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NTNU
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Department of Structural Engineering

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Simplified dynamic analysis of railway bridges and methods to obtain an overview of critical bridges in a segment analysis

An evaluation of parameters, calculation methods, and reasonable assumptions.

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TITLE:

Simplified dynamic analysis of railway bridges and methods to obtain an overview of critical bridges in a segment analysis.

BY:

Andreas Røyksund and Emil Tofte Røhne

SUMMARY:
The main purpose of this thesis is to study the possibility to perform dynamic analyses of multiple railway bridges with limited input. Upgrading railway lines for high speed trains is common, and being able to get an overview over critical bridge lengths and train velocities without performing complicated analysis on every single bridge would be useful.

Nerk is a new German software meant to provide an overview of response based on bridge type and bridge length. It uses assumptions from the Eurocode in terms of damping and eigenfrequency, simplifies the train load as a set of constant moving forces and assumes all bridges to be simply supported. This is used to construct 3D plots with response as a function of train velocity and bridge length. Back-end programming of Nerk has been performed in MATLAB to study individual parts of the software, which is used to evaluate the assumptions and possible alternatives. An extensive number of bridges are used for the assessment of Nerk and for the development of an alternative. Further on complex systems and more detailed bridge models are studied to assess the sufficiency of the analysis.

The results show that the assumptions in Nerk are highly conservative, and that the response obtained is hard to interpret and use for further analysis. The Roehnsund method is introduced as an alternative to assume mass and stiffness, and gives results closer to analytical response using exact properties. A MATLAB script using Roehnsund method and calculation model from Nerk is developed and tested, making it simple to analyze a set of bridges, construct 3D-plots and perform better analyses. Generally, the shape of response against velocity (MAC) is good for the calculation method used, while the value of the response is not consistently close to correct (Mean ratio above 1). Hence the results obtained are evaluated as good for obtaining an overview of critical lengths and velocities, but more detailed calculations should be made for these cases. Additionally, the mean ratio grows for bridges that deviate from the assumption regarding single span and simply supported, generally causing more conservative results.

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Preface

This thesis is made as a completion of the master education in Structural Design at the Norwegian University of Science and Technology (NTNU).

Several persons have contributed academically and with motivational support to this thesis. First of all will we thank our supervisor at the university Nils Erik Anders Rønnquist for his time, input and feedback during the master period. We would also like to thank Aziz Kameran for his interest in the thesis and input on behalf of Bane NOR when needed, and Daniel Cantero for great support in understanding the topic of dynamics and developement of MATLAB-scripts.

Finally, we want to give a special thank you to Emil's roommates for letting us use the kitchen as an office and Jonas Lindgård for motivation and companionship during the COVID-19 period.

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Abbreviations

CDF	=	Cumulative Distribution Function
CFS	=	Closed form solution
DER	=	Decomposition of Excitation at Resonance
EC	=	Eurocode (NS-EN 1991-2)
ERRI	=	European Rail Research Institute
ETR500Y	=	Elettro Treno Rapido 500Y
FEM	=	Finite element method
IoE	=	Interpolation of Excitation
MAC	=	Modal Assurance Criterion

Chapter 1

Introduction

Railway traffic has been important for the development of infrastructure for many years. It is an essential part of The European Commissions goal to cut 60 % of carbon emissions from transport by 2050, where a key goal is to make 50 % of medium to long distance road traffic waterborne or railway traffic [17]. With the population growth in and around cities, a better railway service with faster and longer trains will increase peoples desire to use the train service. The InterCity project by Bane NOR in Norway has a goal to build two track railway with increased train velocity of 250 km/h between Oslo and several of the surrounding cities by 2034 [15]. This will increase the number of departures and decrease the travel time, and satisfy peoples demand for a better railway service, while working towards the 60 % goal by The European Commission.

With faster trains, many existing railway lines have to be rebuilt because of the curvature demands under high velocities. An important aspect of where new lines will be located is if existing railway bridges can resist the increased dynamic load from higher train velocity. It is useful to perform a simple calculation to get an overview of which bridges will resist the increased load and which will need more detailed calculations to evaluate if they need improvements or to be rebuilt.

Many authors have studied the effect of moving loads on bridges and derived exact solutions for bridge deck response, e.g. Timoshenko(1926)[16] and Fryba(1996)[10]. European Rail Research Institute(ERRI) formulated in 1999 a simplified method for calculation of bridge deck response called the Decomposition of Excitation at Resonance-method, DER-method [1], where the aim was to develop an approximate method that should be simple to apply directly using hand calculation or spreadsheets. Chopra [6] derived the numerical Interpolation of Excitation(IoE)-method, as an approach to solve moving load problems. The DER- and IoE-method is the basis for a new German program for dynamic analysis of railway bridges called Nerk.

1.1 Formulation of the problem

The goal of this thesis is to study the possibility of evaluating the dynamic response for a number of railway bridges in a simple way with limited input. A German software called Nerk has been introduced as a tool for analysing many bridges at the same time, and is therefore used as a base. The calculations done in Nerk are evaluated along with the assumptions that are made in order to be able to analyse a segment of bridges. The formulation of the problem can be explained in 4 points:

1. Getting to know Nerk. First by getting to know the software and its functions, terminology and user interface. Further on by back-end programming it to study how the different calculations are made and be able to make modifications.

2. Evaluate Nerk in entirety. This means doing a sensitivity analysis on the determining factors in Nerk and on the assumptions done to be able to run simplified analysis.

3. Develop a new model based on the improvement potential in Nerk. It should be able to perform the same analyses as Nerk in addition to a specific segment analysis, making it an alternative to Nerk.

4. Use the model to perform an analysis of several bridges, and evaluate the model based on existing solutions, better calculations and known parameters.

1.2 Method

The script developed in this thesis is constructed with the software Nerk as a basis, which causes the base assumptions to be the same. Nerk evaluates all bridges as single spanned, simply supported 2D beam models. Nerk in entirety and all assumptions are validated by back-end programming the software in MATLAB, and studying the effects of all parameters participating in response calculation.

Calculation of response from the Eurocode, further referred to as EC, is considered sufficient, making the focus how bridge parameters can be assumed and used to obtain conservative but realistic results. Further on is the calculation method evaluated in entirety, and it is discussed whether it is applicable for bridges that deviate from the base assumptions regarding supports and spans. The main requirement regarding dynamics in EC is acceleration response, which make it the main output focused on in this thesis. The acceleration requirements according to EC are $3.5\frac{m}{s^2}$ for railway bridges with ballast, and $5\frac{m}{s^2}$ for railway bridges without ballast. Moment is also evaluated in some cases, as it is easier obtainable in some cases.

The thesis introduces what is further referred to as the Roehnsund method. It is presented as an alternative to the Nerk assumptions. EC presents a big variation of eigenfrequencies as a function of length, which gives high variation of response. Hence, the purpose of the Roehnsund method is to narrow down this frequency span by adding a couple of simple inputs to the frequency function.

1.3 Structure of the thesis

Chapter 2 describes the theory behind dynamic response on railway bridges. Modal parameters and the equation of motion is defined to begin with, before going in to how resonance occurs and is calculated, and explaining how train loads can be the cause of resonance. Finally different methods for calculating the response is presented, and how results can be evaluated in terms of error and deviation.

Chapter 3 describes how the results of the thesis are obtained from a practical point of view. It explains which results are meant to be obtained, which calculation methods are used in which parts of the thesis, how the calculation models are built up and how the developed script is made.

Chapter 4 presents results obtained corresponding to the parts described in Chapter 3. It gives an overview of how the different calculation methods provides results, and the sensitivity of the parameters evaluated.

Chapter 5 uses the results and the methods that provides them to make comparisons and evaluations. The main focus of the chapter is to show how calculation methods gives different quality in results, both in terms of interpretation and precision.

Chapter 6 and 7 summarises and presents the most important findings, in addition to improvement potential in terms of calculations and further analysis.

Chapter 2

Theory

2.1 Dynamic response of railway bridges

2.1.1 Modal parameters

Assuming a uniform simply supported beam of length L with bending stiffness EI and evenly distributed mass m, the following partial differential equation of motion governs the vertical displacement $v(x,t)$ [13]:

$$EI\frac{\partial^4 v}{\partial x^4} + m\frac{\partial^2 v}{\partial t^2} = 0. \tag{2.1}$$

The physical vertical displacement $v(x,t)$ can be expressed as a sum of modal contributions:

$$v(x,t) = \sum_{n=1}^{N_{modes}} \phi_n(x) q_n(t) \qquad N_{modes} \to \infty, \tag{2.2}$$

where $\phi_n(x)$ is the mode shape for mode n at location x, and $q_n(t)$ is the generalised coordinate of mode n. When inserting Equation 2.2 into Equation 2.1, the mode shape can be derived as:

$$\phi_n(x) = sin(\frac{n\pi x}{L}). \tag{2.3}$$

The modal mass m_n and natural frequency ω_n are obtained from the same expression:

$$m_n = \int_0^L m\phi_n(x)^2 dx = \frac{mL}{2}, \tag{2.4}$$

and

$$\omega_n^2 = \big(\frac{n\pi}{L}\big)^4 \cdot \frac{EI}{m} = \frac{k_n}{m_n}. \tag{2.5}$$

Introducing modal damping ratio ζ_n and applying load to the Equation 2.1, the system of uncoupled differential equations are obtained as a function of the modal parameters:

$$\ddot{q}_n(t) + 2\zeta\omega_n\dot{q}_n(t) + \omega_n^2 q_n(t) = \frac{p_n(t)}{m_n}, \tag{2.6}$$

where p_n is the modal load.

2.1.2 Vibrations of a simply supported bridge

To find a relation between velocity of a moving load and magnitude of vibration, Museros et al. [14] solved Equation 2.6 for a constant moving load on a simply supported beam without damping. The *n*th modal amplitude of deflection when the load is on the bridge, is given as

$$q_n(t) = -\frac{2P}{mL\omega_n^2}\frac{1}{1-K_n^2}[sin(K_n\omega_n t) - K_n sin(\omega_n t)], \quad 0 \le t \le \frac{L}{V}, \quad K_n \neq 1, \tag{2.7}$$

$$q_n(t) = -\frac{2P}{mL\omega_n^2}\frac{1}{2}[sin(\omega_n t) - \omega_n t cos(\omega_n t)], \quad 0 \le t \le \frac{L}{V}, \quad K_n = 1, \tag{2.8}$$

where the non-dimensional speed K_n is defined as the ratio between the load- and natural frequencies:

$$K_n = \frac{\Omega_n}{\omega_n} = \frac{n\pi V}{\omega_n L_{br}} = \frac{VL_{br}}{n\pi}\sqrt{\frac{m}{EI}} = \frac{K_1}{n} \tag{2.9}$$

For practical purposes, only $K_n \leq 1$ is of interest, since the largest possible value of K_1 for existing bridges is around 0.55[14]. Hence, only Equation 2.7 is considered.

Free vibration of the beam begin when the load has passed the beam. The free vibration generated depends on the solution of the forced vibration in Equation 2.7 at the time the load leave the bridge. The amplitude of free vibration is divided by the static contribution, which yields normalised amplitude as

$$R_n = \frac{K_n\sqrt{2}}{1-K_n^2}\sqrt{1 - cosn\pi cos\frac{n\pi}{K_n}}. \tag{2.10}$$

The normalised amplitude of free vibrations is given for the first bending mode in Figure 2.1. Since the largest possible value of K_1 is around 0.55, the peak at $K_1 \approx 0.75$ is not included in practice.

The peaks in Figure 2.1 represent which values of K_1 that gives the largest free vibrations. Conversely, the zero-values represent values where free vibrations cancel out.

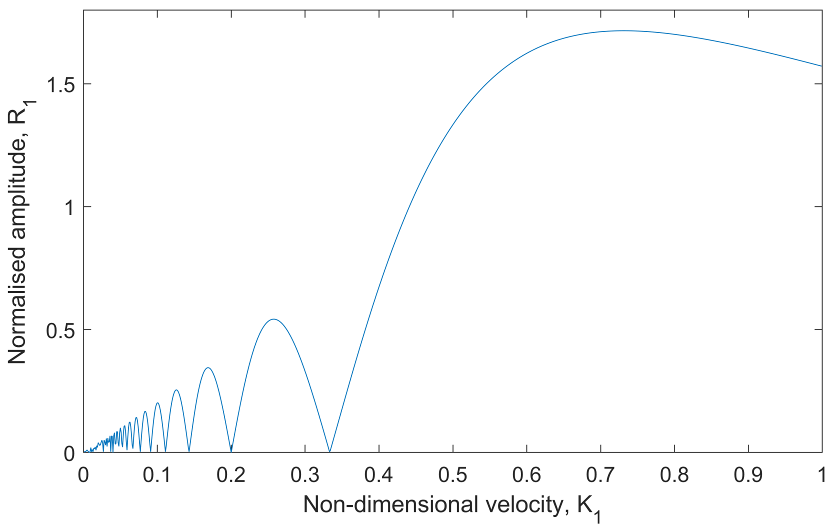


Figure 2.1: Normalised amplitude of free vibration for the first bending mode

2.1.3 Resonance and cancellation

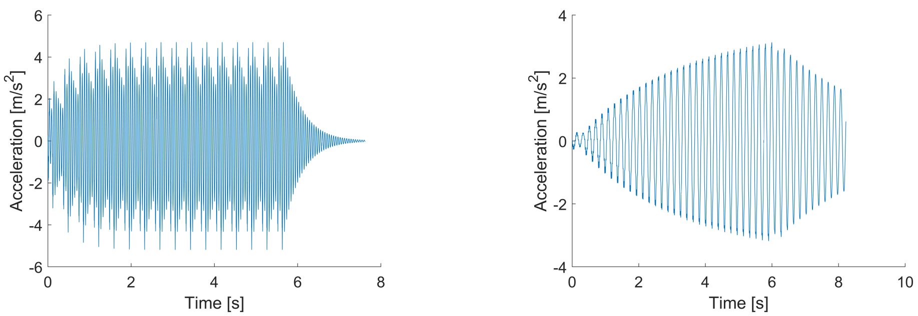
Resonance occurs when the free vibration from loads are added. This happens when the eigenfrequency of the bridge is a multiple of the load frequency. The velocities at which resonance can occur are called *critical velocities*, V_{cr}. The critical velocities for a bridge with natural frequency f_n can be found from Equation 2.11, where *n* and *k* denotes which mode number and resonant velocity the critical velocity applies to. L_c is the carriage length.

$$V_{cr} = \frac{L_c f_n}{k}, \qquad (n, k = 1, 2, 3...) \tag{2.11}$$

The amplitude of the resonance is depending on how the critical velocities coincide with velocities from Equation 2.9. Maximum resonance occur when V_{cr} coincide with one of the velocities creating maximum free vibration, and resonance can be cancelled out if V_{cr} coincide with one of the velocities that cancel out free vibrations.

A series of consecutive loads with intermediate distance of 20 meters is used to display amplification of response for two bridge lengths, shown in Figure 2.2. The resonance amplification is more visible for the longest bridge as the duration of the free vibration is longer, meaning several axle-loads contributes to the resonance.

Both plots show resonance in the first mode. The eigenfrequency increases quadratic with mode number, meaning the resonance velocities are significantly higher for higher modes. Hence higher modes is less important when considering short bridges with train velocities lower than 300 km/h.



(a) Resonance for a short bridge (6m) with first eigenfrequency 19.07 Hz. k=7 corresponds to 196 km/h.

(b) Resonance for a long bridge (30m) with eigenfrequency 5.3 Hz. k = 2 gives 191 km/h.

Figure 2.2: Two different resonance cases.

2.2 Train loading

Figure 2.3 shows a 2D-model of a train crossing a railway bridge, idealized as a simply supported beam. Every carriage has two bogies, with two wheels/axles each. The figure shows the relevant lengths on the train when considering dynamic loading on a bridge.

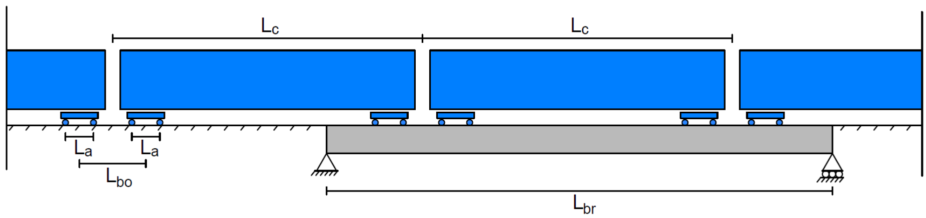
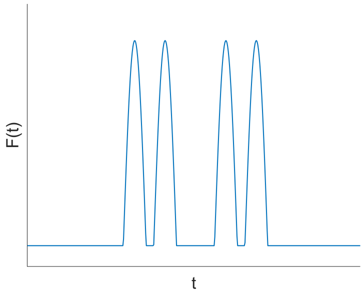
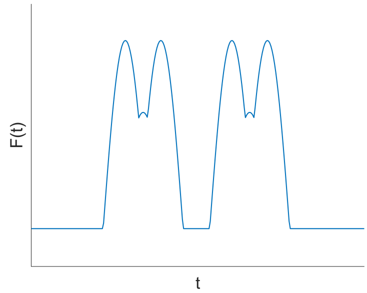


Figure 2.3: Train setup on bridge with relevant lengths.

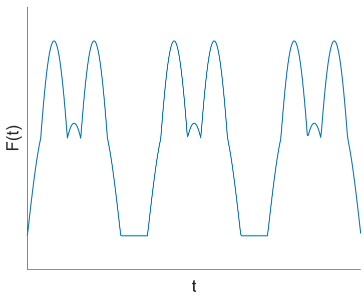
The loading over time can be described as shown in Figure 2.4 for the case shown above, depending on the relations between the different lengths shown in the figure. It is visible that there are several combinations of length-relations that can cause a periodic loading, and therefore also dynamic response. The most relevant loading time-periods are the time between single axles and the time between the axle groups. Figure 2.4 shows that the loading from a single wheel is only visible as a periodic load when only one wheel is located at the bridge at a time, $L_{br} < L_a$. The same applies for the bogies, $L_a < L_{br} < L_{bo} - L_a$. In reality few existing bridges are shorter than $L_{bo} - L_a$, but many bridges are shorter than the carriage length. Since the carriage length usually is uniform within a train, the axle groups can apply a periodic loading which can cause resonance. Considering bridges that are longer than the carriage like in Figure 2.4d and 2.4e several axle-groups will be acting simultaneously and the periodic loading will have smaller contribution to the response.



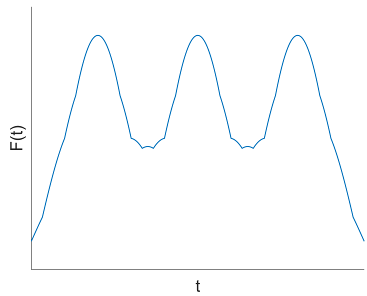
(a) Bridge length shorter than axle-distance.



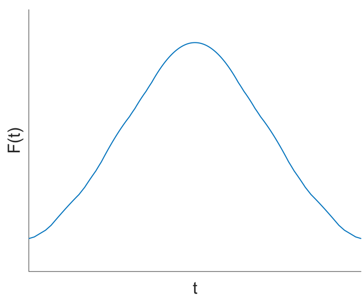
(b) Bridge length between axle-distance and bogie-distance.



(c) Bridge length between bogie-distance and carriage length.



(d) Bridge length longer than carriage length but shorter than the train.



(e) Bridge length longer than the train.

Figure 2.4: Loading for different relations between bridge length and load distance.

2.3 Analytical description of loading

In the case of a moving load P_0 with velocity v applied on the structure, the load $P(x,t)$ can be expressed as:

$$P(x,t) = \begin{cases} \delta(x - vt)\ P_0 & 0 \leq t \leq t_d \\ 0 & \text{otherwise,} \end{cases} \tag{2.12}$$

where $t_d = \frac{L}{v}$ is the time it takes for the load to cross the beam, and δ is the Dirac delta function which defines the physical location of the load. It can be shown that the modal load for the situation above can be expressed as [13]:

$$p_n(t) = \int_0^L \phi_n(x) P(x,t) dx = \begin{cases} \phi_n(vt)\ P_0 & 0 \leq t \leq t_d \\ 0 & \text{otherwise,} \end{cases} \tag{2.13}$$

where $\phi_n(x)$ is the mode shape. The mode shape for a simply supported beam is shown in Equation 2.3.

In the case of a convoy of moving load, superposition can be used to obtain the total load when assuming linear behaviour:

$$p_n(t) = \sum_{k=1}^{N_{loads}} \begin{cases} p_k \cdot \phi_n(vt - d_k) & 0 \leq vt - d_k \leq L \\ 0 & \text{otherwise.} \end{cases} \tag{2.14}$$

Here, p_k is the force from load number k in the convoy and d_k is the distance from the first load in the convoy to p_k. It is important to take into account that the modal load is zero when $(vt - d_k) < 0$ and when $(vt - d_k) > L$, meaning when the load has not entered the bridge and when the load has left the bridge respectively.

With this description the modal load can be described explicitly for any convoy of moving loads. Having time series for the load makes it possible to calculate the response with a variation of numerical methods.

2.4 Calculation of response

There are several methods for calculating response from a moving load. A few of them are evaluated in this thesis: Decomposition of Excitation at Resonance (DER), Interpolation of Excitation (IoE) and Closed form solution (CFS). The former use the frequency domain to calculate which velocities or frequencies can be critical. The last two both generate a function of response (deflection/velocity) against time, that can be modified to give acceleration or moment. The difference between the last two methods is that IoE is a numerical time step method for finding response based on the load function and initial conditions, while CFS gives an explicit expression for finding deflection for a simply supported beam as a function of time.

2.4.1 Interpolation of Excitation

IoE is a numerical calculation tool for finding response based on time series of loading, which means it is highly applicable for a convoy of moving loads, or a train. The IoE method uses linear interpolation of the load between two time instances (t and $(t + \Delta t)$) to find the response. It is based on knowing initial conditions and a complete time series of the load, and calculating the deflection- and velocity response at time $i + 1$ with Equation 2.15 and 2.16.

$$q_{n(i+1)} = Aq_{n(i)} + B\dot{q}_{n(i)} + Cp_{n(i)} + Dp_{n(i+1)}. \tag{2.15}$$

$$\dot{q}_{n(i+1)} = A'q_{n(i)} + B'\dot{q}_{n(i)} + C'p_{n(i)} + D'p_{n(i+1)}. \tag{2.16}$$

Chopra [6] derived the equations and factors, where A, B, C, D, A', B', C' and D' are all functions of modal frequency (ω_n) and -damping (ζ_n).

According to Chopra IoE-method gives a close to exact solution as long as the time step is sufficiently small. Chopra does not provide a specific way of finding it, but a normal way of evaluating the time step for step-by-step calculations is finding the relationship between the time step and the period. The smaller ratio (higher i)- the more accurate solution.

$$\frac{\Delta t}{T_{min}} = \frac{1}{i}, \tag{2.17}$$

Equation 2.17 shows how it is possible to determine the time step by finding the minimum period T_{min} and choosing the number of steps i per period wanted. The shortest natural time period for the bridge is:

$$T_{min} = \frac{1}{f_N} = \frac{1}{N^2 f_0}, \tag{2.18}$$

where f_N is the natural frequency of the bridge for the highest mode and N is the number of modes considered. In addition to the natural frequency of the bridge an excitation frequency corresponding to the load can be found for comparison with the bridge frequency. The excitation frequency is given as:

$$f_{exc} = \frac{v}{\lambda}, \tag{2.19}$$

where λ is the wavelength of the load. By using the highest value of the bridge frequency and the excitation frequency the lowest period to consider becomes:

$$T_{min,relevant} = \frac{1}{max\{\frac{v}{\lambda_{min}}; N^2 f_0\}}. \tag{2.20}$$

This yields the following expression for the necessary time step:

$$\Delta t_{max} = \frac{T_{min}}{i} = \frac{1}{i \cdot max\{\frac{v}{\lambda_{min}}; N^2 f_0\}}. \tag{2.21}$$

2.4.2 Closed form solution for a moving load

CFS for response from a moving load provided by Fryba [10] can be used as an exact alternative to numerical methods. Fryba provides an explicit expression for deflection on a simply supported beam with a constant moving force. This expression can be derivated with respect to time to find velocity and acceleration of the beam, or with respect to position to find moment at any time instant. The time step does not affect the solution directly when using the CFS, it only affects the smoothness of the curve when plotting. It may affect the solution if the max value is wanted and the peaks are not recorded with the chosen time step.

The main issue with the CFS is that it is computationally expensive. This is because programs like MATLAB requires time to transform a variable (time) to a vector of numbers.

2.4.3 Decomposition of Excitation at Reconance-method

European Rail Research Institute (ERRI) developed the approximate DER-method[1] on the basis of precise methods. The first simplifications was to ignore inertia effects and only study the first bending mode, as this allegedly gave satisfying results. Further on, the bridge deck was reduced to a single degree of freedom system, the response was decomposed to a Fourier series and only a term corresponding to resonance, $n\omega \rightarrow \omega_0$, was used.

With these simplifications, maximum acceleration of the bridge deck could be obtained directly from the given equation:

$$\Gamma_{max} = Ct \cdot A \left(\frac{L}{\lambda} \right) \cdot G_{Aggr}(\lambda, \zeta, L). \tag{2.22}$$

Ct and A is a constant and an influence line that contain information about the bridge. G_{Aggr} is called the train *spectrum*, and contains information about the trains contribution to the response.

Two simplifications were made for the train spectrum, first to eliminate bridge length from G_{Aggr} to get a simplified spectrum G, then to eliminate damping from G to get what is defined as *signature*, S_0.

Aggressiveness, $A \cdot G_{Aggr}$, includes an influence line with the spectrum to amplify results based on the ratio between L and λ.

Signature, spectrum and aggressiveness are all functions of wavelength λ, which is defined as $\lambda = \frac{v}{n \cdot f} = \frac{v}{f_0}$. Signature and spectrum are showing which wavelengths that potentially get high response. The influence line in aggressiveness wrongly eliminates response at $\frac{L}{\lambda} = 1.5, 2.5, 3.5....$ ERRI suggest to calculate response as the maximum of DER-results and using dynamic increment. The method with dynamic increment is calculated by multiplying the static response with a factor called dynamic increment.

2.5 Error calculations

2.5.1 Modal Assurance Criterion

MAC is a tool to measure correlation between two vectors given as [3]:

$$MAC(X,Y) = \frac{|X^T Y|^2}{(X^T X)(Y^T Y)}, \tag{2.23}$$

where X and Y are the two vectors considered. A MAC-value close to unity indicates that the two vectors are very similar with only a factor difference, and a MAC-value closer to zero indicates that the two vectors are not correlated.

2.5.2 Mean Ratio

Mean ratio is calculated as the mean value of the ratio between equivalent elements in two vectors:

$$Mean\ Ratio = \frac{\sum_{i=1}^{N} \frac{X_i}{Y_i}}{N}, \tag{2.24}$$

where N is number of elements in the vectors, and X_i and Y_i is the values in each vector in position *i*. A mean ratio above 1 indicate that vector X has higher values than vector Y, and opposite if it is below 1.

As mean ratio and MAC represent different comparisons, they must be considered together in the error evaluation.

2.6 Regression analysis

The Least squares method, or least square approximation, is a method for fitting a line to a set of data. For a linear function, that means minimising the function in Equation 2.25 with respect to a and b. $y = a + bx_i$ is the linear function to optimise. The method can also be generalised for use with nonlinear relationships [2].

$$SS_{resid}(a,b) = \sum_{i=1}^{N}(y_i - (a + bx_i))^2 \tag{2.25}$$

The regression can be assessed by calculating the coefficient of determination R^2, a measure that assesses the ability to predict or explain an outcome in a regression setting [11]. R^2 of 0.8 indicates 80 % of the variation in the outcome has been explained by the regression. Further on the percentage value will be referred to as the cover of variance . The coefficient of determination is given as [11]:

$$R^2 = 1 - \frac{SS_{resid}}{SS_{total}}. \tag{2.26}$$

SS_{resid} is the sum of squared residuals from the regression as shown in Equation 2.25, while SS_{total} is the sum of squared differences from the mean of the dependent variable.

Chapter 3

Method

Both the software studied, Nerk, and the developed script are meant as analytical tools for assessing existing bridges and their capability to resist new train types and/or an increase of train velocity. The main evaluation considered is acceleration response, as the Eurocode provides explicit requirements for acceleration. It is preferable for the methods to be conservative. In this case, conservative results means too high accelerations, as more bridge lengths and velocities would be characterised as critical. Being conservative is not an absolute requirement since both methods are meant to give an overview, meaning results that are similar to analytical results could be more valuable than extremely conservative results.

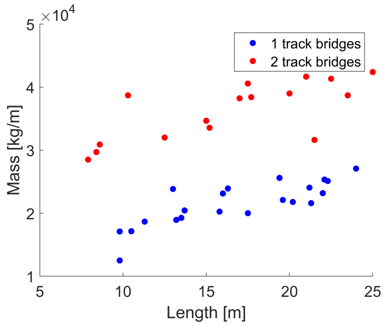
In order to make good evaluations, several trains and a number of bridges have been analysed. These are used to assess Nerk both in terms of its inputs and assumptions. Based on the results are a new script and a method for describing bridges developed, and finally the underlying calculation methods and assumptions have been evaluated.

3.1 Description of bridges

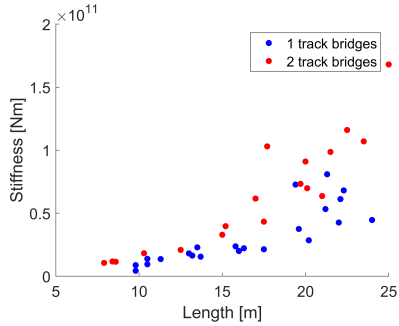
Several sets of bridges have been used for different purposes in this thesis. For evaluating sensitivity of different parameters and assumptions in Nerk as well as assessing modified calculations, comprehensive and easily available sets of bridges were preferable. Two such bridge sets were found in an report by Domenech et al. [8], called European database, and a report by Andersson[4], called Swedish database. The European database contains 40 one and two track prestressed concrete bridges. The span lengths are from 6 meters to 35 meters, and the bridges are supposed to cover a large spectra of simply supported bridges. The mass and stiffness for the bridges is shown in Figure 3.1a and 3.1b. The Swedish database contains 278 bridges, and consists of one track steel-concrete bridges, and one and two track prestressed- and reinforced concrete bridges. The database cover all bridges on four different segments of the Swedish railway network, and does not only contain simply supported bridges, but also bridges with rotational stiffness in supports and multi-span bridges. The mass and stiffness of the bridges from the Swedish database is presented in Figure 3.1c-3.1f.

In addition to the European and Swedish database, a Norwegian database was produced of existing Norwegian railway bridges. The parameters were calculated from design drawings. These bridges were used to evaluate assumptions and calculations executed.

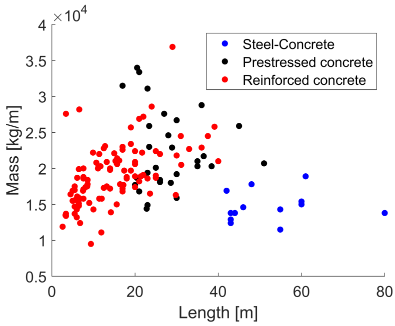
The three databases are given in Appendix A. The Swedish database consists of bridge number 1-278, the European database 301-340 and the Norwegian database contains number 401-408.



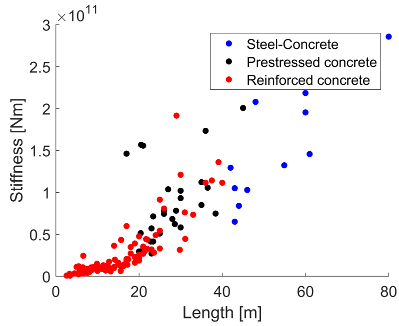
(a) Distributed mass for prestressed concrete bridges in the European database.



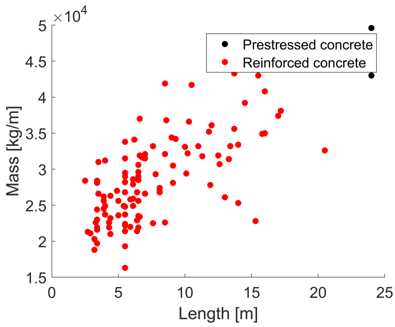
(b) Stiffness for prestressed concrete bridges in the European database.



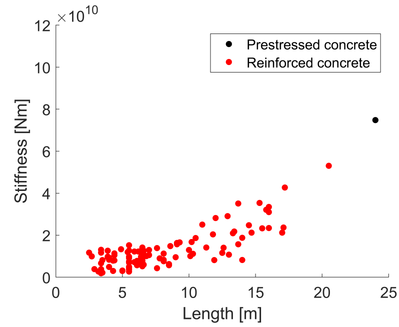
(c) Distributed mass for 1 track bridges in the Swedish database.



(d) Stiffness for 1 track bridges in the Swedish database.



(e) Distributed mass for 2 track bridges in the Swedish database.



(f) Stiffness for 2 track bridges in the Swedish database.

Figure 3.1: Distributed mass and stiffness for bridges in the European((a)-(b)) and Swedish database((c)-(f)) for 1 and 2 tracks.

3.2 Description of trains

EC propose different train compositions for structural analyses of bridges [9]. A dynamic analysis is required when the train velocity exceeds 200 km/h, and two high speed load models(HSLM) has been designed to provoke dynamic response in bridges. HSLMA is used for most bridge types and lengths, and has been used for most analyses in this thesis. The different HSLMA setups is given in Figure 3.2 and Table 3.1. HSLMB should be applied for analyses of simply supported bridges with span length under 7 meters. This type has not been considered in this thesis, even though short bridges are evaluated.

HSLMA trains are supposed to be used in design when the type of high speed train is not defined. This is because response from HSLMA trains will cover most of real high speed trains [7].

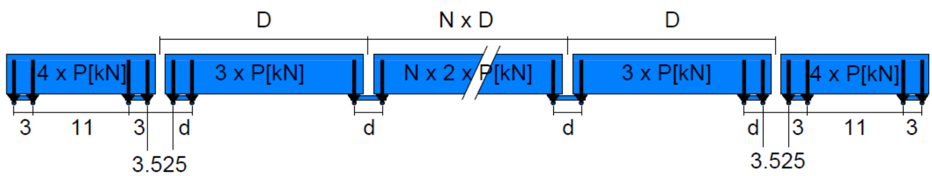


Figure 3.2: HLSMA setup from Eurocode [9] with dimensions in meter. Parameters is given in Table 3.1.

Universal HSLM-train	Number of intermediate coaches, N	Couch length D [m]	Bogie axle spacing d [m]	Point force P [kN]
A1	18	18	2,0	170
A2	17	19	3,5	200
A3	16	20	2,0	180
A4	15	21	3,0	190
A5	14	22	2,0	170
A6	13	23	2,0	180
A7	13	24	2,0	190
A8	12	25	2,5	190
A9	11	26	2,0	210
A10	11	27	2,0	210

Table 3.1: Dimensions for the different HSLMA-trains from the Eurocode.

Another option for dynamic analysis is to use real trains from EC or existing train setups like the Italian ETR500Y[12]. This setup has been used for some of the analyses in this thesis. The difference between the trains is mainly the location of the axles and the loading. ETR500Y has larger axle-load for the locomotive than the passenger carriages, while HSLMA-trains have the same axle load for the entire train.

The choice of train used in the evaluations made in this thesis is not essential, as long as it

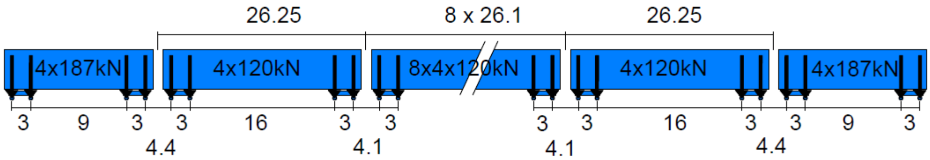


Figure 3.3: Setup of ETR500Y. All dimensions in meter.

represents the loading from a train. The quality of the final segment analysis is dependent on train loads that are verified as sufficient for design purposes.

3.3 Nerk

Nerk is a new software that makes it possible to evaluate a number of bridges with limited input. It uses assumptions from the Eurocode to estimate damping based on bridge type and -length, and eigenfrequency based on the bridge length. EC introduces an upper and lower limit of eigenfrequency for all bridges only dependent on bridge length. Nerk defines three frequency functions within these limits, corresponding to the upper and the lower limit, and one in the middle. In addition Nerk adds two frequency functions, equal to 0.8 times the lower limit and 1.2 times the upper. In total Nerk provides 5 frequency functions, meaning Nerk is supposed to give a conservative analysis of the bridges and cover a wide spectra of eigenfrequencies. These frequency functions will be referred to, where Frequency function 1 corresponds to 1.2 times upper EC limit, and Frequency function 5 to 0.8 times lower limit.

The main outputs from Nerk is a 3D plot with bridge length on the x-axis, train velocity on the y-axis and max acceleration on the z-axis, and ”Single TSC”- which gives response in terms of deflection, acceleration and moment over time for a specific bridge- and load case. This means the user has to decide train velocity, train type, bridge length and bridge type in addition to one of the frequency-functions. In the 3D plot the user can chose whether he would like an envelope of the eigenfrequencies or observe one of the frequency functions. In both calculations Nerk uses the IoE-method to numerically find moment and acceleration as a function of time.

Other possible outputs are signature S_0, spectrum G and aggressiveness. Aggressiveness can be visualized as a 3D-plot with λ and L as parameters, or a 2D-plot where either bridge length or wavelength is constant. Wavelength can be substituted with train velocity depending on frequency function. The advantage with these outputs is their independence of bridge properties as input.

In order to evaluate Nerk, it was necessary to understand how it works and how the inputs are used to compute an analysis. The first part of that process was done by experimenting with different inputs to observe how it affected the results with guidance from the user manual. When a basic understanding of the output was achieved, simple train models with few axles were created to see what influence difference axle distances and -loads had on the output.

Because Nerk was not fully developed, some parameters were impossible to change or not possible to vary enough. To get a complete understanding of Nerk, it was necessary to recreate the calculations behind Nerk, also called back-end programming. This was done in MATLAB and made it possible to see how the results varied with different parameters and to evaluate the sensitivity of the results depending on the inputs.

3.4 Assumptions in Nerk

3.4.1 Calculation method

CFS [6] is, as described in 2.4, time demanding for calculations in MATLAB, hence Nerk uses the numerical IoE method. As it is useful to verify that IoE is precise in terms of acceleration and moment response it was compared to similar results obtained with the CFS for 60 bridge lengths between 2 and 40 meters and velocities between 150 and 300 km/h.

Cumulative distribution function(CDF) of the error was created to observe the magnitude of the general error. This procedure was performed for both acceleration and moment response. The error may differ as derivation can affect the results. In step-by-step calculations the error has a tendency of increasing with numerical differentiation [6].

Both methods (IoE and CFS) is based on modal superposition, and mode shape must be assumed to perform the calculations. Nerk assumes all bridges are simply supported in order to simplify the mode shape assumption, known from Equation 2.3.

Moment response calculated with IoE was compared with measurements from Møstadbekken (bridge number 408 in Appendix A) to compare theoretical calculations with practical measumrements. The measurements was done for two different trains with a low velocity, given in Appendix A.

3.4.2 Numerical time step

In order to calculate the modal response with the IoE-method is it necessary to choose a sufficiently small time step (see 2.4.1).

Nerk uses an assumed minimum value equal to 4 meters for the wavelength in Equation 2.21, as well as a necessary number of steps per period equal to 20. This yields the following final expression for the time step calculated by Nerk:

$$\Delta t_{max} = \frac{1}{20 \cdot max\{\frac{v}{4m}; N^2 n_0\}}, \tag{3.1}$$

where v is the train velocity, n_0 is the first eigenfrequency of the bridge and N is the number of modes chosen by the user.

3.4.3 Bridge parameters

Distributed mass

Nerk assumes a distributed mass of all bridges as 10 tons per meter.

Natural frequency

EC [9] estimates the first natural frequency of any railway bridge as a function of the length as shown in Figure 3.4 [9]. Nerk uses five frequency function to cover the spectra of possible eigenfrequencies (see 3.3).

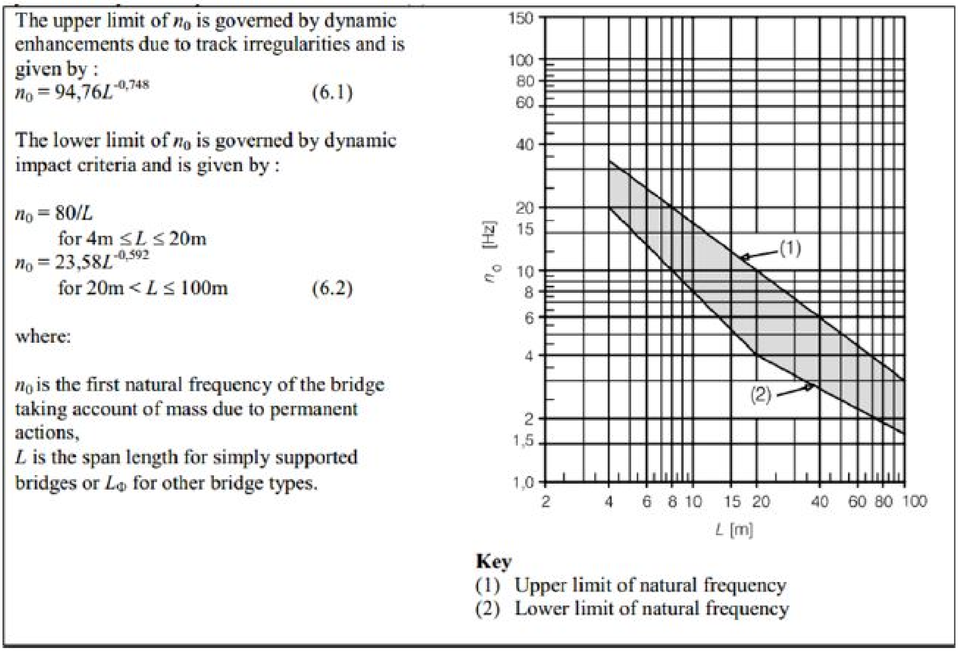


Figure 3.4: Limits of first natural frequency n_0 as a function of L

It should be noted that the frequencies obtained from Figure 3.4 is in Hz and need to be multiplied with 2π in order to find $\omega_0[rad/s]$.

When ω_0 is known it is possible to find an expression for eigenfrequencies for higher modes as well by using the relation given in Equation 2.5:

$$\omega_n = n^2 \cdot (\frac{\pi}{L})^2 \sqrt{\frac{EI}{m}} = n^2 \cdot \omega_0 \tag{3.2}$$

Damping

EC gives an approach for calculating lower bound estimates of damping on railway bridges as a function of length and bridge type, shown in Table 3.2 [9].

Bridge type	ζ Lower limit of percentage of critical damping [%]	
	Span L $<$ 20m	Span L $\geq$ 20m
Steel and composite	ζ = 0,5 + 0,125(20-L)	ζ = 0,5
Prestressed concrete	ζ = 1,0 + 0,07(20-L)	ζ = 1,0
Filler beam and reinforced concrete	ζ = 1,5 + 0,07(20-L)	ζ = 1,5

Table 3.2: Values of damping to be assumed for design purposes according to the Eurocode

Nerk, and the rest of this thesis, refer to steel and composite as bridge type 1, prestressed concrete as bridge type 2 and reinforced concrete as bridge type 3

EC also claims that dynamic vehicle-bridge mass interaction tends to reduce peak response at resonance for spans shorter than 30 meters. This can be taken into account by adding additional damping $\Delta\zeta$, which is a function of the bridge length.

$$\Delta\zeta = \frac{0,0187L - 0,00064L^2}{1 - 0,0441L - 0,0044L^2 + 0,000255L^3} \quad [\%] \tag{3.3}$$

The additional damping can only be applied for bridges with span length shorter than 30 meters. It should be noted that the additional damping suggested by the Eurocode is evaluated as poor in a research by Arvidsson [5].

Nerk uses EC damping in the TSC-calculation with bridge type and bridge length as inputs and the possibility of adding additional damping for the user. The program assumes that damping-ratio ζ is equal for all modes.

3.5 Sensitivity analysis

In order to make it possible to evaluate the assumptions made in Nerk and do a segment analysis was it necessary to perform a sensitivity analysis on the parameters from Nerk. The following parameters were evaluated:

- Distributed mass Δm
- First natural frequency n_0
- Number of time steps per period
- Number of modes

For the simplicity of the sensitivity analysis, default bridge parameters from Nerk were used for mass, time steps per period and number of calculation modes, while first natural frequency corresponding to Frequency function 3 and damping corresponding to bridge type 2 was used. All calculations were made with ETR500Y train composition to shorten the calculation time without decreasing the quality of the results much.

Distributed mass, first natural frequency, number of time steps per period and number of modes where evaluated with the other parameters kept constant. Distributed mass was also evaluated with stiffness constant instead of the first natural frequency. This was done because the contribution from mass to resonant velocity and response is different depending on whether stiffness or first natural frequency is constant.

8 fictive bridges with lengths from 2 to 20 meters and velocities from 100 to 300 km/h were used in the analysis of distributed mass, first natural frequency and number of time steps per period, and four longer bridges up to 50 meters were added to analyse number of modes.

3.6 Roehnsund method

The assumptions and the method used in Nerk gives a very conservative analysis, unless the user knows the exact properties of the bridges he wants to analyse. If that is the case, other scripts or programs can be used to evaluate the bridges and obtain good results. It would be useful to specify the properties, and Roehnsund method is therefore proposed for narrowing down the spectrum of eigenfrequencies to consider and to obtain less conservative results.

The most important properties when calculating bridge response for a moving load are mass and stiffness, which determines eigenfrequency. Hence was it natural to try to obtain more specific mass and stiffness estimates depending on other, easy available, bridge inputs.

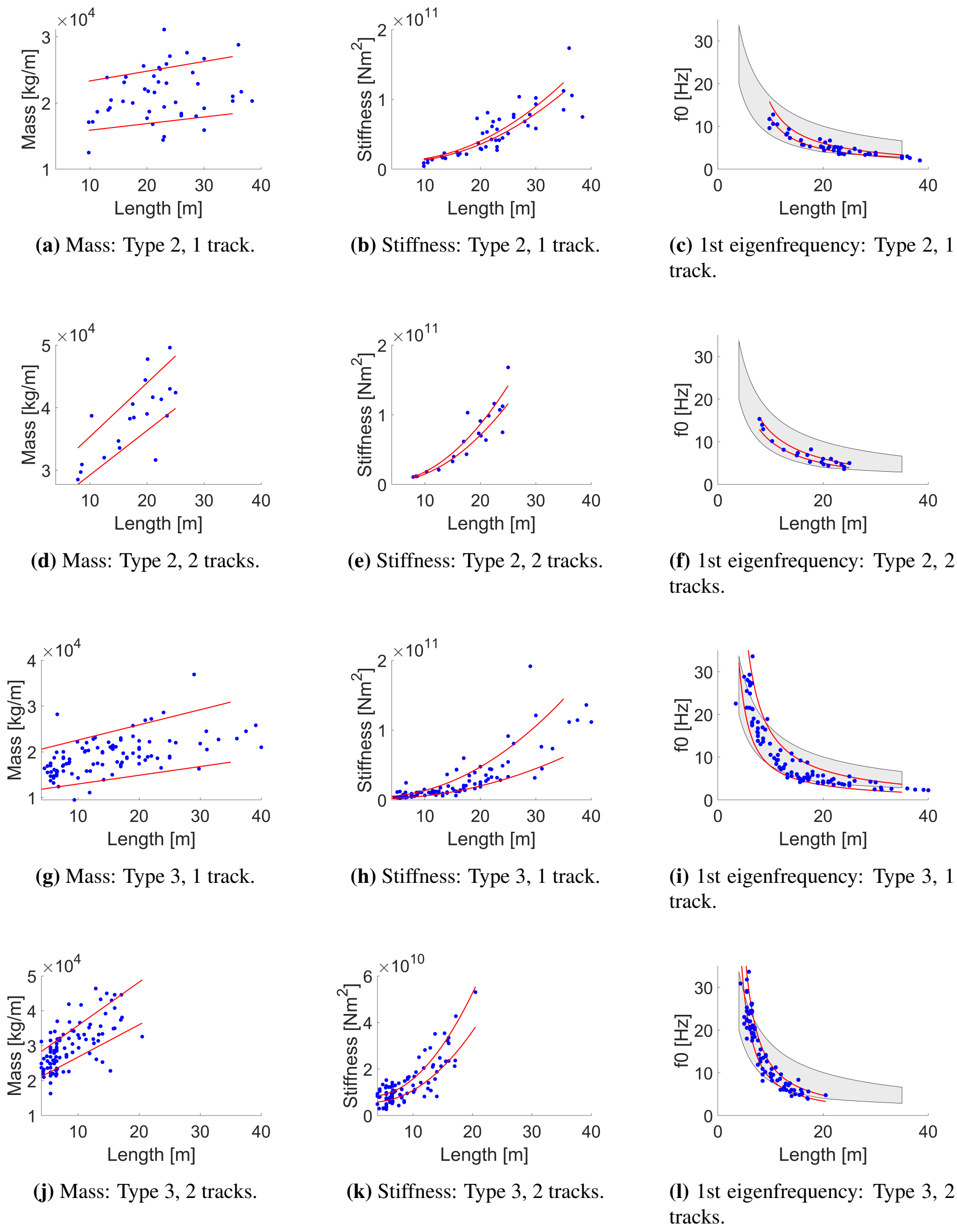
The properties of the bridges shown in Figure 3.1a and 3.1b indicate that it is possible to separate one- and two-track bridges in terms of mass and stiffness. Adding the Swedish database [4] makes it natural to separate between different bridge types as well (steel and composite, prestressed concrete and reinforced concrete). Bridge type is a practical input for finding mass and stiffness, as it is already an input for damping according to EC [9]. Roehnsund method is thus developed based on all bridges from the Swedish and the European database.

Using linear regression to estimate mass and quadratic regression to estimate the stiffness for bridge sets including specific number of tracks and bridge type, and then adding a percentage deviation in both direction gives the fields shown in Figure 3.5. Note that bridge type 1 is not included due to insufficient number of type 1-bridges in the databases.

The size of the spectra are adjustable by changing the accepted cover of variance, which determines the deviation in the spectra. As default 90% is used, meaning 90% of the variance is covered in the mass- and stiffness-fields made. The upper and lower limit for the eigenfrequency-spectra are made by using (i) the lower stiffness and the upper mass, and (ii) the upper stiffness and lower mass respectively.

The grey area in the frequency plots show the Eurocode estimation of first eigenfrequency for all bridge types. From this dataset is it visible that it is possible to reduce the considered area by adding bridge type and number of tracks as inputs. It also shows that there are several bridges with first analytical eigenfrequency outside the Eurocode-estimate.

Further on Figure 3.6b shows how the method is applied to obtain the area that is meant to represent all bridges of a specific type. The four combinations of extremums of mass and stiffness are considered to make the colored curves in Figure 3.6a. The envelope curve is computed using the max response of the cases and the regression lines.



(a) Mass: Type 2, 1 track.

(b) Stiffness: Type 2, 1 track.

(c) 1st eigenfrequency: Type 2, 1 track.

(d) Mass: Type 2, 2 tracks.

(e) Stiffness: Type 2, 2 tracks.

(f) 1st eigenfrequency: Type 2, 2 tracks.

(g) Mass: Type 3, 1 track.

(h) Stiffness: Type 3, 1 track.

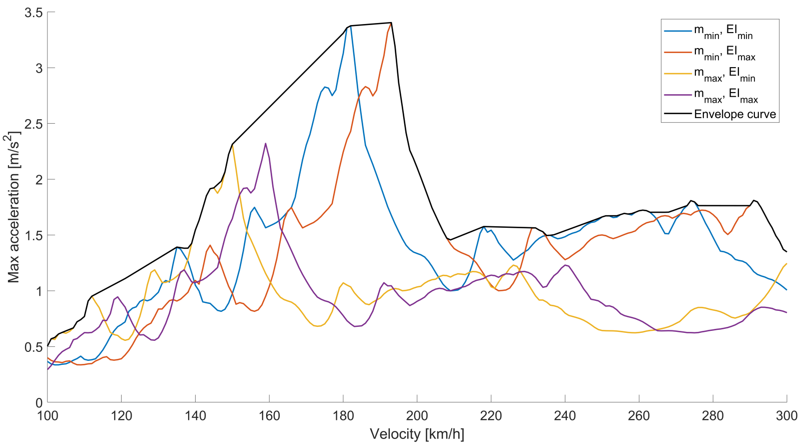
(i) 1st eigenfrequency: Type 3, 1 track.

(j) Mass: Type 3, 2 tracks.

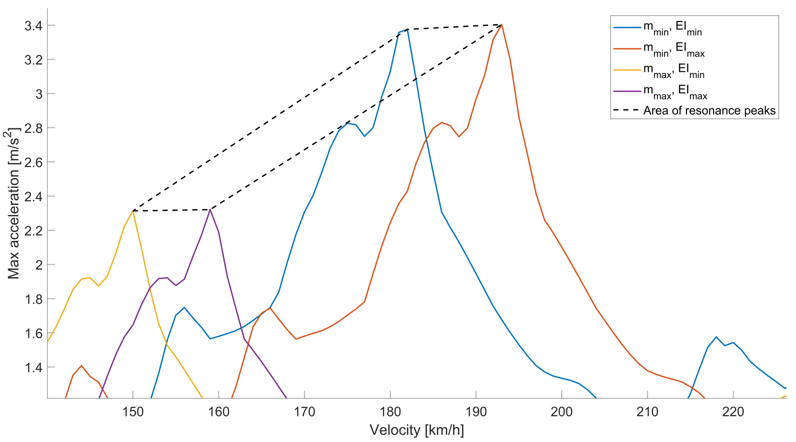
(k) Stiffness: Type 3, 2 tracks.

(l) 1st eigenfrequency: Type 3, 2 tracks.

Figure 3.5: Regression of mass and stiffness which yields the frequency spectrums shown in (c), (f), (i) and (l).



(a) Typical Velocity-Response obtained with Roehnsund method.



(b) The 4 highest peaks from (a) yields the dotted line representing the field corresponding to peaks from all eigenfrequencies within the spectra.

Figure 3.6: Visualisation of Velocity-Response and how the envelope in Roehnsund method is made.

3.7 Development of script

A script is developed such that the user has different options in terms of calculation method and output. It is considered important to be able to run calculations based on the different types of assumptions (Nerk or Roehnsund) or with known properties (analytical response), and to be able to analyse different bridges without running a complete 3D-analysis.

Calculation options:

- Nerk assumptions - using frequency functions by choice.
- Roehnsund method - using regression based on the bridge type and number of tracks.
- Using known bridge parameters (stiffness, mass and bridge type).

Output options:

- Single TSC: Response against time, maximum response, and calculated eigenfrequency and damping for a specific load case.
- Velocity Response plot: Max acceleration against velocity for a specific bridge length and bridge type. It plots the envelope of frequency functions, or limits according to Roehnsund method.
- 3D plot of max acceleration against bridge length and velocity for a specific bridge type. The limits in terms of length are based on the bridge database if Roehnsunds method is used, and user specified if Nerk assumptions are used.

Figure 3.7 shows a flowchart on how the script is used. The left side of the flowchart describes the steps performed by the user. Firstly the user decides which calculation method to use, then the necessary input has to be implemented for the chosen method in addition to input specific for the calculation. The Single TSC does also have some options regarding output. Finally the user runs the script, which makes calculations based on the method chosen and the inputs.

There are three choices regarding calculations: Single TSC is the simplest, Velocity response is more complicated, and the 3D plot-script is the most computational demanding of the three. The flowcharts shown in Figure 3.8, 3.9 and 3.10 show how the MATLAB-functions behind the calculations work. They show that all calculations must find modal response a different amount of times depending on number of modes, trains, bridge lengths etc. in the calculation. The flowcharts give a good indication on the difference in calculation time and the comprehensiveness of the script, as Velocity-Response runs numerous Single TSC's and 3D plot runs a number of Velocity-Response-scripts.

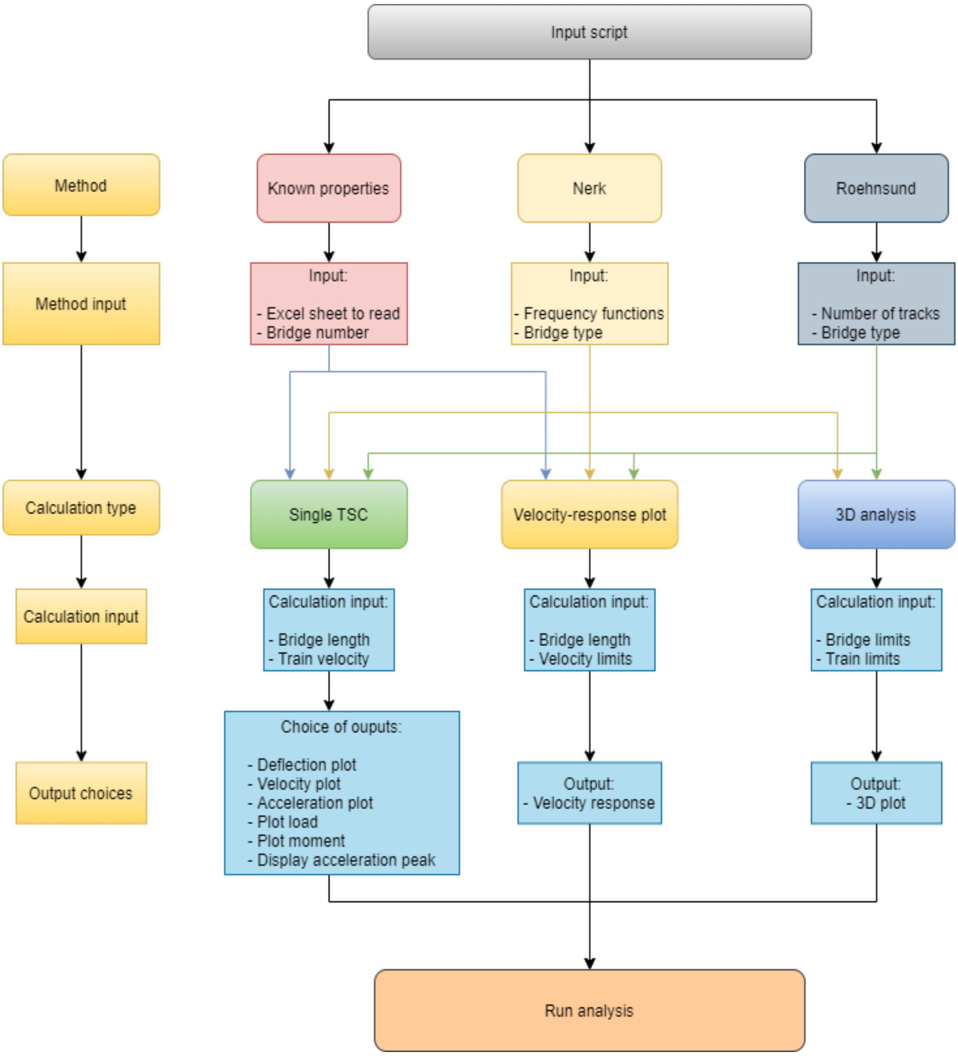


Figure 3.7: Flowchart describing the options in the script.

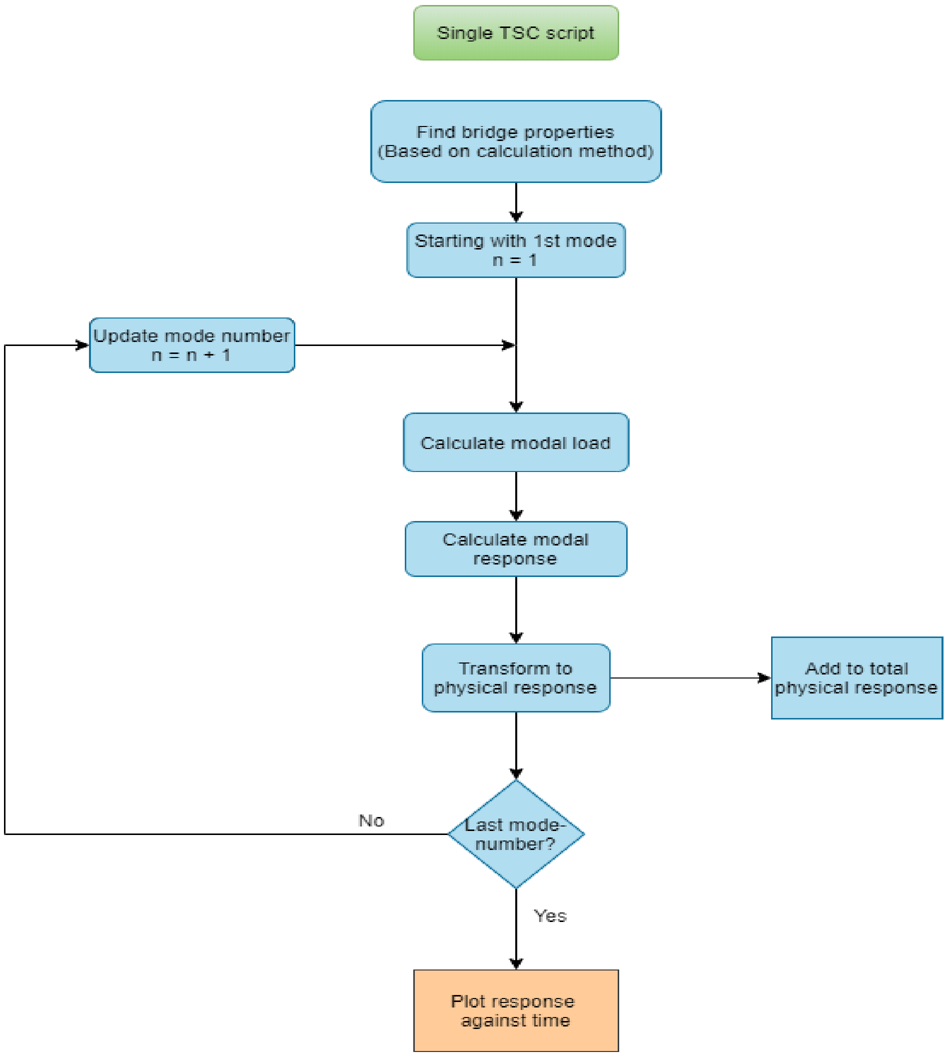


Figure 3.8: How Single TSC is calculated.

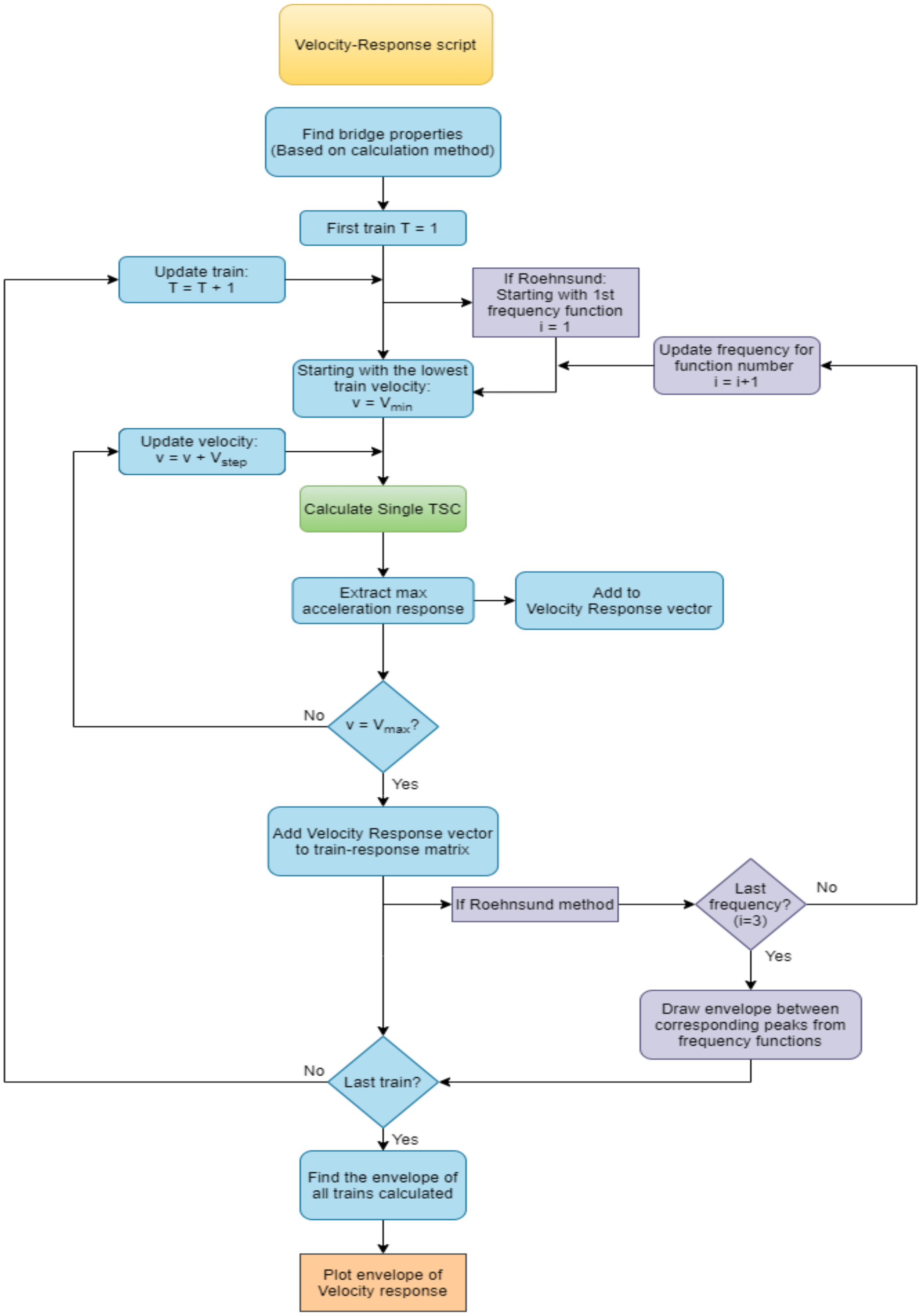


Figure 3.9: How Velocity-Response is calculated.

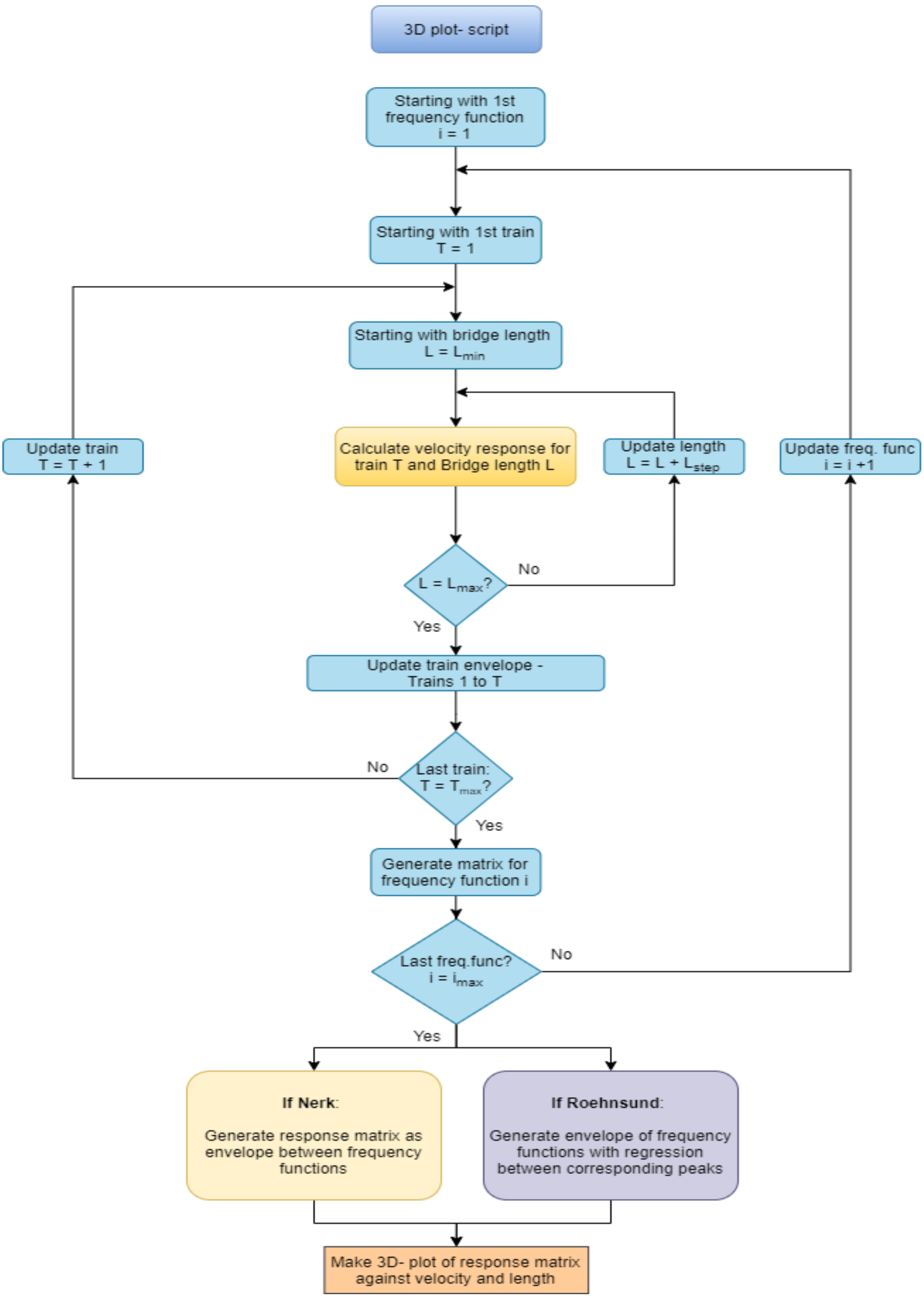


Figure 3.10: How 3D plot is made.

3.8 Segment analysis

In order to perform a segment analysis, a number of bridges must be calculated at the same time. 3D plots are not the best option for such a task considering not all the bridges necessarily would be of the same bridge type. Velocity-Response plots are easy to interpret and less time demanding than 3D plots, and is therefore a reasonable output. Hence the segment analysis is performed with a modification of the script described in 3.7, with less options in terms of input and output. The only input is an Excel-sheet with properties for the bridges to analyse, and the output is Velocity-Response plots for each bridge.

The Excel inputs includes bridge number, number of tracks, bridge type, length of the bridge, mass and stiffness. If mass and/or stiffness is unknown the user can insert the number zero and the Roehnsund method is applied to estimate the unknown parameter including uncertainty, and producing an envelope of the response as shown in 3.6.

To evaluate the segment analysis script, it was tested for seven bridges with (i) known properties, (ii) mass and stiffness from Roehnsund method and (iii) Nerk assumptions. This was done to evaluate the differences regarding calculation methods.

3.9 Assessment of results

In addition to the sensitivity analysis performed for evaluating single parameters was an assessment of quality of the results executed. The results obtained using Roehnsund method or Nerk assumptions was compared and evaluated against analytical response, using known mass and stiffness. As the analytical response was obtained using the same calculation method and some of the same assumptions (EC damping, modal superposition, simply supported beam), the quality of these assumptions is discussed by comparing them to alternatives.

Real time strains obtained with measuring instruments were obtained for a Norwegian bridge with known density, E-modulus and cross section as well as train properties corresponding to the load. These were used to calculate stiffness and distributed mass and to produce calculated analytical moment response. The strain was transformed to moment, and evaluated against the analytical response. Comparing analytical results with measured response is the best way for evaluating the sufficiency of the calculation as long as the measuring instrument are good enough.

It is normal to model bridges as simply supported in simplified analyses like in this thesis, but it is not representative for all bridges. Many bridges has rotational stiffness in the supports or they have multiple spans, which affects the response. These effects are studied and discussed after the presentation of results to validate if the results obtained are representative in reality.

MAC-values and mean ratios (see 2.5) are the main tools used to evaluate deviation and error in response obtained.

Chapter 4

Results

In this chapter will results from the sensitivity analysis be presented. Additionally results obtained by using known properties, Nerk assumptions and the Roehnsund method are displayed and compared to some degree. The three frequency functions covering EC-spectra are mainly used to perform calculations with Nerk assumptions (Frequency function 2, 3 and 4).

Bridge type is an input for all calculation methods used, as damping is based on bridge type according to EC [9]. Since the European and the Swedish database does not include a sufficient number of bridges corresponding to bridge type 1 (steel and composite) no results from Roehnsund method using this bridge type is presented.

4.1 Nerk

Figure 4.1 show 3D-plot corresponding to the output obtained by using the same assumptions as Nerk. It shows an envelope for bridge type 2 (Prestressed concrete) of all HSLMA-trains and the three frequency functions covering the limits in EC. The only differences from the output in the software is the additional cumulative envelope in Nerk and the color spectrum. Figure 4.1 is red where the acceleration is above 5 $\frac{m}{s^2}$, and orange for acceleration between 3.5 and 5 $\frac{m}{s^2}$ to make it easy to visualize where the acceleration is above critical values.

Nerk has the possibility to generate a cumulative envelope which does not descend with increasing velocity. It plots the maximum value of (i) the calculated max response for the specific velocity and (ii) the maximum response of lower velocities for current bridge length.

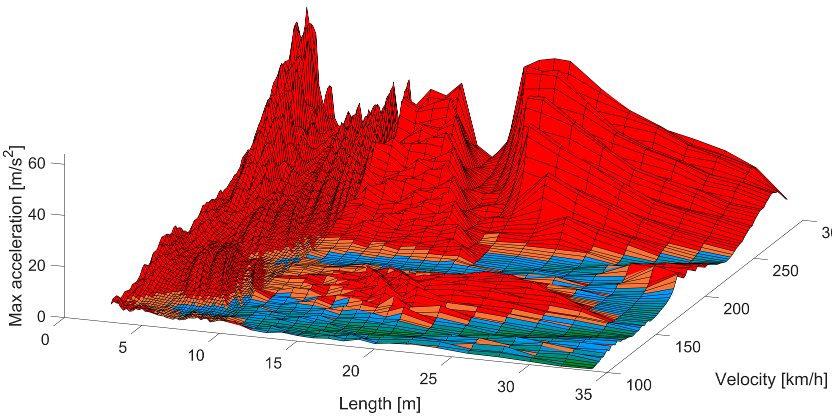
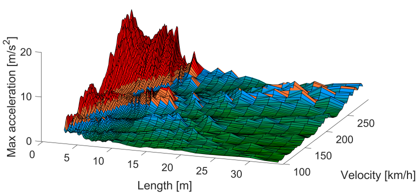


Figure 4.1: Envelope of acceleration for bridge type 2 using frequency functions 2 to 4, for all 10 HSLMA-trains.

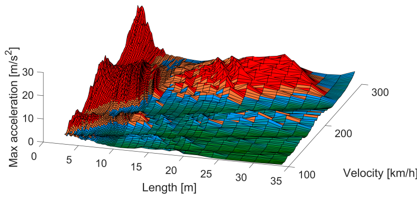
Figure 4.1 is a result of the envelope of Frequency function 2, 3 and 4. The specific results for each of the frequency functions are shown in Figure 4.2. It shows that choice of frequency function highly affects the results. Frequency function 2 corresponds to bridges with high stiffness. These bridges are less exposed to critical response than bridges with lower eigenfrequency, which gets lower critical velocities.

Figure 4.3 shows an envelope calculated the same way as Figure 4.1, but for bridge type 3 (Reinforced concrete). The only difference when using Nerk assumptions is the damping function, which yields higher damping for bridge type 3 than for bridge type 2 (see 3.2).

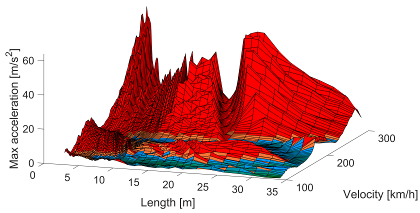
The deviation in response calculated with bridge type 3 compared to type 2 is shown in Figure 4.4. It shows that the choice of bridge type has bigger impact for longer bridges where the deviation is about 30 percent. This is a highly significant difference, meaning it



(a) Frequency function 2.



(b) Frequency function 3.



(c) Frequency function 4.

Figure 4.2: Max acceleration against length and velocity for Frequency function 2, 3, 4 for bridge type 2.

is important to choose correct bridge type.

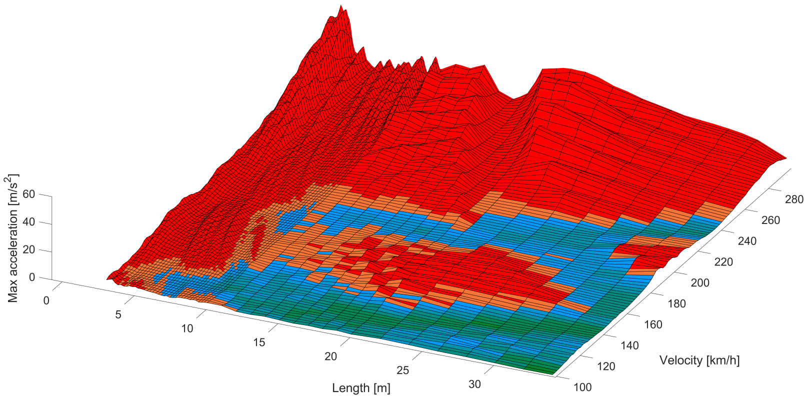


Figure 4.3: Envelope of acceleration for bridge type 3 using frequency functions 2 to 4 for all HSLMA-trains.

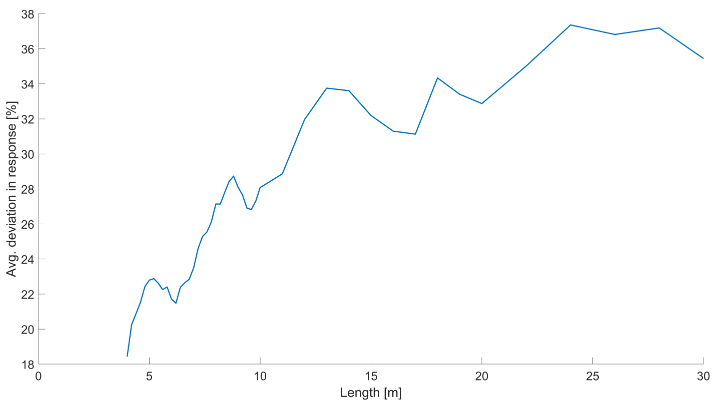


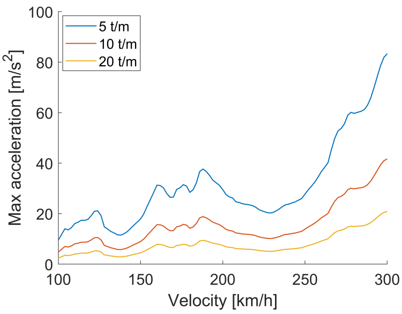
Figure 4.4: Deviation between two different damping functions (Bridge type 2 vs type 3) against bridge length.

4.2 Sensitivity analysis

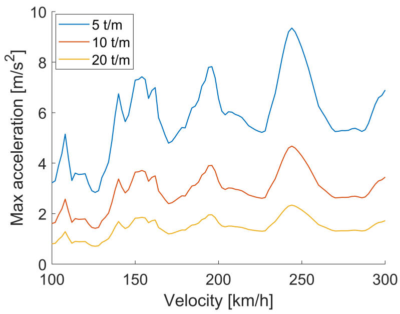
4.2.1 Distributed mass

Figure 4.5 shows max acceleration at the center of the bridge for two different bridge lengths with different distributed masses. Equation 2.7 shows that acceleration is inversely proportional with mass as long as eigenfrequency is not a function of mass, which is the case in the figure.

Conversely, if eigenfrequency is dependent on mass, max accelerations for different masses will be as in Figure 4.6. In this case both max response and resonance velocity is mass dependent. Increasing mass result in lower resonance velocity and lower response.

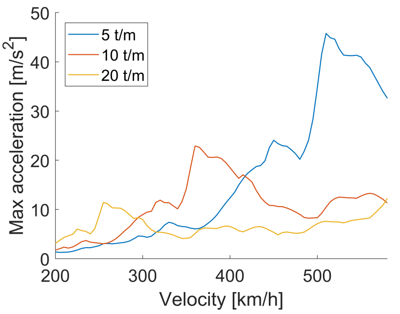


(a) Bridge length 2 meters.

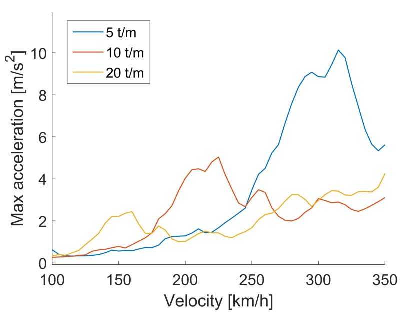


(b) Bridge length 8 meters.

Figure 4.5: Max acceleration against train velocities with constant eigenfrequency.



(a) Bridge length 4 m.

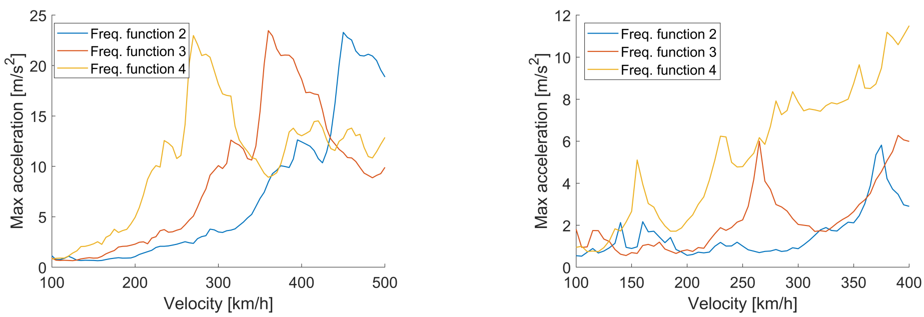


(b) Bridge length 6 m.

Figure 4.6: Velocity-Response calculated with 5, 10 and 20 t/m with constant stiffness.

4.2.2 First natural frequency

Figure 4.7 shows how the max acceleration depends on eigenfrequency. The figure shows that the plots have the same shapes but they are moved along the x-axis, so higher natural frequency results in higher resonant velocities. Figure 4.7b shows that assuming too high natural frequency (assuming Frequency function 2) may have big consequences as it would underestimate the response in general if the analytical frequency corresponds better with Frequency function 4.



(a) Bridge length 4 meters.

(b) Bridge length 16 meters.

Figure 4.7: Max acceleration against train velocities for two different bridge lengths.

Table 4.1 shows how estimated eigenfrequency varies with frequency functions used, and how the choice affects the results. Note that it is not necessarily the highest acceleration value that are compared but peaks that can be identified with the other frequency functions. The table shows that the peak values are nearly the same, but they correspond to different velocities, higher frequency gives higher critical velocity. Additionally the difference in velocity grows with increasing bridge length.

Bridge nr.	Length	Eigenfrequency 2			Eigenfrequency 3			Eigenfrequency 4		
		f_0 [Hz]	Resonant acc. [m/s^2]	Resonance vel. [km/h]	f_0 [Hz]	Resonant acc. [m/s^2]	Resonance vel. [km/h]	f_0 [Hz]	Resonant acc. [m/s^2]	Resonance vel. [km/h]
1	2	56.42	15.17	205	48.21	15.17	175	40	15.28	150
2	4	33.6	7.46	290	26.8	7.27	230	20	7.17	175
3	6	24.81	7.59	290	19.07	7.8	225	13.33	7.59	155
4	8	20	6.93	270	15	6.32	205	10	6.93	135
5	10	16.93	2.8	230	12.46	2.63	170	8	2.3	110
6	13	13.91	1.94	325	10.03	1.9	235	6.15	1.89	145
7	16	11.91	5.81	375	8.46	6	265	5	5.11	155
8	20	10.08	3.7	320	7.04	3.65	220	4	3.65	125

Table 4.1: Resonance values for Frequency function 2, 3, 4.

As increasing natural frequency with constant mass is the same as increasing bridge stiffness, similar results would be obtained if a sensitivity analysis of the stiffness was executed.

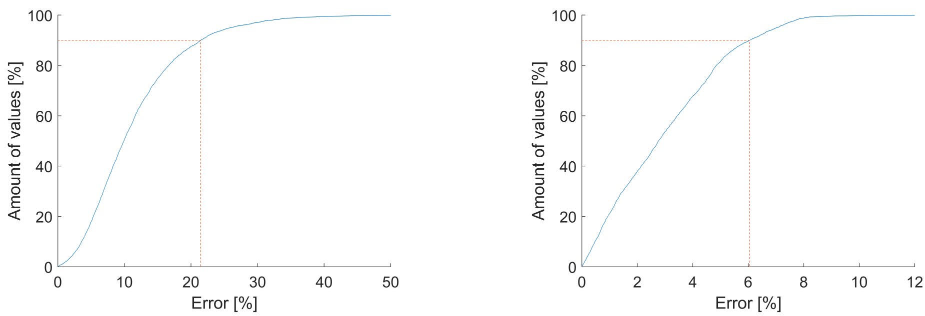
4.2.3 Number of modes

3D plots for bridges were calculated with for 1, 3 and 5 modes. The error between the results of 1 and 3, and 3 and 5 modes is shown in Figure 4.8 and 4.9. The plots show that the difference between 1 mode and 3 modes is large compared to the difference between 3 modes and 5 modes, which indicates that three modes is a minimum to obtain good results.

Calculations were made individually for a set of bridges with 1, 3, 5 and 7 bridge modes. Figure 4.10 shows Velocity-Response plots for two bridges with lengths 2 and 16 meters. The max response is generally higher for higher modes. The mean error values of the calculated bridges is given in Table 4.2. Number of modes used in the calculations yields the same shape of the Velocity-Response functions, as the MAC-values are very close to 1, but there is a large mean ratio of 13.69% from one to three modes compared to three to five and seven modes. It can be noted that the mean ratio is higher for shorter bridges, 19.66% for lengths 2-12 meters from 1 to 3 modes, and lower for longer bridges, 5.75% for lengths 16-50 meters.

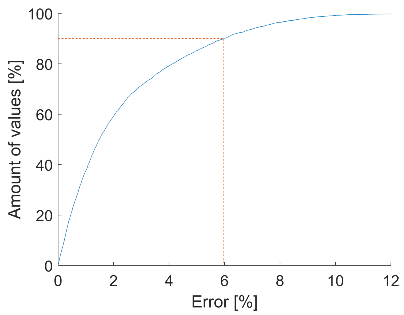
	Modes compared		
	1 vs 3	3 vs 5	5 vs 7
MAC	0.9947	0.9995	0.9999
Mean Ratio	13.69%	2.33%	0.61%

Table 4.2: Mean values of comparison between modes

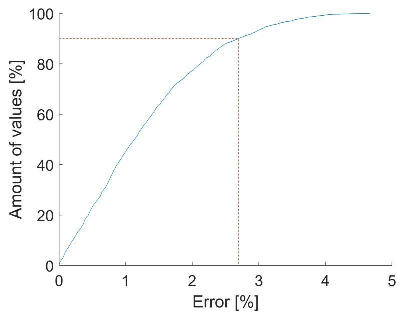


(a) Acceleration error. **(b)** Moment error.

Figure 4.8: CDF of the error for 1 mode vs 3 modes.

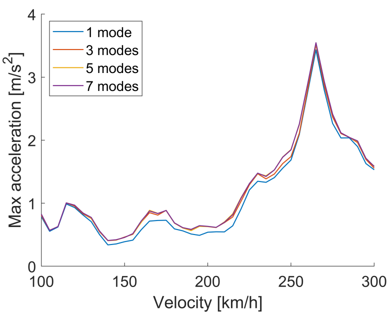


(a) Acceleration error.

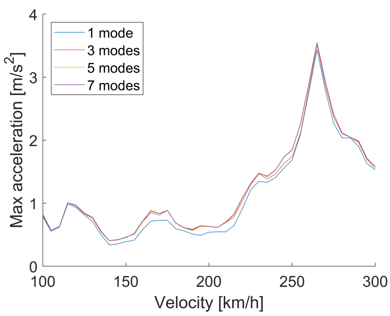


(b) Moment error.

Figure 4.9: CDF of the error for 3 modes vs 5 modes.



(a) Bridge length 2 m.



(b) Bridge length 16 m.

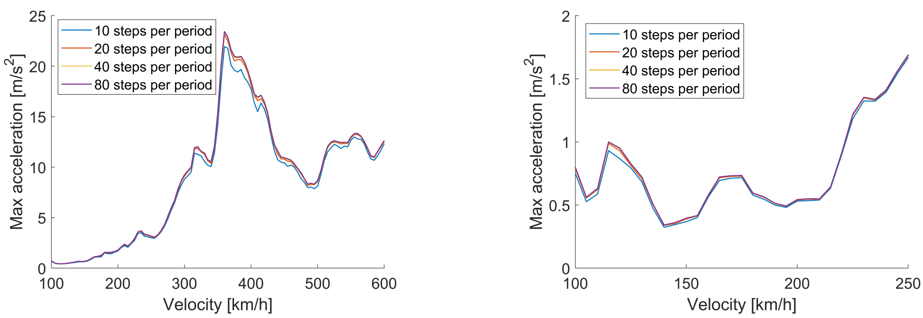
Figure 4.10: Velocity-Response calculated with 1, 3, 5 and 7 modes.

4.2.4 Number of time steps per period

Calculations made with 10, 20, 40 and 80 time steps per period is shown for two bridges in Figure 4.11. The results are very similar with any of the used number of time steps as the MAC-values in Table 4.3 show, but the results are slightly lower with 10 steps compared to 20 steps with a mean ratio of 2.35%. The ratio decreases with increasing number of time steps, meaning the results are converging towards what can be assumed to be correct.

	Number of steps compared		
	10 vs 20	20 vs 40	40 vs 80
MAC	0.9999	1.0000	1.0000
Mean Ratio	2.35%	0.64%	0.17%

Table 4.3: Mean values of comparison between number of steps per period

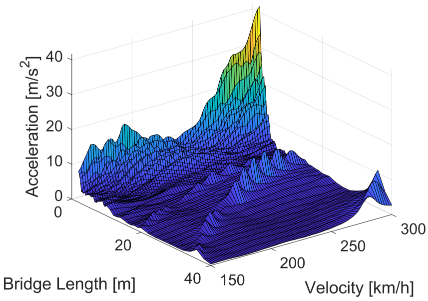


(a) Bridge length 4 m. **(b)** Bridge length 16 m.

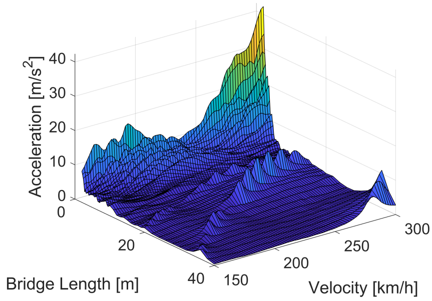
Figure 4.11: Velocity-Response calculated with 10, 20, 40 and 80 steps per period.

4.3 Calculation method

Max response calculated with IoE and the CFS is shown in Figure 4.12 and 4.13.

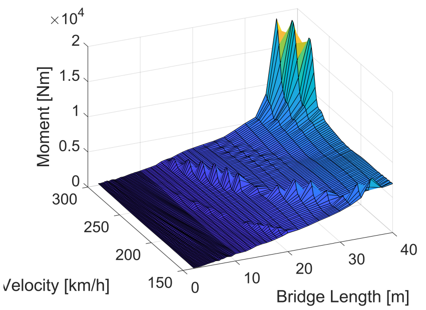


(a) Acceleration from IoE method.

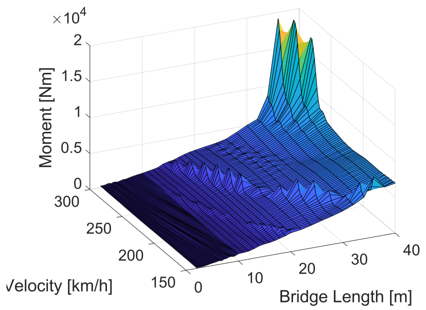


(b) Acceleration from CFS method.

Figure 4.12: Acceleration from the two methods.



(a) Moment from IoE method.



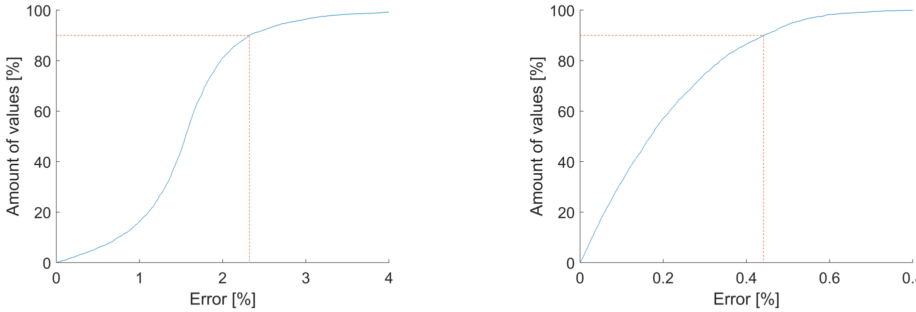
(b) Moment from CFS method.

Figure 4.13: Moment from the two methods.

The error in the IoE method is shown as a cumulative distribution plot in Figure 4.14 for acceleration and moment. It shows that 90 percent of the points have less than 2.5 percent error for acceleration response and less than 0.5 percent for moment response.

Similar plots for different span lengths were made to observe if short bridges were more affected than long bridges or the other way around, but no difference worth mentioning was noticed.

Measurements of response of bridge number 408 in Appendix A were compared to results calculated with IoE with dimensions from design drawings. Values for mean ratio and MAC is given in Table 4.4.



(a) Error in acceleration. **(b)** Moment error.

Figure 4.14: Error in moment and acceleration for IoE-method versus the closed form solution.

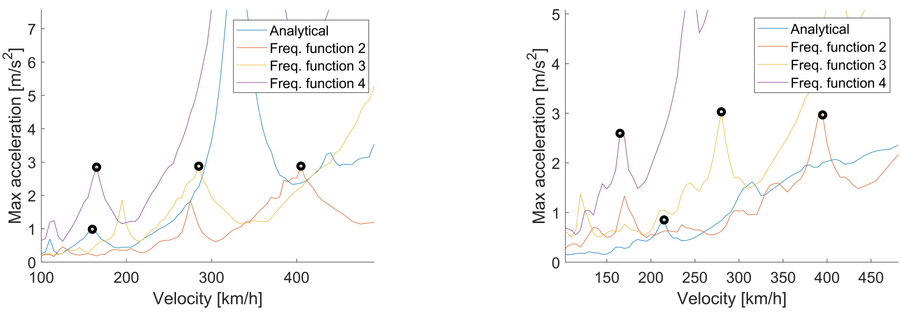
	Train 1	Train 2
MAC	0.9730	0.9133
Mean Ratio	389.87%	349.93%

Table 4.4: Mean values of comparison between measurements and IoE-calculations.

4.4 Analytical data vs Nerk assumption

The use of Nerk assumptions was compared to using known properties in calculation of Velocity-Response for 9 bridges from the Euopean database. The bridges are a selection covering a wide spectra of lengths. MAC-values and mean ratio from the comparison is given in Table 4.5.

Response for two of the bridges is shown in Figure 4.15. The peaks marked in each of Figure 4.15a and 4.15b are representing the same resonance number. The difference in peak acceleration is caused by difference in mass and the difference in resonant velocity is caused by different natural frequencies. For bridge number 322 the first eigenfrequency calculated analytically and with Frequency function 4 is almost the same. This results in similar resonance velocity, and the error of 265% is mainly caused by the mass ratio of 2.71. The MAC-value of 0.965 verify that these results have the similar shapes.



(a) Bridge number 322.

(b) Bridge number 328.

Figure 4.15: Velocity-Response calculated with Frequency function 2, 3 and 4 compared to analytical response.

		Frequency function 2			Frequency function 3			Frequency function 4		
Bridge number	Analytical freq. [Hz]	Freq. [Hz]	Mean ratio	MAC	Freq. [Hz]	Mean ratio	MAC	Freq. [Hz]	Mean ratio	MAC
2	9.5	17.19	0.5236	0.6545	12.67	0.9144	0.8643	8.16	1.5961	0.9403
6	8.1	13.91	1.0152	0.8325	10.03	1.8317	0.9405	6.15	4.4577	0.7771
11	5.7	11.91	0.6542	0.8577	8.46	1.1399	0.8543	5	3.1334	0.5662
19	4.4	9.39	0.6065	0.7955	6.58	1.4362	0.3306	3.78	3.3578	0.524
22	3.5	8.79	0.543	0.657	6.19	1.6528	0.3506	3.59	2.6453	0.965
23	15.2	20.19	1.819	0.8267	15.16	2.929	0.9911	10.13	6.7613	0.7698
28	6.8	12.5	1.3282	0.9368	8.92	2.3443	0.9301	5.33	6.3555	0.6855
34	6	10.08	1.6299	0.6793	7.04	2.731	0.859	4	9.657	0.3456
40	5	8.53	1.7805	0.5027	6.02	4.4699	0.3198	3.51	11.0612	0.421

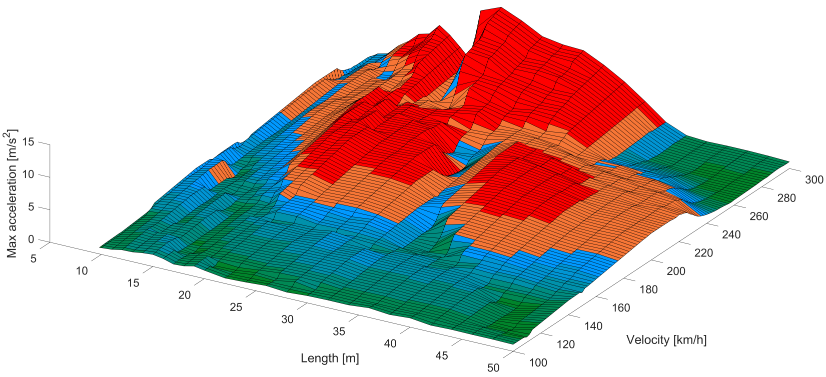
Table 4.5: Response with analytical eigenfrequency versus frequency functions.

4.5 Results from Roehnsund method

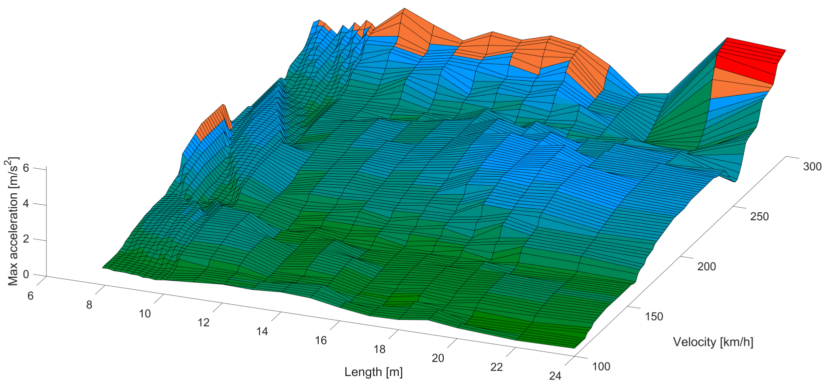
Figure 4.16 and 4.17 shows 3D plots obtained using Roehnsund method instead of Nerks frequency functions for bridge type 2 and 3, with both one and two tracks. It should be noted that the response when using the Roehnsund method is generally lower than with the three frequency functions from Nerk. It is visible by comparing the 3D plots with the ones obtained from Nerk (Figure 4.1 and 4.3) that critical response is obtained at more velocities by using Nerk assumptions.

According to Roehnsund method critical response is obtained at lower velocities for bridge type 3 than bridge type 2. The damping is higher for bridge type 3, which should result in lower response, but according to the bridge data bridge type 3 has lower mass and needs to be explained with a wider spectra of stiffness, causing more possible resonance and higher response.

The response is in general lower for the two-track bridges than for one-track. Figure 3.5 shows that the Roehnsund method gives similar eigenfrequency-spectra for bridges of the same type, but the mass is higher for two-track bridges, which results in lower response.



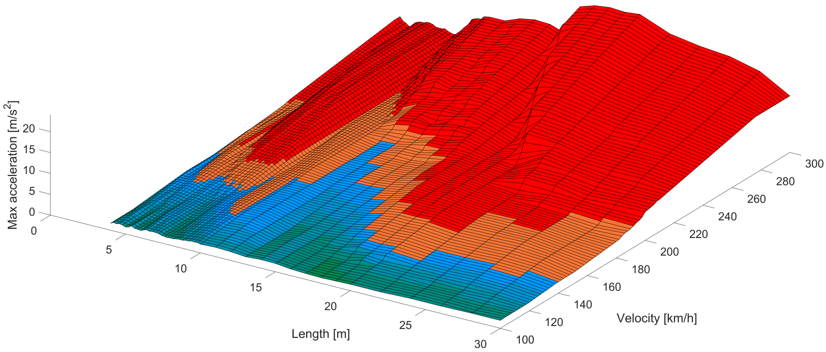
(a) Bridges type 2, 1 track



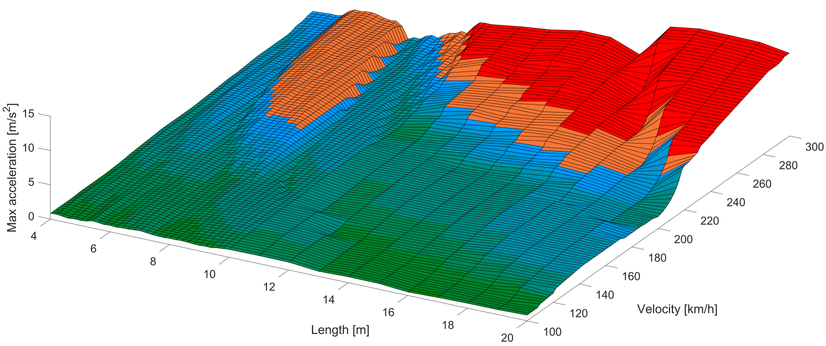
(b) Bridges type 2, 2 tracks

Figure 4.16: Roehnsund method used to obtain 3D plots of bridges of type 2.

Figure 4.18 shows Velocity-Response plots from two of the HSLMA trains (HSLMA 3 and HSLMA 10) for bridge number 401 (form the Norwegian database) in Appendix A. The three colored plots represents the limits in eigenfrequencies that produce the black envelope-curve. Figure 4.19 shows the envelope of the single envelopes from all 10 HSLMA-trains for the same bridge, which is the same curve as the one produced for the corresponding bridge length (28 meters) in the 3D-plot for type 2, 1 track bridges.

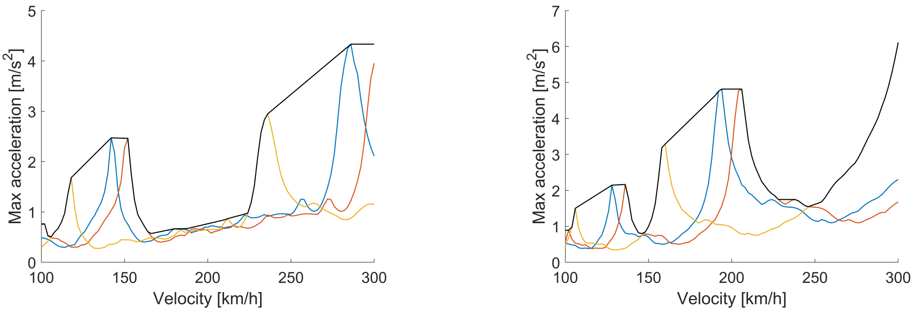


(a) Bridges type 3, 1 track



(b) Bridges type 3, 2 tracks

Figure 4.17: Roehnsund method used to obtain 3D plots of bridges of type 3.



(a) HSLMA 3

(b) HSLMA 10

Figure 4.18: Velocity-Response plots with Roehnsund method for two HSLMA trains and bridge nr 401.

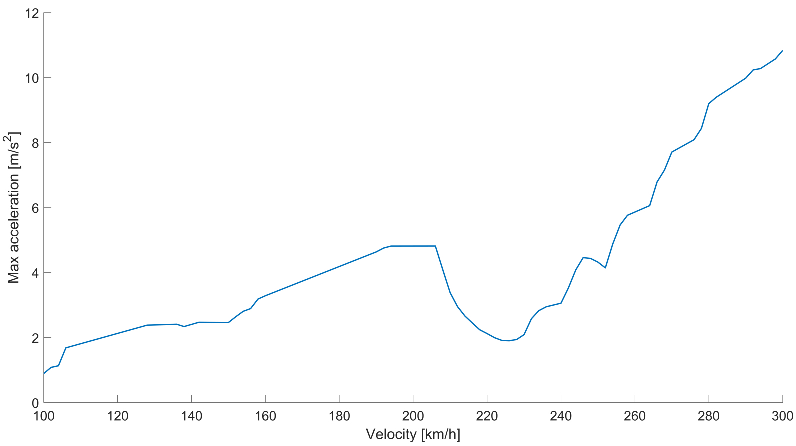


Figure 4.19: Envelope of all 10 HSLMA-trains for bridge number 401.

Chapter 5

Discussion

5.1 Nerk discussion

5.1.1 Signature, spectrum and aggressiveness

The train signature calculation creates a plot that shows potentially critical wavelengths for each train. The forces are representing a sum of all axle loads combined with the effect of the axle composition at each wavelength.

Figure 5.1 show that train spectrum will have similar shape as train signature and the peaks will be located at the same wavelengths. Nevertheless, the amplitude of each peak and the relative difference between the peaks is different from the signature.

There are errors associated with the DER-method. One of them is that the resonance criterion, $n\omega = \omega_0$, is not always satisfied because the frequency of the train passage is not always a whole multiple of the natural frequency and the acceleration might be overestimated. Another is that the influence line wrongly eliminates bridge response for $\frac{L}{\lambda} = 1.5, 2.5, 3.5...$, as shown in Figure 5.2. This problem is not considered in Nerk, since acceleration is not calculated directly in the program with this method. ERRI suggest to calculate the acceleration as maximum of the DER-method and dynamic increment [1]. However, the peak values calculated with the DER-method is higher than with dynamic increment.

The DER-method was validated, and has proved to be good in resonance zones [1]. Response tend to be slightly overestimated when dynamic increment is applied, especially for short bridges with wavelength longer than bridge length. Results from the method seem to be better for lower damping values.

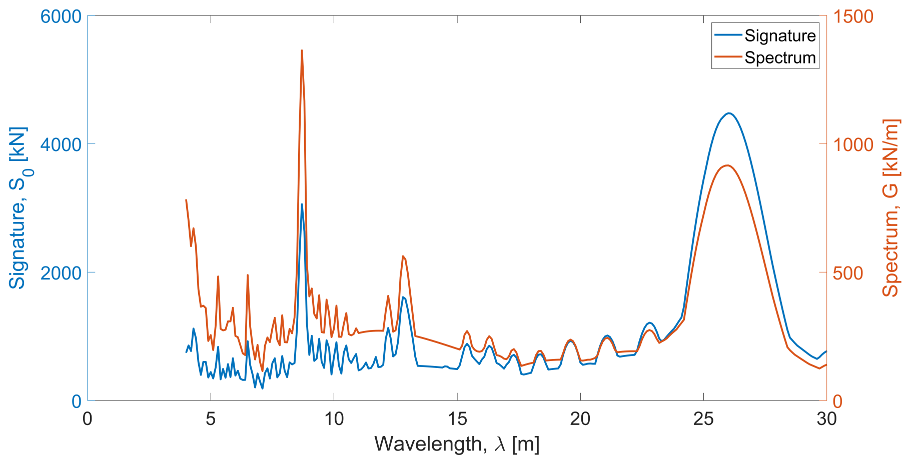


Figure 5.1: Signature and spectrum for ETR500Y train.

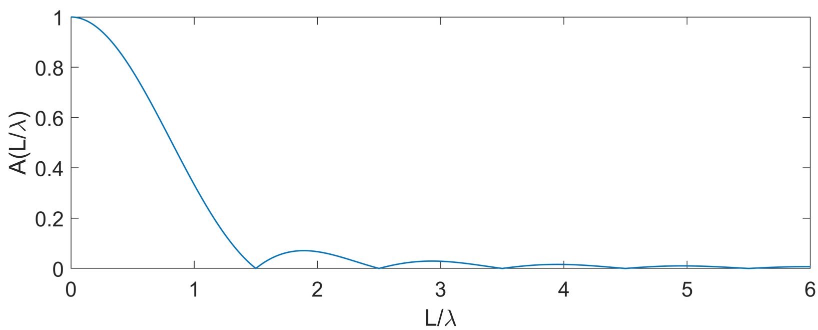


Figure 5.2: Influence Line

A possible additional output would be to plot aggressiveness with bridge length as input and velocity and natural frequency as variables, see Figure 5.3. If maximum speed on a new bridge is known, an estimation of minimum natural frequency for the bridge can be found. In the same way, if the natural frequency of the bridge is known, potential critical velocity for the train can be found. This could be useful if the bridge properties of an existing bridge is known or a new bridge is designed.

5.1.2 TSC

The bridge mass affects bridge deck acceleration in two ways. Lower mass gives higher acceleration at resonance, but also higher natural frequency of the bridge, hence higher resonance speed. EC solves this by recommending calculation of a lower mass limit for

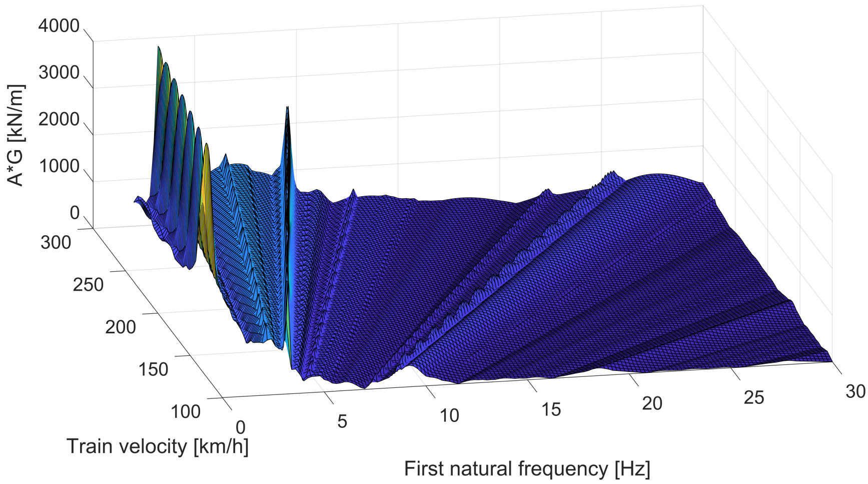


Figure 5.3: Aggressiveness calculated for ETR500Y train with bridge length 6 meter and first natural frequency and velocity as variables.

maximum resonance acceleration and an upper limit with low estimation of stiffness to calculate the lowest resonance speed.

The distributed mass in Nerk is 10 t/m (a low estimation of mass), which results in high resonance response. Instead of using an upper limit of mass to find lowest possible resonance speed Nerk applies the frequency spectra defined with the frequency functions, independent of mass. This method ensures conservative results in terms of resonance speed as long as the natural frequency is not significantly lower than the lowest frequency function in use.

Even though there are bridges with lower natural frequency than the lower estimate from EC the tendency is that Nerk overestimates the response. Conservative calculations is preferable, but over-conservative calculations yields results difficult to interpret. Figure 4.1 and 4.3 indicates that critical response is obtained at low velocities for all bridge lengths, meaning even if the objective is to get an overview of response is it impossible to know which cases to study more detailed.

Figure 4.7 gives an indication on another problem with Nerk. By using the three frequency functions displayed in the figure (Frequency function 2, 3 and 4) the whole frequency spectra from EC is supposed to be covered. The problem is that bridges with frequencies between two of the frequency functions will not be represented in a good way, considering the peaks will be placed between the frequency function-peaks. The alternative in Nerk is to apply the cumulative function, which prevents the response from decreasing with increasing velocity. This is also a bad representation, as it in fact will represent all eigenfrequencies higher than the lowest frequency function chosen, not just the frequencies up to a certain limit. The reason is that assuming eigenfrequency independent of mass causes the peaks to have the same response-value. Additionally, peaks for velocities lower than minimum is not included in the cumulative function.

A better solution to the cumulative problem would be to draw straight lines between the peaks from the frequency function representing the lowest eigenfrequency to consider and the peaks corresponding to the same resonance number from the highest eigenfrequency, as shown in Figure 5.4. In that case the response would decrease for velocities higher than resonance velocity for the highest frequency and lower than the following peak for the lowest frequency. The figure is a modification of Figure 4.7a, with a dotted line between the peak from Frequency function 4 (lowest eigenfrequency) and the peak from Frequency function 2 (upper eigenfrequency). After the last peak the envelope would follow Frequency function 2 until Frequency function 4 arises toward its next peak. Hence all frequencies in the frequency spectra could have been included without calculating the response from Frequency function 3.

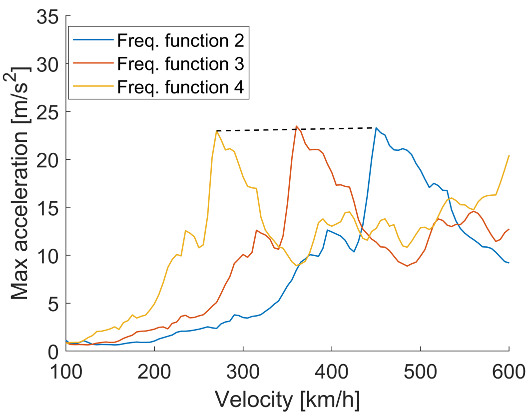


Figure 5.4: Response calculated with three frequency functions corresponding to Nerk assumptions for a 4 meter long bridge with bridge type number 2, along with a proposed envelope line.

Being aware of what happens when applying the cumulative function is very useful, as it can save the user for a lot of calculation time. If the idea is to evaluate the cumulative function the user might as well calculate response from the lowest frequency function to consider only, which would use only one third of the computational time compared to using 3. If the proposed alternative had been available it would be necessary to run two calculations, one for the lowest frequency function and one for the highest, which would also save computational time compared to calculating with three or five frequency functions.

5.2 Input assumptions

Both Nerk and Roehnsund method is based on EC assumptions. EC estimates an upper and lower limit for the first eigenfrequency as functions of length, and it also estimates damping as a function of length. The sufficiency of these assumptions are not discussed in this thesis, but how to determine other inputs are.

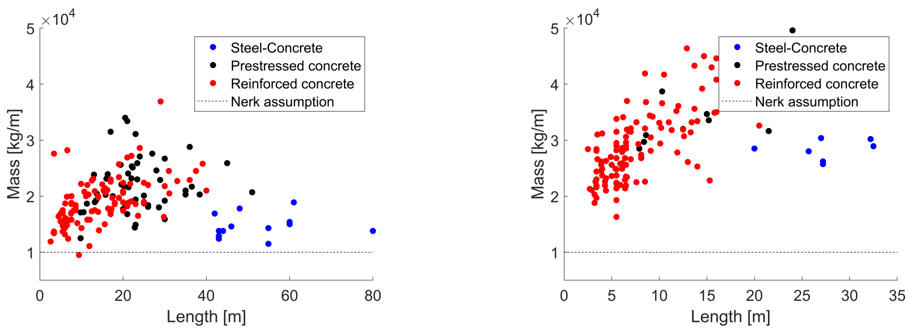
Distributed mass

4.2 shows how the determination of distributed mass affects the results. When keeping the eigenfrequency constant the acceleration is inverse proportional with the mass, meaning it is conservative to use a low mass. If the eigenfrequency is not kept constant, but as a function of stiffness and distributed mass, the resonance velocity will increase as the mass decreases. The EC [9] states:

"(2) Two spesific cases for the mass of the structure including ballast and track shall be considered:
- a lower bound estimate of mass to predict maximum deck accelerations...
- an upper bound estimate of mass to predict the lowest speeds at which resonant effects are likely to occur..."

This means it is not necessarily conservative to use a single low mass to find maximum response, as low mass yields higher resonance speeds.

As described in 3.4 Nerk uses 10 tons per meter as distributed mass for virtually all bridges. Figure 5.5 shows that this is lower than all data used in this thesis.



(a) One track bridges. **(b)** Two track bridges.

Figure 5.5: Distributed masses from the database plotted with the Nerk-assumption of 10 t/m.

The problem is that it could cause over-conservative results, and many bridges with sufficient capacity can be considered too weak. The assumption also makes it harder to determine a specification of the natural frequency, as it is a function of stiffness and mass.

A proposed alternative in this thesis is a linear regression of mass with deviation, dependant on bridge type and number of tracks on the bridge. A simple line would not be very conservative considering the spread of the points and the EC statement that both an upper and lower limit of mass should be considered. Adding deviation that are made to cover a significant part of the variation of the points is a possible solution.

Eigenfrequency and frequency functions

When finding eigenfrequency of a bridge, three methods are considered in this thesis: (i) Assuming eigenfrequency dependent on length, (ii) Calculating eigenfrequency as a function of assumed mass and stiffness, and (iii) Using known mass and stiffness to calculate analytical eigenfrequency.

(i) The method where eigenfrequency is assumed as a function of only length is provided by EC. The positive aspect of this method is that EC in general is a conservative and reliable source. The main issue is that it may give over-conservative results if all frequencies between the upper and lower limit are considered possible. An example of this is shown in Figure 4.1, where the critical train velocity is lower than 150 km/h for all bridge lengths. The second issue is which frequency function to use when specification of properties is wanted without knowing mass and stiffness.

(ii) The alternative where mass and stiffness is assumed is also mentioned in EC. It is stated that an upper and lower limit of mass should be considered, and that a lower bound estimate of the stiffness should be used in order to obtain lowest resonance speed. An upper limit of stiffness is also considered in this thesis, as it makes it easier to make a representative envelope curve of the response. If this method is applied in a good way it should be possible to narrow down the eigenfrequencies to consider compared to (i). The main issue is that it may be non-conservative if the mass- and stiffness approximations are not representative. The estimations used in this thesis are shown in Figure 3.5, which applies Roehnsund method and shows that the variance in mass and stiffness are very different depending on bridge type. It should be mentioned that the bridge data used is limited, and that using more bridges as input could provide better mass- and stiffness estimates. As described in 4.5 this method provides less conservative results, which makes it a good solution as long as it is not underconservative.

(iii) The method where mass and stiffness are known for the bridge is mainly applied to evaluate the other two methods, as the objective is to find a way of analyzing bridges with limited input. It is considered exact compared the other two, even though parameters such as damping and stiffness in supports could be improved to obtain better results.

An important aspect when discussing this is that the analytical approach for calculating the first eigenfrequency assuming mass and stiffness is based on the bridge being simply supported. Especially for short bridges this assumption normally yields too high response, as the supports often has some rotational stiffness.

Number of modes

Number of modes is an input for both 3D plot, Velocity-Response and Single TSC. This will affect the precision of the calculations. More modes will give a higher precision, but higher modes will have smaller and smaller contribution to response. When comparing 1, 3, 5 and 7 modes in the calculation of max acceleration, the main resonance velocities are the same for all modes, but the maximum acceleration increase from one to three modes and almost stabilises when increasing to five and seven modes.

More modes in TSC will give better results, but is computationally more expensive. By increasing number of modes by a factor n, calculation time increases with a factor of n^2. The increase in calculation time when changing number of modes from one to three in Single TSC is not significant, as the calculation time is short in any case. Hence minimum three modes should always be used in Single TSC to get as correct time history and maximum values of response as possible. Velocity-Response and 3D plots calculates response for many lengths and velocities, and an increase in number of modes will have a significant impact on the calculation time. The goal with these calculations is to figure out at which bridge lengths and velocities resonance and large responses may occur, and then perform better calculations for these parameters. One mode will therefore in most cases give satisfying results.

Number of time steps per period

Computational time increases linearly with the number of time steps per period. Short calculation time is preferable, but getting sufficient precise results is most important. Table 4.3 shows how the results varies with increasing number of time steps. 10 time steps per period gives 2.35% error compared to 20, while 20 steps provides under 1 percent error vs 40, meaning 10 is considered sufficient in most cases but 20 could be used for more exact calculations.

Damping

EC damping is applied for all calculations performed in this thesis, and for all calculation methods possible to perform with the developed script. The user also has the possibility to include additional damping according to EC. Arvidsson has studied train-trach-bridge-interaction, comparing it to an MF model to obtain additional damping [5]. She discovers that the lower bound of her calibrated additional damping is well below what EC suggests. The calculated damping is still above the initial damping from EC. As the purpose of both Nerk and Roehnsund is to give a conservative overview of bridge response, it is reasonable to avoid using additional damping in the calculations.

5.3 Roehnsund method

As an improvement of the assumptions from Nerk with low distributed mass and frequency functions, Roehnsund method was developed to reduce the spectrum within upper and lower frequency limit from the Eurocode. The regression of mass and stiffness with cover of variance creates a new spectrum for frequency. This is reduced compared to EC, and the difference in equivalent resonant velocities will be smaller. Nevertheless, the frequency spectrum generated (European and Swedish) is still located within the Eurocode spectrum.

The envelope of equivalent peaks generated by Roehnsund method includes all possible combinations of mass and stiffness. Conversely, Nerk calculates response for the selected frequency functions and create an envelope that only cover those frequencies.

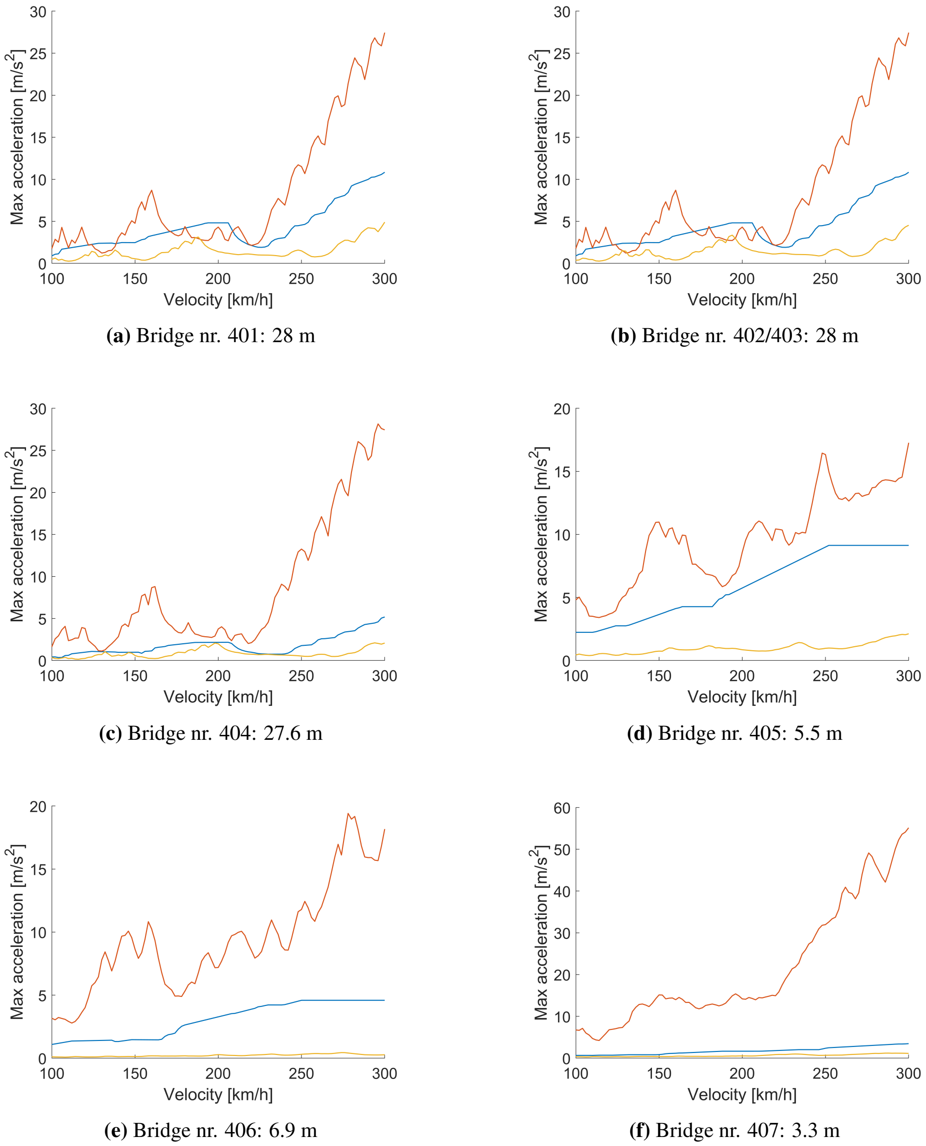
One of the main aspects of Nerk is to keep the number of inputs to a minimum. Hence, it was important in the development of Roehnsund method to do the same. The masses from the database, shown in Figure 5.5, is generally higher for 2 tracks than 1 track, and to add number of tracks as an input for the estimates of bridge mass and stiffness increased the accuracy of the calculations.

Roehnsund method was applied on the bridges in the Norwegian database, which was not included in the development of the method. An envelope of all HSLMA-trains was generated, and the acceleration response was compared to analytical response and results obtainable from Nerk. Figure 5.6 shows that Roehnsund method generally give less conservative results than Nerk, but still conservative compared to the analytical response. Bridge number 405, 406 and 407 are shorter than the others, and have low analytical response compared to Roehnsund method for all lengths. A better view of response calculated analytically and with Roehnsund method for bridge 406 and 407 is given in Figure 5.7.

While the frequency spectrum is limited with Roehnsund method, virtually all masses from the European and Swedish bridge database, thus also the mass function generated, are higher than the Nerk assumption for all bridge types and lengths, shown in Figure 5.5. One of the objectives for the Roehnsund method was to limit the frequency spectrum, while still keeping the results conservative. Figure 5.6 shows that this seem to be achieved, since the response is lower than with Nerk assumptions but still conservative compared to analytical response.

The spectra from Roehnsund method can easily be narrowed down by changing the accepted cover of variance for mass and stiffness. 90 % is default and is the only variance cover tested, but lowering the cover would make the results less conservative. This option has to be studied further as decreasing the cover means the risk for underestimation response increases. The cover of variance for eigenfrequency is generally much higher than the cover for mass and stiffness, as it is based on their extreme values.

It should be noted that bridge nr. 406 in Figure 5.6 has 4 tracks, while the results from Roehnsund method is obtained by assuming 2 tracks, and could be the reason why the



(a) Bridge nr. 401: 28 m

(b) Bridge nr. 402/403: 28 m

(c) Bridge nr. 404: 27.6 m

(d) Bridge nr. 405: 5.5 m

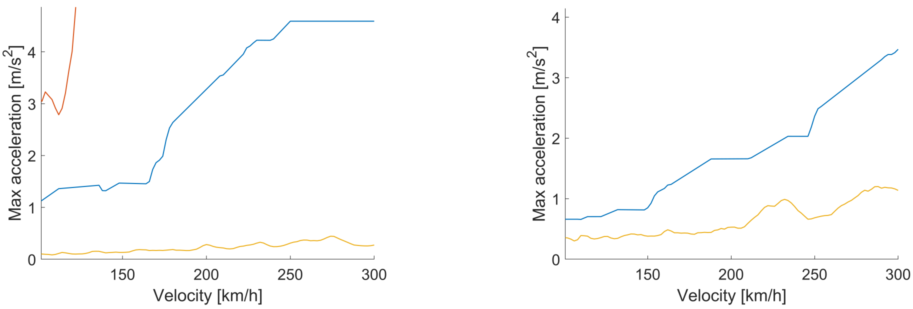
(e) Bridge nr. 406: 6.9 m

(f) Bridge nr. 407: 3.3 m

Figure 5.6: Nerk (red), Roehnsund (blue) and analytical (yellow) results compared.

error is large compared to the other bridges. Bridge nr. 404 is longer than all bridges with the same properties from the databases, which increases the probability of error, but in this case the results from Roehnsund method is good.

It was not possible to make a regression for steel-concrete bridges because of the lack of data available. A more comprehensive database would improve the statistical basis for the



(a) Bridge nr. 406: 6.9 m

(b) Bridge nr. 407: 3.3 m

Figure 5.7: Roehnsund (blue) and analytical (yellow) results compared.

regression, and there might be better methods for calculating regression lines for the mass and stiffness spectrum.

More inputs could have made Roehnsund method more accurate, but also more comprehensive and it would require more knowledge to use. Roehnsund method is the only option for narrowing down the frequency spectrum evaluated in this thesis, and it is not given that it is best option. Based on the results is it considered a good alternative to Nerk.

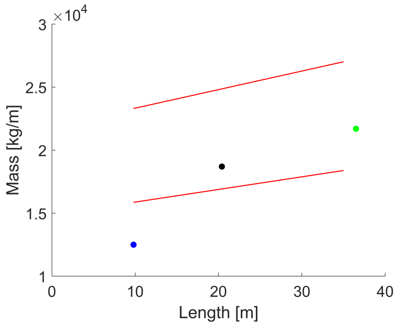
5.3.1 Bridges with mass and stiffness close to boundaries

Figure 3.6 shows the mass and stiffness estimates made with Roehnsund method. As the estimate only cover 90% of the variance, some bridges have mass and/or stiffness located outside the estimates, and 6 of these are evaluated in Figure 5.8, 5.9 and 5.10.

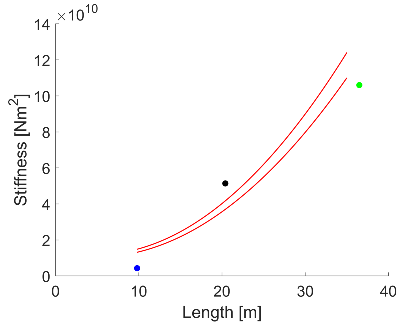
Three bridges of type 2 with one track are shown in Figure 5.8. The response curves show that both Nerk assumptions and Roehnsund method can give under conservative results for some velocities. It is also visible that Roehnsund method gives higher response than Nerk for some velocities, and that the Roehnsund method gives response significantly closer to the analytical response in general. It should be noted that for bridge number 302 Roehnsund method gives significantly low response compared to analytical for high velocities, which could be critical.

One bridge of type 3 with one track is displayed in Figure 5.9. Bridge number 85 has significantly lower mass than the lower mass estimate, which causes the response to be higher at one peak. Roehnsund method is generally on the conservative side, as it covers a large spectra of eigenfrequencies.

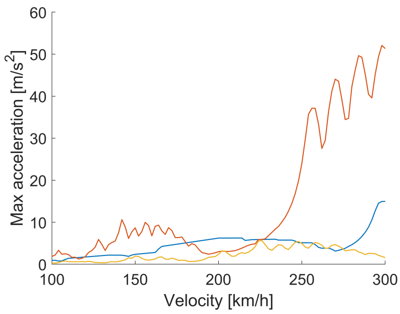
Two bridges with low mass and stiffness is shown for bridge type 3 with two tracks in Figure 5.10. This causes the resonance velocity to be low, and Roehnsund method provides results very close to analytical (barely below and above for the two bridges).



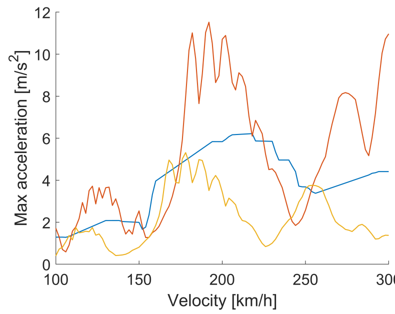
(a) Type 2-one track bridges, and values for bridge nr. 133 (black), 176 (green) and 302 (blue).



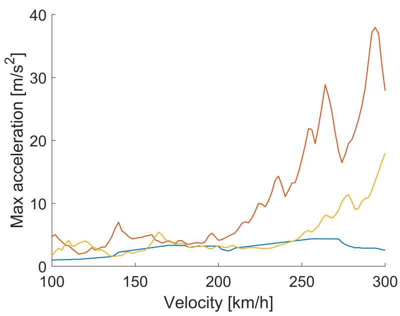
(b) Type 2-one track bridges, and values for bridge nr. 133 (black), 176 (green) and 302 (blue).



(c) Bridge number 133.



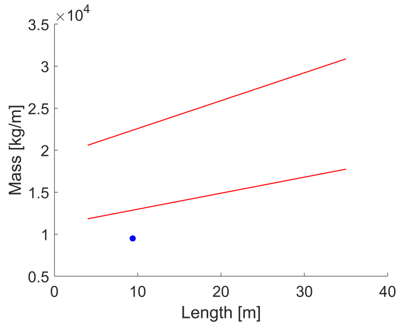
(d) Bridge number 176.



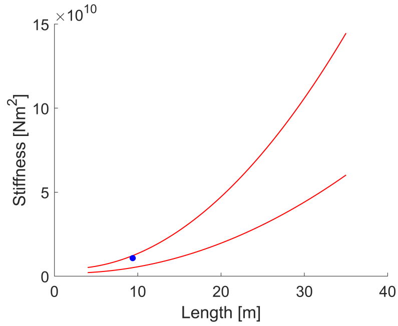
(e) Bridge number 302.

Figure 5.8: Type 2, one track bridges that does not fit well with the regression, and velocity response with Roehnsund method (blue), Nerk (red) and analytical response (yellow).

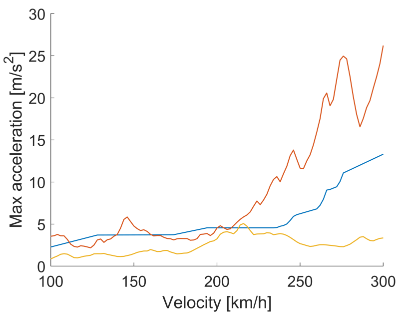
These figures emphasise that Roehnsund method has to be used to obtain an overview over critical velocities. The velocities where Roehnsund method estimates response just below critical acceleration should be analysed more thoroughly. The method is not perfect in terms of being conservative for all velocities, but it gives results significantly closer to the



(a) Type 3-one track bridges, and values for bridge number 85.



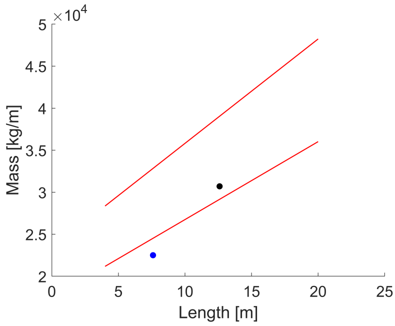
(b) Type 3-one track bridges, and values for bridge number 85.



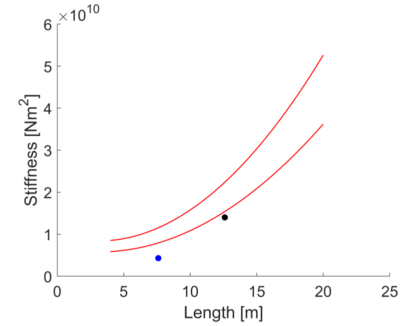
(c) Bridge number 85.

Figure 5.9: Type 3, one track bridge that does not fit well with the regression, and velocity response with Roehnsund method (blue), Nerk (red) and analytical response (yellow).

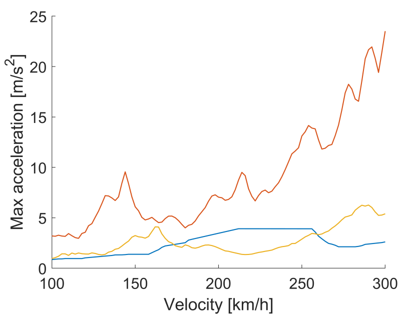
analytical response than Nerk, hence provide a better overview of the response. A way to make it slightly more conservative would be to multiply the envelope with a safety factor.



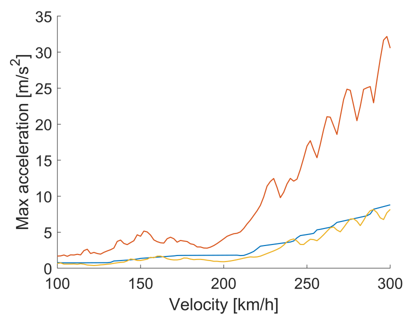
(a) Type 3-two track bridges, and values for bridge nr. 192 (blue), 252 (black).



(b) Type 3-two track bridges, and values for bridge nr. 192 (blue), 252 (black).



(c) Bridge number 192.



(d) Bridge number 252.

Figure 5.10: Type 3, two track bridges that does not fit well with the regression, and velocity response with Roehnsund method (blue), Nerk (red) and analytical response (yellow).

5.4 Quality of results

All results found in this thesis are calculated with certain assumptions. The bridges are considered simply supported and single spanned, EC damping is applied, max response is assumed to occur at mid span and a moving force-model is used for the calculation of response. These assumptions is to keep the model simple and number of inputs to a minimum.

If the bridge model should have been improved, modal parameters of the bridge including mode shapes had to be calculated using a FEM-approach. This was done by Andersson [4], while dynamic response was still calculated using modal superposition. To improve the load model, moving mass- or moving system-models could be applied. Many authors have studied this, including Arvidsson [5]. These improvements would give better results, but it would add more uncertainties to the input parameters and increase the complexity of the model and the calculation time.

Another minor improvement could be to evaluate the response at several points on the bridge. All calculations peroformed are assuming max response at mid span of the bridge. This is generally the case, and therefore a reasonable assumption, but in special cases can the response be significantly higher at other points. Museros et al. [14] presents a case where the response at $0.65L$ is 30% higher than at mid span. Nevertheless, including other points in the calculations is considered unnecessary, as such cases are exceptions and it would increase the calculation time significantly.

5.4.1 Rotational stiffness in supports

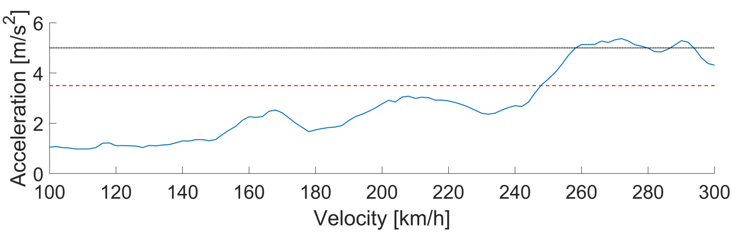
4 spans from bridges in the Norwegian database were created as beam models in Autodesk ROBOT, a finite element structural analysis program, to study the effect of changing the supports from simply supported to include different rotational stiffness and to be fixed. All calculations were done for HSLMA 1 with velocity 250 km/h. Frequencies and acceleration for the bridges is given in Table 5.1. An increase in rotational stiffness increased the first natural frequency, and the maximum acceleration was decreasing for stiffer supports.

In the report by Andersson [4], rotational stiffness in supports are included in the calculation of bridge response. Velocity-Response for bridge number 98 and 135 calculated as simply supported is shown in Figure 5.11, while Figure 5.12 show response of the same bridges calculated with rotational stiffness in the supports by Andersson. The assumption of simply supported bridges is conservative in the way that it overestimate accelerations, but it may give a wrong picture of which velocities high response may occure. The first natural frequency of bridge 98 and 135 is 68 and 10 Hz respectively when calculated as simply supported, and 99 and 16 Hz in Anderssons calculations. These results substantiate the observations from ROBOT in Table 5.1 that rotational stiffness increase natural frequency and reduce response.

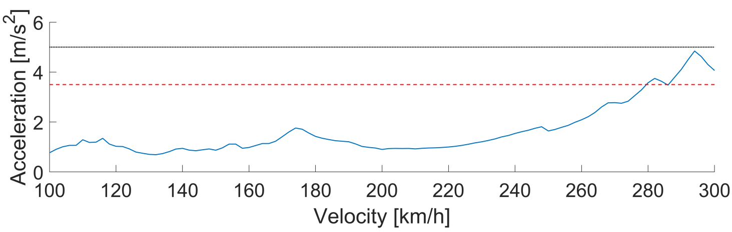
Bridge number	Span length [m]		Simply supported	$10^4\,\frac{\text{kNm}}{\text{deg}}$	$10^5\,\frac{\text{kNm}}{\text{deg}}$	$10^6\,\frac{\text{kNm}}{\text{deg}}$	$10^7\,\frac{\text{kNm}}{\text{deg}}$	Fixed support
400	28	f_0	3.7	3.82	4.62	6.95	8.19	8.39
		Max acc. [m/s^2]	0.438	0.427	0.273	0.167	0.176	0.161
400	21	f_0	6.58	6.74	7.87	11.84	14.44	14.92
		Max acc. [m/s^2]	0.498	0.495	0.419	0.212	0.165	0.154
403	18.6	f_0	8.62	8.73	9.61	13.88	18.44	19.54
		Max acc. [m/s^2]	0.303	0.282	0.22	0.11	0.053	0.055
404	5.5	f_0	44.89	46.38	56.58	85.06	99.35	101.65
		Max acc. [m/s^2]	0.687	0.664	0.428	0.229	0.178	0.178

Table 5.1: Natural frequencies and maximum acceleration calculated in ROBOT with different rotational stiffness in the supports.

Including rotational stiffness as an input in Velocity-Response calculation or segment analysis could be possible, but a finite element method would have to be implemented to calculate modal properties. Since keeping number of inputs down is a goal of the scripts, the addition of the rotational stiffness as an input would not be preferable.

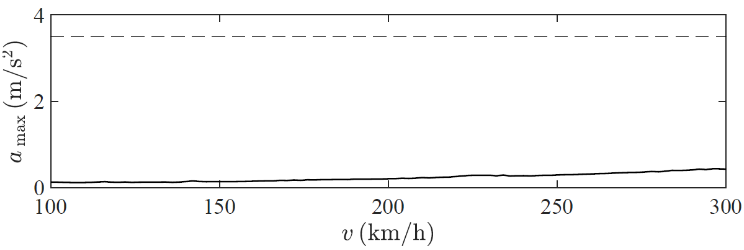


(a) Bridge nr. 98: 3.3 m. Max acceleration is 5.4 m/s^2

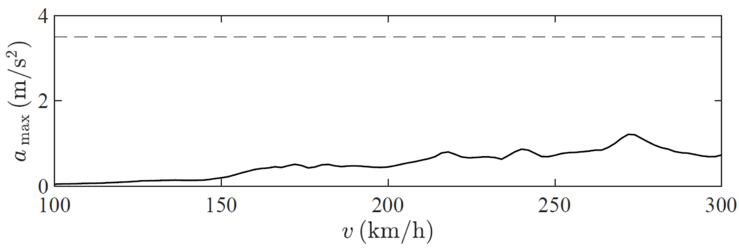


(b) Bridge nr. 135: 9 m. Max acceleration is 4.8 m/s^2

Figure 5.11: Velocity-Response calculated for bridge number 98 and 135 as simply supported.



(a) Bridge nr. 98: 3.3 m. Max acceleration is 0.4 m/s^2



(b) Bridge nr. 135: 9 m. Max acceleration is 1.2 m/s^2

Figure 5.12: Velocity-Response calculated for bridge number 98 and 135 by Andersson [4].

5.4.2 Multiple spans

Nerk and Roehnsund method are not capable of calculating multiple spans for bridges, each span have to be considered individually as simply supported beams. When each span is considered as a one-span beam, adjacent spans affects the rotational stiffness of the supports, and from the previous section, accelerations can be reduced.

Natural frequencies is higher when multiple spans is considered, thus higher response will occur when calculating response for each span as one simply supported span. On the other hand, multiple span beams will have more higher order modes at lower frequencies than a single span beam, and higher order modes may contribute more to the total response.

For 4 multi-span bridges from the Swedish database, Table 5.2 show that maximum acceleration calculated analytically will be higher for each span than the total maximum for the multi-span bridge calculation by Andersson [4].

5.4.3 End over-sail

Some bridges are designed with an over-sail from the end support. The end over-sail is short compared to the main span, and are designed as short beams with low rotational and

Bridge number	Bridge type	Number of tracks	Mass [kg/m]	Stiffness [N/m^2]	Span length [m]	Max acc. analytically [m/s^2]	Max acc. by Andersson [m/s^2]
14	3	1	22700	$7.34 \cdot 10^{10}$	24 33	3.49 6.6	2.2
15	3	1	18200	$2.69 \cdot 10^{10}$	15 18 21	3.35 5.52 17.38	1.9
18	2	1	28800	$1.73 \cdot 10^{11}$	33 36	4.09 4.01	1.5
78	3	2	29400	$1.01 \cdot 10^{10}$	4.8 10.1	1.98 7.11	0.7

Table 5.2: Max acceleration for each span of 4 bridges in the Swedish database calculated as simply supported and max acceleration when calculated as a multispan bridge.

vertical stiffness from the soil or backfill. In a report by Andersson [4], it is shown that end over-sail have large impact on the maximum bridge response. Modifications of the beam model was done to include end over-sail and reduce bridge response.

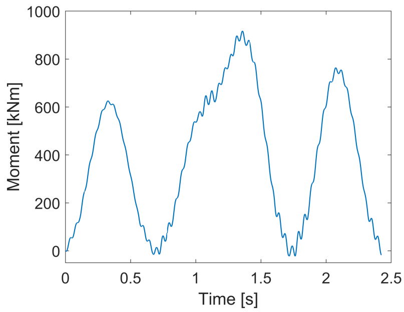
5.4.4 Calculation method

The IoE method is shown to be good compared to accurate solutions from the CFS, given in Figure 4.14. Because numerical differentation with respect to time is applied to obtain acceleration, the error of acceleration response is higher than for moment response.

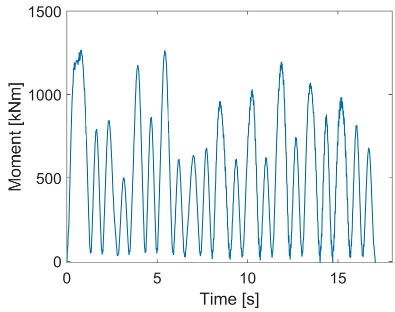
Properties from bridge 408 in Appendix A and the 2 train setups from Appendix B were used as input to get moment response with IoE, which was compared to measurements. The results are given in Figure 5.13. The time series look similar, but higher response is obtained with IoE-calculation. This is substantiated by the MAC-values and mean ratio from Table 4.4.

Mass and stiffness of Møstadbekken bridge is approximated from design drawings, and can be inaccurate. This will not affect the moment calculation with IoE-method, but could be the reason why the measurements are lower than IoE since moment is estimated from strain measurements by using the estimated stiffness. The inaccurate mass and stiffness would have larger impact on acceleration response if that was included in the comparison.

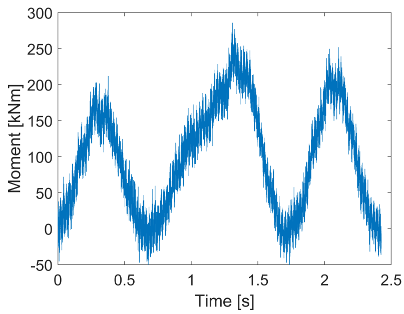
In theoretical calculations of shorter bridges, wind loading is usually not implemented, as it is dominated by white noise with low frequency. The white noise can be seen on the measurements in Figure 5.13. This is more relevant for longer bridges, as eigenfrequency is decreasing with increasing length.



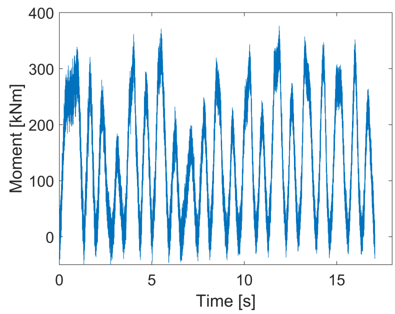
(a) IoE-calculation of moment from Train 1.



(b) IoE-calculation of moment from Train 2.



(c) Measurements of moment from Train 1.



(d) Measurements of moment from Train 2.

Figure 5.13: IoE-calculation((a) and (b)) and measurements((c) and (d)) of moment on bridge 408 from Appendix A, subjected to trains from Appendix B.

5.4.5 Load models

HSLMA trains are used in the calculations in 5.3. This will give a good, but conservative overview of where response may occur when type of high speed train for the bridges is not specified. It is shown in a report by Diachenko [7] that an envelope of HSLMA trains will cover response from many types of real high speed trains. High response might be calculated at wrong velocities for existing bridges. It is not possible to properly optimise design of new bridges when correct type of high speed train is not known.

The moving force-model used in this thesis is a simple and conservative way of calculating bridge response. For moving mass, inertia effects would be included, and for moving system, train-bridge-interaction or train-track-bridge-interaction would be included. The moving system-models include springs to represent the interaction between train carriage, train boogie, train wheels, tracks, ballast, bridge and soil. Both moving mass- and moving system-models demand more inputs and increased complexity of the calculations. Arvidsson states that the presence of these models increase the energy dissipation from the bridge, and the bridge response is generally reduced [5]. Even though response is reduced, there are uncertainties related to the assumed spring stiffnesses, that will give uncertainties in the response.

Chapter 6

Conclusion

The formulation of the problems in this thesis can briefly be summarised in understanding and evaluating assumptions and methods in Nerk, looking into possible adjustments and their effects, and finally develop a comparable alternative and evaluating its sufficiency.

Nerk and the developed script combined with Roehnsund method are both evaluated as good analytical tools for obtaining an overview of response based on train velocities and bridge lengths. Nevertheless is it very important for the user to be familiar with their limitations and advantages, especially for Nerk have some key limitations been identified that determines the results obtained. The consequences of underestimating the response is that critical bridges are not defined as critical, which may lead to failure. The consequences of being over conservative is that the analysis does not provide any information, as it regards to many bridges as critical.

Improvement potential in Nerk

Nerk is considered a nice software in terms of how to use it and user interface, but it could be easier to discover plots, i.e. by rotating and zooming. The idea to give an overview of critical bridges when considering new trains or higher velocities is considered good, but results in this thesis shows that there are improvement potential. The problems are summarised in four points:

- **The response calculation is highly conservative.**

  The main reason for the conservative response is the low estimate of distributed mass. Only one of the 318 bridges considered in this thesis has lower distributed mass than what is assumed in Nerk. The frequency spectra acquired with frequency functions in Nerk is very big, which in addition to the low mass assumption makes the results very conservative.

- **It is hard to interpret the results obtained.**

  Nerk is supposed to give a brief overview of critical velocities for different bridge lengths. The problem is related to the overestimation of response, as it is difficult to figure out which velocities to study more thoroughly. Additionally it gives an envelope which either include the peaks from the considered frequency functions and not eigenfrequencies in between, or a cumulative function that gives a bad picture of the potentially critical velocities.

- **It is difficult to make sensible adjustments or further analysis.**

  If the user has located bridges that seem more critical than others, it would be necessary to make further analysis. Nerks alternative for detailed calculations is Single TSC, where the user can specify the frequency function corresponding best with the eigenfrequency of the bridge and obtain time series of acceleration, deformation and moment. Even though the user has to specify bridge type, bridge length, loading and frequency function, it is not possible to change the mass, hence Nerk will still overestimate the response for most bridges.

  Improvements are discussed in this thesis, see 5.4. The simplest would be to add the option of defining mass and stiffness manually in the Single TSC to obtain analytical response instead of Nerk assumptions.

- **Usage of the DER method requires good understanding of dynamics and how the method works.**

  The DER-method, which produces almost half of the output in Nerk, is hard to interpret, unless the user has a very good understanding of how wavelengths from loads are produced and how it affects response. The method could be useful to evaluate different train compositions, e.g. if a new train was designed. The results gives an idea of critical wavelengths, which can be transformed to velocity or natural frequency of the bridge. The calculations are not sufficient, meaning further calculations must be made to figure out if critical acceleration and moments are exceeded.

  Considering the idea behind Nerk, a simpler way of analysing output from DER-method would be a good modification. An example is a 3D plot as the one in Figure 5.3. It gives a more intuitive representation of where resonance may arise, since it is easier to have an understanding of natural frequency and velocity than wavelengths of moving loads.

Roehnsund method and its advantages

The combination of the developed segment analysis script and the Roehnsund method is meant as a proposed solution to some of the problems associated with Nerk. To summarise its advantages:

- The bridge parameters needed for an IoE-calculation to be completed is length, bridge type, mass and stiffness. Length and bridge type is straight forward to acquire from design drawings, while mass and stiffness is more demanding and uncertain to calculate. When the easy accessible parameter number of tracks is included, mass and stiffness estimations are possible to obtain from length, bridge type and number of tracks.

- In order to obtain less conservative results Roehnsund method gives an estimate of mass and stiffness with number of tracks as input in addition to the existing inputs in Nerk. The eigenfrequency spectra is for all bridge types considered in this thesis, located within the spectra provided by the Eurocode, hence narrowing the velocities for which resonance occurs.

  The results show that it is possible to narrow the spectrum of frequencies to consider without making the calculations or the required knowledge more complicated. The results are in general conservative, but there are some exceptions. To make the results more conservative, a larger cover of variance could be applied or the results could be multiplied with a safety factor.

- Roehnsund method provides an envelope of results that cover all possible combinations of mass and stiffness within the frequency spectra. Since the frequency spectra is smaller and the response is lower than calculated in Nerk, it gives a better picture of where resonant velocities may occur and the magnitude of response.

- The segment analysis made it possible to run detailed analysis on one or several bridges at once, both Single TSC and velocity response. Here, mass and stiffness can be set manually or estimated using Roehnsund method. Input parameters in general are easy to vary, and it is possible for the user to evaluate different scenarios.

The calculations in both the developed script and Nerk are performed with IoE and modal superposition assuming all bridges to be simply supported, which causes imprecise results in some cases. Bridges that deviate from the assumption, by for example having rotational stiffness in supports or multiple spans, have other mode shapes causing bigger deviation in eigenfrequencies. This generally reduces the MAC value and the response decreases compared to simply supported bridges (Mean ratio increases). Methods for performing more exact calculations for these kind of bridges would be a natural supplement and improvement for the developed script and for Nerk.

Steel bridges are not evaluated with the developed script and Roehnsund method, as the bridge data obtained included an insufficient number of this specific bridge type. The results presented could have changed significantly if the scatter in mass and stiffness for steel bridges was big. Considering that Roehnsund method only uses 2 bridge types as basis it should be researched further whether the method is applicable in general.

Chapter 7

Future studies

During the work with this thesis several possible topics and analysis has been identified. The base topic of obtaining an overview of response on bridges with train loads has a lot of potential, and some suggestions to further work is listed underneath:

- Construct a better bridge database that includes more bridges of several types. Such a database could be used for further investigation of Roehnsund method or validating other calculation methods.

- Investigate other ways of narrowing down the frequency spectra compared to the EC. For example by looking at construction types, more specific bridge types etc.

- A highly discussed topic regarding dynamic response on railway bridges is damping. A research by Arvidsson [5] claims damping is related to mass ratio (ratio between train carriage and bridge or train bogie and bridge) and that the EC damping is imprecise. Implementing and analysing different functions for damping and their effects in simplified analysis would be interesting.

- Short bridges seem to experience very high accelerations with Nerk assumptions and Roehnsund method compared to results with more advanced calculations [4]. As the 3D plots presented in this thesis show that the short bridges are the most critical when increasing train velocity, it would be interesting to investigate in detail and find out how the analysis could be modified for more precise response.

- Modal superposition has been used for all calculations in this thesis. Using FEM-approach could open several opportunities, e.g. adding rotational stiffness in the supports, constructing mode shapes or analysing multispan bridges. As the calculation time would probably increase compared to doing modal superposition and the input would have to be more detailed, the FEM-approach could be used for doing more exact calculations for specific bridges.

- Calculations in this thesis has only been evaluated against one set of measurements, and practical measurements is the best way to validate theory. Making several comparisons would be useful to assess in what degree simple analysis fits with measurements.

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Appendix A

The bridges properties used in this thesis are given in this Appendix. Bridge number 1-278 is the Swedish database, 301-340 is the European database and 401-408 is the Norwegian database.

Swedish database

Bridge number	Bridge type	Number of tracks	Longest span [m]	Stiffness [Nm^2]	Mass [kg/m]
1	3	1	37.5	1.14E+11	24500
2	1	1	48	2.08E+11	17800
3	3	1	8.6	7.34E+09	17800
4	2	1	25	5.10E+10	20100
5	1	1	80	2.86E+11	13800
6	2	1	23	2.72E+10	19400
7	3	1	8.6	8.43E+09	18400
8	3	1	6.6	5.44E+09	15700
9	3	1	21.7	4.43E+10	22200
10	3	1	7.6	7.07E+09	18600
11	3	1	12.8	1.74E+10	23100
12	3	1	7.6	8.98E+09	20000
13	1	1	42	1.29E+11	16900
14	3	1	33	7.34E+10	22700
15	3	1	21	2.69E+10	18200
16	1	1	60	2.18E+11	15000
17	3	2	3.4	1.90E+09	21900
18	2	1	36	1.73E+11	28800
19	3	1	31	7.62E+10	24500
20	2	1	35	1.12E+11	20300
21	1	1	61	1.46E+11	18900
22	3	2	7.8	8.98E+09	29300
23	3	1	10.7	1.06E+10	20800
24	2	1	45	2.01E+11	25900
25	3	1	30	1.21E+11	21800
26	3	1	6.5	3.26E+09	14800
27	3	1	18	2.72E+10	18600
28	3	1	40	1.12E+11	21000
29	3	1	3.4	2.45E+09	13700
30	3	1	7.6	5.17E+09	17100
31	3	1	3.4	2.45E+09	13400
32	3	1	15.3	1.31E+10	21040
33	3	1	11.1	8.70E+09	20000
34	3	1	36	1.12E+11	22900
35	3	1	20	4.76E+10	22400
36	3	1	39.1	1.36E+11	25800
37	3	1	25	5.44E+10	22400
38	3	1	17	1.90E+10	22600
39	1	1	60	1.95E+11	15400
40	3	1	17	5.98E+10	23000

Swedish database

Bridge number	Bridge type	Number of tracks	Longest span [m]	Stiffness [Nm^2]	Mass [kg/m]
41	3	2	3.4	3.26E+09	24400
42	3	2	8.1	7.62E+09	27400
43	3	1	7.6	4.08E+09	15800
44	3	1	11.7	8.98E+09	20300
45	1	1	43	1.05E+11	12400
46	3	1	22	3.26E+10	27200
47	3	1	6.5	3.81E+09	15900
48	3	1	24	4.90E+10	28600
49	3	1	15.6	1.22E+10	20700
50	1	1	43	1.05E+11	13800
51	2	1	23	5.71E+10	31100
52	3	1	4.5	3.54E+09	16400
53	3	1	14	9.79E+09	20000
54	3	2	14.7	2.12E+10	45000
55	3	1	5	3.26E+09	15500
56	3	1	16.2	1.20E+10	21100
57	1	1	46	1.03E+11	14600
58	3	1	15.7	1.22E+10	21400
59	2	1	21	1.56E+11	33400
60	3	1	15.7	4.35E+10	20700
61	2	1	30	1.02E+11	26700
62	3	1	15.7	4.33E+10	20600
63	3	1	13.2	9.25E+09	20900
64	3	1	21.4	2.99E+10	18900
65	3	1	5.5	3.54E+09	14700
66	2	1	35	8.50E+10	21000
67	3	1	29	1.91E+11	36900
68	2	1	21	3.20E+10	16800
69	3	1	19	2.29E+10	25700
70	2	1	17	1.46E+11	31500
71	3	1	7.6	6.26E+09	18800
72	2	1	20.5	1.57E+11	34000
73	3	1	6.5	3.54E+09	15100
74	2	1	27	1.04E+11	27600
75	2	1	51	2.86E+11	20700
76	3	1	8.1	5.17E+09	17400
77	3	2	5	2.99E+09	25600
78	3	2	10.1	1.01E+10	29400
79	2	1	28.6	6.22E+10	18000
80	3	1	2.6	9.00E+08	11900

Swedish database

Bridge number	Bridge type	Number of tracks	Longest span [m]	Stiffness [Nm^2]	Mass [kg/m]
81	2	1	23	3.20E+10	14900
82	3	1	14.2	1.98E+10	13900
83	2	1	22.8	4.15E+10	14400
84	3	1	11.9	1.22E+10	11100
85	3	1	9.4	1.08E+10	9500
86	3	2	6.2	1.22E+10	34100
87	3	2	5.5	1.27E+10	33800
88	3	2	6.6	1.09E+10	31900
89	3	2	6.8	1.01E+10	31600
90	3	2	16	3.10E+10	40800
91	3	2	5.5	6.99E+09	31500
92	3	2	8.5	6.26E+09	32100
93	3	4	6.5	1.54E+10	68200
94	3	2	17.1	2.37E+10	44600
95	3	2	3.5	2.18E+09	26600
96	3	2	8.5	5.85E+09	41900
97	3	2	5.5	4.54E+09	29000
98	3	1	3.3	2.99E+09	13500
99	3	2	5.5	3.86E+09	16300
100	3	1	6.6	2.45E+10	28200
101	3	1	12.5	1.24E+10	22800
102	3	2	5.5	4.54E+09	24800
103	3	2	14	1.88E+10	33400
104	3	2	13.7	3.51E+10	43300
105	3	2	6.1	7.62E+09	27900
106	3	2	6.6	1.27E+10	37000
107	3	1	17	2.06E+10	16800
108	3	2	6.1	5.71E+09	24900
109	3	2	6.1	6.26E+09	25900
110	3	2	8.6	1.48E+10	36800
111	3	2	6.5	8.98E+09	29000
112	3	2	10.5	1.87E+10	41700
113	3	1	3.4	7.60E+08	27600
114	3	2	6.1	7.78E+09	28600
115	2	2	24	7.48E+10	43000
116	3	2	13.7	1.57E+10	35600
117	3	2	5.5	4.95E+09	26800
118	3	1	22	4.38E+10	19100
119	3	2	7	1.30E+10	31500
120	0	2	54	1.19E+12	34200

Swedish database

Bridge number	Bridge type	Number of tracks	Longest span [m]	Stiffness [Nm^2]	Mass [kg/m]
121	3	2	7.6	1.39E+10	33200
122	2	1	20	2.96E+10	17700
123	3	1	5.6	4.90E+09	15600
124	3	1	6.5	5.14E+09	15600
125	3	1	9.9	1.50E+10	22200
126	3	1	6.1	5.60E+09	16100
127	3	1	11.2	1.11E+10	20200
128	3	1	6.1	7.53E+09	18700
129	3	2	9.3	1.66E+10	34200
130	1	1	44	8.40E+10	13800
131	3	2	16	3.35E+10	44600
132	3	2	5.5	2.69E+09	22400
133	2	1	20.4	5.14E+10	18700
134	3	1	6.5	1.07E+10	19900
135	3	2	9	9.52E+09	34400
136	3	2	10.3	1.12E+10	36600
137	3	6	6.5	1.02E+10	74900
138	3	3	5.4	5.25E+09	40500
139	3	1	6	3.59E+09	14700
140	3	2	3.4	2.64E+09	23000
141	3	1	6	5.28E+09	13200
142	1	1	54.9	1.32E+11	11500
143	1	1	54.9	2.82E+11	14300
144	3	4	6.4	1.39E+10	46300
145	3	2	16	2.34E+10	35000
146	3	3	6	1.19E+10	32400
147	3	2	7	1.05E+10	26600
148	3	2	15.5	2.33E+10	43000
149	3	1	19	3.92E+10	23400
150	3	2	12	2.82E+10	36100
151	3	2	12.9	2.91E+10	46400
152	3	2	5.5	9.15E+09	29500
153	3	2	6.5	1.30E+10	30600
154	3	2	4	8.34E+09	31200
155	3	2	14.5	2.48E+10	39200
156	3	2	6.5	7.92E+09	29600
157	3	2	12.5	1.16E+10	31900
158	3	2	5	3.18E+09	23600
159	2	1	38.4	7.48E+10	20300
160	0	1	2.1	4.20E+09	22100

Swedish database

Bridge number	Bridge type	Number of tracks	Longest span [m]	Stiffness [Nm^2]	Mass [kg/m]
161	3	1	5.5	1.20E+10	17000
162	3	1	5	1.13E+10	16900
163	3	1	5.5	1.20E+10	17500
164	3	1	8.5	1.07E+10	17500
165	0	2	2.5	4.60E+08	23200
166	3	2	3.5	8.16E+09	31000
167	3	1	7.5	5.52E+09	16900
168	3	1	18	3.48E+10	18800
169	3	1	21	3.55E+10	26900
170	3	1	13.2	5.88E+09	17500
171	3	2	7	1.06E+10	32100
172	3	1	11.4	1.32E+10	22000
173	3	1	5.5	2.37E+09	13700
174	1	1	43	6.51E+10	12900
175	3	1	14	3.65E+10	19800
176	2	1	36.5	1.06E+11	21700
177	3	3	7.7	1.30E+10	41400
178	3	2	11.3	1.41E+10	31800
179	3	1	15	1.63E+10	22100
180	3	2	11.8	2.04E+10	35200
181	3	2	5.9	7.18E+09	25800
182	3	1	6.8	3.89E+09	12400
183	2	2	24	1.12E+11	49600
184	3	2	10	1.30E+10	33100
185	2	1	28	6.83E+10	24600
186	3	2	20.5	5.30E+10	32600
187	3	2	14	8.21E+09	25300
188	3	2	8.1	8.30E+09	26800
189	3	2	3.2	2.72E+09	18800
190	0	2	2.6	5.39E+09	106500
191	3	2	8.5	5.71E+09	22600
192	3	2	7.6	4.30E+09	22500
193	3	1	23.6	2.83E+10	16500
194	3	2	3.2	2.72E+09	20300
195	3	2	6.6	5.98E+09	23400
196	3	2	11.9	8.16E+09	27800
197	3	2	5.5	4.90E+09	22200
198	3	2	17.2	4.27E+10	38100
199	3	2	4.9	1.33E+10	27000
200	3	1	17.2	1.39E+10	17700

Swedish database

Bridge number	Bridge type	Number of tracks	Longest span [m]	Stiffness [Nm^2]	Mass [kg/m]
201	3	2	9.1	1.63E+10	30500
202	3	2	3.9	9.52E+09	24500
203	3	1	20	2.08E+10	20600
204	3	2	9.1	1.57E+10	28100
205	3	1	16.4	1.41E+10	17800
206	3	2	3.9	9.79E+09	25600
207	3	2	6.4	1.25E+10	26800
208	3	2	6.4	8.98E+09	22900
209	3	1	17	1.61E+10	17200
210	2	1	26	7.82E+10	18400
211	3	1	25	9.14E+10	19000
212	2	1	26	7.45E+10	18100
213	3	1	20	1.88E+10	17400
214	2	1	30	9.32E+10	19200
215	3	2	17	2.12E+10	37400
216	3	1	25	3.32E+10	18700
217	3	2	6.5	1.41E+10	28500
218	3	1	17.4	2.04E+10	19700
219	3	2	5.5	9.79E+09	21400
220	3	2	3.4	1.17E+10	28400
221	3	2	2.5	1.17E+10	28400
222	3	2	5.4	1.20E+10	24900
223	3	2	13.3	2.09E+10	31400
224	2	1	23.4	7.14E+10	23000
225	3	2	3.9	1.26E+10	26200
226	3	2	4.3	7.89E+09	21900
227	3	1	26	8.08E+10	22000
228	3	2	13.4	2.18E+10	33200
229	3	2	5.5	1.52E+10	28200
230	3	2	5.9	1.22E+10	22000
231	3	2	6.5	9.52E+09	21900
232	3	2	15.8	3.21E+10	34900
233	3	1	20	2.20E+10	18900
234	2	1	28.9	7.82E+10	22900
235	3	1	11.4	1.20E+10	15800
236	3	5	10.1	2.26E+10	81800
237	3	5	3.3	9.08E+09	43600
238	3	1	31.1	4.46E+10	20500
239	3	1	29.8	3.16E+10	16300
240	3	1	20	2.55E+10	20900

Swedish database

Bridge number	Bridge type	Number of tracks	Longest span [m]	Stiffness [Nm^2]	Mass [kg/m]
241	3	2	6.4	9.79E+09	21400
242	3	1	23	3.18E+10	18600
243	3	2	3.4	1.31E+10	28100
244	3	2	4.4	1.05E+10	26300
245	3	2	4.3	2.99E+09	22600
246	3	2	3.3	2.99E+09	22600
247	3	2	4.4	8.02E+09	21000
248	3	2	2.7	9.93E+09	21300
249	3	2	4.4	1.13E+10	23200
250	3	2	11	2.51E+10	33200
251	3	2	10.2	1.68E+10	32200
252	3	2	12.6	1.40E+10	30700
253	3	2	4	4.90E+09	24900
254	3	1	15.6	6.26E+09	15000
255	3	2	4	9.30E+09	23700
256	3	2	2.9	3.86E+09	21100
257	3	1	6	6.45E+09	14300
258	3	2	5.5	3.62E+09	19300
259	3	2	3.4	3.83E+09	19700
260	3	1	9	1.10E+10	15200
261	3	1	10	6.45E+09	14300
262	3	1	10	6.45E+09	14300
263	3	1	12	6.85E+09	15800
264	3	2	5.5	7.48E+09	23700
265	3	2	6.5	6.94E+09	25600
266	3	1	19	1.63E+10	19700
267	3	1	14	1.12E+10	17200
268	3	1	12.5	9.96E+09	17300
269	3	2	3.4	6.66E+09	21600
270	3	2	8.1	1.13E+10	27000
271	3	2	13	1.07E+10	26100
272	3	3	6.4	1.49E+10	48600
273	0	2	6.1	6.31E+09	69300
274	3	2	6.5	5.22E+09	23500
275	2	3	29.2	9.35E+10	60700
276	3	2	15.3	3.54E+10	22800
277	2	1	30	5.83E+10	15900
278	2	1	23.4	4.15E+10	25900

European database

Bridge number	Bridge type	Number of tracks	Longest span [m]	Stiffness [Nm^2]	Mass [kg/m]
301	2	1	9.8	8.79E+09	17100
302	2	1	9.8	4.31E+09	12500
303	2	1	10.5	1.38E+10	17143
304	2	1	10.5	9.49E+09	17143
305	2	1	11.3	1.36E+10	18667
306	2	1	13	1.81E+10	23846
307	2	1	13.2	1.64E+10	18939
308	2	1	13.5	2.29E+10	19259
309	2	1	13.7	1.55E+10	20438
310	2	1	15.8	2.37E+10	20253
311	2	1	16	2.00E+10	23125
312	2	1	16.3	2.22E+10	23926
313	2	1	17.5	2.14E+10	20000
314	2	1	19.4	7.27E+10	25600
315	2	1	19.6	3.75E+10	22100
316	2	1	20.2	2.85E+10	21782
317	2	1	21.2	5.33E+10	24057
318	2	1	21.3	8.09E+10	21596
319	2	1	22	4.26E+10	23182
320	2	1	22.1	6.12E+10	25339
321	2	1	22.3	6.81E+10	25112
322	2	1	24	4.46E+10	27083
323	2	2	7.9	1.06E+10	28500
324	2	2	8.4	1.17E+10	29700
325	2	2	8.6	1.15E+10	30900
326	2	2	10.3	1.82E+10	38700
327	2	2	12.5	2.09E+10	32000
328	2	2	15	3.29E+10	34667
329	2	2	15.2	3.97E+10	33553
330	2	2	17	6.16E+10	38235
331	2	2	17.5	4.33E+10	40571
332	2	2	17.7	1.03E+11	38418
333	2	2	19.7	7.33E+10	44416
334	2	2	20	9.10E+10	39000
335	2	2	20.1	6.98E+10	47761
336	2	2	21	6.36E+10	41667
337	2	2	21.5	9.86E+10	31628
338	2	2	22.5	1.16E+11	41333
339	2	2	23.5	1.07E+11	38700
340	2	2	25	1.68E+11	42400

Norwegian database

Bridge number	Bridge type	Number of tracks	Longest span [m]	Stiffness [Nm^2]	Mass [kg/m]
401	2	1	28	1.02E+11	27657
402	2	1	28	1.02E+11	25920
403	2	1	28	1.02E+11	25920
404	2	2	27.6	1.69E+11	43209
405	3	1	5.5	1.87E+10	23359
406	3	4	6.9	1.44E+11	68085
407	3	2	3.3	1.47E+10	34594
408	1	1	13	6.48E+09	1221.1

Appendix B

The train setups used in measurements of bridge number 408 is shown in this Appendix

Train 1	
Distance from first axle [m]	Point load at each axle [kN]
0	114.9
2.52	114.9
16.42	106.3
18.96	106.3
24.04	163.5
26.61	163.5
40.52	139.8
43.04	139.8

Train 2			
Distance from first axle [m]	Point load at each axle [kN]	Distance from first axle [m]	Point load at each axle [kN]
0.0	197.7	179.0	149.1
2.7	197.7	180.9	149.1
7.7	204.7	184.7	116.5
10.4	204.7	186.5	116.5
14.9	151.7	198.6	109.7
16.8	151.7	200.4	109.7
29.1	139.0	212.6	124.7
31.0	139.0	214.4	124.7
43.3	129.6	218.1	156.5
45.1	129.6	219.9	156.5
48.9	61.0	232.0	111.2
50.8	61.0	233.9	111.2
62.8	88.1	245.9	160.2
64.7	88.1	247.8	160.2
76.8	126.8	251.5	174.0
78.6	126.8	253.4	174.0
82.5	156.6	265.4	133.4
84.3	156.6	267.3	133.4
96.6	151.9	279.4	162.6
98.5	151.9	281.2	162.6
110.8	156.2	285.1	141.7
112.7	156.2	286.9	141.7
116.6	156.1	299.2	155.0
118.4	156.1	301.1	155.0
130.8	107.3	313.4	140.4
132.6	107.3	315.3	140.4
144.9	60.2	319.1	135.6
146.7	60.2	320.9	135.6
150.7	91.4	333.0	147.0
152.5	91.4	334.8	147.0
164.8	119.5	346.9	122.6
166.7	119.5	348.8	122.6

Appendix C

Attached as digital Appendix, called ”Appendix C Developed scripts”.

The main scripts are the ones starting with ”A_”, which are:

- A_Input_script Letting the user run Single TSC, Velocity-Response or 3D plot with either Roehnsund, Nerk or known properties (using the Excel-sheet called E_Known_prop_bridges”.
- A_Segment_analysis Doing a segment analysis based on the Excel sheet attached called ”E_Segment_analysis”.

The last Excel-sheet called ”E_Fullstendig_brodatabase” is the database for which Roehnsund method is based on. Running calculations with Roehnsund method demands such a database for making the regression.