen til en andregradsfunksjon på formen $f(x) = ax^2 + bx + c$ har bunnpunkt i $(3, -4)$ og går gjennom punktet $(2, -3)$. Finn verdiene av a, b og c.

Oppgave 6

En bedrift skal produsere en sylinderformet boks med bunn og lokk som skal inneholde 1 liter. Sidekantene er laget i et materiale som koster 3 kr/cm^2 mens lokket og bunnen er laget i et materiale som koster 5 kr/cm^2.

a) Tegn en skisse som viser boksen.

b) Vis at kostnaden ved produksjon av en boks kan være:

$$K(x) = 10\pi \cdot x^2 + \frac{6000}{x}$$

der $K(x)$ er kostnaden i kroner ved produksjon av en sylinder med radius x cm.

Bedriften ønsker å produsere boksen rimeligst mulig, altså med minst mulig bruk av materiale.

c) Finn radien og høyden i den boksen som er rimeligst å produsere.

d) Finn kostnaden ved å produsere denne boksen.

Oppgave 7

En tennisball skytes loddrett oppover. Høyden til ballen, $h(t)$, er etter t sekunder gitt ved funksjonen

$$h(t) = -5t^2 + 40t + 2$$

Startfarten til kula er 40 m/s.

a) Når er ballen på sitt høyeste punkt? Hvor stor fart har ballen i dette punktet?

b) Finn $h'(t)$. Hva forteller denne funksjonen oss?

c) Finn $h'(5)$. Hva forteller svaret oss?

d) Løs likningen $h'(t) \geq 10$ og forklar hva svaret forteller.

Oppgave 8

Den deriverte av funksjonen f er gitt ved

$$f'(x) = x^2 - x - 6$$

a) Drøft monotoniegenskapene til funksjonen f.

b) Avgjør om de stasjonære punktene på grafen til f er toppunkter eller bunnpunkter.

Oppgave 9

I denne oppgaven skal vi ta utgangspunkt i funksjonen $f(x) = -x^2 + 3x$.

a) Beskriv kort hvordan vi ut fra definisjonen til den deriverte kan finne en tilnærma verdi for $f'(5)$.

I neste del av oppgaven skal vi ta utgangspunkt i tabellen:

	x	5			
	$f(x) = -x^2 + 3x$				
Antall	Δx	$f(x)$	$f(x + \Delta x)$	$\Delta f = f(x + \Delta x) - f(x)$	$\Delta f / \Delta x$
0	1,0000000000000	-10,00	-18,0000000000000	-8,0000000000000000	-8,000000000000
1	0,1000000000000	-10,00	-10,7100000000000	-0,7099999999999990	-7,100000000000
2	0,0100000000000	-10,00	-10,0701000000000	-0,0700999999999983	-7,010000000000
3		-10,00			
4		-10,00			
5		-10,00			
6		-10,00			
7		-10,00			
8		-10,00			
9		-10,00			
10		-10,00			
11		-10,00			
12		-10,00			
13		-10,00			
14		-10,00			

b) Lag tabellen ovenfor i et regneark. Bruk formler der du kan. Fyll ut de resterende cellene som mangler innhold.

c) Hvilken informasjon gir tabellen oss? Skjer det noe uventet?

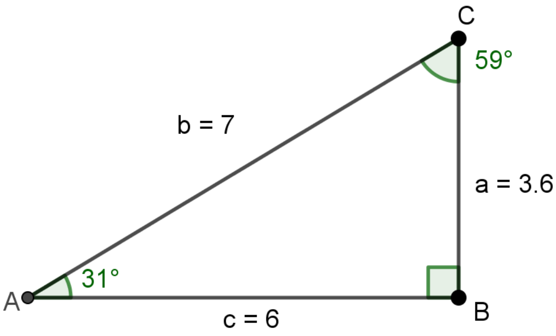
I siste del av programmet skal vi ta utgangspunkt i Python-programmet

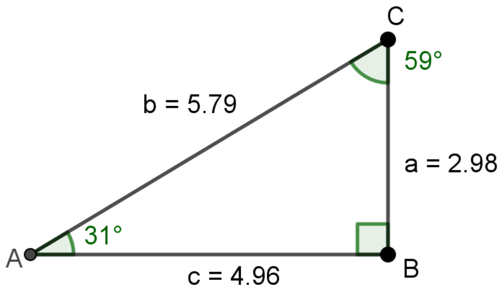
```python
#Definerer funksjonen
def f(x):
    return -x**2 + 3*x


derivert=(f(5.01)-f(5))/0.01

print(f'Tilnærma verdi for den deriverteer er {derivert:.5f}')
```

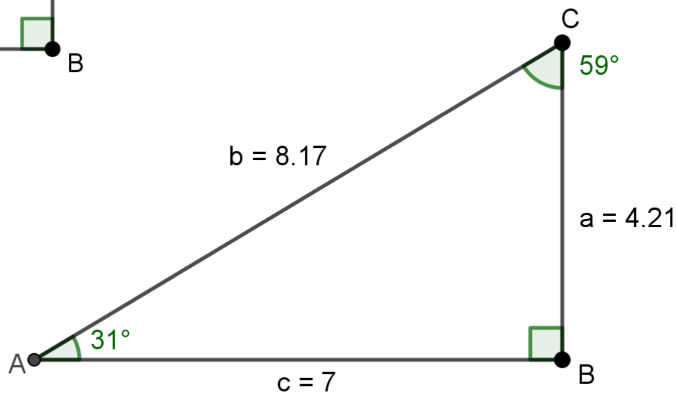
d) Utvid programmet slik at brukeren blir bedt om å angi $\boldsymbol{c}$ og $\boldsymbol{\Delta x}$, der $\boldsymbol{c}$ er $\boldsymbol{x}$-verdien hvor den deriverte ønskes beregnet og $\boldsymbol{\Delta x}$ angir inkrementet.

E Trigonometry tasks

Sinus, cosinus og tangens.





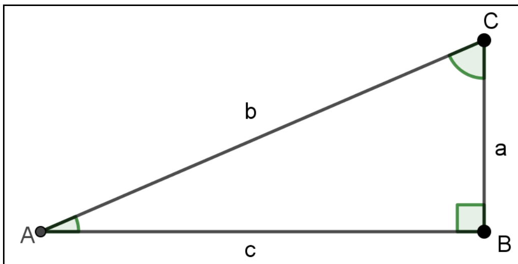


1: Finn $\frac{a}{b}$ i alle trekantene.

2: Finn $\frac{a}{c}$ i alle trekantene.

3: Finn $\frac{c}{b}$ i alle trekantene.

Hva er sammenhengen mellom utregningene i 1, 2 og 3? *Skriv ned 2 spørsmål før og etter du har skrevet svaret ditt. Du skal altså finne 4 spørsmål til denne oppgaven. Tenk gjennom hvordan du kan formulere spørsmålene på en tydelig og klar måte. Disse spørsmålene skal du levere på denne i et Google Skjema som du finner her:* https://forms.gle/4RtriS9SfF1S8MXq6. *Ikke send inn spørsmålene før du har gjort alle oppgavene.*



Siden a er $\angle A$ sin motstående katet.
Siden c er $\angle A$ sin hosliggende katet.
Siden b er hypotenusen.

Hvilken side er $\angle C$ sin motstående katet? ___

Hvilken side er $\angle C$ sin hosliggende katet? ___

$sin\ v = \frac{hosliggende\ katet}{hypotenus}$	$cos\ v = \frac{motstående\ katet}{hypotenus}$	$tan\ v = \frac{motstående\ katet}{hosliggende\ katet}$

Ta utgangspunkt i de tre trekantene øverst og finn:

sin A=	cos A=	tan A=
sin C=	cos C=	tan C=

Bruk kalkulatoren eller CAS til å finn svarene: (Pass på at kalkulatoren er innstilt på Deg.)

sin 31°=	cos 31°=	tan 31°=
sin 59°=	cos 59°=	tan 59°=

Hvilken sammenheng har du funnet? *Skriv ned 2 spørsmål før og etter du har skrevet svaret ditt. Du skal altså finne 4 spørsmål til denne oppgaven. Tenk gjennom hvordan du kan formulere spørsmålene på en tydelig og klar måte. Disse spørsmålene skal du levere på samme Google Skjema som i forrige oppgave.*

F Text on how to pose questions

Hvordan stille spørsmål

Når man skal stille et spørsmål er det viktig å være tydelig og presis i måten man stiller det på. Man må prøve å ha med nok informasjon slik at den som skal svare så enkelt som mulig kan vite hva man mener med spørsmålet. Dette er fordi det ofte kan være flere måter å tolke et spørsmål på, og dermed er det viktig å gjøre klart hva man faktisk ønsker svar på.

Før vinterferien arbeidet dere med en innlevering om vekstfart og derivasjon. Dere hadde blant annet oppgaven:

"Grafen til en andregradsfunksjon på formen $f(x) = ax^2 + bx + c$ har bunnpunkt i $(3, -4)$ og går gjennom punktet $(2, -3)$. Finn verdiene av a, b og c."

Det kan kanskje være fristende å si til læreren at dere ikke forstår oppgaven og trenger hjelp. Dette er selvfølgelig helt greit, men vi trenger mer informasjon enn det. Det er smart å starte et spørsmål med den informasjonen man har forstått. Dette er for å unngå at du får svar på noe du allerede vet. Man kan for eksempel si:

«Jeg klarte å finne a og b med å bruke de to punktene som var oppgitt i oppgaven, men nå sitter jeg litt fast. Har du noen tips til hvordan jeg kan finne c?»

Her får læreren vite at du har funnet a og b, og at det da kun er c du trenger hjelp med å finne. Læreren får også vite hva du har brukt for å finne a og b.

Det er som sagt også viktig å være presise når man stiller spørsmål. En huskeregel kan være å prøve å stille spørsmålet på en sånn måte at man, så langt det lar seg gjøre, ikke trenger å vite hva oppgaven spør om.

Unngå spørsmål som:

«Hvordan finner jeg den tingen?»

«Hva gjør jeg her?»

Si heller:

«Hvordan var det jeg fant et uttrykk for den deriverte med bruk av definisjonen av den deriverte?»

«Hvordan kan jeg gå fram for å finne stigningstallet til tangenten når jeg kun har et bilde av den?»

Ikke vær redd for å stille STORE spørsmål! Selv om du ikke ser hvordan det kan være mulig å finne svar på et spørsmål kan det føre til mye morsom diskusjon og læring.

G Approval form

Thomas Schjem
[redacted]

Trondheim, 15.01.21

Til elever ved [redacted] skole

Anmodning om tillatelse til videoopptak av undervisning og innsamling av elevbesvarelser.

Jeg er en student på lektorprogrammet i realfag ved NTNU og skal gjennomføre en forskning til mitt masterprosjekt. Dette vil innebære å observere deres vanlige undervisning, gjøre noen endringer på hvordan denne fungerer og å gjennomføre en undervisningssekvens og undersøke ulike aspekter ved gjennomføringen. Jeg vil blant annet undersøke hvordan elever bruker spørsmål og muntlige ferdigheter i matematikkfaget, samt hvordan ulike teknikker kan påvirke dette.

For å få så godt dokumenterte data som mulig, er det ønskelig å gjøre videoopptak av ulike undervisningssekvenser. Derfor ber jeg om tillatelse til å kunne gjøre videoopptak, samt samle inn materiale produsert av elever. Det er snakk om videoopptak av opptil 6 timer á 45 min. Forutsetningen for tillatelsen er at alt innsamlet materiale blir behandlet med respekt og blir anonymisert, og at prosjektet ellers følger gjeldende retningslinjer for etikk og personvern. Det er helt frivillig å delta og man kan til enhver tid trekke seg fra deltakelse uten å måtte oppgi noen grunn til det. Dersom du ikke ønsker å delta i forskningen vil du gjennomføre tilsvarende undervisningsopplegg sammen med faglærer i et annet rom. Dette vil ikke bli filmet eller observert. Deltakelse påvirker på ingen måte vurdering i faget eller relasjon til faglærer.

Dine rettigheter
Så lenge du kan identifiseres i datamaterialet, har du rett til:

- innsyn i hvilke personopplysninger som er registrert om deg, og å få utlevert en kopi av opplysningene,
- å få rettet personopplysninger om deg,
- å få slettet personopplysninger om deg, og
- å sende klage til Datatilsynet om behandlingen av dine personopplysninger.

Undervisningen vil bli filmet med to kamera, og det vil bli tatt opp lyd gjennom to lydopptakere. Kameraene vil bli plassert avhengig av hva den gjeldende timen fokuserer på. Alt datamateriell vil bli oppbevart på en kryptert lagringstjeneste med tofaktorautentisering via NTNU. Materialet vil kun bli sett av Thomas Schjem og min veileder for masterprosjektet. I det som presenteres fra prosjektet vil involverte personer bli anonymisert. Innsamlede data vil bli slettet etter at prosjektet er avsluttet, senest 01.12.2021.

Hvis du vil vite mer om dette, eller hva det innsamlede materialet skal brukes til, så er det bare å ta kontakt med en av oss på telefon eller epost (se øverst på siden for detaljer).

Faglig ansvarlig ved NTNU er Yael Fleischmann: tlf.: [redacted]; epost: [redacted].

NTNUs personvernombud er Thomas Helgesen: tlf. [redacted]; epost [redacted].

Hvis du har spørsmål knyttet til NSD sin vurdering av prosjektet, ta kontakt med:

- NSD – Norsk senter for forskningsdata AS på epost (personverntjenester@nsd.no) eller på telefon: 55 58 21 17.

Jeg håper du synes denne forskningen er av verdi, og at du er villig til å være med på den. Vi ber om at svarslippen på neste side fylles ut om hvorvidt du gir eller ikke gir tillatelse til deltakelse i prosjektet.

På forhånd takk!

Vennlig hilsen

Thomas Schjem

Tillatelse

Som del av forskningsprosjektet ber vi om tillatelse til å ta videoopptak av deg under en undervisningssekvens og å bruke besvarelser som du har produsert.

Forutsetningen for tillatelsen er at besvarelser og annet innsamlet materiale blir anonymisert og behandlet med respekt, og at prosjektet følger gjeldende retningslinjer for etikk og personvern.

☐ Jeg samtykker til å bli filmet og gjort lydopptak av i undervisningen. Jeg samtykker til at mine opplysninger behandles frem til prosjektet er avsluttet

☐ Jeg samtykker til at mine arbeider (oppgave ark) i forbindelse med undervisningen blir samlet inn

Dato: ………………………

Elevens fornavn og etternavn: ……………….…………………….…………………...............

Underskrift:

……………………………………………………...

Vennligst returner svarslippen til faglærer så snart som mulig.

Thomas Schjem

Questioning questioning

NTNU
Norwegian University of
Science and Technology