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A Novel Approach to Predicting Interest Rates using PCA & Quantile Regression

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Master's Thesis in Industrial Economics & Technology Management
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Preface

This master thesis is delivered as one of the requirements for the course TIØ4900, a part of a Master of Science degree in Industrial Economics and Technology Management at the Norwegian University of Science and Technology. I would like to thank my supervisors Rita Pimentel and Sjur Westgaard at the Department of Industrial Economics and Technology Management. Furthermore, I would like to thank my supervisor Morten Risstad, Head of FX and Interest Rate Derivatives at Sparebank 1 Markets. I greatly value and appreciate the support and suggestions they offered during this course, and would like to thank them for an excellent final year.

Trondheim June, 2021

Abstract

The aim of this thesis is to investigate the interest rate risk forecasting ability of a novel approach that utilizes well established methods within the field. Interest rate risk managers often employ Value-at-Risk (VaR) estimation techniques to manage risk, amongst others for regulatory purposes. VaR estimation models are continuously being expanded upon in order to provide even more accurate estimations, as this is still considered a statistical challenge. Research in this area has however not yet combined some of the most powerful methods currently being used within interest rate forecasting. This thesis proposes a combination of Principal Component Analysis (PCA) and Quantile Regression (QR) in an approach to predict out-of-sample interest rate changes, one day ahead. The proposed approach, which is named the PCA-QREG model, is applied on U.S. daily Treasury yield curve rates from January 2000 to April 2020. By creating volatility proxies of principal components and applying quantile regression, best-fit coefficients are estimated. These coefficients are further used in predicting the interest rate changes one day ahead at different quantiles. The study finds that the PCA-QREG model offers predictions that are of high accuracy while retaining simplicity in application and interpretability.

Sammendrag

Denne masteroppgaven forsøker å utforske renterisiko predikeringsevnen til en ny modell som benytter seg av kjente verktøy innad i fagområdet. For å kontrollere renterisiko benyttes ofte Value-at-Risk (VaR) estimeringsteknikker, blant annet med hensikt om å oppfylle regulatoriske krav. VaR modeller forbedres stadig vekk i forsøk om å oppnå høyere nøyaktighet i prediksjonene, da dette fortsatt anses som en statistisk utfordring i fagområdet. Tidligere forskning på dette området har derimot ikke enda utforsket hvordan de mest fremtredende fremgangsmåtene som blir brukt innen renterisiko predikering i dag fungerer kombinert, og hvilken påvirkning dette har på resultatene. Denne masteroppgaven foreslår en modell som kombinerer Principal Component Analysis (PCA) og kvantil regresjon for å predikere renteendringer, én dag fremover i tid. Den foreslåtte modellen, heretter kalt PCA-QREG modellen, brukes på daglige U.S. Treasury renter fra januar 2000 til april 2020. Ved å bruke volatilitets proxy av principal components kombinert med kvantil regresjon estimeres det optimale koeffisienter. Disse koeffisientene brukes videre til å predikere renteendringer én dag fremover i tid ved ulike kvantiler. Studien finner at PCA-QREG modellen gir prediksjoner av høy nøyaktighet, samt er rimelig å tolke og anvende.

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Chapter 1

Introduction

Interest rate risk constitutes one of the many financial risks individuals, companies and governments are exposed to. An umbrella definition of interest rate risk can be described as when fluctuations to the interest rate adversely impact an investor or borrower's costs or profits (CPA Australia, 2008). Financial institutions, such as banks, rely heavily on predicting interest rate risk in order to earn profits. However, unexpected changes to the interest rate not only affect banks' earnings, but also threaten their stability, in turn potentially harming economies across the world. As an example, as recent as 2007-2008, during the Financial Crisis one saw the global economy adversely affected by interest rate risk, amongst others. As the World Bank (2017) notes, governments are also prone to difficulties regarding interest rate risk as this often affects loans developing nations hold, which in turn can affect, for example, how much government expenditure that is used on a country's inhabitants. Almeida (2005) describes understanding interest rate risk astutely: "It informs, for different maturities, the cost of borrowing money, being directly related to macroeconomic variables and central bank decisions."

Interest rate risk is commonly deconstructed into four parts by financial institutions (Bank of International Settlements, 2001): repricing risk, option risk, basis risk, and yield curve risk. For all the four components, gaining insight into the interest rate risk enables financial institutions to retain favorable positions and operate with more certainty. Historically, researching the yield curve, also known as the term structure, has been a primary focus within this field. This has varied over the years from attempting to model the entire yield curve, to developing short-rate models, to incorporating macroeconomic information into the models, to varying the restrictions imposed on the models, and more. As Fama (1990) notes, term structure literature is concerned with how to apply current yields to forecast future interest rates, and the risk that the literature attempts to understand relates to understanding the changing rate relationships across the spectrum of maturities. Inherently predicting interest rate risk, particularly yield curve risk in this instance, describes predicting how the future interest rates change across maturities. Forecasting how interest rates change, and thus forecasting the future yield curve is of the utmost importance to certain institutions. Governments depend on understanding the nature of the future yield curve in order to decide monetary policy objectives (Bergo, 2003), regulate the central bank interest rate, and even in order to predict incoming periods of recession (Stojanovic & Vaughan, 1997).

Early forays in the field included applying the PCA (Flury, 1988) to interest rates and interest rate changes such as by Loretan (1997). PCA studies characterize some of the promising attempts at modelling the yield curve based on historic data, and thus modelling the term structure of interest rates. By finding the Principal Components (PC) or risk factors, researchers were able to create models that described the existing interest rates, and captured the variation, well. The components were interpreted by the field as describing the first three components as the slope, level, and curvature of the yield curve, as described by Piazzesi (2010) and Duffee (2013), in an attempt to contextualize the PCs found.

As Hagenbjörk and Blomvall (2019) point out, interest rate risk in the modern perspective increasingly encompasses the idea that the risk may spawn from variations to the term structure of interest rates as well. They further note that measuring interest rate risk through risk factor simulation, such as PCA, is a relatively unexplored area of interest rate risk literature. One of the modern motivations to understanding interest rate risk also lies in financial regulations to banks and institutions, where there is a increasing need to comply to worst case scenario, such as described by Value-at-Risk (VaR) procedures (de Raaji & Raunig, 1998; Sharma, 2012). Examining interest rate risk from this perspective indicates that risk factor models of interest rate risk have a place in interest rate risk modelling and prediction, and that the literature is not yet comprehensive in this area.

Other researchers, such as Gray (1990), synthesized attributes they believed interest rate risk, and other financial data, exhibited. Importantly as the field matured researchers agreed financial series were very commonly not normally distributed, and often leptokurtic. Additionally, it was found that some of financial series typically exhibited volatility clustering, which describes how volatile periods tend to persist before the market returns to normality (Poon, 2005). Other attributes were also revealed sparking a strong interest in the creation of different econometric models.

Further, the non-normal nature of the distribution of interest rates, combined with the need for financial institutions to understand interest rate risk in its extremities prompts the question of whether quantile regression can be applied to interest rate risk. Quantile regression allows researchers to explore how data exhibits different loadings in a regression model, dependent on the quantile being examined. Quantile regression is particularly powerful when data is not normally distributed.

As the Principal Components are well established for capturing the majority of the variation in the interest rates, this study proposes a novel approach to predicting interest rate risk which lies in creating volatility proxies of the Principal Components. The volatility proxies can then be further analyzed using quantile regression to yield predictions, in- and out-of-sample of interest rate changes at different quantiles. Such a model would be beneficial in gaining insight into what risk financial institutions carry, at different quantiles, within the interest rate market. The proposed PCA-QREG model can be implemented on historical datasets with ease, is comprised of interpretable components, and additionally has strong predictive power for future interest rate changes.

A flowchart for the structure of the model is presented below in Figure 1.1:

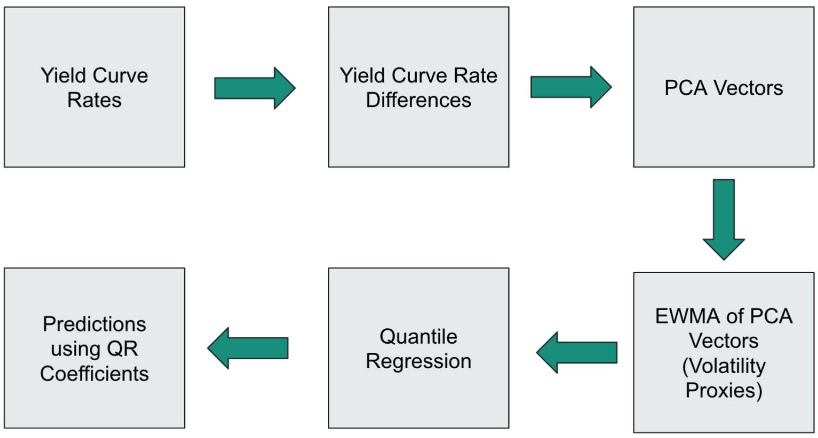

FIGURE 1.1: Flowchart for the PCA-QREG Model

This thesis is organized as follows. In Chapter 2 a literature review is presented. This is followed by a description of the interest rates and the interest rate changes in Chapter 3, and the methodology for applying the model and evaluating the results in Chapter 4. In Chapter 5 the empirical results are presented and discussed. Finally, the study is summarized and concluded in Chapter 6, and further extensions to the study are discussed.

Chapter 2

Literature Review

The literature examined includes fundamental interest rate models and their evolution, in addition to how PCA differs from other models. Further, volatility and risk models commonly used to model interest rate risk are described, after which applications of quantile regression in interest rate risk forecasting are presented.

Forecasting interest rates has several important motivating factors, as mentioned in the previous chapter. However, the methods and techniques applied have varied over the years, and the field has gradually expanded.

Research produced in this field initially began with investigations into understanding the term structure of interest rates, also known as the yield curve. The primary motivation was to model or understand the term structure for different maturities in order to predict how changes to the underlying assets would affect the yield curve (Cox et. al, 1985). Tesler (1966), for example, explored how two different theories, namely expectations theory and liquidity preference theory, explain the determinants of the term structure. Further, Merton (1973) made one of the earliest attempts at modelling the term structure in order to explain the behaviour of interest rates.

As the theoretical and empirical research of the term structure increased in volume, the amount of models attempting to model the term structure began to increase as well (Yan, 2001). Approaches specifying the stochastic development of the entire term structure, while intuitively attractive, imply an increase in model complexity. This has prevented more widespread use of such models (Gibson et. al, 2001).

Models that built on Merton´s work were also generated and attempted to model the development of the instantaneous risk-free rate. These increased with complexity over time. Simple Single Factor Models encompassed more attributes including mean-reversion characteristics using a Gaussian model (Vasicek, 1977) and allowing for the determination of the risk premium (Hull and White, 1993), amongst others. Eventually, these models were expanded on due to criticisms of the Single Factor Models´ simplicity and failure to adhere to empirically identified traits of the interest rates. (Gibson et. al, 2001; Maes, 2004; Lapshin, 2012).

Other avenues of modelling the term structure included the rise of Multi-Factor Models which outperformed Single-Factor Models (Dai and Singleton, 2000), generally, and allowed for the term structure to be modelled with higher complexity, and greater inclusion of stylized facts interest rates exhibited. Parametric approaches, which presuppose a fixed parametric form of the term structure (Lapshin, 2012), were also explored. Most notably the Nelson-Siegel (1987) model, which is parametric with respect to the spot forward rate, has been widely used and expanded upon (Diebold and Li, 2006). One of criticisms of parametric approaches has been that despite the obvious power in producing sensible yield curve models, the models themselves often lacked economic intuition (Lapshin, 2012).

Common to most approaches has been inherent sense of applying predetermined stochastic equations that describe the dynamics of the factors driving the term structure movements (Almeida, 2005), usually with no arbitrage restrictions imposed. However, stochastic equations themselves possess processes that investors are required to predict, which can be done erroneously. Such instances lead to incorrect analyses (Bierwag et. al., 1983). Furthermore, while the Multi-Factor models work well, the components still exhibit correlation between them (Su & Knowles, 2010), resulting in factor relationship risk, another issue within interest rate risk management.

Statistical studies of interest rates found that the yield curve exhibits shifts or changes in its shape that are attributed to a few unobservable factors (Dai and Singleton, 2000). In contrast to other models, PCA has been applied to the term structures of interest rates to determine these factors driving term structure movements. The approach aims to classify and quantify the yield curve movements using historical data, and attempts to produce uncorrelated factors that explain these movements with as much economic intuition as possible (Hull, 2012). PCA describes the yield curve variations by analyzing how much variance a factor contributes to the movements, percentage-wise, and additionally can significantly reduce the dimensionality compared to other models. This proves to be successful as the procedure accounts for the variability existing in the entire dataset (Hull, 2012; Knowles & Su, 2010). Litterman and Scheinkman (1991) found that three factors was an adequate number to describe the movements of the U.S. Treasury term structure. It has since been used in many financial problems, such as risk management (Singh, 1997), portfolio immunization (Barber and Copper, 1996), a benchmark to define the number of factors in dynamic models (Collin-Dufresne and Goldstein, 2002), and in modelling global term structure as Malava (1999), and Novosyolov and Satchkov (2008) do. More recently, Joslin et. al. (2011) applied the PCA procedure in their Gaussian dynamic term structure model when testing the out-of-sample forecasting result. Additionally, Bauer & Rudebusch (2016) utilized the PCA procedure in evaluating the risk factors for their zero-bound dynamic term structure model finding good forecasting performance.

With the intention of providing investors a means to better hedge investments in fixed-income securities, Litterman & Scheinkman (1991) introduced three factors resulting from PCA; level, steepness or slope, and curvature. These factors, in Litterman and Scheinkman´s (1991) paper captured the movements of various Treasury bond yields by more than 99%. Additionally, Phoa (2000) found when looking at U.S. Treasury bond yields that the first factor accounted for 90% of the observed variation in yields, and that the second and third factors were decreasingly important. As described by Heidari & Wu (2003) the level depicts parallel shifts to the yield curve, the slope represents flatter or steeper yield curves when short-term interest rates increase or decrease, and long-term interest rates remain more static, and the curvature explains changes to medium-term interest rates making the yield curve more "humped" or flatter. Naturally, researchers also advocate that macroeconomic variables may affect the dynamics of the yield curve, as Phoa (2000) related the level to be affected by inflation expectations, and the slope to monetary policy changes. However, the understanding of macroeconomic variables in relation to the principal components is not fully comprehensive, and the literature has been conflicted in its nature (Heidari & Wu, 2003).

From a practical perspective, interest rate risk is of importance to investors and portfolio managers as changes to the yield curve often signals the market volatility in the bond market. For this reason, while absolute changes to the yield curve still are of interest, understanding the volatility of interest rates aids in determining the environment surrounding investments in the bond market, as highlighted by Baygun et al. (2000). Furthermore, empirical work by Bliss (1997) and Nath (2012) indicates that while there has been little variation in the yield curves since 1970, the interest rate volatility has not remained as stable. However, as Poon and Granger (2003) point out, volatility is not the same as risk, however it is often used as a proxy or building block towards it. The most common tools applied by financial institutions to predict, analyse and mitigate interest rate risk include: sensitive gap analysis, duration and convexity models, option adjusted spreads and Value-at-Risk (VaR) models (Wang et. al., 2014). VaR models, in particular, are statistical techniques that measure the amount of potential loss within an investment portfolio. They are often used by investment and commercial banks to determine the magnitude and occurrence probability of potential losses. With regards to interest rate risk the VaR models more specifically "assesses financial risk by evaluating the probability of loss that results from stochastic variation of the rate of return" (Trenca & Mutu, 2009). Additionally, as shortly mentioned in Chapter 1, since 1996 the Basel Committee on Banking Supervision have imposed regulatory capital requirements corresponding to VaR estimates that banks need to adhere to.

VaR interest rate risk models tend to be characterized by one of three approaches: Nonparametric, parametric and semi-parametric models. (Engle & Manganelli, 2001). The differences between the models relates to how changes to the portfolio value are estimated. Nonparametric models include using historical simulation or stress scenarios, such as Monte Carlo models, that calculate risk factors and the daily return distribution before calculating the VaR Metric. Parametric approaches are typified by RiskMetrics, Generalized Autoregressive Conditional Heteroscedacity (GARCH)(Engle, 1982; Bollerslev, 1986), and Exponentially Weighted Moving Average (EWMA). These models propose specific parameterisations for the behavior of the interest rates (Trenca & Mutu, 2009), and allow for complete determination of the distribution of changes. Semiparametric models include Danielsson and de Vries´ (1998) Extreme Value Theory which assumes little about the daily interest rates and works well in tail estimation. Another such model is Engle and Manganelli´s (1999) Conditional Auto-Regressive Value at Risk (CAViaR) model which estimates the evolution of the quantile rather than the whole distribution of the portfolio (Trenca & Mutu, 2009).

The historical simulation approach to estimating VaR emerged as one of the most popular methods within the field. Perignon and Smith (2006) conducted a survey showing that 73% of financial institutions employed historical simulation for calculating VaR estimates. Sharma (2012) informs that this method does well for unconditional tests of the VaR estimates, but not the conditional tests. Dowd (2005) mentions that the assumption of independent normally distributed errors in historical simulation approaches are one of the disadvantages of this method. Historically, assuming the interest rate changes followed a normal distribution was widespread practice. This despite empirical results by Mandelbrot (1963) and Fama (1965) indicating otherwise. As mentioned in Chapter 1 in relation to Gray´s (1990) work, it is well-known that financial returns are established to be leptokurtic, and are thus non-normally distributed. Additionally, it has been concluded that they exhibit volatility clustering where days of high volatility tend to be followed by days of high volatility. Models such as the EWMA and the GARCH are more apt in accounting for these stylized facts. The EWMA model proposed by J.P. Morgan´s RiskMetrics department models variance as an exponentially moving average and Engle (1982) and Bollerslev´s (1986) GARCH models a time varying conditional variance. The EWMA model performs well in following rapid changes to volatility, and the GARCH model, which reduces to an EWMA model in special cases, is known as being a powerful model for predictions and being highly customizable.

As Engle (2001) points out, the use of these models with VaR estimations is extremely widespread where volatility of returns are in question. Vlaar (2000) applied a historical simulation model, a Monte Carlo simulation, and a model with GARCH variance specification to estimating VaR values for Dutch bond portfolios. He found that the historical simulation model and Monte Carlo simulation needed very high amounts of data samples in order to forecast well, and that the GARCH variance specification in his variance-covariance method led to some underestimation of the variance. de Raaji & Raunig (1998) found that when comparing VaR estimates from historical simulations and the variance-covariance method with a EWMA variance specified, that the EWMA method captured the volatility clustering present in the foreign exchange rate portfolio being examined. Lopez & Walter (2000) also evaluated foreign exchange portfolios using two covariance-matrix forecast methods with an EWMA specification and a GARCH specification. They found that VaR frameworks with simple specifications, such as the EWMA specification, performed well indicating the additional structure or information other specifications supplied were superfluous in producing accurate VaR estimates.

The aforementioned PCA approach has also been applied within interest rate risk management in certain scenarios. As previously mentioned PCA has the ability to capture large amounts of the variation of a dataset as components that can be linearly combined. Jamshidian and Zhu (1996), as well as Frye (1996), detail how firms can employ PCA within their risk management operations. Loretan (1997) describes how PCA can be best applied by recreating stress scenarios and analyzing them further. By capturing the interest rate changes in a few variables, it is also possible to induce shocks to the historical data. By examining these different scenarios and capturing different quantiles in the different distributions it may be possible to investigate if any existing risk can have adverse effects for strategies hedged against such exposure. Other applications include Hagenbjörk and Blomvall´s (2018) application of Principal Component Analysis on the term structure innovations, thus identifying risk factors, and thereafter modelling the distribution using GARCH models with non-normal innovation distributions. Their approach yields lower Value-at-Risk measurements opposed to other variants that may overestimate the interest rate risk.

Another procedure often applied within interest rate risk management is quantile regression introduced by Koenker and Bassett (1978). The procedure allows for the modelling of chosen quantiles of a response variable against the observed explanatory variables. And so, a quantile of the response variable is expressed as a linear combination of the covariates, and the estimation of the model involves finding the coefficients for that linear combination (Uribe & Guillen, 2020). Generally, it is understood that quantile regression is more proficient in capturing what influences the occurrence of extreme response values. Another important aspect that favours quantile regression when working with financial returns includes that Ordinary Least Squares assumes a normal distribution in the return series, which is not always appropriate (Allen et. al, 2013), and a linear relationship between the variables. As mentioned earlier, stylized facts financial data exhibits include non-normality, and as such quantile regression, which makes no normality assumptions, can be more powerful in evaluating the relationship between interest rate changes and explanatory variables (for instance, interest rate volatility). Quantile regression has often been used in measuring the sensitivity of financial assets to various factors or risks.

A common object of enquiry has been national stock markets´sensitivity to different existing rates, such as exchange rates and interest rates. For instance, Ferrando et. al (2017), and Jiranyakul (2016) investigate the Spanish and Thai stock markets´ sensitivity to interest rates, respectively, yielding more astute understandings of the relevant risk factors. Additionally, Jareño et. al (2018) investigate European insurers´ sensitivity to interest rate movements. Unsurprisingly however, quantile regression has been in large part employed within Value-at-Risk modelling and some degree evaluation, given its previously mentioned strengths. Quantile regression is heavily embedded in the semi-parametric CAViaR (Engle & Manganelli, 1999), one of the currently most popular Value-at-Risk models. Numerous examples exist of the CAViaR model being applied in a similar fashion to the previous examples mentioned. There is ample literature where VaR estimates are constructed for stock assets, exchange assets, commodity markets, and more (Allen & Singh, 2010 ; Yongjian & Peng, 2015; Aloui & Mabrouk, 2011). Another application of quantile regression within risk management has been explorations of utilizing it as an alternative to backtesting (Gaglianone et. al., 2008), however literature in this realm is somewhat scarce. Generally quantile regression has been in large part applied towards analyzing the sensitivity of financial assets with regards to interest rates as a risk management technique.

Throughout the literature however, it is evident that despite PCA being applied in existing term structure models, and being used for stress testing financial positions, the application of PCA within traditional risk management approaches, such as VaR, is as of yet not very well explored. PCA represents a powerful procedure that captures vast amounts of variation embedded in a dataset, and VaR models represent a regulatory and practical necessity for financial institutions to operate within. With this in mind a novel approach for predicting quantiles of interest rate changes is proposed using PCA on U.S. Treasury Yields and transforming the components into volatility proxies that capture volatility clustering well by using EWMA. From this the model estimates out-of-sample VaR predictions at different quantiles using quantile regression.

Chapter 3

Data

3.1 General Description

The U.S daily Treasury yield curve rates [1] with data ranging from January 3rd 2000 to April 14th 2020 are used in this paper. This accounts for 5291 days of observations for each maturity. After discounting the missing values, the remaining number of days included are 5052. This study considers rates from 3-month, 6-month, 1-year, 2-year, 3-year, 5-year, 7-year, and 10-year maturities. The 30-year maturity U.S Treasury rate was omitted due to a significant proportion of missing values.

The different interest rates in the data span are visualized below:

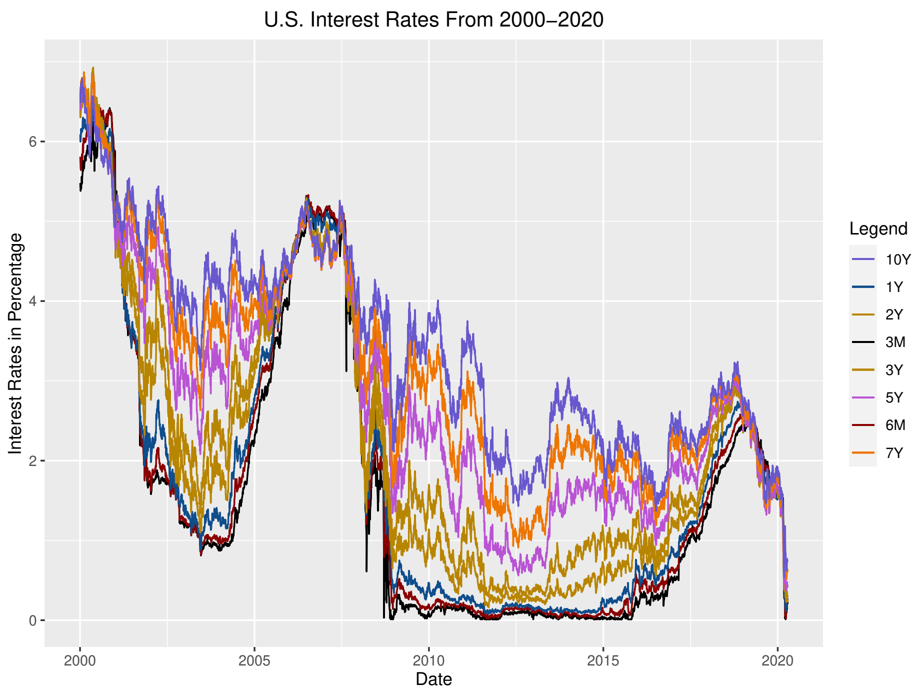

FIGURE 3.1: U.S. Daily Treasury Yield Curve Rates from 2000-2020

Some of the summary statistics of the interest rates are also presented below.

[1] Accessed from https://www.treasury.gov/resource-center/data-chart-center/interest-rates.

TABLE 3.1: Descriptive Statistics of U.S. Treasury Yield Curve Rates from January 2000 to April 2020

Maturity	Min.	1st.Qu.	Median	Mean	3rd.Qu.	Max.
3M	0.01	0.11	1.08	1.67	2.42	6.42
6M	0.02	0.18	1.19	1.78	2.51	6.55
1Y	0.08	0.30	1.33	1.87	2.64	6.44
2Y	0.16	0.68	1.63	2.11	3.09	6.93
3Y	0.28	1.00	1.85	2.33	3.42	6.88
5Y	0.37	1.60	2.48	2.76	3.90	6.83
7Y	0.51	2.02	2.90	3.11	4.13	6.87
10Y	0.54	2.32	3.34	3.41	4.40	6.79

One of the motivations for later applying PCA on the available data is the method´s ability to obtain linearly independent vectors. This is particularly useful in instances where datasets to be analyzed exhibit high levels of correlation. The correlation matrix is shown below.

TABLE 3.2: Correlation Matrix of U.S. Treasury Yield Curve Rates from January 2000 - April 2020

	3M	6M	1Y	2Y	3Y	5Y	7Y	10Y
3M	1.00	1.00	0.99	0.97	0.95	0.89	0.83	0.76
6M	1.00	1.00	1.00	0.98	0.96	0.90	0.84	0.77
1Y	0.99	1.00	1.00	0.99	0.97	0.92	0.86	0.80
2Y	0.97	0.98	0.99	1.00	0.99	0.96	0.91	0.85
3Y	0.95	0.96	0.97	0.99	1.00	0.98	0.95	0.89
5Y	0.89	0.90	0.92	0.96	0.98	1.00	0.99	0.96
7Y	0.83	0.84	0.86	0.91	0.95	0.99	1.00	0.99
10Y	0.76	0.77	0.80	0.85	0.89	0.96	0.99	1.00

This correlation matrix displays high levels of correlation between the different yield curve rate maturities, supporting the use of PCA in order to tackle the high multicollinearity in the dataset.

3.2 Stationarity Tests of Yield Curve Rates

One of the key components of a successful Principal Components Analysis is that the procedure is run on data that is stationary in order to ensure a meaningful resulting covariance matrix. With this in mind some stationarity tests have been applied to the interest rates in order to investigate this attribute.

First, we apply the Augmented Dickey-Fuller (ADF) test (Cheung & Lai, 1995)which has a null hypothesis of non-stationarity in the dataset. The ADF test introduces a certain amount of lags of the dependent variables as regressors in the test equation. We allow the test to automatically include the number of lags based on a default equation. However, to avoid the issue of lag selection we can also test for stationarity using a similar test, the Phillips-Perron (PP) test (Phillips & Perron, 1988) .

The Phillips-Perron test makes a non-parametric correction to the t-test statistic, and therefore works well for unspecified autocorrelation. The null hypothesis for the PP test is also non-stationarity in the dataset. Alternatively, we apply the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test (Kwiatkowski et. al., 1992), which also tests for the level of stationarity. Contrary to the other tests, in this case the null hypothesis is stationarity in the dataset. The tests are conducted on all the interest rates with varying maturities. The results are presented below:

TABLE 3.3: Stationarity Tests of Yield Curve Rates

	ADF p-values	PP p-values	KPSS p-values
P-values 3M	0.729359280277555	0.914556474563918	$<$0.01
Results 3M	Non-Stationary	Non-Stationary	Non-Stationary
P-values 6M	0.729359280277555	0.914556474563918	$<$0.01
Results 6M	Non-Stationary	Non-Stationary	Non-Stationary
P-values 1Y	0.624376565384456	0.912529240552071	$<$0.01
Results 1Y	Non-Stationary	Non-Stationary	Non-Stationary
P-values 2Y	0.539813237076849	0.827158600743331	$<$0.01
Results 2Y	Non-Stationary	Non-Stationary	Non-Stationary
P-values 3Y	0.453316247712882	0.739360811973506	$<$0.01
Results 3Y	Non-Stationary	Non-Stationary	Non-Stationary
P-values 5Y	0.279905276141332	0.520618784984383	$<$0.01
Results 5Y	Non-Stationary	Non-Stationary	Non-Stationary
P-values 7Y	0.11346639148798	0.271099176264885	$<$0.01
Results 7Y	Non-Stationary	Non-Stationary	Non-Stationary
P-values 10Y	0.0236959699615407	0.0505722516716883	$<$0.01
Results 10Y	Stationary	Non-Stationary	Non-Stationary

As the table displays, there is no statistical evidence that the series are stationary for the different yield curve rate maturities. This makes applying the PCA procedure on the yield curve rates unviable. The next section explores whether transforming the interest rates yields data on which PCA is applicable and the results interpretable.

3.3 Yield Curve Rates Changes & Tests

The results from Table 3.3 concluded that the raw interest rates data is not stationary. A common tactic when working with financial series is to difference the data. This tends to induce stationarity in the data, and the simple transformation of data retains the dataset´s interpretability. For this thesis two specific variants of differences are used: relative changes and logarithmic changes. These are both calculated in similar fashion to financial returns. Calculating the relative changes and logarithmic changes should yield two datasets that are stationary. Further details about the relative and logarithmic changes are presented in Chapter 4. The relative changes and logarithmic changes for the different maturities are displayed in Figures 3.2 and 3.3.

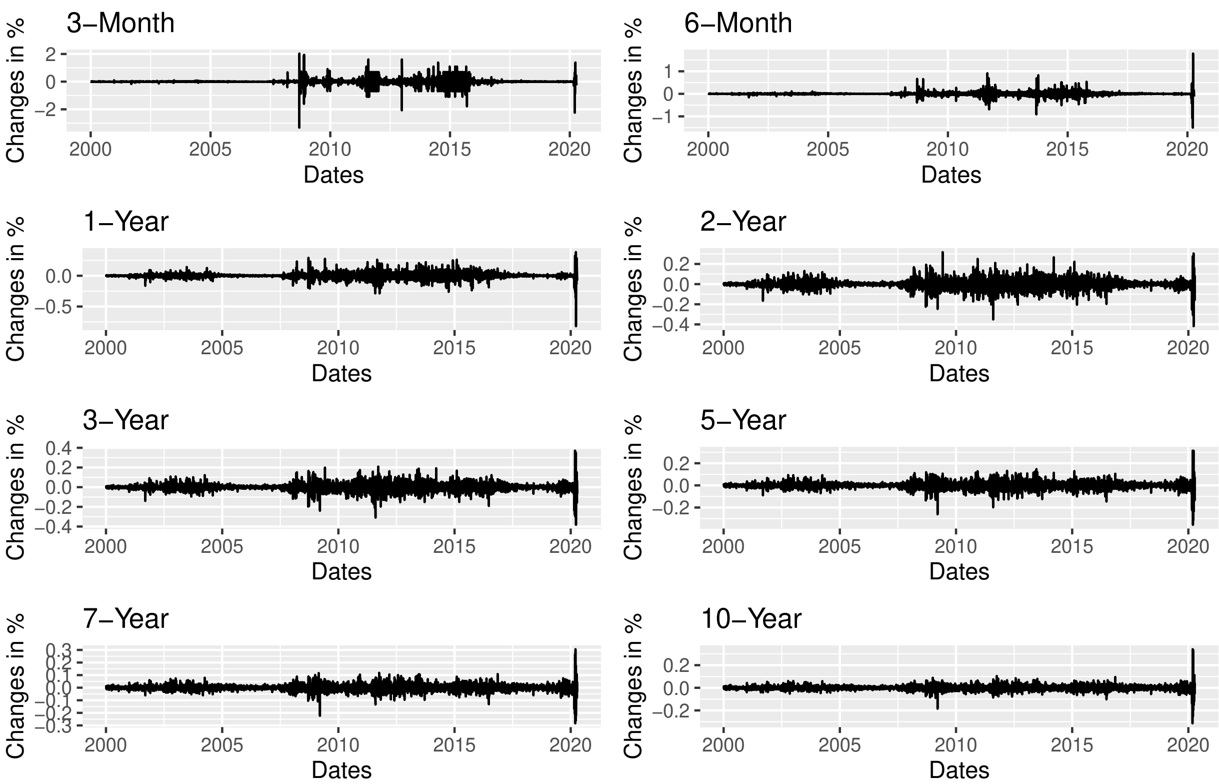

FIGURE 3.2: U.S. Daily Treasury Yield Rate Relative Changes from 2000-2020

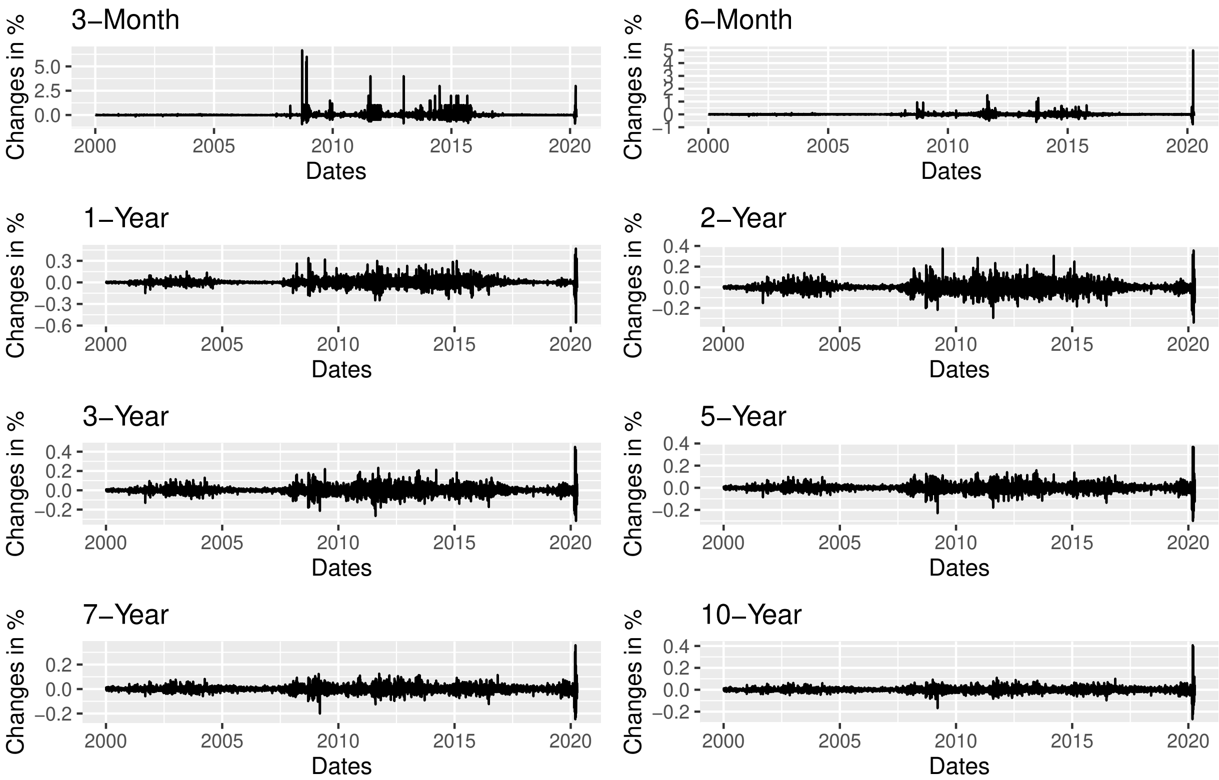

FIGURE 3.3: U.S. Daily Treasury Yield Rate Logarithmic Changes from 2000-2020

Additionally some descriptive statistics about both the relative changes and logarithmic changes are provided:

TABLE 3.4: Descriptive statistics of U.S. Treasury Yield Curve Rate Relative Changes from January 2000 - April 2020

Maturities	Min.	1st.Qu.	Median	Mean	3rd.Qu.	Max.
3M	-0.96	-0.01	0.00	0.02	0.01	6.67
6M	-0.78	-0.01	0.00	0.00	0.01	5.00
1Y	-0.56	-0.01	0.00	0.00	0.01	0.47
2Y	-0.34	-0.02	0.00	0.00	0.01	0.38
3Y	-0.32	-0.02	0.00	0.00	0.01	0.45
5Y	-0.30	-0.01	0.00	0.00	0.01	0.37
7Y	-0.25	-0.01	0.00	0.00	0.01	0.36
10Y	-0.27	-0.01	0.00	0.00	0.01	0.41

TABLE 3.5: Descriptive statistics of U.S. Treasury Yield Curve Rate Logarithmic Changes from January 2000 - April 2020

Maturities	Min.	1st.Qu.	Median	Mean	3rd.Qu.	Max.
3M	-3.33	-0.01	0.00	0.00	0.01	2.04
6M	-1.50	-0.01	0.00	0.00	0.01	1.79
1Y	-0.82	-0.01	0.00	0.00	0.01	0.39
2Y	-0.42	-0.02	0.00	0.00	0.01	0.32
3Y	-0.38	-0.02	0.00	0.00	0.01	0.37
5Y	-0.36	-0.01	0.00	0.00	0.01	0.31
7Y	-0.28	-0.01	0.00	0.00	0.01	0.31
10Y	-0.32	-0.01	0.00	0.00	0.01	0.34

The descriptive statistics, while not yielding necessary information, do show that values for the differencing have their means around zero, and exhibit some more variation outside of the 1st and 3rd quantiles. None of the statistics are alarming nor indicate any cause for further examination.

3.3.1 Stationarity of Yield Curve Rate Changes

Having looked at the relative and logarithmic changes visually and checked some simple statistics, the next step is to conduct the same stationarity tests as in the previous section to the relative and logarithmic changes. The tests will inform on whether the initial transformation of the data was an adequate, or if other manipulations of the data are required.

TABLE 3.6: Stationarity Test Results of U.S. Treasury Yield Curve Rate Relative Changes from January 2000 - April 2020

	ADF p-values	PP p-values	KPSS p-values
P-values 3M	<0.01	<0.01	<0.01
Results 3M	Stationary	Stationary	Non-Stationary
P-values 6M	<0.01	<0.01	0.01357
Results 6M	Stationary	Stationary	Stationary
P-values 1Y	<0.01	<0.01	>0.1
Results 1Y	Stationary	Stationary	Stationary
P-values 2Y	<0.01	<0.01	>0.1
Results 2Y	Stationary	Stationary	Stationary
P-values 3Y	<0.01	<0.01	>0.1
Results 3Y	Stationary	Stationary	Stationary
P-values 5Y	<0.01	<0.01	>0.1
Results 5Y	Stationary	Stationary	Stationary
P-values 7Y	<0.01	<0.01	>0.1
Results 7Y	Stationary	Stationary	Stationary
P-values 10Y	<0.01	<0.01	>0.1
Results 10Y	Stationary	Stationary	Stationary

TABLE 3.7: Stationarity Test Results of U.S. Treasury Yield Curve Rate Logarithmic Changes from January 2000 - April 2020

	ADF p-values	PP p-values	KPSS p-values
P-values 3M	<0.01	<0.01	>0.1
Results 3M	Stationary	Stationary	Stationary
P-values 6M	<0.01	<0.01	>0.1
Results 6M	Stationary	Stationary	Stationary
P-values 1Y	<0.01	<0.01	>0.1
Results 1Y	Stationary	Stationary	Stationary
P-values 2Y	<0.01	<0.01	>0.1
Results 2Y	Stationary	Stationary	Stationary
P-values 3Y	<0.01	<0.01	>0.1
Results 3Y	Stationary	Stationary	Stationary
P-values 5Y	<0.01	<0.01	>0.1
Results 5Y	Stationary	Stationary	Stationary
P-values 7Y	<0.01	<0.01	>0.1
Results 7Y	Stationary	Stationary	Stationary
P-values 10Y	<0.01	<0.01	>0.1
Results 10Y	Stationary	Stationary	Stationary

The stationarity tests indicate that the transformations of the data seems to induce stationarity in the different maturities of the dataset. Explicitly the tests show that there is enough statistical evidence to consider that the changes in Treasury Yield Curve Rates, for all maturities, are stationary, and thus PCA can be run on them yielding interpretable results.

3.3.2 Correlation in Changes

As mentioned earlier, the correlation between the maturities incentivizes the use of PCA. Large multicollinearity in the dataset makes the linearly non correlated principal components very valuable. Next it is verified whether calculating the relative and logarithmic changes of the dataset has impacted the correlation between the maturities. The correlation matrix for the relative changes and logarithmic changes are displayed below in Tables 3.8 and 3.9.

TABLE 3.8: Correlation Matrix of U.S. Treasury Yield Curve Rate Relative Changes from January 2000 - April 2020

	3-Month	6-Month	1-Year	2-Year	3-Year	5-Year	7-Year	10-Year
3-Month	1.00	0.32	0.17	0.07	0.06	0.05	0.04	0.04
6-Month	0.32	1.00	0.33	0.17	0.17	0.14	0.12	0.13
1-Year	0.17	0.33	1.00	0.54	0.53	0.48	0.44	0.41
2-Year	0.07	0.17	0.54	1.00	0.87	0.82	0.76	0.69
3-Year	0.06	0.17	0.53	0.87	1.00	0.93	0.88	0.81
5-Year	0.05	0.14	0.48	0.82	0.93	1.00	0.97	0.91
7-Year	0.04	0.12	0.44	0.76	0.88	0.97	1.00	0.96
10-Year	0.04	0.13	0.41	0.69	0.81	0.91	0.96	1.00

TABLE 3.9: Correlation Matrix of U.S. Treasury Yield Curve Rate Logarithmic Changes from January 2000 - April 2020

	3-Month	6-Month	1-Year	2-Year	3-Year	5-Year	7-Year	10-Year
3-Month	1.00	0.36	0.21	0.10	0.08	0.07	0.06	0.06
6-Month	0.36	1.00	0.42	0.25	0.24	0.20	0.18	0.18
1-Year	0.21	0.42	1.00	0.54	0.53	0.48	0.44	0.42
2-Year	0.10	0.25	0.54	1.00	0.87	0.83	0.77	0.70
3-Year	0.08	0.24	0.53	0.87	1.00	0.93	0.88	0.81
5-Year	0.07	0.20	0.48	0.83	0.93	1.00	0.97	0.92
7-Year	0.06	0.18	0.44	0.77	0.88	0.97	1.00	0.96
10-Year	0.06	0.18	0.42	0.70	0.81	0.92	0.96	1.00

Clearly the correlations between the different vectors of the different maturities have changed after calculating the changes of the yield curve rates.

In order to evaluate the power of PCA on this dataset it is possible to employ two tests that validate the use of it: the KMO (Kaiser-Meyer-Olkin) Measure of Sampling Adequacy (1970) statistic and Bartlett´s Test of Sphericity (1951). The KMO statistic examines to what degree the proportion of variance among variables may be common variance. Initially, it calculates the partial correlation matrix of the changes for each maturity. This matrix is the correlation between maturities without other maturities that may be numerically related. Using this and the original correlation matrix the statistic calculates a number from 0 to 1, where the closer the number is to 1, the more suited PCA is to the dataset.

$$KMO = \left(\frac{\sum_{i} \sum_{j \neq i} r_{ij}^2}{\sum_{i} \sum_{j \neq i} r_{ij}^2 + \sum_{i} \sum_{j \neq i} a_{ij}^2} \right) \tag{3.1}$$

In equation 3.1 r_{ij} and a_{ij} are the entrance (i,j) of the correlation and partial correlation matrices, respectively. Ideally, the partial correlation is low, indicating strong relationships between all maturities and thus the use of PCA. The KMO Statistic for the relative changes and the logarithmic changes are both 0.85. This value is sufficiently high that PCA is still considered a viable procedure to apply on the dataset.

On the other hand the Bartlett Test of Sphericity checks the observed correlation matrix against the identity matrix. More specifically it ascertains whether there is a redundancy between the maturities that can then be summarized with a few components. The null hypothesis of the test is that the maturities are orthogonal, that is, not correlated. The corresponding alternative hypothesis is that the maturities are correlated to the extent that the correlation matrix is significantly different from the identity matrix. For both sets of changes the test yields p-values rounded to zero, thus rejecting the null hypothesis.

After transforming the Yield Curve Rates the KMO Statistic and Bartlett Test of Sphericity confirm that PCA is still a worthwhile procedure to apply.

3.3.3 Squared Changes - Variance of the Residuals

An important assumption of linear regression is homoscedacity; constant variance in the residuals for any given time. One of the ways to examine if this assumption holds for the relative and logarithmic changes created is to visually inspect the squared changes of the maturities. Consider the following equation:

$$S^2 = \frac{\sum_{t=1}^{n} (u_t - \bar{x})^2}{n-1}, \tag{3.2}$$

where S^2 is the variance, u_t is the yield curve change for a given maturity at time i, $\bar{x}$ is the mean of the yield curve change for a given maturity, and n the number of observations. Knowing that the mean of the yield curve changes are close to zero, then the numerator of the equation reduces to the squared changes. Thus, by inspecting the squared changes homoscedacity can either be verified or discarded. The squared relative and logarithmic changes for each maturity are presented in Figures 3.4 and 3.5.

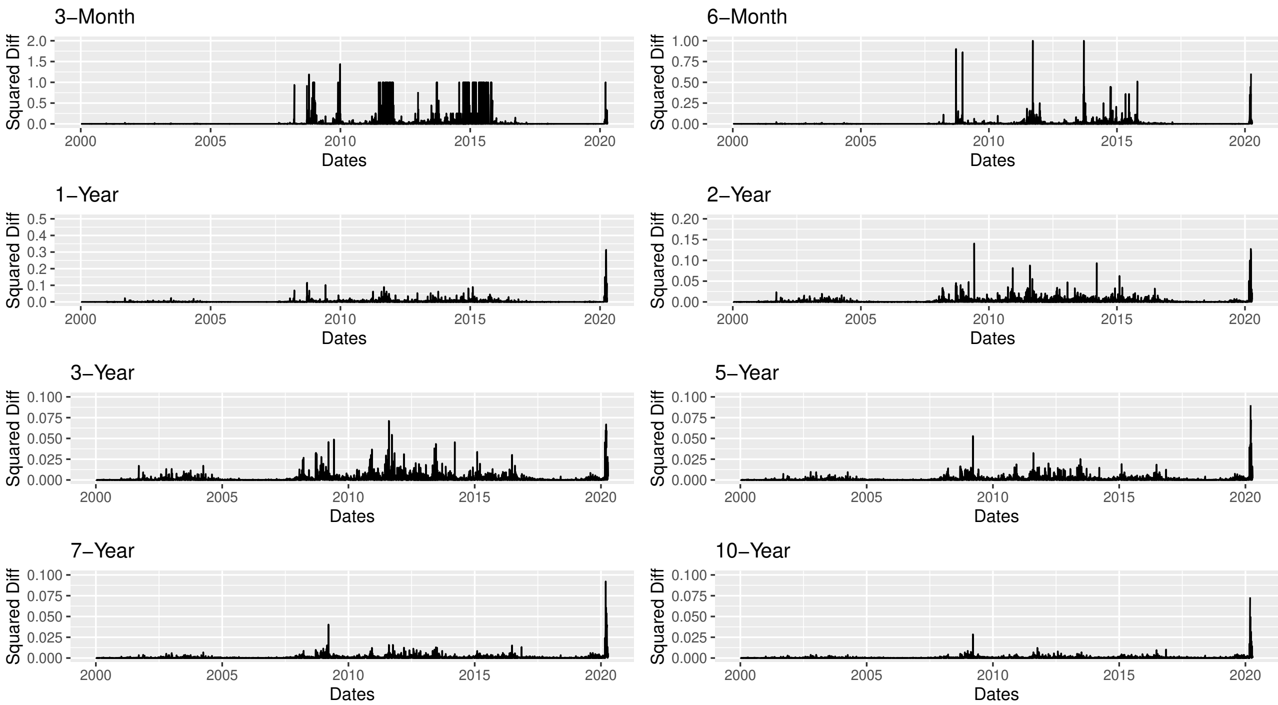

FIGURE 3.4: Squared Relative Changes of U.S. Daily Treasury Yield Rates from 2000-2020

From the plots supplied it can be seen that the squared relative and logarithmic changes vary over the dataset. This implies that the variance of the changes are

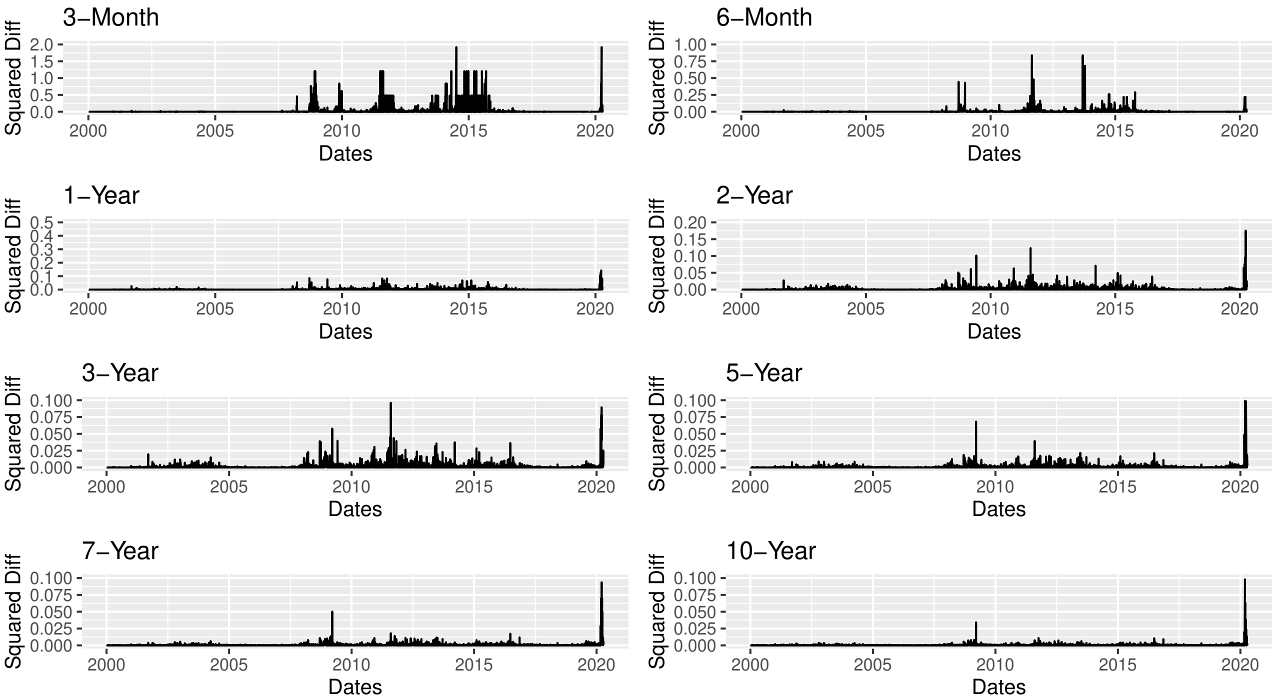

FIGURE 3.5: Squared Logarithmic Changes of U.S. Daily Treasury Yield Rates Square from 2000-2020

not constant and thus one of the assumptions required for linear regression is also violated. With existing multicollinearity and heteroscedacity in the dataset quantile regression is well-suited as none of the attributes mentioned affect its performance.

3.3.4 Normality of Yield Curve Rate Changes

In order to further validate the use of quantile regression it is pertinent to investigate the changes for attributes that quantile regression is known to apply well for. In particular, it is useful to attempt to identify non-normality within the changes as quantile regression yields meaningful results when the distribution is not normal. First, we visually inspect the changes to identify non-normality in the shape of skewness, or kurtosis. Histograms of each variant of the changes, and for each maturity are displayed below with 100 bins in each plot:

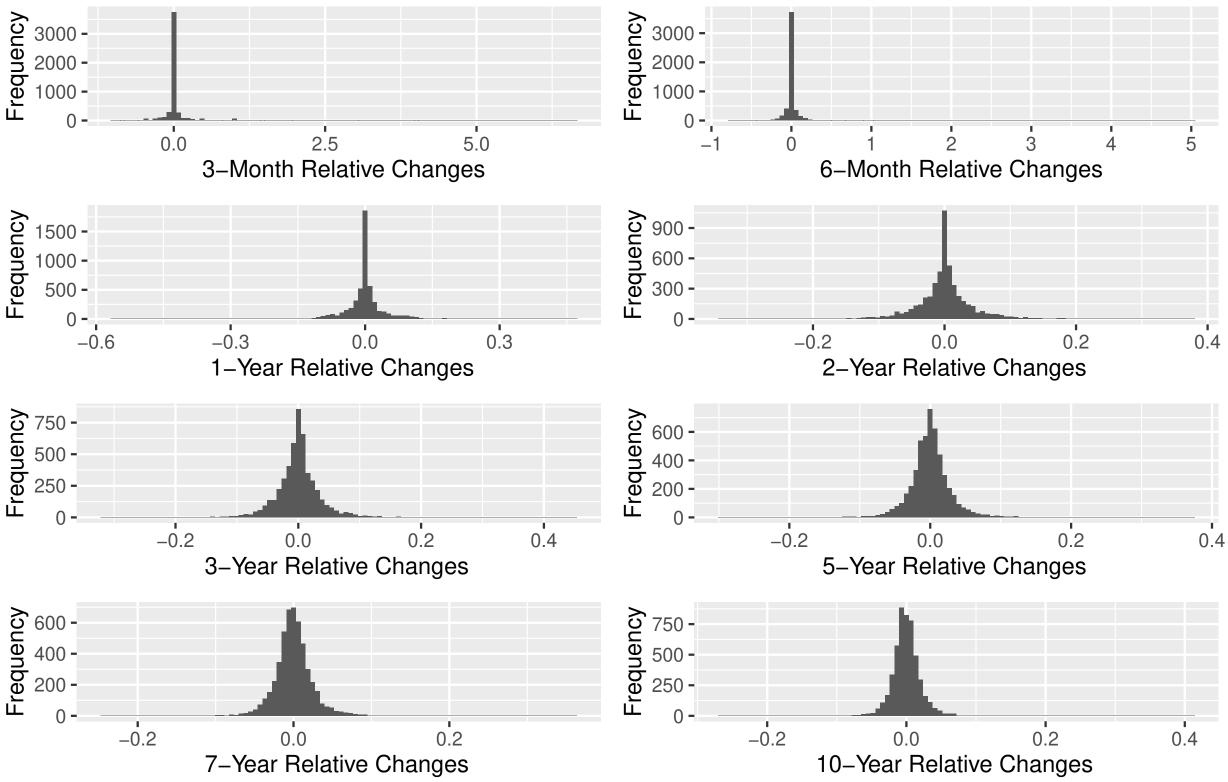

FIGURE 3.6: Histograms of U.S. Daily Treasury Yield Rate Relative Changes from 2000-2020

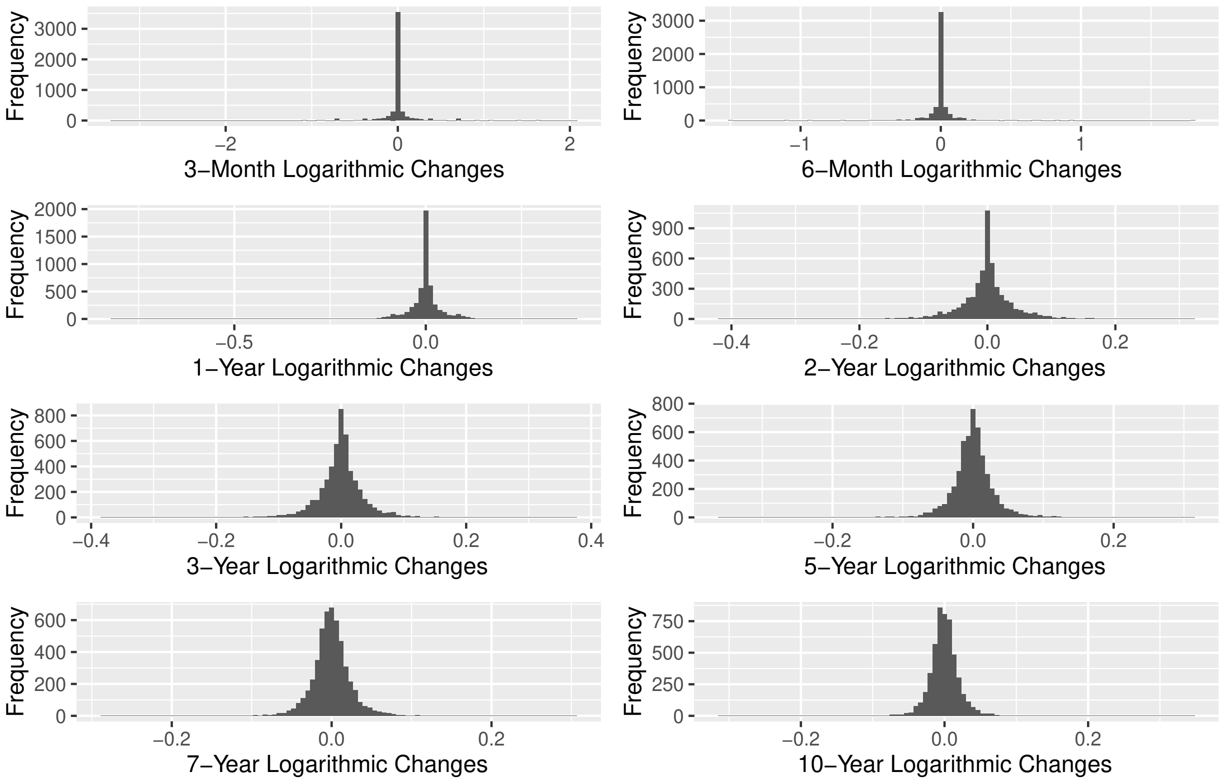

FIGURE 3.7: Histograms of U.S. Daily Treasury Yield Rate Logarithmic Changes from 2000-2020

Visual examination of the histograms indicates that a normal distribution cannot be assumed for the changes in most cases, as the peaks cluster greatly around the mean indicating excess kurtosis. To investigate this further we conduct skewness and kurtosis tests, along with the Jarque-Bera (1980) test to each type of changes and each maturity. The Jarque-Bera test creates a statistic based on sample skewness and kurtosis given as:

$$JB = n\left(\frac{(\sqrt{b_1})^2}{6} + \frac{(b_2 - 3)^2}{24}\right), \tag{3.3}$$

where b_1 and b_2 represent skewness and kurtosis respectively. The null hypothesis of the test is that the data is normal distributed, that is, skewness equal to zero and kurtosis equal to 3. Large values of the statistic reject the null hypothesis of normality. Kurtosis less than 3 (Kallner, 2018) and skewness between -2 and 2 (Kim, 2013) are considered acceptable in order to accept normal univariate distribution.

TABLE 3.10: Normality Test Results of U.S. Treasury Yield Curve Rate Relative Changes from January 2000 - April 2020

	Skewness	Kurtosis	JB χ^2 Statistic
3M Values	11.00	210.47	9431847.84
6M Values	20.93	903.17	172180432.40
1Y Values	0.59	14.95	47404.72
2Y Values	0.39	8.51	15368.25
3Y Values	0.54	12.91	35374.49
5Y Values	0.54	17.53	64967.05
7Y Values	0.52	18.37	71311.41
10Y Values	1.50	51.50	560626.05

TABLE 3.11: Normality Test Results of U.S. Treasury Yield Curve Rate Logarithmic Changes from January 2000 - April 2020

	Skewness	Kurtosis	JB Statistic
3M Values	-0.90	43.80	404817.74
6M Values	0.81	83.85	1481302.37
1Y Values	-0.91	27.50	160051.57
2Y Values	-0.28	8.67	15891.16
3Y Values	-0.25	11.66	28690.27
5Y Values	-0.32	16.32	56190.13
7Y Values	-0.22	16.62	58227.53
10Y Values	0.03	38.38	310243.74

When looking at the relative changes in Table 3.10 it seems that skewness can be considered within normal range for most of the maturities, however values for kurtosis seems to be very high for all maturities. This corresponds well with the conclusion reached based on the previous visual inspection. The Jarque-Bera statistic is very high for all maturities, supporting the observation that the relative changes cannot be assumed to be normally distributed.

Similarly for the logarithmic changes in Table 3.11, skewness values range between values acceptable to conclude a normal distribution, however the kurtosis values indicates high kurtosis. Additionally, the Jarque-Bera statistics are very high also supporting the conclusion that logarithmic changes cannot be assumed to be normally distributed.

After concluding that the relative and logarithmic changes for all maturities are not normally distributed, the study proceeds with applying quantile regression as its characteristics are more optimally suited for non-normally distributed data.

Chapter 4

Methodology

The proposed PCA-QREG model follows the procedure detailed ahead. The interest rate changes are calculated using standard approaches to create daily relative changes and daily logarithmic changes. Using the interest rate changes the PCA procedure is applied to both types of changes separately. The PCA procedure yields three independent components that capture large parts of the variation in the changes. The selection of three components is based on common practice in literature that attempts to optimize the balance between reduction in dimensionality and therefore noise, and accuracy in capturing variations. The choice is validated in Chapter 5. Using the Principal Component vectors volatility proxies of each Principal Component are created using an Exponentially Weighted Moving Average procedure. Finally, the volatility proxies are run through quantile regression against the interest rate changes, yielding best fit coefficients for each quantile investigated. Using these coefficients predictions of the interest rate changes are made for corresponding quantiles, both in-and-out of sample. Ultimately, the accuracy of the predictions are evaluated using well established VaR backtesting tests which test if the predictions compare to their expected success. More in-depth explanations for each step are provided in the sections that follow.

4.1 Yield Curve Rates Changes

In order to make inferences about the volatility of the changes it is useful to clarify how the changes are defined. This study employs daily relative changes:

$$y_t = \left(\frac{P_t - P_{t-1}}{P_{t-1}} \right) \tag{4.1}$$

and daily logarithmic changes:

$$y_t = ln \left(\frac{P_t}{P_{t-1}} \right) \tag{4.2}$$

Where P_t is the interest rate at time t for a certain maturity. The calculation of changes in Equations 4.1 and 4.2 are synonymous with the procedures taken to calculate financial returns.

4.2 Principal Component Analysis

PCA is a statistical procedure of feature extraction that transforms a set of correlated observations into a set of linearly uncorrelated variables denoted as the principal components. As mentioned in Chapter 2, literature indicates that a high proportion of the variation in the interest rates is captured with three principal components, where additional components contribute with more accuracy, but also more noise. This paper also retains only the first three principal components. This is empirically validated to be a sufficient trade-off between accuracy and noise for the relative and logarithmic changes. This is shown in Chapter 5. This procedure yields three vectors X_1, X_2, and X_3, commonly referred to as the level, slope, and curvature respectively (see Chapter 2). The length of the vectors corresponds to the number of time points of changes in the dataset.

4.3 Exponentially Weighted Moving Average

The EWMA is a simple and frequently used volatility model. Where historical standard deviation places an equal weight on all observations in the period and is often ill-suited to volatility estimations within interest rate forecasting, EWMA provides a slightly more complex and accurate, yet still simple approach. The EWMA model calculates the moving average of data series and places more weight on the most recent data points. This aspect of the model is useful in attempting to accommodate for the commonly seen feature of financial series, namely volatility clustering.

The model is described as such:

$$EWMA_{t,i} = \sqrt{X_{t,i}(1-\lambda)^2 + EWMA_{t-1,i}(\lambda)^2}, \tag{4.3}$$

where λ is a constant decided beforehand that determines the weight of the previous observations in the model. As mentioned, financial series are known to have volatility clustering and so high values of λ accommodate for this. In this study the λ is predefined as being 0.97 in order to closely follow volatility developments. First the model requires an arbitrary predefined value for the first data point. The approach in the study is to calibrate the first value based on what order of magnitude the EWMAs calculated for an arbitrary first value possesses. The model is repeated for each principal component vector resulting in three vectors of volatility proxies.

4.4 Quantile Regression

Quantile regression is an extension of standard linear regression that allows for the examination of relationships between variables at tail-ends of distributions. Unlike standard linear regression it makes no assumptions about the distribution of the residuals. This is advantageous when working with data that may be non-normally distributed or that possess non-linear relationships with predictor variables. Financial data is well-known to have these stylistic features, thus validating the use of quantile regression in this field. Employing QR to the volatility proxies of the principal components of the yield curve rate changes is beneficial in order to inspect how changes may be dependent on the level, slope and curvature at different quantiles. Assuming that the changes react uniformly to certain volatility changes in the level, slope and curvature irrespective of the values of the changes may be erroneous. Instead of estimating the conditional mean, and minimizing the Mean Squared Error (MSE), as is done in standard linear regression, Quantile Regression estimates the conditional median, and minimizes the Median Absolute Deviation (MAD). Quantile regression models take the following form:

$$Q_\tau(y_t) = B_0(\tau) + B_1(\tau)x_{t,1} + ... + B_p(\tau)x_{t,p}, t = 1, ..., n, \tag{4.4}$$

where beta coefficients depend on the quantile τ

In this thesis the model is written as:

$$Q_\tau(y_t) = B_0(\tau) + B_1(\tau)EWMA_{t,1} + B_2(\tau)EWMA_{t,2} + B_3(\tau)EWMA_{t,3}, t = 1, ..., n, \tag{4.5}$$

where coefficients $B_n(\tau)$ are estimated.

The quantiles investigated include the lower and higher extremes of 1% and 2%, and 98% and 99%. In addition, increments of 5% are explored from 5% to 95%. In total 23 different quantiles are considered in the quantile regression. The study calculates the coefficients produced by the quantile regression model for two different types of changes, eight different maturities and 23 different quantiles, thus yielding 368 different equations to estimate the coefficients for.

4.5 In-sample Prediction

The quantile regression yields four different coefficients that are the best fit for Equation 4.5 for each quantile, maturity and both variants of calculated changes. Using the estimated coefficients it is possible to calculate estimates of the changes.

$$\hat{y}_{t,\tau} = \hat{B}_0(\tau) + \hat{B}_1(\tau)EWMA_{t,1} + \hat{B}_2(\tau)EWMA_{t,2} + \hat{B}_3(\tau)EWMA_{t,3}, \tag{4.6}$$

where $\hat{y}_{t,\tau}$ is the estimated change at time t for the τ quantile, for a specified variant of the changes, and maturity type.

4.6 Out-of-sample Prediction

By using a subset of the available changes data, running the model and estimating coefficients for the quantile regression equation, it is possible to make a one day ahead prediction, out-of-sample. By defining the length of the subset as the window length, and incrementally varying the training data with one day at a time, creating a rolling window, numerous one day ahead predictions can be made. The equation for the estimator is as follows:

$$\hat{y}_{WL+1,\tau} = \hat{B_0}(\tau) + \hat{B_1}(\tau)EWMA_{WL,1} + \hat{B_2}(\tau)EWMA_{WL,2} + \hat{B_3}(\tau)EWMA_{WL,3} \quad (4.7)$$

Where WL is the time at the end of the current window, and $\hat{B_n}$ are the estimated coefficients for the current window.

As mentioned in Chapter 3, the time the dataset spans is from January 2000 to April 2020. The full dataset is used for the out-of-sample predictions, with the window length set to 10 years. An example of the method is given here, and repeated in the next chapter to ensure clarity.

For instance, with the window length set to 10 years, the first window is from 3rd January 2000 to 3rd January 2010, and an out-of-sample prediction is made for the 4th of January 2010. The following window is 4th January 2000 to 4th January 2010, and the subsequent prediction is for the 5th of January 2010. This is repeated for the whole dataset until the final date, 14th April 2020.

4.7 Testing the Results

In order to test the accuracy of the predictions, Value-at-Risk (VaR) backtesting procedures may be employed. The tests measure the accuracy for the estimated changes against the real changes, at a given quantile, thus assessing the accuracy of the risk model. Two popular tests are the Kupiec (1995) and Christoffersen (1998), which examine the unconditional and conditional accuracy of the results, respectively.

4.7.1 Kupiec´s Unconditional Coverage Test

Kupiec´s Proportion-of-Failure (PoF) test is one of the first VaR backtesting procedures and only examines unconditional coverage. That is, the test checks whether the given VaR level is violated more or less than the significance level allows.

For a given quantile, the test calculates the expected number of times the predicted change should be below the real change. If testing the 1% quantile predictions, the test checks whether roughly only 1% of the real changes are below the predicted changes. Of a total n observations, the number of hits can be denoted as n_1, and for a given quantile, with a corresponding $1 - \tau$ confidence interval, the number of hits is expected to be $\tau * n$. Hits being whether the value of the real change is below the predicted change.

This leads to the two hypotheses that are investigated:

$$H_0 : \pi_{ex} = \pi_{obs} = \frac{n_1}{n} \tag{4.8}$$

$$H_1 : \pi_{ex} \neq \pi_{obs} = \frac{n_1}{n} \tag{4.9}$$

Where π_{obs} is the frequency of the hits, and π_{ex} is the expected probability of hits. It should be noted that in this test, contrary to the setup of many other hypothesis tests, the null hypothesis is the hypothesis we wish for the test to reject. In order to test the significance of the observed number of hits a log-likelihood statistic is used:

$$LR_{Kupiec} = -2ln\left(\frac{{\pi_{ex}}^{n_1}(1-\pi_{ex})^{n_0}}{\pi_{obs}^{n_1}(1-\pi_{obs})^{n_0}}\right), \tag{4.10}$$

where n_0 is the number of misses; $n - n_1$. The test statistic is compared to a Chi-Squared test statistic based on the chosen confidence interval: $1 - \tau$, where ultimately the test rejects the null hypothesis or fails to reject it.

While powerful, one of the shortcomings of the Kupiec Test with regards to testing risk models, is the unconditional nature of it, where it only tests the failure rates, and not the successive occurrence of failures. We know that financial data is often stylized by volatility clustering, and the changes depicted in Chapter 3 were clearly dependent on previous values, thus motivating testing the predictions using a conditional VaR backtesting procedure.

4.7.2 Christoffersen´s Markov Conditional Test

The Christoffersen test, similarly to the previous test, examines to what degree the proportion of hits for the predicted changes against the real changes, are valid. However, the test aims to reveal whether the hits that occurred depend on a hit that occured on the previous day. Ideally the prediction model successfully estimates the changes to the extent that hits for a given quantile do not depend on whether the day before resulted in a hit or not. The null hypothesis is that hits are independent across the time series, and the reverse for the alternate hypothesis. Similarly to the

Kupiec Test, a log-likelihood statistic is calculated using the proportion of consecutive hits, and is compared to a Chi-Squared statistic. One of the weaknesses of the Christoffersen test is the day by day Markov nature of the test. In time series with clustering where the hit of a certain day may be dependent on a hit a week ago, the test will not identify this effect, as it only looks at day to day dependence.

Chapter 5

Empirical Results & Discussion

5.1 Principal Components

Literature indicates that the use of three principal components is sufficient to capturing large amounts of the variation in the dataset, whilst not including too much noise. For the study it is pertinent to validate this hypothesis for the entire dataset. The percentage of variances each principal component captures is shown below:

TABLE 5.1: Percentage of Variance for each Principal Component: U.S. Treasury Yield Curve Rate Relative Changes from January 2000 - April 2020

	PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8
Percentage of Variance	60%	17%	9%	7%	4%	1%	1%	0%
Cumulative Sum	60%	77%	86%	93%	97%	98%	99%	100%

This is also visualized below:

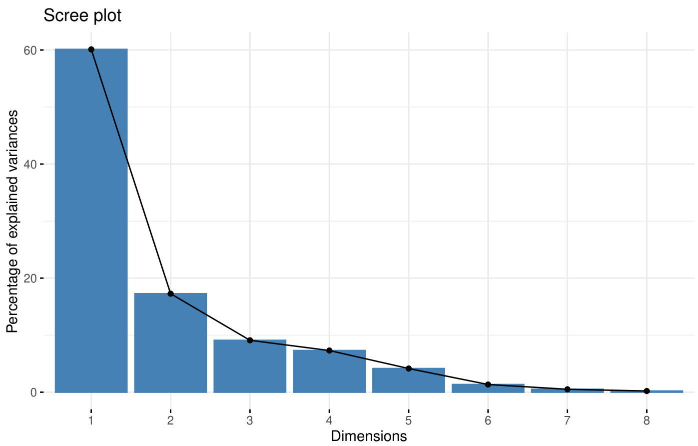

FIGURE 5.1: Percentage of Variation Captured by Principal Components for U.S. Daily Treasury Yield Rate Relative Changes from 2000-2020

The improvement in capturing variation naturally increases with the more Principal Components included, however at three components the captured variation is as high as 86% where further inclusions do not contribute too much to increase the explained variance. The process can be repeated for logarithmic changes:

TABLE 5.2: Percentage of Variance for each Principal Component: U.S. Treasury Yield Curve Rate Logarithmic Changes from January 2000 - April 2020

	PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8
Percentage of Variance	61%	18%	9%	6%	4%	1%	1%	0%
Cumulative Sum	61%	79%	88%	94%	98%	99%	100%	100%

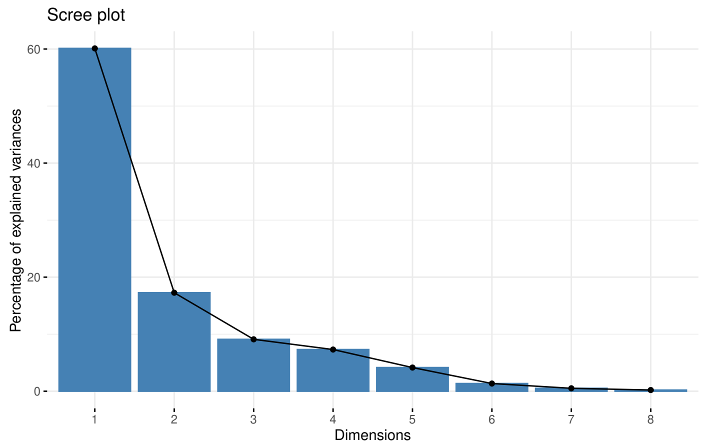

FIGURE 5.2: Percentage of Variation Captured by Principal Components for U.S. Daily Treasury Yield Rate Logarithmic Changes from 2000-2020

Similarly to the relative changes, the logarithmic changes has a high amount of variance captured by the first three principal components. The high variance captured and the only minor improvements by including more dimensions validates the choice of three principal components for the relative and logarithmic changes datasets.

5.2 In-sample Predictions January 2000 - April 2020

The in-sample prediction investigates how well the proposed procedure predicts the yield curve rate changes from January 2000 to April 2020. Following the methodology highlighted in Chapter 4, the in-sample prediction is tested using the Kupiec and Christoffersen tests. In Tables 5.3 to 5.4 the success of the in-sample prediction is visualized. Table 5.3 and 5.4 display the prediction accuracy when examining relative changes and logarithmic changes, respectively. For each in-sample prediction of a given quantile and maturity a color is provided. Red cells indicate predictions that failed both tests, yellow cells indicate predictions which passed only one test, and green cells indicate predictions that passed both tests.

TABLE 5.3: Successful In-Sample Prediction Maturities and Quantiles for Relative Changes from January 2000 to April 2020

Quantiles	3M	6M	1Y	2Y	3Y	5Y	7Y	10Y
1%	Yellow	Yellow	Yellow	Yellow	Green	Green	Green	Yellow
2%	Yellow	Yellow	Yellow	Green	Green	Green	Green	Yellow
5%	Yellow	Yellow	Yellow	Green	Yellow	Green	Green	Green
10%	Yellow	Yellow	Yellow	Green	Green	Green	Green	Green
15%	Yellow	Yellow	Yellow	Green	Green	Green	Green	Green
20%	Yellow	Yellow	Green	Green	Green	Green	Green	Green
25%	Yellow	Yellow	Green	Green	Green	Green	Yellow	Green
30%	Yellow	Yellow	Green	Green	Green	Green	Green	Green
35%	Yellow	Yellow	Green	Yellow	Green	Green	Green	Green
40%	Red	Red	Green	Green	Green	Green	Green	Green
45%	Red	Red	Red	Red	Green	Green	Green	Green
50%	Red	Red	Yellow	Red	Red	Red	Green	Red
55%	Red	Red	Red	Red	Green	Green	Green	Green
60%	Red	Red	Green	Green	Green	Green	Green	Green
65%	Green	Green	Green	Green	Green	Green	Green	Green
70%	Green	Green	Green	Green	Green	Green	Green	Green
75%	Green	Green	Green	Green	Green	Green	Green	Green
80%	Green	Green	Green	Green	Green	Green	Green	Yellow
85%	Green	Green	Green	Green	Green	Green	Yellow	Yellow
90%	Yellow	Green	Green	Green	Green	Yellow	Yellow	Yellow
95%	Yellow	Green	Yellow	Green	Yellow	Green	Green	Green
98%	Yellow	Green	Yellow	Yellow	Yellow	Green	Green	Green
99%	Yellow	Yellow	Green	Yellow	Yellow	Green	Green	Green

TABLE 5.4: Successful In-Sample Prediction Maturities and Quantiles for Logarithmic Changes from January 2000 to April 2020

Quantiles	3M	6M	1Y	2Y	3Y	5Y	7Y	10Y
1%	Yellow	Yellow	Yellow	Green	Green	Green	Green	Green
2%	Yellow	Yellow	Yellow	Green	Green	Green	Green	Yellow
5%	Yellow	Yellow	Yellow	Green	Yellow	Green	Green	Green
10%	Yellow	Yellow	Yellow	Green	Green	Green	Green	Green
15%	Yellow	Yellow	Yellow	Green	Green	Green	Green	Green
20%	Yellow	Yellow	Green	Green	Green	Green	Green	Green
25%	Yellow	Yellow	Green	Green	Green	Green	Yellow	Green
30%	Yellow	Yellow	Green	Green	Green	Green	Green	Green
35%	Yellow	Yellow	Green	Green	Green	Green	Green	Green
40%	Red	Red	Green	Green	Green	Green	Green	Green
45%	Red	Red	Red	Red	Green	Green	Green	Green
50%	Red	Red	Green	Red	Yellow	Red	Green	Green
55%	Red	Red	Red	Red	Green	Green	Green	Green
60%	Red	Red	Green	Green	Green	Green	Green	Green
65%	Green	Green	Green	Green	Green	Green	Green	Green
70%	Green	Green	Green	Green	Green	Green	Green	Green
75%	Green	Green	Green	Green	Green	Green	Green	Green
80%	Green	Green	Green	Green	Green	Green	Green	Yellow
85%	Green	Green	Green	Green	Green	Green	Yellow	Yellow
90%	Yellow	Green	Green	Green	Green	Green	Yellow	Yellow
95%	Green	Green	Yellow	Green	Yellow	Green	Green	Green
98%	Yellow	Green	Green	Yellow	Yellow	Green	Green	Green
99%	Yellow	Yellow	Green	Green	Yellow	Green	Green	Green

Some visible trends can be drawn from the tables presented. It is clear that for both standard and logarithmic changes in-sample predictions often performed poorly for shorter maturities and better for longer maturities. Generally, for the shorter maturities the predictions fail entirely when close to the median. This is also a trend for the longer maturities. For the lowest maturities, three month and six month, predictions are sub-optimal until the 65th quantile. For those same maturities the predictions perform well before reaching the longer quantiles. While generally the predictions seem sub-optimal at the highest quantiles as well, it can be observed that the predictions perform well at the highest quantiles for the longer maturities five year, seven year, and ten year.

From the VaR estimation perspective the estimations of most interest lie at the 1-5% and 95-99% predictions. As described in Chapter 2, these values, particularly the 1% estimation, will hold the most relevance to interest rate risk managers. Both Tables 5.3 and 5.4 indicate that in-sample predictions of the lowest quantiles are most accurate from middle maturities to longer maturities. For relative changes the tables indicate that predictions at the 1-5% quantiles for the two year to ten year maturities are good. Additionally, the most accurate 1% quantile predictions occur between the three year to seven year maturities.

The worse performance of the predictions at shorter maturities could be related to the high variation in the changes at those maturities. When revisiting Figure 3.4 it can be seen that the squared changes are of a higher magnitude for the three month and six month maturities. The higher level of variation and rapid changes may be too high for the model to capture appropriately.

Comparing Tables 5.3 and 5.4, it seems that the logarithmic changes yielded a greater proportion of predictions that passed both the condition and unconditional coverage tests. Of the predictions based on the relative changes 118 of the 184 tests were completely successful. This is in contrast to the 127 prediction sets that passed both tests successfully for predictions based on the logarithmic changes. There seems to be a slight indication that the methodology based on logarithmic changes yielded better results, however the difference between 127 and 118 being relatively small suggests nothing definitive can be concluded. The proportion of prediction sets that failed both tests amounted to 18 and 16 of 184 for Tables 5.3 and 5.4, respectively.

In addition to the tables, plots of in-sample predictions that performs sub-optimally, and others that performs well, are presented below. From the perspective of an interest rate risk manager, in order to predict the VaR estimate predictions the 1% and 99% quantile interest rate change are the most interesting. Which of the 1% or 99% quantile is of interest depends on the whether the manager´s position is long or short. As pointed out earlier, the shorter maturities seem to perform worse compared to the longer maturities. Therefore in Figure 5.3 the three month maturity logarithmic changes predictions are presented at the 1st and 99th quantile, as an example of sub-optimal predictions. Correspondingly, as examples of successful predictions the five year maturity logarithmic changes predicted values are presented for the same quantiles in Figure 5.4.

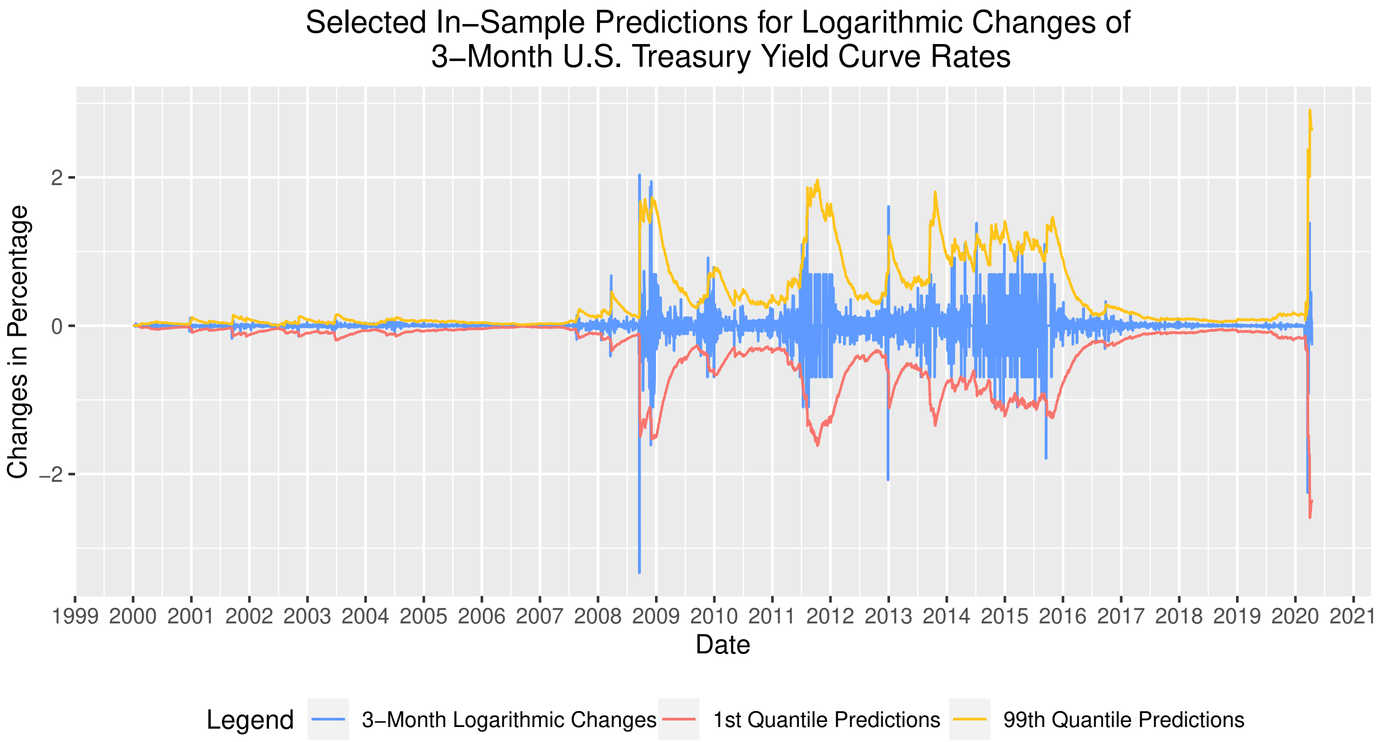

FIGURE 5.3: In-sample Predictions of 3-Month Logarithmic Changes of U.S. Daily Treasury Yield Curve Rates at 1st and 99th Quantile from 2000-2020

In Figure 5.3 the prediction sets do well in retaining an appropriate magnitude of size compared to the original three month logarithmic changes from January 2000 to roughly August 2008. Around August 2008 the original dataset rapidly explodes in magnitude, and while the 1% quantile prediction does not capture the change appropriately, both prediction lines are very reactive to the sudden change. The same characteristic is seen around April 2020. Interestingly, the same adaptability is not seen when the original dataset falls towards its zero mean. This is seen from 2009 to 2010, and June 2011 to roughly March 2012, for instance. This may be due to the fact that the estimated quantile regression coefficients implicitly contain information that neither the 1% nor the 99% quantile tends to lie around zero, and therefore the prediction equation is not tuned to capturing the reversion to the mean, compared to the sudden falls and rises, respectively.

The predictions in Figure 5.3 follow the shape of the original dataset reasonably well, however they follow it the best in periods with low variation. This can be seen from 2000 to June 2007, and 2017 to 2020. In periods with variation the magnitude of the

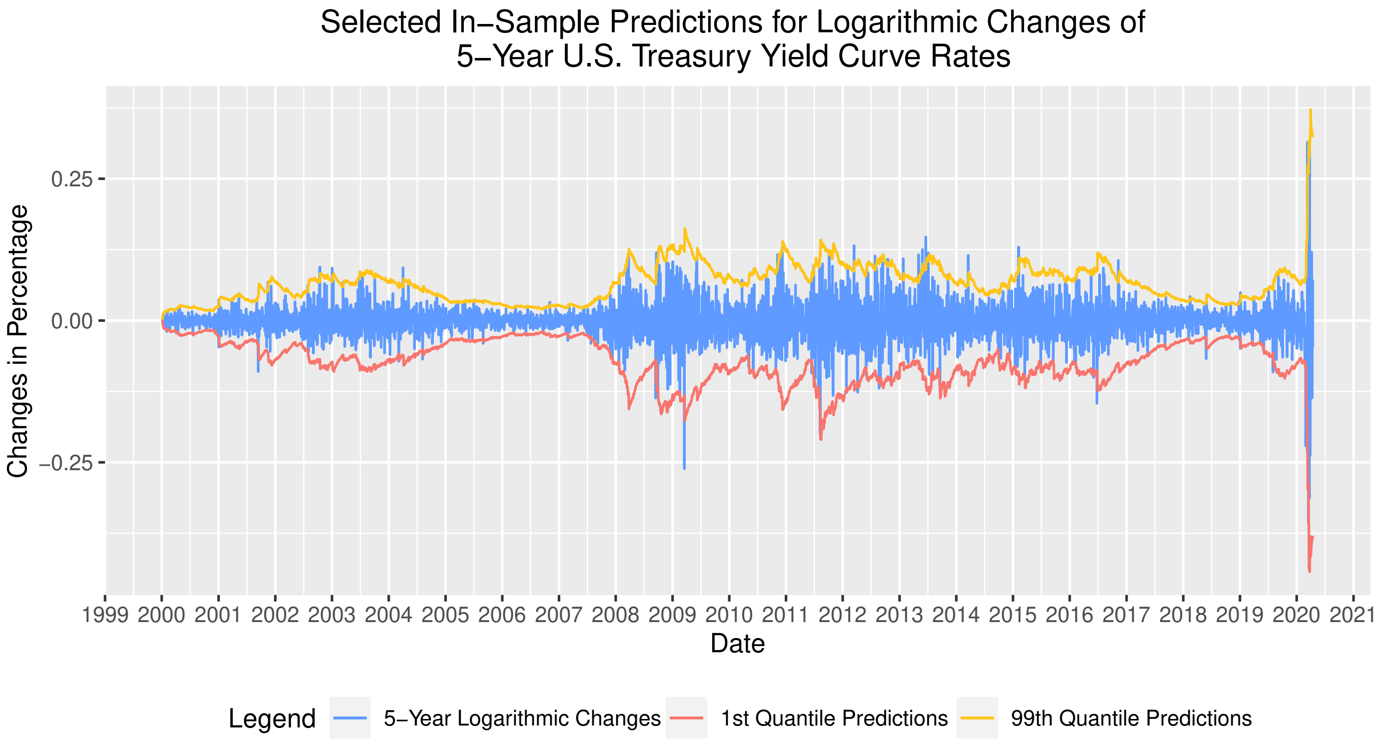

FIGURE 5.4: In-sample Predictions of 5-Year Logarithmic Changes of U.S. Daily Treasury Yield Curve Rates at 1st and 99th Quantile from 2000-2020

original dataset changes so dramatically, and somewhat often. Like from 2009 to 2016, the prediction sets seem to capture the trend of the direction of the original dataset very well, but struggles more with accurate quantile predictions. With large fluctuations in the three month logarithmic changes´ values the quantile regression coefficients seem less optimized for the rapid and large changes. Indicative of this seems to be how the original dataset deviates far more from prediction lines in this turbulent period, seen before 2009, at 2013, and before 2016.

For similar periods the same characteristics identified above are seen for both quantiles in Figures A.1, A.2, A.9 & A.10 in Appendix A, these being plots with statistically the same predictive power.

Comparatively, the prediction sets in Figure 5.4 perform similarly, if not slightly better, at following sudden changes of large magnitude. This is seen just after 2009, around June 2011, June 2016, and around March 2020. The original dataset is relatively stable with values lying between -0.125 and 0.125, for the most part. Due to this it is difficult to evaluate to what extent the prediction lines respond to sudden and dramatic changes, with exception of the sudden fluctuation in March 2020. The predictions in Figure 5.3 and 5.4 are similar however in the somewhat sluggish response to mean reversion after sudden changes. This is seen between 2009 and June 2010, and between June 2016 and roughly April 2017, for the 1% quantile in Figure

5.4. Generally however, the predictions do quite well with large changes presented, where only the 1% quantile line has a major deviation just after 2009.

The prediction lines in Figure 5.4 do however follow the shape of the original dataset very well, doing particularly well from 2005 to 2008, where the observable variation is also quite low. The same is seen from June 2017 onwards. Otherwise, between 2012 and 2017, a period with more variation, the prediction lines still perform relatively well in retaining the overall shape of the dataset. This may be attributed to the fact that the values in the original dataset remain of the same order of magnitude, and thus the prediction lines are better optimized given fewer shocks to the dataset. The seven and ten year maturities perform equally well and display the same traits. These are shown in Figures A.13 and A.14. This is perhaps unsurprising given how similar the original datasets are. The similarity is not only visually identified, as these maturities also exhibit high correlation between each other as seen in Table 3.9.

5.3 Out-of-sample Predictions January 2010 - April 2020

In order to examine the effect varying amounts of training data may have on the proposed model, different sets of dates can be utilized. However, both quantile regression and statistical tests of the predictions require considerable amounts of data in order to have statistically significant results. The trade-off between enough data for training and testing led to the decision of using 10 years of training data at a time to produce one day ahead predictions for 10 years. The set decided upon is from January 2010 to April 2020, as mentioned in Chapter 4, with the length of each rolling window training period specified as 10 years. The out-of-sample prediction is applied using a rolling training window that is 10 years long. This is also described in Chapter 4, however a short example is reiterated here. For instance, the first 10 year window is from 3rd January 2000 to 3rd January 2010, and an out-of-sample prediction is made for the 4th of January 2010. The following window is 4th January 2000 to 4th January 2010, and the subsequent prediction is for the 5th of January 2010. This is repeated for the whole dataset until the final date, 14th April 2020.

In similar fashion to the previous section, tables with the test results are presented below. Tables 5.5 & 5.6 describe the prediction accuracy of out-of-sample predictions based on relative changes and logarithmic changes, respectively. The color of the cells corresponds to the number of accuracy tests passed, identical to the details described in the previous section.

TABLE 5.5: Successful Out-of-Sample Prediction Maturities and Quantiles for Relative Changes

Quantiles	3M	6M	1Y	2Y	3Y	5Y	7Y	10Y
1 %								
2%								
5%								
10%								
15%								
20%								
25%								
30%								
35%								
40%								
45%								
50%								
55%								
60%								
65%								
70%								
75%								
80%								
85%								
90%								
95%								
98%								
99%								

TABLE 5.6: Successful Out-of-Sample Prediction Maturities and Quantiles for Logarithmic Changes

Quantiles	3M	6M	1Y	2Y	3Y	5Y	7Y	10Y
1%								
2%								
5%								
10%								
15%								
20%								
25%								
30%								
35%								
40%								
45%								
50%								
55%								
60%								
65%								
70%								
75%								
80%								
85%								
90%								
95%								
98%								
99%								

Similarly to the trend observed for in-sampling prediction accuracy, it is clear when examining Tables 5.5 & 5.6 that the accuracy of predictions is poor at shorter maturities from three month to one year. As seen in the previous section, at the maturities mentioned, the prediction accuracy when close to the median is poor. Generally, the prediction accuracy reverses to passing both tests around the 75th quantile. As mentioned previously, the poor prediction accuracy at shorter maturities that is almost irrespective of quantile may be attributed to the high variation in the datasets at these maturities.

Further, it seems the predictions at different quantiles for longer maturities including three year to ten year, and partially including two year, are significantly better and perform well. With regards to the 1% quantile estimation the out-of-sample predictions are most accurate for the maturities between two year to seven year, for both Tables 5.5 & 5.5.

With regards to the difference in accuracy dependent on the type of changes, in contrast to the earlier section, the number of prediction sets that passed both the Kupiec and Christoffersen set are very similar. Table 5.5 contains 116 cases that passed both tests opposed to 114 cases in Table 5.6. The two tables look very similar, however naturally differ for some cases. It is difficult to ascertain whether relative or logarithmic changes are preferable for out-of-sample prediction from this. The proportion of prediction sets that failed both tests amounted to 39 and 36 of 184 for Tables 5.5 and 5.6, respectively.

As a supplement to the tables, plots of accurate and sub-optimal out-of-sample predictions are presented. All the displayed prediction sets are based on the logarithmic changes. To exemplify an sub-optimal prediction set, the out-of-sample predictions at the 1st and 99th quantile for the three month maturity are presented in Figure 5.5. The 1st quantile prediction at the three month maturity passed one of the backtests, opposed to the 99th quantile prediction which failed both. To display accurate predictions the 1st and 99th quantile predictions for the five year maturity are presented in Figure 5.6, both of which passed both statistical tests.

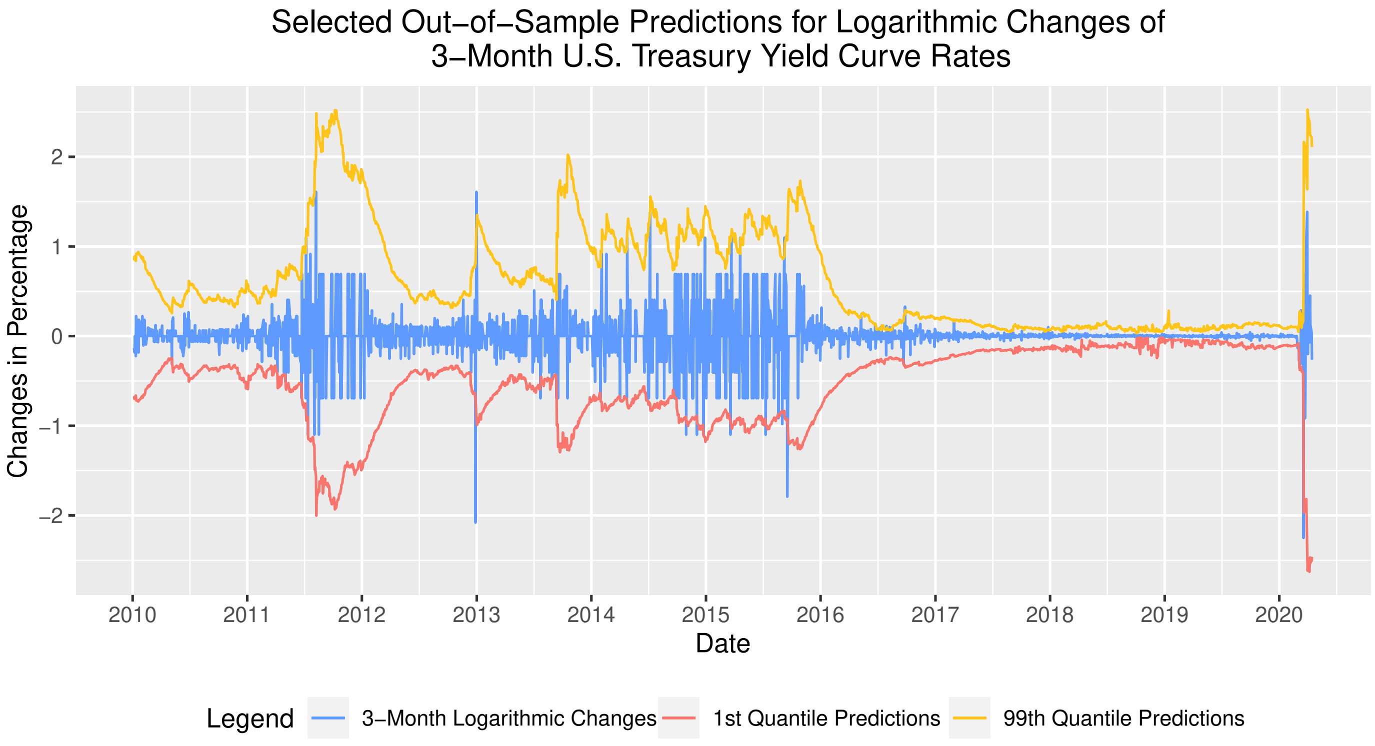

FIGURE 5.5: Out-of-Sample Predictions of 3-Month Logarithmic Changes of U.S. Daily Treasury Yield Curve Rates at 1st and 99th Quantile from 2010-2020

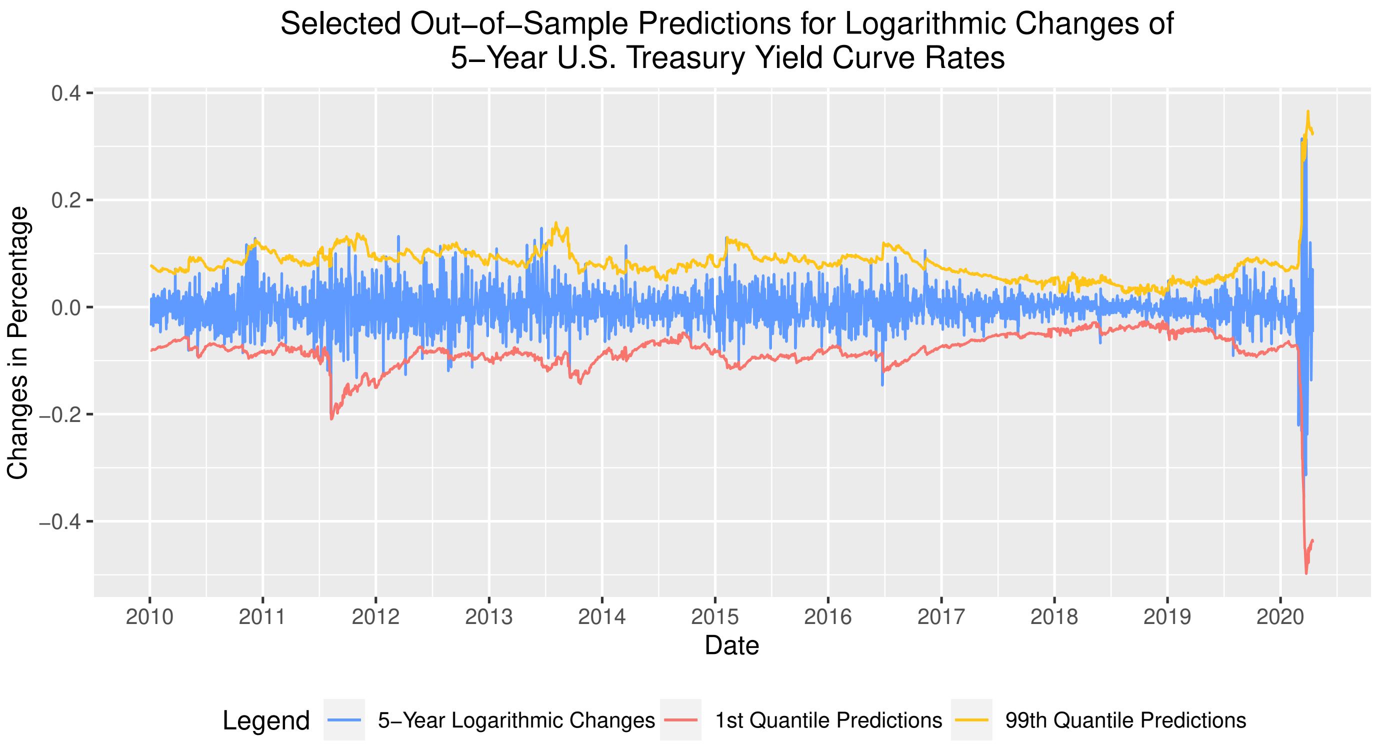

FIGURE 5.6: In-of-Sample Predictions of 5-Year Logarithmic Changes of U.S. Daily Treasury Yield Curve Rates at 1st and 99th Quantile from 2010-2020

The prediction sets in Figure 5.5 perform relatively poorly in following the shape of the original logarithmic changes. In particular, both the 1% and 99% quantile predictions between 2010 and 2014 do not adjust to reasonable values. Extreme examples of this can be seen from approximately June 2011 to August 2012, around August 2013, and around August 2015. Where some deviation for quantile predictions can be expected, in all these instances the quantile predictions have a very large distance from the three month logarithmic changes. For the 99th quantile prediction at August 2013 the predicted value is as much as 2.5 times larger than the maximum value in this period. Visually it is to some extent possible to see that the 99th quantile predictions failed both tests compared to the 1st quantile predictions. Between 2014 and approximately August 2015 the 1% quantile follows the original dataset well, and lies in an acceptable area. On the other hand, the 99% quantile predictions for the same period lie much higher than considered reasonable.

In the previous section the rapid response to sudden changes was mentioned. This is clearly evident in Figure 5.5 as well. Shortly before June 2011 both prediction sets follow the change in magnitude of the original dataset quite well. Similarly to Figure 5.3 at 2013 and April 2020 evidence of the fast reactivity is present again. In the same vein however, the time from fast response to a dramatic change to reverting to lower values closer to the mean is very slow. Understanding these characteristics of the predictions helps explain why the 99% quantile predictions perform worse than the 1% quantile set between 2014 and August 2015. As an example, when looking at June 2014 in Figure 5.5 the 99% quantile prediction has increased as the original logarithmic changes has a spike there. The 99% quantile prediction quickly adjusts, whereas the 1% quantile prediction does not change much as the direction of the change is "unnatural" to it. From this point onwards it is clear that the 1% quantile prediction, which more or less ignores the upwards spike within the historic data, is better able to predict the future values that lie in a similar area. The 99% quantile estimation however, reacting quickly to the new spike fails to predict the future values well, jumping up in magnitude for each new spike, and not coming very close to the area most of the logarithmic changes lie in, for that period.

A possible explanation to this skewed sensitivity may lie within the use of the EWMA to calculate the volatility proxies. The aim of using the procedure was to ensure the proxies captured volatility clustering appropriately. In a sense the EWMA worked well as the prediction lines´ shape follows quite well to the original dataset. However, the sensitivity may be so high that the predictions overreact to the sudden changes. However, on the other hand it is difficult to logically explain why the reversion to values closer to the mean takes a significant amount of time. This is likely not due to the use of EWMA, as logic would dictate equally fast response time when the dataset suddenly reverts to the mean value.

Some of the predictions in Figure 5.5 are also difficult to explain. The jump, and corresponding fall, for both the 99% and 1% quantile prediction around August 2011, August 2013 and September 2015. In each of these cases the prediction values change dramatically without any clear reason. Figures that best exhibit the same tendencies as Figure 5.5 include B.1, B.3, B.9, B.10 and B.11, in Appendix B.

The less turbulent nature of the original five year logarithmic changes in Figure 5.6 explain the higher prediction accuracy for both the quantiles depicted. The low amount of variation allows for the prediction lines to lie close to the original dataset and for the most part lie within a reasonable range. The same rapid response time to sudden changes, and slow reversion as mentioned before, can be seen in Figure 5.6, however the lower magnitude of the changes, and lack of dramatic falls or jumps allow for stable and more accurate predictions. The prediction set seems to follow the direction of the five year logarithmic changes reasonably well. The period with low magnitude in values and few spikes between 2017 and shortly after 2019 displays where the predictions perform the best. The performance accuracy for this maturity can be seen in Figures B.6, B.7, B.8, B.13, and B.14, for the five, seven and ten year maturities. As mentioned in the previous section, the correlation between these maturities´ relative and logarithmic changes was very high in the in-sample analysis, and so it is reasonable to assume this explains their similar predictive power.

When looking at both in-and-out-of-sample predictions the model predicted better values at higher quantiles opposed to lower quantiles. An explanation could be that the changes exhibited negative skewness, thus resulting in a larger distribution of data lying between the mean (zero) and the maximum value. This would in turn allow the PCA-QREG model to better predict one day ahead VaR estimates at higher quantiles. The normality tests displayed in Tables 3.10 and 3.11 indicate however that this logic would only apply for the logarithmic changes, and even the logarithmic changes exhibit very low levels of negative skewness. In reality with the yield curve rates being transformed multiple times through producing relative and logarithmic changes, the PCA and the EWMA, it is difficult to pinpoint the cause of this trend.

Additionally, it is clear that the proposed model does well in capturing sudden changes into the predicted quantile values. However, the model exhibits some trouble in predicting values closer to the mean, particularly after a prior sudden change. This explains the higher accuracy for longer maturities, as the datasets vary less and have fewer jumps/falls as the maturities get longer. This characteristic amongst the predictions may however be reasonable given the model uses time-varying quantile regression coefficients, and that other studies using this approach exhibit similar tendencies such as in the work by Gerlach et. al. (2011) when investigating Bayesian time-varying quantile forecasting for VaR in financial markets.

The variation in the datasets mentioned touches upon the volatility of the different datasets. As mentioned in Chapter 3, the squared changes in Figures 3.4 and 3.5 provide insight into the volatility of the different maturities. The short rates, three month up to one year maturity, have larger magnitudes, and visually can be seen to possess higher volatility and be more time-varying. This indicates more heteroscedacity amongst these maturities, which explains why the proposed model struggles in producing very accurate predictions, both in- and out-of-sample. The reverse case applies to the longer rates, which provides an explanation for the improved accuracy of predictions amongst these maturities.

The EWMA specification may be inappropriate in capturing the heteroscedacity within the shorter maturities, however it seems to function relatively well for the longer maturities. Other volatility proxies for the principal components may be better suited for the short rates, and could improve the accuracy of the predictions. This is mentioned in the next chapter.

Chapter 6

Conclusion & Further Research

This master thesis aims to propose a novel interest rate risk prediction model using well established approaches in the field. The prediction model is evaluated both on in-sample interest rate risk accuracy and out-of-sample one day ahead interest rate risk accuracy. The study aims to examine the prediction model´s strength on U.S. Daily Treasury yield curve rates´ relative changes, using maturities ranging from 3-month to 10-year. The in-sample predictions are calculated for the period from January 2000 to April 2020. The out-of-sample model is trained on data from January 2000 to January 2010, and tested for predictions one day ahead from January 2010 to April 2020. The model applies PCA, EWMA, and Quantile Regression to predict the quantile estimates, and evaluates the accuracy using two popular backtesting procedures; the Kupiec & Christoffersen tests. The study evaluates whether the PCA-QREG model can be applied as a useful procedure for interest rate risk management.

For in-sample predictions the PCA-QREG model produced accurate estimations for higher maturities from 3-year to 10-year, irrespective of Yield Curve Rates differences being standard or logarithmic. The out-of-sample accuracy tests indicated similar results. The predictions for the shorter maturities, such as 3-month to 2-year, were generally inaccurate particularly around the median. This applied for both in-sample and out-of-sample predictions. Managers and regulators often wish to assess the 1% VaR estimates, of which the PCA-QREG model produced consistently accurate predictions in- and out-of-sample for the higher maturities.

When the estimated predictions using the proposed model were overwhelmingly accurate. Roughly 64-70% of the in- and out-of-sample prediction sets for different quantiles and maturities passed both accuracy tests, and only 8-21% failed both.

The overall accuracy of the predictions across the quantiles and maturities indicates that the PCA-QREG model performed very well. The PCA procedure captured sufficient amounts of the variation in the yield curve rates. Additionally, the EWMA method of creating volatility proxies appears to have been successful in capturing the volatility clustering in the dataset. Finally, the quantile regression produces coherent results as the dataset exhibited non-normal distribution patterns. The PCA-QREG model shows strong predictive power for in-sample and one day ahead out-of-sample estimates at the 1% quantile, and it is fundamentally easy to apply using readily available, and well understood procedures.

Further, several avenues of extensions to the study can be considered. The study uses U.S. Treasury yield curve rates, however it might be of interest to investigate for the same results produced here for other assets. The LIBOR rate is a good example. Additionally, the model showed good results for predictions at high maturities. A possible extension to the study would be to see if the predictions of assets at higher maturities perform equally well, for instance by looking at the 30-year maturity.

As mentioned in Chapter 5, the PCA-QREG model and the accuracy testing methods both require large amounts of data to perform reasonably well. For this reason varying the training and testing split in the dataset from 50-50 was not feasible. Performing the methodology described in this study to larger datasets would allow for a closer investigation into how the results are influenced by different splits.

The PCA procedure described in Chapter 4 was applied to all eight maturities, yielding three principal component vectors. An alternative approach to the methodology might entail applying the PCA procedure to fewer or varying amounts of maturities, in order to assess the accuracy changes in the predictions.

In Chapter 5 response time to sudden changes in the changes by the prediction sets was extensively described. A proposed reason for the traits identified was the possibility that the EWMA was aggressively capturing volatility clustering in the dataset, inhibiting the prediction sets to revert to reasonable values in certain cases. The specification may be inappropriate for the shorter maturities, in particular. From this a change to the volatility proxy selection could be considered. Explorations into this aspect can be explored where more sophisticated models can be employed instead, in order to evaluate for any change in prediction accuracy. Models such as GARCH or asymmetric variants of GARCH, such as Glosten-Jagannathan-Runkle GARCH (GJR-GARCH) (Glosten et. al., 1993), or the Asymmetric Power GARCH (APARCH) (Ding et. al., 1991) can be considered. As a next step from this, further research can explore how varying the volatility proxy methodology dependent on the maturity length affects the prediction accuracy as well.

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Appendix A

Additional Plots - In-sample Predictions

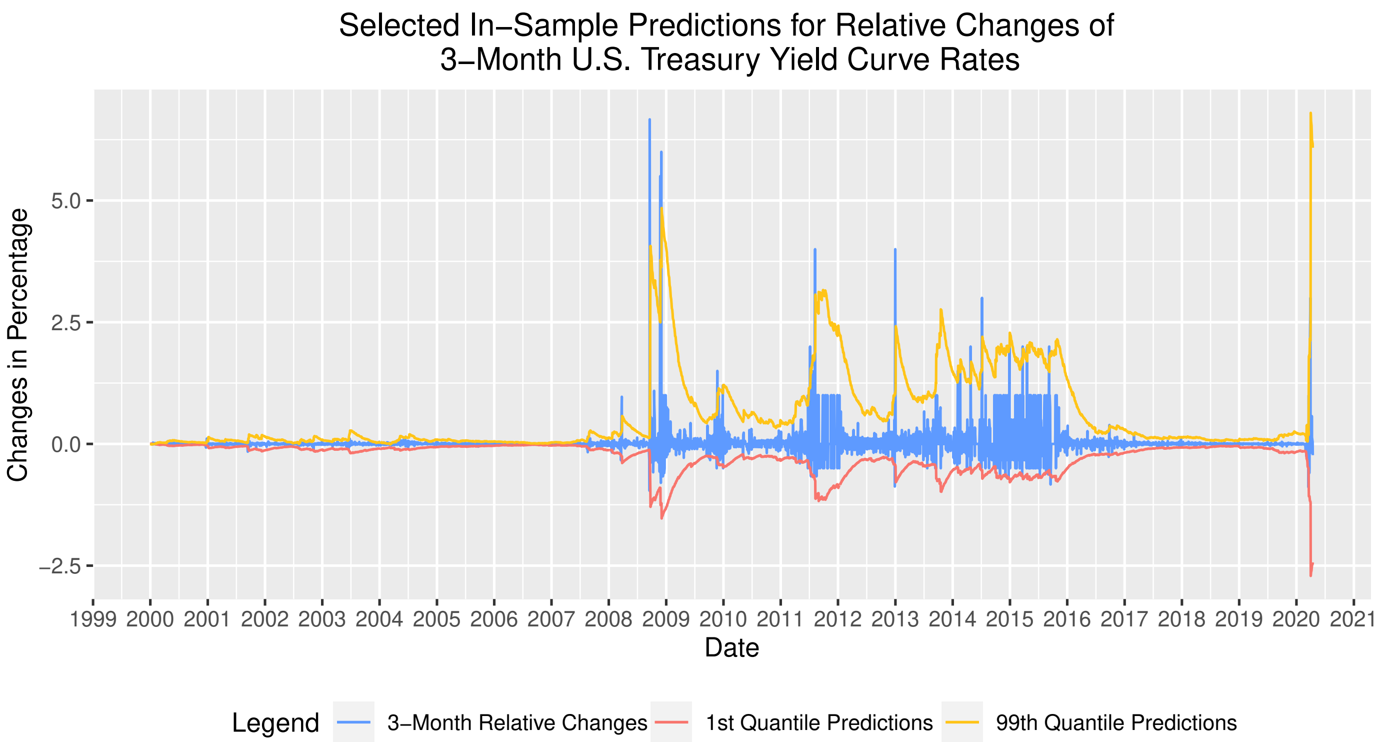

FIGURE A.1: In-sample Predictions of 3-Month Relative Changes of U.S. Daily Treasury Yield Curve Rates at 1st and 99th Quantile from 2000-2020

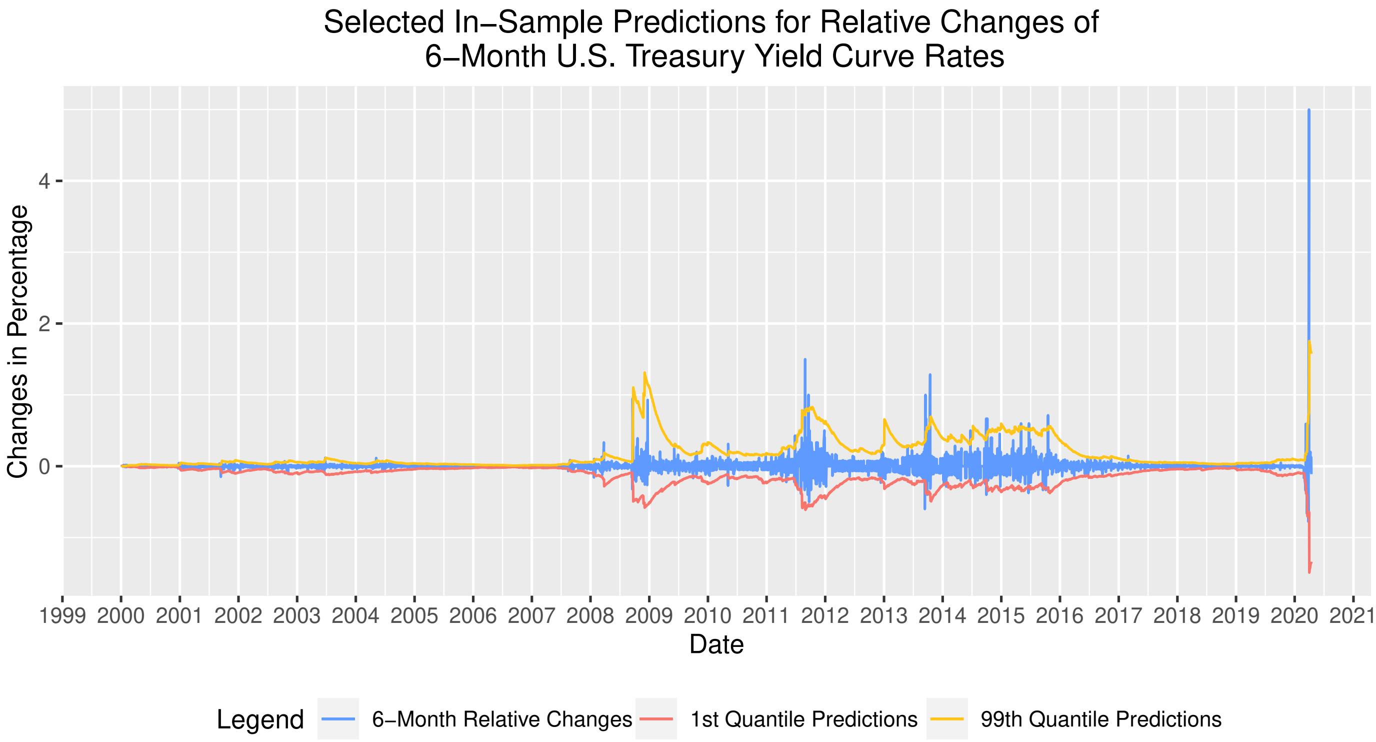

FIGURE A.2: In-sample Predictions of 6-Month Relative Changes of U.S. Daily Treasury Yield Curve Rates at 1st and 99th Quantile from 2000-2020

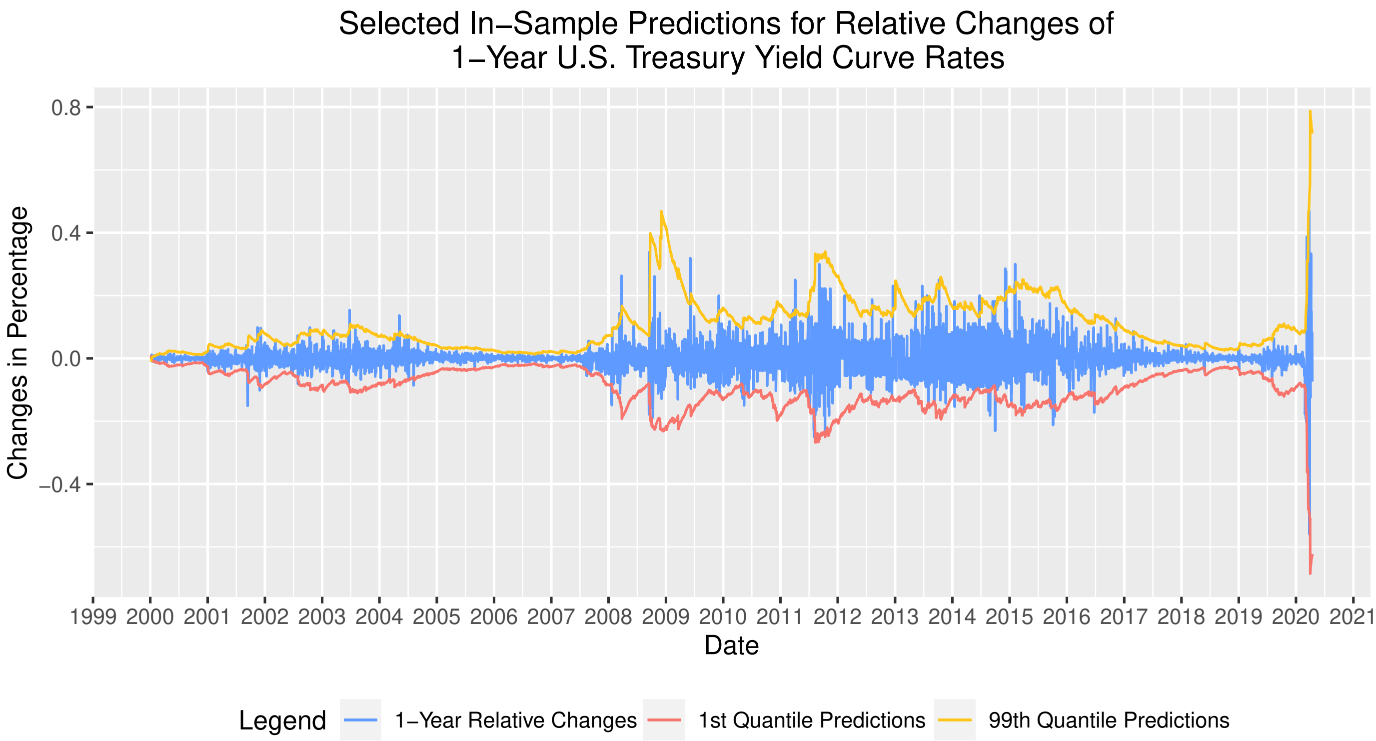

FIGURE A.3: In-sample Predictions of 1-Year Relative Changes of U.S. Daily Treasury Yield Curve Rates at 1st and 99th Quantile from 2000-2020

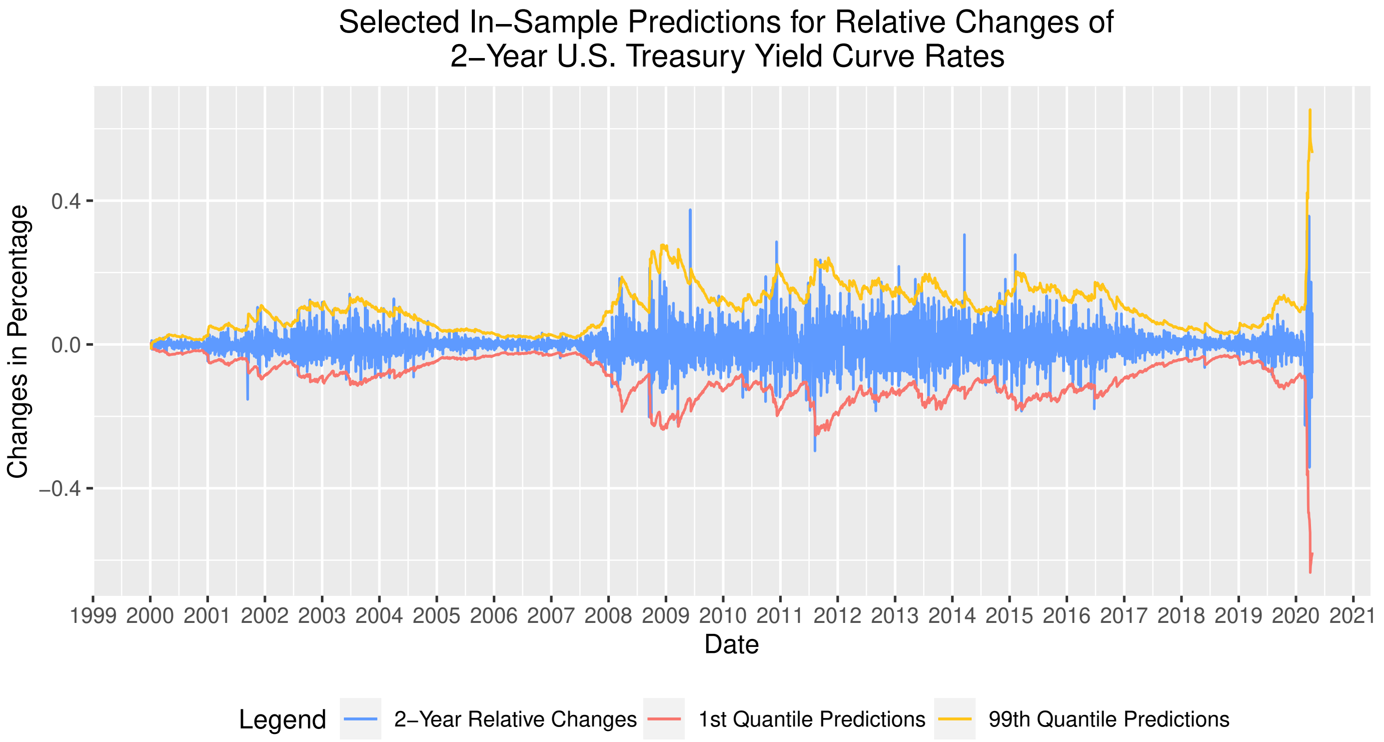

FIGURE A.4: In-sample Predictions of 2-Year Relative Changes of U.S. Daily Treasury Yield Curve Rates at 1st and 99th Quantile from 2000-2020

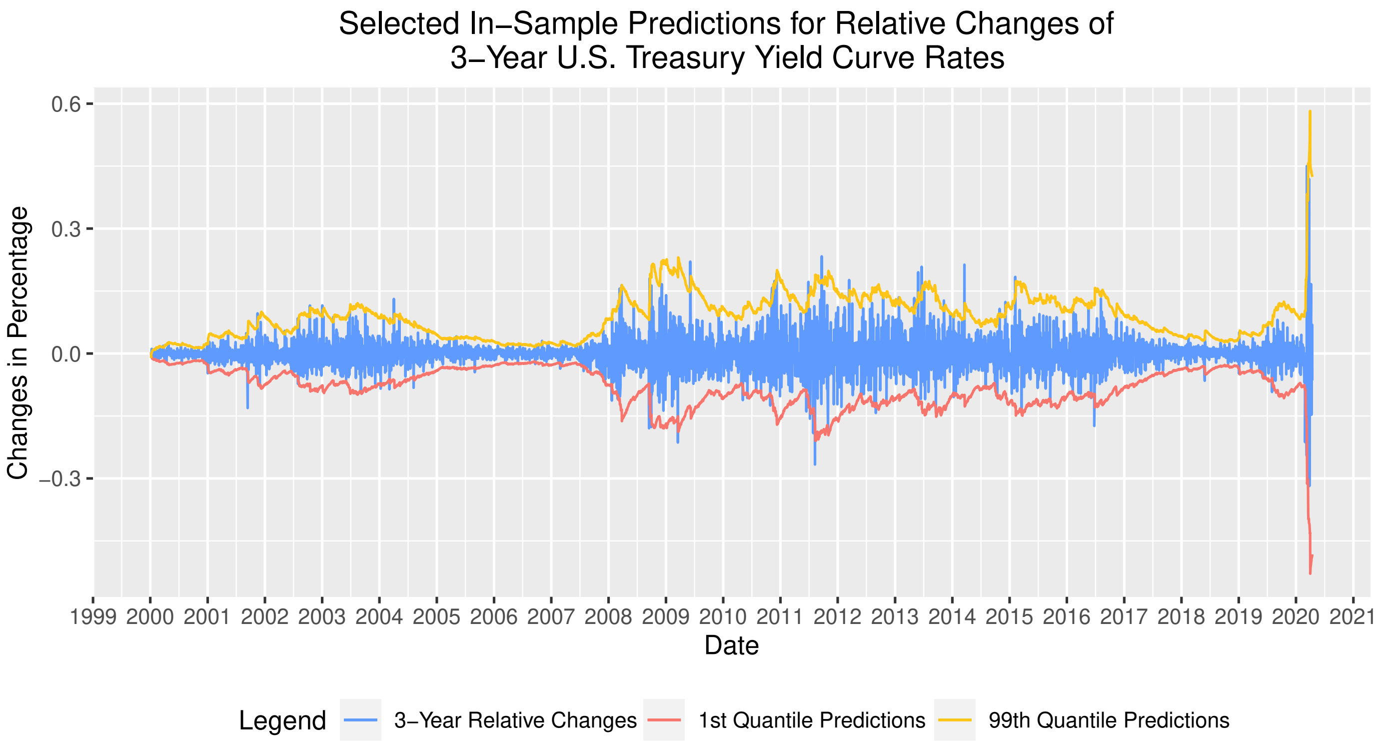

FIGURE A.5: In-sample Predictions of 3-Year Relative Changes of U.S. Daily Treasury Yield Curve Rates at 1st and 99th Quantile from 2000-2020

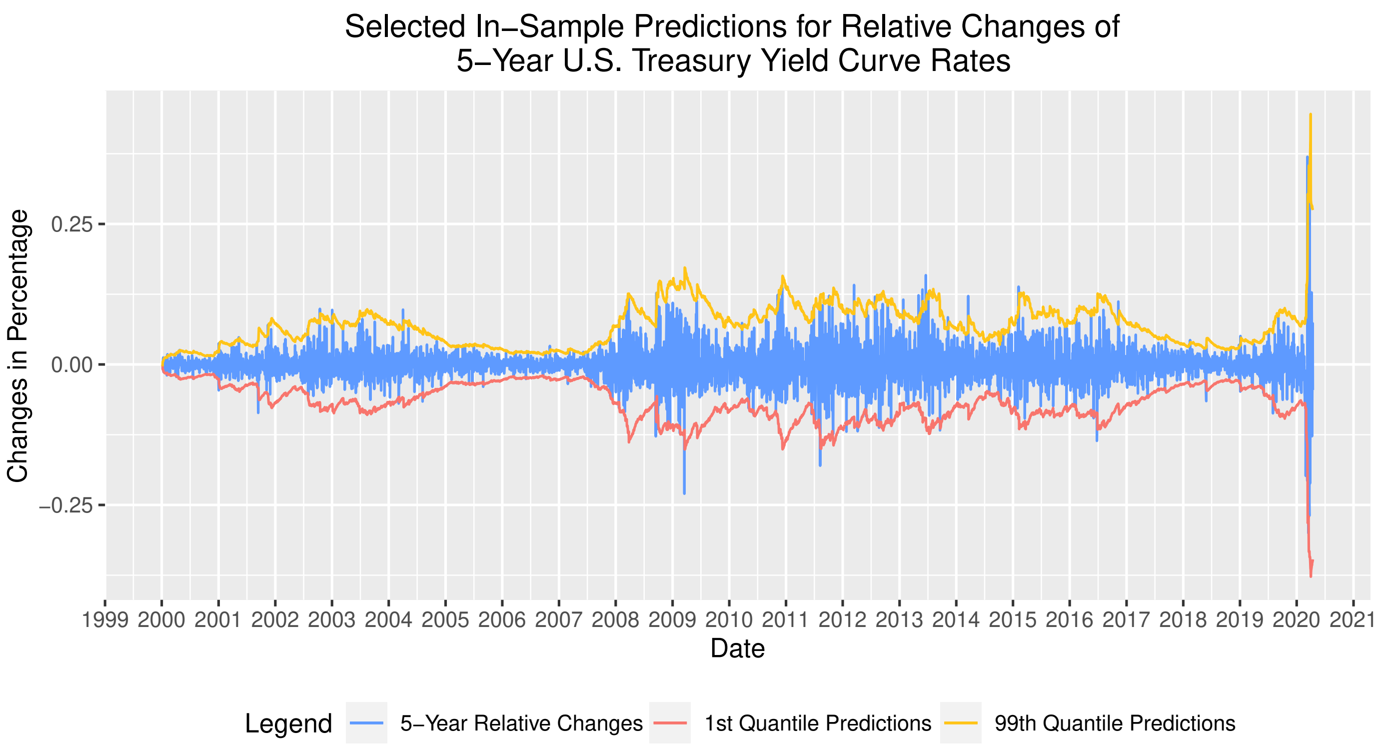

FIGURE A.6: In-sample Predictions of 5-Year Relative Changes of U.S. Daily Treasury Yield Curve Rates at 1st and 99th Quantile from 2000-2020

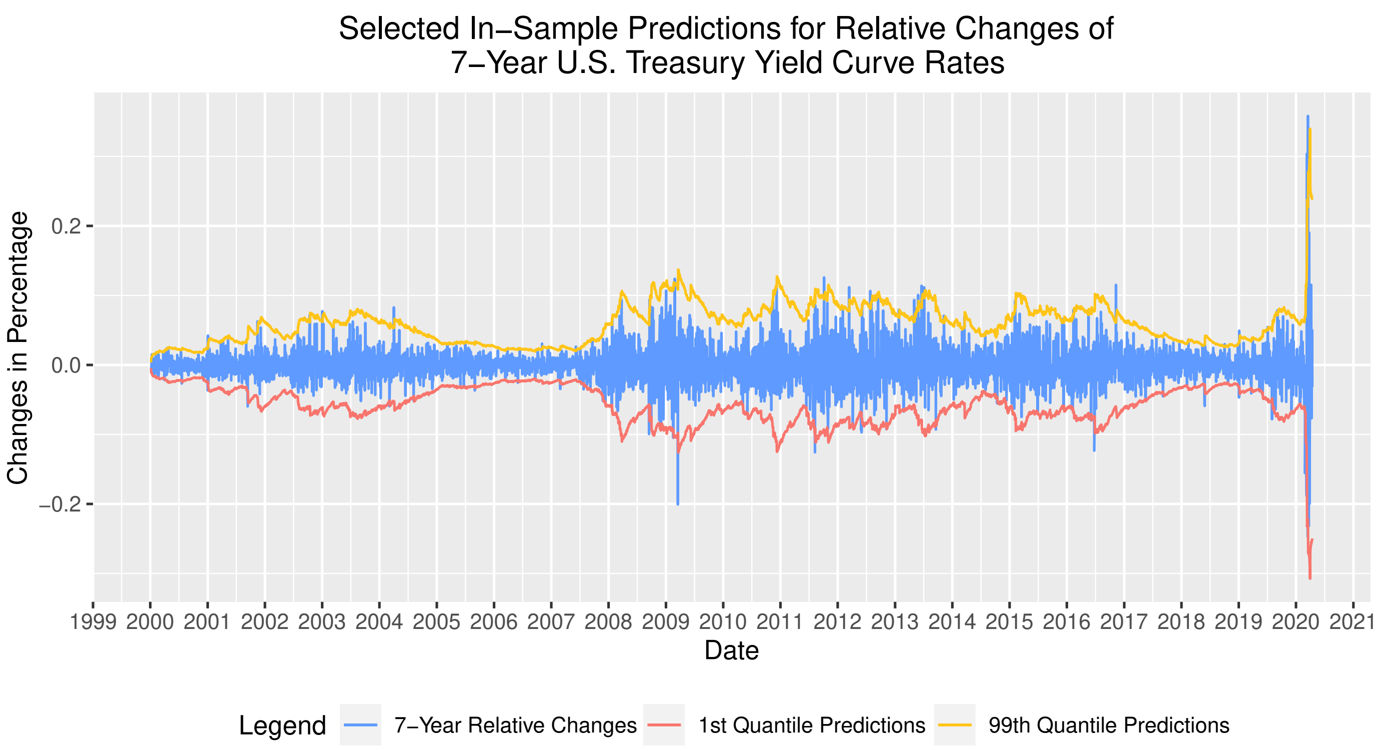

FIGURE A.7: In-sample Predictions of 7-Year Relative Changes of U.S. Daily Treasury Yield Curve Rates at 1st and 99th Quantile from 2000-2020

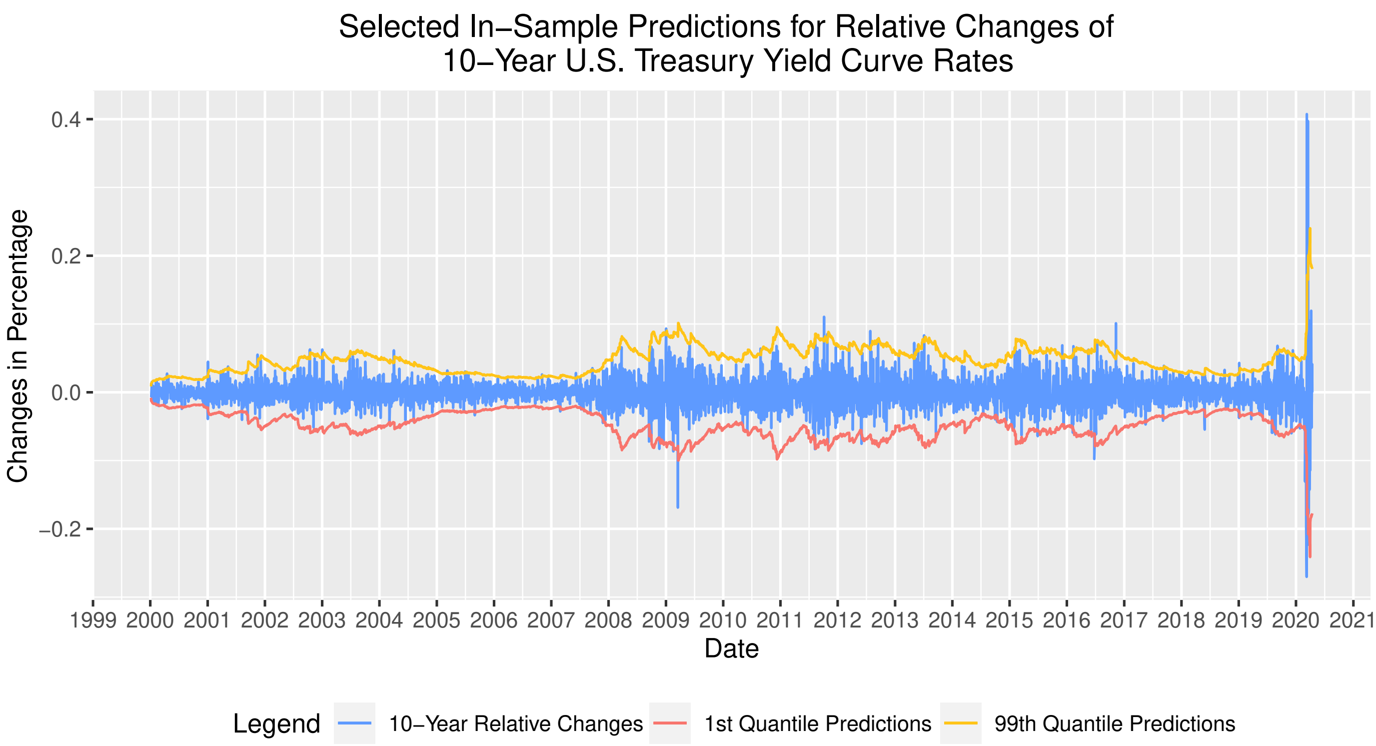

FIGURE A.8: In-sample Predictions of 10-Year Relative Changes of U.S. Daily Treasury Yield Curve Rates at 1st and 99th Quantile from 2000-2020

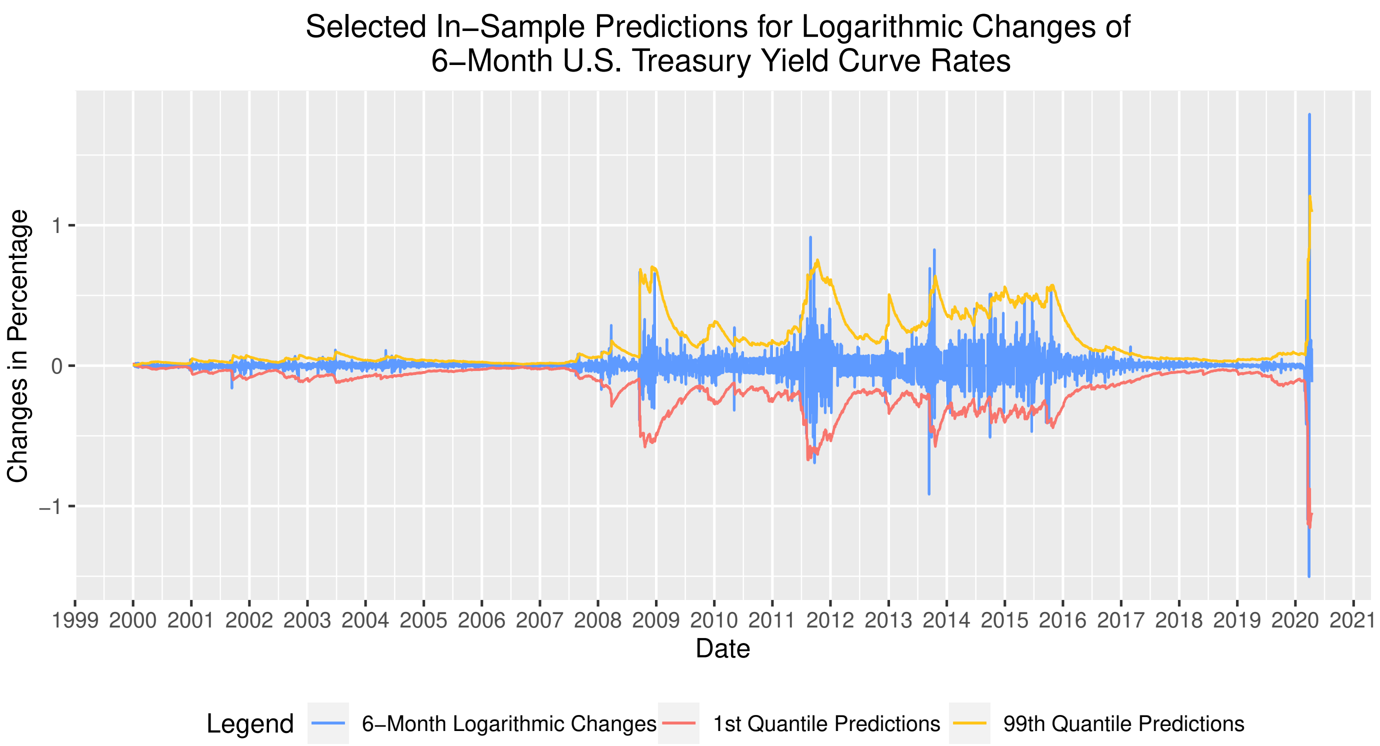

FIGURE A.9: In-sample Predictions of 6-Month Logarithmic Changes of U.S. Daily Treasury Yield Curve Rates at 1st and 99th Quantile from 2000-2020

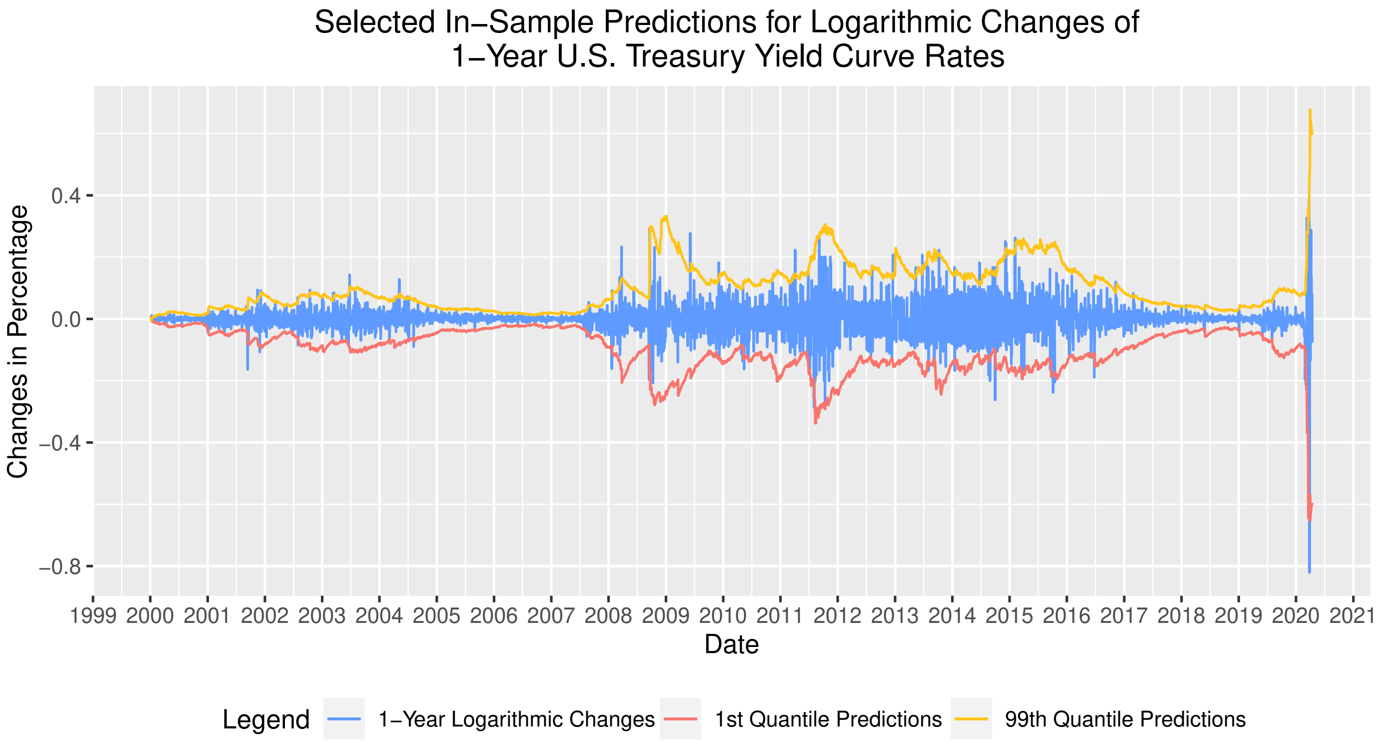

FIGURE A.10: In-sample Predictions of 1-Year Logarithmic Changes of U.S. Daily Treasury Yield Curve Rates at 1st and 99th Quantile from 2000-2020

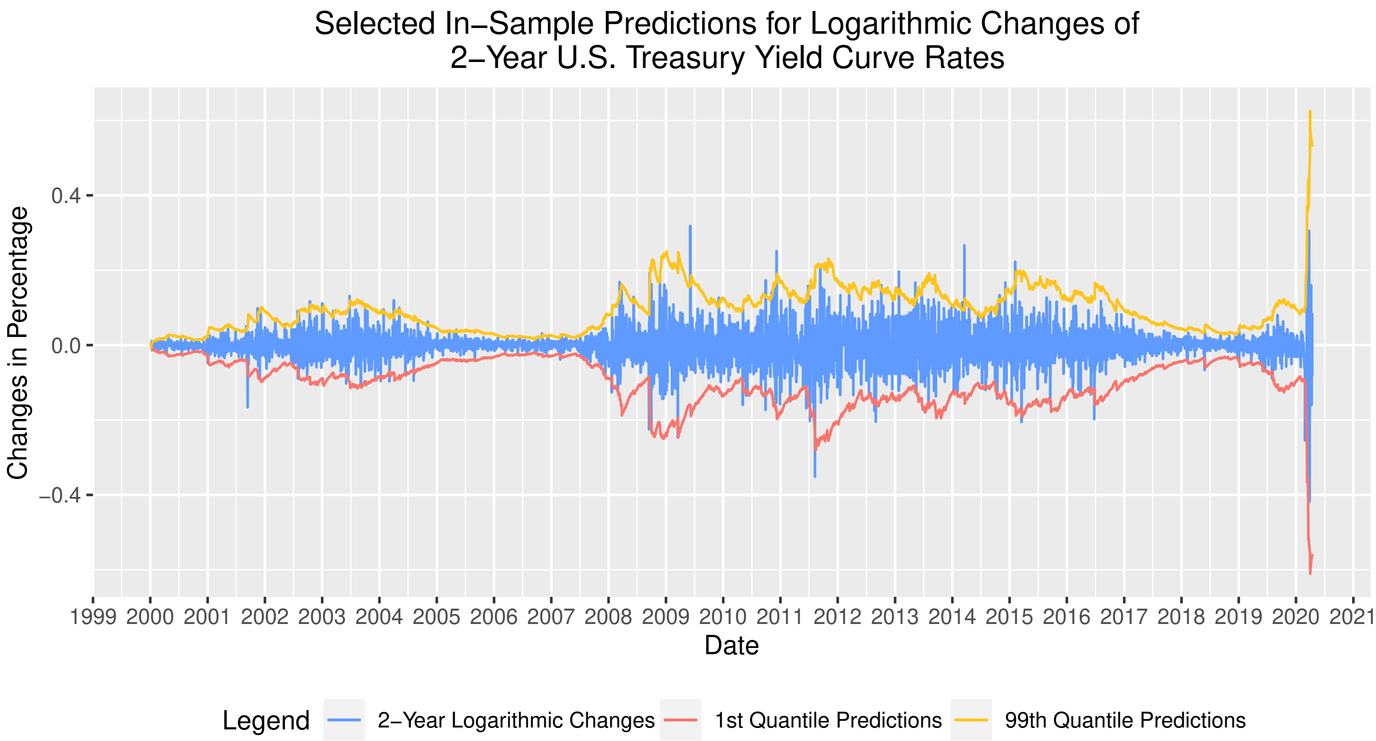

FIGURE A.11: In-sample Predictions of 2-Year Logarithmic Changes of U.S. Daily Treasury Yield Curve Rates at 1st and 99th Quantile from 2000-2020

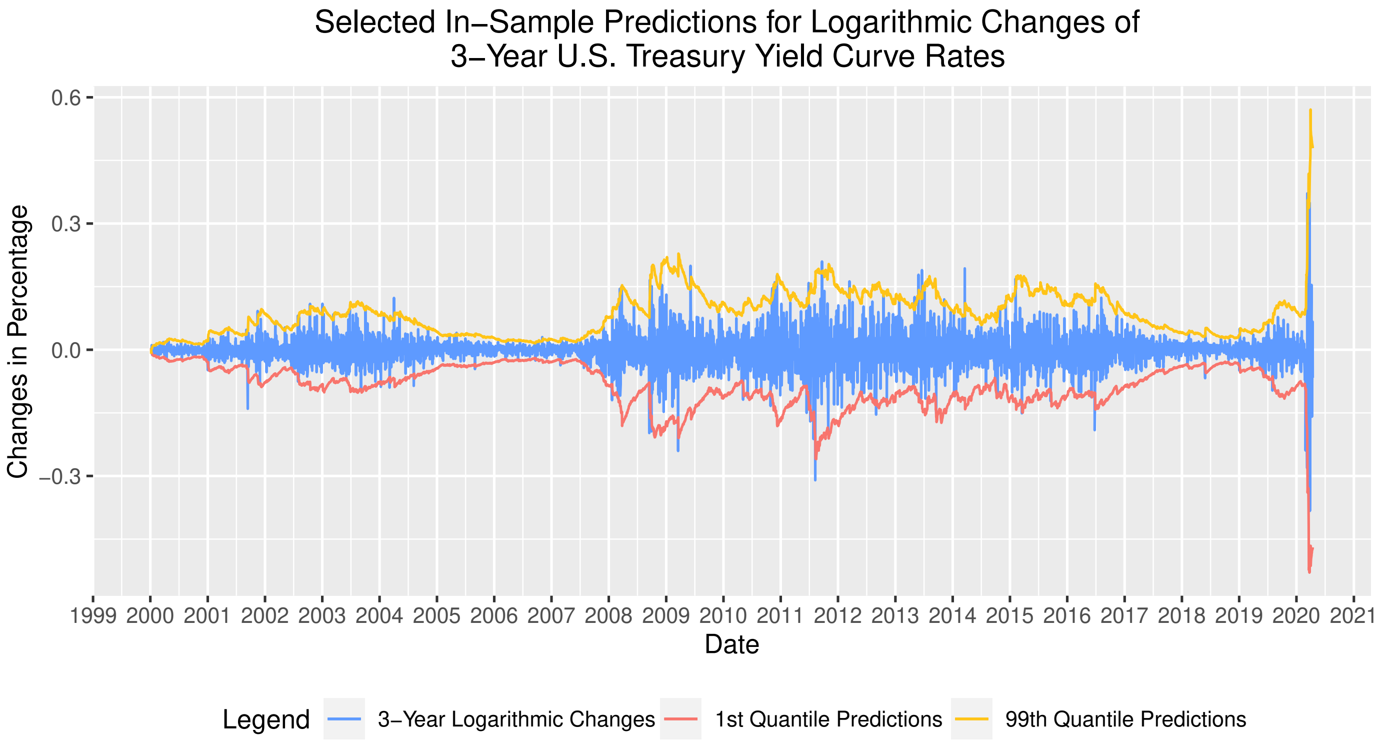

FIGURE A.12: In-sample Predictions of 3-Year Logarithmic Changes of U.S. Daily Treasury Yield Curve Rates at 1st and 99th Quantile from 2000-2020

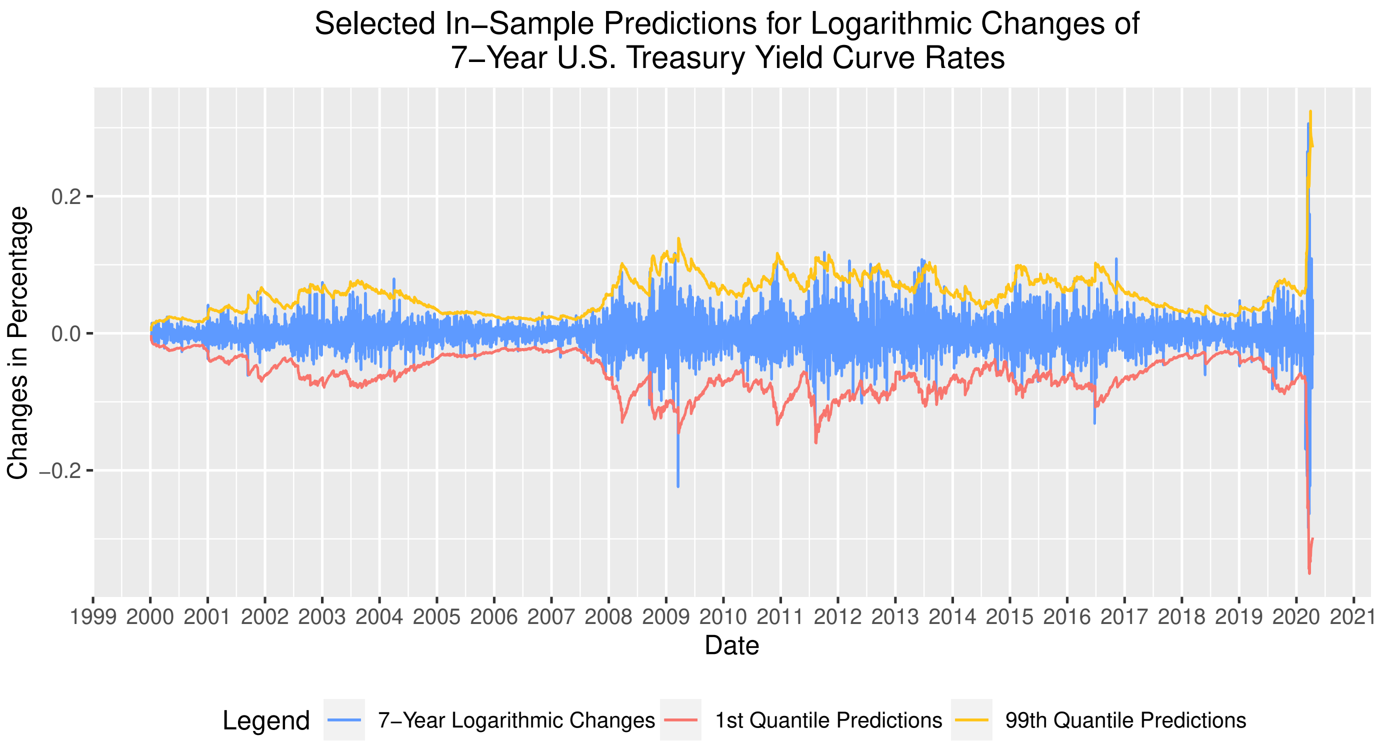

FIGURE A.13: In-sample Predictions of 7-Year Logarithmic Changes of U.S. Daily Treasury Yield Curve Rates at 1st and 99th Quantile from 2000-2020

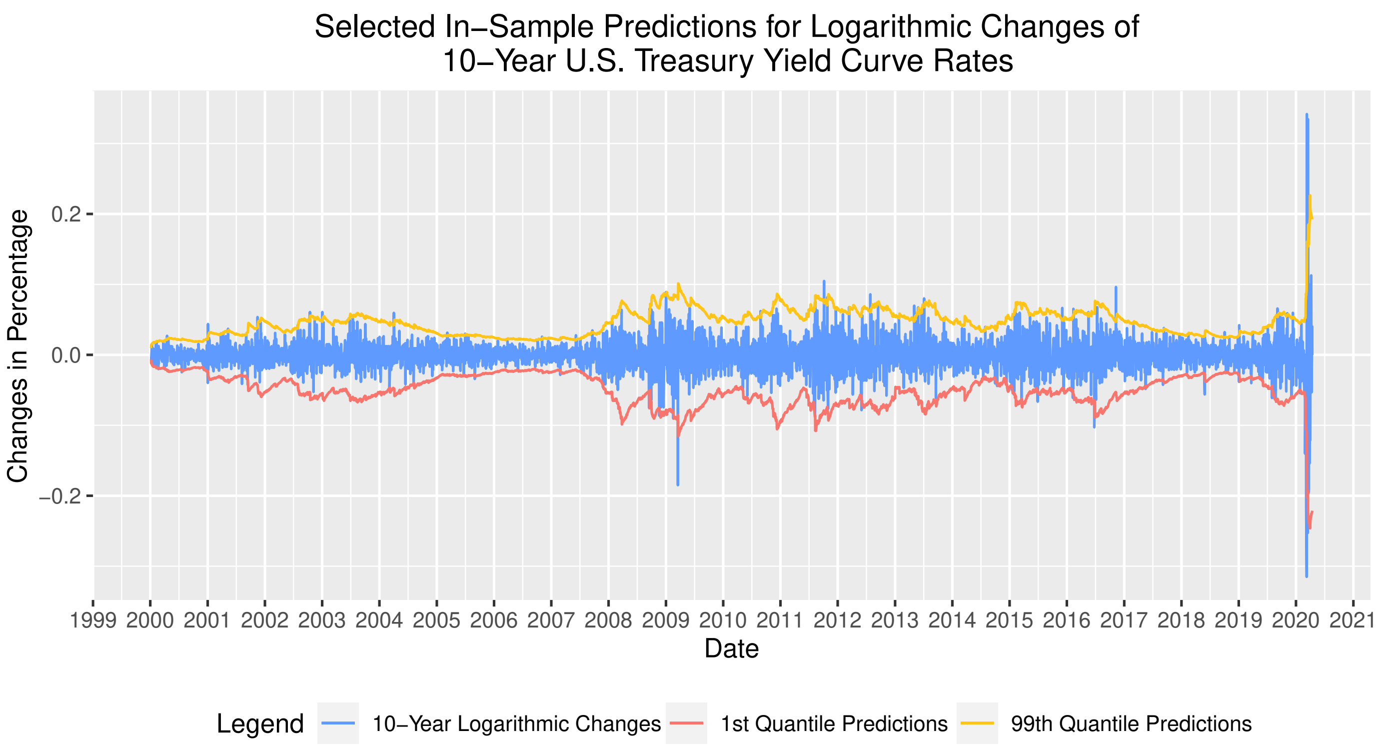

FIGURE A.14: In-sample Predictions of 10-Year Logarithmic Changes of U.S. Daily Treasury Yield Curve Rates at 1st and 99th Quantile from 2000-2020

Appendix B

Additional Plots - Out-of-sample Predictions

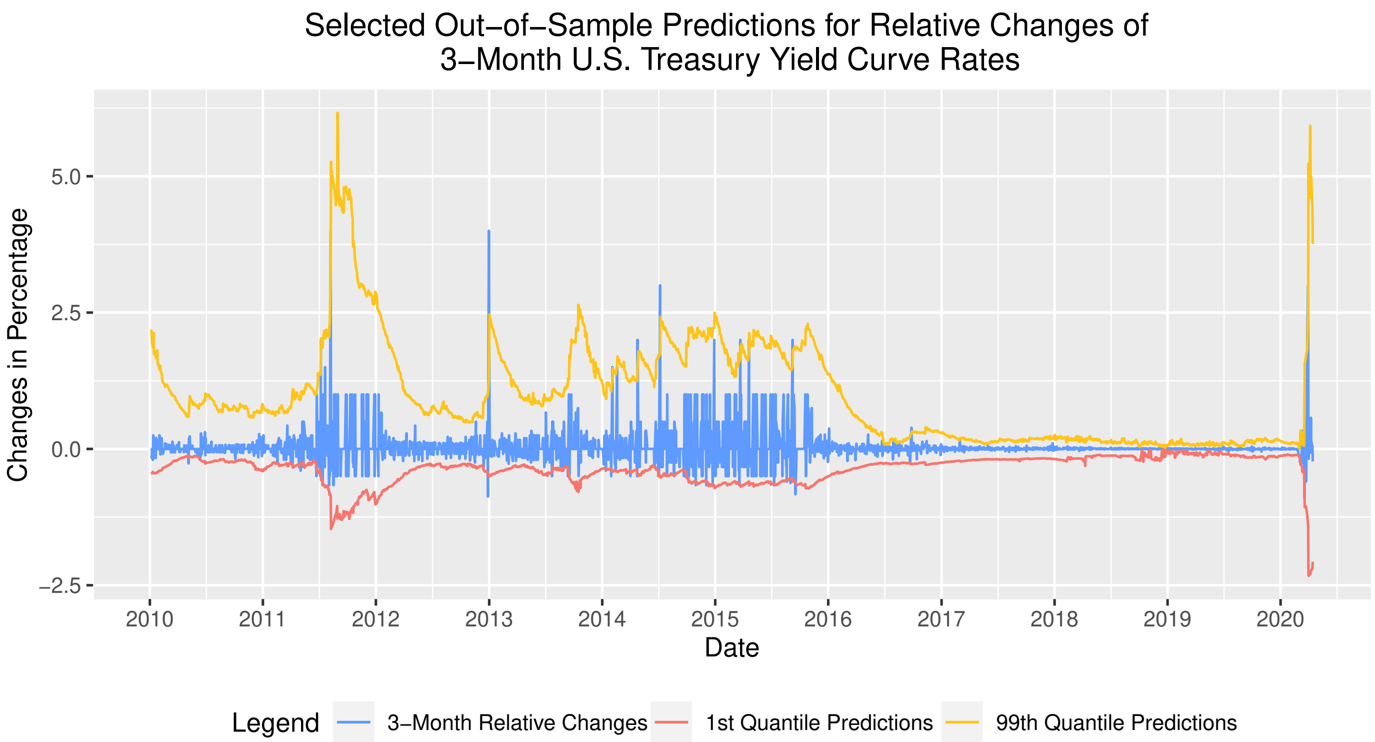

FIGURE B.1: Out-of-Sample Predictions of 3-Month Relative Changes of U.S. Daily Treasury Yield Curve Rates at 1st and 99th Quantile from 2000-2020

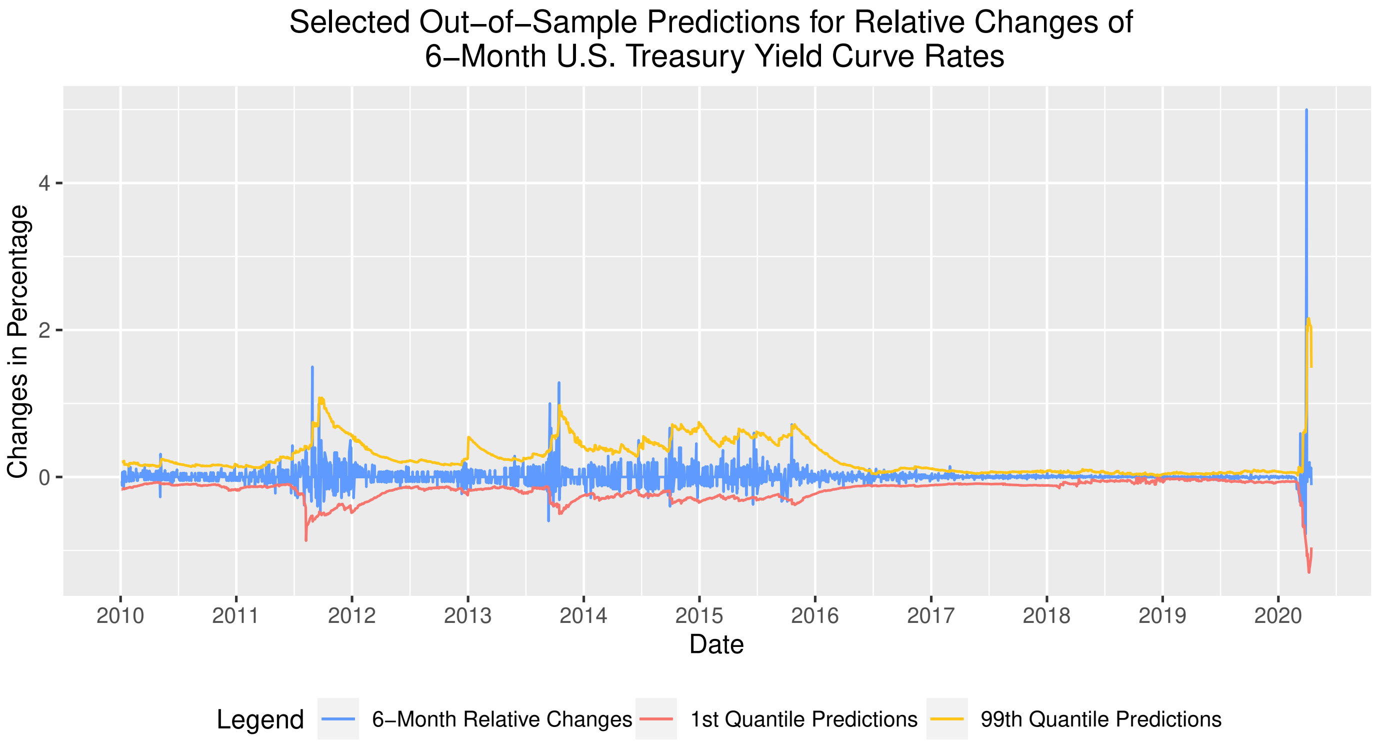

FIGURE B.2: Out-of-Sample Predictions of 6-Month Relative Changes of U.S. Daily Treasury Yield Curve Rates at 1st and 99th Quantile from 2000-2020

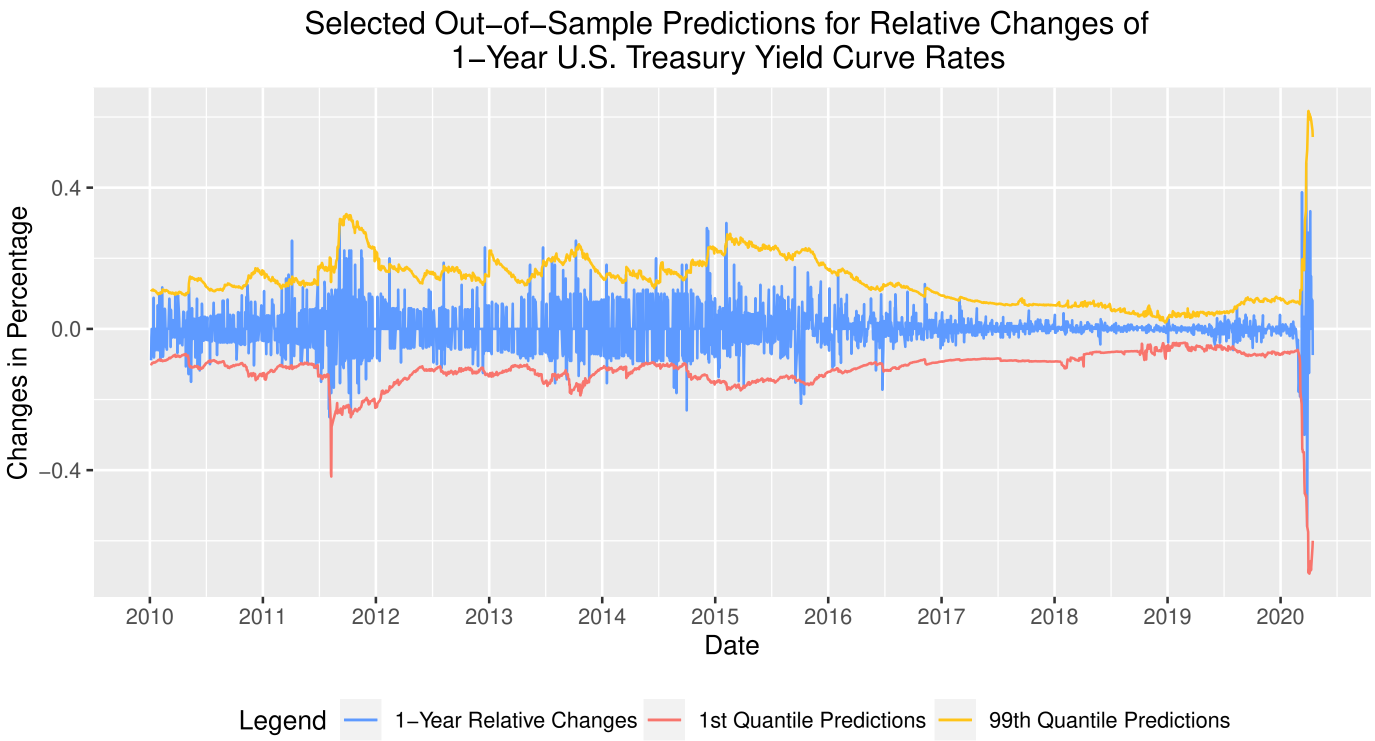

FIGURE B.3: Out-of-Sample Predictions of 1-Year Relative Changes of U.S. Daily Treasury Yield Curve Rates at 1st and 99th Quantile from 2000-2020

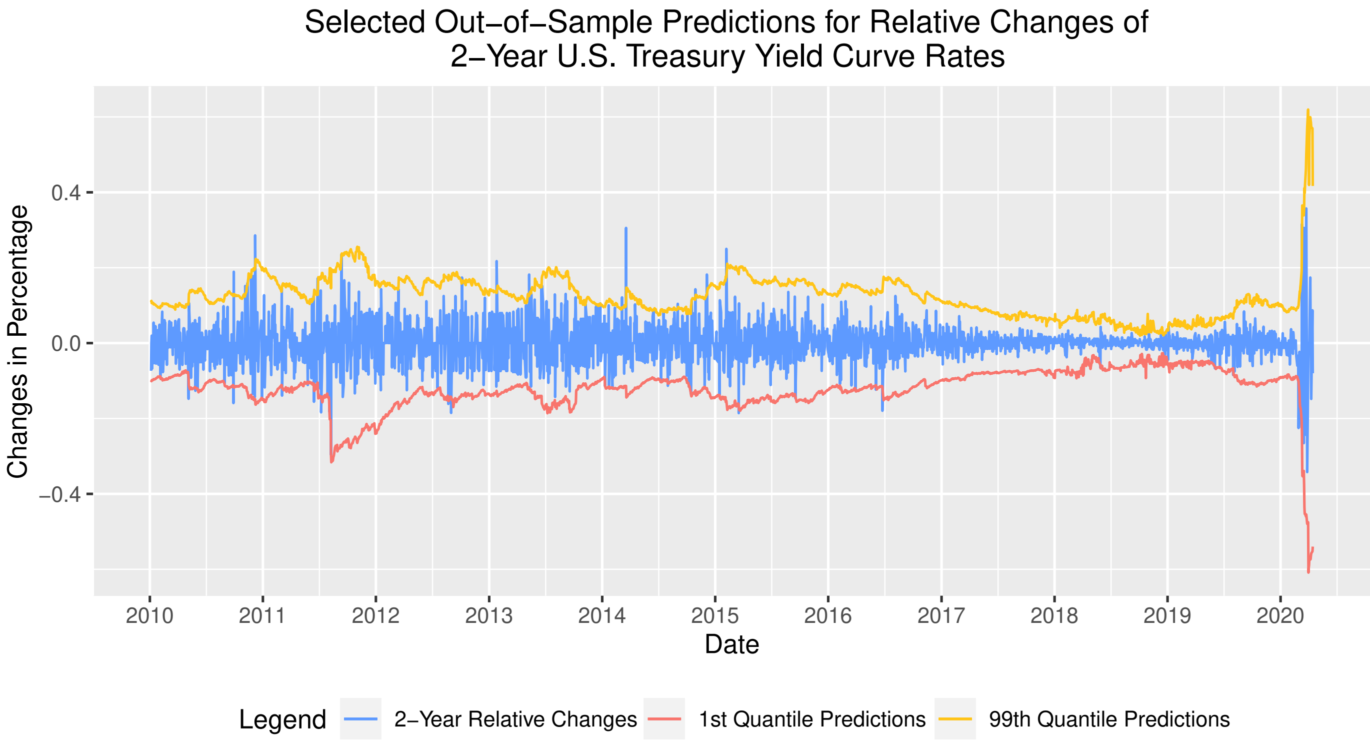

FIGURE B.4: Out-of-Sample Predictions of 2-Year Relative Changes of U.S. Daily Treasury Yield Curve Rates at 1st and 99th Quantile from 2000-2020

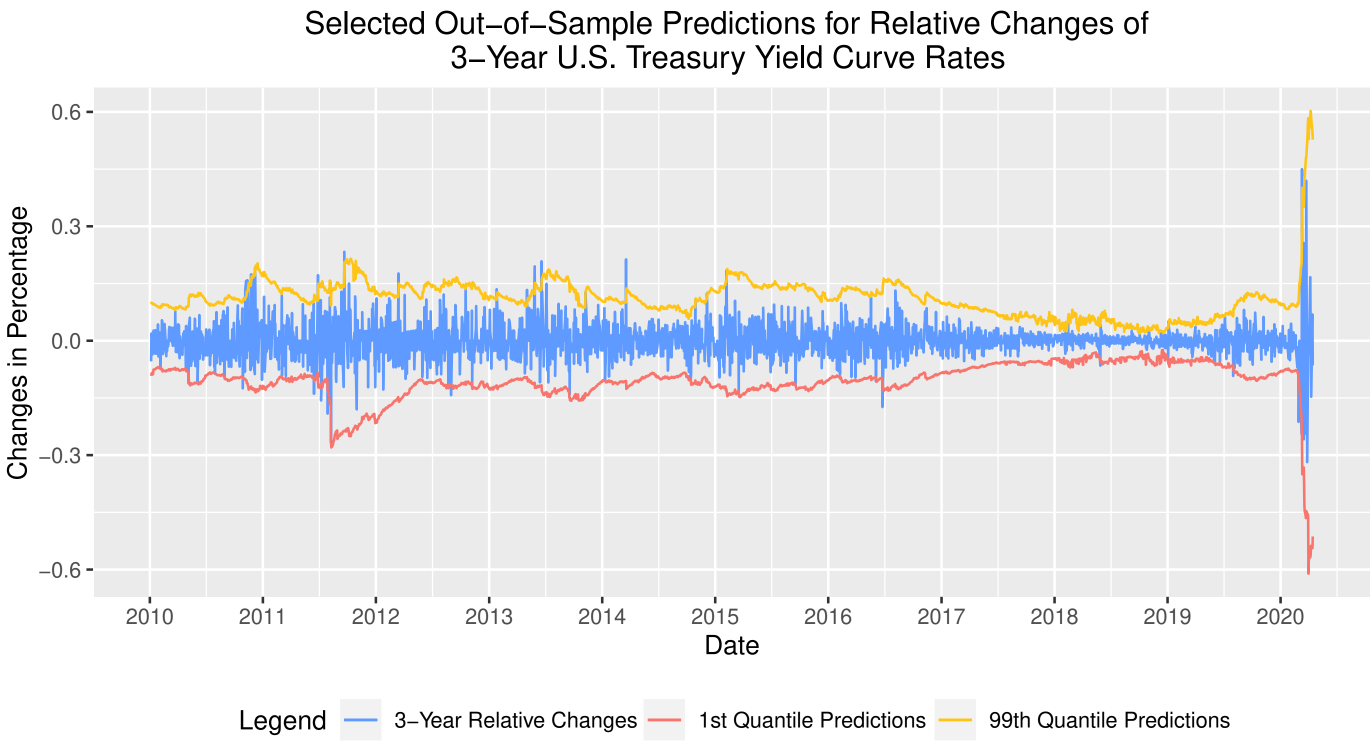

FIGURE B.5: Out-of-Sample Predictions of 3-Year Relative Changes of U.S. Daily Treasury Yield Curve Rates at 1st and 99th Quantile from 2000-2020

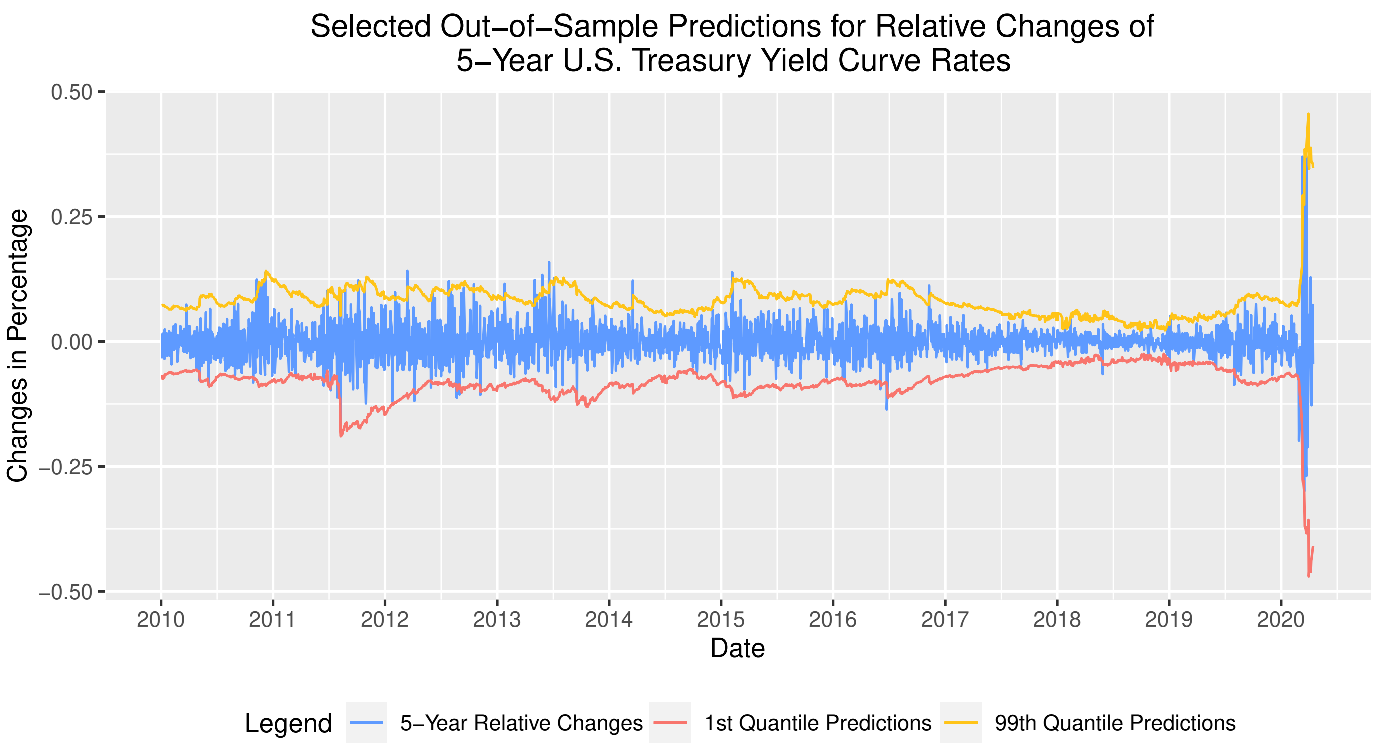

FIGURE B.6: Out-of-Sample Predictions of 5-Year Relative Changes of U.S. Daily Treasury Yield Curve Rates at 1st and 99th Quantile from 2000-2020

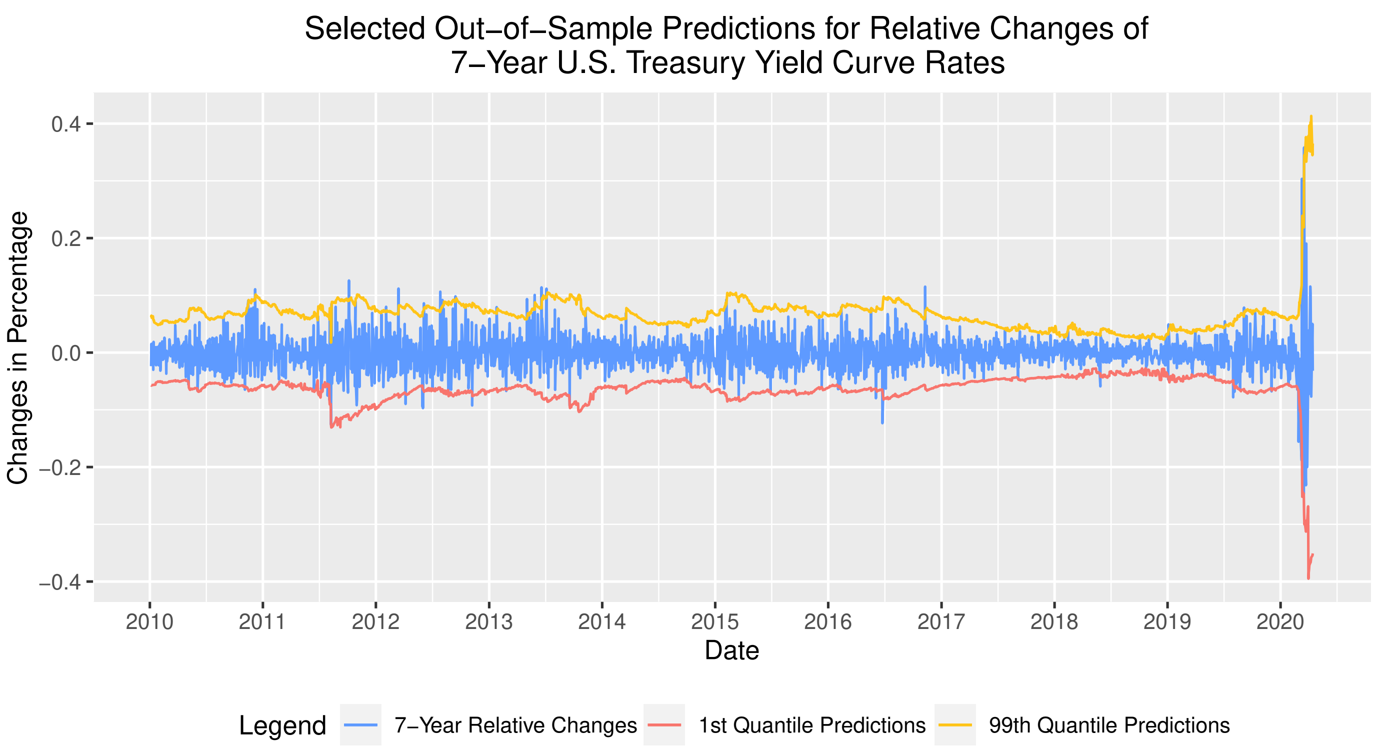

FIGURE B.7: Out-of-Sample Predictions of 7-Year Relative Changes of U.S. Daily Treasury Yield Curve Rates at 1st and 99th Quantile from 2000-2020

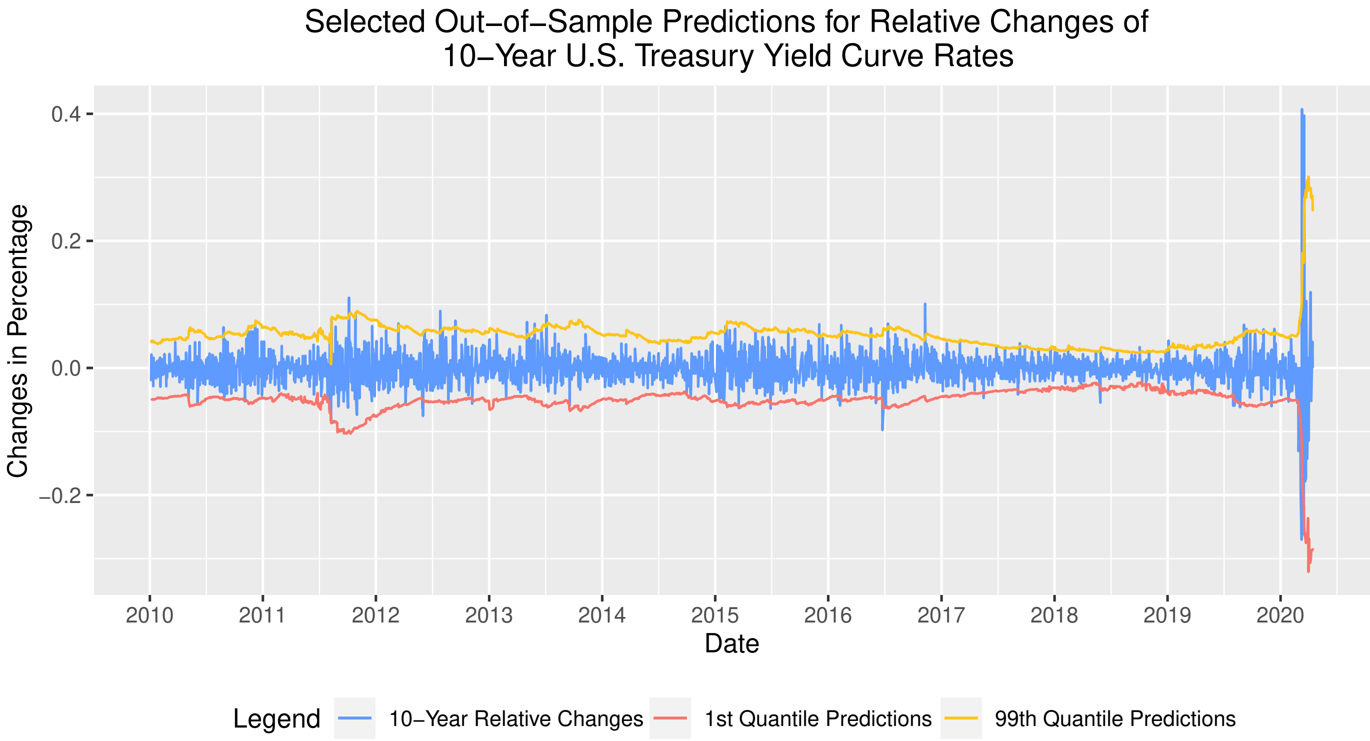

FIGURE B.8: Out-of-Sample Predictions of 10-Year Relative Changes of U.S. Daily Treasury Yield Curve Rates at 1st and 99th Quantile from 2000-2020

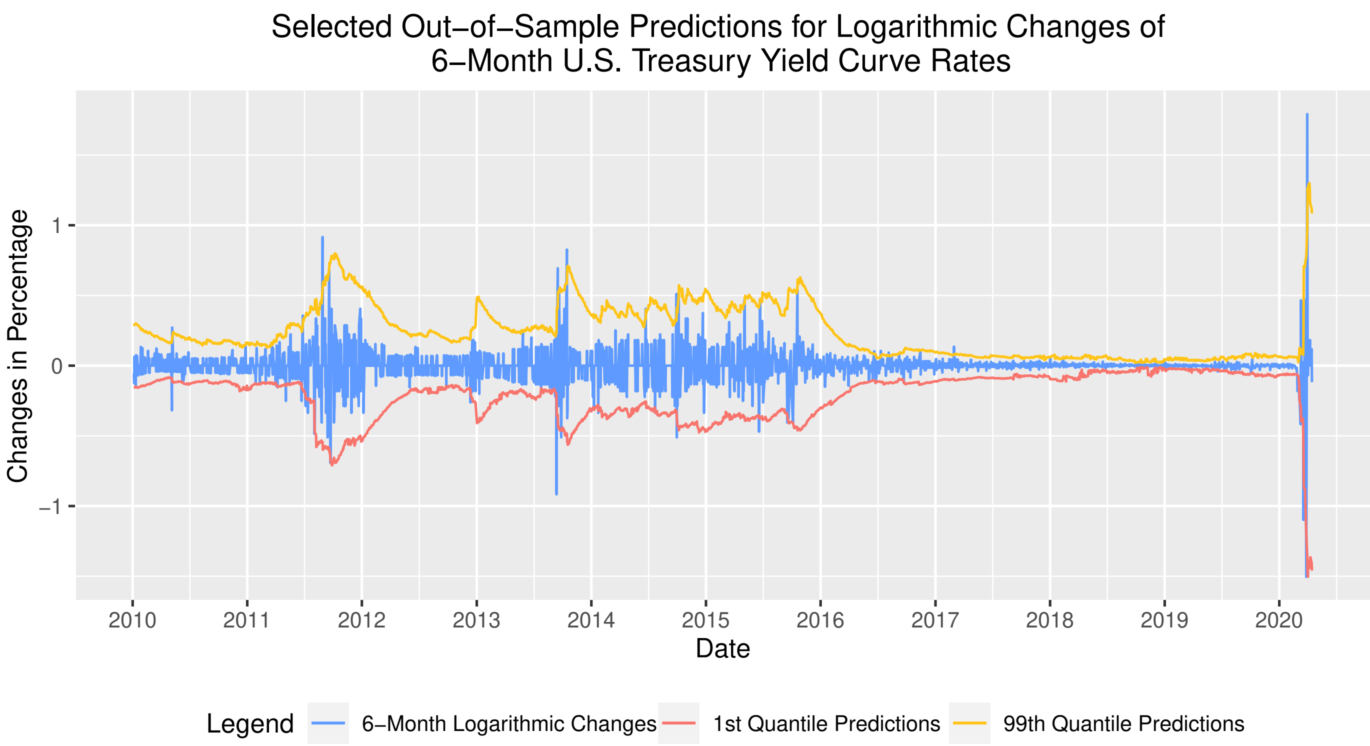

FIGURE B.9: Out-of-Sample Predictions of 6-Month Logarithmic Changes of U.S. Daily Treasury Yield Curve Rates at 1st and 99th Quantile from 2000-2020

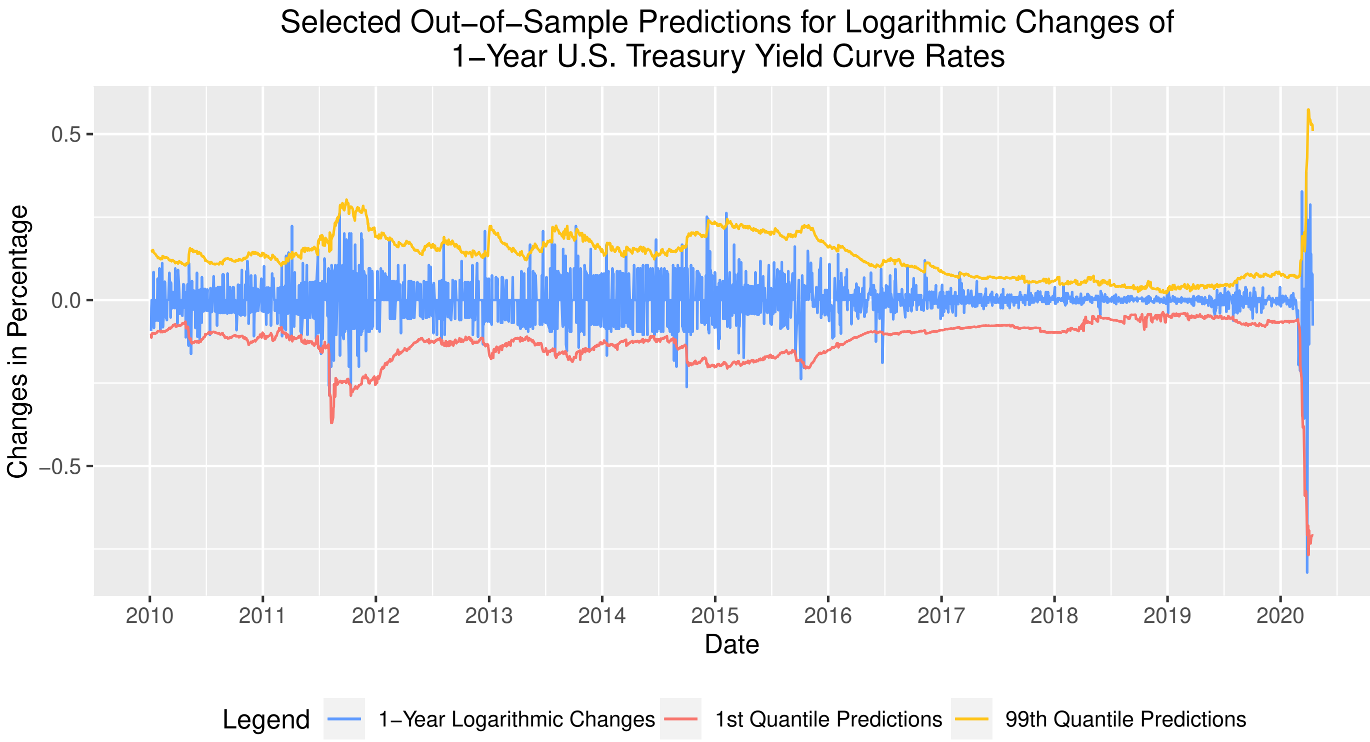

FIGURE B.10: Out-of-Sample Predictions of 1-Year Logarithmic Changes of U.S. Daily Treasury Yield Curve Rates at 1st and 99th Quantile from 2000-2020

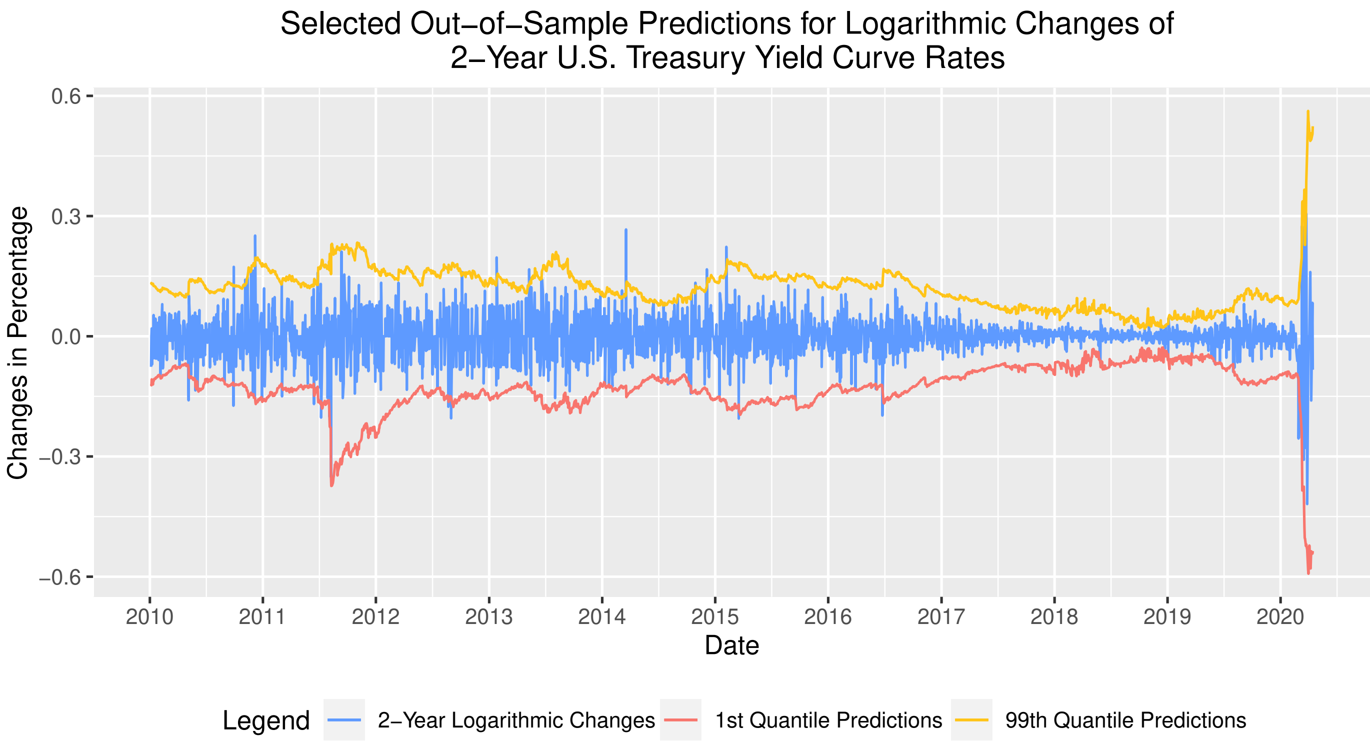

FIGURE B.11: Out-of-Sample Predictions of 2-Year Logarithmic Changes of U.S. Daily Treasury Yield Curve Rates at 1st and 99th Quantile from 2000-2020

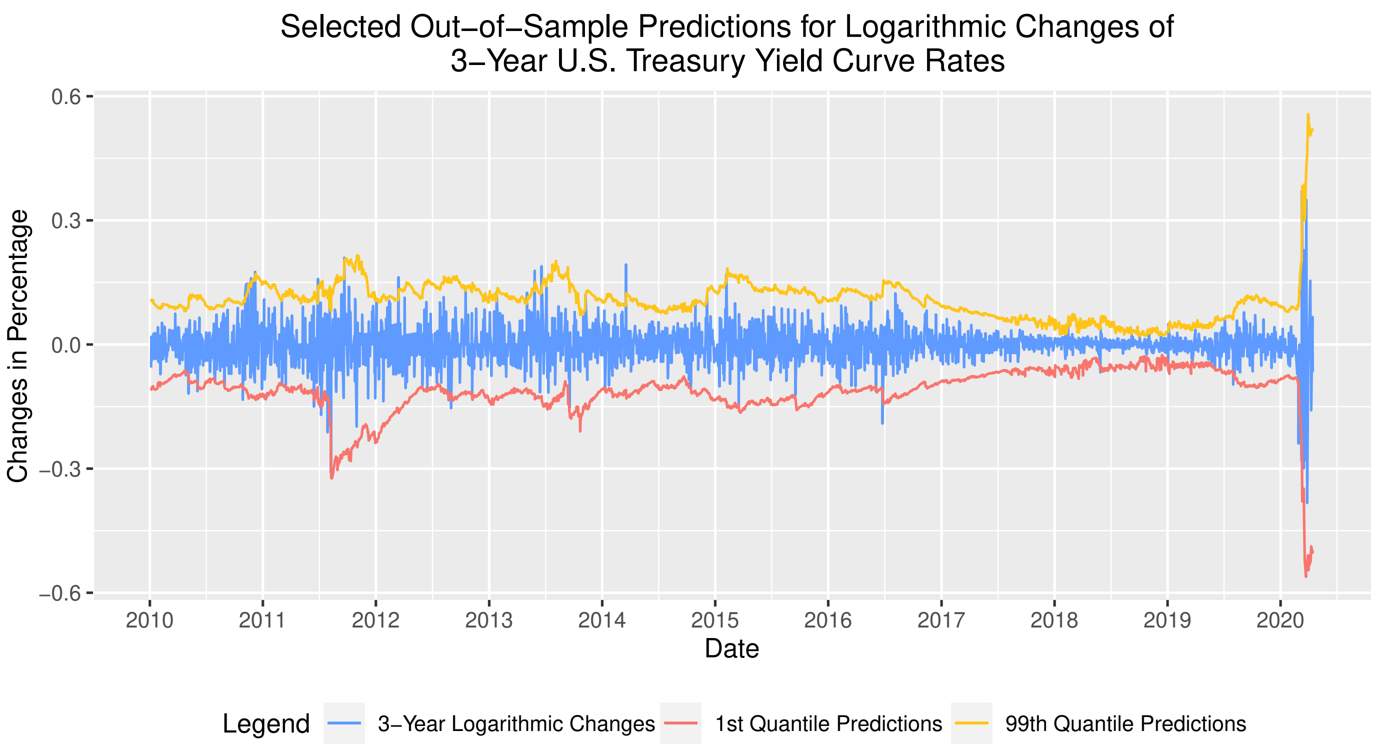

FIGURE B.12: Out-of-Sample Predictions of 3-Year Logarithmic Changes of U.S. Daily Treasury Yield Curve Rates at 1st and 99th Quantile from 2000-2020

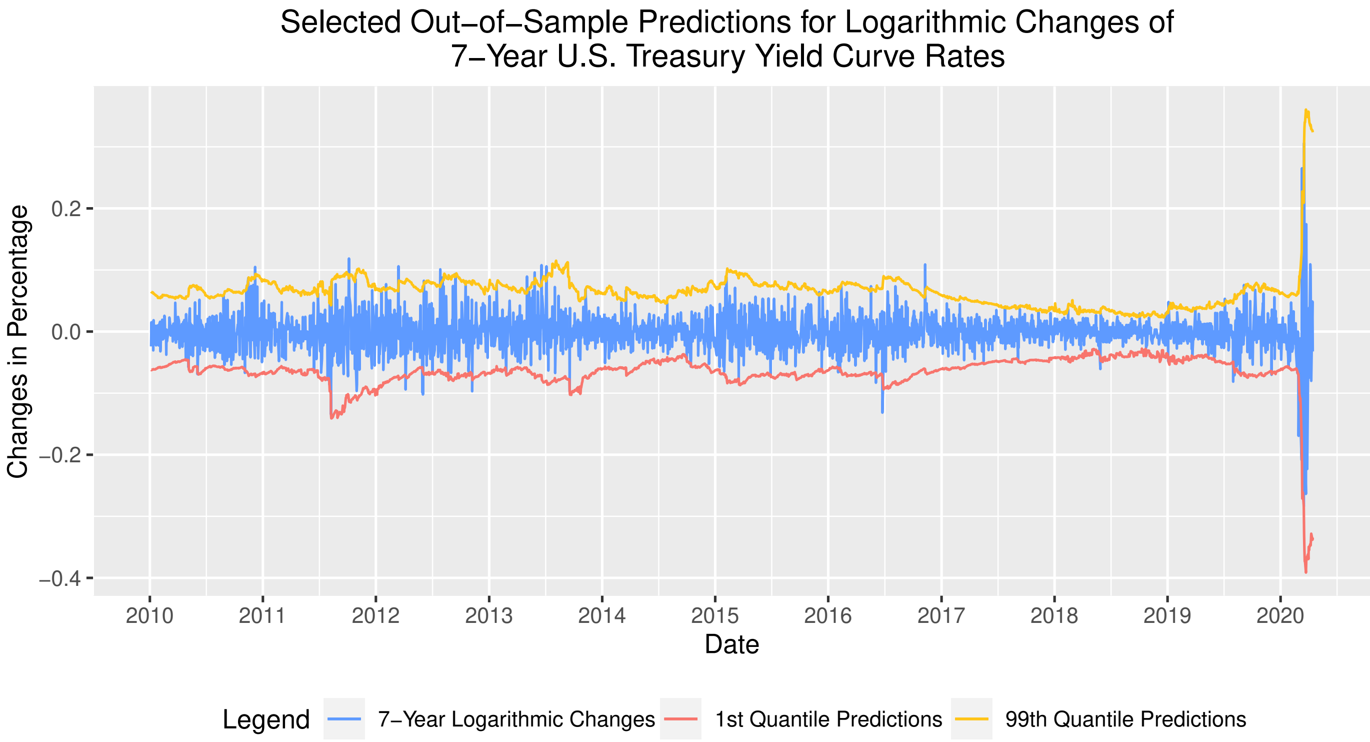

FIGURE B.13: Out-of-Sample Predictions of 7-Year Logarithmic Changes of U.S. Daily Treasury Yield Curve Rates at 1st and 99th Quantile from 2000-2020

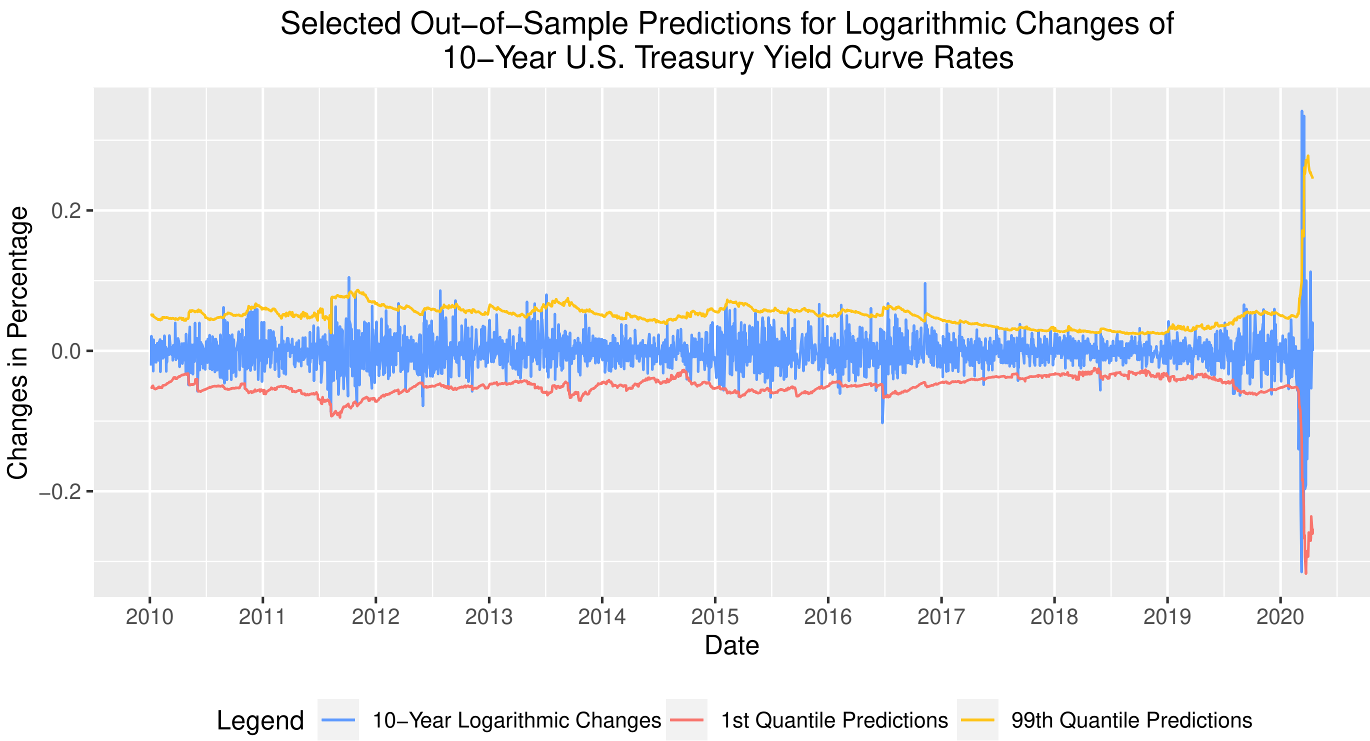

FIGURE B.14: Out-of-Sample Predictions of 10-Year Logarithmic Changes of U.S. Daily Treasury Yield Curve Rates at 1st and 99th Quantile from 2000-2020