

Valuation of the Executive stock options

-An empirical investigation of the valuation models for the executive stock options

by

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Preface

This thesis is the completion of the two-year Master of Science program in Financial Economics at the Norwegian University of Science and Technology (NTNU). The thesis has been written in Microsoft Word. Microsoft Excel with VBA and Matlab have been used extensively.

I would like to thank my supervisor Professor Snorre Lindset for guidance and helpful suggestions.

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Abbreviations

The ESOs The executive stock options

1 Introduction

Why the executive stock options are important?

Stock options give firm executives and employees the right to buy their own firm's shares at a pre-specified strike price and so benefit from a higher share price. It is important device for firms to compensate and incentivize their executives. According to the National Bureau of economic research, 'CEOs of the largest U.S. companies now receive annual stock option awards that are larger on average than their salaries and bonuses combined. In contrast, in 1980 the average stock option grant represented less than 20 percent of direct pay and the median stock option grant was zero' ¹.

Expenses to the executive stock options as a compensation tool such as salary should be disclosed in the firms financial statements as well as being deducted from quarterly earnings just like other compensation costs, such as salary, bonus and stocks.

Executive stock options differ from standard market-traded options in many ways. This fact makes a great challenge to value ESOs.

The ignoring of the special characteristics of executive stock options in valuation process cause dramatic differences in option values. Inadequate disclosure of ESOs makes firms' financial statements not transparent and represent a corporate governance problem – the separation of ownership and control permits executives to take actions that owners would prefer not to.

In this thesis I try to see the differences between the executive stock options and marked traded options and how existing methods for evaluating marked-traded options can be extending to get the "right value" of the executive stock option.

I use "A general framework for evaluating executive stock options" presented by Ronnie Sircar and Wei Xiong (2006). I will refer later to this as the article.

R. Sircar and W. Xiong provide a framework for the joint evaluation of the several special characteristics of the executive stock options and compared results from this with several other methods, namely binomial version of their model, FASB recommendation and Hull-White method.

¹ A. Balls. «Executive stock options»: <http://www.nber.org/degest/dec98/w6674.html>

Section 2 of this thesis describes the important features to include in valuation of the ESOs. Sections 3, 4 and 5 provide valuation methods and results obtained by them. Section 6 concludes the thesis.

2 The important features to include in valuation of ESOs

ESOs differ from standard market-traded options in a number of ways:

Trading restrictions. Executive stock options cannot be sold or hedged. The option recipient either has to exercise them or to forfeit them upon separation from the firm. It is essential to retain executives and to avoid a situation where they could simply receive and immediately resell their options.

Vesting period. They can only be exercised after a certain “vesting” period. In general, the executive stock options are not exercisable when they are granted, but only after a vesting period of one to five years. Once vested, they can be exercised any time (American style) with a long maturity of about 10 years.

Reloading and resetting. The firm typically grants more options when the existing ones are exercised, or fall deep underwater. The firm provides more options after large stock price increases which induce firm executives to exercise their existing options - reloading or after large stock price drops, which cause executives’ existing options to lose much of their value and incentive capacity, firms tend to reset the terms of these out-of-the money options or grant more new options –resetting. Reloading and resetting of stock options represent economic cost to firms. That is why it is important to incorporate reloading and resetting in a valuation framework.

3 FASB method

3.1 FASB recommendations

In October 1995, the Financial Accounting Standards Board published FASB 123 “Accounting for Stock Based Compensation”. This standard recommends companies to value the executive stock options by using a fair-value-based method adopted for some features of the executive stock options. In appendix B of FASB 123 is discussed the value of ESOs in detail.

The executive stock options value can be found by using either Black-Scholes (1973) or Cox, Ross and Rubinstein (1979) binomial tree method with the expected life as the time-to-maturity parameter and then adopted for the possibility of the employee leaving the company during the vesting period.

FASB proposes an expensing approach by simply adjusting the Black-Scholes model, which was initially derived for market-traded options. The FASB method ignores many of the particular characteristics of ESOs.

3.2 The Black-Scholes pricing model

The idea underlying the Black-Scholes pricing model is to create a riskless hedge portfolio by continuously adjusting the proportions of stocks and options in a portfolio.

The option price derived by the Black-Scholes option pricing model rests on a number of assumptions such as how the stock price is distributed and assumptions about the economic environment:

- an efficient market with no riskless arbitrage opportunities, any portfolio with a zero market risk must have an expected rate of return equal to the risk-free interest rate
- the asset price follows a geometric Brownian motion

$$dS_t/S_t = (\mu - q)dt + \sigma dZ_t,$$

where

- μ is the total expected stock return, through both dividend and price appreciation
- q is the dividend yield of the firm's stock,
- σ is the volatility of the stock return,

- Z is a standard Brownian motion.
- trading can take place continuously without any transaction costs or taxes
- short selling is permitted and the assets are perfectly divisible.
- the continuously compounded risk-free interest rate is constant.
- investors can borrow or lend at the same risk-free rate of interest.

The Black-Scholes partial differential equation for an options granted on a dividend paying stock is

$$\frac{1}{2}\sigma^2 S C_{SS} + (r - q) S C_S - r C = 0$$

with the final condition for the call option $C(S,T)=\max(S-K,0)$ and the boundary conditions $C(0,t)=0$, $\lim_{S \rightarrow \infty} C(S,t) = S \exp(-q\tau)$.

The solution of this equation provides the pricing formulae for a European call $C(S,t)$ on a dividend paying stock:

$$C(S,t) = S \exp(-q\tau) N(d_1) - K \exp(-r\tau) N(d_2)$$

where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + (r - q + \frac{\sigma^2}{2})\tau}{\sigma\sqrt{\tau}},$$

$$d_2 = d_1 - \sigma\sqrt{\tau}.$$

$C(S,t)$ = price of the European the call option,

S = current underlying asset (stock) price,

K = strike price,

$\tau = T - t$ is the current annualized time to expiration, where T is the expiration date,

r = the annualized risk – free interest rate,

σ = the annualized standard deviation of underlying asset price,

N = the cumulative distribution function for a standardized normal variable.

3.3 The option valuation according to FASB 123

We are going to evaluate the executive stock option with the parameters in Table 1 (see Appendix A) using FASB 123.

The Black-Scholes price for a 10 years European option on a dividend paying stock is 48.38. According to FASB 123 we have to use the expected life of the option instead of the time to maturity. A vesting period for this option is 2 years and we assume that the expected life of the option is 5 years. That is, the option value is 37.75.

To incorporate the possibility that the option can be forfeited during the vesting period, we use the exit rate 0,2 or 20% during the 2 years vesting period and the option value become:

$$37,75 * 0,8 * 0,8 = 24.16$$

According to FASB method the option value is found to be 24.16.

The question of how well the FASB approach is dealing with the features of the executive stock options still remains. That the ESOs can be exercised before the maturity is incorporated by treating the ESOs as a European option with a finite maturity equal to the expected life of the option. The possibility that option recipient might leave the firm during the vesting period and therefore forfeiting the option is taken into account by adjustment the option value by the exit rate during the vesting period. However, the FASB method does not deal with reloading and resetting at all.

4 An analytical model by Ronnie Sircar and Wei Xiong

4.1 Models provision and underlying assumption

“A general framework for evaluating executive stock options” provided by Ronnie Sircar and Wei Xiong does take into account reloading and resetting as well as other features typical to the executive stock options: vesting period, American style and also considers the trading restriction on executives.

The analytical model based on the following provisions in executive stock options:

- when an option is granted, its strike price K is set equal to the current stock price (at the money)
- the option is American and perpetual
- it can only be exercised after a vesting period of length T years
- reload provision. If the option is exercised in the money, say at time $\tau \geq T$, its holder receives the usual option payoff $S_\tau - K$, together with $\rho_H * K / S_\tau$ new options which have a new strike price S_τ , and a new vesting period of length T years.
- resetting provision. If the stock price falls to a fraction of the strike price, $l * K$ ($0 < l < 1$), the option, either vested or unvested, will be replaced by ρ_L shares of new options with a new strike price $l * K$, a new vesting period of a further T years, and a new resetting barrier at $l^2 * K$.
- the non-tradability constraint. Executives cannot sell their stock options. That makes the options less valuable to them, but less costly for the granting firm. To incorporate this effect, Sircar and Xiong (2006) suppose that the option recipient is subject to an exogenous random shock upon whose arrival executive exits the firm. This shock is modelled as the first jump of a Poisson process with hazard rate λ , which implies that the recipient has expected employment duration of $1/\lambda$ years with the firm. When the shock hits, the option holder cannot sell the option. Instead, she has to exercise the option if it is vested, or forfeit otherwise. Even if she can exercise her current options, she would have to forfeit on reloaded new options. Since the option recipient is not allowed to sell, the exit shock affects the option value and exercising strategy. Although the option recipients cannot sell their options, they can trade other derivative contracts to hedge their stock options. To avoid complication with hedging restriction

in their analytical model, Sircar and Xiong assume that the option recipient can hedge the option.

In the summary, the option program incorporates a vesting period, reloading and resetting, as specified by parameters T , ρ_H , ρ_L and l as well as the non-tradability constraint.

Assumptions:

- the risk-free interest rate r is constant, and the firm's stock price $(S_t)_{t \geq 0}$ is modeled by the following geometric Brownian motion:

$$dS_t/S_t = (\mu - q)dt + \sigma dZ_t, \quad (1)$$

where

- q is the dividend yield of the firm's stock,
- σ is the volatility of the stock price,
- Z is a standard Brownian motion and
- μ is the total expected stock return, through both dividend and price appreciation.

Eq. (1) represents the usual price process that is used in the standard Black-Scholes option pricing model, but there are several restrictions embedded in this process:

- by treating the total stock return parameter μ as a constant independent of firm executives' incentive, there is ignoring of the executives' influence on stock prices. It is assumed here that an efficient stock market where investors have perfect foresight, current stock price would already incorporate the future impact that option grants might bring to the firm. Given that, the current stock price is taken as given and assumed that it evolves according to Eq. (1).

- Eq. (1) specifies a continuous dividend stream in contrast to reality when firms usually pay discrete dividends. By examination this effect, Rubinstein (1995) finds that the valuation of stock options is not sensitive to the assumption about dividend yield.

- It is assumed that a potential dilution effect and the option recipient's information have all been incorporated into the current stock price.

4.2 The option valuation method.

The option valuation method is taken from section 4 in the article.

The options value in the vesting period – $V(S,t;K)$ where

S- the current stock price

t- the time into the vesting period

The options value out of the vesting period – $C(S;K)$

The strike price of the option might change over time due to resetting and reloading.

For a vested option, the optimal strategy is to exercise the option when the stock price S hits a certain level, and this level can be determined by a smooth-pasting condition. When the option is exercised, for each share, the executive will receive $\rho_H * K/S$ shares of new unvested at the money options (each with value $V(S,0;S)$ and reset barrier $l * S$).

If both the stock price and the strike price are multiplied by a positive factor, the value of the option program and the optimal exercise barrier also change by the same proportion (a certain scaling property of the option value). The strike price K is used as a scaling variable for other variables, such as the exercising level and the option value.

$h * K$ – the exercising level, with $h > 1$ to be determined.

$$D = V(S,0;S) / S \tag{2}$$

– the ratio between the value of a new option $V(S,0;S)$ and its strike price S is a constant, which is also to be determined.

4.2.1 Valuation of a vested option

A vested option is similar to an American perpetual option with an additional lower resetting barrier. In the exercise region $S \geq h * K$, its payoff, including the value of the reloaded options that are granted, is

$$C(S;K) = S - K + \rho_H * D * K \tag{3}$$

At the resetting barrier $S = l * K$, the option will be replaced by ρ_L new unvested options with value

$$C(l * K;K) = \rho_L * D * l * K \tag{4}$$

In the continuation region between the lower barrier and the exercise level, $l * K < S < h * K$, $C(S;K)$ satisfies the following ordinary differential equation:

$$0,5\sigma^2 S^2 C_{SS} + (r - q) S C_S - (r + \lambda) C + \lambda \max(S - K, 0) = 0. \tag{5}$$

This is simply the Black–Scholes differential equation for an infinitely lived derivative security, plus the additional term

$\lambda(\max(S-K,0)-C)$,

which captures the effect that the option may be exercised due to the employment shock with a probability λdt over an infinitesimal time period dt . To determine the exercise level h , the smooth-pasting condition is employed

$$C_s(hK;K)=1, \quad (6)$$

which enforces the condition that the option has continuous first derivative at hK .

Proposition 1. The solution $C(S;K)$ of (5), satisfying (3), (4) and (6), is given by

$$C(S;K) = \begin{cases} S + (pHD - 1)K & \text{if } S \geq hK \\ a1K \left(\frac{S}{K}\right)^{k1} + a2 * K \left(\frac{S}{K}\right)^{k2} + \frac{\lambda}{\lambda + q} S - \frac{\lambda}{\lambda + r} K & \text{if } K \leq S < hK \\ b1K \left(\frac{S}{K}\right)^{k1} + b2 * K \left(\frac{S}{K}\right)^{k2} & \text{if } lK \leq S < K \end{cases} \quad (7)$$

where

$$k1 = \frac{1}{\sigma^2} \left[-\left(r - q - \frac{1}{2}\sigma^2\right) + \sqrt{\left(r - q - \frac{1}{2}\sigma^2\right)^2 + 2\sigma^2(\lambda + r)} \right],$$

$$k2 = \frac{1}{\sigma^2} \left[-\left(r - q - \frac{1}{2}\sigma^2\right) - \sqrt{\left(r - q - \frac{1}{2}\sigma^2\right)^2 + 2\sigma^2(\lambda + r)} \right],$$

and the parameters $a1, a2, b1, b2$ and h solve the following algebraic equations:

$$a1h^{k1} + a2h^{k2} - \frac{q}{\lambda + q} h = pHD - \frac{r}{\lambda + r} \quad (8)$$

$$a1k1h^{k1-1} + a2k2h^{k2-1} + \frac{\lambda}{\lambda + q} = 1$$

(9)

$$a_1 + a_2 + \frac{\lambda(r - q)}{(\lambda + q)(\lambda + r)} = b_1 + b_2$$

(10)

$$a_1 k_1 + a_2 k_2 + \frac{\lambda}{\lambda + q} = b_1 k_1 + b_2 k_2$$

(11)

$$b_1 l^{k_1} + b_2 l^{k_2} = pLDl$$

(12)

Eqs. (8)-(12) are independent of K , and can be used to determine the values of a_1, a_2, b_1, b_2 and h for a given D .

4.2.2 Valuation of an unvested option

An unvested option is European, with its value converging to that of a vested option at the end of the vesting period:

$$V(S, T; K) = C(S; K).$$

At the resetting barrier $S = l^*K$, it will be replaced by ρ_L new unvested options:

$$V(l^*K, t; K) = \rho_L * D * l^*K, \quad 0 \leq t < T.$$

Inside these boundaries ($S > l^*K$ and $0 \leq t < T$), the unvested option price $V(S, t; K)$ satisfies the following differential equation:

$$V_t + \frac{1}{2} \sigma^2 S^2 V_{SS} + (r - q)SV_S - (r + \lambda)V = 0 \quad (13)$$

This is similar to the standard Black–Scholes equation except for the additional term $-\lambda V$ which is again generated by the selling restriction: the option recipient may have to forfeit the option upon the arrival of a Poisson employment shock.

An unvested option is similar to a down-and-out barrier option, except that at the resetting barrier the value of the option is equal to that of the new unvested options, instead of zero. Sircar and Xiong adapted the method of images to derive the option value in an analytical form. The following proposition provides V , given the parameters a_1, a_2, b_1, b_2 and h from the valuation of the corresponding vested option.

Proposition 2. Define $g(S)$ on $S>0$ by

$$g(S) = \begin{cases} S + (pHD - 1)K - \left[b1K \left(\frac{S}{K}\right)^{k1} + b2K \left(\frac{S}{K}\right)^{k2} \right] & \text{if } S \geq hK, \\ (a1 - b1)K \left(\frac{S}{K}\right)^{k1} + (a2 - b2)K \left(\frac{S}{K}\right)^{k2} + \frac{\lambda}{\lambda + q}S - \frac{\lambda}{\lambda + r}K & \text{if } K \leq S < hK \\ 0 & \text{if } lK \leq S < K \end{cases} \quad (14)$$

and let $w(S,t;K)$ be the solution of

$$wt + \frac{1}{2}\sigma^2 S^2 w_{SS} + (r - q)S w_S - (r + \lambda)w = 0, \quad (15)$$

in $S>0$ and $0 \leq t < T$, with terminal condition $w(S,T;K)=g(S)$. Then, the value of an invested option, for $S \geq l^*K$, is

$$V(S, t; K) = w(S, t; K) - \left(\frac{S}{K}\right)^{-(k-1)} w\left(\frac{X^2}{S}, t; K\right) + b1K \left(\frac{S}{K}\right)^{k1} + b2K \left(\frac{S}{K}\right)^{k2} \quad (16)$$

with $k=2(r-q)/\sigma^2$ and $X=l^*K$

The V function in Eq. (16) contains the difference of two w functions in ‘images’. Both of these terms are solutions to the differential equation (13), and are designed in an exact way to cancel each other on the resetting boundary. Using the Feynman–Kac representation of the solution to (15), w is given by

$$w(S, t; K) = e^{-(r+\lambda)(T-t)} \left\{ S e^{(r-q)(T-t)} N(d_1^h) - (1 - pHD)KN(d_2^h) \right. \\ \left. + \frac{\lambda}{\lambda + q} S e^{(r-q)(T-t)} \left[N(d_1) - N(d_1^h) - \frac{\lambda}{\lambda + r} K [N(d_2) - N(d_2^h)] \right] \right. \\ \left. + K \left(\frac{S}{K}\right)^{k1} e^{\theta_1(T-t)} [(a1 - b1)N(d_3) - a1N(d_3^h)] \right. \\ \left. + K \left(\frac{S}{K}\right)^{k2} e^{\theta_2(T-t)} [(a2 - b2)N(d_4) - a2N(d_4^h)] \right\}$$

where N is standard normal cumulative distribution function, and

$$\theta_j = k_j \left(r - q + \frac{1}{2} (k_j - 1) \sigma^2 \right), j = 1, 2,$$

$$d_1 = \frac{\log\left(\frac{S}{K}\right) + (r - q + \left(\frac{1}{2}\right) \sigma^2)(T - t)}{\sigma\sqrt{T - t}},$$

$$d_2 = \frac{\log\left(\frac{S}{K}\right) + (r - q - \left(\frac{1}{2}\right) \sigma^2)(T - t)}{\sigma\sqrt{T - t}},$$

$$d_1^h = \frac{\log\left(\frac{S}{(hK)}\right) + (r - q + \left(\frac{1}{2}\right) \sigma^2)(T - t)}{\sigma\sqrt{T - t}},$$

$$d_2^h = \frac{\log\left(\frac{S}{(hK)}\right) + (r - q - \left(\frac{1}{2}\right) \sigma^2)(T - t)}{\sigma\sqrt{T - t}},$$

and

$$d_3 = d_2 + k_1 \sigma \sqrt{T - t}, \quad d_3^h = d_2^h + k_1 \sigma \sqrt{T - t},$$

$$d_4 = d_2 + k_2 \sigma \sqrt{T - t}, \quad d_4^h = d_2^h + k_2 \sigma \sqrt{T - t}.$$

Having found (a1, a2, b1, b2, h) in terms of D following Proposition 1, the formula (16) is then used to give a nonlinear algebraic equation (2) for D.

4.3 Using Excel to get the option value

Table 6 from the article reports the results from the analytical model for ESOs evaluation (analytical VERR model).

Panel A provides the valuations for an option without any reloading or resetting. Panel A uses the following parameters: S=100, K=100, $\sigma=42.7\%$, $q=1.5\%$, $r=4\%$, $\lambda=0.2$, $T_{vest}=2$, $T_{maturity}=10$, $ph=0$, $pl=0$.

To get the values of parameters a1, a2, b1, b2 and h we solve the system of equations (8)-(12) using the Excel Solver. Eqs. (8)-(12) are independent of K, and can be used to determine the values of a1, a2, b1, b2 and h for a given D. Note that value of D does not affect any result here since $ph=0$ and $pl=0$.

Results are in the table:

Table 2: The values of parameters a1, a2, b1, b2 and h found by Excel Solver

Ph	0
Pl	0
D	0
a1	0,006126
a2	0,220725
b1	0,323751
b2	0
h	5,77
l	0

2) Since we are interested to value an unvested option, we use the Proposition 2. and the equation (16).

$$V(S, t; K) = w(S, t; K) - \left(\frac{S}{X}\right)^{-(k-1)} w\left(\frac{X^2}{S}, t; K\right) + b1K \left(\frac{S}{K}\right)^{k1} + b2K \left(\frac{S}{K}\right)^{k2}$$

which takes the form

$$V(S, t; K) = w(S, t; K) + b1K \left(\frac{S}{K}\right)^{k1}$$

Since b2=0 and $X \rightarrow 0$ since $l \rightarrow 0$.

The option value obtained by solving this equation is 27.11 which is almost the same as the result from table 6 (27.12).

Panel B of table 6 provides estimate for an option with an additional reloading feature (ph=1, pl=0).

The option value can be found by using the Excel Solver in the same way as we did in the first case but with one additional condition. Since the values of a1, a2, b1, b2 and h are dependent of D which is determined by equation (2), we have to include equation (2) as the additional condition. There are no other differences from the first case.

The option value is found to be 30.74 (30.75 in the table 6).

Panel C provides further value estimate when an additional resetting feature is introduced (ph=1 pl=1 l=0.6). We are getting the option value in the same way as we did in the second

case but using equation (16) for finding the option value. The result is 32.77 (35.70 in the table 6) and the value of h is 1.64 (1.54 in the table 6).

4.4 Using Matlab to get the option value

The Matlab codes for the ESOs evaluation are given in Appendix B.1 and B.2. Codes are self written based on formulas in the section 4 from the article described above. There are 2 codes. The first one (appendix B.1) is written for the first (no reload no reset) and second cases (reload, no reset) since they both use the evaluation formula in reduced form.

The second one (appendix B.2) is written for the third case where both reloading and resetting provision are included.

The results are:

The first case: $D=0.2710$ V_{min} (the option value)=27.11 and $h=5.7698$

The second case: $D=0.3070$ V_{min} (the option value)=30.75 and $h=1.594$

In the Matlab code for the third case the option value is defined by equation (16):

$$V(S, t; K) = w(S, t; K) - \left(\frac{S}{X}\right)^{-(k-1)} w\left(\frac{X^2}{S}, t; K\right) + b1K \left(\frac{S}{K}\right)^{k1} + b2K \left(\frac{S}{K}\right)^{k2}$$

The results are: $D=0.3280$, V_{min} (the option value)=32.86 and $h=1.6377$.

The ESOs values obtained with the use of both programs Excel and Matlab are almost identical

Table 3: Comparing option values estimated from Excel and Matlab evaluation with estimates from table 6 (Sircar and Xiong 2006).

	Excel	Matlab	Estimates from table 6
Panel A: no reload, no reset			
D	0.2711	0.2710	0.2712
The option value	27.11	27.11	27.12
h	5.77	5.77	5.77
Panel B: reload, no reset			
D	0.3074	0.3070	0.3075
The option value	30.74	30.75	30.75
h	1.60	1.60	1.60

Panel C: reload and reset. Using (16) as given in the article			
D	0.3278	0.3280	0.3570
The option value	32.78	32.86	35.70
h	1.64	1.64	1.54

I could not get the same option value in the third case as the table 6 provided. In my computation the resetting barrier $l=0.6$ is taken as given from the article. Assuming that the resetting barrier is found by solving the equations (8)-(12) where just $p_L=1$ is given and by changing the resetting lever from 0.6 to 0.6484, the option value equal to 35.70 is obtained.

5 Binomial version of an analytical approach

Binomial version of an analytical approach to a valuation ESOs is described in appendix D in the article.

To evaluate how well the results from the analytical approach take into account the specific features of the ESOs Sircar and Xiong presents results from the a fully fledged binomial method which is adapted for their evaluation approach and allows to incorporate a finite maturity for the ESOs. The adapted binomial model does not involve any simplification and is used as a benchmark to evaluate estimates from other models. A fully fledged binomial method description is given in Appendix D in the article and described here in some more details.

5.1 Binomial pricing model

The binomial option pricing model assumes that, over a period of time, the price of the underlying asset can move up or down only by a specified amount - that is, the asset price follows a binomial distribution. Binomial pricing achieves its simplicity by making a very strong assumption about the stock price: at any point time, the stock price can change to either an up value or a down value. In-between, greater or lesser values are not permitted.

The binomial model involves the three step process:

Calculate potential future prices of the underlying asset at expiry and possibly at intermediate points in time.

Calculate the payoff of the option at expiry for each of the potential underlying prices.

Discount the payoffs back to determine the option price today.

The first step in pricing options using a binomial model is to create a tree of potential future prices of the underlying asset.

The price of the underlying asset S changes only at discrete times $t_0=0, t_1=\Delta t, t_2=2\Delta t, \dots, t_n=n\Delta t, \dots, t_N=N\Delta t=T_{\text{exp}}$, where T_{exp} –the expiration date of the ESO including the vesting period of the length T .

S_0 -the stock price today. At any point time, the stock price can change to either an up value or a down value. That is, the stock price over a time interval Δt can move to one of two potential values $S_{11}=S_0*u$ or $S_{10}= S_0*d$ and price $S_{11}>S_0$ and $S_{10}<S_0$.

The other assumption underlying the binomial tree is a risk-neutral world.

In the risk-neutral world, the probability of the stock going up p can be found by assuming that the stock is expected to earn the risk-free rate.

The asset price return follows a Brownian motion (the term μ is replaced by the risk-free rate r)

$$\frac{dS_t}{S_t} = rdt + \sigma dZ_t$$

and the option value V_n is its expected value at $t_{n+1}=(n+1)\Delta t$ discounted by the risk-free interest rate r

$$V_n = E[\exp(-r\Delta t)V_{n+1}]$$

The probability p and the returns u, d should reflect the statistical properties of the continuous random walk, which means that they have to insure that for $\Delta t \rightarrow 0$ the underlying asset S follows the Brownian motion. The parameters p, u and d should give the correct values for the mean and the variance of the underlying asset, i.e.

$$\ln S_{n+1} \sim N(\ln S_n + (r - q - \frac{\sigma^2}{2})\Delta t, \sigma^2\Delta t), \text{ during a time interval } \Delta t.$$

So that, these parameters must solve the following equations:

$$E = p \ln(uS_n) + (1 - p) \ln(dS_n) = \ln(S_n) + (r - q - \frac{\sigma^2}{2})\Delta t$$

$$p(\ln(uS_n) - E)^2 + (1 - p)(\ln(dS_n) - E)^2 = \sigma^2\Delta t$$

The Cox, Ross and Rubinstein (CRR) chose the restriction $ud=1$, and the at time $t_n=n\Delta t$ the stock price is defined as

$$S_{n,j} = u^j d^{n-j} S_0, \quad j = 0, 1, \dots, n \quad n = 1, 2, \dots, N$$

$$u = e^{\sigma\sqrt{\Delta t}}, \quad d = e^{-\sigma\sqrt{\Delta t}},$$

And the risk-neutral probability

$$p = \frac{e^{(r-q)\Delta t} - d}{u - d}$$

$V_{n,j}$ – the ESO value at the node (n,j) , $n=N_v$ –the end of the vesting period.

The ESO price at time zero is V_0 , and the constant $D=V_0/S_0$ which will be determined at the last step.

λ - the risk-neutral termination rate of the executive, which means that in a time period of length Δt , the executive leaves the firm with probability $\lambda\Delta t$ and stays with probability $1-\lambda\Delta t$.

If the option is exercised in the next period, say at time t_{n+1} , and if the option is vested and in the money, then the holder will receive p_H new fresh options worth $p_H S_0 D$ as well as the amount $S_{n+1,j} - K$. If the stock price falls to IK , each option is replaced by p_L new fresh options worth $p_L IK D$.

This leads to the following algorithm for computing V_0 , given an initial guess for D .

Post-vesting: $j=N-1, N-2, \dots, N_v$, away from the lower barrier IK :

$$V_{n,j} = (1 - \lambda\Delta t) \max \left(1_{(S_{n,j} \geq K)} (S_{n,j} - K + \rho_H S_0 D), e^{-r\Delta t} (pV_{n+1,j+1} + (1-p)V_{n+1,j}) \right) + (\lambda\Delta t) e^{-r\Delta t} (p(S_{n+1,j} - K)^+ + (1-p)(IK - K)^+),$$

With values on or below the barrier IK replaced by $\rho_L IK D$.

Pre-vesting: $j=N_v-1, N_v-2, \dots, 0$, away from the lower barrier IK :

$$V_{n,j} = (1 - \lambda\Delta t) e^{-r\Delta t} (pV_{n+1,j+1} + (1-p)V_{n+1,j}) + (\lambda\Delta t) \times 0,$$

With values on or below the barrier IK replaced by $\rho_L IK D$.

In binomial model there were used 2000 periods. It is not possible to perform such estimation manually in Excel. It had to be written code in Matlab (appendix B.3). In the table below, we present the results obtained by our Matlab code with the results from table 6 from the article. They are almost identical.

Table 4: Comparing option values estimated from Matlab with estimates from table 6.

	Matlab	Estimates from table 6
Panel A: no reload, no reset		
The option value	26.37	26.38
Panel B: reload, no reset		
The option value	30.49	30.50
Panel C: reload and reset.		
The option value	35.69	35.71

6 The Hull-White model

The Hull-White model is presented in as a forth estimated method in the table 6.

Hull and White paper (2004) provides a method to estimate the value of the ESOs. They assume that a vested option is exercised whenever the stock price hits a certain constant level, or when the option reaches maturity. But the exact value of this level is left as a free parameter. Sircar and Xiong supply their model with the optimal barrier from analytical model.

The Hull-White method assumes also that the option recipient can leave the firm, and therefore need to forfeit or exercise the option early.

We used the model adapted from a paper by Hull and White that was written in VBA program (taken from Benninga 2008).

Table 5: Comparing values estimated from VBA with estimates form table 6.

	VBA code	Estimates from table 6
Panel A: no reload, no reset		
The option value	26.32	26.35
Panel B:reload,no reset		
The option value	22.79	22.79
Panel C: reload and reset.		
The option value	22.48	22.51

7 Conclusion

In this thesis, on the basis of the analytical framework for evaluation the executive stock options by Ronnie Sircar and Wei Xiong I tried to value the executive stock options by different methods and by comparing them with each other to see how important it is to incorporate the specific characteristics of the executive stock option in the valuation.

As a result of this work there were written two Matlab codes for the analytical approach. There are two different codes since the formula for the cost of the option with reloading and resetting differs from the first two cases.

The option values for the first and second cases almost coincide with the results in the article. The result obtained in the third case differs a lot from the value given in the article. By changing the parameter l which defines the resetting barrier, I obtained the same option value. As l goes from 0,6 to 0,6484, the option value varies from 32, 80 to 35,70. It shows that analytical model is very sensitive to this parameter. Changing resetting barrier has a significant effect on the option value.

Further results, obtained by using the analytical model, are compared with the binomial model. The binomial model is the version of the analytical model and does not have any simplification and incorporate a finite maturity of the ESOs as well as other their features. The binomial model uses 2000 periods. It was not possible to estimate the ESOs value manually in Excel. It was used self written Matlab code. Without any knowledge of programming language it was used a lot of time to write that Matlab code (see appendix B.3).

The difference in the results is extremely low and proves robustness of the analytical approach.

Thus, we can conclude that the analytical model can be successfully applied by firms to determine the value of the executive stock options.

FASB method and Hull-White model takes into account only some of the characteristics of the ESOs. The option values resulting from the application of these methods are very different from the results of the analytical approach.

Analytical approach provided by Sircar and Xiong (2006) also enables us to discuss the effects of the different features on the option value.

Figure 1 illustrates the effect of reloading on the option value. As pH goes up, the option becomes significantly more valuable. It is especially true for options on a low-volatility stock.

For an option on a high-volatility stock, a reload ratio of $pH=1$ would lead to an increase of 12% in the option value. In contrast, for an option on a low-volatility stock, it would lead to an increase of 40% in the option value.

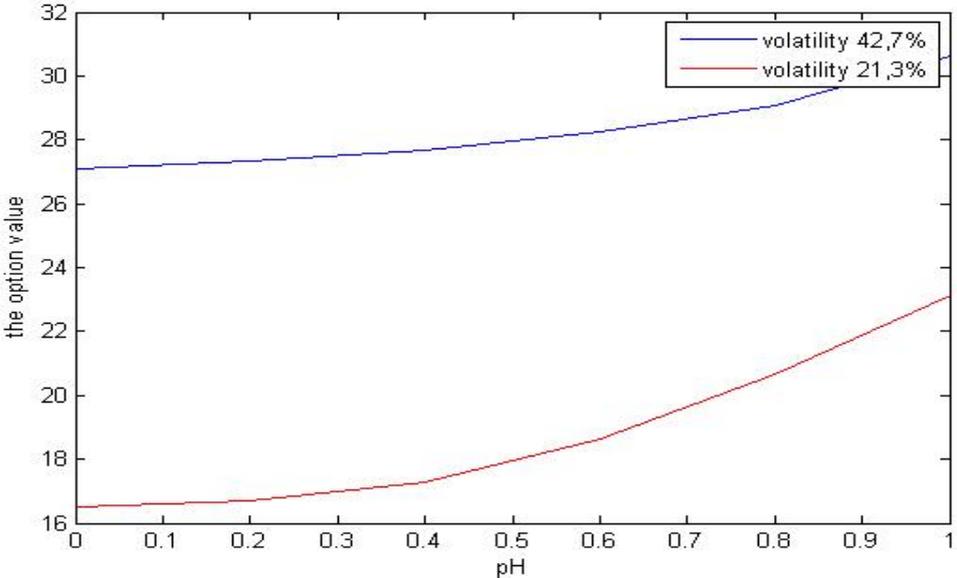


Figure 1: Option value. The following parameters have been used: $T=2$, $l=0$, $pL=0$, $K=100$, $r=4\%$, $q=1.5\%$, $\lambda=0.2$

Figure 2 shows the effect of reloading on the option value for three values of the vesting period: $T=1$, 2 and 3. The option value changes greatly as pH goes up. Figure 2 also indicates that the length of the vesting period is important for the effect of reloading. When the vesting period becomes longer, the value increase caused by the reloads decreases. The results suggest that the interaction between the vesting period and the reloading provision is important for evaluating executive stock options and determining optimal exercising strategies.

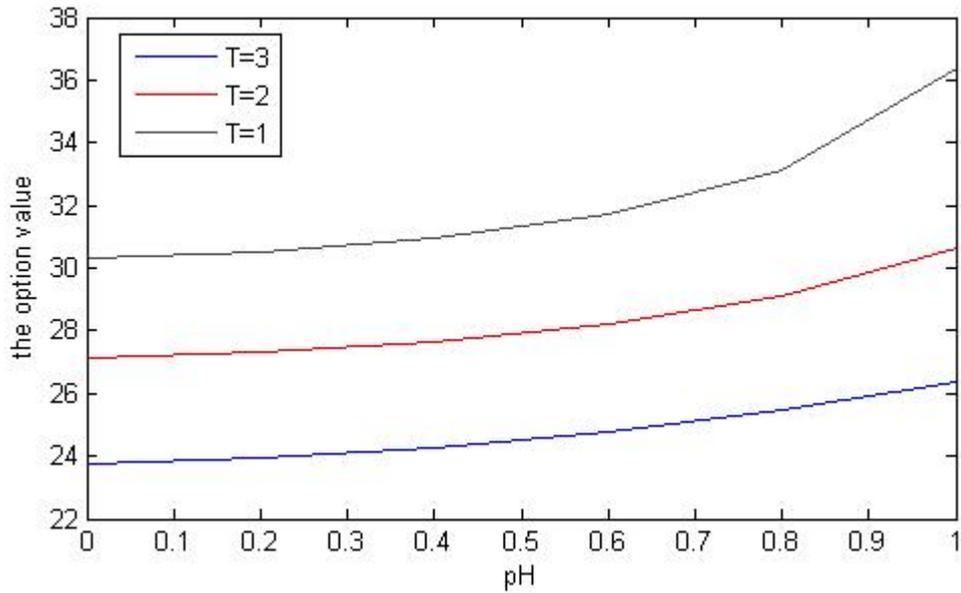


Figure 2: Option value. The following parameters have been used: $l=0$, $pL=0$, $K=100$, $r=4\%$, $q=1.5\%$, $\lambda=0.2$, $\sigma=42.7\%$

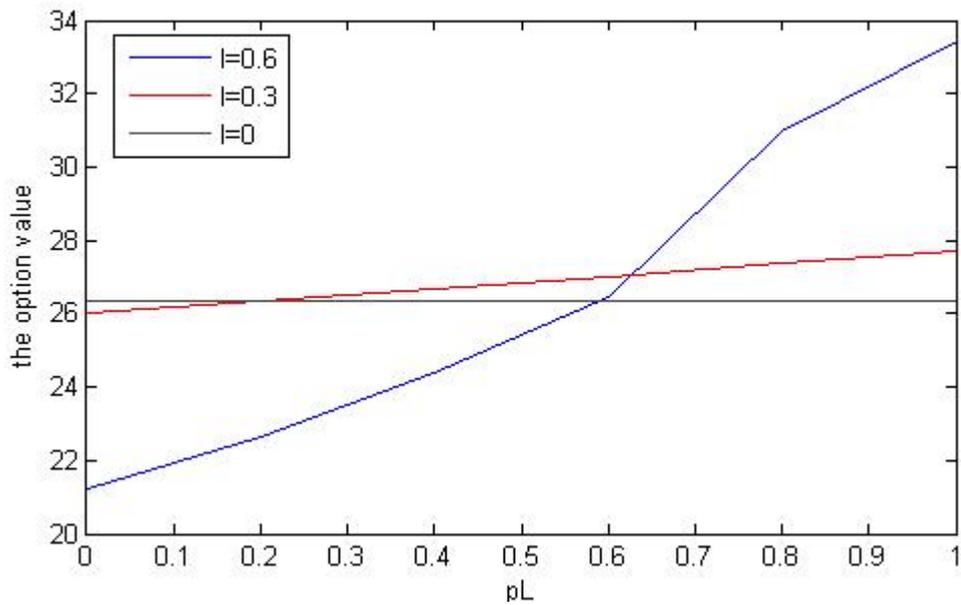


Figure 3: Option value. The following parameters have been used: $pH=0$, $T=2$, $K=100$, $r=4\%$, $q=1.5\%$, $\lambda=0.2$, $\sigma=42.7\%$

Figure 3 illustrates the effect of resetting on valuation of new option grants for three different resetting barriers $l=0$, 0.3 and 0.6 . When l is equal to zero, there is no resetting barrier. In the case $l=0.6$, resetting has a significant effect on option valuation. As pL goes from 0.1 to 1 , the option value varies from 22.6 to 33.7 .

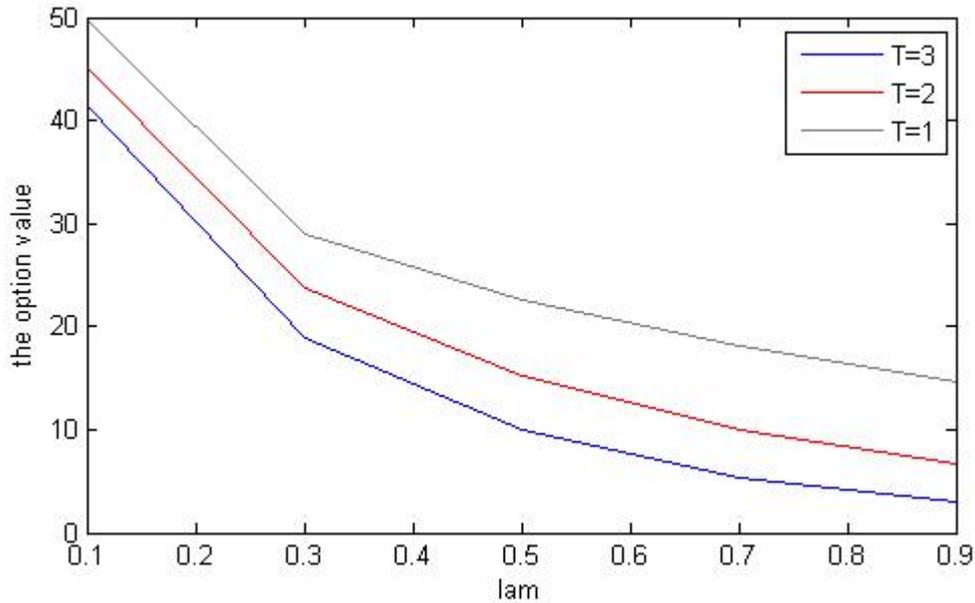


Figure 4: Option value. The following parameters have been used: $pH=1$, $pL=1$; $l=0.6$, $K=100$, $r=4\%$, $q=1.5\%$, $\lambda=0.2$, $\sigma=42.7\%$

The effect of the employment shock on the option value for three different values of the vesting period $T=1, 2$ and 3 is illustrated on figure 4. As the exit rate goes up from 0.1 to 1 , the expected duration goes down from 10 to 1 year. When $T=2$ (middle case), the option value goes from 45 to 7 , reduced by 85% . The dramatic effect of λ on option valuation is caused by the trading restriction that the option recipient is not allowed to sell the option upon his separation from the firm, but has to either exercise the option if it is vested or to forfeit otherwise.

Trading restriction has a significant impact on the option value and this impact depends critically on the length of the vesting period. As the vesting period becomes longer, the reduction in option value caused by a given exit rate becomes even bigger. It shows also the importance of the interactions among different features of executive stock options.

Working on this work, I have gotten a deeper understanding of the methods of valuation options, as well as experience with the program applications Excel and Matlab.

References

- Financial Accounting Standard Board. 1995. FASB 123: Accounting for Stock-Based Compensation.
- R. Sircar, W. Xiong. 2007. "A general framework for evaluating the executive stock options", *Journal of Economic Dynamics&Control* 31(2007) 2317-2349
- McDonald, R. L. (2013): *Derivatives Market*. Pearson Education, Inc.
- S. Benninga, (2008): *Financial modeling*. Massachusetts Institute of Technology
- S. Benninga, (2011): *Principles of finance with Excel*. Oxford University Press, Inc,
- F. Black, M. Scholes. 1973. "The pricing of options and corporate liabilities", *Journal of political economy*, volume 81, issue 3 (May-Juni): 637-654
- J.C. Cox, S. Ross, and M. Rubinstein. 1979. "Option pricing: a simplified approach", *Jornal of Financeal Economisc*, volume 7 (September): 224-264
- J. Hull, A. White. 2004. "How to value employee stock options", *Financial analysts jornal*, volume 60, no 1 (January/February): 114-119

A Option date

Table 1: The options parameters²

The stock price, S	100
The strike price, K	100
Volatility, σ	42,7%
Dividend yield	1,5%
Interest rate	4%
Exit rate	0,2
Time to maturity	10
Vesting period	2

Table 6: Comparing option values estimated from different models²

Panel A: Vesting period, early exit (no reload or reset)				
Panel A and B use the following parameters: S=100, K=100, $\sigma=42,7\%$, $q=1,5\%$, $r=4\%$, $\lambda=0,2$, $T_{vest}=2$, $T_{maturity}=10$, $\rho_H=0$, $\rho_L=0$				
	VERR (binomial)	VERR (analytical)	FASB method	Hull-White
Option value	26,38	27,12	24,17	26,35
Panel B: Vesting period, early exit, reload (no reset)				
	VERR (binomial)	VERR (analytical)	FASB method	Hull-White
Option value	30,5	30,75	N/A	22,79
Panel C: Vesting period, early exit, reload and reset				
This panel uses the following parameters: S=100, K=100, $\sigma=42,7\%$, $q=1,5\%$, $r=4\%$, $\lambda=0,2$, $T_{vest}=2$, $T_{maturity}=10$, $\rho_H=1$, $\rho_L=1$				
	VERR (binomial)	VERR (analytical)	FASB method	Hull-White
Option value	35,71	35,7	N/A	22,51
aThe value is based on a barrier lever of $h=5.77$ from the VERR analytical model				
bThe value is based on a barrier lever of $h=5.77$ from the VERR analytical model				
cThe value is based on a barrier lever of $h=5.77$ from the VERR analytical model				

² The tables are taken from "A general framework for evaluating the executive stock options", Sircar and Xiong (2006)

B Matlab and VBA codes

B.1 The analytical approach provided by Sircar and Xiong (2006)

The Matlab code is self written based on the formulas in Section 4 A general framework for evaluating executive stock options for both the options with and without reload provision

```
clear all;
S=100;
r=0.04;
K=100;
sig=0.427;
q=0.015;
T=2;
t=0;
lam=0.2;

pH=1;
pL=0;
l=0.00001;

k1=1/sig^2*(-(r-q-0.5*sig^2)+sqrt((r-q-0.5*sig^2)^2+2*sig^2
*(lam+r)));
k2=1/sig^2*(-(r-q-0.5*sig^2)-sqrt((r-q-0.5*sig^2)^2+2*sig^2*
(lam+r)));

D=0.29
diff=10;

while diff>0.00001
syms a1 a2 h b1 b2 real;
equ=solve(a1*h^k1+a2*h^k2-q*h/(lam+q)-
pH*D+r/(lam+r),a1*k1*h^(k1-1)+a2*k2*h^(k2-1)+lam/(lam+q)-
```

```

1,a1+a2+lam*(r-q)/(lam+q)/(lam+r)-b1-b2
,a1*k1+a2*k2+lam/(lam+q)-b1*k1-b2*k2,b1*l^k1+b2*l^k2-
pL*D*1,a1,a2,h,b1,b2);
h=equ.h;
a1=equ.a1;
a2=equ.a2;
b1=equ.b1;
b2=equ.b2;

teta1=k1*(r-q+0.5*(k1-1)*sig^2);
teta2=k2*(r-q+0.5*(k2-1)*sig^2);
d1=(log(S/K)+(r-q+0.5*sig^2)*(T-t))/(sig*sqrt(T-t));
d2=(log(S/K)+(r-q-0.5*sig^2)*(T-t))/(sig*sqrt(T-t));
d1h=(log(S/(h*K))+(r-q+0.5*sig^2)*(T-t))/(sig*sqrt(T-t));
d2h=(log(S/(h*K))+(r-q-0.5*sig^2)*(T-t))/(sig*sqrt(T-t));
d3=d2+k1*sig*sqrt(T-t);
d3h=d2h+k1*sig*sqrt(T-t);
d4=d2+k2*sig*sqrt(T-t);
d4h=d2h+k2*sig*sqrt(T-t);

e=exp(1);
w=e^(-(r+lam)*(T-t))*(S*e^((r-q)*(T-t))*normcdf(d1h)-(1-
pH*D)*K*normcdf(d2h)+lam*S*e^((r-q)*(T-t))*(normcdf(d1)-
normcdf(d1h))/(lam+q)-lam*K*(normcdf(d2)-
normcdf(d2h))/(lam+r)+K*(S/K)^k1*e^(teta1*(T-t))*((a1-
b1)*normcdf(d3)-a1*normcdf(d3h))+K*(S/K)^k2*e^(teta2*(T-
t))*((a2-b2)*normcdf(d4)-a2*normcdf(d4h)));

V2=w+b1*K*(S/K)^k1;

diff=eval((V2/100-D));

D=eval(V2/100);

```

```
end;
```

```
x=sprintf('D=%0.4f Vmin=%0.4f\n h=%0.4f',D,eval(V2),eval(h));  
disp(x);
```

B.2 The analytical approach provided by Sircar and Xiong (2006)

The Matlab code is self written based on the formulas in Section 4 A general framework for evaluating executive stock options for the options with reload and resetting provision.

```
clear all;  
S=100;  
r=0.04;  
K=100;  
sig=0.427;  
q=0.015;  
T=2;  
t=0;  
lam=0.2;  
  
pH=1;  
pL=1;  
l=0.6;  
  
k1=1/sig^2 *(-(r-q-0.5*sig^2 )+sqrt((r-q-0.5* sig^2  
)^2+2*sig^2 *(lam+r)));  
k2=1/sig^2 *(-(r-q-0.5* sig^2 )-sqrt((r-q-0.5* sig^2  
)^2+2*sig^2* (lam+r)));  
  
D=0.32;  
diff=10;
```

```

while diff>0.00001;
syms a1 a2 h b1 b2 real;

equ = solve(a1*h^k1+a2*h^k2-q*h/(lam+q) -
pH*D+r/(lam+r),a1*k1*h^(k1-1)+a2*k2*h^(k2-1)+lam/(lam+q)-
1,a1+a2+lam*(r-q)/(lam+q)/(lam+r)-b1-b2
,a1*k1+a2*k2+lam/(lam+q)-b1*k1-b2*k2,b1*1^k1+b2*1^k2-
pL*D*1,a1,a2,h,b1,b2);
h=equ.h;
a1=equ.a1;
a2=equ.a2;
b1=equ.b1;
b2=equ.b2;

teta1=k1*(r-q+0.5*(k1-1)* sig^2 );
teta2=k2*(r-q+0.5*(k2-1)* sig^2 );
d1=(log(S/K)+(r-q+0.5* sig^2)*(T-t))/(sig*sqrt(T-t));
d2=(log(S/K)+(r-q-0.5* sig^2)*(T-t))/(sig*sqrt(T-t));
d1h=(log(S/(h*K))+(r-q+0.5* sig^2)*(T-t))/(sig*sqrt(T-t));
d2h=(log(S/(h*K))+(r-q-0.5* sig^2)*(T-t))/(sig*sqrt(T-t));
d3=d2+k1*sig*sqrt(T-t);
d3h=d2h+k1*sig*sqrt(T-t);
d4=d2+k2*sig*sqrt(T-t);
d4h=d2h+k2*sig*sqrt(T-t);

e=exp(1);

w=e^(-(r+lam)*(T-t) )*(S*e^((r-q)*(T-t)) * normcdf(d1h)-(1-
pH*D)*K*normcdf(d2h)+lam*S*e^((r-q)*(T-t))* (normcdf(d1)-
normcdf(d1h)) /(lam+q) - lam*K*(normcdf(d2)-
normcdf(d2h))/(lam+r) +K*(S/K)^k1* e^(teta1 *(T-t) )* ((a1-
b1)*normcdf(d3)-a1*normcdf(d3h))+K*(S/K)^k2* e^(teta2*(T-
t))*((a2-b2)*normcdf(d4)-a2*normcdf(d4h)));

```

```

XX=1*K;
w1=e^(-(r+lam)*(T-t)) * ((XX^2/S)*e^((r-q)*(T-t)) *
normcdf(d1h) - (1-pH*D)*K*normcdf(d2h) + lam*(XX^2/S)*e^((r-q)*(T-
t)) * (normcdf(d1) - normcdf(d1h)) / (lam+q) - lam*K*(normcdf(d2) -
normcdf(d2h)) / (lam+r) + K*((XX^2/S)/K)^k1 * e^(teta1 *(T-t) ) *
((a1-b1)*normcdf(d3) - a1*normcdf(d3h)) + K*((XX^2/S)/K)^k2 *
e^(teta2*(T-t)) * ((a2-b2)*normcdf(d4) - a2*normcdf(d4h)));

kk=2*(r-q)/sig^2;
V=w - (XX/S)^( (kk-1) ) * w1 + b1 * K * (S/K)^k1 + b2 * K * (S/K)^k2;

diff=abs(eval((V/100-D)));

D=eval(V/100);
end;
x=sprintf('D=%0.4f Vmin=%0.4f\n h=%0.4f',D,eval(V),eval(h));
disp(x);

```

B.3 The binomial version of the analytical approach provided by Sircar and Xiong (2006)

The Matlab code is self written based on the formulas in Appendix D A general framework for evaluating executive stock options.

```

clear all;

S=100;
r=0.04;
K=100;
sig=0.427;
q=0.015;
T=10;
Vest=2;
lam=0.2;

```

```

n=200;
up=exp(sig*sqrt(1/n))

down=exp(-sig*sqrt(1/n))

% piUp=(exp((r-q)*(1/n))-down)/(up-down)
piUp=(exp(r*(1/n))*exp(-q*(1/n))-down)/(up-down)
piDown=1-piUp
D=0.4;
pH=0; %reload
pL=0; %reset
l=0.0000000006;
diff=10;

Ss=zeros(T*n,T*n);
Opt=zeros(T*n,T*n);

while abs(diff)>0.000000001
for i=0:1:(T*n)
    for j=0:1:(i)
        Ss(i+1,j+1)=S*(up^j)*down^(i-j);
    end
end

for i=0:1:(T*n)
    Opt(T*n+1,i+1)=max(Ss(T*n+1,i+1)-K,0);
end

for i=T*n-1:-1:0
    for j=0:1:(i)
        if (i>=Vest*n)&&(Ss(i+1,j+1)>1*K)
            if (Ss(i+1,j+1)<K)
                c1=0;
            else
                c1=(Ss(i+1,j+1)-K+pH*K*D);
            end
        end
    end
end

```

```

    if (Ss(i+2,j+2)-K)>0
        c2=(Ss(i+2,j+2)-K);
    else
        c2=0;
%       c2=(Ss(i+2,j+2)-K);
    end

    if (Ss(i+2,j+1)-K)>0
        c3=(Ss(i+2,j+1)-K);
    else
        c3=0;
%       c3=(Ss(i+2,j+1)-K);
    end

        Opt(i+1,j+1)=(1-lam/n)*max(c1,exp(-
r/n)*(Opt(i+2,j+2)*piUp+piDown*Opt(i+2,j+1)))+lam/n*exp(-
r/n)*(piUp*c2+piDown*c3);

    end
    if (i>=Vest*n)&&(Ss(i+1,j+1)<=1*K)
        Opt(i+1,j+1)=pL*1*K*D;
    end
    if (i<Vest*n)&&(Ss(i+1,j+1)>1*K)
        Opt(i+1,j+1)=(1-lam/n)*exp(-
r/n)*(piUp*Opt(i+2,j+2)+piDown*Opt(i+2,j+1));
    end
    if (i<Vest*n)&&(Ss(i+1,j+1)<=1*K)
        Opt(i+1,j+1)=pL*1*K*D;
    end
end
end
end
diff=D-Opt(1,1)/100
D=Opt(1,1)/100;
end
Opt(1,1)

```

B.4 The Hull-White method for evaluation ESOs

The VBA code is taken from Benninga (2008)

```
Function ESO(Stock As Double, X As Double, T As Double, Vest
As Double, _
    Interest As Double, Sigma As Double, Divrate As Double, _
    Exitrate As Double, Multiple As Double, n As Single)

    Dim Up As Double, Down As Double, R As Double, Div As
Double, _
    piUp As Double, piDown As Double, Delta As Double, _
    i As Integer, j As Integer

    ReDim Opt(T * n, T * n)
    ReDim S(T * n, T * n)
    Up = Exp(Sigma * Sqr(1 / n))
    Down = Exp(-Sigma * Sqr(1 / n))
    R = Exp(Interest / n)
    Div = Exp(-Divrate / n)
    piUp = (R * Div - Down) / (Up - Down) 'Risk-neutral up
probability
    piDown = (Up - R * Div) / (Up - Down) 'Risk-neutral down
probability

    'Defining the stock price
    For i = 0 To T * n
        For j = 0 To i ' j is the number of Up steps
            S(i, j) = Stock * Up ^ j * Down ^ (i - j)
        Next j
    Next i

    'Defining the option value on the last nodes of tree
    For i = 0 To T * n
        Opt(T * n, i) = Application.Max(S(T * n, i) - X, 0)
    Next i
```

```

'Early exercise when stock price > multiple * exercise
after vesting
For i = T * n - 1 To 0 Step -1
For j = 0 To i
If i > Vest * n And S(i, j) >= Multiple * X Then _
    Opt(i, j) = Application.Max(S(i, j) - X, 0)
If i > Vest * n And S(i, j) < Multiple * X Then _
    Opt(i, j) = ((1 - Exitrate / n) * (piUp * Opt(i + 1, j
+ 1) + _
    piDown * Opt(i + 1, j)) / R + Exitrate / n * _
    Application.Max(S(i, j) - X, 0))
If i <= Vest * n Then Opt(i, j) = (1 - Exitrate / n) * _
(piUp * Opt(i + 1, j + 1) + piDown * Opt(i + 1, j)) / R

Next j
Next i

ESO = Opt(0, 0)
End Function

```