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TITLE:

Parametric Study of Bridge Concepts and Cable Net
Conceptual Design of a Footbridge Crossing Tønberg Canal

Parametrisk utforming av brokonsept med kabelnett
Utforming av ny gangbro over kanalen mellom Tønsberg og Kaldnes

BY:

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SUMMARY:
This thesis project comprises a preliminary footbridge design concept, intended for the quay areas of Tønsberg and Kaldnes, in Vestfold and Telemark County (Norway). With an engineering approach, the project investigates the usefulness of a cable-net bridge using tension membrane technology and includes simplified design analysis and self-frequency controls. The aim of the project is to create a bridge concept that has a stable, practical, and efficient construction, as well as a bridge that blends in with the cityscape and offers inhabitants a positive tectonic experience. The thesis reports on the contextual background and relevant bridge concepts. However, the primary focus is the structural analysis of the cable-net bridge concept. The analysis discusses the superstructure; thus substructure and bascule elements are not covered in depth. The technology of light-weight roof structures started in the early 1950s – after decades of monumental architecture – and became well known during the 1970s. Yet, light-weight bridges, particularly without pylons, is less common. Cable-nets are prestressed steel structures with non-linear behaviour. Although, it is understood due to the aeroelastic nature of the deck, it is important to investigate the aerodynamic effects of the aeroelastic structure as well, but in this thesis the study has been limited to Human Induced static and cyclic loading only.

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CARRIED OUT AT: Department of Structural Engineering, NTNU, Gløshaugen.

Abstract

This thesis project comprises a preliminary footbridge design concept, intended for the quay areas of Tønsberg and Kaldnes, in Vestfold and Telemark County (Norway). With an engineering approach, the project investigates the usefulness of a cable-net bridge using tension membrane technology and includes simplified design analysis and self-frequency controls. The aim of the project is to create a bridge concept that has a stable, practical, and efficient construction, as well as a bridge that blends in with the cityscape and offers inhabitants a positive tectonic experience. The thesis reports on the contextual background and relevant bridge concepts. However, the primary focus is the structural analysis of the cable-net bridge concept. The analysis discusses the superstructure; thus substructure and bascule elements are not covered in depth.

The background for the project is a recommendation in an intercity-transportation scheme (2019) for a new footbridge with tilting function between the quay areas, Kaldnes west and Tollboden in Tønsberg. The channel is about 250 meters wide and 7 meters deep, with soft ground conditions. The bridge concept involves a 200-meter-long and 12-meter-wide cable-net bridge, which carries a 6-meter-wide timber grid shell for pedestrian traffic. This is a quite novel concept as tension bridges usually use tall towers to hang the structure. This report focuses mainly on the construction of the cable-net and and a wooden grid. Other key areas such as tilt function and foundation are not addressed in the report but are included where relevant to the other components.

A cable-net is a tension membrane structure with non-linear behaviour. Although, it is understood due to the aeroelastic nature of the deck, it is important to investigate the aerodynamic effects of the aeroelastic structure as well, but in this thesis the study has been limited to Human Induced static and cyclic loading only. The parameter-controlled software Grasshopper is used in the design and modelling work, and Karamba is used as a built-in FEM solver. The bridge weight is 1.64 tonnes/m; and is about 20% lighter than the London Millennium Footbridge as reference point. This cable net also achieves high lateral eigenfrequencies through transverse cables. Such values (2,5 Hz) reduce potential issues concerning lateral vibrations – which was the Achille's Heel of the Millennium Bridge.

The report concludes that this bridge concept has a lot of potential, but it is deemed unsuitable for the canal area in Tønsberg. The bridge deck itself is of academic interest and should be explored further. At the same time, the city's need for a bridge with tilt function, combined with the weak anchoring possibilities in the canal, makes the outlined cable-net bridge impractical and overly complicated. On the other hand, for areas with accessible rock footings, this bridge concept may be ideal.

Sammendrag

Dette masterprosjektet er en forstudie for konstruksjon av ny gangbro over kanalen mellom Tønsberg brygge og Kaldnes, i Vestfold og Telemark fylke. Prosjektet har en ingeniørfaglig inngang, og består av en konseptstudie for kabelnett bro, samt forenklede konstruksjonsanalyser og kontroll av egenfrekvenser. Prosjektets mål er å utvikle et brokonsept som har en trygg, praktisk og effektiv konstruksjon, og som passer inn i bybildet og gir innbyggerne en positiv tektonisk opplevelse. Bakgrunnen for prosjektet er innstillingen i Tønsbergs transportplan (2019) om ny gangbro med vippefunksjon mellom kaiområdene, Kaldnes vest og Tollboden. Kanalen er om lag 250 meter bred og 7 meter dyp, med bløte grunnforhold.

Konseptet innebærer et 200 meter langt og 12 meter bredt kabelnett, som bærer et 6 meter bredt brodekke i heltre. Dette er et nytt og lite utprøvd konsept da strekk-broer vanligvis bruker høye tårn for å henge konstruksjonen på. Denne rapporten fokuserer i hovedsak på konstruksjonen av kabelnettet og tregitteret. Andre sentrale områder som vippefunksjon og fundamentering behandles ikke i rapporten, men trekkes inn der det er relevant for de andre komponentene. Et kabelnett er forspente konstruksjoner med ikkelineær oppførsel. På grunn av broens aero-elastiske karakter, er undersøkelser av aerodynamisk effekt høyst relevant. Rapporten avgrenser seg likevel til statiske- og periodiske laster forårsaket av menneskelig ferdsel.

Den parameterstyrte programvaren Grasshopper er brukt i design- og modelleringsarbeidet, og Karamba er brukt som innebygd FEM-solver. Egenvekten av broen er 1,64 tonn/m, og er om lag 20 % lettere enn referansepunktet, London Millennium Footbridge. For øvrig er broens naturlige egenfrekvenser i tverrgående retning utenfor det kritiske domenet, mens den i vertikal retning tilfredsstiller komfortkravet for tillatte akselerasjoner.

Rapporten konkluderer med at dette brokonseptet har mye potensial, men at det ikke passer for kanalområdet i Tønsberg. Brodekket i seg selv er av akademisk interesse og bør utforskes nærmere. Samtidig gjør byens behov for en bro med vippefunksjon, kombinert med de svake forankringsmulighetene i kanalen, at den skisserte kabelnettbroen vil være upraktisk og komplisert. På den andre siden er konseptets innovasjon godt egnet for områder med faste forankrings muligheter og grunne sjøforhold.

Acknowledgement

This thesis concludes a Master of Science Degree at the Norwegian University of Science and Technology, Department of Structural Engineering. The work has endured six months from mid-Januar to June 2020. This spring has been different as the University closed early March. Despite challenging circumstances, this work has been an interesting adventure. My sincere gratitude go to Bunji Izumi for jumping straight into a supervising role on my work from May and towards the end – without your contribution, the advanced parametric modelling would not reach the heights it did. I also thanks Anders Rønnquist for his passion and optimise toward my ambitious on advanced engineering topics. Thank you Raj Janmejay for being an internationally recognised bridge engineer who cares for my performances. I also thank Haldis Sandøy Nærum for nice discussions on architectural topics at an early phase.

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Chapter 1

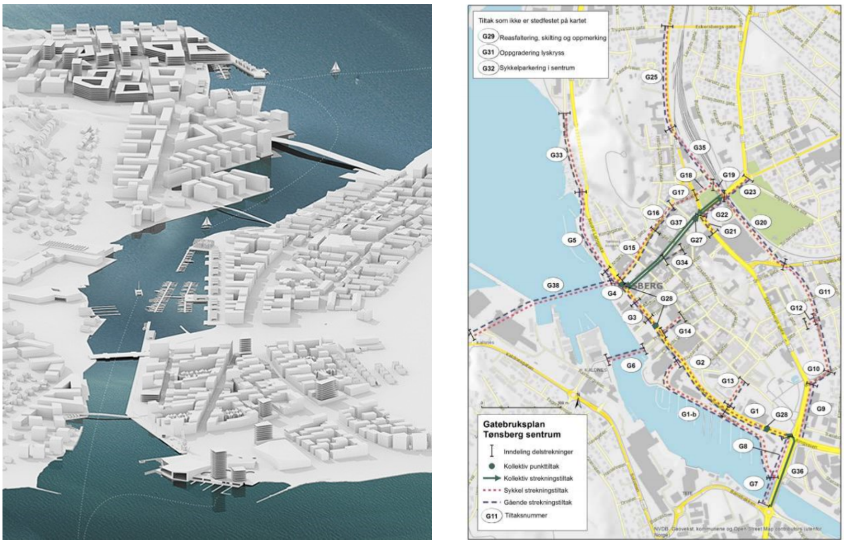
Introduction

This chapter introduce the study conducted in this thesis. The study aim to develop a bridge design relevant to the contextual location, Tønberg, and also relevant to the scientific field at hand; A master thesis within conceptual design and structural engineering. The thesis is delivered at the Norwegian University of Science and Technology, Gløshaugen. This chapter covers:

- Contextual *background*: why this study is needed
- *Problem definition*: description of the technical problems at hand
- *Objectives:* what the study intend to solve
- *Overview:* a reading instruction of the following chapters

1.1 Background

Vestfold County published in September 2019 a new city transportation scheme, aiming to improve pedestrian mobility around Tønsberg and reduce the demand for driving trough and around the city promenade. Their report, named *Interkommunal kommunedelplan for gange- sykkel og kollektivtransport*, from here on called 'city-scheme' was conducted by representatives from the county, Tønsberg and Færder municipality, along with a few technical consultants from the engineering sector; Statens Vegvesen, Norconsult [8]. The city scheme recommends a new footbridge to be constructed 200 m west for the existing footbridge, making a 250-300 m wide crossing from Kaldnes west to Tollbodbrygga (Tønsberg). The new link is shown in figure 1.1 and marked as G38 in figure 1.1b. The city blocks at the top of figure 1.1a show a layout of residential investments that is likely to occur in the near future – as the area is currently a decaying industrial zone.



(a) Illustration: Dyrvik Arkitekter [9] **(b)** Current [G6,G7] and proposed [G38] link [8]

Figure 1.1: Sketch and plan drawing of future Tønsberg-Kaldnes links

As of today, the channel has two crossings. One is the old city bridge ('Kanalbroen' 1957), a bascule bridge of 90 m with four lanes and a narrow pedestrian path. This bridge is located furthest out in the channel, marked as G7 in figure 1.1b. Secondly is 'Kaldnes bridge' (2005), a 130 m long, 3 m wide bascule footbridge marked as G6 in figure 1.1b, two thirds into the channel at a narrow location. A new link at the channel's west end will strengthen the pedestrian mobility in an attractive and urban areas along the bay with the possibility to cross the channel at both ends [8].

Kaldnes has developed significantly over the last 20 years. The first residential blocks was constructed in 2003 [10]. In the 20th century, Kaldnes was a busy industrial seat with a successful shipyard. Figure 1.2 illustrate this shift; from an industrial era to a post-industrial society with shops and residential apartments near the sea.

(a) Kaldnes shipyard (1899-1994) [11]

(b) Residential development at Kaldnes [12]

Figure 1.2: Kaldnes; once a shipyard industry, now a residential area

In large, the political environment in Vestfold supports the idea of a new link between Nøtterøy and Tønsberg. Although some other options were considered (visible in figure 1.1a), the report recommend a new footbridge at the channel's west end by arguing for the socio-economic pros of investing towards the least populated side of both banks. For this reason, this thesis will address a structural concept for a footbridge that is applicable for a 250-300m long crossing as described in the city scheme.

1.2 Problem definition

Bridge engineering is a technical field engineers would spend a whole carrier to master, therefor this thesis focus on some selective goals related to bridge design, such as:

- Develop a bridge concept, relevant to Tønsberg.
- Seek a light-weight design while secure serviceability limits on dynamic behaviour.

The prominent challenge is to develop a structural concept that is statically plausible, and possible to model and analyse with the parametric design tools at hand. Secondary, the societal aspect of this study; seek a bridge concept suitable to Tønsberg: the oldest city in Norway yet a post-industrial city today. That means a concept capable to address Tønsberg's traditional identity and also explore a modern look. Last concern is an light-weight design that satisfy general requirement toward public use.

The channel between Nøtterøy and Tønsberg is the contextual location for this study. At site, the channel is both wide and shallow with 250-300 m in width and rangeing five to ten meters in depth. Furthermore, the site has poor ground conditions with soft clay stretching far from solid rock. Such properties are significant to the range of good structural options. In large, long stretching bridges requires strong foundations to their abutments and piers. For those bridges, soft ground would lead to a high use of structural concrete. For design criteria, the city scheme specify the following two requirements (page 73 [8]):

- The bridge need a minimum width of 6 m for pedestrian traffic.
- The bridge need an opening mechanism to allow for passing sailboats.

Although the existing bascule bridges all tilts vertical direction (with help of a motor engine), outlines for horizontal, as well as vertical, rotation ought to be addressed.

1.3 Objectives

This report aim to develop a structural concept for a footbridge with use of parametric design tools.

The *superstructure* (bridge deck) is the main focus. In other words a structural design of *substructure* that is, pile caps, abutments and piers will not be included. The bascule bridge element will neither be covered due to time constrains. In order to achieve a relevant study, material use and usage ought to be addressed somehow. However, a complete bridge design depends upon advanced technical skills, sets of empirical data and experience form similar bridge projects. Therefore, this thesis do not seek to optimise all numbers toward the bridge's weight, structural utilisation of members, design cost or CO_2 emissions. The objectives here is rather to commence a concept study from an architectural angle, then build a parametric model of the chosen concept(s) and lastly discuss the solution at hand – structurally, practically, and its relevance to the public.

1.4 Overview of the report

The layout for this report is inspired from a guide on technical writing published by KTH, Institute of Electrical Engineering, Stockholm [13]. The body of this report consist of the following five chapter-blocks:

- **Presentation of Fields** – theory chapter
- **Bridge Study** – sketches and design philosophy
- **Models** – study with simplified models
- **Case study** – theory applied on specific design scenarios.
- **Closure** – summary, conclusions, and directions for future work

Presentation of Fields

Chapter 2, *Presentation of Fields*, present a theoretical background on the topics and tools used in this thesis. Unlike the *background* in section 1.1, which cover the contexts of the study, presentation of fields stay technical. Present relevant software tools, design requirements from European standards and service limits for dynamic behaviour.

Bridge Study

Chapter 3, *Bridge Study*, describe a subjective approach used to evaluate different bridge concepts at an initial phase. That include thoughts on aesthetic needs and purpose followed by early sketches and structural descriptions of each idea. Furthermore this chapter evaluate the bridge concepts and decide on which to proceed with, for further study.

Models

Chapter 4, *Models*, describe the computational methods applied in this thesis. This chapter include simplified models for two specific bridge concepts, covering prominent inputs and outputs as well as discussing obstacles along the way.

Case Study – cable net Bridge in Tønsberg

Chapter 5, *Case study*, present the practical implementation of theory and methods discussed in chapter 2. The study relates to a specific structural concept determined in chapter 4, and include three main parts:

- **Background** – a short description of the case study and location.
- **Design process** – iterations with set of inputs and problem solving.
- **Results** – presentation of the concluding results.

Closure

Chapter 6, *Closure*, summarise the thesis in large and point out directions for future work. The following setup is used:

- **Summary** – include background and justifications for the study, summary of completed works and which objectives that has been fulfilled.
- **General conclusions and recommendations** – summary of earlier conclusions, and a discussion of the implications of the results on the society.
- **Conclusions from case study** – discussion of specific conclusions to the case study; in which extent they are applicable to their *cases*.
- **Future studies** – a highlight of issues and alternative ideas which deserve further studies.

Chapter 2

Presentation of Field

This chapter provide a technical background of the tools and European guidelines applied in this thesis.

2.1 Use of digital software

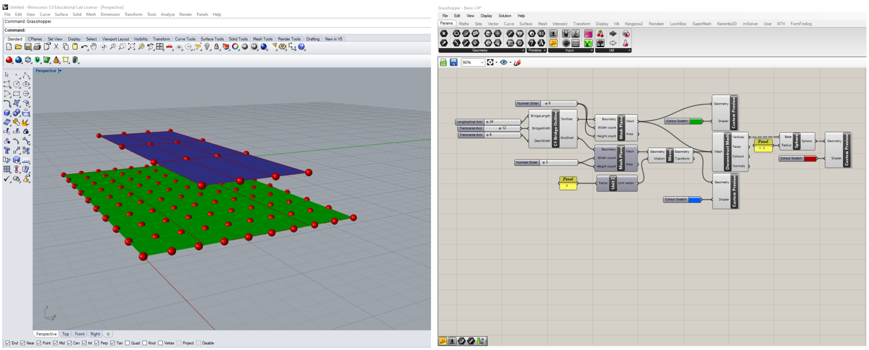
This section introduce the software used for this thesis. That is drafting, graphical editing, and structural analysis tools available through university licenses issued by the Institute of Structural Engineering.

2.1.1 Rhino 5,0

Rhino (Rhinoceros) is an design modelling software from Robert McNeel & Associates. Rhino is capable to 3D-model all sorts of intricate curves and surfaces directly, or through an interface like Grasshopper. Grasshopper is used in this project.

2.1.2 Grasshopper

Grasshopper is a tool serving as a graphical algorithm editor in rhino. Grasshopper consist of parameters, components and wires which connect the two. The parameters store data such as geometry (curve, surface, point and others) and numbers While a components perform a operation according to its input, such as dividing a curve into segment or constructing geometry from a vector or corner points [14]. In a nutshell, a typical workflow in grasshopper starts with a parameter (data) serving as the input in a component, the component do an operation and create a new data set which could be draught in Rhino or used for further operations within grasshopper. This ability make grasshopper an intuitive graphical algorithm editor, often called a *visual programming* tool.



(a) View port, Rhino 3D **(b)** Canvas, Grasshopper

Figure 2.1: Grasshopper define geometry draught in Rhino

C# / Python

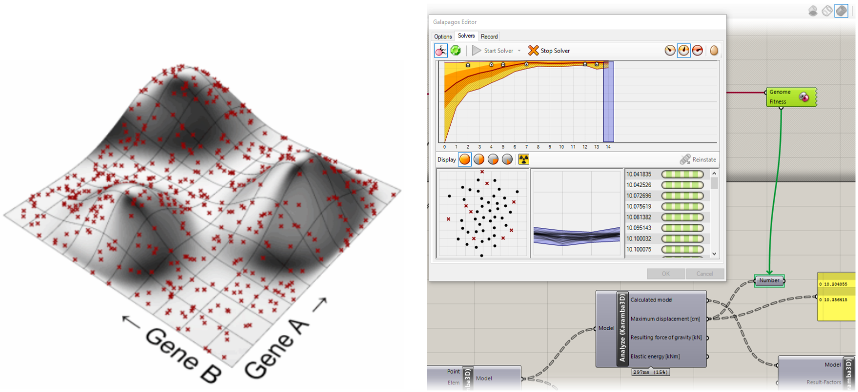
Interpreted programming language such as *C#* and *python* are available in grasshopper. Such customised component as useful to work with lists of data, calculations and creating generic geometry.

2.1.3 Karamba 3D

Karamba 3D is a structural analysis software with a FEM-package compatible grasshopper and its plugin's. Karamba therefor combine parametric design with numerical structural analysis (FEM). Karamba can analyse membranes, frames and most structural elements with sufficient accuracy in order to be useful at an preliminary design phase. [15]

2.1.4 Galapagos

Galapagos Evolutionary solver is an optimisation tool applicable in grasshopper. Its component require two inputs in order to operate; a *fitness* function and a *genome* variable. Galapagos use an algorithm that solve the function for every 'gene', collect the results,'genomes', and plot them as points in which shape a 'fitness' landscape and represent all possible outcomes for the give input. See illustrated example in figure 2.2a



(a) Illustration of a *fitness landscape* **(b)** Conveyed fitness algorithm, Galapagos Editor

Figure 2.2: Galapagos Evolutionary solver

Galapagos iterate between genomes as a process to reach the maximum or minimum output value. The 'optimal fitness' (min or max value) is determined by a simple click in the editor following the option tab visible in figure 2.2b.

2.2 Loading according to European Norm (EN)

This section describe relevant loads for the structural analysis conducted in this thesis. In engineering, universal standards is notoriously used to determine loads and their magnitudes for all sorts of structures. The European Norm (EN), or the *Eurocode*, provide such guidance in Europe and therefore used in this study.

A footbridge shall in addition to self weight, hold external loads from pedestrians, wind actions, snow and service vehicles. The relevant EN-publications are:

- Eurocode 1: Actions on structures (EN 1991)
  - Part 2: Traffic loads on bridges (NS-EN 1991-2:2003+NA:2010)
  - Part 3: General actions - Snow loads (NS-EN 1991-1-3:2003+A1:2015+NA:2018)
  - Part 4: General actions - Wind actions (NS-EN 1991-1-4:2005/AC:2010)
- Eurocode 0: Basis of structural design (NS-EN 1990:2002+A1:2005+NA:2016)

An overview of the uniformly distributed loads (UDL) are listed in table 2.1 (see section E.1 for calculations). Load combinations are according to EN-1990 , with service actions described in section 2.3.

EN 1991	Section	Description	Load	Direction
Part 2	4.5.3	Load Model 4 (crowd)	$Q_k = 5,0kN/m^2$	Vertical
Part 3	Tabel NA.4.1(901)	Snow Load, Tønberg	$S_k = 4,0kN/m^2$	Vertical
Part 4	National Annex	Wind Load	$q_{p,z} = 0,81kN/m^2$	Vertical
Part 4	National Annex	Wind Load	$q_{p,y} = 1,17kN/m^2$	Horizontal

Table 2.1: Relevant characteristic loads

2.3 Action on footbridge, structural model

This section cover relevant load effects for footbridges according to the Norm – EN 1991-2 section 5.2 [2].

2.3.1 Traffic Loads: Vertical models

For footbridges; three vertical load models ought to be considered.

- Compact crowd: expressed by an uniformly distributed load (UDL), q_{fk}.
- Maintenance load: expressed by one point load, Q_{fwk}.
- Service Vehicle: expressed by a group of point loads, Q_{ser}.

Compact crowd, UDL

If a continuous dense crowd is likely, or explicitly specified in a scheme, an uniformly distributed load (UDL) shall act on footbridges to express the static effect of a pedestrians. Such load is listed as *Load Model 4* in table 2.1. However, for cases where Load Model 4 is not required (long footbridge in suburban areas), $q_{fk}[kN/m^2]$ can be determined from equation 2.1.

$$2,5 \leq q_{fk} = 2,0 + \frac{120}{L + 30} \leq 5,0 \qquad (2.1)$$

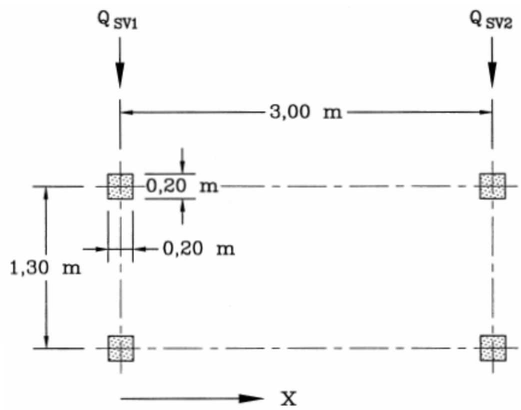
where L [m] is the loaded bridge length.

Maintenance load

For local effects, a maintenance load (Q_{fwk}) of 10 kN ought to be applied on the bridge, acting on a square surface of 100 cm^2. However, for bridges where a service vehicle apply, Q_{fwk} is not necessary to consider.

Service vehicle

For bridges subjected to service vehicles (maintenance car, snow plough, ambulance), their static effect must be considered as a service load, (Q_{serv}). If no explicit vehicle type is specified, nor any permanent obstacle block the path for vehicles the enter the bridge, figure 2.3 apply: $Q_{serv} = Q_{Sv1} + Q_{SV2} = 120kN$. The load model is described as figure 5.2 in EN1991-2 and illustrate two axle loads of 80 kN and 40 kN, which act on a surface of 0,04 m^2. The loads are separated by a 3 m wheel base and spaced 1,3 m transversely.



x : Bridge axis direction
$Q_{sv1} = 80$ kN
$Q_{sv2} = 40$ kN

Figure 2.3: Load Model for Service or Accidental Vehicle [2]

2.3.2 Traffic loads, horizontal model

For footbridges, a horizontal force Q_{flk} acting along the bridge alignment shall be included in the static model with a characteristic value of:

$$Q_{flk} = max(0,1 * q_{fk}; 0,6 * Q_{serv}) \tag{2.2}$$

Q_{flk} shall ensure the horizontal stability along the alignment, and ought to act together with the controlling vertical load in eq. 2.2. However, Q_{flk} is not to act together with Q_{fwk} (5.4.(3) [2]).

2.3.3 Groups of traffic loads on footbridges

Characteristic traffic loads in horizontal and vertical direction shall combine with non-traffic loads when relevant, making separate and exclusive load action on the bridge. Figure 2.4, illustrate two groups of traffic loads in their characteristic state, F_k. Note that figure 2.4 applies for footbridges if service vehicle is likely, thus the omit of Q_{fwk} (maintenance loading).

Load type		Vertical forces		Horizontal forces
Load system		Uniformly distributed load	Service vehicle	
Groups of loads	gr1	F_k	0	F_k
	gr2	0	F_k	F_k

Figure 2.4: Grouping of traffic loads [2]

Furthermore, for footbridges without roofing, traffic actions are not presumed to act in concert with the controlling wind or snow load – nor shall thermal actions act in concert with controlling wind actions. For implementation of loads, load models described in this section shall be applied within the bridge width and length in order to achieve the worst case scenario.

2.4 Verification of deformations and vibration, SLS

Vibrations induced from pedestrian traffic may influence a bridge design a great deal. If pedestrian exert forces with a similar frequencies as to the eigenfrequencies of the bridge, resonance will occur. Such accelerations affect peoples comfort, and ought to be check against serviceability requirement. This section present two norms related to dynamic verification and much used in Europe: the European Norm and Hivoss Guidelines.

2.4.1 Human induced vibration, EN

This content is based upon the theory presented i EN 1990, A2 [16]. The relevant design checks for a footbridge could vary due to the traffic admitted to each bridge during its service life. However, the following design situations ought to be considered:

- a group of 8 to 15 people walking normally
- a stream of pedestrians in high numbers $>>$ 15 walking normal
- Special events gathering large numbers of people standing still or moving rapidly.

Although pedestrians may induce a various set of frequencies, depending on whether they run, jump or walk normally, the Eurocode define some mean frequencies due to periodic loading from normal pedestrian use, such are:

- Walking; 1–3 Hz, vertical direction
- Walking; 0,5–1,5 Hz, horizontal direction
- Group of joggers; 3 Hz, vertical direction

The EN also address aspects of *bridge-traffic interaction*, where people may synchronise their stepping with the vibration, and boost the response even further, the number of people who participate in such resonance is somewhat random. For more than ten people, the correlation seem to decrease [17]. Thus in general, the prominent parameter for resonance is the eigenfrequency of the bridge. For this reason, the EN specify that eigenfrequencies (corresponding to horizontal, vertical and torsional vibrations) ought to be obtained from an adequate structural models.

2.4.2 Human induced vibrations; HIVOSS

This section cover a design guide from HIVOOS on dynamic behaviour of footbridges according to their publication; *Design of Footbridges - Guidelines* [7]. HIVOSS, being short for Human Induced Vibration of Steel Structures, from now on written as 'Hivoss', studies the effects of human vibrations on footbridges with use of sophisticated simulations.

Hivoss establish guidelines to lead bridge designs within the viable domain of serviceability. That is, ensuring peak accelerations induced by pedestrians streams are within the limits of comfort.

Step-wise design procedure; Hivoss

The remaining part of this section cover the essential steps according to the guide.

1. Evaluation of natural frequencies, f_i
2. Check of critical range of natural frequency
   (i) If OK; Finish
   (ii) Otherwise; proceed
3. Assessment of design situation
   (a) Assessment of structural traffic classes
   (b) Assessment of comfort classes (i.e. check acceleration, a_{limit})
4. Assessment of structural damping
5. Evaluation of acceleration a_{max} for *each* situation
6. Check for lateral lock-in (i.e. $a_{max} \leq a_{limit}$)
   (i) If OK; Finish
   (ii) Otherwise; proceed
7. Control of vibration (i.e modification of mass, frequency and additional damping devises)

Evaluation of natural frequencies, f_i

f_i ought to be obtained by finite element method (FEM).

Check of critical range of natural frequencies

Hivoss set the critical range for f_i [Hz] to be:

- For vertical and longitudinal vibrations:

$$1,25 \leq f_i \leq 2,3$$

- For lateral vibrations:

$$0,5 \leq f_i \leq 1,2$$

Footbridges within the range $\in [2,5 \leq f_i \leq 4,6]$ might be subjected by resonance from a 2^{nd} harmonic of pedestrian loads. However, such vibration is rarely addressed in literature as a likely concern [18]

Assessment of design situation

Bridge design is much about determine the predominant design situation which may occur. Design situations are sets of physical conditions expressing real load conditions that may take place during a given time interval. A specific design situation apply a given traffic density, together with a comfort requirement to which it shall fulfil. These design sets are significant to the dynamic requirement of the bridge. The expected human traffic on footbridges naturally depend on its location, therefore the design situations are customised for what is relevant. However, on an opening day, a bridge may be fully packed despite being located at the county side. Figure 2.5 define each design situation by an expected traffic class. Each class described with its characteristics and pedestrian stream (density). Note, that pedestrian formations, processions or marching soldiers are not included in these general traffic classification – but may need additional consideration [7].

Traffic Class	Density d (P = Person)	Description	Characteristics
TC 1	group of 15 P; d=15P/bl	Very weak traffic	15 single persons (b=width of deck; l=length of deck)
TC 2	d= 0.2 P/m²	Weak traffic: 	Comfortable and free walking, Overtaking is possible, Single pedestrians can freely choose pace
TC 3	d= 0.5 P/m²	Dense traffic: 	Significantly dense traffic, Unrestricted walking, Overtaking can intermittently inhibit.
TC 4	d= 1.0 P/m²	Very dense traffic: 	Freedom of movement is restricted. Uncomfortable situation, obstructed walking, Overtaking is no longer possible.
TC 5	d= 1.5 P/m²	Exceptional dense traffic	Very dense traffic and unpleasant walking. Crowding begins, one can no longer freely choose pace.

Figure 2.5: Traffic classes; Hivoss [3]

For the comfort classes, criteria for human comfort are given by acceleration limits in the footbridge. Hivoss recommend four classes of comfort, shown in figure 2.6.

Comfort level	Degree of comfort	Acceleration level vertical	Acceleration level horizontal a_{limit}
CL 1	Maximum	< 0.50 m/s²	< 0.10 m/s²
CL 2	Medium	0.50 – 1.00 m/s²	0.10 – 0.30 m/s²
CL 3	Minimum	1.00 – 2.50 m/s²	0.30 – 0.80 m/s²
CL 4	Unacceptable discomfort	> 2.50 m/s²	> 0.80 m/s²

Figure 2.6: Comfort classes for levels of acceleration [3]

Assessment of structural damping

Damping effect the amplitude from vibrations induced by pedestrians and wind. Damping, being the energy dissipation within the structure, derives from construction materials, supports/bearings, but also non-structural elements such as surfacing and parapets/handrails. In general, the amount of damping leans on the magnitudes of vibrations; higher amplitudes create greater friction between structural and non-structural elements and bearings. Also, for light-weight bridges, wind velocities can influence damping in bridges further than pure elastic behaviour. Wind generate what is called *aerodynamic* damping, and high wind speeds generate higher aerodynamic damping [7]. Yet, these phenomena lay within the science of *aeroelasticity*, most used by aeronautical engineers in airplane design [19], and relevant for wind studies. However, not for pedestrian induced vibrations. In general, estimation of damping in bridges is an intricate thing, and much easier to estimate once a bridge stand constructed and ready to be tested.

Determination of peak acceleration

There is several ways to determine the acceleration of a bridge. a_{max} shall be calculated for each design situation with a given damping ratio. Due to damping estimations, a_{max} become a predicted value. HIVOSS recommend three methods to estimate acceleration:

- Spectral method; Empirical method, based on simulations – give results with little calculations and relevant for preliminary design.
- SDOF method; Simplified method – reduce the system to a single harmonic, easily examined
- FEA method; Advanced method – Finite element investigations demanding suitable FEM software.

A main objective for this thesis is conduct a preliminary design of a chosen bridge concept. For this purpose, a spectral analysis is adequate, and therefor described more in detail.

Response Spectra Method

This method base upon a comprehensive study of response induced by different pedestrian streams. Pedestrian loads are stochastic – loads with random probability. The dynamic properties of bridges carry uncertainties, and so do this system response. Thus, "The aim of a spectral design method is to find a simple way to describe the stochastic loading and system response that provide design values with a specific confidence level."[7]

Given premises for this spectral evaluation are:

- mode shapes are sinusoidal/sine waved
- mass distribution is uniform in longitudinal direction
- no modal couplings stand
- mean step frequency ($f_{s,m}$) from pedestrian traffic befalls with natural frequencies of bridge, f_i

- structural behaviour is linear elastic

'Peak acceleration' from the response system is used as design values for this method. and obtained by equation 2.3.

$$a_{max,d} = k_{a,d} * \sigma_a \tag{2.3}$$

where $k_{a,d}$ is peak factor and σ_a is the standard deviation of acceleration/response. Both factors are based upon empirical data conducted through Monte Carlo simulations; numerical time step simulations of various pedestrian streams and bridge geometries [7]. Relevant data sets and equations for determine peak accelerations are included in Annex **??**

For evaluation of each design situation, a peak acceleration is calculated, and must satisfy the relevant comfort level, as shown in figure 2.6.

Lateral lock-in

Lateral lock-in is a phenomenon where all structural damping vanish and high response are produced all of a sudden. This may be triggered by a certain number of pedestrians, or for certain acceleration; $a_{lock-in} =\in [0.1, 0.15] m/s^2$. The critical number of pedestrians is defined by equation 2.4

$$N_{lock-in} = \frac{8\pi\xi m^* f}{k} \tag{2.4}$$

Hivoss recommend either approach to check for lateral lock-in, as they both have shown to be accurate in recent tests [7].

Control of vibration

If a bridge design fail to comply with her relevant design situation, further measures ought to be considered, including modification of: mass, frequency, structural damping. Alternatively, additional damping devices may be installed – as done for several bridges, such as *Mjømnesundbrua* and *London Millennium Footbridge*.

Chapter 3

Bridge Study

This chapter is a subjective approach used to evaluate the bridge concepts developed at an early phase. That include thoughts on aesthetics, collections of sketches, and background for each idea. Furthermore, this chapter conclude on which bridge concept relevant further study.

The chapter has the following setup:

- Design Philosophy – relevance
- Structural Concepts – bridge theory
- Discussion – concept evaluation

3.1 Design philosophy

This section describe three attributes which define this 'design philosophy' That is, aesthetic character, structural relevance to Tønsberg, and originality.

3.1.1 Need for beauty

In the 11st century BC, the Roman architect and military engineer, Vitruvius Pollio, wrote the first recognised litterateur on architecture. Here he argued how all buildings ought to process three attributes: *Firmitas, Utilitas and Venustas*, latin for strength, utility, and beauty. Those principles has been adopted repeatedly ever since up until the modern era.

Today, most project claim to seek aesthetic qualities in their design – thus architects play the lead in most major design processes. However, some philosopher and scholars seem to degree upon the focus toward aesthetics today. For instance, the English philosopher Sir Roger Scruton (1944-2020) has criticised post-modern culture for decades. Scruton argues in his BBC documentary, *Why Beauty Matters (2009)*, that buildings erected merely for their utility are soon to become useless [20]. Arguing; when these 'modern' buildings reach some age, the public lose interest towards them and therefore end up useless – because they are ugly and only constructed for utilitarian use. On the contrary, ”if beauty come first, the result will be useful forever”. As a known conservative, he point out classic architecture as the field that possess the missing key of modern architecture by saying:

> We see this in traditional architecture with its decorative details. Ornaments liberate us from the tyranny of the useful and satisfy our need for harmony. They remind us that we have more than practical needs. We are not just governed by animal appetites, like eating and sleeping, we have spiritual and moral needs too and if those needs go unsatisfied, so do we.

Scruton address the consequences of previous architects such as Louis Sullivan (1856-1924). Sullivan being famous for his thesis *form follows function*, a way of thinking that has inspired later generations including his pupil, Frank Lloyd Wright – another pioneer for modern style. The idea that form follows function ask for a consent between a structure's physical form and its intended function to serve. Scruton declare Sullivan's thesis as the staring point of were the society went wrong and henceforth the society have embrace the utilitarian qualities and place the *beauty* second in line.

However, for bridges, aesthetic qualities are important for the similar reasons as to buildings. Yet, this thesis argues that a bridge's need for beauty in order to stay useful to the public stand in slight different – but not in opposite – to buildings. Different because the value of a bridge is protected by society's need for infrastructure. A bridge serve as a link over troubled ground and a quality hard to replace. An easy example could be the Kaldnes footbridge in Tønsberg. Kaldnes bridge is rapidly used despite her lack of decorative details nor impressive engineering. On the other hand, a bridge like The Clifton Suspension Bridge in Bristol, serve as an enduring icon to the city and brings solely thousands to Clifton every day [21]. Hence, bridges with an innovative character along aesthetic qualities stretch far beyond the measured of simply utility and durability.

> ...Yet it is also true, most profoundly true, that in the most pure aesthetic emotion (as in so many other things in life) simplicity is a virtue. Hence, beauty is now sought within a minimum of elements: all of them essential. [22]

Although aesthetics is a philosophical term, for most sorts of design – and especially structural design – beauty is reveal through solving their tasks by as few and essential elements as possible [23]. Henceforth, this thesis seek simplicity and coherent tectonics as virtues in the design process. Given Webster's Dictionary's definition, Tectonics is “the science or art of construction, both in relation to use and artistic design” [24]. Furthermore, the German architect, Peter Behrens on tectonics: “it refers not just to the activity of making the materially requisite construction that answers certain needs, but rather to the activity that raises this construction to an art form.”[25]

Architectural background

Tønsberg is the oldest city in Norway with history dated back to 872 A.D. [26].

The city experienced her 'golden' area in the middle ages and was one of the three Norwegian cities included in the trade alliance, Hanseatic League [27]. Still there are visible ruins around the city from ancient churches, wooden houses and larger stone structures symbolising the history. In 2008 and 2012 a foundation (Nytt Osebergskip) build, using autentic construction methods, a copy of two original vikings chips which King Harald V launched during an opening ceremony. In regard to Tønberg, its presumable that structures 'natural' of materials such as stone, steel and timber is much compatible with the local public – and their feelings towards aesthetics.

3.1.2 Structural context

A crossing from Kaldnes to Tollboden (Tønsberg) may vary in length depending on the preferred bridge alignment. The shortest feasible option is 220 m, and a more curved alignment would need about 300 m, 350 m topmost. Never the less, a crossing of such length, is significant and require good engineering judgement. Also, the channel's depth ranges from 7 to 12 m, which is fairly shallow. That is relevant for the costs of constructing piers. The ground is informed [orally by Statens Vegvesen] to consist of soft clay. Such conditions are unfavourable for certain bridges that demand strong pile caps and anchoring possibilities, such as arch, suspension, and cable stay bridges.

3.1.3 Parametric outlook

Abstract shapes are becoming increasingly frequent in the urban landscape. Architects like Zaha Haid is an example of thus. Abstract and uncommon design may rely on new and empowered digital drafting tools. Parametric design is a field entering the industry more and more – although the field stretch back to the architect Luigi Moretti (1907-1973) [28]. Software involved with parametric modelling offer possibilities for iterative structural optimisation, parallel to early design processes of form. This leads to a common platform for both engineers and architects to work; combined in the search for better design solutions.

This thesis recognise parametric modelling as an innovative branch with attributes most relevant for the engineering field at hand – *conceptual design*. For this reason, bridge concepts suited for parameter controlled design is regarded favourable.

3.2 Structural concepts

This section present the leading structural concepts from a brainstorming phase, conducted together with co-student Haldis S. Nærum. The chosen concepts are:

- Arches – statically independent
- Stressed ribbon – external tendons
- Net – tension membrane

Each concept is discussed and link to the design philosophy described in section 3.1. A few additional concepts are also presented with a sketch and a short comment. However, excluded from the concepts analysis as they either lacked an innovative character, or on the contrary, strike as somewhat ostentatious.

3.2.1 Arches

Arch bridges is an ancient construction technique traced back to the Ponte Sant'Angelo bridge in Rome from 134 A.D. Due to the easy access of masonry, old arch bridges are found widely across the world – for instance, Zhaozhou Bridge in China from 605 A.D [29]. For this bridges, the arch itself, carry the vertical loads by compression, a compression strut forms a line from static equilibrium by axial and bindings stress. For a *perfect* arch, theoretical that is, the compression strut act along geometric centre line of section, creating a uniform load distribution and no bending forces. However, such an ideal design would in practise be unpractical and unsafe considering real service conditions; live loads. Also, geometric imperfections and *kinking* forces (horizontal reaction at the boundary) may disturb the load path and create moments and instability.

Sketch

An outline of an arch concept is presented in figure 3.1.

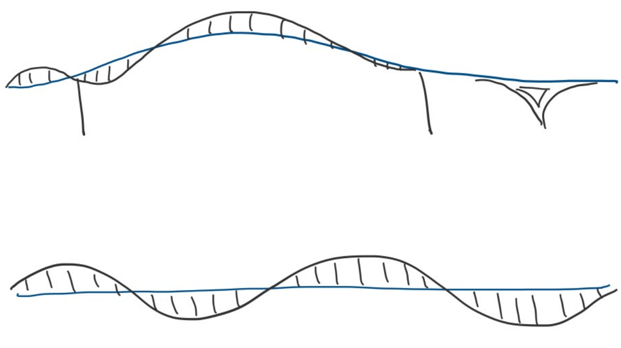


Figure 3.1: Concept: Lohse arch

Structurally

The concept is a continuous and statically independent arch bridge, with a unsymmetrical shape. The idea is to introduce hinges where the arch cross the deck and transmit horizontal forces onto the deck; similar to a *tied arch*. A tied arch reduce the horizontal pressure on the foundation and preferable in Tønsberg with soft ground conditions. The unsymmetrical shape is inspired by typical stressed ribbon bridges with under external ties. For the bascule element, a stiff frame is suggested at the rightmost pier, rotate in the xy-plane.

Aesthetics

By defining simplicity a virtue in aesthetic quality, this concept describes the principle for an arch that except from the hanging deck, bascule part and substructure consists of one element: a continuous arch. That leads to an presumably elegant design – according to the Spanish engineer E. Torroja at least [22]. Also, arches similarly with rainbows seem to create curiosity the people, and might as Scruton stress: “liberate us ... and satisfy our need for harmony”.

Parametric outlook

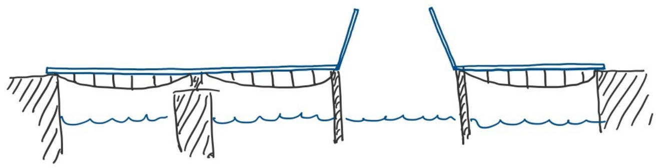
Although arch bridges is an old construction method, modern software [such as Midas Engineering] can model arch bridges with deck and hangers useful outcome and support bridge designers a great deal. For instance, arched geometry can be modelled by many short straight beam-elements, to be analysed by FEM. Other parameters such as spacing of hangers and optimisation of arch and deck geometry is applicable within nowadays parametric and engineering design tools [29].

3.2.2 Stressed ribbon

Stressed ribbon (SR) bridges offer a light and slender shape yet, a strength capable to span 150 meters [30]. The strength is obtained from prestressing the bridge deck with external or internal ties/tendons –or both combined. SR bridges are comparable with suspension bridges minus the pylons. There exist a wide range of SR bridges; having creative shapes, clever use of materials and good architectural qualities. However, due to their dynamic properties SR bridges is mainly used to carry pedestrian traffic [31]. The longest span is achieved with concrete and prestressed tendons, yet Islambard Brunel made SR-viaducts for the *Great Western* railway out of timber in the 1850s! [21].

Concept

Figure 3.2 show a SR design with two alternative layouts for the bascule element: vertical tilting and horizontal rotation.



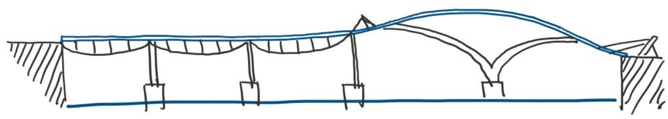


Figure 3.2: Concept: Stressed ribbon

Structurally

The SR-bridge use external tendons as the bearing of the superstructure. External ties sagging underneath the deck provide high ribbon stress by its lever arm. Also, with the help of compression struts, the torsional strength increase and become favourable to restrain wind forces. For maintenance, external tendons are easier to access. However, for a structure this close sea water, external tendons would require substantial protection. Figure 3.2 also present two bascule alternatives: A motor pulling each element in the xz-plane, and a 'tied-arch' concept rotating horizontally.

Aesthetically

This concept describes the principle for a classic stress ribbon bridge with a slender profile. It may offer a tectonic expression that is easy for the public to understand; including a range of material options. For the bascule part, the tied-arch element is self-anchored thus sophisticated. Simple solutions to structural challenges are much in compliance with the design philosophy at hand.

Parametric outlook

SR-bridges open for a wide range of structural solutions. Stiffness may come from wires along the handrail or traditional tendons external or internal inside plastic ducts – in case of concrete. However, modern parametric tools allow engineers to experiment with compression struts and tension ties in an open environment. This lead to unique and tailored SR designs with an innovative character.

3.2.3 Tensile membrane

A tensile membrane is *form-active* system which responding to external loading by reshaping itself in order to stay efficient as a tension structure. For thin structures with low deformation *Membrane Theory* applies; bending and twisting moments can be neglected in stress analysis. This lead to an efficient structure with all its capacity set for axial stress.

Tensile membranes was surly invented by spiders. Yet, fabric membranes for human use – tents – has brought shelter since ancient times. J.S Dorton Arena in Raleigh, North Carolina became the first large cable net structure to be built in 1953 [4]. Dorton Arena was engineered by the Norwegian pioneer Fred Severud – who also engineered the cable net roof at Madison Square Garden in New York in 1968 [32]. Tensile membranes as an engineering discipline was enlightened to a higher extent through the work of Frei Otto. The multi-talented scholar has inspired countless of architects and engineers across the world and considered on of the greatest in the field [5]. Otto established Institute of Lightweight Structures in Stuttgart, and the city remains a seat for sophisticated engineering to date. For instance, the International Garden Exhibit 1993 was held in Stuttgart. There, six light weight bridges was presented and still in use [33]. The Lodz footbridge is a rare cable net bridge which carry an external bridge soffit with use of compression bars.

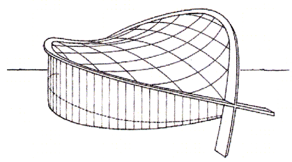
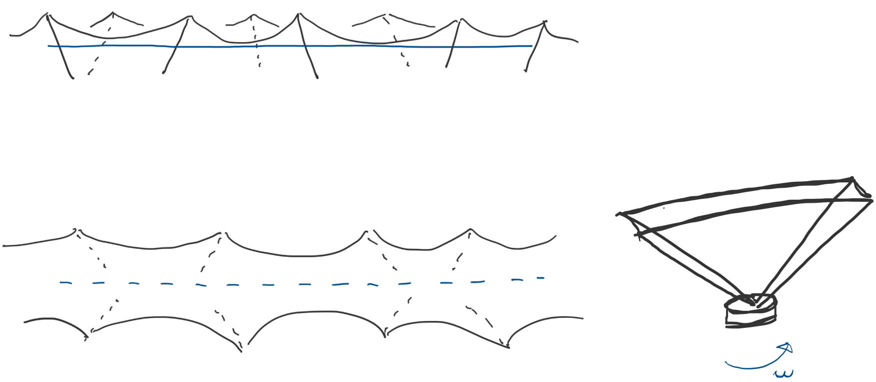


Figure 3.3: Dorton Arena [4]

Concept

Figure 3.4 illustrate a three-dimensional tensile membrane carried by inclined piers, unevenly spaced.



(a) Plan and elevation sketch

(b) Bascule element

Figure 3.4: Concept: Tensile membrane

Structurally

This membrane concept explore anticlastic surfaces as shown in figure 3.5 – concave and convex curvature for 'warp' and 'weft' direction. Together, the curves acts as a stable and stiff surface which may look like a *saddle*. Such shapes can serve as structural membrane by fabric and stressed edge cables, or cable net; both illustrated in figure 3.6b. For large structures, cable net is more efficient. Figure 3.6a show a grid of interconnected tensile cables hinged to prestressed edge cable; and if anchored at different elevation, the double-curved saddle shape is achieved.

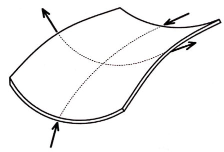
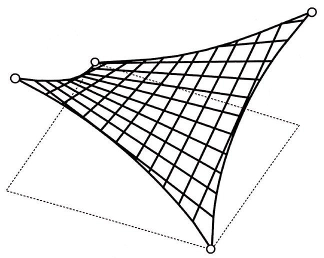
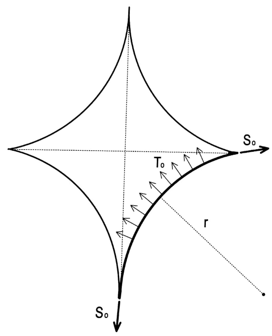


Figure 3.5: A *saddle*



(a) Anticlastic cable net



(b) Prestressed fabric membrane

Figure 3.6: Tensile membranes, 3D [5]

Aesthetically

Tensile membranes open the door to elegant engineering solutions and some material options; Wires combined with coated fabrics or structural glass for instance. Also, the connection of cables to masts and foundations carry a high visual prominence and a potential asset to the tectonic performance. For example, anchorage joints may lay at ground level for people to see, and even touch, thus a detail to be thought through.

Originality

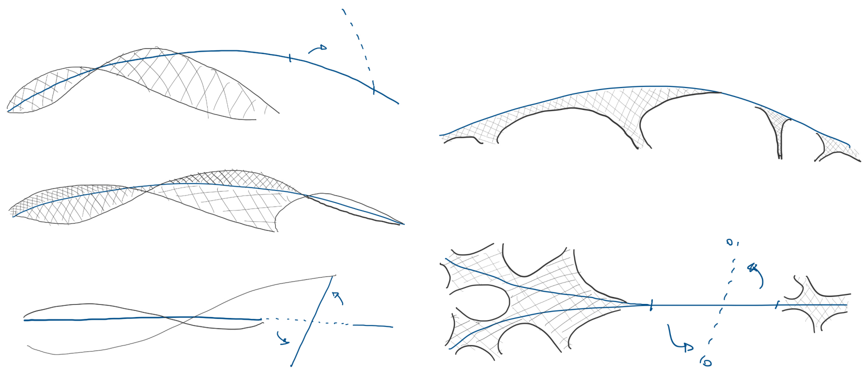
Tensile membranes as roofs exists all over the world and known light weight structure. Yet for bridges, tensile membranes barely exist. Prestressed cable nets offers the highest strength and therefore the natural option in case of a bridge. The Lodz footbridge in Stuttgart is the only cable net bridge known to this study however, that bridge is a short inland overcrossing, not a spatial structure at sea. For this reason, a cable net bridge at this site is original in all sense of the term.

3.2.4 Remaining ideas

This section list the design concepts excluded from further analysis due to a lack of originality or relevance to parametric modelling.

Compression Membrane

A form-passive shell system.



(a) Twisting Shell

(b) *Organic* Shell – a leaf

Figure 3.7: Concept: Compression membrane

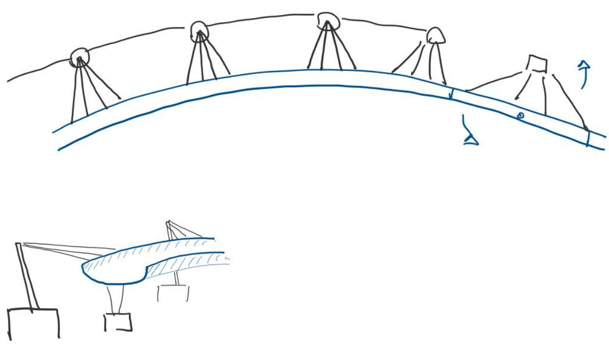


Figure 3.8: Concept: Hanging shell

Balanced Cantilever

A concept similar to traditional balanced cantilever bridges – typically raised by segmental construction with structural concrete.

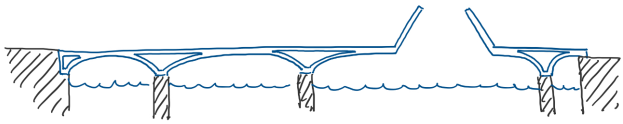


Figure 3.9: Concept: Balanced cantilever

Cable Stay Bridge

A classic CSB design, inspired by the artist Vebjørn Sand and his work on the Ypsilon bridge (CSB) in Drammen, Norway.

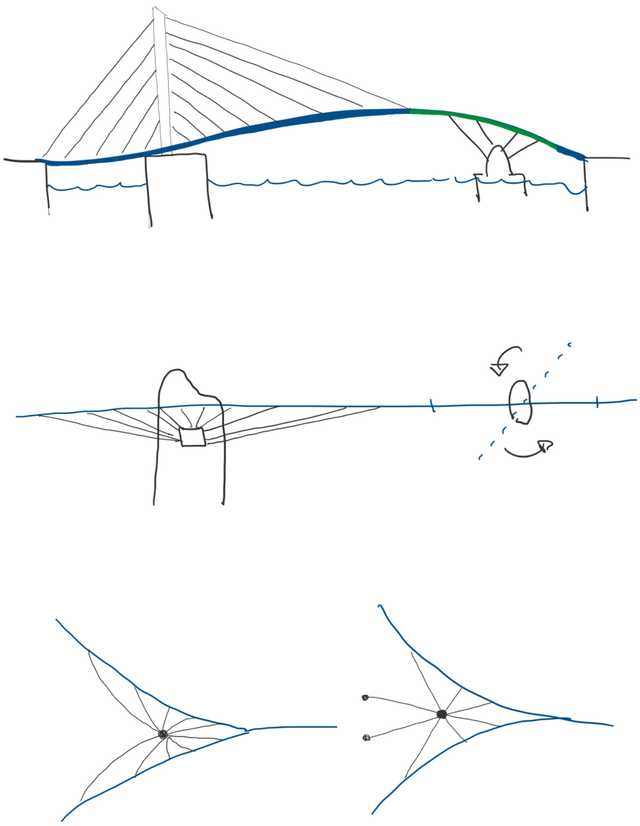


Figure 3.10: Concept: Cable stay

3.3 Discussion

This section discuss concept analysis, both in a wide relation to the construction industry, and also specifically for this evaluation.

Concept evaluation is an analytic field; choosing the right concept are according to Knut Samset the leading attribute which differ successful project from the unsuccessful. Samset being a NTNU Professor, and author of *Early Project Appraisal* (2010), stress why a successful project must inhere a high tactical as well as strategic performance. Tactical performance is a question of delivering the project outputs (cost, quality, time) as planned –'doing things the right way'. While strategic performance is the worth and benefit of the project in a long-term perspective (relevance, effectiveness, sustainability), basically 'doing the right thing'. Furthermore, Samset address the paradox of how the public measure success only in terms of tactical performance, rather than the strategic [34].

However, due to the context of this thesis, a popper concept analysis lay beyond the scope of this report. An outline of a proposed assessment is presented in annex **??** . The concept evaluation will therefor be controlled by academic interests and structural considerations.

Decision making

All three bridge concepts discussed in section 3.2, are fairly relevant to site in regard of the channel's length and depth. However, arch and tension structures require strong foundations for anchorage. So in terms of the substructure, the ground conditions in Tønsberg may be unpractical. Yet, as this is a academic thesis, not intended for public use, the risk of exploring a concept that is new to the field, and without any referencing examples, is low in comparison to paid consultant work. With that in mind, tension membranes as a subject to footbridges is worth a study. Therefor, cable nets is chosen as the governing subject henceforth.

Chapter 4

Models

This chapter describe the computation and analysis methods applied in this thesis. The parametric design methods are implemented on two simplified models; a tension membrane and a stressed ribbon structure, both with use of cable nets.

4.1 Need for simplified modelling

There are surely several ways to create a plausible bridge design. Empirical data is gathered for all sorts of bridge types and often used to predict a reasonable preliminary design. An example is shown in Figure 4.1 with a plot of 221 simply supported truss bridges constructed in the UK between 1978 and 1993. Each bridge is plotted with regard to deck weight (unit ton-force/sqm) and span lengths (m). Such data in handy for engineers if relevant to ones project. However, these data clusters are only accurate for lengths within the main scope – as for this case, spans ranging 50 to 90 meters.

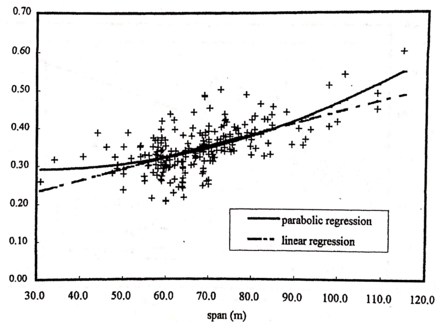


Figure 4.1: Truss Bridges; 1978-1993

Without empirical data at hand, a preferred way to develop a structural concept is through a simplified model – physically or digitally. In this chapter two simplified model is created digitally and analysed. By doing so, and evaluated reasonably, an expanded bridge model can be modelled responsibly.

4.2 Basic models, parametric modelling

This section describe methodically the computation process a simplified parametric models. That include *form finding* methods, FEM analysis and geometric optimisation. Figure 4.2 sketches the two bridge concepts at hand; a wide cable net supporting a grid shell, and a stressed ribbon deck with external tendons and compression struts. The rightmost figure sketch a section-view of the SR idea. The sketch illustrate how the net acts as tension ties transversely, and connected with the stressed bridge surface. Ultimately the basic models shall be enlarged to a wider footbridge model described in chapter 5; Case Study.

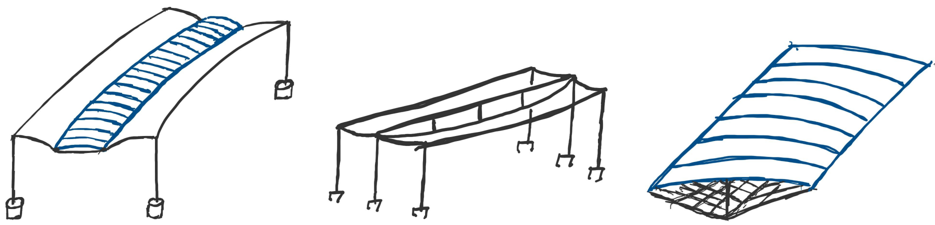


Figure 4.2: Sketch of simplified bridge models

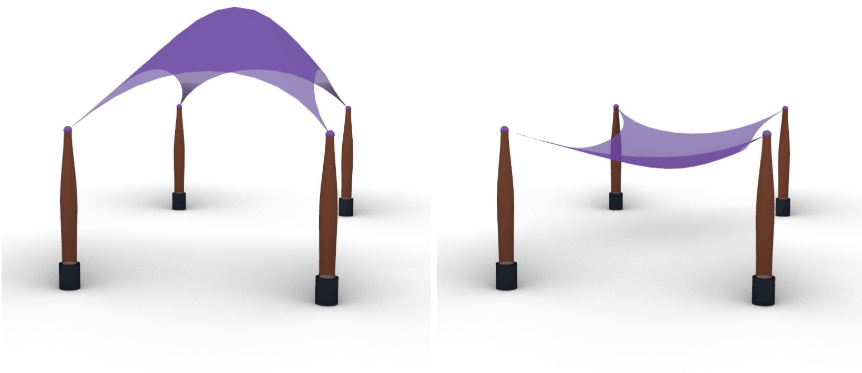
Form Finding – dynamic relaxation

The parametric model is initiated in grasshopper with a C# component creating the basic outline; length and width. From that geometry, a mesh is defined and projected vertically with a given depth. For here, form finding and structural analysis can start.

Form finding is governing term for a set of geometric optimisation methods. Kangaroo is a plug-in made for Grasshopper with components that implement algorithms for dynamic relaxation – a form finding method. The following four Kangaroo-components are used to achieve an optimised shape of tension and compression membranes:

- Load – force vector to initiate shape. In case of a tension of compression membrane: Negative z-component leads to a concave shape, whilst a positive value give a convex shape.
- Length line – length of curves in mesh, an adjustable factor used for tightening or loosening membrane.
- Anchor points – define anchor points with a given pull strength.
- Solver – optimise the form with respect to input; geometry, force vector, anchor points and line lengths.

To illustrate, figure 4.3 compare two membranes with different inputs. In figure 4.3a, a positive z-component is applied as force vector, in addition to a line-lengths equal 1,0 (no line reduction). This input lead to a highly curved compression membrane. On the other side, figure 4.3b use a negative force vector and a line-length of 0,75 (25% line reduction) which produce a tighter tension membrane. Note: in figure 4.3, a *mesh to surface* component is used and the lines are hidden for visual purposes.

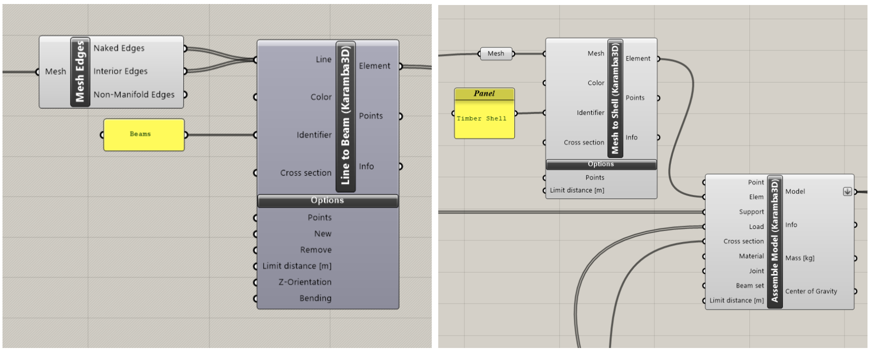


(a) Positive force-vector with no line reduction **(b)** Negative force-vector with 25% line reduction

Figure 4.3: Form finding illustrated in Rhinoceros

Structural analysis – FEM

Karamba have a user-friendly interface. To illustrate, let us categorise the two membranes in figure 4.3, in the structural software. Assuming the tension membrane in figure 4.3b ought to become a net-structure; the component *line to beam* is used. The component use straight lines as input, in this case, a deconstructed mesh. Karmaba locate where the lines connect and produce beam-elements as output. Also, under the 'Options' tab, the element can be further defined: if for instance the boolean 'bending' parameter is switched off, the elements will no longer resist moments – useful for cable net design. Whilst for figure 4.3a, assuming a shell membrane is relevant, the *MeshToShell* component may be used. This produce shell elements with patch/element properties computed for a quadratic – or triangular – mesh. Furthermore, the shell elements are *pinned* together, unless the node distance is lower than a given minimum tolerance (as defined in the option tab), in which Karamba make *fixed* connections. However, there are other differences between the beam and shell components: For the plane shell elements, Karamba assume a constant strain/stress 'curve' and neglect shear deformation (Timoshenko theory). Whilst for the beam-elements, Karamba apply Euler-Bernoulli theory [15].



(a) *LineToBeam* component

(b) *MeshToShell* component

Figure 4.4: Structural component for net and shell membrane

For structural analysis, Karamba is a useful FEM-software to a certain extent. With material properties and boundary conditions defined, Karamba run analysis for given load case. Typical output may be:

- Vibrations – eigenmodes and natural frequencies
- Reaction forces at the supports
- Stress values and member utilisation
- Maximum displacements

Parameters for structural modelling

For stressed ribbon bridges with external tendons, the curvature and prestress of tendons effects the structural capacity – and therefore disused in this section. In figure 4.5, a catenary chain is used to define the external tendon; which provide the bearing strength in the longitudinal direction. Yet, figure 4.5 also illustrate two different inputs of this chain length.

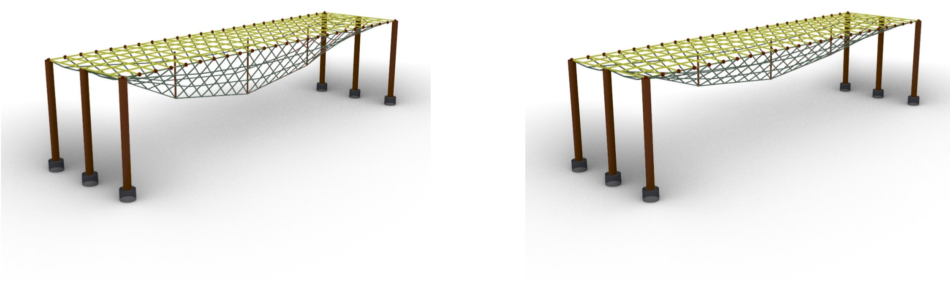


Figure 4.5: Design cases for two catenary chain lengths

For a cable under a uniformly distributed load (on horizontal projection), a parabolic curve is achieved. With such geometry, simple equilibrium considerations can be applied from mechanics, as illustrated in figure 4.6. Figure 4.5 illustrate two different cable depths (sag), marked as f in figure 4.6. By rearranging the parabolic equation for y=f, the support reaction become $S_0 = qL^2/8f$ – implying that low chain height bring high horizontal forces. Yet the vertical component is constantly $q/2$, therefor the total cable force with Pythagoras become increased for tendons with low sag. This principle is also valid for other tension structures, and a practical example of this is presented in annex D.

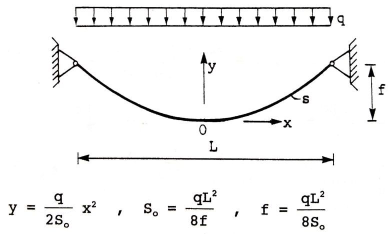


Figure 4.6: Cable forces for UDL

4.3 Recommendations for case study

This chapter has discussed topics on parametric modelling and behaviour of cable structures. Furthermore, a small case study with a simplified beam-net model was conducted and presented in appendix D.

Unfortunately, the stressed ribbon sample with a supporting cable net did not converge in Karamba. Compression occur several places which lead to buckling even under self weight – with beam elements the model converge yet ineffectively. For this reason, the case study will focus on the concept shown in figure 3.4; investigating the possibility of a grid shell serving as the footpath, and carried by a *light* cable net.

Chapter 5

Case Study – Cable Net Bridge in Tønsberg

This chapter cover a case study that investigate the feasibility to design a footbridge with cable net as superstructure, and with a grid shell as footpath. This chapter is divided in seven blocks:

- Background
- Initial Design iterations
- ULS analysis
- Serviceability Considerations
- Topics of practical considerations
- Results
- Discussion

5.1 Background

This study builds on a concept evaluation, and lessons learnt from previous modelling concluded ion this thesis.

To start out with a small model, helps to determine whether a bridge idea is feasible, as a concept for given location, and a structure.

The importance of good inital conditions (resonable mesh geometry, inputs of structural members) was a lesson learnt from the parametric study. There is not time or computer power at hand to obtain useful results without determine a set of parameters manually – and those ought to be somewhat sensible.

The following subsection, present some key parameters that influence results and outcomes of the remaining study.

The cable net

The mesh used for cable net is sized, 1 m by 1.618 m in transverse and longitudinal axis respectively. In comparison, Frei Otto used a square grids 0,75 m for his Olympic area [5]. Yet, to implement form-finding on a long and narrow structure, such as this, a square mesh would mean homogeneous tightening in x and y direction which lead to an undesirable shape, too narrow. For this reason, Da Vinci's golden ratio is chosen, and seem adequate. Also, rectangular mesh is used rather than triangular mesh. Although triangular mesh give stiffer models, for practical aspects of prestressing, rectangular mesh is chosen. The net width is somewhat random, yet 12 meters is decided upon an engineering – and aesthetic – hunch.

The grid shell

The mesh for the grid shell is sized similarly to cable net (1 m by 1.618 m). The shell width is 6 m, as specified through by design requirements (section 1.2). Also, the shell is only supported at boundaries thus, span 6 m. However, shall this design turn out deficient, vertical compression bars may be introduced as additional support – or mesh dimensions could be scaled down. This timber grid is supposed to act similar to a shell (mainly exposed to axial forces), therefore, 'homogeneous' glulam is to prefer. The choice of surfacing is not addressed yet, a light material is recommended for architectural and structural purpose. On that basis, a surface layer of 0,5 kN/sqm is applied (based on litterateur for wood-element [35]). In comparison, the smallest concrete hollow section (h=200) weigh 2,0 kN/sqm.

Glued laminated timber (glulam) consist of thin wood lamellas glued together along their wide faces. As each lamella is flexible, curved member is possible to achieve and a cheep option compared to other structural materials [36]. Glulam is more moisture stable than ordinary timber as they are delivered dry (12%), and with great sections. Also, glulam offer a higher mean strength as twigs and other flaws are reduced. Glulam sections are delivered in two types: homogeneous (h) or composite (c). The homogeneous is the high end type where all lamellas are of the high strength, hence an effective option for axial forces. The composite type have a lower quality of the internal boards and therefore become most suitable to bending actions.

The cables

The cable design is based on litterateur on structural cable systems [35]. On this basis, spiral and locked-coil cables as the internal and edge members (see figure 5.1). For simplicity, PFEIFER's product brochure on tension members [1] is used as database – and provide real dimensions, prestress calculations and tested strength values.

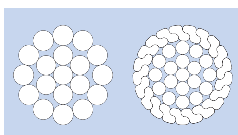


Figure 5.1: Spiral & Locked coil [1]

Prestressing

In general, prestressing is used to prepare a structure on a future load. By prestressing cables, geometric stiffness is achieved – and a necessity in structural use. Prestressing cable nets is rather a complicated matter, and may require much post-tensioning before reaching desired strength.

For pragmatic reasons, the net is modelled in groups depending on their direction and loading exposure. The prestress force is an percentage of the ultimate limit strength (UTS) or the allowed tensile limit. A *python* script is used to calculate the equivalent *elongation* in the cables (depending in their loading, sag, and lengths), an the elongation is used as input in Karamba.

Computation

All computational input is applied in grasshopper where, Kangeroo is used for *form-finding*, and Karamba is used for FEM analysis. The cable net and grid shell is both sized 1 m by 1.618 m, and shaped in Kangeroo with a line reduction of 25% (from original length) and default a strength ratio of 100.

5.1.1 Referenced structures

The German philosopher J. Goethe said:

> ..."self-knowledge comes from knowing other men"

The same may apply for bridges; in order to have perspectives on a result or a design solution, other similar bridges (empirical data) is valuable. Yet, for this concept there is close to none of such. Two bridges with bits of the same philosophy is described here.

Lodz Footbridge, Stuttgart. Lodz is an overcrossing from 1992. The concept is a cable net as superstructure which carry an external footpath though compression bars. Their concept is relevant for study with regard cable net details and anchoring. Figure 5.2 show the bridge form a corner angle. Yet, to receive structural data on this bridge was difficult. For those reasons, Lodz bridge stay as an inspirational source, for tectonic attributes.

Figure 5.2: Lodz Footbridge: cable net

London Millennium Footbridge

Millennium bridge open June 2000, and is a 325 m crossing River Thames just west of Westminster. Arup and Foster & co. design the bridge after winning a comprehensive tendering.

Millennium bridge is good reference for this case study for two reasons:

- The bridge is a light-weight tension structure with, similar site conditions to Tønberg (length, depth, soil).
- The bridge has been a case study for human induced vibrations and dynamic behaviour, due to unexpected lateral movements in service (at opening day).

Figure 5.3 show the bridge from two angles.

Figure 5.3: London Millennium Footbridge: plan and elevation view [6]

Through contact with engineer Roger R. Smith and Chris Wise who work on the bridge design – and retrofit, has provided useful insight for construction sequence and structural features. While further readings is included in appendix C, the most essential data is included in table 5.1. Note, the natural frequencies F_i are from original design (without additional damping system).

London Millennium Footbridge		Comment
Bridge width [m]	8	between cables
Deck width [m]	4	footpath
Pier height [m]	5	bridge height
No. span [#]	3	north, main ,south
Span length [m]	81,144,108	north, main, south
Fi_lateral [Hz]	1.0, 0.5, 0.8	north, main, south
Selfweight [t/m]	2,0	in total
Cables [mmˆ2]	80800	8Ø120

Table 5.1: Structural features: London Millennium Footbridge

5.2 Initial design steps

This section cover the first iterations conducted for this study. That include a 90 m model, and an enlarged 200 m model; both studied with three alternative pier layout.

Bridge model, 90 m

Relevant bridge models for this section is shown in figure 5.4, with spans equal to 45, 30, 22.5 meter.

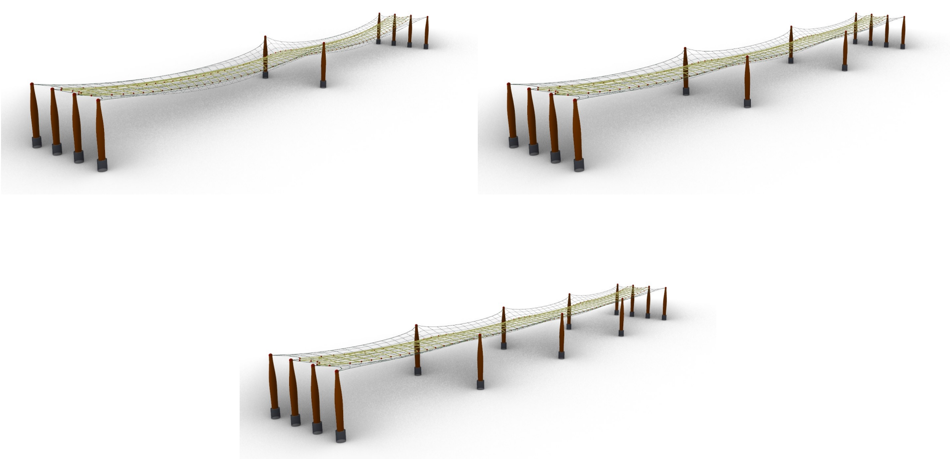


Figure 5.4: Alternative pier layout

1^{st}Iteration

For the initial design, spiral cable type, PG 90 ($A = 572mm^2$), and locked coil cable type, PV 240 is used for the internal net and edge cables. Furthermore, a prestress level equal 20% of tensile limit Z_R, d is applied, and UDL in SLS is used for simplicity. The most relevant data is presented in the tables below.

Iteration 1, prestess 20%	***Cable description***			
Net weight [kg]	*Edge cable*	*Cable along shell*	*Cable net x-dir*	*Cable net y-dir*
9605	PV240	PG90	PG90	PG90
Span [m]	*Deflection [mm]*	*Limit value span/200*	*Reaction x-vector [kN]*	*Reaction y-vector [kN]*
22,5	840	112	776	695
30,0	1078	150	948	712
45,0*	1952	225	1194	411

Table 5.2: 1.st iteration; structural members, deflection & support reactions
*Design case buckled

Some explanations:

- Column *limit value* (in table 5.2) of 'span/200' is picked from the European Norm on timber bridges [[37]], and used as the limiting values for deflections. However, the cable net do not really 'care' for these values, however, the limit might be relevant for the grid shell – which experience the same movements.

- Column *Reaction*, (in table 5.2) present the peak [horizontal] reaction force at the supports (top of pier).

- These results are induced by SLS loading, and prestress of 20 % design limit. Yet, the eigenfrequencies are calculated without variable loads, hence: prestress, self-weight of net, grid shell, and surfacing, plus an additional load of 15 kg at every *net-to-edge* joints (where the net connects to the edge cable).

Span [m]	*Eigenfrequency [Hz]*	*Cable description*	*Tension [kN]*	*Normal stress [mPa]*
22,5	1.30	Edge cable	560	340
	1.40	Along shell	284	490
	1.50	Cable net x-dir	267	490
	1.59	Cable net y-dir	370	630
30	1.87	Edge cable	530	330
	1.11	Along shell	382	660
	1.15	Cable net x-dir	309	530
	1.36	Cable net y-dir	419	630
45*	0.73	Edge cable	814	320
	0.77	Along shell	657	1150
	0.85	Cable net x-dir	361	620
	1.14	Cable net y-dir	694	770

Table 5.3: 1.st iteration; frequencies & cable actions
*Design case buckled

Results

Table 5.3 show the four lowest eigenfrequencies for each design case. From modal displacement, these are all vertical oscillations. From literature (in chapter 2), critical frequency range 1,2–2,3 Hz in vertical direction. In that case, none of these layouts seem convincing furthermore, the cables are oversized, leading to an unfavourable mass. However, the prestress is low, and so are the stiffness – so much can change.

Obstacles

For the 45m span alternative, the cable net buckled under given SLS loading. Buckling, being a secondary effect caused by compression, is an unexpected phenomenon for a tension membrane yet, it occur. The failure break down to conditions given in the Kangeroo components for dynamic relaxation (form-finding method). With a certain deflection/sag, the mesh fail hold their position, and the solution become unstable / do not converge.

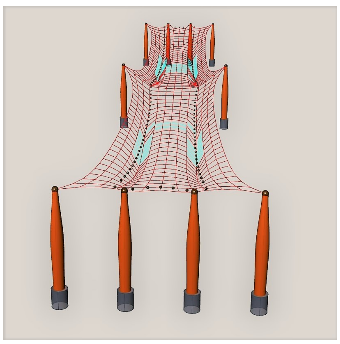


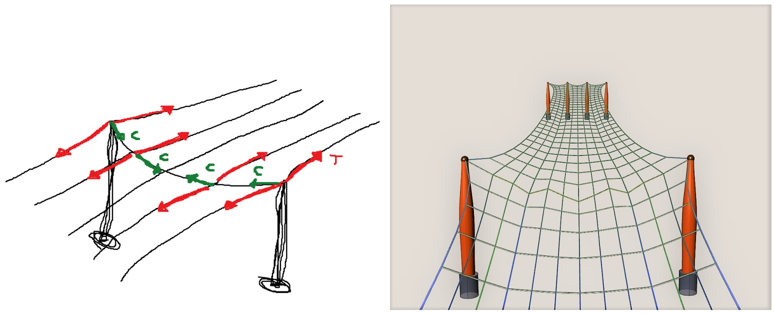
Figure 5.5: Buckled members

Discussion

A *quick-fix* to overcome this buckling issue is by change the dynamic relaxation limits and allow for a longer sag of the net. However, to compromise the 'architectural' shape is rash move. The obvious problem here is the wide deflection values – an effective measure would be to increase the prestress.

Obstacles after 2nditeration

In order to overcome the buckling mode as shown in figure 5.5, an increased prestress is necessary. However, by doing so, a new concern arise. Although high prestress certainly works fine during a 'heavy' load case, the net mush also work for none service loads, or worst, a case with wind from underneath lifting the net upwards. Given the shaped net, stresses in the longitudinal cables initiate a compression strut in the transverse cable between the pier – as sketched in figure 5.6a. For heavy loading, gravity control, yet with cases for upward lift the net buckles for prestress exceeding a third of allowed design force. Figure 5.6b illustrate the concern.



(a) Compression struts　　**(b)** Cable folds

Figure 5.6: Buckling issues in cable net

Discussion

Given the difficulty to reach a prestress suitable for all load magnitudes, some measures ought to be done. The prestress required to hold the displacement within a reasonable range for heavy loading, is not compatible with minimum loads. Consequently, a compression member placed transversely between the piers is recommendable – if not unavoidable. With such reinforcements, the problem may be solved, and the model is 'safe' to scale into a more realistic size.

Bridge model; 200 m

Henceforth, the model shall equal 200 meters. That is about the full length of the crossing (if considering an independent bascule part of fifty meters). With the previous issues on the two-span option in mind, a two-span layout for this model is particularly of interest. However, a compression member must be installed.

Introduce compression strut

When introducing a circular hollow section (CHS 70x3 [38]) between the middle piers, the prestress may reach 45% of design limit before buckling – without a beam it yields at 15 %. However, the current cable design is not suitable for the 100 m span; given an maximum prestress of 100%, the net still deflect 5,5 m at mid-span, while the limit is 0,5 m; the net-cables must enlarge. The results from the first iteration is covered in appendix D.1.

2nd Iteration

In this iteration, the prestress equals 90% of design limit. Table 5.4 and table 5.5 present the most relevant data for the two-span case.

Iteration 1, SLS-loading		***Cable description***		
Struc. weight [kg]	*Strut @ pier*	*Edge/shell cable*	*Cable net x-dir*	*Cable net y-dir*
52050	HFRHS-300x20	PV360	PV240	PG90
Span [m]	*Deflection [mm]*	*Limit value span/200*	*Reaction x-vector [kN]*	*Reaction y/z-vector [kN]*
100	2120	500	6330	999/1566

Table 5.4: 2nd iteration: structural members, deflection & support reactions

Span [m]	***Eigenfrequency [Hz]***	***Cable description***	***Axial [kN]***	***Normal stress [mPa]***	***Utilisation***
	1.35	Edge cable	2674	1070	74
	1.36	Along shell	2622	1230	85
100	1.40	Cable net x-dir	2035	910	63
	1.43	Cable net y-dir	370	640	60
	1.57	HFRHS-section	1865/-794	120	34

Table 5.5: 2nd iteration: frequencies, beam and cable actions

Discussion

A note for table 5.4 is the structural weight which is high compared with 1st iteration. The net has increased from 9050 kg to 45178 kg in pure steel. In addition *point-masses* from the timber shell, connection components and surfacing is included in this interaction. This add roughly 7000 kg on top of the 45 tons.

From the peak *normal-axial-stress* values, the cables are reasonably utilised by comparing to yield stress (1440 mpa for locked coil, and 1546 mPa for spiral cables). However, only two load cases are used for these iterations – and without load factors. The following step shall therefor address more critical load cases for specific parts of the net, in ULS. Thus it is likely that current dimensions may not restrain.

5.3 Load cases; UDL

This section investigate the structural feasibility of a cable net serving as a spatial pedestrian bridge. To determine those questions, four load cases are considered, and possibly decisive. However, due to relevance and time restrains, some load considerations is neglected: accidental, and temperature load. Furthermore, snow loads is also removed in order to prevent moving point loads from the service vehicle. Cables do generally poorly under heavy points loads [35] – so a much appreciated measure. This leads however, to a requirement of heated in the deck during winter.

Known statics for continuous beams (for two, three, and four span) is used in approaching this analysis (see figure D.2 for further illustration). Karamba is not a preferred software if lots of load combinations ought to be considered, nor cases of moving loads (for such work influence lines could be a pragmatic tool). The goal of this analysis is an close estimate of load scenarios which is likely to be decisive for the cable structure. The load combination used is group 1 from table 2.4, and four load cases are considered:

- #1: Uniform loading, full length
- #2: Unsymmetrical loading, along width
- #3: Uplift, full length
- #4: Unsymmetrical loading, along length

Structural input

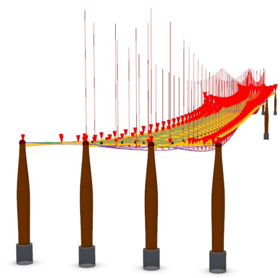
Cable types and sizes used are presented in table 5.6. All cables are corrosion protected with galfan coat with an E-modulus of 160 000 mPa. The PV-type is *locked coil* cables (two or three layers) while PG-type consist of 61 spiral strands (see figure 5.1 for illustration).

Cable properties	Edge/shell cable	Net cable x-dir*	Net cable y-dir**
Cable type	PV490/VVS-3	PV240/VVS-2	PG125/1x61
CS Area [sqmm]	3390	1650	769
Breaking load [kN]	4890	2380	1189
Tension limit [kN]	2964	1442	721
Yield stress [mPa]	1442	1442	1546

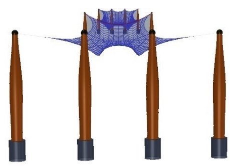
Table 5.6: Cable properties. *longitudinal, **transverse [1]

5.3.1 Load case #1

Load case #1 include uniformly distributed load (UDL) along the full length and width of deck, as shown in figure 5.7a. This case would challenge the middle pier, the beam connecting the piers as well as the edge cables closest to the supports. Deformation is shown in figure 5.7b.



(a) Loaded net #1



(b) Deformed net #1 [scaling= 1]

Figure 5.7: Load case #1 for two-span model

Results

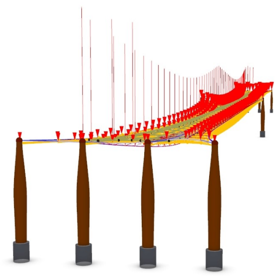
Output from this load case is shown in table 5.7. Peak deflection was measured to 1.91 m.

Load Case # 1 ULS	Axial force [kN]	Normal stress [mPa]	Characteristic strength [mPa]	Utilisation of char. yield strength
Edge Cable	3588	1040	1442	72%
Along shell	3520	1100	1442	76%
Net x-dir	2010	1220	1442	85%
HFRHS	2500	180	355	51%

Table 5.7: Outputs; Load case #1 for two-span model

5.3.2 Load case #2

Load case #2 is an unsymmetrical load case where live loads from wind and pedestrians act on one side of the width, and transferred along one edge. This scenario may create highest local stress on the 'along shell' cable. Load and deformation is shown in figure 5.8.



(a) Loaded net #2

(b) Deformed net #2 [scaling= 1]

Figure 5.8: Load case #2, for two-span

Results

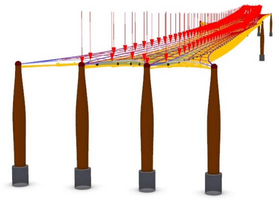
Output from this load case is shown in table 5.8. Greatest deflection was measured to 1.71 m.

Load Case # 2 ULS	Axial force [kN]	Normal stress [mPa]	Characteristic strength [mPa]	Utilisation of char. yield strength
Edge Cable	3403	540	1442	37%
Along shell	3448	1170	1442	81%
Net x-dir	1935	1070	1442	74%
HFRHS	930	380	355	107%

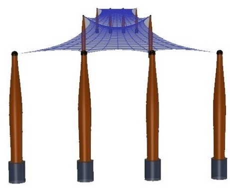
Table 5.8: Outputs; Load case #2

5.3.3 Load case #3

Load case #3 address a scenario with no live loads in the direction of gravity, instead wind from underneath lifting the structure and create peak compression in the RHS beam member connecting the piers. Wind actions are shown as yellow 'line-load' in figure 5.9a, and deformation is shown in figure 5.9b.



(a) Loaded net #3



(b) Deformed net #3 [scaling= 1]

Figure 5.9: Load case #3, for two span model

Results

Output from this load case is shown in table 5.9. Peak deflection was measured to -1.05 m (upwards).

Load Case # 3 ULS	Axial force [kN]	Normal stress [mPa]	Characteristic strength [mPa]	Utilisation of char. yield strength
Edge Cable	2590	750	1442	52%
Along shell	2320	660	1442	46%
Net x-dir	1590	960	1442	67%
HFRHS	-1147	65	355	18%

Table 5.9: Outputs; Load case #3 for two-span model

5.3.4 Load case #4

Load case #4 aim to create a worst case scenario for the longitudinal cables along at back-spans. The basis for this combination is illustrated in figure 5.10. Loads and deformation is shown in figure 5.10.

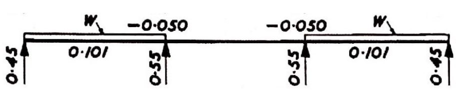
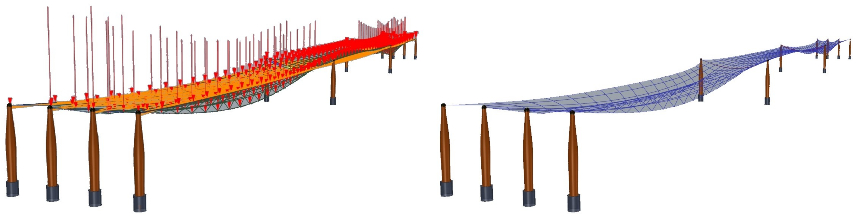


Figure 5.10: LC#4: Asymmetric load along length



(a) Loaded net **(b)** Deformed net [scaling= 2.5]

Figure 5.11: Load case #4, for three-span model

Results

Output from this load case is shown in table 5.10. Peak deflection was measured to 0.85 m.

Load Case # 4 ULS	**Axial force [kN]**	**Normal stress [mPa]**	**Characteristic strength [mPa]**	**Utilisation of char. yield strength**
Edge Cable	3390	1000	1442	69%
Along shell	2845	900	1442	62%
Net x-dir	1845	1110	1442	76%
HFRHS	770	135	355	38%

Table 5.10: Outputs; Load case #4, for three-span model

5.3.5 Discussion

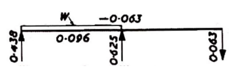


Figure 5.12: Alternative LC#4

The cable net deal alright with load magnitudes, as shown in table 5.11. In retrospect, LC #4 should rather be executed on a two-span layout (shown in figure 5.12) as the span-length surly controls for 100 m vs 67 m cases. However, the deflections are not within a suitable domain anyhow. LC#1 report deflection of 1,91 m, and would even been higher if figure 5.12 was applied. With the presumed deflection limit of span/200 (0,5 m) greater measured mush be conducted in a SLS analysis. Yet, structurally, the cable net is adequate. Regarding the HFRHS-300x20 beam, load case #3 only achieve 18% utilisation of yield stress during compression, however, buckling is the real concern. Karamba consider secondary effects and with a tube thickness of for example 10 mm, local buckling would occur hence 20 mm. Regarding LC #2, the RHS beam exceeds the yield stress due to moment about the y-axis. This issue may be solved by expand the thickness (10 mm), or increase the height.

Utilisation [%]	Case #1	Case #2	Case #3	Case #4
Edge Cable	72	37	52	69
Along shell	76	81	46	62
Net x-dir	85	74	67	76
HFRHS	51	107	18	38
Deflection [m]	1,91	1,71	-1,05	0,85

Table 5.11: Overview; Characteristic utilisation and displacement

5.3.6 Grid shell design

GL 32h is used for analysis and the input is given in figure 5.13.

```
Material: Glulam '32h' applies
to elements: 'Timber grid';
E1:1110[kN/cm2] E2:46[kN/cm2]
G12:85[kN/cm2] nue12:1.62
G31:0.38[kN/cm2] G32:0.38
[kN/cm2] gamma:4.22[kN/m3]
alphaT1:5.0E-6[1/C°]
alphaT2:5.0E-6[1/C°] fy1:3.2
[kN/cm2] fy2:0.05[kN/cm2]
```

Figure 5.13: Material input, karamba

'Homogeneous' glulam is selected based on literature books ([35], [36]). Also, the cross section (90x90) is used for the first iteration which is the smallest of the commonly used cross-sections for Norwegian glulam. The structural inputs (in karamba) are listed in figure 5.13. The values originate form EN 1194.

Glulam strength class		GL 32h
Bending strength	$f_{m,g,k}$	32
Tension strength	$f_{t,0,g,k}$ $f_{t,90,g,k}$	22,5 0,5
Compression strength	$f_{c,0,g,k}$ $f_{c,90,g,k}$	29 3,3
Shear strength	$f_{v,g,k}$	3,8
Modulus of elasticity	$E_{0,g,mean}$ $E_{0,g,05}$ $E_{90,g,mean}$	13 700 11 100 460
Shear modulus	$G_{g,mean}$	850
Density	$\rho_{g,k}$	430

Figure 5.14: EN 1194; units [mPa] & $[kg/m^3]$

Design process

For this analysis, two load cases are applied:

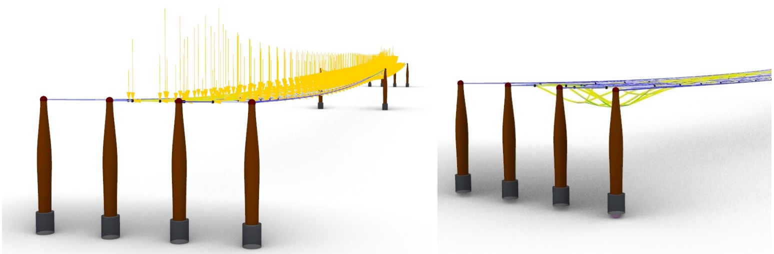
- #1: Uniform loading, full length

- #2: Unsymmetrical loading, along width

These load cases are similar as in section 5.3.

Load Case #1

Uniform loading over the length and width, as shown in figure 5.15a.



(a) Symmetric UDL

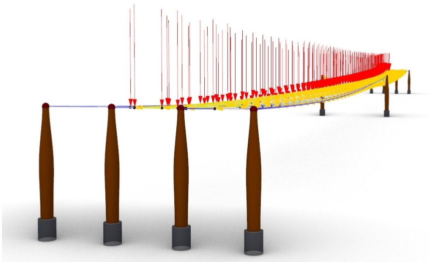
(b) Inadequate form

Figure 5.15: Load case #1

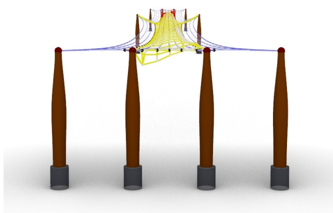
For this load case the grid shell do well for the major parts of the span – where the grid shell works as a compression membrane. However, at abutments the grid is no longer curved, but straight. This results in failure and illustrated in figure 5.15b.

Load Case #2

Uniform loading at half the width of deck; shown in figure 5.16a.



(a) Asymmetric UDL



(b) Side way deformation

Figure 5.16: Load case #1

The grid shell do poorly for these loads. The cross sections is neither sufficient near the abutments nor the mid spans. The unsymmetrical deformation is illustrated in figure 5.16.

Discussion

The failure shown in figure 5.15b is understandable when the grid's shape no longer allow for the efficient *membrane theory* to apply. Three feasible solutions are:

1. Replace the critical grid arches with steel tubes.

2. Prevent crowd loads at the critical parts of the grid (at both ends).

3. Decrease mesh geometry (density the grid pattern)

Text and figures of these measures are included in section D.3 and concludes with: *Seven rows of HFRHS 180x6.3 tubes to be installed at each end, stretching at least ten meters into the span.*

Concluding results

Results from this study is presented in table 5.12, with utilisation values which reflecting on both the characteristic, and design strengths of GL 32h. As table 5.12 imply, load case #1 and #2 seem pretty indifference to the timber members, yet for deflection, load case #2 doubles the magnitude. For the steel elements, load case #1 controls. In retrospect, load case # 4 (as in fig. 5.12) should have been considered. however, the outcome would likely be pretty similar – having in mind the minor difference between case #1 and #2.

Load Cases	Members / Area [mm^2]	Normal Stress [mPa]	Strength [mPa]	Utilisation f_ed/f_y	Utilisation f_ed/f_d	Deflection [mm]
#1/ #2	*GL (180x115) A= 20700*	18.5 / 18.7	fy=32 fd=23	58 / 58	80 / 81	90 / 189
#1/ #2	*RHS (180x6.3) A= 4320*	255 / 142	fy=355 fd=338	72 / 39	75 / 41	30 / 30

Table 5.12: Results, grid shell analysis

5.4 Serviceability, design requirements

This section focus on the practical aspects of this bridge study. It address the service limits with respect to deflection of the superstructure and dynamic behaviour.

5.4.1 Deflection analysis

As demonstrated earlier, deflection of tensile membranes depend on loading, service loads and prestress. In reality, its even more complicated. cable net are *aeroelastic* structures which change character depending on shape, and loading however, matters of non-linearity and aerodynamics are outside this scope. For these reasons, the net deflection is not so interesting, yet the the grid shell is. The grid shell is influence by the moving bridge alignment, so the service limit applied in the net-design, is in practise controlled by the grid shell. Henceforth, the service limit for timber footbridges is adapted: $span/200$ [37].

Even though deflection limits are assumed, the behaviour of the bridge and the construction sequence of the superstructure is essential in order to apply these principles logically. For the superstructure, the cable net must be installed and prestressed before the grid shell can be constructed. For some structures, pre-tensioning and post-tensioning are both possible to execute. However, for this cable net, after the grid shell is installed, the possibility to post-tension is not there (by doing so, the grid shell breaks). Therefore, the net's prestressed state become the initial condition for the grid shell; and at loading, the deformation become the difference between the initial state and the loaded state (*method*: deformation is calculated by karamba for each LC, a component store the data and deflection is calculated through list operations in grasshopper).

Load estimates

For this SLS-analysis, load case #1, #2, and #3 as illustrated in figure 5.17 is applied on the 'two', 'three', and 'four' span layouts. These may produce the greatest deflection (change in cable sag) and thus the critical scenario. This section cover an iterative process for each design option. Yet, the results are presented in section D.4.

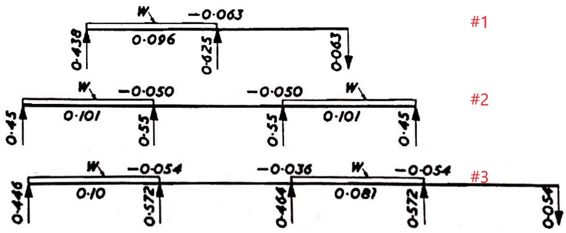


Figure 5.17: Relevant load cases, 1-3

On advice from Raj Janmenjay, a prominent bridge designer for Jacobs Engineering, prestressing levels in cables are determined as a function of the ultimate tensile strength (UTS) (breaking load / sectional area). Henceforth, prestressing is described as a percentage of UTS. Normal prestress levels lay at 20-35% UTS.

Discussion

These deflection iterations lead to large increase of cable dimensions, yet none of the span-alternatives brought convincing results. The two-span model deflected 860 mm with a prestress of 50% of UTS (an required 85% UTS to reach 500 mm). The three and four span models fulfilled the service limits with 50% UTS, however only with a small margin (90-95%) against allowed tensile limit. Consequently, this study may proceed with the 67 m and 50 m span alternatives, and exclude the two-span model for reasons related to deflection. In order to proceed further with the two-span model, double-layers of cables ought to be considered at the critical places.

The deflection limit of span/200 is challenging for the cable structure. Cables net are mentioned really *aero-elastic* structure with large deflection and behave non-linearly. While, this analysis exclude time-history, and not long term effects are included (prestress losses). In other words, Karamba is not quit adequate for thorough net design. With that in mind, a simplified analysis of deflection is conducted, and the target of span/200 was assumed for pragmatic reasons – with the grid shell in mind.

5.4.2 Dynamic analysis

This section consider the natural frequencies (eigenfrequencies), of the cable net structure. Some conservative approximations are made: The natural frequencies retrieved by karamba only account for the cable net itself plus added modal mass to reflect the weight of grid shell and surfacing. Although in reality the grid shell also provide stiffness in which, puch the frequencies toward 'better' values. While higher mass lower the frequencies.

The frequency domain is evaluated according to the European Norm and Hivoss guidelines for footbridges. If relevant, peak accelerations are estimated by spectral analysis (see section 2.4. Hivoss specify the following critical range of eigenfrequencies (f_i in Hz):

- For vertical and longitudinal vibrations:

$$1,25 \leq f_i \leq 2,3$$

- For lateral vibrations:

$$0,5 \leq f_i \leq 1,2$$

Acceleration limits

In cases where f_i lay within critical range, a control of peak accelerations is needed. Figure 5.18 illustrate the four comfort classes defined by Hivoss. *CL 4* is unacceptable in service.

Comfort level	Degree of comfort	Acceleration level vertical	Acceleration level horizontal a_{limit}
CL 1	Maximum	< 0.50 m/s²	< 0.10 m/s²
CL 2	Medium	0.50 – 1.00 m/s²	0.10 – 0.30 m/s²
CL 3	Minimum	1.00 – 2.50 m/s²	0.30 – 0.80 m/s²
CL 4	Unacceptable discomfort	> 2.50 m/s²	> 0.80 m/s²

Figure 5.18: Hivoss acceleration limits [7]

Cable net

Cables used for this dynamic analysis is shown in table 5.13. An observation during the deflection analysis (section **??**) was high stresses in the transverse cables near the abutments. For that reason, doubled lay PG 125 are used at both ends, closest to abutment.

Cable properties	Edge/shell cable	Net cable x-dir*	Net cable y-dir**
Cable type	PV2000/VVS-3	PV1450/VVS-3	PG125/1x61
CS area [sqmm]	13900	10100	769
Breaking load [kN]	20000	14500	1189
Tension limit [kN]	12121	8788	721
Yield stress [mPa]	1442	1442	1546

Table 5.13: Cable properties. *longitudinal, **transverse

Three span model

Table 5.14 list dynamic properties for mode 1-4, given 67 m span layout and prestress of 50% UTS.

Mode no. [#]	Eigenfrequency [Hz]	Modal mass [kg]	Mode shape (Oscillation)
1	1.835	74779	Vertical
2	1.846	40553	Vertical
3	1.863	93430	Lateral + Torsion
4	1.885	82225	Vertical

Table 5.14: Dynamic behaviour, cable net 3-span

Figure 5.19 show the first vertical and lateral+torsional mode (scaled by 5,2).

(a) Mode 1, vertical oscillation

(b) Mode 3, lateral oscillation

Figure 5.19: Mode 1 and 3, for three span model

Accelerations

Given the eigenfrequencies in table 5.14, peak accelerations shall be assessed for relevant modes – and controlled against comfort limits. In this case that, the critical modes are either mode #1 or mode #2.

Dynamic input				**Peak acceleration** $[m/s^2]$			
Mode [#]	*F_i [Hz]*	*Modal mass [kg]*	*Red. coef.*	*d=0.2* $[p/m^2]$	*d=0.5* $[p/m^2]$	*d=1* $[p/m^2]$	*d=1.5* $[p/m^2]$
1	1.835	74779	1,0	1,19	1,87	2,15	2,03
2	1.846	40553	1,0	2,19	3,47	3,97	3,76

Table 5.15: Peak accelerations; three-span cable net

Table 5.15 show the peak acceleration with spectral analysis (see calculations in section E.2). The reduction factor, ψ equals 1,0 as the frequency lay between 1,7 and 2,1 (most critical range). The damping ratio, ξ is assumed as 2%, which is not insignificant. Damping is a complicated matter, and this ratio is determined upon advise from bridge expert and designer Raj Janmejay from Jacobs Engineering. For single cable structures such as a suspension bridges, a normal ξ is 0,5%. However, this bridge has a net with cables in both directions and with *cable cross clamps* at each intersection. Those connectors provide stiffness however, as the transverse cables are small compared to the longitudinal, the ratio is reduced with 1% from an originally advised ratio of 3%.

Mode #2 in table 5.15 show values above the maximum limit of 2,5 m/s^2 for all cases of *dense* pedestrian streams ($d > 0.5 person/m^2$). This imply that the bridge cause unacceptable discomfort in service – unfortunately.

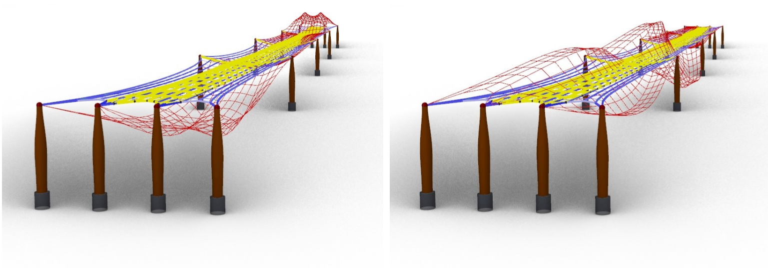
Four span model

Table 5.16 list dynamic properties for mode 1-4, given 50 m span layout and prestress of 50% UTS.

Mode no. [#]	Eigenfrequency [Hz]	Modal mass [kg]	Mode shape (Oscillation)
1	2.321	62263	Vertical
2	2.372	59366	Vertical
3	2.485	28163	Vertical
4	2.507	57842	Lateral + Torsion

Table 5.16: Dynamic behaviour, cable net four-span

Figure 5.20 show the first vertical and lateral+torsional mode (scaled by 5,2)



(a) Mode 1, vertical oscillation

(b) Mode 4, lateral oscillation

Figure 5.20: Mode 1 and 4, for four span model

Accelerations

Given the natural frequencies in table 5.16, no check of peak acceleration is needed as all the modes go clear from the critical domain. For values above 2,3 the reduction factor, ψ

equals zero. Hence, uncomfortable peak acceleration is not likely to occur – according to Hivoss's empirical data.

Grid shell

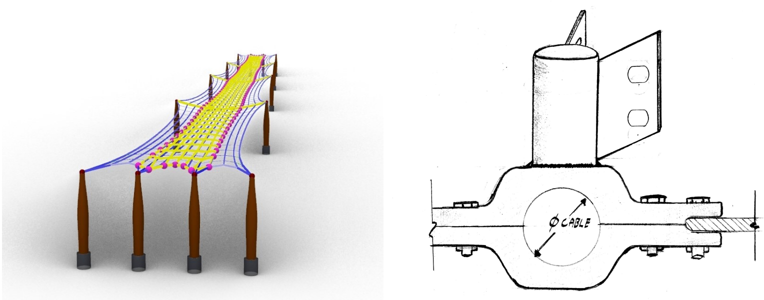
For the grid shell, dynamic vibrations is not a concern due to the many supports along the net. Yet a short discussion is included in section D.5.

5.5 Structural components

This section include two design suggestions related to joints and anchoring. This section is somewhat pragmatic by pointing in directions of similar engineering solutions, solved before.

Shell to net joint

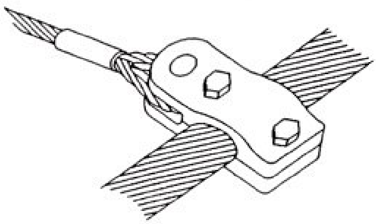
At every 1.62 m along the grid shell, there is an steel joint clamped to the locked coil cable (with a diameter of 140 mm), as marked in figure 5.21a. In addition to connect the longitudinal and transverse cables, the joint also anchor the grid shell and its kinking force. A customised joint is sketched in figure 5.21b, to illustrate a possible arrangement.



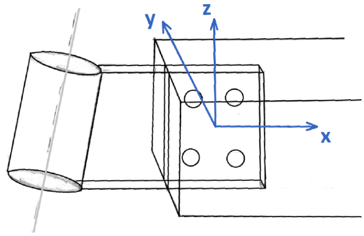
(a) Locations, pink bullets   **(b)** Customised joint, sketch

Figure 5.21: Shell to cable net connection

The customised joint spring out from two components more commonly used, and illustrated in figure 5.22.



(a) Cable to edge cable

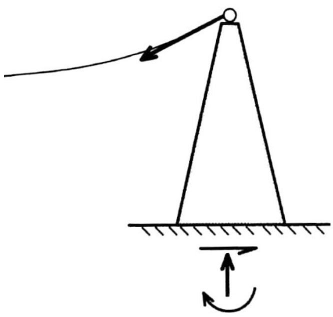


(b) Grid shell link [39]

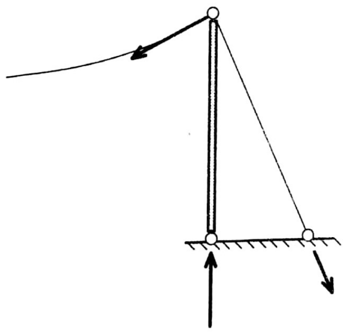
Figure 5.22: Inspirational joint types

Cable anchoring

Cantilever piers or guyed masts are two common ways to anchor a cable with a large horizontal component. A cantilever pier, shown in figure 5.23a, can be used within a reasonable height. Yet, for tall piers place at water level or in a pit, an cantilever pier might be extravagant. In such cases, a guyed mast is a good solution, given the ground conditions allow the tension tie as shown in figure 5.23b.



(a) Cantilever pier



(b) Guyed mast

Figure 5.23: Anchor principles [5]

For this study, the ground conditions is uncertain. Therefor guyed mast might be demanding – depending on the force magnitudes. For the four span outline with 50 m pier spacing, the highest transverse support reaction (y-dir) is 2500 kN (ULS). The anchor force also depends on the angle of guy. By further assuming an 25° angle between mast and guy, the tension tie become $2500/cos(90-25) = 5915kN$; the PV 1100 locked coil cable (∅ 100 mm) do a breaking load of 11100 kN and a tension limit of 6121 kN. Furthermore, for the mast, the peak vertical force from superstructure is 772 kN, that plus the vertical component of the tie become $772 + 5915 * sin(90-25) = 6130kN$ for the strut.

Such magnitudes would suggest a steel or concrete pier. (If compared with timber, No.2 impregnated GL 32h member of the largest dimensions 180x400 [35], we reach load carry capacity of 1900 kN when including buckling effects and service class 3.)

On this this reason a steel mast is recommended. Figure 5.24 show two guyed masts in steel used for tension structure, Elbsteg Herrenkrug footbridge (cable stay) and Lodz footbridge (cable net).

(a) Lodz Footbridge, Stuttgart 1992 [40] **(b)** Elbsteg Herrenkrug, Magdeburg 1998[1]

Figure 5.24: Inspirational guyed masts, tension structures

For the abutments, the reaction forces are predominantly horizontal. The two inner anchor supports experience a horizontal force of 28,5 MN in ULS, while the Millennium bridge experience about 30 MN [41] on their abutments. Such magnitudes require substantial anchoring and massive concrete piles. The Millennium bridge anchored their cables down to the 3m pile cap by strut and tie modelling at each side [41]. This thesis propose doing exactly the same.

5.6 Results

This section tabulate the most relevant structural output for the concluding bridge design with 50 m pier spacing. Karamba is used as FEM-solver and provide the outputs for this section.

General overview

Table 5.17 give a general overview of the bridge's specifications. The prestress is optimised to 42% of UTS and lead to deflections and vertical accelerations barley underneath the service limits (Hivoss guidelines and the European Norm for pedestrian timber bridges).

Concluding Structural Outputs		Limit*	Comment
Bridge width [m]	12	-	
Deck width [m]	6	6	City-scheme
Bridge height [m]	4	-	
Bridge span [m]	50	-	
Selfweight [t/m]	1,64	-	
Deflection [cm]	24,7	<25	EN 1995
Fi_vertical [Hz]	2,13	>2,3	check a_max
a_max_vertical [m/s^2]	2,39	<2,5	OK.
Fi_lateral [Hz]	2,13	>1,3	OK.
a_max_lateral [m/s^2]	-	<0,8	n/a

Table 5.17: Result outputs; prestress 42% UTS. * miscellaneous design limits

Figure 5.25 and 5.26 show the a full outline of the design proposal including footpath, handrail and substructure. These details are included for visual purposes as structural design is not conducted. The substructure is outlined with existing bridges in mind; abutments and piles are scaled after London Millennium Footbridge.

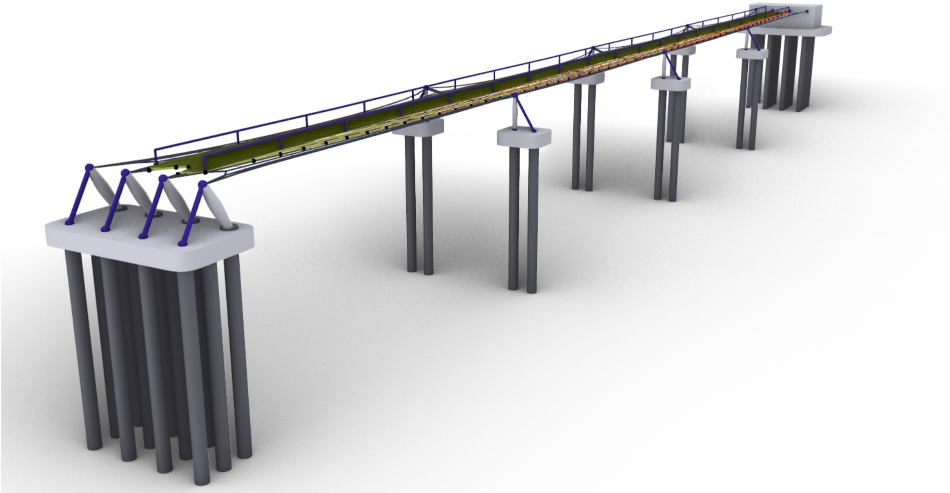


Figure 5.25: Proposed bridge design, with piles

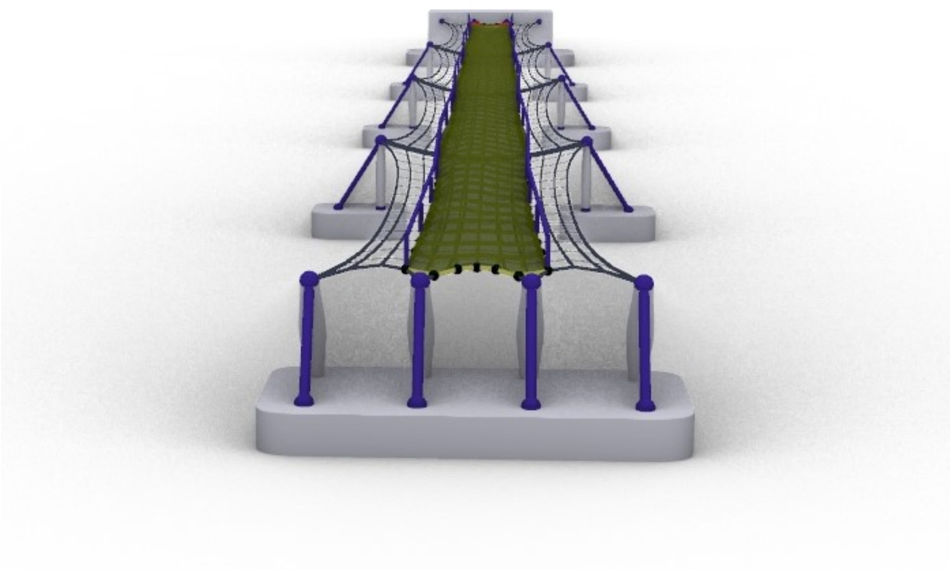


Figure 5.26: Proposed bridge design

Superstructure

Structural outputs of proposed cable net and prestress are presented in table 5.18.

Members	Prestress [kN]	Axial force [kN]	Tension limit [kN]	Normal stress[mPa]	UTS [mPa]	Utilisation of UTS	Load Case #
Edge cable	8400	9140	12121	630	1442	44%	1
Cable@shell	8400	9986	12121	710	1442	49%	2
Net x-dir	6900	8250	8788	775	1442	54%	4
Net y-dir	-	695	721	1100	1546	71%	0
RHS-beam	-	-3972	-	93	355	26%	3

Table 5.18: Concluding utilisation, cable net

A description of controlling load cases are given in table 5.19.

Controlling Load Cases				
#0	*#1*	*#2*	*#3*	*#4*
Self weight	UDL, full length	Uneven loading along width	Self weight + upward wind	Uneven loading along length
-	see fig. 5.27 and 5.7	see fig. 5.8	see fig. 5.9	see fig. 5.27 and 5.11

Table 5.19: Prominent load cases, 50 m bridge span

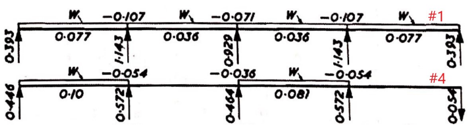


Figure 5.27: Illustrated load case, #1 and # 4

Chapter 6

Closure

This chapter summarise the thesis in large and point out directions for future work. The chapter has four sections:

– **Summary** – background and justifications for the study.

– **General conclusions and recommendations** – summarise conclusions, and discuss implications of the results onto society.

– **Conclusions from case study** – reflections on concluding results and relevance to other cases.

– **Future studies** – highlight issues and alternative ideas that deserve further study.

6.1 Summary

The context of this study is the channel dividing Tønsberg and Nøtterøy, south of Norway. In September 2019, Vestfold county proposed a *city transportation scheme* that recommend a new footbridge connecting the banks – 250 m apart. Yet, a precise axis was not described. By correspondence with technical representatives for the scheme, a thesis that address a concept study of a new footbridge was discussed and warmly welcomed. That lay the basis of this report and problem statement.

The work consists of:

- A conceptual bridge study – feasibility study of different structural concepts, and its contextual relevance.
- Parametric modelling of two structural concepts; stressed ribbon bridge, cable-net bridge.
- Case study for a 200 m long cable-net bridge.

The objectives, described in section 1.3, scaled the scope of this project within the superstructure. After a feasibility study and work with parametric models, a concept with cable-nets was found. The concluding net structure fulfil the design objectives and achieve a light-weight design. Although, it is understood due to the aeroelastic nature of the deck, it is important to investigate the aerodynamic effects of the aeroelastic structure as well, but in this thesis the study has been limited to Human Induced static and cyclic loading only. Dispite these simplifications, a generic design was not fully achieved. For instance, a heated deck is required to prevent snow loads and moving service vehicles, – given Karamba's incompatibility with *time-history analysis* / moving loads. Another design flaw are the horizontal anchoring at sea, in order to open for a bascule function.

6.2 General recommendation

This study leads to several thoughts and conclusions, both of general and specific matters. Structural features with an appealing tectonic look was a key objective in the design process. The proposed design, as shown in figure **??**, offer a readable structure for the public eye, on the other hand, in regard of aesthetics the bridge may posses the elegance that characterise the beautiful bridges around the world. By increasing the span, the net could sag deeper and possibly improve the visual look with a more prominent parabolic curvature.

In regard to structural relevance to site, this concept struggle on some aspects. In large, to anchor high horizontal forces in soft soil and urban areas is not advisable (due to Newton's third law of motion). This issue is stressed further by being a bascule bridge, since the anchoring take place at sea. The idea of tension structure, without pylons, serving as a footbridge is rare alone – even without a bascule part. Two similar structures are:

- London Millennium Footbridge (Millennium bridge) crossing River Thames. It use longitudinal cables to carry a stiff bridge deck however, the bridge struggled with lateral movement (*wobbling*), due to her lateral eigenfrequencies [41].
- Lodz Footbridge in Stuttgart. It use cable-net to carry an externally stiff steel deck. Yet, this bridge is in-land, with a wide net anchored several places in short intervals.

Ultimatly, this bridge is not considered highly relevant for its site, not because of the technical innovation or aero-elastic character, but because Tønberg ask for a bascule bridge. The difficult anchoring might explain why no such bridge exist at this scale – as far as this report is concern. Instead, this concept is better in areas with strong ground conditions and shallow water, so that long spans are not required.

6.3 Conclusion from case study

The case study started with a 90 m model and was expand into a 200 m model. Several obstacles came along to be solved. The bridge has been analysed in a simplified way by considering no moving structural nor dynamic loads – for such analysis Karamba is not recommendable.

However, a detailed bridge design was never the main objective for this study. The goal for this thesis was to develop a bridge concept, structurally plausible, light-weight and suitable to Tønsberg. The cable-net was the main scope, secondly the timber grid shell.

Cable net

The controlling parameter for the net design was deflection, a limit of span/200. The limit apply for Norwegian pedestrian timber bridges in serviceability, and was implemented on the net design as it carry a timber shell. One major simplification in the analysis is the uncoupled consideration of the net and shell, structurally. No additional stiffness from the shell are included in the net design – only contribution from mass. This make an conservative consideration since stiffness improve the structural and dynamic performance.

The concluding net design is excessive with large cables and high prestress levels. Recommended prestess for tendons according to bridge designer Raj Janmejay ranges 25% to 30% of UTS. This cable-net apply 42%, however under ultimate loading no cable exceed the tensile limit of ca 60% UTS (given from manufacturer). Also, the longitudinal cables have large dimensions. in comparison, London Millennium bridge use No. 8 ∅120 locked coil cables, turning $80800mm^2$ in section area. While this net used No. 4 ∅140 cables along edges of net and gridshell, in addition to N0. 8 ∅120 spaced in between, resulting $136400mm^2$ in an area of steel. That is cs 70% more, however by including transverse axis, this net only use No.1 ∅36 spiral stand at spacing's 1,62 m. Hence, the total weight of the superstructure become 1,64 tons/m, whilst the Millennium bridge weigh 2,0 t/m [41]. For this reason, a light-weight design is achieved. Although in the sense of cost, cables are expensive – and so are constructing pile caps at sea. So the cost of this construction may be higher- However, if comparing further, Arup did retrofit their bridge with installing comprehensive damping systems; tuned mass dampers and viscus damping along the full length as a consequence of lateral eigenfrequency below 1,3 Hz (presumably 0,8 Hz) [41]. This net have a lateral eigenfrequency of 2,5 Hz, which is outside critical domain. The use of transverse cables producing geometric stiffness while staying light, leads to comfortable lateral eigenfrequencies. Yet, ultimately the introduction of a RHS-beam between the piers is the most decisive measure in turning this concept plausible.

This report propose a bridge design with 50 m span. That is a modest length and shortest of the three spatial alternatives considered. The 50 m (four span) alternative is proposed because the 67 m (three span) alternative gave unacceptable vertical accelerations from spectral analysis. Two ways to improve such limitations are:

- Introduce cables in double layers, similarly as the Olympic station in Munich and Lodz Footbridge in Stuttgart. Such measures increase the applicable prestress for which reduce deflection. Yet, for the eigenfrequencies, the effect is more uncertain, though the damping ratio may increase some small bits with stiffer cable connectors however, such matters a never factual until the bridge is constructed.

- Reduce the net width; a narrow net is stiffer.In case of reducing the width (from 12 m) to 9 m, the first vertical eigenfrequency increase with ca 0,3 Hz.

Timber Grid Shell

The grid shell is design in glulam 32h, yet near the abutments, steel members is recommended. Steel is necessary at the first 10-12 m at each end where the curvature (yz-plane) is close to zero. However, in reality, this issue is more a question of 'form-finding' than practical detailing. In the model, the grid shell have anchor points along the transverse edge/end – which in not necessary. With more time at hand, this detailing could be solved in grasshopper and led to a thorough grid design.

6.4 Future studies

There are no complete solution to engineering problems – always details to improve or alternative ideas to test. This thesis aimed to design a footbridge from a conceptual angle, and study structural plausibility of a bridge concept not known at forehand. In order for this bridge concept to be truly competitive against existing and well established bridge structures, further studies is required. The next steps ought to be:

Although, it is understood due to the aeroelastic nature of the deck, it is important to investigate the aerodynamic effects of the aeroelastic structure as well but in this thesis the study has been limited to Human Induced static and cyclic loading only

- Import the parametric model into a comprehensive structural analysis software capable run time-history analysis, and combine the grid shell with the cable net to one structural element. Only then may the aerodynamic effects of the aeroelastic structure be fully examined, and long-term effects included. Also, in order to design sufficient expansion joints, change in strain due to temperature must be considered. LARSA 4D is a leading software much used for these purposes and may thus be recommended.

- Secondly, light-weight structures ought to process long stretching spatial abilities. A wider study on how to increase this span is needed. That may include a reduction of the bridge' width; 12 meter is a bit extravagant in a structural sense and from an environmental perspective. Also, an asymmetric pier layout is worth investigating as it might reduce the need for compression members between the piers.

- At last, a proper study of surfacing, footpath and handrail is necessary. Such details are important to the tectonic performance, and deserve sufficient consideration. In case of the Millennium bridge, much resources was invested to develop the geometry, and setting-out of architectural components. They research different material options and eventually assembled a physical *mock-up* models of the deck [42].

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Appendices

A Proposed Concept Analysis

This section address a concept analysis of the bridge designs outlined in section 3.2. The method is inspired by the literature writen by Prof. Knut Samset in the book *Early Project Appraisal* (2010).

The purpose of this analysis is to evaluate the different design concepts with some analytic measures. This concept analysis will emphasis and compare each design concept's performance in regard to:

- Aesthetics – simplicity and coherent tectonics as design virtues.
- Historical context – relevant to Tønberg's identity and material preference
- Structural context – relevant to the channel's width, depth and ground condition
- Originality – relevant as an object for parametric modelling

Score - Spider Diagrams

To concertise this concept analysis, sets of spider diagrams is used with a scoreline ranging $\in [-3, 3]$, where 0 is a neutral performance. The amount of area enclosed in each diagram hint towards the 'total' relevance for that specific concept.

	Aesthetics	*Historical context*	*Structural context*	*Innovative character*
Shell	0	-2	0	2
BCB	2	2	1	0
CSB	1	0	-3	-2
Arch	0	1	-2	0
SRB	2	2	2	1
Net	1	1	1	3

Table A.1: Score table

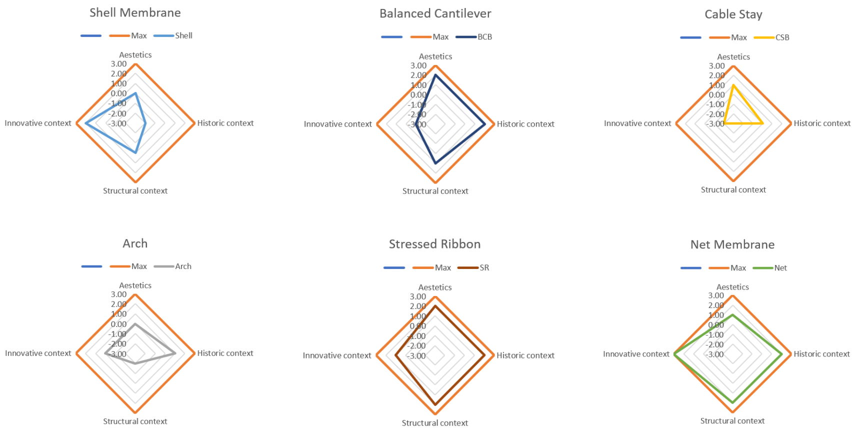

Figure A.1: Concept evaluation – Spider Diagrams [figure to be improved] write this properly

B Tension structure with beam elements

This section cover a design case of a small tension structure with a few simplified loads. The structure consists of pinned beam elements (with moment resistance) in a mesh quadratic mesh pattern, creating a net. Furthermore, a grid shell is also included to symbolise the pedestrian path however, the shell in not included in the analysis.

Table B.1 show an overview of the key input used. The vertical load – and horizontal loading on shell – parts are defined as 28 points loads at 1 m spacing – symbolising the joints connecting the net to footpath. The magnitudes of the points loads are not shown in table B.1, however they are calculated easily by an C# component using the estimated vertical loads, surface of shell and number of joints.

Design Input	**Size [m]**	**Weight [kg]**	**Supports #**	**Members [mm]**	**Vert. load [kN/sqm]**	**Hor. load [kN/sqm]**
Net (S355)	12x24	15130	4	Rec 50x70	N/A	1
Shell (GL30h)	6x24	366	28	60x80	12	1

Table B.1: Designs inputs for basic net

Although, loads and material properties are presumed, there are parameters for the dynamic relaxation yet to be determined. In theory, form finding can optimise the membrane to work solely in tension or compression however, the resulting shape of such design might be unpractical. Furthermore, the level of tightening (input strength) can produce some conflicting engineering concerns:

- If tight (high strength) net:
  - The net experience solely tension however, the stiffness attracts large axial forces which may exceed the capacity.
  - High horizontal reaction at the anchor points (top of column), leads to high moments in pier and pile caps.
- If loose (low strength) net:
  - The net experience both tension and compression forces and may exceed the total capacity.
  - unacceptable shape for service – overly curved net

For this reason, a few parameters ought to be determined manually as starting points for further iterations. After bits of trial and errors, the line reduction (determine size of net) is set to 40 %. The tables below present key outputs for the net considering a tightness strength factor of 2, 4 and 8, where 2 is low and 8 is high strength. These numbers are not the main interest; what interesting is the change in reaction forces and stress levels due to the different forms.

For design case in table B.2, a max displacement were measured to 14,5 cm downwards.

Reaction Force (x,y,z)	**Compression Force [kN]**	**Axial Force [kN]**	**Axial Utilization**	**Member Utilization**
119	-194	544	0.682	1.236
45	-147	486	0.432	0.992
38	-147	486	0.386	0.946
	-142	364	0.354	0.715
	-142	364	0.289	0.603

Table B.2: Design output for net in low tension; strength = 1,0

For design case in table B.3, a max displacement were measured to 17,5 cm downwards.

Reaction Force (x,y,z)	**Compression Force [kN]**	**Axial Force [kN]**	**Axial Utilization**	**Member Utilization**
341	-1	729	0.535	0.91
128	-1	674	0.426	0.602
38	-1	674	0.402	0.576
	-1	537	0.358	0.575
	-1	536	0.329	0.537

Table B.3: Design output for net in medium tension; strength = 3,0

For design case in table B.4, a max displacement were measured to 22,5 cm downwards.

Reaction Force (x,y,z)	**Compression Force [kN]**	**Axial Force [kN]**	**Axial Utilization**	**Member Utilization**
562	-1	970	0.738	0.963
212	-1	929	0.597	0.792
38	-1	929	0.578	0.792
	-1	752	0.577	0.782
	-1	752	0.561	0.769

Table B.4: Design output for net in high tension; strength = 5,0

For the reaction forces, listed as x,y,z vectors, the first value, x, represent the longitudinal direction. Furthermore, the member utilisation levels are compared against Navier's equation for stress over beam sections, $N/A + M_z/I_z + M_y/I_y$. The table present the five highest force levels and utilisation levels however, the member with highest axial force in not necessarily the members with highest total utilisation.

Figure B.1 illustrate the design cases presented in table above.

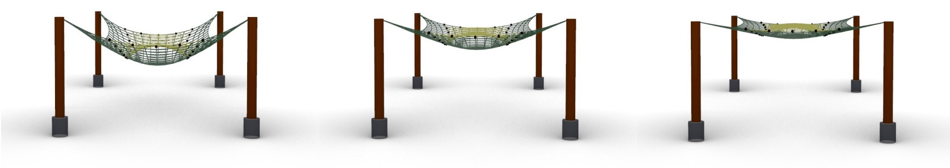

Figure B.1: Design case for tension strengths; low, medium, high

An interesting take from this experiment is the net's improvement from loose to a medium tight form. Although the horizontal reaction vector more than doubles, the absent of compression stress in the net provide a more favourable utilisation of the members, and the contribution of moments about local y and z is highly reduced. Yet, for the tightest design case, the disadvantage of stiff structures appears. By overdoing the tightening – or as illustrated in figure 4.6 with a low sag height f – the horizontal reaction forces shoot, as well as axial forces in the net. Consequently, the high tension case is a less favourable with respect to displacement, stress in members and reaction forces.

C London Millennium Bridge

The following descriptions is received by (MIStructE) Roger R. Smith, Partner and Head of Structural Engineering in Foster + Partners.

> [On the cable design].. The locked coil cables were manufactured by Bridon ropes. They were pulled across the river and placed in a higher level than their final geometry. Then the deck elements were placed in prefabricated 16m long sections, and the cables deflected into their finished position. Contractor Monberg Thorsen carried out the detailed calculations for the installation sequence. Bridge dead load is 2T/m.

Also form an article written by Smith (et-al) [41] describe further:

> [From Synopsis]The lateral force exerted by pedestrians on the moving deck surface is found to be related to the movement. The results show that the phenomenon is not related to the technical innovations of the bridge and that the same phenomenon could occur on any bridge with a lateral frequency below about 1.3Hz loaded with a sufficient number of pedestrians.
>
> [On superstructure] The bridge structural diagram is that of a shallow suspension bridge,where the cables are as much as possible below the level of the bridge deck to free the views from the deck.Two groups of four 120mm diameter locked coil cables span from bank to bank over two river piers.The lengths of the three spans are 81m for the north span,144m for the main span between the piers and 108m for the south span.The sag of the cable profile is 2.3m in the main span, around six times shallower than a more conventional suspension bridge structure. Fabricated steel box sections,known as the transverse arms, span between the two cable groups every 8m.The 4m wide deck structure comprises two steel edge tubes which span onto the transverse arms.The deck itself is made up of extruded aluminium box sections which span between the edge tubes on each side.Other finishes such as the lighting and handrail are also fixed onto the edge tubes. The groups of cables are anchored at each bank. Each abutment is founded on a 3m reinforced concrete pilecap anchored by a group of 2.1m diameter reinforced concrete piles.There are 12 piles on the north bank and 16 on the south bank, where the site is constrained and the pilecap shorter in consequence.The river piers themselves comprise a steel 'V' bracket fixed to a tapering elliptical reinforced concrete body which is founded on two No.6m diameter concrete caissons..
>
> [On the cable design] The cables form the primary structure of the bridge and have a very shallow cable profile, as described above. Ribbon bridges have similar shallow profiles, but are typically single spans, allowing the cables to be anchored directly to substantial stiff abutments.The stiff abutments help limit the live load deflections. The Millennium Bridge has some of the characteristics of a ribbon bridge, but is unusual in having multiple spans.The intermediate river piers are quite slender and cannot provide stiffness comparable to that of a massive abutment.This means the spans interact,making

the behaviour of the structure more complex than that of a single span bridge. For example, if only the central span was loaded, the outer spans would deflect upwards. An initial series of parametric analyses, using the simplest models that could still meaningfully represent the non-linear response of the bridge, helped to understand the bridge behaviour and set realistic targets for parameters such as the cable sizes, and the pier and abutment stiffnesses.The behaviour of each span is driven by the stiffness of the structural system at the extremities of the span.The system providing stiffness is either an abutment or the combination of the cable stiffness of the adjacent span with the stiffness of the pier.These studies quantified the relative stiffnesses of the adjacent span cables and of the pier structures,demonstrating that the cable stiffness provided about 80such a termination.Hence the main restraint stiffness to the central span came from the outer span cables and from the abutments, not directly from the piers. It was therefore important that variations in the abutment stiffnesses would not result in similar variations of the bridge deflections.The parametric studies enabled limiting foundation stiffnesses to be selected to achieve this requirement.

D Excessive case-study material

D.1 Initial design steps

1st Iteration

For this iteration the prestress is increased to the limit yet much of the structural input is kept from the 2nd iteration in section **??**. Table D.1 and table D.2 present the most relevant data for the two span design case.

Iteration 1, prestess 100%, SLS		***Cable description***		
Net weight [kg]	*Strut @ pier*	*Edge cable*	*Cable net x-dir*	*Cable net y-dir*
9050	CHS127x3.6	PV150	PG75	PG125
Span [m]	*Deflection [mm]*	*Limit value span/200*	*Reaction x-vector [kN]*	*Reaction y-vector [kN]*
100	5560	500	2715	472

Table D.1: 1.st iteration; structural members, deflection & support reactions

Span [m]	***Eigenfrequency [Hz]***	***Cable description***	***Tension [kN]***	***Normal stress [mPa]***	***Utilisation My/Mz***
	1.42	Edge cable	1535	1300	n/a
	1.52	Along shell	1220	1680	n/a
100	1.60	Cable net x-dir	740	1290	n/a
	1.76	Cable net y-dir	183	230	n/a
	1.84	CHS-section	2027	2360	6,4/0,2

Table D.2: 1.st iteration; frequencies, beam and cable actions

Discussion

From table D.1 and D.2 the deflection past five meters, which is unacceptable, the eigenfrequencies is also within a critical range however, such matters are amendable with a better prestress configuration. Equally, the stress levels in the cables supporting the footpath/shell is above yield strength of 1550 mPa yet, amendable with larger cables. The CHS beam on the other hand, experience high axial stress as well as shear and moment forces. Such magnitudes are problematic, as the purpose of the beam is to prevent buckling in the net for a load case that lift upwards. A tensile force of 2027 kN, or more exact 2800 kN in ULS, require a sectional area of 6000 sqmm to prevent axial failure given S355 steel. In

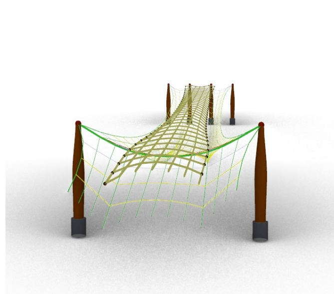

Figure D.1: Obstacle

including a shear stress and and bending moment, it appears that no rectangular hollow section according to NTNU's academic booklet [38] is sufficient (see annex for calculations). Thus a customised tube-section would be required for the two-span alternative.

D.2 From: *ULS analysis*, 5.3

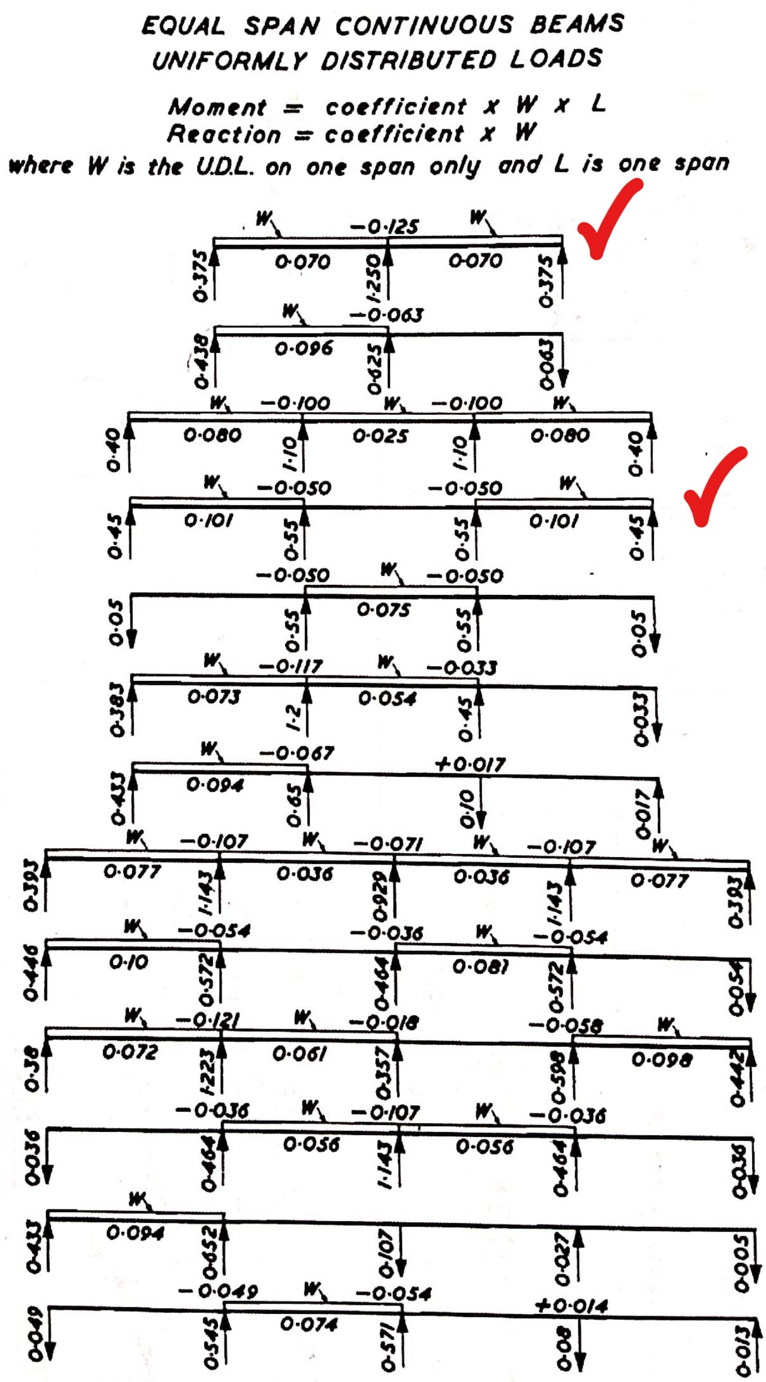

Figure D.2: Diagrams; cont. beams

D.3 From: *Grid shell design*, 5.3.6

Three feasible solutions are:

1. Replace the critical grid arches with steel tubes.

2. Prevent crowd loads at the critical parts of the grid (at both ends).

3. Decrease mesh geometry (density the grid pattern)

1. By replacing the first arch members with steel and allow the crowd load to stretch the full length, a HFRHS 140x80x8 tube (A= 3230 sqmm) is adequate for the normal axial stress. Such section would lead to a peak normal stress of 180 mPa, roughly 60% of the design limit. For the GL90x90 sections, such replacements would have to reach 20 m into the span before the stress limits ground below design limit of $k_m * f_k/\gamma = 23$mPa. On the other hand, if changing the sections to 135x115 (hxb), a 90% increase, this disturbed zone is reduced down to ten meters – halved. Figure D.3 illustrate this zone as the distance between abutment piers and the green bullets.

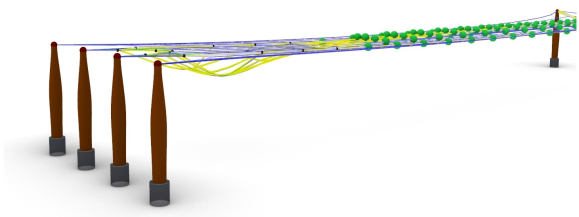

Figure D.3: Disturbed grid zone; 20 m

2. The solution to 'pull in' the bridge, or in another way prevent crowd loading to act at first critical areas of the deck, is most likely not economical. In case of the 90x90 sections, such minimum length would be 17 m, as illustrated in figure D.4. Yet again, if enlarging the sections to 115x135, is no-zone reduces to 12 meters.

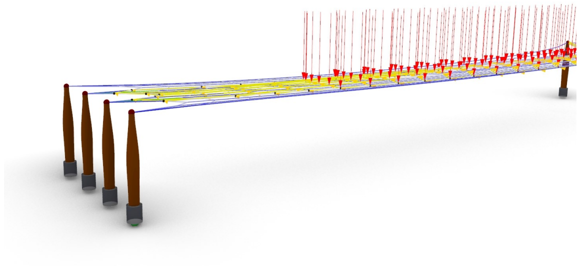

Figure D.4: Prohibited area of loaded; 17 m

3. by increasing the mesh, the grid layers are closer and would strengthen the structure. However, due scope and time constrains, this alternative will not be studied.

From these few iterations it is clear that the 90x90 cross section is not suitable in any of the cases. By bits of trial and error, a 180x115 section seem the most effective in the combination of mainly compression, though bending moments closest to the ends. Based on the investigation, introducing steel tubes for the first ten meters from the abutments is preferred. From an economical perspective, 'wasting' 12 meters of bridge at both ends is hard to defend. Therefor, seven rows of HFRHS 180x6.3 tubes at each end is proposed.

D.4 From: *Serviceability requirements* (D.3)

From deflection analysis with load cases described in figure 5.17. yet, the three span layout is presented more in depth.

Three span model

The input in table D.3 come from the load study in section 5.3. For those iterations the net did fine structurally, however for serviceability limit and safety, the net design was not adequate. A prestress 90% of the allowed tensile limit is not safe, in addition, a deflection of nearly two meters is not compatible with a pedestrian bridge. The target for this subsection is to reach a deflection within 330 mm. Hence, a significant increase of cable dimensions is needed to fulfil this goal.

Iteration #1 SLS	**Prestress: 30% of UTS**	**Cable description**		
Weight [kg]	*Deflection [mm]*	*Edge & @Shell*	*X-dir.*	*Y-dir.*
133,600	1870	PV 490	PV 240	PG 90

Table D.3: Deflection verification; iteration #1

Table D.4 show a great reduction in deflection, yet the longitudinal cables require further enlargement to reduce the deflection another 60%. Alos, the highest stresses were discovered in the transverse net cables (about 75% of UTS), therefore they are increased to PG 125.

Iteration #2 Prestress: 30% UTS	**Cable net propperties**		
Cable description	Edge & @Shell	X-dir.	Y-dir.
Type	PV 1220	PV 810	PG 125
Normal Stress [mPa]	500	570	580
Utilisation (UTS)	35%	40%	38%
Deflection [mm]	870		
Weight [kg]	224,600		

Table D.4: Deflection verification 3-span; iteration #2

For the 3rd iteration the longitudinal cables are increased to the limit. PV 2000 is among the greatest cable locked coil cables in the market, having a breaking load of 20 000 kN. The regular net cables are sat to PV 1450 which is also a massive dimension with a breaking load of 14 500 kN. Furthermore, the prestress is increased to 40% of the breaking load (UTS). This lead to a deflection of 410 mm which is still more than span/200, thus another iteration is necessary to reach the minimum limit.

Iteration #3 Prestress: 40% UTS	**Cable net propperties**		
Cable description	Edge & @Shell	X-dir.	Y-dir.
Type	PV 2000	PV 1450	PG 125
Normal Stress [mPa]	660	690	970
Utilisation (UTS)	46%	48%	63%
Deflection [mm]	410		
Weight [kg]	321,880		

Table D.5: Deflection verification 3-span; iteration #3

The 4th iteration are within the deflection limit, with a prestress of 50% of UTS. Such initial stress above the recommended levels, though it may seem an utilisation round 60% of UTS is okay, it is not necessarily so. In case of the PV 2000 for instance, 57% of UTS equal 94% of the allowed tensile limit (12121 kN). For this reason, a prestress of 50% is not ideal.

Iteration #4 Prestress: 50% UTS	Cable net propperties		
Cable description	Edge & @Shell	X-dir.	Y-dir.
Type	PV 2000	PV 1450	PG 125
Normal Stress [mPa]	830	840	1150
Utilisation (UTS)	57%	58%	74%
Deflection [mm]	332		
Weight [kg]	321,880		

Table D.6: Deflection verification 3-span; iteration #4

Two-span model

For the two-span model, the deflection limit is 500 mm, and load case #1 shown in figure 5.17 is used as it bring the greatest sag. Table D.7, present the output with 50% prestressing. 860 mm above the limit, hence another increase of initial stress is needed.

Iteration #1 Prestress: 50% UTS	Cable net propperties		
Cable description	Edge & @Shell	X-dir.	Y-dir.
Type	PV 2000	PV 1450	PG 125
Normal Stress [mPa]	780	800	1070
Utilisation (UTS)	54%	55%	70%
Deflection [mm]	860		
Weight [kg]	320,380		

Table D.7: Deflection verification 2-span; iteration #1

Table D.8 present an input which lead to exactly on the minimum limits. On the other hand, the stress limits in the cables are far beyond the acceptable range.

Iteration #2 Prestress: 85% UTS	Cable net propperties		
Cable description	Edge & @Shell	X-dir.	Y-dir.
Type	PV 2000	PV 1450	PG 125
Normal Stress [mPa]	1350	1370	1800
Utilisation (UTS)	94%	95%	116%
Deflection [mm]	500		
Weight [kg]	320,380		

Table D.8: Deflection verification 2-span; iteration #2

Four-span

For the four-span model, the highest allowed deflection is reduced to 250 mm. Load case #3 (in figure 5.17) is used for this analysis. Table D.8, present the first iteration with 50% prestressing. 210 mm is below the limit, so there is no need for further iterations in that regard. However, utilisation levels around 60 % and 70 % are as disused earlier, not ideal.

Iteration #1 Prestress: 50% UTS	**Cable net propperties**		
Cable description	Edge & @Shell	X-dir.	Y-dir.
Type	PV 2000	PV 1450	PG 125
Normal Stress [mPa]	840	850	1160
Utilisation (UTS)	58%	59%	75%
Deflection [mm]	210		
Weight [kg]	325,330		

Table D.9: Deflection verification 4-span; iteration #1

D.5 From *Grid Shell*

For the grid shell, dynamic vibrations is not a concern due to the many supports along the net. Table D.10 present frequencies outside the critical range according to Hivoss. Yet more importantly, the figure D.5 show the first mode shape, and by inspection these frequencies lead to no real concern.

Weight grid [kg]	**Span [m]**	**Modal mass [kg]**	**Eigen-frequency [Hz]**
7556	100	6666	2,63
	67	5636	2,76
	50	4661	2,95

Table D.10: Results; Vertical gridshell vibrations

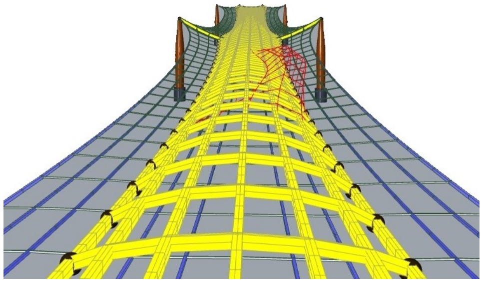

Figure D.5: Unlikely modal oscilliation

Discussion

The dynamic study concluded in this section is rather simplified. In order to conduct fully adequate dynamic analysis with moving loads and time history, a different software would be needed. The scope has been to evaluate dynamic properties in an pragmatic manner.

The results received by karamba and through the response spectra analysis clearly differentiate the two design alternatives. The three span model gave many mode shapes within the critical domain which furthermore produced unacceptable peak accelerations – even high a high damping ratio. In order to satisfy the service limits for this model, addition of damping is necessary. Such measured would be viscous or tuned mass dampers in the net or dampers installed at the piers and abutments.

Whilst for the four span model, the frequencies obtained lay outside the critical range and is therefore not a subject to further investigations – an easy chose one might say.

E Code

E.1 Characteristic live loads, wind

```
%% Wind magnitudes [EN 1991 1-4; bridges]

% general input
B= 12; %bridge width, conservatively
L= 250; % Full crossing length, approx
v_b= 24; %m/s char. wind speed
c0= 2.5; %terrain class, 0, height 5m EN[4.5]
ro= 1.25; %air density

%cals for vertical z-dir.
c_ez= 0.9; %[8.3.3 (1)]
c= c0*c_ez;
A= B*L; %mm^2, bridge surface
F_wz= 0.5* ro* v_b^2*c*A/1000; %kN force z-direction [8.3.3]
qk_v= F_wz/A %charateristic vind load


% cals for horizontal y-dir.
q_p0= 800; %Nm^2
d_tot= 1.2+1.5; %horizontal projection, handrail + cable sag(table 8.1)
ratio= B/d_tot;
c_fx_0= 1.3; %figure 8.3
c= c0*c_fx_0; % eq(8.2)
A_ref_x= d_tot*L;  %reference area
F_w= 0.5* ro* v_b^2*c*A_ref_x/1000; %kN force horizontal,[8.3.2] eq(8.2)
qk_h= F_w/A_ref_x %characteristic wind load
```

E.2 Spectral analysis

```
%% input Spectral evaluation
prompt= 'Eigenfrequency?';
modal= 'Modal mass?';
fi = input(prompt);
m_star = input(modal);
%% calculations according to HIVOSS
%variables
B= 6; %width
L= 200; % length

%constants
Kf= [0.012 0.012 0.007 0.00334];
C= [2.95 2.95 3.7 5.1];
Ka_95= [3.92 3.92 3.8 3.74];
a= 10e-3*[-7 60 7.5; -7 60 7.5; -7 56 8.4; -8 50 8.5];
b= 10e-3*[0.3 -4 -100; 0.3 -4 -100; 0.4 -4.5 -100; 0.5 -6 -100.5];

%variables
d= [0.2 0.5 1 1.5]; %streams densities
xi= 0.02; %damping ratio
ps= 0.85; % reduction factor, 0.85 for fi= 2,13

n=zeros(1,4);
k1 = zeros(1,4);
k2 = zeros(1,4);
sigma_f2 = zeros(1,4);
sigma_max = zeros(1,4);

for i = 1:4 %design cases
    n(i) = d(i)*(L*B);
    k1(i) = a(i,1)*fi^2 + a(i,2)*fi + a(i,3);
    k2(i) = b(i,1)*fi^2 + b(i,2)*fi + b(i,3);
    sigma_f2(i) = Kf(i)*n(i);
    temp =sqrt(C(i)*sigma_f2(i)/(m_star^2) * k1(i)*xi^k2(i));
    sigma_max(i) = ps * Ka_95(i) * temp;
end
disp('Maximum accelerations:')
disp(sigma_max*1000)
```