Master's thesis

NTNU
Norwegian University of Science and Technology
Faculty of Economics and Management
Dept. of Industrial Economics and Technology Management

Andreas D. Brynildsen
Håkon Andreas Hyttedalen

# Negotiations Concerning Speculative Real Estate Investments

## Development of a practical autonomous negotiation model

Master's thesis in Industrial Economics and Technology Management
Supervisor: Verena Hagspiel
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NTNU
Kunnskap for en bedre verden

*One may never be able to predict or to simulate in laboratory setting all the aspects of complex real-world negotiation, but there is no question as to the value of applying decision-theoretic concepts: analysis can help.*

- Howard Raiffa, *The Art and Science of Negotiation*, 1982.

# Preface

This thesis is written as the final part of the Master of Science degree in Industrial Economics and Technology Management at the Norwegian University of Science and Technology (NTNU), with specialization in Financial Engineering.

The authors of this thesis, Andreas D. Brynildsen and Håkon Andreas Hyttedalen, wanted to extend their recent contribution to the Norwegian speculative real estate market by creating a negotiation model. By combining their passion for real estate with their academical background and software skills, they were able to create a practical tool for real estate investors, and derive an optimal bidding strategy for a seller in the Norwegian market. To the best of the authors' knowledge, they were the first to do so.

This thesis was conducted in collaboration with the Norwegian real estate investing firm Securum Eiendom AS, providing valuable insight from the industry.

Norwegian University of Science and Technology

Trondheim, June 2021

Andreas Brynildsen

Andreas D. Brynildsen

Håkon Andreas Hyttedalen

# Acknowledgements

We would first and foremost like to thank our supervisor, Prof. Verena Hagspiel at the NTNU. Her input has been vital for the outcome of this master's thesis. Next, we would like to thank our industry partner Securum Eiendom AS for teaching us about the industry and providing us with much appreciated feedback. A special thanks goes to CEO Odd Hyttedalen for sharing his expertise and experiences within real estate investing and negotiations. Without his input, we would not have been able to parameterize our model as we did. Further, we would like to thank the author of "Psychological pricing in mergers & acquisitions using game theory", Prof. Nishkant Agarwal at the University of Perth for valuable explanations of his paper. Additionally, we thank Prof. Dr. Gabrielle Wanzenried from Institute of Financial Services in Zug, Switzerland, who inspired and first introduced game theory to the authors of this paper. Lastly, we would like to thank our families for being supportive and understanding. Especially Hanne Nicole Briedis Hyttedalen, who recently married one of the authors of this paper, for being tolerant, patient, and allowing late night working on this thesis.

A.D.B & H.A.HY


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# Abstract

Negotiations are a crucial part of the real estate business, yet the presence of practically applicable negotiation tools in the game theory literature is scarce. More recently, progress within computer science has allowed the development of autonomous negotiation models by utilizing artificial intelligence and machine learning. However, even with all the computational power in the world, existing models fall short in overcoming some of the fundamental practical negotiation issues, particularly relevant for speculative real estates; namely, establishing a seller's reservation price and optimal initial counteroffer. In this thesis, we overcome these practical issues by developing an autonomous negotiation model, which seeks to assist a real estate investor with the objective to maximize profit from a property in the scope of a bilateral bargaining game. In the presented model, we combine existing machine learning algorithms used in autonomous negotiation models with real options valuation techniques. To the best of our knowledge, we are the first to develop a practical negotiation tool for this purpose.

Together with a well-established Norwegian real estate investment firm, we parameterize the model to the characteristics of the Norwegian market and identify, by simulating millions of games, the best strategy in terms of payoff. Then, we use our findings from the simulations in a recent +100 MNOK deal to see how this strategy would have performed in real-life negotiation and to obtain general insights for this market.

***Keywords*** – Real Estate Negotiation, Bilateral Game Theory, Bargaining Game, Automated Negotiation, Bayesian Learning, Real Options Valuation, Profit Maximization.

# Sammendrag

Forhandlinger er en avgjørende del av eiendomsbransjen, men tilstedeværelsen av praktisk anvendbare forhandlingsverktøy i spillteorilitteraturen er knapp. I senere tid har fremskritt innenfor datateknologi muliggjort utviklingen av autonome forhandlingsmodeller, ved å ta i bruk kunstig intelligens og maskinlæring. Likevel, selv med all datakraft i verden, så klarer ikke dagens modeller å overkomme noen av de fundamentale praktiske forhandlingsutfordringene, spesielt relevant for spekulative eiendommer; nemlig, å bestemme en selgers reservasjonspris og optimale åpningsmotbud. I denne avhandlingen løser vi disse praktiske utfordringene ved å utvikle en autonom forhandlingsmodell, som skal hjelpe en eiendomsinvestor med målet om å maksimere profitt av en eiendom gjennom et bilateralt forhandlingsspill. I den presenterte modellen kombinerer vi eksisterende maskinlæringsalgoritmer brukt i autonome forhandlingsmodeller, med realopsjonsverdivurderingsteknikker. Så vidt vi vet, er vi de første til å utvikle en praktisk forhandlingsmodell for dette formålet.

Sammen med et veletablert norsk eiendomsinvesteringsselskap, parameteriserer vi modellen vår til kjennetegnene på det norske markedet, og identifiserer gjennom å simulere millioner av spill, den strategien som gir høyest belønning. Deretter bruker vi funnene våre fra simuleringene i en nylig +100 MNOK avtale for å se hvordan denne strategien ville ha utspilt seg i en ekte forhandling, og for å få generell innsikt i dette markedet.

***Nøkkelord*** – Eiendomsforhandling, Bilateral Spillteori, Forhandlingsspill, Automatisert Forhandling, Bayesisk Læring, Realopsjonsverdivurdering, Profittmaksimering.

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# 1 Introduction

"There is no worse feeling than when you propose an offer and the buyer immediately accepts it with a smug smile" (real estate investor Odd Hyttedalen, personal communication April 12, 2021). In real estate negotiations, a seller's reservation price represents the least they are willing to sell a property for, and should correspond to the best alternative to a negotiated agreement (BATNA). Oppositely, the buyer's upper limit is their reservation price. A mistaken judgement of the opponent's reservation price could lead to a far smaller payoff that what could have been achieved. Or maybe even worse, either the seller or the buyer gets too greedy and initiate the negotiations too far from a realistic price, resulting in no deal taking place.

The motivation behind this thesis is to assist an investor with the objective to maximize profit from negotiations concerning a speculative real estate investment. For the purpose of this thesis, we define speculative real estate investment as investments related to properties with a significant unlocked value from a set of mutually exclusive prospective development projects[1].

In this thesis, we develop an autonomous negotiation model, intended to be used as a decision tool supporting an investor selling a speculative real estate. The modelling approach both extends and combines existing models and techniques found in the fields of real options valuation, game theory, and Bayesian machine learning. In particular, we address the situation where only two parties are involved, frequently referred to as bilateral negotiation games. However, to the best of our knowledge, even the most advanced negotiation models (Nash (1950b), Roy (1989), Zeng & Sycara (1997), Agrawal & Chari (2009), Sim et al. (2009), Baarslag et al. (2013), Saha et al. (2013) and Williams et al. (2013), among others) lack a scientific method of overcoming the practical issues of establishing an agent's reservation price and initial offer. Moreover, probabilistic machine learning algorithms often require numerous negotiation rounds, in contrast to what is generally observed in real-life. In this thesis, we overcome these practical issues by using real options valuation techniques and developing a framework allowing the investor to

[1] An example of a speculative real estate investment includes the purchase of a small house with a belonging larger piece of land located in a city center, with a prospective project to change the zoning classification from residential to a high-rise commercial building.

parameterize the model to reflect their business environment.

Consequently, the problem statement in this thesis is:

**Problem Statement**:

*Develop an autonomous negotiation model which overcomes the existing models' practical limitations in a bilateral negotiation game, to maximize profit for an investor selling a speculative real estate.*

The result is a model which incorporates Time-dependent tactics (T) in a constructed Realistic environment (R) in the real estate business. Further, it is Autonomous (A), meaning the output proposes bids generated by itself in a negotiation. Additionally, it is developed from a Practical (P) point of view, and foremost intended to be applied to speculative Properties (P). The developed model, referred to as TRAPP, provides the possibility to:

1. Determine a seller's reservation price.
2. Generate an optimal initial offer for a seller, based on both the seller's and the buyer's reservation price.
3. Simulate a realistic environment, by allowing the user of the model to easily parameterize the model to the market they operate in.
4. Determine which strategy a seller should adopt in a negotiation, based on simulations in a virtual laboratory setting.

Moreover, in collaboration with the Norwegian real estate investment company Securum Eiendom AS, we parameterize our model to reflect the Norwegian speculative real estate market, and obtain general insights related to the outcome of different strategies. Through millions[2] of simulations, we derive the optimal strategy in terms of achieving the highest expected profit for a seller in this market.[3] In addition, we validate this strategy in a recent negotiation process where Securum sold a property.

[2]We simulate three types of opening bids with 11 different strategies 10,000 times with and without the possibility to dynamically change strategy, resulting in a total of $3 \cdot 11^2 \cdot 10,000 = 3,630,000$ simulations, before the sensitivity analysis.

[3]Note that previous literature like Sim et al. (2009) for instance, derive optimal strategies in terms of percentage number of deals and lowest average number of rounds before a deal takes place.

In this thesis, the words *agent* and *player* are used interchangeably, and refer to a seller or buyer, which make up all the players involved in this bilateral bargaining game. In general terms, an *investor* is used as a synonym for either a seller or buyer. However, in this thesis, we often refer to the investor as a seller and the *opponent* as a buyer. Furthermore, the words *offer* and *bid* are used interchangeably and we assume that both agents propose bids/offers in the negotiations.

The remainder of this thesis is organized as follows: In Section 2 Background & Related Literature, we provide an overview of relevant literature conducted in the past and describe the game assessed in this thesis. Next, the methodology and modelling approach is presented in Section 3 Model. Further, we parameterize the model to fit our industry partner and their business environment in Section 4 Empirical Results before we look at a real-life case study to test our model in Section 5 Case Study. Finally, our conclusions, general insights, and suggestions for further research are presented in Section 6 Conclusion. Extended derivations and an overview of the parameters we use can be found in the Appendix.

# 2 Background & Related Literature

In this section, we present the relevant literature for real options valuation of speculative real estate investments and explain how game theory is applicable in real estate negotiations. First, we set the scope of the speculative real estate investment process we are assessing. Next, we present a method of establishing an agent's reservation price. Remember that a seller's reservation price is the least they are willing to sell the property for, while for the buyer, it is the maximum they are willing to pay. Finally, the characteristics of the game considered in this thesis are presented and placed in the context of game theory.

## 2.1 Scope of the Investment Process

Together with our industry partner, we classify the Norwegian speculative real estate investment market in four sequential stages of decision making. Stage 0 is where the investor decides to purchase the property or not, followed immediately by Stage 1 when the investor chooses to wait or act. If the investor chooses to act, they are faced with three alternatives: Develop the property, sell it on the open market, or enter into negotiations. Figure 2.1 depicts an overview of the stages, accompanied with an explanation of the color coding provided in Table 2.1.

| **Color Coding** | |
|---|---|
| Black | Out of scope for both theses. |
| Gray | Addressed in Brynildsen & Hyttedalen (2020). |
| White | Addressed both in Brynildsen & Hyttedalen (2020) and in this thesis. |
| Green | Addressed in this thesis. |

**Table 2.1:** Explanation of the color coding used in Figure 2.1.

The black area is out of our scope in both theses, while the gray areas are assessed in Brynildsen & Hyttedalen (2020) and related to this thesis. Act at Stage 1 and sell at Stage 2 are assessed in both papers. The main focus in this thesis, is the *negotiation* with a single buyer of a speculative real estate investment in Stage 2.

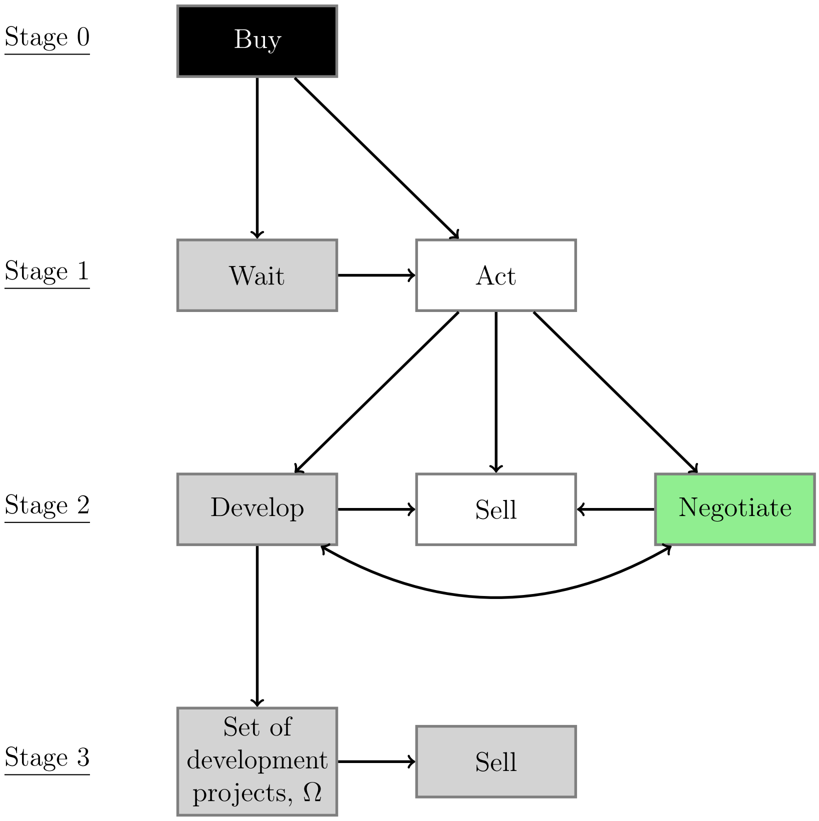



**Figure 2.1:** Overview of an investor's different stages of decision making related to a speculative real estate investment.

## 2.2 Valuation of a Speculative Real Estate Investment

The real options valuation (ROV) model presented in Brynildsen & Hyttedalen (2020) quantifies the value of a speculative real estate investment with mutually exclusive development projects. In addition, it provides a belonging set of optimal decisions to be made in order to maximize the expected payoff *today*. Today refers to the point in time when the property is evaluated. The purpose of the model is to derive an optimal decision policy in terms of what the investor should do at each point in time with the objective to maximize profit of a property. The general steps in the value chain of a speculative real estate investment for an investor, are illustrated in Figure 2.2. It should be noted that the expected payoff, $F^*(0)$, in Figure 2.2 calculated by the ROV-model, reflects the expected profit based on inputs from a particular investor. For another investor, it is plausible to assume that a different expected payoff would have been obtained. $V_0(n)$ is the value of the property at time $n$. The value of a project $i \in \Omega$, where $\Omega$ is the set of all development projects, is $V_i(n)$ at time $n$ if the project is successfully developed. Development must be completed within time $\tau$, $t_{min,i} \leq \tau \leq T$, where $t_{min,i}$ is the minimum time the development of project $i$ takes, and $T$ is the time until the last project expires.

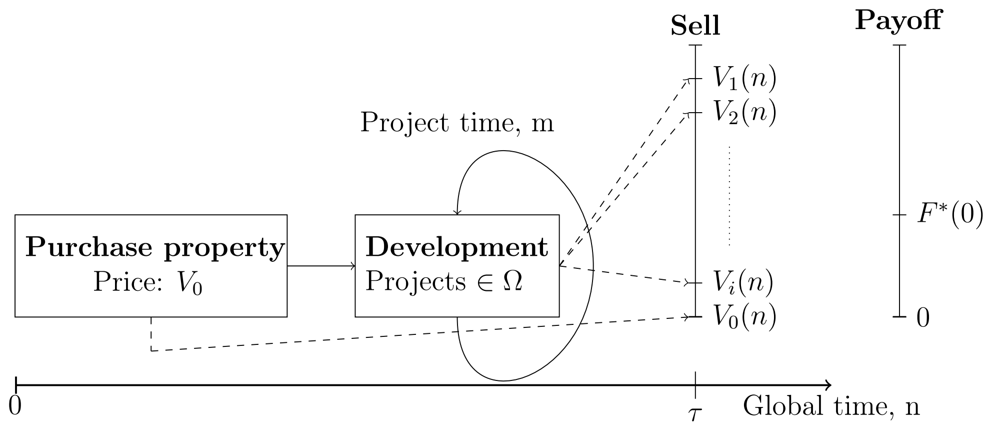



**Figure 2.2:** Simplified representation of the value chain of a speculative real estate investment.

A speculative real estate investment usually refers to a property with several prospected development projects. The investor has to provide the following input parameters and the anticipation of how they will change over time: (1) Today's property value[4] and (2) net holding costs. Moreover, the investor has to provide the project specific development details in each time step which include (3) the cost of continuing to attempt development, (4) the probability of a successful development, and eventually (5) what the property is worth to the investor if a project is successfully developed. The output from the model is a real options value of the speculative real estate investment and a corresponding optimal decision policy. A full overview of the notation used in the ROV-model is found in Table A1.1 in Appendix.

There are particularly two aspects of the ROV-model that are essential for further reading of this thesis. (1) The output is investor-specific. In particular, this implies that different investors with different development projects and input parameters will obtain distinct real options values due to their personal skills, network, and experience, among other aspects. In this thesis, the real options value reflects the assessment made by the investors at Securum. (2) The output is reflecting the real options value evaluated today. To exemplify,

[4]We use the purchasing price as the property value if it is evaluated a short amount of time after it is acquired.

the probability of a successful development could rise due to unforeseen circumstances like a change in political control in the specific municipality the property is located. Additionally, we assume that the investor is both rational and risk-neutral. If the investor has the possibility to obtain a higher payoff, this opportunity will be chosen.

The novelty of the ROV-model presented in Brynildsen & Hyttedalen (2020) is that it quantifies the value of a property by incorporating the options to defer, convert, abandon and keep. Additionally, the option to develop the property is included. Moreover, the modelling approach makes it easy for an investor with basic software skills[5] to use it in practice. Hence, by using the ROV-model, the investor is able to evaluate the property of interest more accurately and establish a reservation price which corresponds to the best alternative to a negotiated agreement (BATNA) for that particular investor.

## 2.3 Game Theory

The negotiation game for a speculative real estate investment is described in this subsection. Furthermore, existing literature concerning these types of games is presented before we summarize our contributions to the literature.

Game theory "provides general mathematical techniques for analyzing situations in which two or more individuals make decisions that will influence one another's welfare." (Myerson, 1991, p. 1). In this thesis, we consider game theory from the perspective of human economic behavior[6]. The origin of game theory, in the scope of this thesis, roots back to the work in Zemerlo (1913), Borel (1921), von Neumann (1928) and von Neumann & Morgenstern (1944) which all developed two-person game concepts, among other things, in the field today known as modern game theory. In the 1950s, the Nash equilibrium was developed in Nash (1950b), and laid the foundation for the extensive research within game theory in the following years. In this thesis, we use Nash's pioneering work related to strategic thinking in our negotiation model. We address repeated games and in particular the case of a two-player bargaining game.

In recent years, game theory has been applied in a wide range of practical cases to help managers with their decision making. For instance, the work in Lindstädt & Müller (2009)

[5]It only requires a basic level of skill in Microsoft Excel.
[6]Examples of other perspectives include political, psychological and sociological behaviors.

and Gittins (2012) substantiate the usefulness of game theoretical concepts in real-life situations. The most relevant game theory models for our case are found in Zeng & Sycara (1997) and Sim et al. (2009), which both develop autonomous negotiation models. In Zeng & Sycara (1997), Bayesian machine learning is used to find an agent's optimal strategy, and in Sim et al. (2009), this model is extended to include the realistic possibility for an agent to withdraw from the negotiation. Throughout this thesis, we elaborate why the work in these papers are important for our modelling approach.

In game theory models applied to negotiations, the term *zone of agreement* (ZoA) is an essential concept (Zeng & Sycara, 1997). The ZoA is defined as the overlap between the highest price the buyer is willing to pay and the lowest value the seller will accept, known as the reservation prices. The buyer's reservation price, $RP^B$, has to be higher than the seller's reservation price, $RP^S$, for the ZoA to exist. Contrary, in a situation where $RP^B < RP^S$, a ZoA does not exist and a deal will never take place. Note that even in situations where a ZoA exists, it is not guaranteed that a deal will occur. For instance, if either the seller or the buyer is under a time pressure, the agent with the shortest deadline might eventually be forced to walk away before an agreement is reached. A visualization of the ZoA and the buyer's and seller's reservation price are shown in Figure 2.3.

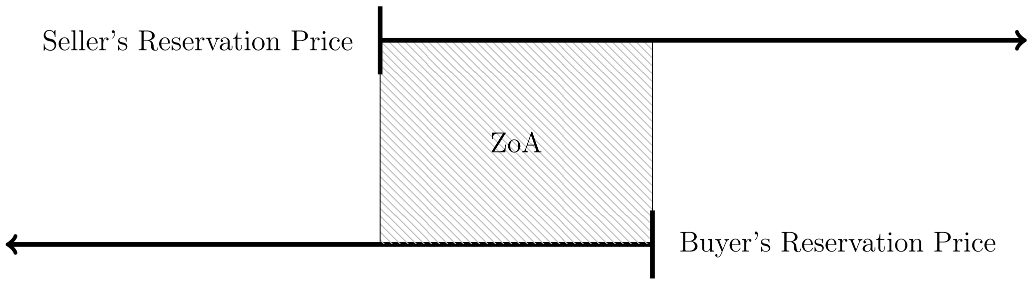



**Figure 2.3:** The zone of agreement (ZoA) is recognized as the area between the seller's and buyer's reservation price.

In this thesis, we base the seller's reservation price on the best alternative to a negotiated agreement (BATNA). We argue that the value of BATNA is, for a speculative real estate, the real options value obtained by using the ROV-model developed in Brynildsen & Hyttedalen (2020). The relationship between the seller's reservation price and the option value, $F^*(0)$, from Figure 2.2 is expressed as follows: $RP^S = F^*(0) + V_0$. This expression is obtained with the assumption that we are assessing a rational and risk-neutral investor. The least acceptable offer should be equivalent to what the investor can expect to obtain from developing the property. Any sale at a price below the expected value from developing the property would be irrational. In the reminder of this section, we focus on the negotiation game for a speculative real estate investment.

### 2.3.1 The Game Considered in this Thesis

The game in this thesis can be viewed as a discrete two-person (bilateral) non-zero-sum bargaining game, first properly introduced by Nash (1950a). However, in contrast to Nash (1950a), the moves in our game are played sequentially. Consequently, both agents, denoted $A$, have perfect information, although the game itself consists of incomplete information. Furthermore, there are no restrictions on the number of rounds played.

We consider the following situation: A real estate investor, denoted seller, $S$, possesses a property with the objective to maximize profit. In the scope of our modelling approach, we assume that the investor can make a profit in only two general ways: (1) Increase the market value of the property by either developing it now or later as described in Figure 2.2 with expected payoff calculated using the ROV-model developed in Brynildsen & Hyttedalen (2020).[7] The investor may keep the property until it is potentially valued higher due to a positive market growth. The second alternative is to (2) sell the property through a bargaining game. In the latter alternative, we assume that the negotiation process consists of only one other agent, the buyer, $B$. Furthermore, the final payoff from the game is calculated as the difference between the settlement price, $P^*$, and the agents' respective reservation price, $RP^B$ and $RP^S$. The payoffs, $U^S$ and $U^B$, are visualised in Figure 2.4. In this thesis, we assess speculative real estate investments in the Norwegian market. Hence, the payoffs are given in NOK higher than the seller's reservation price, or below the buyer's reservation price.

[7]For information, the latest market trend in the Norwegian market shows that waiting before development is currently not of any value (L'Orsa & Eggen, 2021).

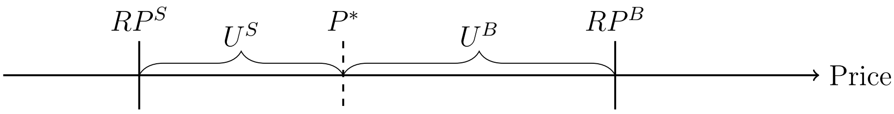



**Figure 2.4:** Illustration of the final payoff achieved for each agent at settlement price, $P^*$.

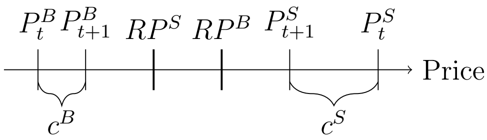
To provide a better understanding of the payoff structure in each round in our game, we address some well-known game theory concepts. The payoff structure from the bargaining game described above can be compared to what is known as an iterated prisoner's dilemma (IPD) (Kendall et al., 2007) and as the "peace-war game" (Shy, 1995). In particular, it has similarities with the "donation game"[8] where both players might offer the other player a benefit, $b$, at a personal cost, $c$. In our case, a benefit for the seller, $b^S$, corresponds to an increased offer by an amount of $c^B$ by the buyer, and vice versa for the buyer, only that it corresponds to a decreased offer from the seller. Figure 2.5 displays the unrealized[9] payoff matrix from the IPD, in terms of a donation game. In the figure, we set a constant, $\alpha$, to be strictly greater than one, assuming that both agents value the act of getting closer to an agreement more than the absolute value of a reduced or increased offer by the seller or the buyer, respectively. The highest payoff for both agents is when they defect while the opponent cooperates. The worst outcome from a round is when the agents cooperate while the opponent defects. When both agents defect, their payoffs are zero. On the other hand, when both agents cooperates, they receive a payoff of $b^A - c^A > 0$. Moreover, it should be noted that since $b^B > (b^B - c^B) > 0 > -c^B$, the game is in fact a prisoner's dilemma in the strong sense, and $2(b^B - c^B) > (b^B - c^B)$ indicates that the game could qualify as an IPD (Axelrod, 1984).

[8]The exact payoff structure in our game differ slightly from the donation game. The purpose of the analogy to the donation game is to highlight the intuition for the payoff structure.

[9]The agents do not receive any payoff until an offer has been placed in the ZoA.

Payoff Matrix

| | | Seller: Cooperate | Seller: Defect |
|---|---|---|---|
| Buyer | Cooperate | $b^S - c^S$ / $b^B - c^B$ | $b^S$ / $-c^B$ |
| Buyer | Defect | $-c^S$ / $b^B$ | $0$ / $0$ |




$$b^S = c^B \cdot \left[\alpha + \frac{c^B}{c^S}\right], \alpha > 1, \text{when } c^S \neq 0$$
$$= c^B, \alpha > 1, \text{when } c^S = 0$$

$$b^B = c^S \cdot \left[\alpha + \frac{c^S}{c^B}\right], \alpha > 1, \text{when } c^S \neq 0$$
$$= c^S, \alpha > 1, \text{when } c^S = 0$$

**Figure 2.5:** Payoff matrix from an iterated prisoner's dilemma donation game.

The bilateral game in our case can be summarized by the following eight characteristics. (1) The game is non-cooperative, meaning that the buyer and seller are assumed not to make alliances. (2) The payoff structure makes it a non-zero-sum game, although a gain for one agent is a direct loss for the other agent at the final agreement price. (3) The game is played sequentially where the player moves every other time. (4) The players have perfect information about the previous actions taken by the opponent. Nevertheless, (5) the game is incomplete as the payoff of the opponent in unknown.[10] (6) As a result of (4) and (5), the game is combinatorial, with no straightforward approach of finding an optimal strategy. (7) The game is not restricted to a finite number of rounds, classifying the game as infinitely long. Lastly, (8) the game is discrete with a finite number of players and possible moves.

### 2.3.2 Existing Negotiation Model Literature

In the existing literature, several automated negotiation models have been developed to help the agents adopting to their opponent in multiple ways (Zeng & Sycara (1997), Ren & Anumba (2002), Sim et al. (2009), Agrawal & Chari (2009), Williams et al. (2011), Chen & Weiss (2013), Baarslag et al. (2013), Saha et al. (2013), Williams et al. (2013), and Yu et al. (2013), among others). In general, the agents adapt their opponent, $\overline{A}$, by estimating the opponent's reservation price, $RP^{\overline{A}}$, deadline, $\tau^{\overline{A}}$, or a combination of these,

[10] In particular, the opponent's reservation price and deadline are unknown.

and adjusts their own strategy according to the estimate(s) obtained. In our model, we estimate both the opponent's reservation price and deadline by building further on the algorithms presented by Roy (1989), Zeng & Sycara (1997) and Sim et al. (2009).

First, we define the term *learning*. In Zeng & Sycara (1997, p. 36), a "sequential decision making negotiation model that is capable of learning" is developed. Here, learning refers to updating an estimation of the opponent's reservation price in each round based on the standard Bayesian updating rule (Baarslag et al., 2016). Despite the fact that the game in Zeng & Sycara (1997) being almost identical to our case, the model proposed, named Bazaar, is not adequate for our purpose due to two main reasons. First, (1) the creators of Bazaar do not take into consideration that the players might have a deadline where they walk away from the negotiations. Consequently, an agreement is always reached when applying the Bazaar-model as long as the buyer's reservation price is higher than the seller's, $RP^B > RP^S$. This is a highly important factor to include in our case, as we anticipate that the opponent can eventually withdraw if the negotiation drags out, even if a zone of agreement (ZoA) exists. In 50 % of the cases we encounter that a ZoA does not exist, as we discuss further in our parameterization of the model in Section 4 Empirical Results. Second, (2) the Bazaar-model requires a domain knowledge about the conditional probability distribution of the opponent's expected offer, given a reservation price, denoted $P(P^{\overline{A}}|RP^{\overline{A}})$. In real estate negotiations, this knowledge is rarely known in the exact details as required by the Bazaar-model, according to our industry partner. Hence, we want to relax this restriction in our model. Moreover, from our perspective, if the opponent's offer, $P(P^{\overline{A}}|RP^{\overline{A}})$, is known prior to the negotiations, there would not be a need for a comprehensive model to estimate the opponent's reservation price.

A model that overcomes the two restrictions in the Bazaar-model highlighted above, is found in Sim et al. (2009). In this model, named BLGAN, $P(P^{\overline{A}}|RP^{\overline{A}})$ is assumed to follow a normal distribution with a standard deviation of one, $\sigma^2 = 1$. The mean value is obtained using a formula which assumes that "initially, it is very likely for an agent to generate a proposal that is far from its reservation price. As time passes, it will generate a proposal that is closer to its reservation price." (Sim et al., 2009, p. 201). The situation where the agents concede monotonically towards their reservation price corresponds well with our situation, and we use this modeling approach in Section 3 Model. In addition, BLGAN has a procedure to estimate the opponent's deadline. However, it requires initial offers from both agents as input parameters, and an agent is first capable of learning[11] after receiving two offers from the opponent. From a practical point of view, the agents have to somehow decide on their initial offer. Consequently, a method for suggesting an optimal initial offer is a feature we implement in our model.

A closed formula for obtaining an optimal offer when the offer from the opponent given a reservation price, $P(P^{\overline{A}}|RP^{\overline{A}})$, follows certain probability distributions, has been derived in Roy (1989). In our model, we use the formula for the optimal offer strategy derived in this paper, when the hypotheses of the opponent's reservation price can be modelled with either a triangular or uniform probability distribution. This is further described in Section 3 Model.

To summarize, our main contributions to the game theory literature in this thesis, are that we:

1. Develop a tool that is able of establishing the reservation price for an investor in the speculative real estate market that can be incorporated in a bilateral bargaining game with incomplete information.
2. Determine the agents' initial offer based on both their own and their opponent's reservation price.
3. Provide empirical evidence of practical usage of game theory in the real estate business through a real-life case study.

[11] Learning in BLGAN refers to estimation of both the opponent's reservation price and deadline.

# 3 Model

The model presented in this section, hereafter referred to as TRAPP[12], is a practical negotiation tool supporting an investor in the speculative real estate market. The aim of the model is to provide the investor with an optimal bid in each round of the negotiations, including the initial offer. We find the optimal offer in each round by simulating a seller with several different bidding approaches, and compare the average values obtained. For instance, we incorporate the possibility to learn about the opponent in order to choose a dynamic bidding strategy. In this section, we first present the modelling approach, followed by a description of the different strategies. Further, methods to estimate the opponent's reservation price, deadline and strategy are presented. Lastly, we look at different approaches the investor can base the initial offer, strategy and deadline on, referred to as *modes*. Discussion of the procedures are provided throughout the text.

## 3.1 Model Setup in TRAPP

The parameters used in TRAPP are listed in Table 3.1. In TRAPP, we consider a bilateral negotiation game between a seller, S, and a buyer, B, referred to as agents, A. We assume that both agents choose their own reservation price, $RP^A$, before the game begins, and that they retain this reservation price throughout the game. In reality, the players might change their reservation price for any reason, as time passes. However, this is beyond the scope of the modelling approach in TRAPP and addressed in Section 6 Conclusion as further research. We consider the best alternative to a negotiated agreement (BATNA) in this case to be the output from the ROV-model developed in Brynildsen & Hyttedalen (2020) and consequently the seller's reservation price in the negotiations. Similar to an agent's reservation price, the upper time limit before an agent walks away from the negotiation, $\tau^A$, is determined prior to the game and is assumed to be constant. In real-life negotiations however, human beings may act irrational and emotional in negotiations and change both their reservation price and deadline throughout the game. However, behavioral psychology is out of the scope for this model.

It is assumed that the seller possesses information regarding the range in which the buyer's

[12] The abbreviation TRAPP is explained in Section 1 Introduction.

| **Indices** | |
|---|---|
| B | Buyer |
| S | Seller |
| A | An agent, $A \in N$ |
| $\overline{A}$ | The other agent, $\overline{A} \in N$ |
| t | Negotiation round, $t \in \{0, \ldots, T\}$, $T = \min\{\tau^B, \tau^S\}$ |
| i | Hypothesis of the buyer's RP. $i \in \{\widetilde{RP}_l^B, ..., \widetilde{RP}_h^B\}$ |
| **Sets** | |
| $N = \{B, S\}$ | Set of players in the game |
| $H = \{\widetilde{RP}_l^B, ..., \widetilde{RP}_h^B\}$ | Set of hypotheses of the buyer's RP |
| $\Lambda = \{0.1, 0.2, 0.33, 0.5, 0.67,$ $1, 1.5, 2, 3, 5, 10\}$ | Set of strategies used by the seller |
| **Parameters** | |
| $\beta$ | Dummy variable. Equals 1 for seller, and 0 for buyer |
| $\lambda_t^A$ | An agent's strategy at $t$, $A \in N$ |
| $P_t^A$ | An agent's offer at $t$, $A \in N$ |
| $P^*$ | Settlement price |
| $\tau^A$ | An agent's time limit in a game, $A \in N$ |
| $\widetilde{\tau}_t^{\overline{A}}$ | An agent's estimate of the opponent's time limit at $t$ |
| $RP^A$ | An agent's reservation price in a game, $A \in N$ |
| $\widetilde{RP}_t^{\overline{A}}$ | An agent's estimate of the opponent's $RP$ at $t$ |
| $\widetilde{RP}_l^B$ | Seller's estimation of $RP^B$ lowest value |
| $\widetilde{RP}_*^B$ | Seller's estimation of $RP^B$ most probable value |
| $\widetilde{RP}_h^B$ | Seller's estimation of $RP^B$ highest value |
| $U^A$ | An agent's payoff at $P^*$, $A \in N$ |

**Table 3.1:** The parameters used in TRAPP.

reservation price is found. Hence, the seller estimates an upper and lower limit of the buyer's reservation price, denoted $\widetilde{RP}_h^B$ and $\widetilde{RP}_l^B$, respectively. It is assumed that the buyer's true reservation price is within this range. In addition, it is possible for the seller to specify a most probable estimate of the opponent's reservation price, $\widetilde{RP}_*^B$.

In TRAPP, we need to provide an estimate of the opponent's deadline, $\widetilde{\tau^A}$ in the first two rounds of the game, identical to the estimation procedure in Sim et al. (2009). If an agent does not specify this explicitly before the game begins, we assume for the first two rounds that the buyer's deadline is equal to the seller's. Furthermore, both players' initial offer take place in the first round. The moment the first player receives a second offer from the opponent, the second round starts. This continues for the remainder of the negotiation.

## 3.2 Negotiation Tactics

Several classifications of negotiating tactics are found in the existing literature. In Baarslag et al. (2016), two categories of negotiation tactics are presented: Time- and behavioral-dependent. The difference is that time-dependent tactics are based on the agent's own time limit referred to as deadline, $\tau^A$, while behavioral-dependent tactics are subject to the opponent's bidding behaviour. In this thesis, we only consider time-dependent tactics, assuming that the agents propose offers independent of their opponent's behaviour. The agents generate their bids following a time-dependent tactic, where it is assumed that the buyer's bids are strictly increasing and the seller's bid strictly decreasing. This bidding pattern is frequently found in most negotiation processes in real estate, although it in some situations occurs that a bid from an agent deviates from this traditional pattern (Agarwal & Zeephongsekul, 2011). In particular, we adopt the same formula for generating an agent's offer as in Sim (2005) and Sim et al. (2009) in TRAPP,

$$P_t^A = P_{t-1} + (-1)^{\beta} \cdot |RP - P_{t-1}| \cdot \left(\frac{1}{\tau - t - 1}\right)^{\lambda^A}, \tag{3.1}$$

where $\beta = 1$ when the agent is the seller and $\beta = 0$ for the buyer.

Further, two types of strategies within time-dependent tactics that we use in this thesis, are presented in Baarslag et al. (2016). The first strategy type is called *boulware*. By following this strategy, the agents concede slowly in the beginning and bid their reservation price only at their time limit, denoted $\tau^A$. In a scenario where both agents follow an extreme boulware strategy, the agents will keep on bidding their initial offer until one agent reaches the last round $\tau^A$, and eventually bids the reservation price, $RP^A$. Secondly, an agent could be a *conceder*. Oppositely from an extreme boulware strategy, the agents concede towards their reservation price much sooner with an extreme conceder strategy.

Additionally, a third term called *linear* is introduced in Sim et al. (2009) where the agent concedes linearly. This strategy falls between the conceder and boulware strategy. According to our industry partner, this is the most likely bidding strategy observed for buyers in their everyday business. By only considering the three strategies presented until this point, we end up with three types of investors: extreme conceder, extreme boulware, or linear. However, this would not be representative in real-life since there are different degrees of the conceder and boulware strategies. To allow for sufficient granularity of the strategies, both Sim et al. (2009) and Baarslag et al. (2016) introduce a concession factor, denoted $\lambda^A$, allowing the agents to provide their own willingness to settle the deal quickly, i.e. different degrees of boulware and conceder. A $\lambda^A$ greater than one implies a boulware strategy and less than one implies a conceder strategy. When $\lambda^A = 1$, it corresponds to the linear strategy. The strategy types for different $\lambda^A$ are summarized in Table 3.2.

| $\boldsymbol{\lambda^A}$ | **Strategy Type** |
|---|---|
| $< 1$ | Conceder |
| $= 1$ | Linear |
| $> 1$ | Boulware |

**Table 3.2:** Strategy types for different strategy concession factors, $\lambda^A$.

In this thesis, we implement seven different types of strategies for the buyer, $\lambda^B$, and 11 different strategies in the set $\Lambda$ for the seller, $\lambda^S$, in TRAPP to include a greater specter of agents. The reason for choosing a fewer set of strategies for the buyer is that we focus on the seller in this thesis. Nevertheless, we find that seven strategies for the buyer is sufficient for the purpose of this thesis to simulate a realistic business environment for our industry partner.

To illustrate how an offer made by the seller is dependent on $t$ for different strategies, $\lambda^S$, Figure 3.1 is presented. In this numerical example, the seller's reservation price, $RP^S$,

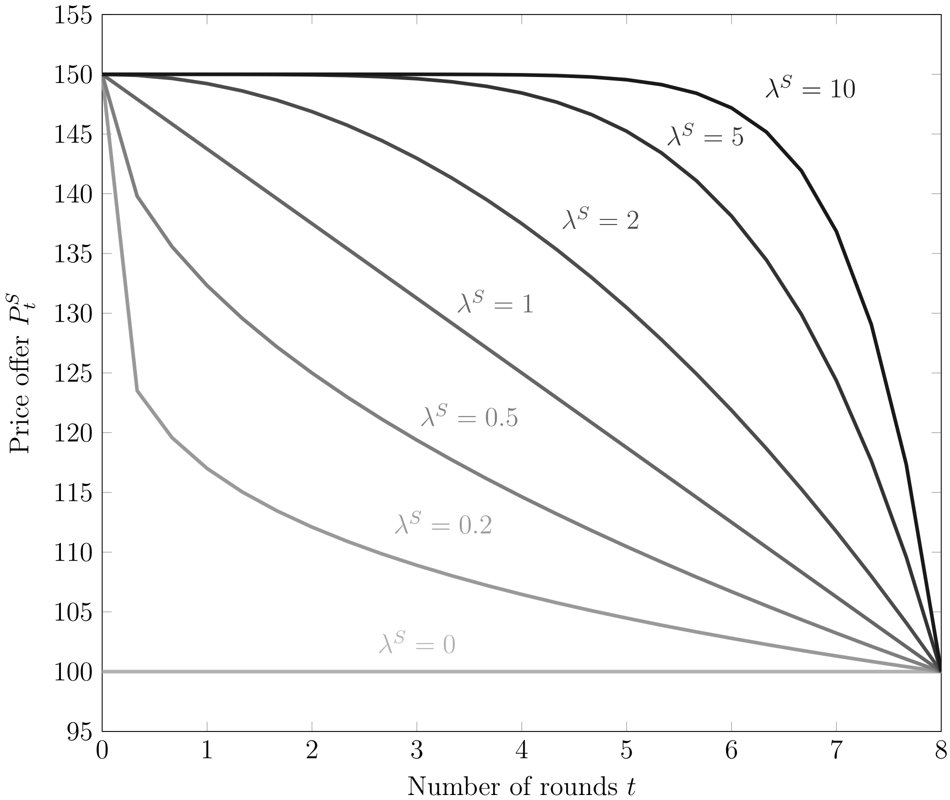



**Figure 3.1:** Illustration of a seller's concession pattern for different strategies, $\lambda^S$. Values of $\lambda^S \in \{0, 0.2, 0.5, 1, 2, 5, 10\}$ in ascending order from the bottom of the plot with a reservation price of 100 for the seller, initial offer at 150, and a deadline of eight rounds.

is set to 100 while the seller's initial offer, $P_0^S$, is 150. The maximum number of rounds the seller will participate in, is set to eight. The lines above $\lambda^S = 1$ are recognized as boulware strategies while the strategies below $\lambda^S = 1$ are conceders.

In reality, the agents might change their strategy during a game. In TRAPP, we implement this possibility for both agents, and refer to it as *learning mode*. Hence, we let the agents' strategy, $\lambda^A$, depend on the current round, denoted by $\lambda_t^A$. This allows the agents to switch to a different strategy in each round, which is a more realistic representation of real-life negotiations, according to our industry partner. Note that the agents only change their strategy if they adopt the learning mode. Otherwise, for the purpose of this thesis, their strategy is assumed to remain fixed throughout the game. Additionally, it should be noted that the agents cannot learn until round two due to the lack of information obtainable from an initial offer. After receiving two offers, the agents can, based on estimations of their opponent's deadline and reservation price, change their strategy $\lambda_t^A$.

The objective for both agents is to choose the strategy that leads to the highest expected payoff at round $t$. Two theorems applicable in the case of perfect and complete information are derived in Sim et al. (2009); one to obtain the optimal strategy for the buyer and one for the seller. Recall that in our case, we have perfect yet incomplete information. However, as we will see later in this section, we can use these theorems with estimations of the unknown parameters. Initially, to derive these two theorems, we use the formula for the price offer from both agents in order to find the optimal strategy. Note that in the case of complete and perfect information, the agent obtains an optimal strategy independent of $t$. The general equation is expressed by Sim et al. (2009) as:

$$P_t^A = P_0^A + \left(\frac{t}{\tau^A}\right)^{\lambda^A} \cdot (RP^A - P_0^A), \tag{3.2}$$

where $0 \leq \lambda^A \leq \infty$.

$P_0^A$ denotes the initial price offer from an agent. When the strategy, $\lambda^A$, goes to zero in this equation, the price offer goes towards the agent's reservation price, $RP^A$. This represents an extreme conceder strategy. Oppositely, when $\lambda^A$ goes to infinity, the agents will continue to offer their initial price offer until they reach their deadline, $t = \tau^A$. This is the extreme case of a boulware strategy. The challenge for the agents is to choose which strategy to play in round $t$, $\lambda_t^A$.

In TRAPP, we assume that both agents eventually concede to their reservation prices. Additionally, we assume that the reservation prices do not always overlap and create a zone of agreement (ZoA). However, every agent has to believe that a deal can take place when entering into negotiations. Otherwise, the agent would leave immediately to not waste time. Consequently, by applying the formulas used to derive the optimal strategy we assume that a ZoA exists. In order for a buyer to ensure that a deal takes place, an offer higher than the seller's reservation price before the seller's deadline must be submitted. This can be expressed as $RP^S \leq P_{\tau^S}^B$, and by implementing this condition in Equation 3.2, we end up with

$$RP^S \leq P_0^B + \left(\frac{\tau^S}{\tau^B}\right)^{\lambda^B} \cdot (RP^B - P_0^B). \tag{3.3}$$

The buyer obtains the highest possible payoff when the proposed offer is equal to the seller's reservation price. Consequently, we derive that the optimal strategy for the buyer is equal to

$$\lambda^B = \frac{\ln\left(\frac{RP^S - P_0^B}{RP^B - P_0^S}\right)}{\ln\left(\frac{\tau^S}{\tau^B}\right)}. \tag{3.4}$$

Contrarily, the seller has to make an offer lower than the buyer's reservation price before the buyer's deadline, and the condition $RP^B \geq P^S_{\tau^B}$ must hold. By using Equation 3.2 and the same calculations as for the buyer's optimal strategy, we obtain the seller's optimal strategy as the expression

$$\lambda^S = \frac{\ln\left(\frac{P_0^S - RP^B}{P_0^S - RP^S}\right)}{\ln\left(\frac{\tau^B}{\tau^S}\right)}. \tag{3.5}$$

Regardless of which strategy the other agent is adopting, if either the seller or buyer is using the optimal strategy shown in Equations 3.4 and 3.5 and an agreement is obtainable, a settlement will take place. However, in these equations we assume that the agent knows the opponent's reservation price, $RP^{\overline{A}}$, and deadline, $\tau^{\overline{A}}$. Both of these parameters need to be estimated in order to obtain the agent's optimal strategy. The next subsections present how this can be done, using Bayesian machine learning.

## 3.3 Estimation of the Opponent's Reservation Price

If an agent knows the opponent's reservation price, $RP^{\overline{A}}$, the best response in our case is to propose an offer at this value and receive the maximum possible payoff after only one round. In reality, the agents usually do not accept an initial offer although it equals their reservation price. Two possible reasons being that the agent who turned down the offer (1) believes that it would be possible to negotiate an even better price, or (2) reassesses the reservation price and updates it according to the initial received offer (Raiffa, 1982). In TRAPP, these situations are beyond the scope of the modelling approach. Hence, an offer placed within the ZoA is modelled to be accepted by either agent regardless of which round it is proposed. Nevertheless, for the purpose of this thesis, an accurate estimation method for the opponent's reservation price is unquestionably valuable to an any agent. An estimation of the opponent's reservation price, $\widetilde{RP}^{\overline{A}}$, is required both in the learning mode and to derive an optimal initial offer. We base TRAPP on the algorithm developed by Sim et al. (2009) for estimating the opponent's reservation price. However, we extend the method by allowing the investor to provide the model with a prior knowledge about the opponent's deadline and reservation price. This procedure for estimating the reservation price is based on Bayesian learning, where the agent first needs to specify a finite number of hypotheses addressing possible reservation prices the opponent may have. Figure 3.2 is an extension of Figure 2.3 and illustrates how the seller can obtain such hypotheses by assuming a lowest and highest limit for the buyer's reservation price, denoted by $\widetilde{RP}_l^B$ and $\widetilde{RP}_h^B$, respectively. In TRAPP, we distribute the hypotheses with equal spacing for practical reasons.[13]

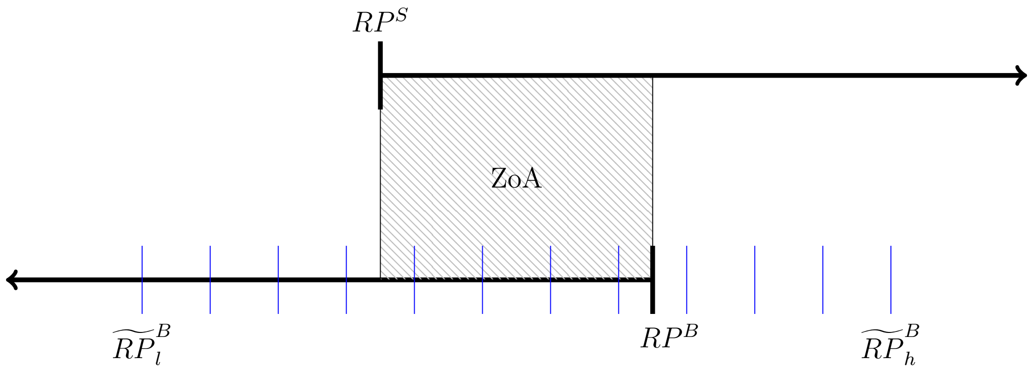



**Figure 3.2:** Illustration of a seller's hypotheses of a buyer's reservation price. The hypotheses are marked with blue vertical lines.

Initially, at $t = 0$, a probability distribution must be assigned to the hypotheses. We allow the seller to choose between two probability distributions in TRAPP: (1) A uniform distribution, implying that the seller has no opinion on what the buyer's reservation price might be other than between the two limit values, $\widetilde{RP}_l^B$ and $\widetilde{RP}_h^B$. (2) A triangular distribution which allows the seller to specify a belief of the buyer's most probable reservation price, denoted as $\widetilde{RP}_*^B$. This feature extends the approach in Sim et al. (2009), which only incorporates an uniform distribution. An illustration of the possible distributions are shown in Figure 3.3.

[13] In special cases when an agent has strong beliefs that the opponent's reservation price can only take a few possible values, it might be necessary to allow for unequal spacing between the hypotheses. This is disregarded in this thesis.

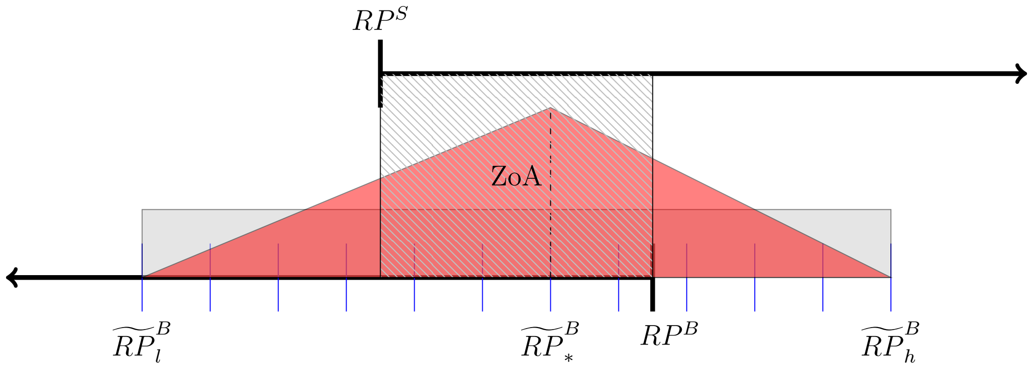



**Figure 3.3:** Illustration of a light gray colored uniform and a red colored triangular probability distribution assigned to hypotheses of a buyer's reservation price, shown as blue vertical lines.

Next, the seller updates the belief about the buyer's reservation price by using the initial probability distribution of the hypotheses together with the offer received from the buyer. We denote the prior probability of the $i$th hypothesis of the buyer's reservation price as $P(RP_i^B)$ and $P(P_t^B|RP_i^B)$ as the conditional probability that the buyer will offer $P_t^B$ given that the true reservation price is $RP_i^B$.

Obtaining an adequate estimation of the conditional probability is often found to be the most difficult step in Bayesian learning, as concluded in several papers (Roy (1989, p.599) and (Baarslag et al., 2016, p.861), among others). In Zeng & Sycara (1997, p.39), this issue is avoided by assuming a prior knowledge: "Usually in our business, people will offer a price which is above their reservation price by 17%, which can be represented by a set of conditional statements". In our opinion, if this knowledge is known in such details, there would be no need for a comprehensive negotiating model. In TRAPP, we compute the conditional probabilities using the same procedure as found in Sim et al. (2009), by assuming the conditional probability to be normally distributed, $P(P_t^{\overline{A}}|RP_i^{\overline{A}}) \sim \mathcal{N}(\mu_i, \sigma^2 = 1)$. We obtain $\mu_i$ by using

$$\mu_i = RP_i^{\overline{A}} \cdot [1 + (-1)^\beta \cdot \alpha(t)] \tag{3.6}$$

, which assumes that the agents initially propose bids that are further away from their reservation price rather than later in the game. For the seller, $\beta = 1$ while $\beta = 0$ for the buyer. In Equation 3.6, $\alpha(t)$ is a recursive formula given by

$$\alpha(t) = \begin{cases} |1 - P_t^{\overline{A}} \cdot [1 + (-1)^{\beta} \cdot \alpha(t-1)] & \text{when } t > 0, \\ \left|1 - \frac{P_0^{\overline{A}}}{P_0^{A}}\right| & \text{when } t = 0. \end{cases} \tag{3.7}$$

The concept in Equation 3.6 is further illustrated with a numerical example, depicted in Figure 3.4. Note that in this case, we set $P(P_t^B|RP_i^B) \sim \mathcal{U}$ when $t = 0$, represented by the blue horizontal line. The probability distribution shifts towards the buyer's latest offer as $t$ increases, and corresponds well with what we can anticipate in our game, as described in Subsection 2.3.1. Probability is shown on the y-axis and the buyer's reservation price on the x-axis.

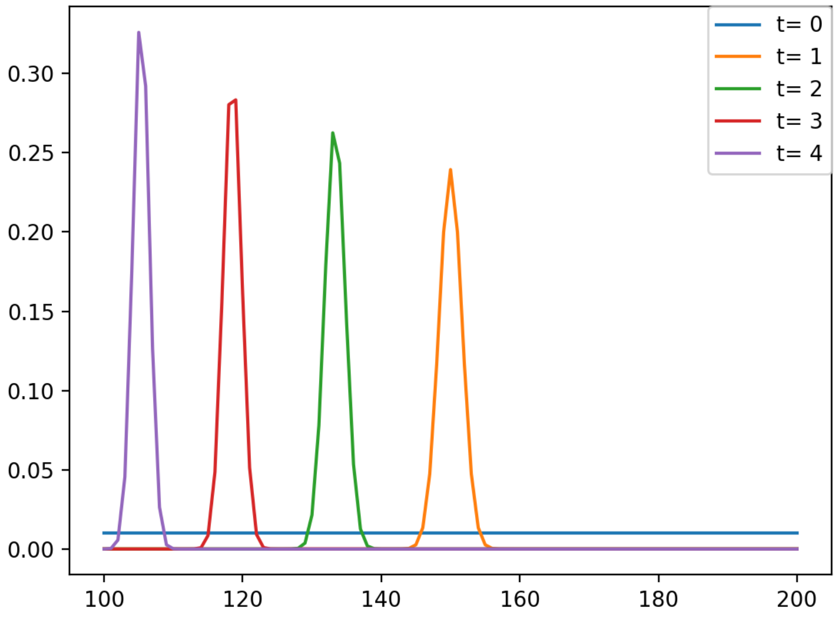



**Figure 3.4:** Probability distributions of $P(P_t^B|RP_i^B)$ when receiving $P_t^B = 90$ at different rounds, $t$. Numerical values used include $P_{t-1}^B = P_0^B = 80$, $P_0^S = 150$, $\widetilde{RP}_l^B = 100$, and $\widetilde{RP}_h^B = 200$.

The final step in estimating the opponent's reservation price is to update the probability for the different hypotheses, using the Bayesian updating formula as shown in Equation 3.8. Then, the estimation of the agent's reservation price, $\widetilde{RP}_t^{\overline{A}}$, is calculated as the weighted average, given by Equation 3.9:

$$P(RP_i^{\overline{A}}|P_t^{\overline{A}}) = \frac{P_{t-1}(RP_i^{\overline{A}}) \cdot P(P_t^{\overline{A}}|RP_i^{\overline{A}})}{\sum_{i=\widetilde{RP}_l^{\overline{A}}}^{\widetilde{RP}_h^{\overline{A}}} P_{t-1}(RP_i^{\overline{A}}) \cdot P(P_t^{\overline{A}}|RP_i^{\overline{A}})}, \tag{3.8}$$

$$\widetilde{RP}_t^{\overline{A}} = \sum_i RP_i^{\overline{A}} \cdot P(RP_i^{\overline{A}}|P_t^{\overline{A}}). \tag{3.9}$$

The full procedure for estimation the opponents' reservation price, $RP^{\overline{A}}$ at round $t$ is summarized in Algorithm 1.

**ALGORITHM 1:** Bayesian learning procedure for estimating $RP_t^{\overline{A}}$

**Input** : $\mathbf{H}$, P($\mathbf{H}$), t, $P_t^{\overline{A}}, P_{t-1}^{\overline{A}}, \beta, P_0^{\overline{A}}, P_0^{A}$

**Output :** Class: $\widetilde{RP}_t^{\overline{A}}$ and updated $P(\mathbf{H})$

1 **forall** *hypothesis in* ***H*** **do**
2     **if** *hypothesis is below/above offer received from buyer/seller* **then**
3         P(hypothesis) = 0 `/* reject the hypothesis */`
4     **else**
5         **if** $t$=*0* **then**
6             Assign initial distribution `/* uniform or triangular */`
7         **else**
8             Calculate $\mu_i$ using Equation 3.6
9             Calculate $P(P_t^{\overline{A}}|RP_i^{\overline{A}}) \sim \mathcal{N}(\mu_i, \sigma^2 = 1)$
10         **end**
11     **end**
12 **end**
13 **if** $t = 0$ **then**
14     $P(\mathbf{H}) \longleftarrow P(\mathbf{H})$
15     Calculate $\widetilde{RP}_t^{\overline{A}}$ using Equation 3.9
16 **else**
17     **forall** *hypothesis in* ***H*** **do**
18         Update $P(hypothesis)$ using Equation 3.8 `/* conditional probability obtained in line 9 */`
19         Calculate $\widetilde{RP}_t^{\overline{A}}$ using Equation 3.9
20     **end**
21 **end**

## 3.4 Estimation of the Opponent's Deadline

In TRAPP, the agents' only possible threat is to withdraw from the negotiations, resulting in no deal taking place and a payoff of zero to both agents. The threshold for when an agent decides to leave the negotiations is modelled as a deadline represented by a predetermined and fixed number of rounds the agent is willing to exchange bids. We denote an agent's deadline as $\tau^A$, as presented in Section 2 Background & Related Literature. For instance, if $\tau^B = 7$, the buyer is determined to leave the negotiations after round $t = 7$ if an agreement has not yet been reached. In the case of no deal, the buyer is forced to offer $P_7^B = RP^B$ for a deal to happen if a zone of agreement exists. Additionally, this final offer restriction can be obtained in more general terms by letting $\lim_{t \to \tau^A}$ in Equation 3.2, which yields $P_{\tau^A}^A = RP^A$. Without any deadline limitations, an obvious strategy for the agents in order to maximize payoff would be to start with an initial offer far from their reservation price, $RP^A$, and then approach $RP^A$ linearly and monotonically with small increments. With this strategy, it will undoubtedly take a great number of rounds before an offer is placed within the zone of agreement.

> *For example, if the last rejected offer for the block of shares for sale was \$100 million, the next offer could conceivably be just \$1 more than \$100 million. Such ridiculous offer strategies are not observed in the marketplace since negotiations would drag on forever and become prohibitively costly. Hence, a minimal concession strategy is neither feasible nor optimal in the marketplace.*
>
> (Roy, 1989, p.597)

Such a bidding pattern is rarely to be observed in real estate negotiations as the agents are far more likely to terminate negotiations before reaching an agreement. For illustrative purposes, Figure 3.5 displays an inefficient bidding pattern where both agents adopt a linear strategy with marginal concessions. Eventually, the buyer decides to walk away from the negotiation.

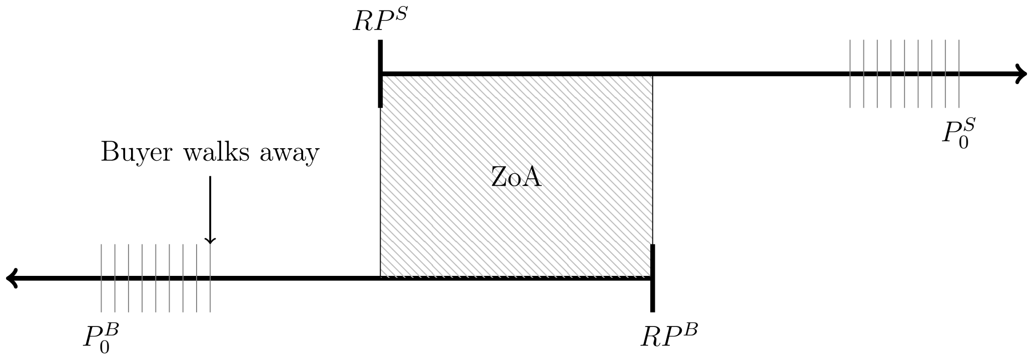



**Figure 3.5:** Illustration of an ineffective bargaining game where the buyer gets tired of playing and eventually walks away.

The optimal concession strategy for an agent depends on the opponent's deadline, $\tau^{\overline{A}}$. As shown in Subsection 3.2, the optimal response for the agents when they have a longer deadline than their opponent, $\tau^{A} > \tau^{\overline{A}}$, is to be boulware. Oppositely, when $\tau^{A} < \tau^{\overline{A}}$, the agent is better off being a conceder. Hence, an important part in this game is to estimate the opponent's deadline in order to propose the best offer.

For the learning mode in TRAPP, we implement the same the procedure for estimating $\tau^{\overline{A}}$ as found in the model developed in Sim et al. (2009). Algorithm 2 shows all the steps.

**ALGORITHM 2:** Procedure for estimating $\tau^{\overline{A}}$

**Input** : t, $P_t^{\overline{A}}, P_{t-1}^{\overline{A}}, P_{t-2}^{\overline{A}}, N$
**Output** : $\widetilde{\tau}_t^{\overline{A}}$

Find $\widetilde{RP}_t^{\overline{A}}$ using **Algorithm 1**.
**for** $A \in N$ **do**
  **if** $A == B$ **then**
    **if** $P_t^S \geq P_{t-1}^S$ ***or*** $P_{t-1}^S \geq P_{t-2}^S$ ***or*** $P_{t-1}^S = \widetilde{RP}_t^S$ ***or*** $P_{t-2}^S = \widetilde{RP}_t^S$ **then**
      **return** $\widetilde{\tau}_{t-1}^S$ `/* return previous estimation of deadline */`
    **end**
  **else**
    **if** $P_t^B \leq P_{t-1}^B$ ***or*** $P_{t-1}^B \leq P_{t-2}^B$ ***or*** $P_{t-1}^B = \widetilde{RP}_t^B$ ***or*** $P_{t-2}^B = \widetilde{RP}_t^B$ **then**
      **return** $\widetilde{\tau}_{t-1}^B$ `/* return previous estimation of deadline */`
    **end**
  **end**
**end**
Construct a set of three equations, using equation 3.2 on the rounds $t-2$, $t-1$, and $t$
Substitute $\tau_t^{\overline{A}}$ with $\widetilde{\tau}_t^{\overline{A}}$ and $RP^{\overline{A}}$ with $\widetilde{RP}_t^{\overline{A}}$, and solve for $\widetilde{\tau}_t^{\overline{A}}$
**return** $\widetilde{\tau}_t^{\overline{A}}$

## 3.5 Adjusting the Seller's Strategy

In the case of perfect and complete information where deadlines and reservation prices are known, the optimal strategy for a seller is given by Equation 3.5. With the estimation of the buyer's reservation price at round $t$, $\widetilde{RP}_t^B$, and deadline, $\widetilde{\tau}_t^B$, explained in Subsections 3.3 and 3.4, respectively, the seller adjusts the strategy based on these estimates. The seller's optimal strategy in round $t \; \forall \; t > 2$ is given by Equation 3.10:

$$\lambda_t^S = \left| \frac{\ln\left( \max\left[0, \left(\frac{P_{t-1}^S - \widetilde{RP}_t^B}{P_{t-1}^S - RP^S}\right)\right]\right)}{\ln\left(\frac{\widetilde{\tau}_t^B - t - 1}{\tau^S - t - 1}\right)} \right| \tag{3.10}$$

Finally, the seller proposes the next offer by using Equation 3.1 introduced in Subsection 3.2 with the seller's strategy at round $t$, $\lambda_t^S$, found by Equation 3.10.

## 3.6 Modes

To simulate the Norwegian speculative real estate market reliably, we test TRAPP for a seller in different *modes*. In this subsection, we explain the distinct characteristics for the following modes of the seller: Learning, initial price offer and strategy. Next, these modes are compared in Section 4 Empirical Results and eventually used in a real-life case in Section 5 Case Study.

In the beginning of this section, we introduced the term learning. Recall that the agents are defined as learners if they change their strategy during a game, based on estimations of their opponent's reservation price and deadline, denoted $\widetilde{RP}_t^{\overline{A}}$, and $\widetilde{\tau}_t^{\overline{A}}$, respectively. Further, the agents are first able to adopt the learning mode in round two, after receiving two offers. The motivation for learning is to maximize the expected payoff obtainable in that particular round. Nevertheless, since the learning procedure is based on offers received from the opponent, an initial strategy needs to be chosen for the first two rounds, referred to as the *pre-learning phase*. In TRAPP, we allow the seller to be either a learner or non-learner. Experiments where either none, one or both agents learn are presented in Zeng & Sycara (1997)[14]. Since we are considering negotiations from the perspective of

[14]In this paper, *learning* includes only the procedure of estimating the opponent's reservation price.

the seller in this thesis, we mainly focus on the learning mode from a seller's perspective. However, we incorporate the possibility for either agent to be a learner. In Zeng & Sycara (1997), a non-learning agent follows a linear strategy ($\lambda^A = 1$) throughout the game. We incorporate the buyers with seven possible strategies and the seller with 11 in the set $\Lambda$ to better reflect the real-life setting. This is further discussed in Subsection 4.1.

Furthermore, we implement three different methods for the seller to generate the initial offer. First, an opening offer $x$ % above the seller's own reservation price: $P_0^S = 1.x \cdot RP^S$. In Subsection 4.2.1, we perform a sensitivity analysis on $x$. Next, we include the two types of probability distributions presented in Subsections 2.3.2 and 3.3: a uniform and triangular distribution. These distributions generate the initial offer depending on a probability of the opponent's reservation price. For both distributions, closed formulas can be derived for the offer that yields the highest expected payoff in the first round. For the seller, the optimal offer when using a uniform distribution is found to be

$$P_0^S = \begin{cases} \frac{RP^S + \widetilde{RP}_h^B}{2} & \text{if } P^* \in [\widetilde{RP}_l^B, \widetilde{RP}_h^B], \\ \widetilde{RP}_l^B & \text{if } P^* < \widetilde{RP}_l^B, \\ \widetilde{RP}_h^B & \text{if } P^* > \widetilde{RP}_h^B, \end{cases} \tag{3.11}$$

while the optimal offer when using a triangular distribution is expressed as

$$P_0^S = \frac{4\widetilde{RP}_h^B + 2RP^S - \sqrt{(-4\widetilde{RP}_h^B - 2RP^S)^2 - (4 \cdot 3 \cdot (2\widetilde{RP}_h^B RP^S))}}{6}, \tag{3.12}$$

when $P_0^S > \widetilde{RP}_*^B$. When $P_0^S \leq \widetilde{RP}_*^B$:

$$P_0^S = \frac{\frac{-(2RP^S + 4\widetilde{RP}_l^B)}{(\widetilde{RP}_h^B - \widetilde{RP}_l^B)(\widetilde{RP}_*^B - \widetilde{RP}_l^B)} - \sqrt{\left(\frac{(2RP^S + 4\widetilde{RP}_l^B)}{(\widetilde{RP}_h^B - \widetilde{RP}_l^B)(\widetilde{RP}_*^B - \widetilde{RP}_l^B)}\right)^2 - 4\delta\left(\frac{-3}{(\widetilde{RP}_h^B - \widetilde{RP}_l^B)(\widetilde{RP}_*^B - \widetilde{RP}_l^B)}\right)}}{\frac{-6}{(\widetilde{RP}_h^B - \widetilde{RP}_l^B)(\widetilde{RP}_*^B - \widetilde{RP}_l^B)}}, \tag{3.13}$$

where

$$\delta = \frac{(\widetilde{RP}_*^B)^2 - 2\widetilde{RP}_l^B RP^S - 2\widetilde{RP}_l^B \widetilde{RP}_*^B}{(\widetilde{RP}_h^B - \widetilde{RP}_l^B)(\widetilde{RP}_*^B - \widetilde{RP}_l^B)} + \frac{(\widetilde{RP}_h^B)^2 - 2\widetilde{RP}_h^B \widetilde{RP}_*^B + (\widetilde{RP}_*^B)^2}{(\widetilde{RP}_h^B - \widetilde{RP}_l^B)(\widetilde{RP}_h^B - \widetilde{RP}_*^B)}.$$

The derivations of Equations 3.11, 3.12 and 3.13 are found in Appendix A2 and A3. For illustrative purposes, Figure 3.6 presents a numerical example, showing how the optimal initial offer for a seller depends on the initial probability distribution assigned to the hypotheses. Using the uniform distribution yields an initial offer, $P_0^S$, at 160 and an estimation of the buyer's reservation price, $\widetilde{RP}_t^B$, at 175, while the triangular distribution proposes 164[15] as $P_0^S$ and $\widetilde{RP}_t^B$ to be 177.

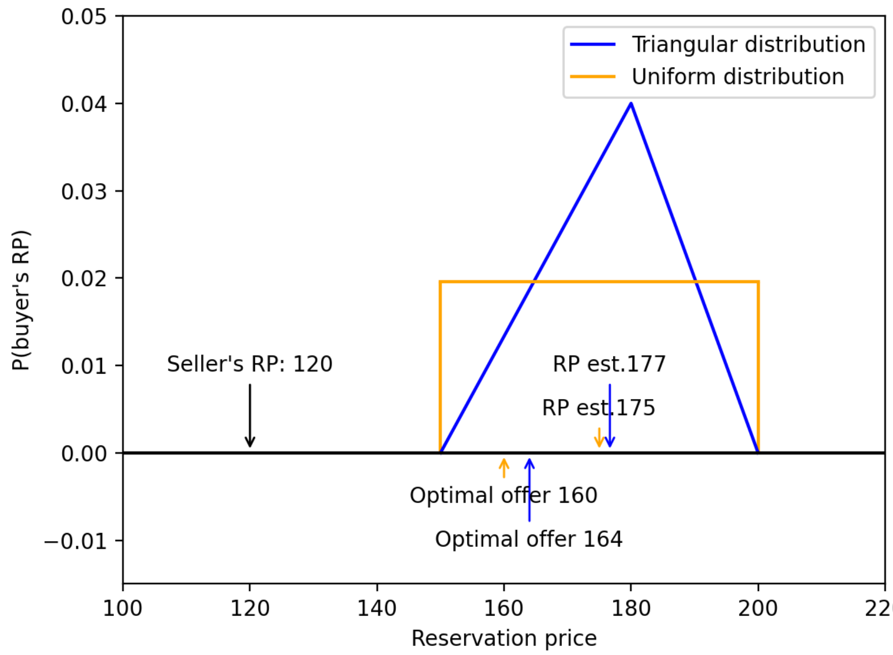



**Figure 3.6:** Optimal offer for a seller when a uniform or triangular distribution is applied. Numerical values used include $RP^S = 120$, $\widetilde{RP}_l^B = 150$, $\widetilde{RP}_h^B = 200$, and $\widetilde{RP}_h^* = 180$.

In addition to the learning mode and different initial offer procedures, we include 11 strategies for the seller, $\lambda^S$ in the set $\Lambda$ as mentioned earlier in this section. We have included five boulware, five conceder and a linear strategy in the set, where $\Lambda = \{0.1, 0.2, 0.33, 0.5, 0.66, 1, 1.5, 2, 3, 5, 10\}$ are all the values we simulate and compare with each other.

[15] Rounded to nearest integer. The exact value is 164.494897, rounded to seven significant figures.

The different modes are summarized in Table 3.3, where the average payoff in each mode is to be compared through simulations in the next section. The results of the simulations are found in Table 4.1. Note that TRAPP can easily be extended to include a broader set of modes if desired. For the purpose of this thesis, we find it sufficient to use the modes listed in Table 3.3.

| $\boldsymbol{P_0^S}$ | $\boldsymbol{\lambda^S}$ | **Learner?** |
|---|---|---|
| $1.x \cdot RP^S$ | $\Lambda$ | Yes/No |
| Triangular | $\Lambda$ | Yes/No |
| Uniform | $\Lambda$ | Yes/No |

**Table 3.3:** The different modes incorporated in TRAPP.

# 4 Empirical Results

In this section, we evaluate the different modes described in Subsection 3.6. The setup of the simulations we present below is designed to reflect the business environment of our industry partner, Securum Eiendom AS. Consequently, the insights we obtain from the simulations can be applied directly into supporting Securum in real-life negotiations, which is made clear in Section 5 Case Study. In order to obtain relevant results for another company and other businesses, the parameterization in this section needs to be revised. For future use, Securum should update their perception of the market.

Ideally, the optimal strategy is the one that simultaneously yields the highest average payoff, uses the shortest time to reach an agreement, and provides the highest likelihood of a deal taking place. However, we find that there exists no strategy that maximizes all these three measurements at the same time with our parameterization of the Norwegian speculative real estate market. For instance, by playing a very conceding strategy, a deal takes place in a few number of rounds. However, at the cost of giving up some payoff. Since we assume Securum to be a profit seeking company, we refer to the *optimal strategy* as the strategy which yields the highest average payoff. Thus, a strategy which achieves the highest percentage of number of deals or the lowest average number of rounds before a deal takes place, is not considered as the optimal strategy in this thesis.[16] Nevertheless, we discuss all these three performance measurements in this section.

A fundamental part for any agent in a negotiation, is to decide what strategy to play, as assessed in Subsection 3.2. Being too boulware may result in no agreement, while following a too conceder strategy often yields a lower payoff than what could have been achieved. Additionally, in TRAPP we include the possibility for an agent to switch between different strategies in each round by adopting the learning mode, described in Subsection 3.6. Furthermore, another important part of the strategy which does not relate to the conceding pattern, is how to come up with the initial offer. We look at three different methods for this as written in Subsection 3.6: (1) $x$ % above the seller's reservation price, or (2) based on a uniform or (3) triangular probability distribution of the hypotheses of the buyer's reservation price.

[16]Unless it additionally achieves the highest average payoff.

The first part of this section describes our parameterization process of TRAPP, before we present the results obtained from simulations.

## 4.1 Parameterization of TRAPP

In order to achieve results that reflect Securum's business environment applicable in Section 5 Case Study, it is important to establish a relevant basis for the simulations. Together with Securum, we constructed a pool of 10,000 fictive buyers with different profiles reflecting Securum's business environment, whom all played against a seller using TRAPP in different modes. Figure 4.1 shows the parameterization process. Further description of the parameters are found in Table 3.1.

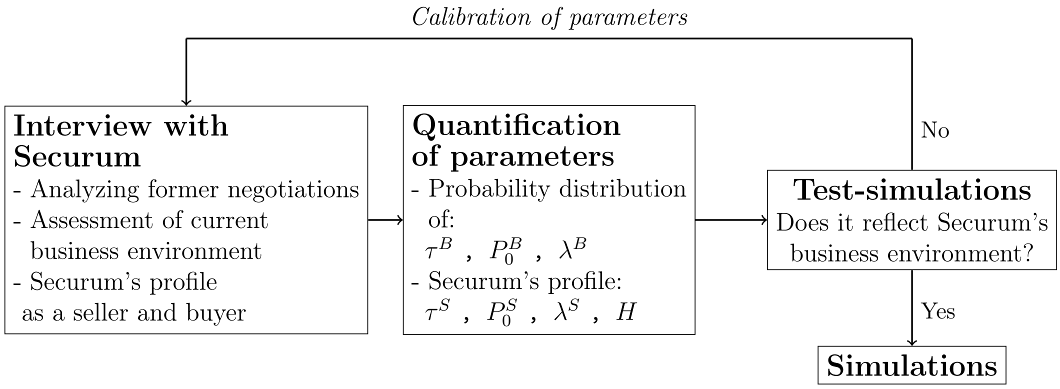



**Figure 4.1:** Overview of the steps in the parameterization process.

The employees of Securum have decades of experience within the area of negotiating in the Norwegian market of speculative real estate investments. During these years, they have encountered a lot of different buyers from a specter of cultures. In order to simulate reliable scenarios and obtain helpful empirical results for Securum, we discussed the buyers' deadline, initial price offer and strategy with them.

We used parts of the CISSE[17]-methodology presented in Dias et al. (2017) to construct subjective probability distributions of the parameters. Considering that Securum is a relatively small company in its industry, the information obtained was based on only a handful of employees. Consequently, since the information is based on only a few people, it is uncertainty in the estimation of the probabilities (Vick, 2002). However, the parameterization of TRAPP fits the employees at Securum as well as it can if collect the

[17] Characterize, Identify, Sentence, Select & Estimate

data in an unbiased matter. We used the same scales for the different interviews, and avoided vague definitions open for interpretation[18], and focused solely on questions with numerical answers inspired by the methodology used in Haase et al. (2013).

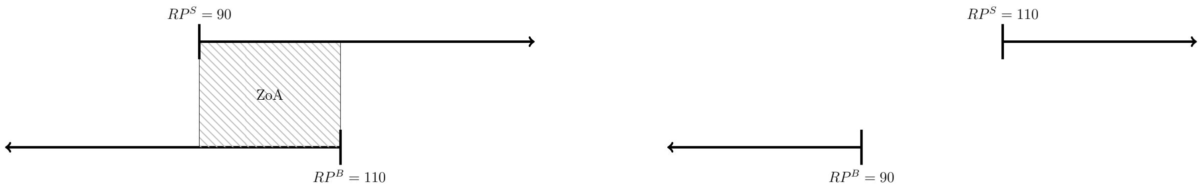
Using the CISSE-methodology, we needed to determine which market we wanted to construct a subjective probability distribution for. In our case, we assess the Norwegian speculative real estate market. Moreover, the data needs to be characterized (Dias et al., 2017). Characterization means identifying what we want to construct a subjective probability distribution of, within the market, referred to as factors. The factors to be identified in this thesis, are an estimation of the buyers' deadline, initial offer as a percentage of the buyers' own reservation price, and strategy. The next step is to identify whom to collect data from and from which events. For the purpose of this parameterization, we interviewed employees at Securum about their encounters with speculative real estate investments and negotiations concerning these types of projects from both a seller and buyer's perspective.[19] For simplicity of our modelling, we choose discrete distributions. In interviews, we found it useful to crosscheck data obtained with previous negotiations involving Securum to secure as accurate parameters as possible for the parameterization. This procedure is followed throughout this section.

Since we want to create a practical model applicable to the daily life business for our industry partner, we have to account for a non-existing zone of agreement (ZoA), as mentioned in Subsection 2.3.2. According to Securum, a deal does not take place in about 50 % of all negotiations they are involved in, due to an absent ZoA. We assume, based on input from Securum, that the ZoA is a maximum of 10 % in both directions away from the property value. Further, a typical price for a speculative real estate for Securum, is in the size of 100 MNOK. We use this as a basis in the simulations. Hence, both agents' reservation price in the bargaining game are simulated in the following way to obtain the features provided by Securum:

First, a range from 90 to 110 with discrete intervals of one is constructed. Next, both the buyer and seller are randomly assigned a reservation price from this range using a uniform probability distribution. With this set-up, a ZoA does not exist in 50 % of the cases, and the largest ZoA is 20 MNOK when $RP^S = 90$ and $RP^B = 110$. Both of these

[18] For instance, expressions like "rather likely" can be interpreted differently by different investors.
[19] Exact data from previous projects is excluded for confidentiality reasons.

cases are illustrated in Figure 4.2.




**(a)** The largest possible ZoA. **(b)** An example of a non-existing ZoA.

**Figure 4.2:** The largest possible ZoA and an example of a non-existing ZoA in the simulations.

The practical and novel implementation where a ZoA does not exist, is not found in Sim et al. (2009), Agarwal & Zeephongsekul (2011), and Baarslag et al. (2016), where the simulations are constructed such that a ZoA exists in every game.

### 4.1.1 The Buyers' Deadline

Securum estimates the probabilities of the buyers' deadline, as seen in Figure 4.3. A deadline of four rounds is the most likely scenario, while a buyer with a deadline of six rounds is rarely observed in real-life. If the negotiation goes beyond five rounds, Securum claims that the parties often leave the room as enemies. For illustrative purposes, we include an analogy presented by Securum; Imagine you as a seller are negotiating with a potential buyer about a house you own. Then, you receive a fifth and possibly a sixth offer below your reservation price. Most people would probably get tired of that particular buyer and terminate the negotiation process. This reflects the situations found in corporate contexts as well, according to Securum. Additionally, the probability of a deadline of two and five rounds are approximately equal, while of 30 % the buyers have a deadline of three rounds.

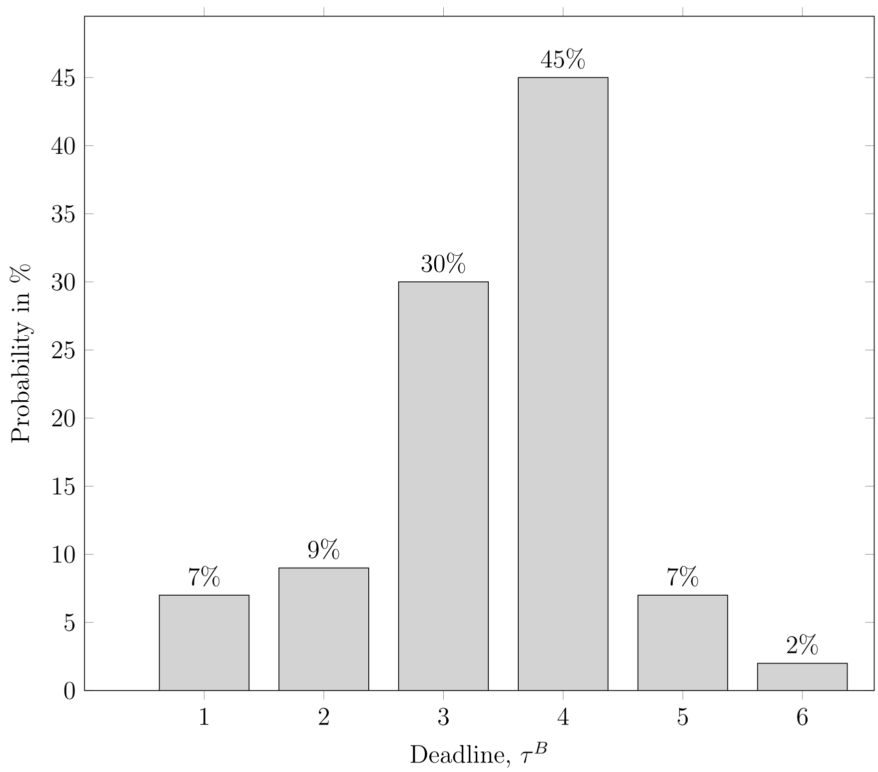



**Figure 4.3:** Securum's estimations of the buyers' deadline, $\tau^B$.

### 4.1.2 The Buyers' Initial Offer

A buyer is far more likely to make an initial bid that is a higher percentage away from the buyer's own reservation price compared to a seller's opening offer, according to Securum. Their perception is that the buyer often is a lot more optimistic of getting a better deal than the seller. Note that Securum never know the true reservation price of a buyer. Nevertheless, they are able to provide us with estimations of the initial offer as a percentage of the reservation price because they (1) buy properties themselves and have insights from a buyer's mindset and (2) have negotiated with a lot of different types of buyers.

> *Usually, a buyer's initial offer is 10 % below his absolute limit, while a seller's counterbid is 5 % above his reservation price in average. My experience is that a buyer in general bids a higher percentage away from his reservation price than a seller.*

Odd Hyttedalen (personal communication April 12, 2021)

Based on the input from Securum, we assume that the most probable scenario is that the buyers open with a bid 90 % below their reservation price. The most extreme case for the buyers' opening bid is set to 60 % below their reservation price, which Securum expects to happen in 0.5 % of the cases. In 10 % of the negotiations, the opening bid from the buyers is 95 % of their reservation price, $RP^B$. Securum claim that it never occurs, from their experience, that the opening bid is higher than 95 % of $RP^B$. All the remaining probabilities are shown in Figure 4.4.

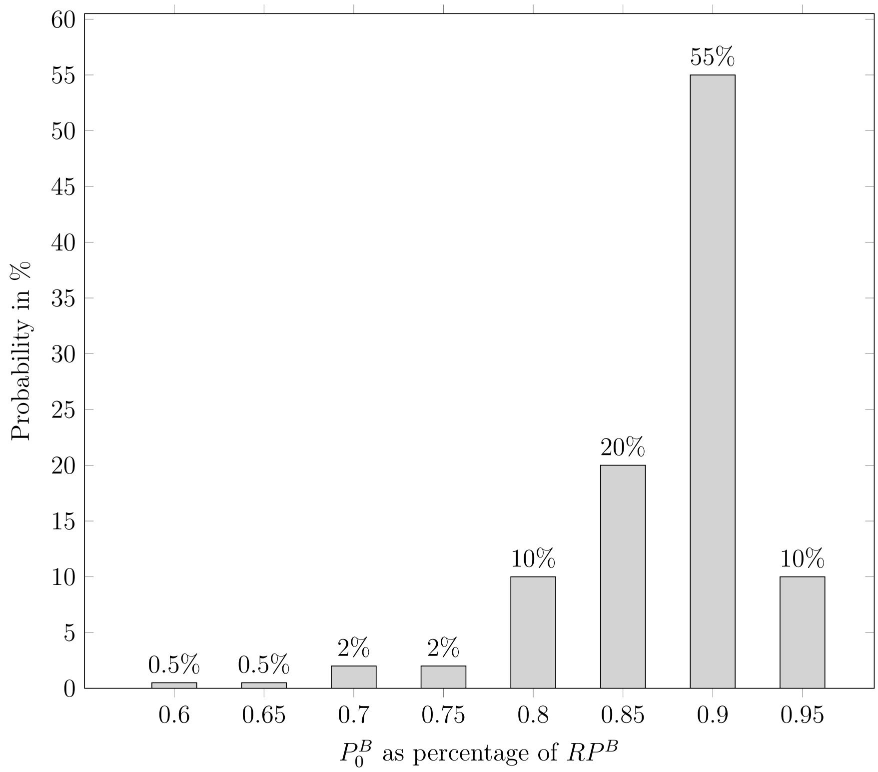



**Figure 4.4:** Securum's estimations of the buyers' initial price offer, $P_0^B$.

### 4.1.3 The Buyers' Strategy

We model the buyers' strategy to take a value from a set of seven different strategies: three conceder, three boulware, and a linear strategy, as previously mentioned in Subsection 3.2. We interviewed Securum by selecting an initial offer from a buyer at 90 MNOK and a reservation price of 100 MNOK, and asked them what they would expect the buyer's following offers to be until an agreement was reached, and the probability of each outcome. Note that we disregard Securum's reservation price for the purpose of this example to solely focus on the buyer's behavior. Next, we calculated which strategy the buyer had based on this input from Securum.

With this procedure as a basis, Securum estimated a 60 % chance that the buyer has a linear strategy. This strategy implies equal increments between the buyers' successive offers. Additionally, they emphasized that a buyer is more likely to take a boulware strategy than a conceder. They explained that the main reason is that a buyer is more likely to lose face by conceding at an early stage than a seller. Remember that the buyers with a boulware strategy has small increments to begin with, before conceding towards their reservation price at the end of their deadline, $\tau^B$.

> *It is far more likely that a buyer has a linear strategy because they are testing limits as they bid. Furthermore, some buyers usually concede a lot more at the end of the game rather than in the beginning.*
>
> Odd Hyttedalen (personal communication April 12, 2021)

Figure 4.5 depicts the full probability distribution of the buyers' strategy.

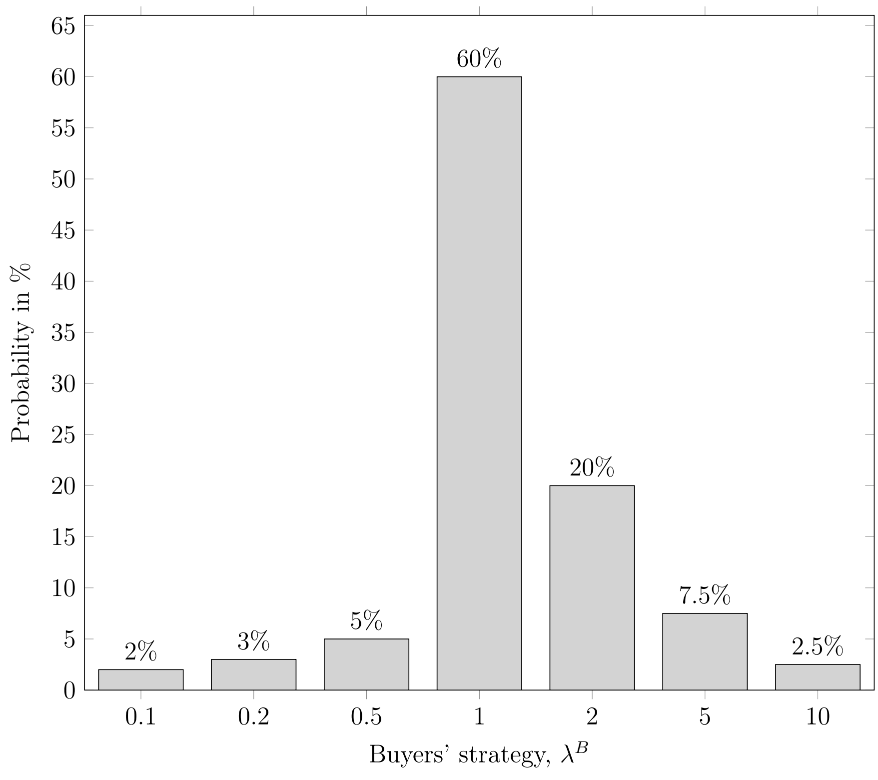



**Figure 4.5:** Securum's estimations of the buyers' strategy, $\lambda^B$.

## 4.2 Simulations Results

The aim of the simulations that follow is to emulate bilateral bargaining games between our industry partner as a seller, and a prospective buyer. We do this by simulating games where we pick a random buyer in each game with the characteristics from the parameterization described in the previous subsections. Next, we adjust the modes of the seller to find the optimal policy. We simulate each mode 10,000 times to obtain the policy that achieves the highest average payoff.

### 4.2.1 Sensitivity on Securum's Policy

In discussions with Securum, we learned that a typical opening bid for a seller is about 5 % higher than the reservation price. They requested us to use this value for comparison, as they found this value to be most interesting to them. Nevertheless, we conduct a sensitivity analysis to see how the average payoff might change when the opening bid is a function of the seller's reservation price, i.e.: $P_0^S(x) = x \cdot RP^S$. For simplicity, the learning mode is excluded from this sensitivity analysis as it turned out to have a negligible impact on the final result.

The optimal initial offer for a seller is found to be at 10 %[20] above the seller's own reservation price, $P_0^S = 1.10 \cdot RP^S$, represented by the blue line at the top in Figure 4.6. The seller's optimal strategy is to be a conceder with $\lambda^S = 0.33$, for $5\% \leq x \leq 15\%$. When the initial offer is 20 % or more above the seller's reservation price, the optimal policy is to adopt an even more conceding strategy with $\lambda = 0.1$. The black line represents Securum's normal practice of 5 % above their own reservation price. The reason for why the average payoff eventually drops when opening bids get far above the seller's reservation price ($x \geq 20\%$), is that in these cases a deal takes place on fewer occasions.

[20]This is the optimal initial offer among the values we include in this sensitivity analysis. We conclude from the results observed in Figure 4.6 that the exact optimal initial offer must lie between 7 % and 15 % higher than the seller's own reservation price, $RP^S$.

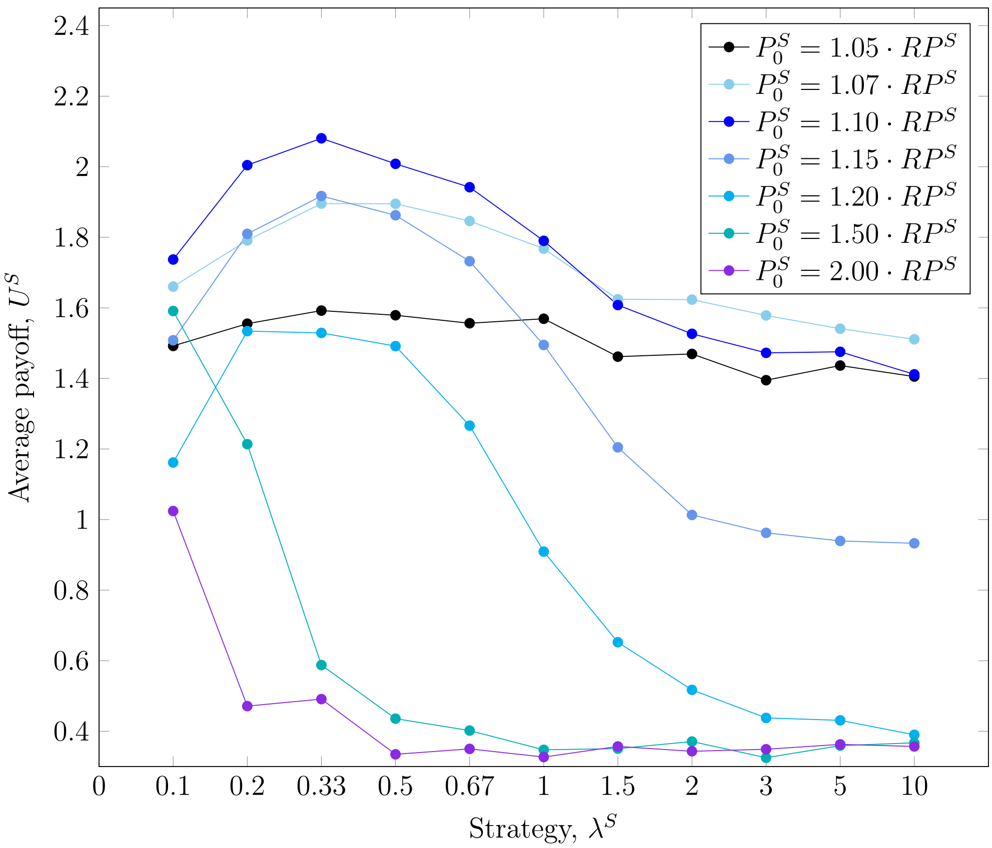



**Figure 4.6:** Average payoff, $U^S$, for all strategies, $\lambda^S \in \Lambda$, with different opening bids, $P_0^S$, based on the seller's own reservation price, $RP^S$.

To summarize, our results show that a seller who base the initial offer on the reservation price, $RP^S$, should propose it 10 % above $RP^S$. For this initial offer method, the seller should adopt a non-learning mode, as depicted in Figure 4.6. Nevertheless, Securum argued that they still consider an initial price offer 5 % higher than their own reservation price as their best practice. In the next subsection, we go further into detail on this initial offer policy. The remaining policies are included in the final results in this section.

### 4.2.2 Securum's Current Initial Price Offer Policy

In this subsection, we go further into detail on Securum's initial price offer policy, $P_0^S = 1.05 \cdot RP^S$. This subsection serves three purposes. First, we find the optimal strategy[21], and see how it performs in two other measures: Number of rounds before an agreement is reached and percentage of deals reached. In parallel, we analyze the impact for a seller adopting the learning mode. Finally, we illustrate with a numerical example

[21]Recall that we define optimal strategy as the strategy which yields the highest average payoff.

how a seller proposes offers, depending on mode.

The results from the simulations show that when a seller generates the initial offer 5 % above the seller's own reservation price, the optimal strategy without the possibility to learn, is to be a conceder with $\lambda^S = 0.33$. If the seller learns, the optimal strategy is still to be a conceder, slightly changing the initial strategy to $\lambda^S = 0.5$ before the learning phase begins. The expected payoff from all 11 strategies are shown in Figure 4.7. Solid line represents a seller without the possibility to learn, while the dashed line corresponds to a seller adopting the learning mode. Additionally, we observe that the learning mode is more valuable in terms of payoff for the boulware strategies, while it does not seem to have a significant impact on the conceder strategies when $\lambda \leq 1$.

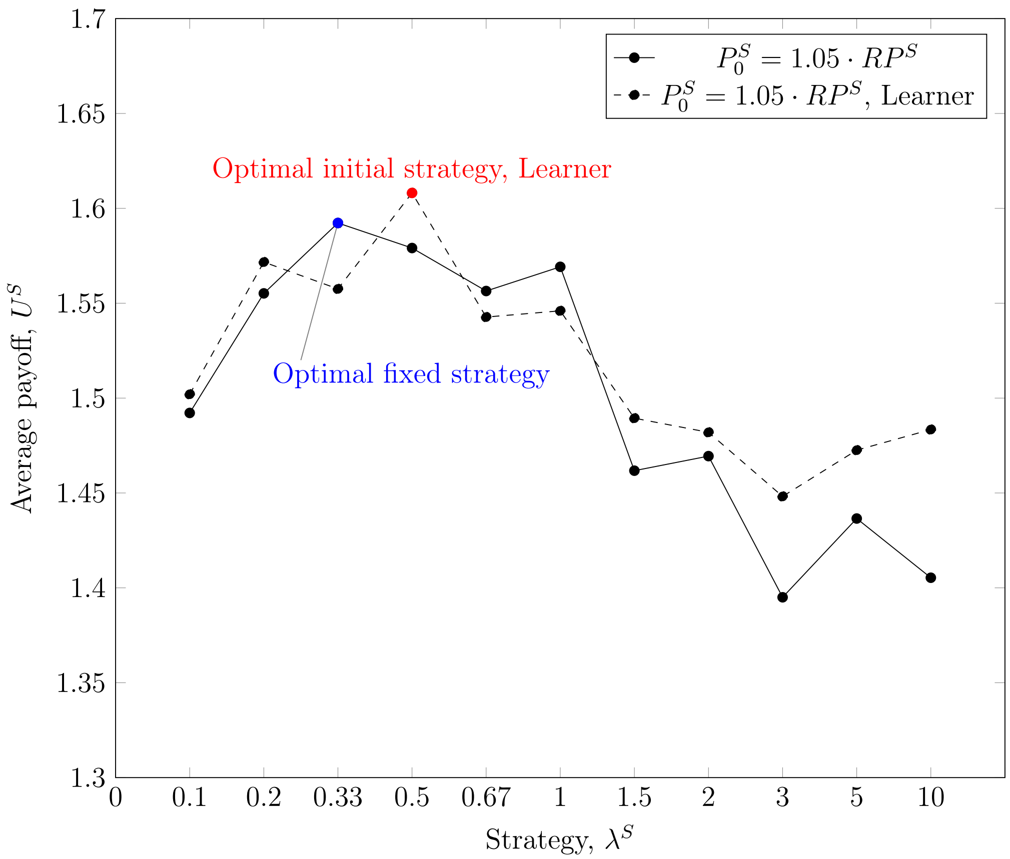



**Figure 4.7:** Average payoff, $U^S$, for a seller with an initial offer 5 % higher than the seller's own reservation price in a learning and non-learning mode.

Looking at the average number of rounds before a deal takes place, we see from Figure 4.8 that the more conceding the strategy is, the faster an agreement is reached. This is what we would expect since a seller with a conceding strategy gives way faster than a boulware seller, and consequently moves more rapidly towards the zone of agreement

(ZoA). Moreover, the possibility to learn seems to have a positive impact only for the boulware strategies, similar to the results obtained for the average payoff.

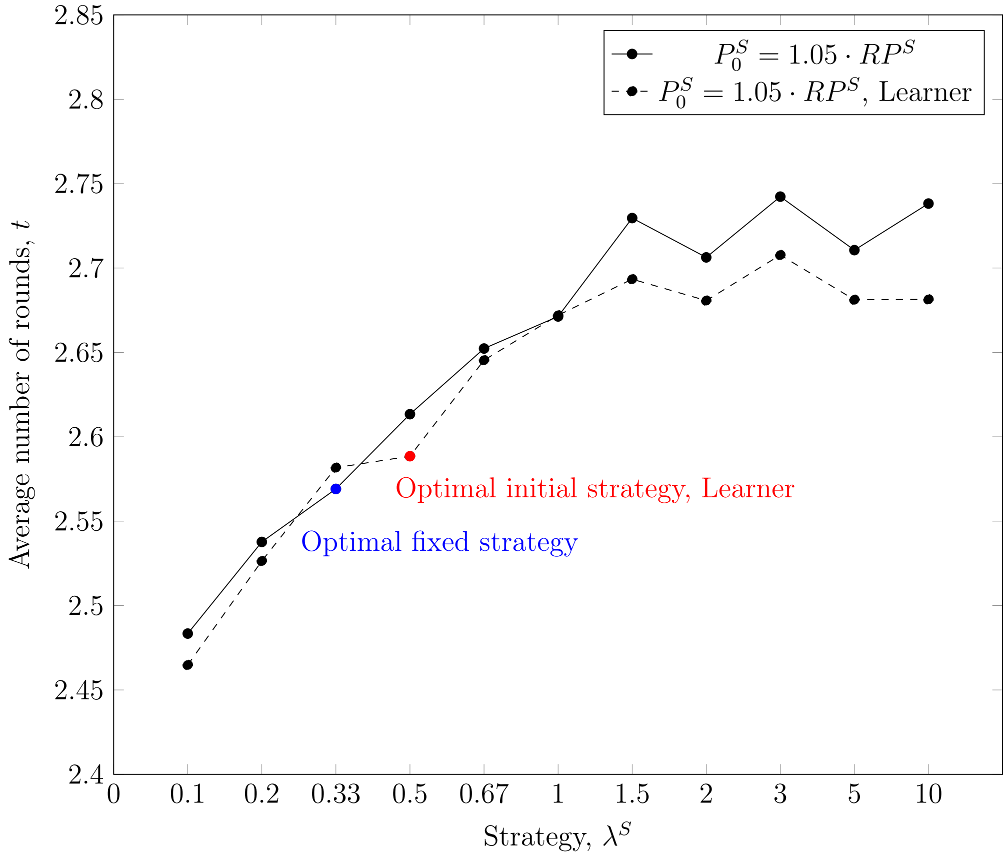



**Figure 4.8:** Average number of rounds for a seller with an initial offer 5 % higher the seller's own reservation price in a learning and non-learning mode.

In the simulations, there are two scenarios which cause the buyer and seller to never reach an agreement. Either one of the agents' deadline has been reached, or a ZoA does not exist as depicted in Figure 4.2b. The simulations are constructed such that a non-ZoA occurs in 50 % of the negotiations to reflect Securum's business environment, as described in Subsection 4.1. Hence, the maximum percentage of deals a strategy can obtain from the simulations is 50 %. Any percentage points below this threshold is caused by the combination of the agents' deadline and strategy. Figure 4.9 shows the percentage of agreements reached for all the different strategies. As we would have expected, the percentage of agreements reached decreases with an increase in the strategy, $\lambda^S$. Note that the learning mode seems to converge to about 34 % of agreements reached for the boulware strategies.

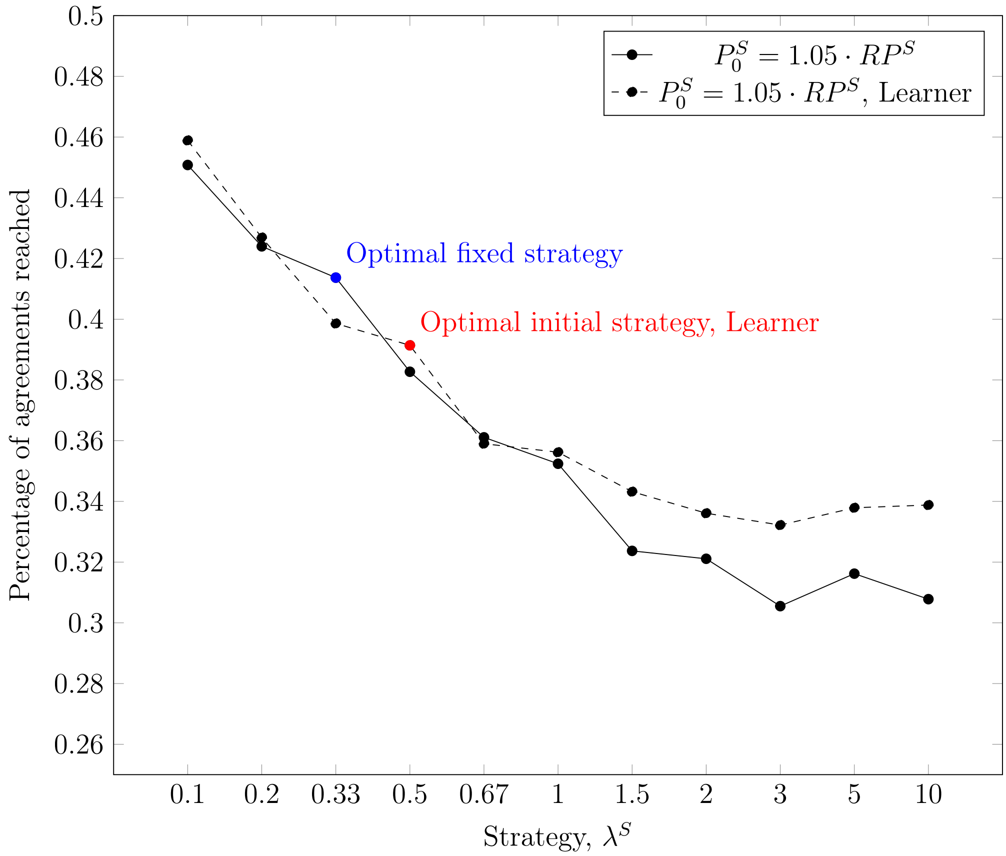



**Figure 4.9:** Percentage of agreements reached for a seller with an initial offer 5 % higher than the seller's own reservation price in a learning and non-learning mode.

A numerical example showing how the seller would generate offers based on the optimal strategy for both the non-learning and learning mode is presented in Figure 4.10. Due to practical and visual reasons in Figure 4.10, the different strategies in learning mode is reduced from eleven to five.[22] With a maximum number of rounds set to five, we end up with $1,331$[23] different strategy paths a seller adopting the learning mode can choose from. The complexity increases exponentially along with the number of rounds an agent has the possibility to learn, and number of strategies. Remember that the seller is first capable of adopting the learning mode after receiving two offers.

[22]Figure 4.10 includes the strategies 0.33, 0.5, 1 , 2, and 3, while in TRAPP, we additionally include 0.1, 0.2, 0.67, 1.5, 5 and 10.

[23]$11^{(5-2)} = 1,331$

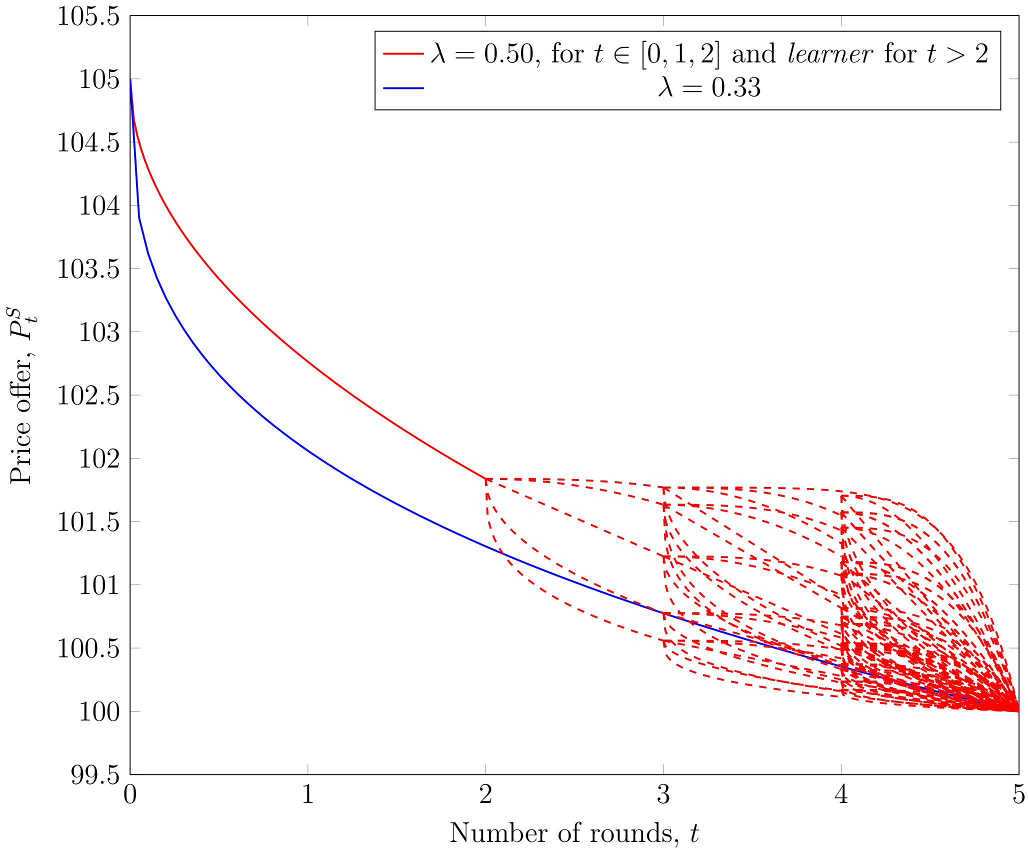



**Figure 4.10:** Example showing how a seller would propose offers in each round, $t$, with the optimal strategies found in Figure 4.7. Numerical values used include the seller's reservation price, $RP^S = 100$ and deadline, $\tau^S = 5$. Possible learning strategies included are $\lambda^S \in \{0.33, 0.5, 1, 2, 3\}$.

### 4.2.3 Optimal Initial Offer and Strategy

Next, we present the result obtained when the seller generates the initial offer based on an estimation of the buyer's reservation price, $\widetilde{RP}^B$, instead of basing it on the seller's own reservation price, $RP^S$. For comparison, we include Securum's current policy of $P_0^S = 1.05 \cdot RP^S$ for the remainder of this section. The estimation is based on some prior knowledge about the hypotheses of the buyer's reservation price, expressed as either a uniform or triangular probability distribution. The optimal initial offer by using a uniform probability distribution is then calculated by using Equation 3.11, whereas we use Equations 3.12 and 3.13 to calculate the optimal initial offer using a triangular probability distribution. This is further described in Subsection 3.6, while the full derivations are found in Appendix A2 and A3. Figure 4.11 shows that proposing the initial offer, $P_0^S$, based on either a uniform or triangular assessment of the hypotheses of the opponent's reservation price yields a significantly higher average payoff than using $P_0^S = 1.05 \cdot RP^S$. The triangular distribution outperforms the uniform distribution which in turn is a better strategy than $P_0^S = 1.05 \cdot RP^S$.[24] The optimal strategy is found to be among the conceder strategies ($\lambda^S < 1$) for all three initial offer methods.

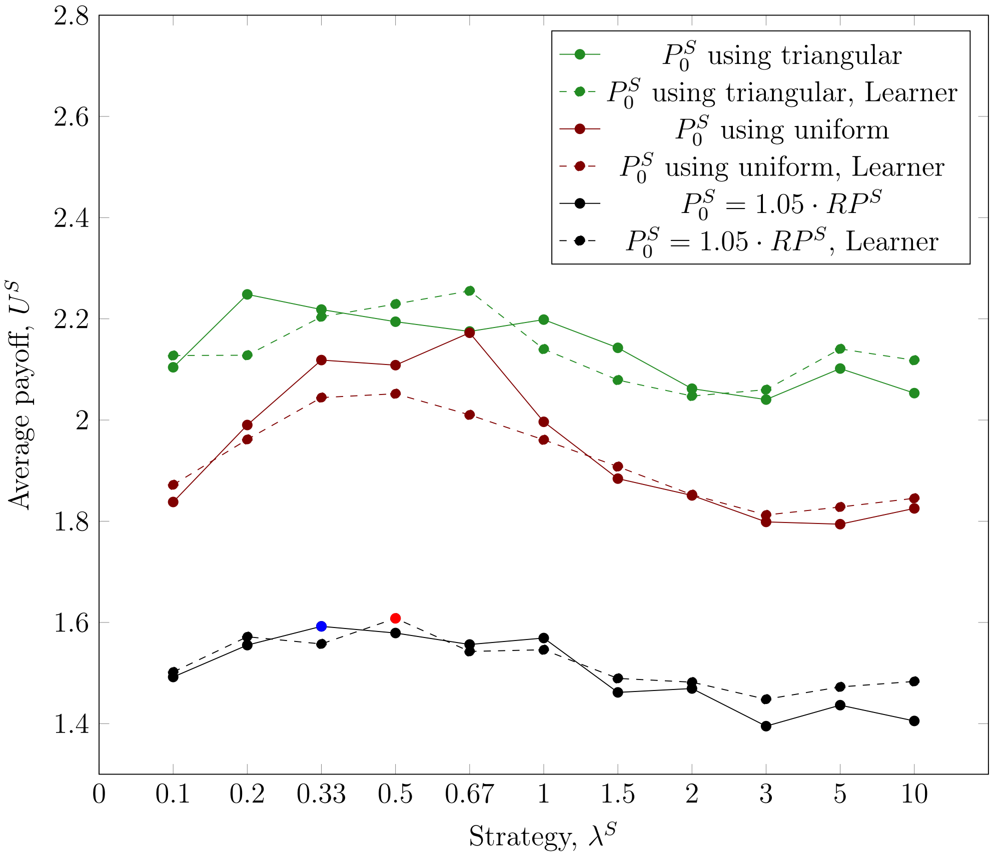



**Figure 4.11:** The average payoff, $U^S$ for the three methods of proposing an initial offer, $P_0^S$, in both learning and non-learning mode.

In the results we obtain using the triangular distribution shown in Figure 4.11, we assume that a seller is able to provide a 100 % accurate estimate of the buyer's reservation price: $\widetilde{RP}_*^B = RP^B$. However, this is an unrealistic assumption in most of the negotiations Securum take part in. Hence, Figure 4.12 is presented to see how sensitive the output from using the triangular distribution is. We find that the seller achieves the highest payoff, $U^S$, for all strategies by generating an initial offer, $P_0^S$, based on a triangular distribution assessment as long as the seller's estimation of the buyer's reservation price is more than 90 % accurate. Once the accuracy is below 80 %, using a uniform distribution

[24]Note that the triangular and uniform distribution achieve a higher average payoff for all strategies compared to $P_0^S(x) = x \cdot RP^S$, for all values of $x$. The numerical values are added in Table 4.1.

and generating $P_0^S$ based on Equation 3.11, results in a higher average payoff for nearly all strategies. Compared to the initial offer based on the seller's own reservation price, $P_0^S = 1.05 \cdot RP^S$, the seller achieves a slightly higher average payoff for the conceder strategies once the estimation when the triangular distribution is more than 50 % accurate. For the boulware strategies, however, the seller obtains approximately the same payoff for the triangular distribution and $P_0^S = 1.05 \cdot RP^S$, when the triangular is estimated 50 % accurately.

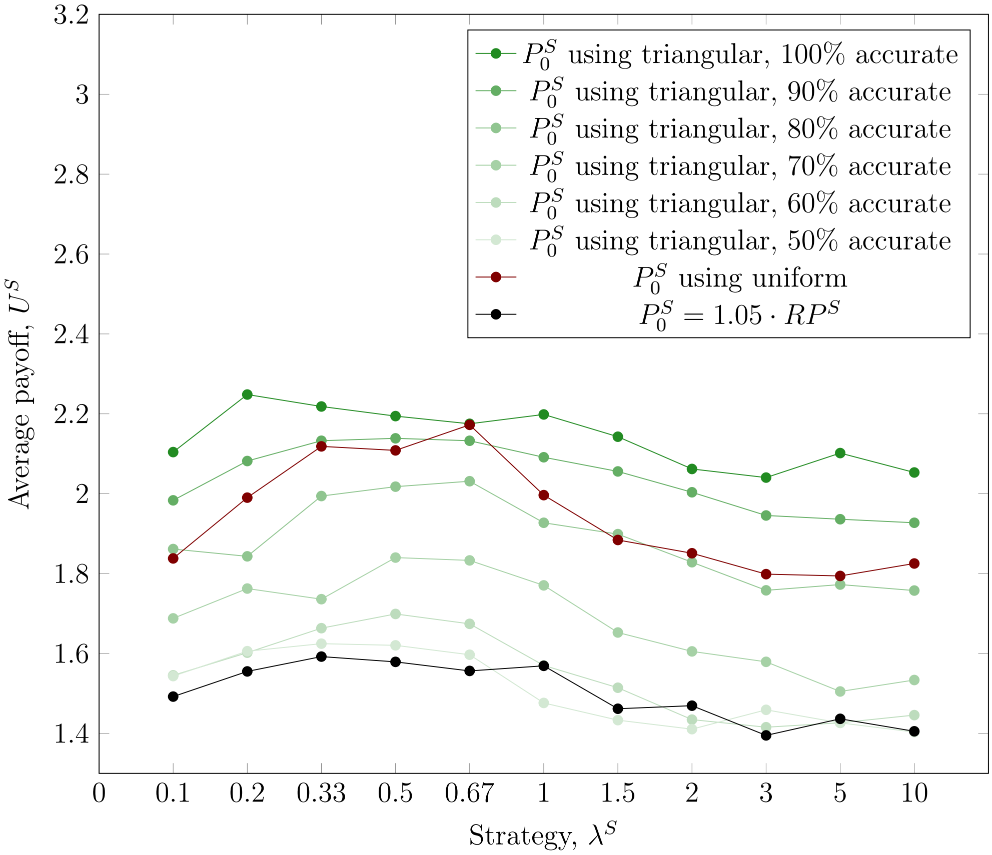
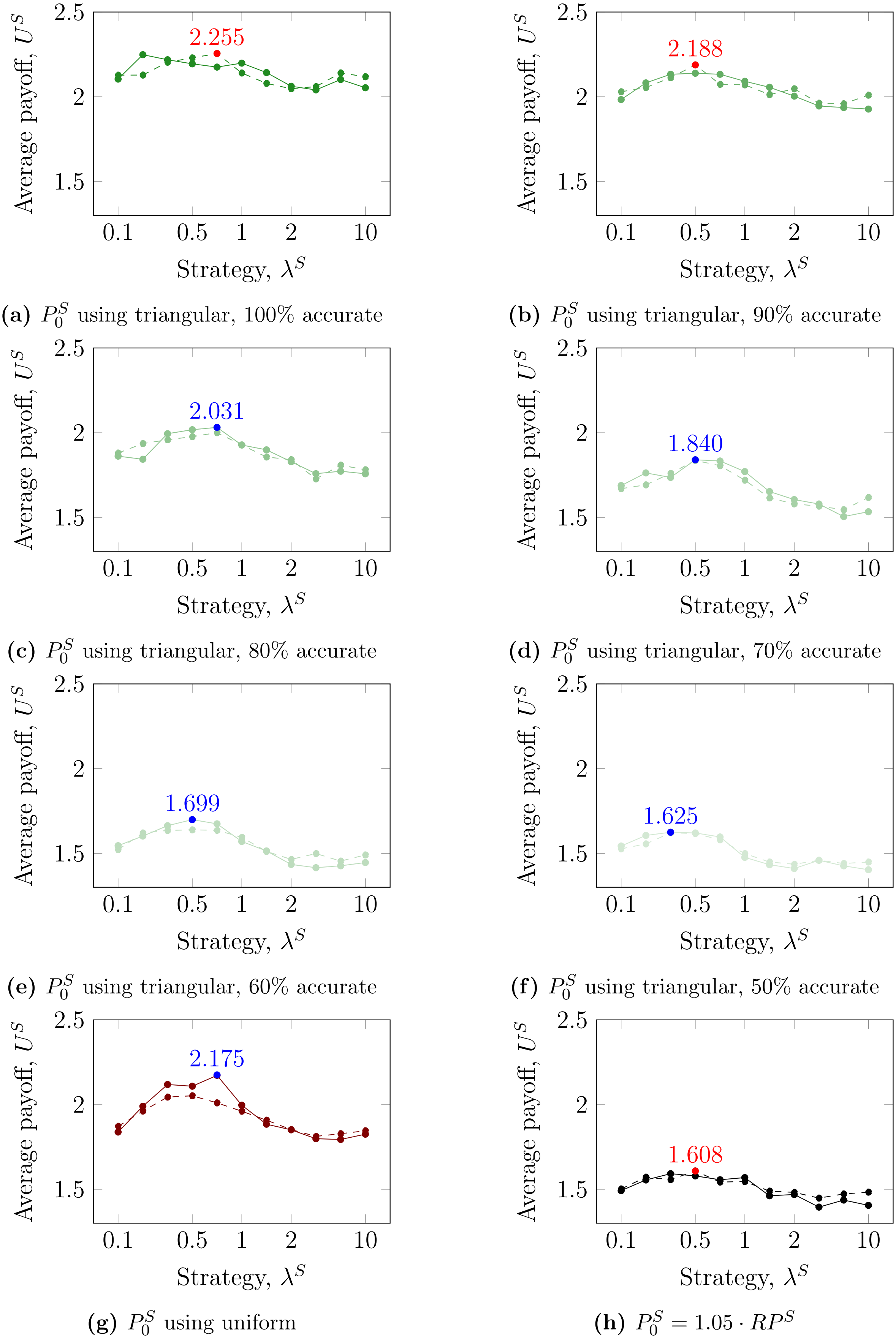



**Figure 4.12:** Average payoff, $U^S$, when conducting sensitivity on the triangular distribution compared to an initial offer, $P_0^S$, using a uniform distribution and 5 % above the seller's own reservation price, $RP^S$, for all strategies, $\lambda^S \in \Lambda$.

By including the learning mode in results obtained in Figure 4.12, we get the eight subfigures presented in Figure 4.13.




(a) $P_0^S$ using triangular, 100% accurate
(b) $P_0^S$ using triangular, 90% accurate
(c) $P_0^S$ using triangular, 80% accurate
(d) $P_0^S$ using triangular, 70% accurate
(e) $P_0^S$ using triangular, 60% accurate
(f) $P_0^S$ using triangular, 50% accurate
(g) $P_0^S$ using uniform
(h) $P_0^S = 1.05 \cdot RP^S$

**Figure 4.13:** Average payoff, $U^S$, for different modes. The average payoff for the optimal strategy in each mode is highlighted in red (learner) or blue (non-learner).

## 4.3 Summary of the Simulations

The optimal strategies for the different modes are summarized in Table 4.1 and ranked with the highest average payoff at the top, and the lowest at the bottom. We have added all the offers based on the seller's own reservation price, $RP^S$, for comparison. By using a triangular distribution with a 100 % accurate assessment of the buyer's reservation price, $RP^B$, yields the highest average payoff. Here, the optimal strategy was found to implement the learning mode with $\lambda = 0.67$ in the pre-learning rounds.

| | Optimal strategy | | Avg. payoff |
|---|---|---|---|
| $P_0^S$ mode | $\lambda^S$ | Learner? | $U^S$ |
| Triangular, 100 % accurate | 0.67 | Learner | 2.255 |
| Triangular, 90 % accurate | 0.50 | Learner | 2.188 |
| Uniform | 0.67 | Non-learner | 2.175 |
| $1.10 \cdot RP^S$ | 0.33 | Non-learner | 2.081 |
| Triangular, 80 % accurate | 0.67 | Non-learner | 2.031 |
| $1.15 \cdot RP^S$ | 0.33 | Non-learner | 1.917 |
| $1.07 \cdot RP^S$ | 0.33 | Non-learner | 1.896 |
| Triangular, 70 % accurate | 0.50 | Non-learner | 1.840 |
| Triangular, 60 % accurate | 0.50 | Non-learner | 1.699 |
| Triangular, 50 % accurate | 0.33 | Non-learner | 1.625 |
| $1.05 \cdot RP^S$ | 0.5 | Learner | 1.608 |
| $1.50 \cdot RP^S$ | 0.1 | Non-learner | 1.591 |
| $1.20 \cdot RP^S$ | 0.2 | Non-learner | 1.534 |
| $2.00 \cdot RP^S$ | 0.1 | Non-learner | 1.024 |

**Table 4.1:** The optimal strategy, $\lambda^S$, for the different methods of proposing an initial offer, $P_0^S$, sorted by the highest average payoff, $U^S$, at the top.

In general, the conceder strategies outperform the boulware strategies in terms of average payoff for all the modes. We find that the main part of the average payoff from the conceding strategies stems from the high percentage of deals obtained. As seen in Figure 4.14, the blue line shows that the average payoff *if* a deal takes place, increases as the seller's strategy becomes more boulware. However, the trade-off for being too boulware is a loss in the number of deals taking place. The product of the average payoff achieved when a deal takes place and the percentage of deals obtained for a given strategy, results in the black line which we recognize as the solid line from Figures 4.7 and 4.13h.

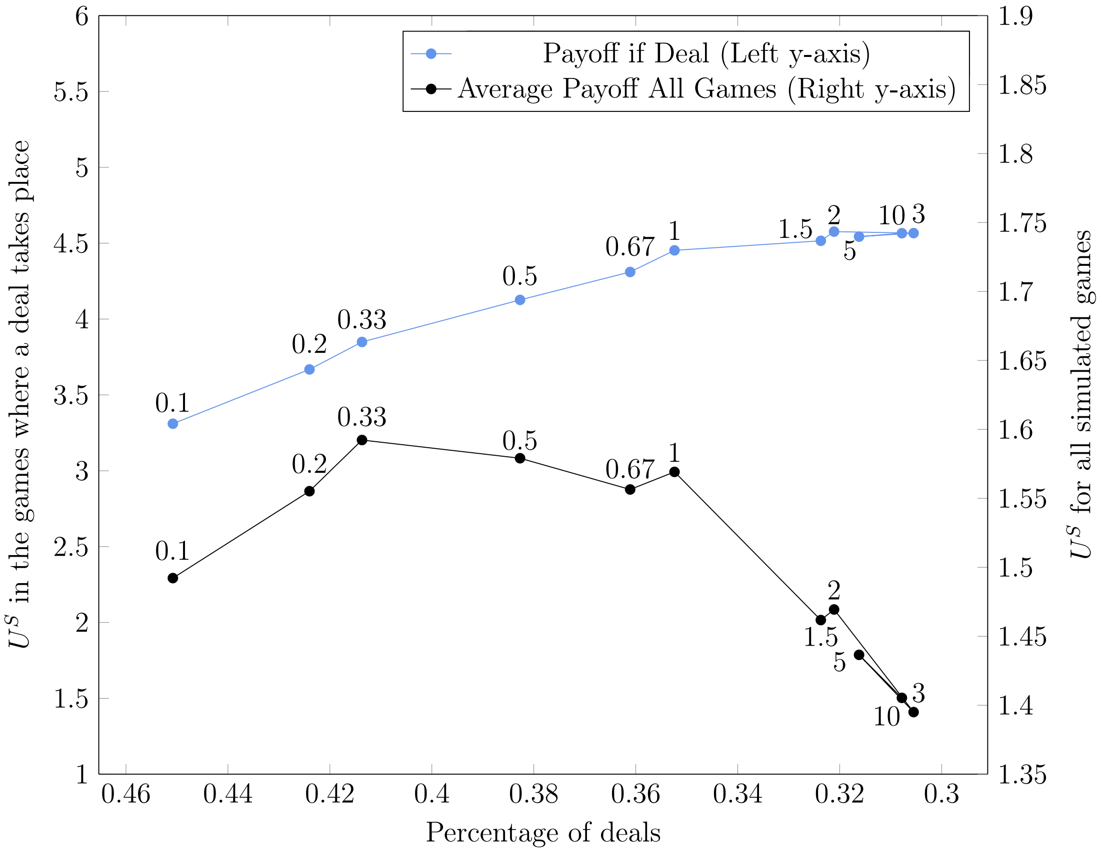



**Figure 4.14:** Average payoff, $U^S$, if a deal takes place is shown on the left y-axis and the total average payoff on the right y-axis. The strategies are displayed as data labels above the lines.

By comparing the different methods for generating the seller's initial offer, $P_0^S$, our simulations show that the seller obtains a higher average payoff when basing $P_0^S$ on an estimation of the buyer's reservation price, $\widetilde{RP}^B$, rather than on the seller's own reservation price, $RP^S$, i.e., $U^S\Big(P_0^S(\widetilde{RP}^B)\Big) > U^S\Big(P_0^S(RP^S)\Big)$. This makes sense as the seller can, in the cases where the buyer proposes the very first offer, adjust the hypotheses of the buyer's reservation price due to the additional information gathered throughout the game, seen as step 2 and 3 in Algorithm 1, even before proposing the initial offer.

Implementing the learning mode is found to have little to none real improvement for all strategies in our simulations. In previous work like Zeng & Sycara (1997) and Sim et al. (2009) on the other hand, the learning mode is found to have a significant effect. Our main deviation from previous Bayesian machine learning models applied to negotiations, is the implementation from a practical standpoint. In Sim et al. (2009) for instance, a typical number of rounds played in the simulations is about 50. In our case, the agents negotiating about a speculative real estate, usually have a deadline between two and five rounds. Hence, with only a few offers exchanged in each simulated game, there is a limited amount of information to learn from. In addition, remember that an agent does not start learning until round two due to the lack of information after having received only an initial offer. Therefore, the learning mode becomes less valuable in our case.

# 5 Case Study

The purpose of this section is to apply our findings from Section 4 Empirical Results to see how our model performs in a real bargaining game. We compare the results we obtain using our model with the outcome obtained by Securum in the actual negotiations, to see if their strategy could have been improved. Moreover, the model's limitations, discovered through this case study, are addressed to understand their implications in a real-life setting.

Securum acquired a speculative real estate, hereafter referred to as *Globusgården*, in 2012. In 2020, a buyer approached them, initiating a negotiation process. After two rounds of offer exchanges, a deal took place and Securum sold the property through negotiations instead of continuing with their development plans.

This section is organized in three main parts. First, we address the development projects Securum had for Globusgården and use the real options valuation (ROV) model developed in Brynildsen & Hyttedalen (2020) to derive the real options value as a method of establishing the reservation price, $RP^S$. Next, we present the real-life negotiation between Securum and the buyer before we compare the actual payoff with the output from TRAPP. Finally, based on this case study, we discuss the performance and limitations by using TRAPP to generate a seller's offers.

## 5.1 Globusgården

Globusgården is a property located in the city center of Drammen, Norway. Today, the building hosts retail stores, offices, and restaurants. Securum acquired the property in 2012, and classified it as a speculative real estate investment, as they wanted to apply for a change in the zoning at the municipality's office. Globusgården is one of Drammen's most venerable buildings, and when Securum announced their plans for the building, a political debate arose about what Securum should be allowed to do with it. Some politicians were happy that Securum wanted to renew the historical building, while others wanted to preserve it.


**Figure 5.1:** Picture of Globusgården in Drammen, used with permission from Securum.

Securum had identified three prospective development projects for Globusgården. We will now assess these with the ROV-model presented in Brynildsen & Hyttedalen (2020) and derive the optimal decision policy found using this model, along with the corresponding real options value in order to establish a reservation price for Securum.

## 5.2 Real Options Valuation Model

In the case of selling Globusgården through a negotiation, we argue that the optimal reservation price for Securum as the seller should equal the purchase price of the property plus the real options value at the time a sale is taking place. A sale below this value would yield a lower payoff than what Securum could expect from the continuation of development, and would be irrational. Recall that we assume a risk-neutral and rational investor. An offer above this value should be accepted as it is higher than the expected payoff value from development. Hence, we use the ROV-model presented in Brynildsen & Hyttedalen (2020) to obtain the real options value and the optimal decision policy, corresponding to Securum's best alternative to a negotiated agreement (BATNA). The specifications of the property provided by Securum are summarized in Table 5.1.

| $V_0(0)$ | $C^H(n)$ | $r_f$ |
|---|---|---|
| 55 MNOK | 0 NOK | 10 % |

**Table 5.1:** Overview of the values of the Globusgården-project.

Securum acquired the property for 55 MNOK in 2012. They assumed the property value to increase by approximately 3 % each year, resulting in a property value of about 70 MNOK in an undeveloped state in 2020, the year they ended up selling the property. Additionally, the property generated a yearly rental income of 4 MNOK. However, the financial- and maintenance costs were approximately equal to the rental income, resulting in net holding costs, $C^H$, of zero per year for Securum. The discount rate, $r_f$, used by Securum is 10 %[25].

Securum had outlined three development projects, $i = \{1, 2, 3\}$, for Globusgården: (1) A four storey building intended as offices for the county, hereafter referred to as *Fylkesmannen*. (2) A 13 storey office and apartments building sketched by MAD Architects, hereafter referred to as *MAD*.


**Figure 5.2:** Architect drawing of project 2, referred to as MAD. Picture used with permission from Securum.

(3) Finally, a futuristic 34 floor hotel projected to be one of the tallest skyscrapers in Norway, sketched by the local architect Trond Martens, hereafter referred to as *Martens*.

[25]As for Securum, this is a typical assumption in the speculative real estate investment business.


**Figure 5.3:** Illustration of project 3, referred to as Martens. Picture used with permission from Securum.

For each of these three projects, the new property value, $V_i(n)$, would be significantly higher than the purchase price, $V_0(0)$, if they would succeed and get their development plans approved. The specifications of each project are summarized in Tables 5.2 and 5.3. Note that Securum believed the new property value of a developed project, $V_i(n)$, to be the same independent of what time, $n$, it would be successfully developed. Consequently, we set $V_i(n) = V_i$ as seen in Table 5.2. Both the minimum time until the project is successfully developed, $t_{min,i}$, and the time increment between a possible successful development, $\Delta t_i$, are 1 year for all the development projects. $M_i$ denotes the maximum number of years Securum could attempt project $i$. A detailed explanation of every parameter used in the ROV-model is found in Table A1.1 in Appendix.

| Project | $i$ | $V_i$ | $t_{min,i}$ | $M_i$ | $\Delta t_i$ |
|---|---|---|---|---|---|
| Fylkesmannen | 1 | 80 MNOK | 1 year | 3 year | 1 year |
| MAD | 2 | 135 MNOK | 1 year | 8 year | 1 year |
| Martens | 3 | 225 MNOK | 1 year | 3 year | 1 year |

**Table 5.2:** Overview of project details for the three projects outlined by Securum.

| | | **m** | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Project** | **Variable** | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Fylkes- | $C_1(m)$ | 250' | 250' | 250' | 250' | - | - | - | - | - |
| mannen | $P_1(m)$ | 0 % | 10 % | 20 % | 20 % | - | - | - | - | - |
| MAD | $C_2(m)$ | 1 000' | 750' | 750' | 750' | 750' | 750' | 750' | 750' | 750' |
| | $P_2(m)$ | 0 % | 5 % | 15 % | 15 % | 10 % | 10 % | 10 % | 5 % | 5 % |
| Martens | $C_3(m)$ | 500' | 750' | 750' | 750' | - | - | - | - | - |
| | $P_3(m)$ | 0 % | 5 % | 15 % | 10 % | - | - | - | - | - |

**Table 5.3:** Overview of development project details for Securum at Globusgården. Numbers in NOK ('000).

In Table 5.3, $m$ denotes the number of years the project has been attempted. The cost of attempting to develop, $C_i(m)$, and the probability of a successful development, $P_i(m)$, are shown for all $m$. Note that $P_i(m) = 0\%$ when $m = 0$ since the minimum time until a successful development is $t_{min,i} = 1 \; \forall \; i$.

Securum considered a 13 year long timeline for this investment. They started with Martens for two years before converting to MAD and were planning to keep attempting this project for a total of eight years. Lastly, Securum would try Fylkesmannen for the remaining three years. If none of the projects had been successfully developed by the end of 2025, they were planning to sell the property in the same state as in 2012 and embrace the sunk development costs.

The main novelty in the ROV-model is that it incorporates four different types of options, in addition to the option to develop the property, simultaneously: deferment, abandonment, conversion and keep. The corresponding optimal decision policy for Securum is presented in Table 5.4. Here, *Attempt* refers to development of the specified project and *Sell* implies to use the option to abandon the project. $F^*(n)$ is the total option value in time $n$.

| $n$ | Year | Project to choose | Decision | $F^*(n)$ |
|---|---|---|---|---|
| 0 | 2012 | Martens | Attempt | **42,189.87** |
| 1 | 2013 | Martens | Attempt | 40,483.01 |
| 2 | 2014 | MAD | Attempt | 23,360.37 |
| 3 | 2015 | MAD | Attempt | 20,884.89 |
| 4 | 2016 | MAD | Attempt | 17,553.76 |
| 5 | 2017 | MAD | Attempt | 16,983.30 |
| 6 | 2018 | MAD | Attempt | 16,322.79 |
| 7 | 2019 | MAD | Attempt | 15,557.94 |
| 8 | 2020 | MAD | Sell | 14,672.35 |

**Table 5.4:** Optimal path derived for Globusgården. Real options values are stated in NOK ('000).

The decision policy we obtain by the ROV-model is almost identical to the decision policy Securum decided on in 2012. The only difference is that the output from the model suggests that Securum should sell the property in 2020 regardless of the possibility to try MAD for another year or convert to the Fylkesmannen-project. The real options value in 2020 comes from the increased property value of 3 % that Securum anticipated in 2012. The additional property value from the purchasing price in year 2012, is equivalent to the abandonment option. The reservation price in 2020 is assessed to be approximately 70 MNOK according to the output from the ROV-model, which corresponds to the estimated property value in that period. Hence, at this point in time, the expected payoff from the remaining development options are worth less than to simply sell the property, taking into account the discount rate used by Securum. The optimal decision policy along with the property value, option value and reservation price (RP) are depicted in Figure 5.4.

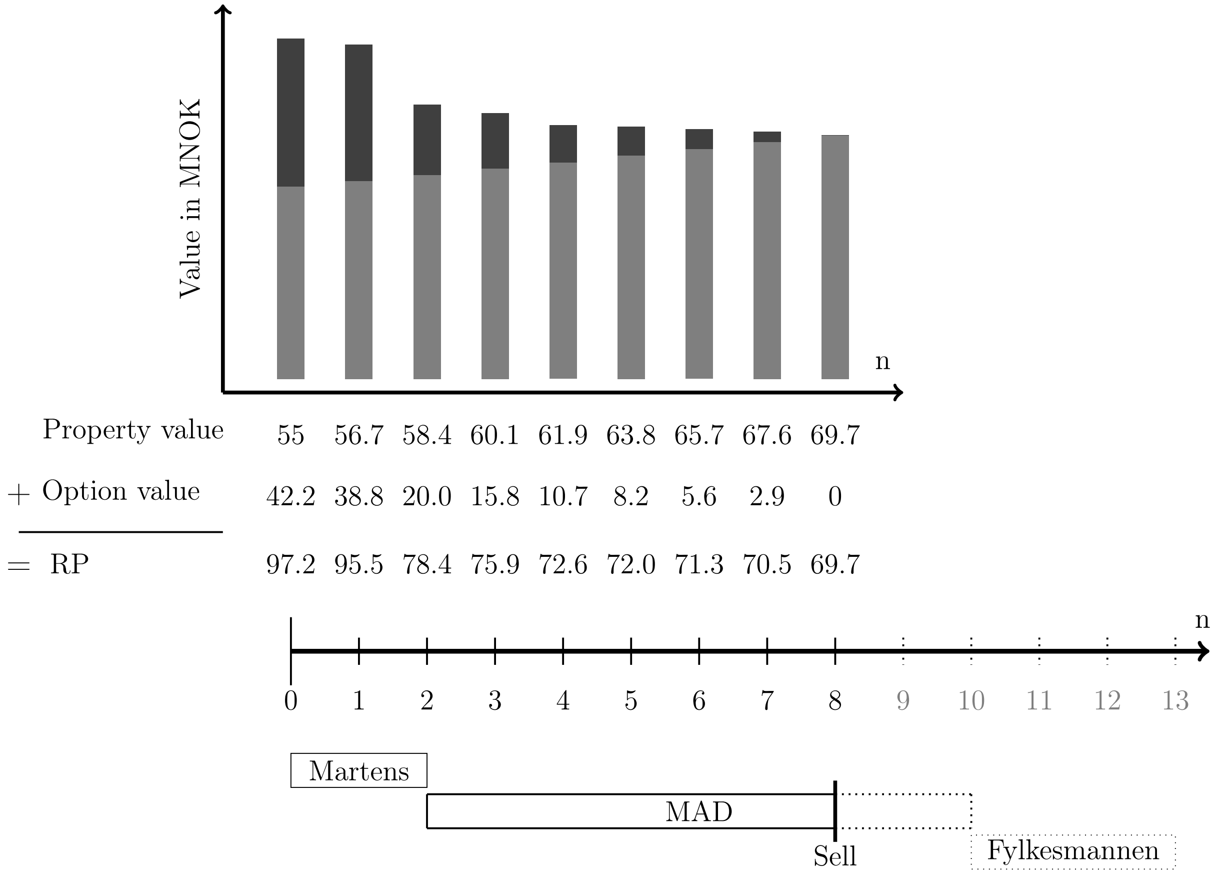
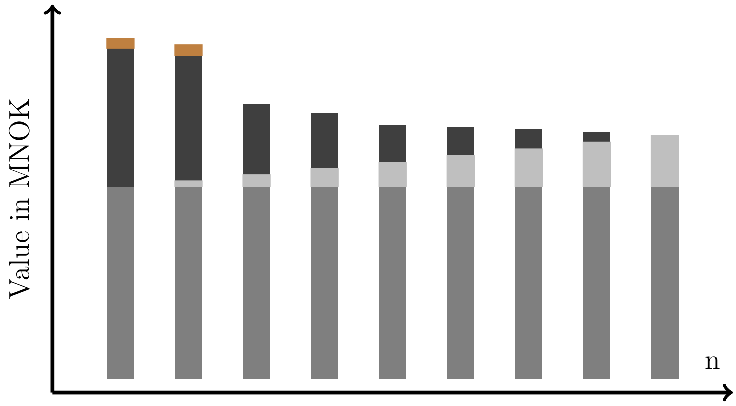



**Figure 5.4:** The optimal decision policy (solid Gannt chart) with property value, option value and reservation price (RP) in each period in time, $n$. Dotted Gannt chart represents Securum's original decision policy after n = 8. Numbers in MNOK.

As mentioned earlier, the novelty of the ROV-model is that it incorporates four different option in addition to the option to develop the property. Figure 5.5 extends Figure 5.4 and shows that the option to develop (dark gray bar) is absent when n = 8, since the discount rate used by the company is greater than the underlying growth of the property. Consequently, the best alternative is to sell the property and obtain the abandonment option value of 14.7 MNOK. The values of the options to defer and keep are zero in this particular case. As mentioned, the value of the option to abandon is in this case the aggregated increase in property value each year, colored light gray in Figure 5.5. Marked with a brown color, the value from the conversion option is present when n equals 0 and 1. This comes from the possibility to convert from the Martens-project to the MAD-project. Once the conversion happens, in n = 2, the option is exercised and is of no value for the rest of the negotiation. Lastly, the bottom gray bar chart is the purchase price for Securum in 2012, which equals $V_0(0)$.




| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| Conversion | 3.0 | 3.5 | - | - | - | - | - | - | - |
| + Development | 39.2 | 35.4 | 20.0 | 15.8 | 10.7 | 8.2 | 5.6 | 2.9 | - |
| + Abandonment | - | 1.6 | 3.4 | 5.1 | 6.9 | 8.8 | 10.7 | 12.6 | 14.7 |
| + Purchase Price | 55 | 55 | 55 | 55 | 55 | 55 | 55 | 55 | 55 |
| = *Total Value* | 97.2 | 95.5 | 78.4 | 75.9 | 72.6 | 72.0 | 71.3 | 70.5 | 69.7 |

**Figure 5.5:** Reservation price (BATNA) divided up into the different options and purchase price.

Summarized, we find that the reservation price for Securum should be approximately 70 MNOK when they entered into the negotiation in year 2020 ($n = 8$). In the next subsection, we will see how this value corresponds to the real-life negotiation.

## 5.3 The Globusgården Negotiation

Eight years after the purchase of Globusgården and since the first development process began, a buyer approached Securum with an offer to buy the property. An initial offer turned into negotiations, which eventually ended in a deal. We begin this subsection with an overview of how the negotiation evolved, before we reconstruct the negotiation using the optimal mode found in Subsection 4.2 for Securum as the seller. A brief outline of the main bidding events is summarized as follows:[26]

**December 1st, 2020** Securum is approached by another real estate company interested in buying Globusgården.

[26]Note that this is meant to serve only as an overview of the bids exchanged. Involvement with brokers and internal meetings, for instance, are not included in this description.

**December 5th, 2020** The buyer proposes an initial offer: 106 MNOK.

**December 6th, 2020** Securum proposes a counteroffer: 112 MNOK.

**December 18th, 2020** The buyer proposes a new offer: 108 MNOK.

**December 21th, 2020** Securum proposes a counteroffer: 110.5 MNOK.

**December 22nd, 2020** The buyer accepts Securum's latest offer at 110.5 MNOK.

The negotiation dance is depicted in Figure 5.6, and will be revisited in the analysis of the bidding sequence.

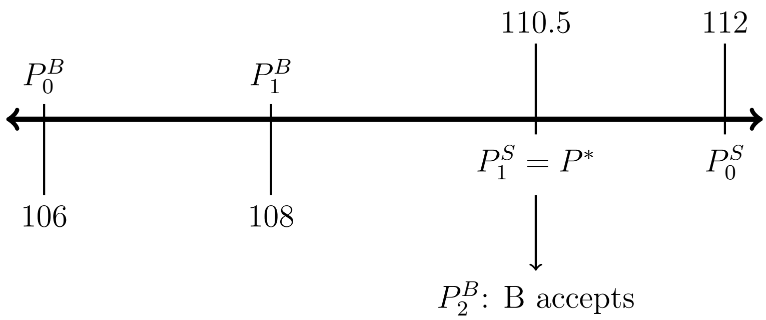



**Figure 5.6:** The actual negotiation dance between Securum and the buyer of Globusgården.

In order to assess the usefulness of TRAPP as a supporting tool in a negotiation process, we apply it to the Globusgården negotiation. This is done by reconstructing the negotiations, using the optimal mode found in Subsection 4.2 to generate Securum's offers. In addition, we analyze how the negotiation dance looks like if Securum proposes their initial offer before the buyer. Our main challenge in the simulation of this negotiation is to reconstruct the buyer's counteroffers as a response to the new offers proposed when using TRAPP. Moreover, the opponent's true reservation price, deadline, and strategy are unknown. Even after the negotiations have been terminated, the agents are likely to keep their own reservation price and deadline confidential (Raiffa, 1982, p. 130). To solve this issue, scenario based offers from the buyer beyond the two actual offers are generated using the bidding history from the actual negotiations combined with the assumptions of the buyer's profile from Subsection 4.1.

The first step in order to simulate the negotiation dance using TRAPP, is to establish Securum's reservation price. In our methodology, we argue that the best alternative to a negotiated agreement (BATNA) for a speculative real estate equals the sum of the property value and the option value, and should be used as the seller's reservation price, $RP^S$. The option value can be estimated using the ROV-model developed in Brynildsen & Hyttedalen (2020) as explained in Subsection 5.2. Using this methodology, $RP^S$ is found to be approximately 70 MNOK, as seen in the previous subsection. However, in discussions with Securum, they told us that $RP^S = 70$ MNOK is far below what they considered as their reservation price in this negotiation. They agreed that there was a minimal value left of the development option as seen from the output by the ROV-model illustrated in Figure 5.4. However, Securum argued that they had some knowledge about the buyer's purchasing power, and the fact that the buyer approached them, influenced how they determined their reservation price.

Further, Securum estimated their reservation price in this case to be somewhere between 108 and 109 MNOK. For the purpose of this case study, we use the average value of this estimate, $RP^S = 108.5$ MNOK. Hence, rather far from the output from the ROV-model at 70 MNOK. The subjective assessment of the probability distributions based on the seller's intuition and the exclusion of market risk, make the output from the ROV-model to serve only as an indication for Securum when assessing a property, as discussed in Brynildsen & Hyttedalen (2020). Nevertheless, in talks with Securum, they agreed that their BATNA for Globusgården in 2020, if only considering the alternatives displayed earlier in this section, had a value of about 70 MNOK. However, we discover through this case study, that our definition of BATNA may be inadequate in the case where the owner of a property possesses valuable information about the prospective buyer. Odd Hyttedalen from Securum puts it this way:

> *If we had decided to abandon this project by actively trying to sell it on the market, I guess we would eventually accept a price around 70 MNOK. But that does not mean that we would have listed it for an asking price at 70 MNOK! For instance, I believe we could have managed to sell parts of the property to investors, valuing it a great deal above 70 MNOK. Maybe around 100 MNOK but it is hard to say.*

Odd Hyttedalen (personal communication April 30, 2021)

Hence, if Securum had approached the market and wanted to sell the property, they would have had a reservation price of around 70 MNOK. This corresponds very well with our definition of BATNA as the sum of property value and option value obtained using the ROV-model in Brynildsen & Hyttedalen (2020). However, in the case of Globusgården, Securum had no intention of selling the property at the time when they were approached by the buyer. However, they explained that "everything is for sale, at the right price." (personal communication, employee from Securum, April 30, 2021).

Next, we asked Securum what they considered, at the time prior to receiving the initial offer, to be a reasonable estimate of the buyer's reservation price. They estimated a lower limit of $\widetilde{RP}_l^B = 100$ MNOK and a best guess of the buyer's highest upper limit at $\widetilde{RP}_h^B = 120$ MNOK, resulting in the hypotheses depicted as blue vertical lines in Figure 5.7.

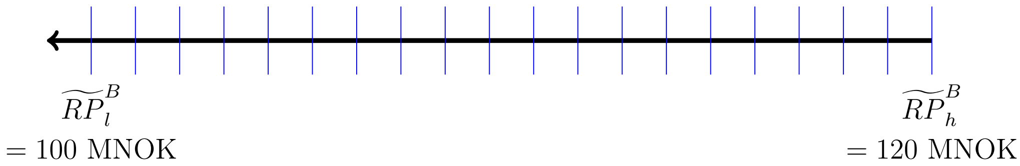



**Figure 5.7:** Securum's hypotheses of the buyer's reservation price before receiving an initial offer, $P_0^B$.

As a buyer would always prefer the lowest possible settlement price, it may seem odd to be bounded by a lower limit. Nevertheless, an estimation of the lower limit reservation price, $\widetilde{RP}_l^B$, is not really necessary in cases where the buyer is the first to propose an offer, as this offer automatically is updated as the lowest feasible limit.[27] Securum received an initial offer at $P_0^B = 106$ MNOK, and all the initial hypotheses below this value are consequently rejected. The updated set of hypotheses is illustrated in Figure 5.8.

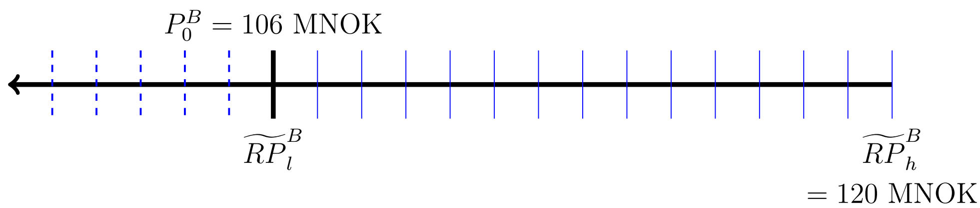



**Figure 5.8:** Securum's hypotheses of the buyer's reservation price after receiving the first offer. The rejected hypotheses are represented by the dashed lines.

Next, we assign a probability distribution to the hypotheses illustrated in Figure 5.8, by

[27] This procedure is illustrated in line two and three in Algorithm 1.

asking Securum if they had an opinion on the most likely hypothesis:

> *I can't really say that we had any better estimation of the buyer's reservation price other than the interval between now 106 MNOK and 120 MNOK. If I had to give you my best guess of an exact value in that interval, I would say the center point: 113 MNOK. But I am not even 25 % sure of this.*
>
> Odd Hyttedalen (personal communication April 29, 2021)

Due to the fact that Securum is unable to estimate the buyer's reservation price with an accuracy equal or higher than 90 %, we do not use the triangular probability distribution for generating Securum's initial offer, $P_0^S$. These results are summarized in Table 4.1. Securum's best response at this point is to base their initial offer on a uniform distribution of the hypotheses of the buyer's reservation price. Moreover, the optimal strategy is to play a conceder strategy with $\lambda^S = 0.67$ and not adopt the learning mode. Consequently, we use Equation 3.11 to calculate the optimal initial offer for Securum, as a response to the buyer's initial offer at 106 MNOK:

$$\begin{aligned} P_0^S &= \frac{RP^S + \widetilde{RP}_h^B}{2} \\ &= \frac{108.5 + 120}{2} \\ &= \mathbf{114.25}. \end{aligned}$$

Now, the buyer either accepts the current offer at 114.25 MNOK, leaves the negotiation, or proposes a new offer using Equation 3.1. Hence, as long as the current offer from Securum is above the buyer's reservation price, the buyer either withdraws from the negotiations if the deadline has been reached, or generates a counteroffer independent of Securum's latest offer since we assume a non-learning buyer. As depicted in Figure 5.6, the buyer proposed a counteroffer at $P_1^B = 108$ MNOK. The buyer bids solely based on the buyer's own reservation price with a time-dependent strategy, and we consequently assume the buyer's second offer to be the same in this reconstruction as in the actual negotiation.

Considering that Securum's optimal strategy is a constant conceder strategy with $\lambda^S = 0.67$, we use Equation 3.2 to generate Securum's next offer:

$$P_1^S = 114.25 + (\frac{1}{5})^{0.67} \cdot (108.5 - 114.25)$$
$$= \mathbf{112.30}.$$

Based on the bidding history seen in Figure 5.6, we know that the buyer turned down Securum's counteroffer at 112 MNOK. Consequently, we assume that the buyer turns down the offer at $P_1^S = 112.30$. Again, the buyer now either proposes a new offer or withdraws from the negotiations if the deadline has been reached.

At this point, however, the bidding history from the real-life negotiations stops due to the fact that the buyer only proposed two offers. Values like the buyer's true reservation price, $RP^B$, and deadline, $\tau^B$, remain unknown. Going forward, we use scenario based offers to imitate the anticipated behavior of the buyer as precise as possible. Still, we obtain useful information from the actual negotiation dance to draw some conclusions; We know that $RP^B$ must lie in the region between the buyer's last actual offer and Securum's last rejected counteroffer. Hence, $RP^B \in [P^* = 110.5, P_1^S = 112)$, and we use the center value as an approximation for $RP^B$. Thus, $RP^B \approx (110.5 + 112)/2 = 111.25$. For the time limit, $\tau^B$, we know that it must be greater or equal to the number of rounds played in the real game, namely three[28]. Based on the buyers' profile constructed together with Securum in Subsection 4.1, we assume that the maximum deadline for a buyer is six rounds. Hence, we conclude that $\tau^B$ is either three, four, five, or six. Given the procedure described above, we derive the bids the buyer can propose in the following rounds, depending on $\tau^B$, as seen in Table 5.5. Note that $P_2^B$ corresponds to the buyer's third offer, as the indexing starts at zero with the initial offer, $P_0^B$.

[28]The third round began and terminated when the buyer accepted Securum's counteroffer at 110.5 MNOK.

| $\tau^B$ | $P(\tau^B \mid \tau^B > 2)$ | $\lambda^B$ | $P_2^B$ | $P_3^B$ | $P_4^B$ | $P_5^B$ |
|---|---|---|---|---|---|---|
| 3 | 0.357 | 1.39 | 111.25 | *withdraw* | *withdraw* | *withdraw* |
| 4 | 0.536 | 0.88 | 110.08 | 111.25 | *withdraw* | *withdraw* |
| 5 | 0.083 | 0.70 | 109.68 | 110.50 | 111.25 | *withdraw* |
| 6 | 0.024 | 0.60 | 109.46 | 110.12 | 110.71 | 111.25 |

**Table 5.5:** Predicted bids from the buyer, $P_t^B$, depending on the buyer's deadline, $\tau^B$.

We see from Table 5.5 that the buyer's third offer, $P_2^B$, is greater than Securum's reservation price, $RP^S$, independent of which deadline the buyer has: $P_2^B > RP^S \; \forall \; \tau^B$. Consequently, Securum accepts this offer and the negotiation terminates in round three. The negotiation dance if Securum adopts the optimal mode found in Subsection 4.2 is shown in Figure 5.9.

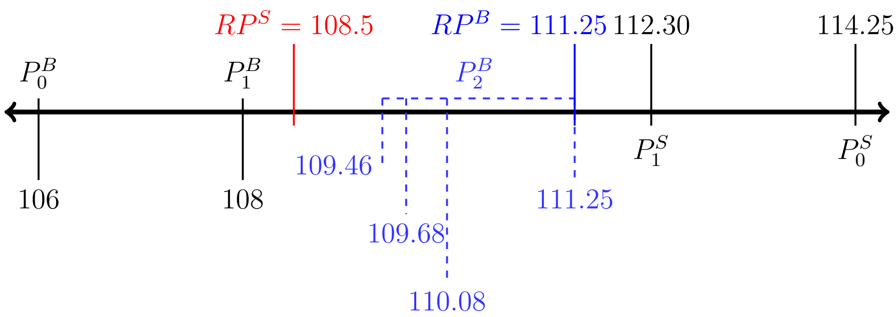



**Figure 5.9:** The negotiation dance if Securum had applied TRAPP with the optimal mode found in Subsection 4.2. Blue vertical dashed lines represent possible settlement values.

Using the conditional probabilities for the buyer's deadline, $P(\tau^B|\tau^B > 2)$, from Table 5.5, we calculate the expected payoff for Securum:

$$\begin{aligned} E[U^S] &= \sum (P_2^B - RP^S) \cdot P(P_2^B) \\ &= (111.25 - 108.5) \cdot 0.357 \\ &\quad +(110.08 - 108.5) \cdot 0.536 \\ &\quad +(109.68 - 108.5) \cdot 0.083 \\ &\quad +(109.46 - 108.5) \cdot 0.024 \\ &= \mathbf{1.95}. \end{aligned}$$

The expected payoff is slightly lower than the payoff achieved in the real-life negotiations: $110.5 - 108.5 = 2.00$, shown in Figure 5.6. Nevertheless, the payoff $U^S = 1.95$ we obtain when using the optimal mode shown in Table 4.1 implies that Securum got $1.95/(111.25 - 108.5) \approx 71$ % of the possible payoff to be shared between the agents, given that the approximation of the buyer's reservation price is correctly estimated.

"There were definitively talks within the company if we should suggest a price to the buyer after they approached us." (personal communication, employee from Securum, April 30, 2021). We will now analyze the payoffs in the scenario when Securum propose the very first offer. When Securum were approached by the prospective buyer of Globusgården, it was a possibility that Securum could have initiated the bidding. Recall that it went four days from the day the initial contact was made until the buyer proposed an offer. The actual negotiation dance would then have been as illustrated in Figure 5.10.

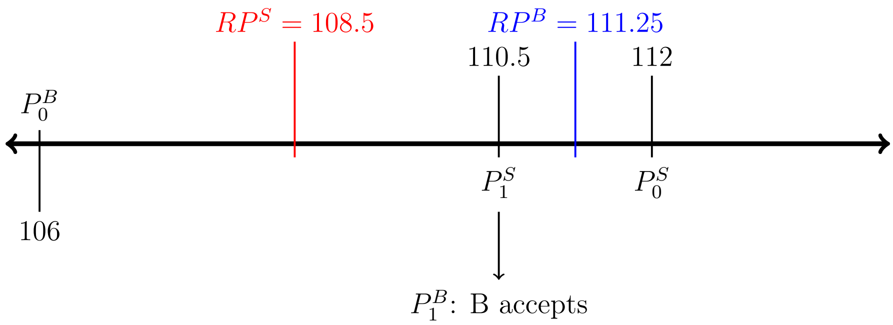



**Figure 5.10:** The actual negotiation dance if Securum had initiated the bidding.

Note that the bids in this case are equivalent to the bids exchanged when the buyer initiated the negotiation dance, only proposed in a different order. The reason is that we assume the agents to generate bids independent of their opponent's offers (non-learning mode), and only by following its own time-dependent tactic. Hence, with only two real counteroffer to base Securum's bidding on, our best estimation is to assume that they follow this pattern in the case where they make the very first offer. An interesting extension to our model, however, would be to include behavioral-dependent tactics. The difference from time-dependent is that the agents' own bidding is subject to the opponent's behaviour, as explained further in Subsection 3.2. For instance, if the buyer adopts a boulware strategy, it is plausible to believe that the seller's response is to do the same.[29]

[29]The strategy of mimicking the opponent's strategy is known as the "tit-for-tat" strategy in behavior-dependent strategy literature (Baarslag et al., 2016).

The implementation of this feature in TRAPP is, however, out of the scope of this thesis and requires further research.

The actual negotiation dance if Securum would have initiated the bidding is shown in Figure 5.10 and results in the same settlement price, 110.5 MNOK, and payoff, 2.00 MNOK, when the buyer proposed the first offer. This is expected since we assume that both Securum and the buyer are non-learners. However, when we use the optimal mode using uniform distribution for the initial offer with a strategy of $\lambda^S = 0.67$, shown in Table 4.1, to generate Securum's offers, and let them propose their initial offer before the buyer, the negotiation dance drags on for three whole rounds before the buyer accepts. This negotiation dance is depicted in Figure 5.11.

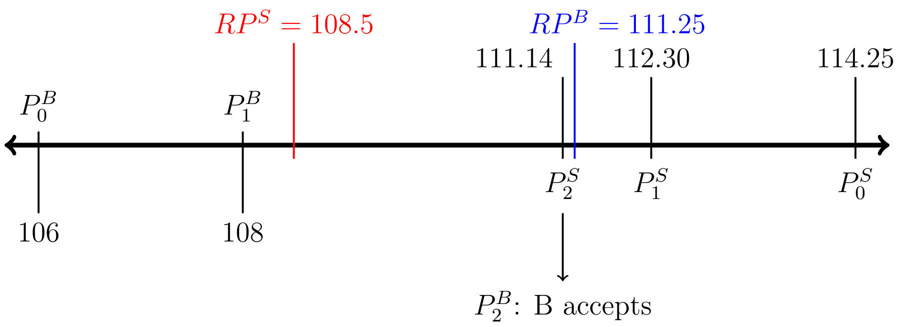



**Figure 5.11:** The negotiation dance if Securum had initiated the bidding and applied TRAPP with the optimal mode shown in Table 4.1.

The negotiation dance in Figure 5.11 results in a payoff, $U^S$, close to the maximum for Securum in this game if the estimates are conducted correctly, yielding $U^S = 111.14 - 108.5 = 2.64$, which corresponds to 96 % of the zone of agreement (ZoA).

All the different negotiation dances shown in Figures 5.6, 5.9, 5.10, and 5.11 are summarized for comparison in Figure 5.12.

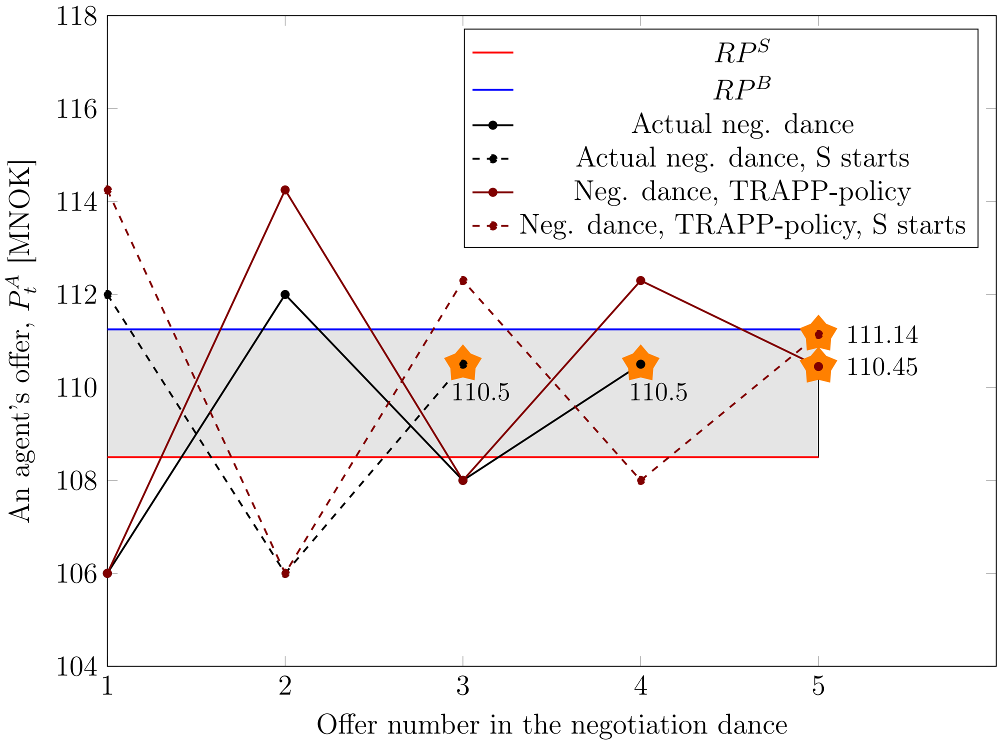



**Figure 5.12:** Overview of all the possible negotiation dance scenarios. The orange star indicates the settlement in each scenario.

To sum up, we find that using TRAPP to generate Securum's offers adopting the optimal mode presented in Table 4.1, is very satisfactory in terms of achieving a high payoff for Securum in the Globusgården-negotiation. Using the concession pattern generated by TRAPP, our analysis show that Securum would have obtained 71 % of the ZoA when the buyer proposed the very first offer. This is slightly less than what Securum achieved in the actual negotiations. In the scenario where Securum had initiated the negotiation dance, as much as 96 % of the ZoA would have been allocated in favour of Securum, by following the policy obtained using TRAPP. Note that a prerequisite for this to be true, is that our estimates of the buyer's reservation price and strategy are the actual values. Moreover, it is assumed that both agents choose a fixed reservation price and deadline prior to the negotiation. We acknowledge that this is rather unlikely in practice, and this issue is addressed as further research in Section 6 Conclusion. Furthermore, the limited amount of information we gain from the actual negotiation dance and the fact that we are assessing human beings with feelings, intuition, and their own negotiation technique, makes it difficult to replicate accurate scenarios. For us to be able to reconstruct a buyer 100 % credibly, would require a lot more insight. This could have been done by conducting interviews of the buyer, for instance. However, for the purpose of this case study, we focus on Securum and how they could have achieved the highest payoff in the negotiation from their perspective, with the information they had in the beginning of the negotiation.

Throughout this thesis, we assume that each agent always accepts an offer within the ZoA. However, if the opening bid from either agent lands in the ZoA, it is almost certain that a higher payoff can be obtained by rejecting that offer and continue the negotiations. In the parameterization of the Norwegian speculative real estate market, Securum estimate a buyers' initial offer to be a maximum 95 % of their own reservation price.[30] Consequently, a minimum of a five percentage points higher prospected settlement price than a buyer's initial offer is assumed certain if a ZoA exists. In the case of Globusgården, the buyer's initial offer was anyhow not within ZoA. However, if Securum had used the best alternative to a negotiated agreement (BATNA) value derived by the ROV-model at 70 MNOK as $RP^S$, they would have, by following the acceptance rule in TRAPP, accepted the buyer's initial offer at 106 MNOK. Recall that the actual settlement price was 110.5 MNOK, and the issue of accepting the initial offer without negotiating becomes trivial. However, remember that Securum estimate that the buyer has a deadline of one round in 7 % of the negotiations. Therefore, the initial offer should not necessary be rejected every round either. Anyhow, the possibility to dynamically adjust the reservation price and to reject initial offers within the ZoA are suggested as interesting extensions to TRAPP.

[30] Further details regarding the parameterization of the buyers' initial offer are found in Subsection 4.1.2.

# 6 Conclusion

In this thesis, we develop a new model to support an investor in the Norwegian speculative real estate market with the objective to maximize the profit from a single property. In particular, it complements the real options valuation (ROV) model of a speculative real estate investment developed in Brynildsen & Hyttedalen (2020) by incorporating the possibility to enter into negotiations. An autonomous negotiation model, called TRAPP, is developed and designed to help the investor in a discrete, bilateral non-cooperative and non-zero-sum bargaining game with incomplete yet perfect information. In this game, the agents propose offers in a sequential order for a theoretically infinite number of rounds. This thesis focuses specifically on the **practical implementation** from a seller's perspective. Moreover, together with our industry partner, the Norwegian real estate investment company Securum Eiendom AS, a recent negotiation case is assessed to see what insights we can obtain by applying TRAPP in a real-life bargaining game.

The modelling approach is an extension of existing models and techniques found in the fields of ROV, game theory, and Bayesian machine learning. Nevertheless, this thesis differentiates itself from existing work in several ways.

The main novelty is that we are, to the best of our knowledge, the first to develop a model that implements ROV techniques to overcome some important practical limitations encountered in earlier game theory based negotiation models.

1. **Defining the agents' reservation prices.** A sine qua non in previous negotiation models (Nash (1950b), Roy (1989), and Sim et al. (2009), among others), is that the reservation prices are given. However, in real-life negotiations, the owner of a speculative real estate has to derive this value somehow. If they choose a reservation price above the value of their best alternative to a negotiated agreement (BATNA), they face the risk of not reaching an agreement and consequently end up with a lower payoff than what could have been achieved. Oppositely, if they choose a reservation price below the value of their BATNA, they might end up selling the property with a profit lower than their expected payoff from the next best alternative. By using the ROV-model presented in Brynildsen & Hyttedalen (2020), we derive the BATNA-value for a rational and risk-neutral investor who possesses a speculative

real estate.

2. **Generating an optimal initial offer.** Similar to the requirement of the reservation price, a commonly found assumption in numerous negotiation models up to the current date (Zeng & Sycara (1997), Agrawal & Chari (2009), Baarslag et al. (2013), and Yu et al. (2013), among others) is that the agents' initial offer is given. In practice, this is found to be a challenging decision problem for the investor. From a seller's perspective, they want the highest price a buyer is willing to pay. However, if they start off by proposing a counteroffer too far from the opponent's reservation price, there is a fair chance that a deal will not take place. We obtain a seller's optimal initial offer by simulating two main methods a seller can use to generate the opening bid: A function of the seller's own reservation price, or based on an estimation of the buyer's reservation price.

3. **Simulating a realistic environment.** We develop a framework that lets the user of the model specify the agent's business environment. The reason for including this feature is due to the fact that an optimal strategy in a real-life setting depends greatly on the possible players an agent could face in a negotiation. For instance, in Sim et al. (2009) they conclude that agents adopting a Bayesian machine learning algorithm to dynamically change strategy in each round, achieve much higher average utilities. However, in their experimental setting, the agents' profile was randomly selected. Using the same algorithm at the Norwegian speculative real estate market, we find that dynamically changing strategy throughout the game has a negligible effect. Hence, a practical model requires the possibility for the user to specify the characteristics of their particular business environment to obtain relevant results.

By simulating several million negotiations in a virtual laboratory setting, we obtain general insights into how an investor with a speculative real estate investment in the Norwegian market should act in order to maximize the expected payoff from a negotiation. Note that the results are based on the assumption that our industry partner's assessment of the market characteristics is valid. The general insights we obtain are:

1. **Initial offer.** We find that the optimal initial offer for an agent depends on how accurate they can estimate the opponent's reservation price. If the agent can provide a 90 % or better estimate of the opponent's reservation price, a triangular probability

distribution should be assigned to the hypotheses of the opponent's reservation price and then use the closed formula derived in Roy (1989) to generate the initial offer. If the agent cannot provide an estimation at this accuracy, the next best alternative is to assign a uniform probability distribution. Lastly, an opening bid based on the seller's own reservation price is included. We find that the optimal opening bid for a seller in the Norwegian market using this method is to propose an initial offer 10 % higher than their own reservation price. However, this method still achieves a lower payoff than the uniform probability distribution method. Furthermore, if they propose an initial offer above 10 % of their reservation price, we find that a deal often does not occur, and the average payoff consequently drops.

2. **Strategy.** In this thesis, we include three types of time-dependent tactics, characterizing how quickly an agent concedes to their reservation price. For a seller, the conceder strategies are found to outperform the linear which in turn outperforms the boulware strategies.

3. **Learning mode.** We find that the Bayesian learning procedure developed in Sim et al. (2009), where an agent changes their strategy based on their estimate of the opponent's reservation price and deadline, to be of little impact in real-life negotiations. This is due to the fact that in speculative real estate negotiations, only a few rounds are played before an agreement is reached or not. Hence, it is only a limited amount of bids to learn from.

For further research, we suggest the following extensions to the model we have developed to be assessed:

1. **Behavior type.** We find that the main problem of Bayesian learning in real-life negotiations when only incorporating time-dependent tactics, is the limited amount of bids exchanged, and consequently limited data to learn from. As an approach of overcoming this issue, we believe including behavior-dependent tactics in the model could potentially increase the value when applying learning mode.

2. **Flexible reservation prices and deadlines.** For an even more realistic simulation model, the agents should be able to change their reservation price and deadline during a game. For instance, new information could arise during the negotiation

phase and an agent may wish to adjust their reservation price and deadline in either direction.

3. **Multilateral negotiation.** In this thesis, we only consider the negotiation process between one seller and one buyer. Although this reflects the real-life situation in many cases for the speculative real estate market studied in this paper, it cannot be excluded that in other cases it might involve multiple potential buyers. In such cases, the bargaining game transforms into a type of auction game. To make TRAPP useful in a broader setting, we suggest to extend the model to include multilateral negotiations in future work.

We acknowledge that "one may never be able to predict or to simulate in laboratory setting all the aspects of complex real-world negotiation" (Raiffa, 1982, p.6). However, our work in this thesis substantiates Howard Raiffa's statement that "there is no question as to the value of applying decision-theoretic concepts: analysis can help.". We developed a negotiation model designed for speculative real estate investments and found an optimal bidding policy for a seller in the Norwegian market. This policy was verified through a real-life negotiation.

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# Appendix

## Content of Appendix

# A1 Input parameters and notation used in the real options valuation model

| **Indices** | |
|---|---|
| i | Project. Where $i \in \{1, \ldots, I\}$, I: Total number of projects |
| j | Time steps waited before attempting |
| n | Global time step. $n \in \{0, \ldots, N\}$ |
| m | Project time step |
| **Sets** | |
| $\Omega = \{1, \ldots, I\}$ | Sets of all projects |
| **Parameters** | |
| $\Delta t_i$ | Length of one time step for project $i$ |
| T | Number of time steps until the last project expires |
| $\mathrm{t}_{min,i}$ | Min. possible time to reach successful state for project $i$ |
| $\mathrm{r}_f$ | Risk-free rate of return |
| $\mathrm{M}_i$ | Number of time steps until expiration for project $i$ |
| $\mathrm{V}_0(n)$ | Value of undeveloped property at time step $n$ |
| $\mathrm{V}_i(n)$ | Value of property when successfully developed project $i$ at time step n |
| $\mathrm{C}_i^A(m, n)$ | Cost of attempting project $i$ at project time $m$ and global time $n$ |
| $\mathrm{C}^H(n)$ | Cost of owning the property in the period between $n$ and $n + 1$ |
| $\mathrm{P}_i^u(m)$ | Probability for successfully development of project $i$ between time $m$ and $m + 1$ |
| $\mathrm{F}_{i,j}(n)$ | Option value at time step $n$ for project $i$ waited $j$ time step(s) to first attempt |
| $\mathrm{F}_{i,j^*}(n)$ | Optimal steps waited, $j$, for project $i$ at time $n$ |
| $\mathrm{F}^*(n)$ | Optimal option value at time step $n$ for the compounded option |

**Table A1.1:** Notation for the parameters used in the real options valuation model in Brynildsen & Hyttedalen (2020).

# A2 Derivation of optimal offer using a triangular distribution

Derivation of seller's optimal offer, when a triangular distribution has been assigned to the hypothesis.

Expected payoff:

$$\Pi_s(P^*, RP^S) = \begin{cases} \int_{P^*}^{\widetilde{RP}_h^B} \frac{2(\widetilde{RP}_h^B - RP^B)(P^* - RP^S)}{(\widetilde{RP}_h^B - \widetilde{RP}_l^B)(\widetilde{RP}_h^B - \widetilde{RP}_*^B)}\, dRP^B & \text{if } P^* > \widetilde{RP}_*^B \\ \int_{P^*}^{\widetilde{RP}_*^B} \frac{2(RP^B - \widetilde{RP}_l^B)(P^* - RP^S)}{(\widetilde{RP}_h^B - \widetilde{RP}_l^B)(\widetilde{RP}_*^B - \widetilde{RP}_l^B)} + & \\ \int_{\widetilde{RP}_*^B)}^{\widetilde{RP}_h^B} \frac{2(\widetilde{RP}_h^B - RP^B)(P^* - RP^S)}{(\widetilde{RP}_h^B - \widetilde{RP}_l^B)(\widetilde{RP}_h^B - \widetilde{RP}_*^B)}\, dRP^B & \text{if } P^* \leq \widetilde{RP}_*^B \end{cases}$$

When $P^* > \widetilde{RP}_*^B$:

$$\begin{aligned} \Pi_s(P^*, RP^S) = \frac{1}{(\widetilde{RP}_h^B - \widetilde{RP}_l^B)(\widetilde{RP}_h^B - \widetilde{RP}_*^B)} \Big[ & 2P^*(\widetilde{RP}_h^B)^2 - 2RP^S(\widetilde{RP}_h^B)^2 - (\widetilde{RP}_h^B)^2 P^* \\ & + (\widetilde{RP}_h^B)^2 RP^S - 2\widetilde{RP}_h^B P^{*2} + 2\widetilde{RP}_h^B RP^S P^* + P^{*3} - P^{*2} RP^S \Big] \end{aligned} \tag{0.1}$$

$$\begin{aligned} \frac{\partial \Pi_s}{\partial P^*} = \frac{1}{(\widetilde{RP}_h^B - \widetilde{RP}_l^B)(\widetilde{RP}_h^B - \widetilde{RP}_*^B)} \Big[ & 2(\widetilde{RP}_h^B)^2 - (\widetilde{RP}_h^B)^2 \\ & - 4\widetilde{RP}_h^B P^* + 2\widetilde{RP}_h^B RP^S + 3P^{*2} - 2P^* RP^S \Big] \end{aligned} \tag{0.2}$$

Applying the first-order condition for a maximum and obtain:

$$P^* = \frac{4\widetilde{RP}_h^B + 2RP^S - \sqrt{(-4\widetilde{RP}_h^B - 2RP^S)^2 - (4 \cdot 3 \cdot (2\widetilde{RP}_h^B RP^S))}}{6} \tag{0.3}$$

When $P^* \leq \widetilde{RP}_*^B$:

$$
\begin{aligned}
\Pi_s(P^*, RP^S) = {} & \frac{1}{(\widetilde{RP}_h^B - \widetilde{RP}_l^B)(\widetilde{RP}_*^B - \widetilde{RP}_l^B)}\Big[ - P^{*3} + P^{*2} RP^S \\
& + 2\widetilde{RP}_l^B P^{*2} - 2\widetilde{RP}_l^B RP^S P^* + (\widetilde{RP}_*^B)^2 P^* \\
& - 2\widetilde{RP}_l^B \widetilde{RP}_*^B P^* - (\widetilde{RP}_*^B)^2 RP^S + 2\widetilde{RP}_l^B RP^S \widetilde{RP}_*^B \Big] \\
& + \frac{1}{(\widetilde{RP}_h^B - \widetilde{RP}_l^B)(\widetilde{RP}_h^B - \widetilde{RP}_*^B)}\Big[(\widetilde{RP}_h^B)^2 P^* - (\widetilde{RP}_h^B)^2 RP^S \\
& - 2\widetilde{RP}_h^B P^* \widetilde{RP}_*^B + 2\widetilde{RP}_h^B RP^S \widetilde{RP}_*^B + (\widetilde{RP}_*^B)^2 P^* - (\widetilde{RP}_*^B)^2 RP^S \Big]
\end{aligned} \tag{0.4}
$$

$$
\begin{aligned}
\frac{\partial \Pi_s}{\partial P^*} = {} & \frac{1}{(\widetilde{RP}_h^B - \widetilde{RP}_l^B)(\widetilde{RP}_*^B - \widetilde{RP}_l^B)}\Big[ - 3P^{*2} + 2P^* RP^S \\
& + 4P^* \widetilde{RP}_l^B - 2\widetilde{RP}_l^B RP^S + (\widetilde{RP}_*^B)^2 - 2\widetilde{RP}_l^B \widetilde{RP}_*^B \Big] \\
& + \frac{1}{(\widetilde{RP}_h^B - \widetilde{RP}_l^B)(\widetilde{RP}_h^B - \widetilde{RP}_*^B)}\Big[(\widetilde{RP}_h^B)^2 - 2\widetilde{RP}_h^B \widetilde{RP}_*^B + (\widetilde{RP}_*^B)^2 \Big]
\end{aligned} \tag{0.5}
$$

Applying the first-order condition for a maximum and obtain:

$$
P^* = \frac{\frac{-(2RP^S + 4\widetilde{RP}_l^B)}{(\widetilde{RP}_h^B - \widetilde{RP}_l^B)(\widetilde{RP}_*^B - \widetilde{RP}_l^B)} - \sqrt{\left(\frac{(2RP^S + 4\widetilde{RP}_l^B)}{(\widetilde{RP}_h^B - \widetilde{RP}_l^B)(\widetilde{RP}_*^B - \widetilde{RP}_l^B)}\right)^2 - 4\delta\left(\frac{-3}{(\widetilde{RP}_h^B - \widetilde{RP}_l^B)(\widetilde{RP}_*^B - \widetilde{RP}_l^B)}\right)}}{\frac{-6}{(\widetilde{RP}_h^B - \widetilde{RP}_l^B)(\widetilde{RP}_*^B - \widetilde{RP}_l^B)}} \tag{0.6}
$$

Where,

$$
\delta = \frac{(\widetilde{RP}_*^B)^2 - 2\widetilde{RP}_l^B RP^S - 2\widetilde{RP}_l^B \widetilde{RP}_*^B}{(\widetilde{RP}_h^B - \widetilde{RP}_l^B)(\widetilde{RP}_*^B - \widetilde{RP}_l^B)} + \frac{(\widetilde{RP}_h^B)^2 - 2\widetilde{RP}_h^B \widetilde{RP}_*^B + (\widetilde{RP}_*^B)^2}{(\widetilde{RP}_h^B - \widetilde{RP}_l^B)(\widetilde{RP}_h^B - \widetilde{RP}_*^B)}
$$

# A3 Derivation of optimal offer using a uniform distribution

Derivation of seller's optimal offer, when a triangular distribution has been assigned to the hypothesis.

Expected payoff, assuming that $RP^B \in [\widetilde{RP}_l^B, \widetilde{RP}_h^B]$:

$$\begin{aligned}
\Pi_s(P^*, RP^S) &= \int_{\widetilde{RP}_h^B}^{P^*} \frac{P^* - RP^S}{\widetilde{RP}_h^B - \widetilde{RP}_l^B}\, dRP^B \\
&= \frac{P^{*2} - P^* \cdot RP^S - P^* \cdot \widetilde{RP}_h^B + RP^S \cdot \widetilde{RP}_h^B}{\widetilde{RP}_h^B - \widetilde{RP}_l^B}
\end{aligned}$$

Applying the first-order condition for a maximum and obtain:

$$P^* = \begin{cases} \frac{RP^S + \widetilde{RP}_h^B}{2} & \text{if } P^* \in [\widetilde{RP}_l^B, \widetilde{RP}_h^B] \\ \widetilde{RP}_l^B & \text{if } P^* < \widetilde{RP}_l^B \\ \widetilde{RP}_h^B & \text{if } P^* > \widetilde{RP}_h^B \end{cases}$$

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Science and Technology