Reusing waste heat from apartment greywater in a CO2 flooded evaporator

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Abstract. *Harnessing heat from greywater in apartment buildings can reduce energy consumption and the environmental impact. In this paper, a CO2 heat pump with a flooded evaporator will use greywater from 10600m^2 of apartments to produce domestic hot water. The greywater has a temperature of approximately 30$°C$ and the apartments produce an estimated 24645 kg per day. To increase heat transfer from the greywater to the CO_2, the pipe will be coiled, and to avoid fouling inside the pipe due to impurities, high velocity is a criterion. With an inner pipe diameter of 12mm and a wall thickness of 1,5mm, velocities below 3,4m/s will not deliver sufficient power to the evaporator. For higher velocities, there is not enough greywater to operate the heat pump for the desired 20 hours and a backup coil is needed. Using a single coil is a more efficient solution than dividing it into several coils due to pressure drop and coil length needed.*

Keywords: *Greywater, Carbon Dioxide, Flooded Evaporator, Coil, Heat Transfer.*

1. Introduction

To meet the challenge of more energy-efficient buildings, 4th generation district heating is part of the solution. Present district heating uses water as a carrier and often has a supply temperature of below 100$°C$. The general development trend is to lower supply temperature that in turn decreases grid losses. Another advantage is the possibility to connect with sources like solar thermal, geothermal, waste heat from industrial processes and other low quality/temperature heat sources [7]. The waste heat source that will be examined in this paper is greywater. When hot water is used for showering, washing dishes or clothes, it enters the drain with a high temperature which is a major source of inefficiency [8]. Utilizing this water can reduce the carbon footprint and energy demand for buildings. To avoid water with too many impurities, showers, dishwashers and washing machines will be the sources for the greywater. The water will be used in a flooded evaporator and a model has been developed in Python to calculate the heat transfer for velocities ranging from 3-6m/s.

The heat pump this evaporator is designed for has a gascooler capacity of 60 kW and operates 20 hours every day to provide the necessary amount of hot water. The evaporator must supply 41,8 kW for this to be possible.

2. Theory

A flooded evaporator ensures a 100% liquid refrigerant to the evaporator and the exiting refrigerant is 50-80% gas [10]. A forced flow evaporator uses a pump or an ejector to move the flow. For a flooded system to work it needs a receiver to separate the liquid from the gas. A flooded system does not achieve any superheat and therefore more power can go towards evaporation.

2.1. Heat transfer

at transfer from a pipe to a surrounding fluid will depend on the convection coefficient of the fluid in the pipe and the conduction coefficient of the pipe material. The energy transferred by convection is given by

$$Q = hA(T_s - T_\infty) \tag{1}$$

where T_∞ is the fluid temperature and T_s is the surface temperature, h (W/m^2K) is the convection coefficient and A is the area. For conduction, Fouriers's law is used and defined below where k (W/mK) is the thermal conductivity and the temperature difference is on either side of the wall [11].

$$Q = -kA\frac{dT}{dx} \tag{2}$$

In general, heat transfer in a system can be expressed as a function of temperature difference, the overall heat transfer coefficient, U, and the area.

$$Q = UA\Delta T = \frac{\Delta T}{R_{tot}} \tag{3}$$

Thermal resistance is a method to evaluate the heat transfer through a material or fluid. It is given in the unit of K/W and can be either put in series or parallel. The equations for conduction and convection resistance in a radial system is shown in equation 4 and 5 respectively. r_2 is the outer diameter of the pipe, r_1 is the inner diameter and L is the pipe length.

$$R_{th,cond} = \frac{ln(r_2/r_1)}{2\pi Lk} \tag{4}$$

$$R_{th,conv} = \frac{1}{h2\pi rL} \tag{5}$$

Heat transfer over a radial surface is given by,

$$Q_r = \frac{2\pi Lk}{ln(r_2/r_1)}(T_1 - T_2) = \frac{T_1 - T_2}{R_{th,cond-cyl}} \tag{6}$$

Equation 6 can be rewritten to find heat transfer per unit length of tube and by using the thermal resistance in the material. Before determining the convection coefficient it is necessary to examine how the nature of the flow will impact the convectional heat transfer. Flow types can be categorized as laminar, turbulent, and transitional, where transitional is when the flow transitions from laminar to turbulent. Laminar flow is characterized by smooth layers or streamlines moving parallel to each other while turbulent is a distorted flow that creates eddies and vortices [16].

When a fluid is flowing past a solid body, the flow is affected by it. This creates a region that has a variable velocity that is located between the flow and the body surface which is called a boundary layer [6]. The thickness of the boundary layer, δ is defined as the distance from the wall where the velocity is within 1% of the freestream velocity and is usually very thin in comparison to the other dimensions of the flow. The boundary layer thickness divided by its characteristic length can be expressed as a function of the Reynolds number. This is the flow boundary layer.

$$\frac{\delta}{x} = f(Re_x) \tag{7}$$

The Reynolds number is a way to predict the flow pattern, i.e. if the flow is laminar or turbulent. Usually, flow in a pipe is laminar when the Reynolds number is below 2100 but this depends on the nature of the flow such as if it is coiled [4]. The critical Reynolds number is the point when the flow transitions and can be defined as

$$Re_{critical} = \frac{\rho D u_{avg,crit}}{\mu}. \tag{8}$$

$u_{avg,crit}$ is the transitional value of the average velocity. The thermal boundary layer is different from the flow boundary layer and is inversely proportional to the Nusselt number $Nu_x = \frac{x}{\delta_t'}$. The Nusselt number is a non-dimensional unit of measurement to show the ratio of convective to conductive heat transfer of a fluid and can be used to find the convection coefficient, h.

$$Nu_x = \frac{hx}{k_f} \tag{9}$$

For a turbulent water flow, the convective heat transfer is significantly larger than the conductive heat transfer which means a large Nusselt number. The thermal boundary layer for such a flow will, therefore, be thin. For laminar flow, the convection coefficient will be lower and this is a consequence of the boundary layer that is created because of the smooth fluid flow. The general equation for the Reynolds number is shown in equation 10.

$$Re_x = \frac{\rho u x}{\mu} \tag{10}$$

Prandtl number is the ratio between momentum diffusivity to thermal diffusivity. ν is the kinematic viscosity and α is the thermal diffusivity. Thermal diffusivity is a measure of the rate of heat from the hot end of a material to a colder end and has the unit m^2/s. The thermal boundary layer divided by the flow boundary layer can be expressed as a function of the Prandtl number and the Nusselt number can in turn be a function of the Prandtl number. This is important for determining the convection coefficient.

$$Pr = \frac{\nu}{\alpha} \tag{11}$$

2.2. Forced convection relations coil

When the pipe is a coil, the geometry of this coil is important for its heat transfer properties. When the pipe is curved there will be centrifugal forces in the fluid which causes a secondary flow that circulates outward into the core region of the pipe and forms a pair of symmetric vortices. Since there is both a main and secondary flow, the maximum velocity is shifted outwards from the pipe center. The secondary flow creates an additional convective heat transfer between the fluid and the wall which improves the heat transfer. These differences are particularly prevalent in laminar flows [4].

The secondary flow has a stabilizing effect on the laminar flow which means that the transition from laminar to turbulent happens at a higher Reynolds number. For a coil the critical Reynolds is defined as in equation 12

$$Re_{crit} = 2300 \left[1 + 8,6 \left(\frac{d}{D} \right)^{0,45} \right] \tag{12}$$

For a laminar flow, $Re < Re_{crit}$ in a coil the Nusselt number the Schmidt equation (13) can be used. This equation has a deviation of $\pm 15\%$ compared to the measured values of some coils. The Prandtl number is evaluated at mean fluid temperature and the Pr_w is the Prandtl number at tube wall temperature. The factor $\left(\frac{Pr}{Pr_w}\right)^{0,14}$ is there to account for the temperature dependence of the physical properties.

$$\begin{gathered} Nu = 3,66 + 0,08\left[1 + 0.8\left(\frac{d}{D}\right)^{0,9}\right] Re^m Pr^{1/3}\left(\frac{Pr}{Pr_w}\right)^{0,14} \\ m = 0,5 + 0,2903(d/D)^{0,194} \end{gathered} \tag{13}$$

There is a transitional region for heat transfer when the Reynolds number is above critical but below fully developed flow at $Re = 2,2 \cdot 10^4$. Measurements for a turbulent/fully developed flow were done with air and water and equation 14 has a deviation of $\pm 15\%$ when compared. The friction factor, f, has a correctional factor $\left(\frac{\mu_w}{\mu}\right)^{0,27}$ where μ_w is the dynamic viscosity at wall temperature and μ is the dynamic viscosity at mean fluid temperature.

$$\begin{aligned} Nu &= \frac{(f/8)RePr}{1 + 12,7\sqrt{f/8}(Pr^{2/3} - 1)}\left(\frac{Pr}{Pr_w}\right)^{0,14} \\ f &= \left[\frac{0,3164}{Re^{0,25}} + 0,03\left(\frac{d}{D}\right)^{0,5}\right]\left(\frac{\mu_w}{\mu}\right)^{0,27} \end{aligned} \tag{14}$$

When the flow is in the transit region of heat transfer, $Re_{crit} < Re < 2,2 \cdot 10^4$. $Nu_l(Re_{crit}$ is the Nusselt number for laminar flow, equation 13, if $Re = Re_{crit}$. $Nu_t(Re = 2,2 \cdot 10^4)$ is Nusselt number at turbulent flow if $Re = 2,2 \cdot 10^4$ in equation 15

$$\begin{gathered} Nu = \gamma Nu_l(Re_{crit)} + (1-\gamma)Nu_t(Re = 2,2 \cdot 10^4) \\ \gamma = \frac{2,2 \cdot 10^4 - Re}{2,2 \cdot 10^4 - Re_{crit}} \end{gathered} \tag{15}$$

2.3. Boiling

When the temperature on the surface of a body exceeds the saturation temperature at a given pressure, boiling will occur. There are two boiling processes that are possible: Pool boiling is when the liquid is at rest and free convection occurs, while forced boiling is when the liquid is moving [11].

The heat flux given in W/m^2 from the surface to the fluid is,

$$q_s^{''} = h(T_s - T_{sat}) = h\Delta T_e \tag{16}$$

where T_s is the surface temperature and T_{sat} is the saturation temperature of the fluid. For nucleate boiling the heat flux can also be estimated using the Rohsenow correlation.

$$q_s^{''} = \mu_l h_{fg}\left[\frac{g(\rho_l - \rho_v)}{\sigma}\right]^{1/2}\left(\frac{C_{p,l}\Delta T_e}{C_{s,f}h_{fg}Pr_l^n}\right)^3 \tag{17}$$

h_{fg} is the latent heat of vaporization in kJ/kg, μ_l is the dynamic viscosity of the liquid in Ns/m^2, ρ_l and ρ_v is the density of liquid and vapor respectively. σ is the surface stress tensor in N/m, $C_{p,l}$ is the specific heat of liquid, $C_{s,f}$ is the Rohsenow coefficient is constant and an empirical correction for surface-fluid condition. The exponent n is also an found empirically for common surface-fluid combinations.

3. Method

To find the heat transfer of a coil in the flooded evaporator with high velocity, a model was developed in Python. The pipe dimensions had to be chosen. To avoid a too high mass flow rate a pipe diameter of 15mm was chosen and according to Lister Tube the standard wall thickness for such a pipe is 1,5mm [13]. The set properties are listed in table 1

An assumption is that the temperature of the inner pipe wall has about the same temperature as the water flow. This is because of a negligible boundary layer due to the high Reynolds number due to the high velocity.

Table 1. Flooded evaporator dimensions and assumptions

Pipe dimensions	
Inner diameter [d]	0,0012 m
Outer Diameter [d_o]	0,0015m
Distance between coils [h]	0.03 m
Number of turns [n]	25
Various properties	
Cp	4,19 kJ/kgK
Temperature at inlet [T_{in}]	30°C
Density [ρ]	997 kg/m^3
Conduction coefficient stainless steel (AISI 304) [k]	14,9 W/mK
Temperature of CO_2 [T_{CO_2}]	273 K
Inlet water temperature [T_i]	303,15K
Minimum water outlet temperature [T_o]	273.15
Water velocity [V]	3-6 m/s

The velocity for GW has to be high to avoid fouling in the heat exchanger pipe and a range from $3m/s$ to $6m/s$ with an interval of $0,2$ was tested. The required length of the pipe is not known and therefore the calculations are done for different sections with a temperature drop of $1K$ per section until it reaches equilibrium with the external CO_2. The mass flow rate was calculated using the velocity with the following formula:

$$\dot{m} = V\pi r^2\rho \tag{18}$$

To avoid a large evaporator, the diameter winding of the coil was chosen to be 0,3 m. Coil heat transfer relations were then used to find the convection coefficient for the coil, equations 13, 14 and 15. For the coil, the convection coefficient for external CO_2 was accounted for. First, the temperature of the outer pipe surface was calculated using equation 6, solving for the T_2. Choosing the Rohsenow coefficient was based on a paper by Tao Ding et al. [2]. For CO_2 a $C_{s,f}$ of $0,0049$ and $n = 1,7$ was used. This was for n-pentane-Lapped cooper. Copper is not a material used for this heat exchanger and the results were not realistic with this value. The value for n-pentane-polished copper was used instead which also matches the coefficient used for water and stainless steel. $C_{s,f} = 0.0154$ and $n = 1,7$ was used for this heat exchanger [1] [5].

The properties of liquid CO_2 at $35bar$ are shown in table 2

Table 2. Thermal properties of CO_2 at 35 bar, 0°C,[3], [9], [14], [15], [12]

h_{fg}	$234,5kJ/kg$
μ_l	$105,4 \cdot 10^{-6}\ pas$
ρ_l	$926,78kg/m^3$
ρ_v	$98,145kg/m^3$
σ	$5,57 \cdot 10^{-3}N/m$
$C_{p,l}$	$2,47kJ/kgK$
$C_{s,f}$	$0,0154$
n	$1,7$
Pr	$2,38$

The properties for CO_2 was evaluated based on several sources listed in the table description. Heat flux for boiling was then calculated using the Rohsenow correlation, equation 17, where the excess temperature is the temperature difference from the surface of the pipe to the CO_2. The convection coefficient of the boiling CO_2 was found using equation 16 and used to find the thermal resistance. Since all the thermal resistances was found, but coil length was still unknown, an iterative approach was used to find the new length.

The pipe was then divided into several pipes to see which solution was better. Velocity is still a criterion, and therefore the mass flow rate through all the pipes combined was chosen to be the same as for a single pipe and the diameter was reduced.

3.1. Overall heat transfer coefficient

The overall heat transfer coefficient was found using equation 3 and then solve for U. To assure realistic values the coefficient was calculated for all the sections cumulative. Every section was added together and the average temperature difference from the water to the CO_2 was used as the pipe got longer. For example, for the whole pipe, the ΔT value is $15°C$ because the inlet is $30°C$ and the outlet is $0°C$ and the CO_2 is kept at $0°C$.

4. Results

The results for the flooded evaporator will be divided into results from a single coil and results where it is divided into several pipes. With a total production of 24645 kg/day assuming the heat pump will run for 20 h/day the mass flow rate can be 1232,25 kg/h or 0,34229 kg/s. The inner diameter is 12 mm, and using equation 18 and solving for the velocity show that for the GW supply to be sufficient the water can have a velocity of up to 3,035 m/s. Before examining the results from the flooded evaporator it is necessary to look at the boiling convection coefficient and how it relates to the heat flux from the pipe shown in figure 1. The convection coefficient increases with the heat flux increase however, it is lower than the heat flux by a factor of 10.

The minimum temperature that is possible to reach in the coiled pipe is $3°C$ before the sectional length became excessive, see figure 2a. For all velocities, the temperature drops more rapidly in the first section of the pipe due to the higher temperature difference between water and CO_2. From figure 2b, it is apparent that the velocity should be above 3,4m/s to have a safety margin for reaching the threshold of 41,821 kW which leads to a mass flow rate of above 0,38 kg/s.

How the overall heat transfer coefficient will evolve in the pipe is shown in figure 3. The mass flow rate or velocity has an impact on the coefficient as well as the overall area. Both the convection coefficient of the water and the boiling decrease with the length of the pipe due to the temperature drop of water and the reduced temperature difference of the outer wall of the pipe. The thermal resistance of the wall is also increased due to the increased length. The results start at the outlet of the first pipe section and end when the minimum temperature of $3°C$ is achieved.

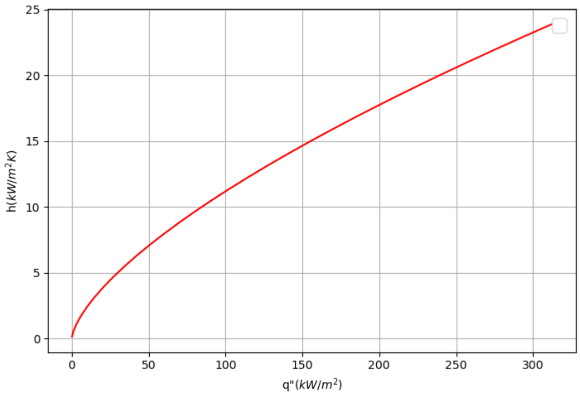
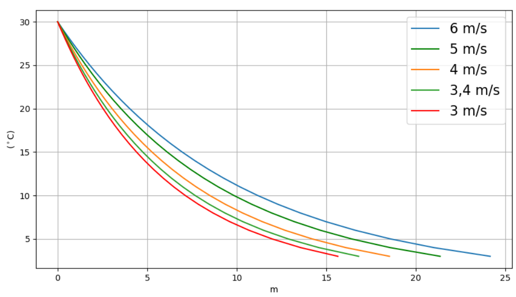
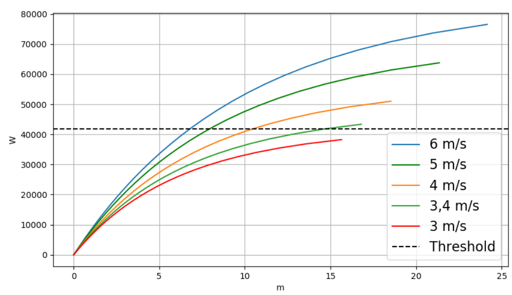


Figure 1. Convection coefficient for boiling plotted against the heat flux for the flooded evaporator



(a) Temperature drop over pipe length



(b) Delivered power from the single coil

Figure 2. Comparison of the water temperature drop in the pipe and the equivalent power of the evaporator

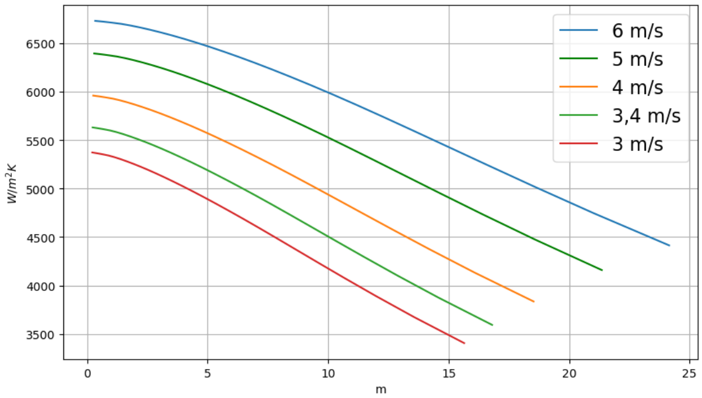
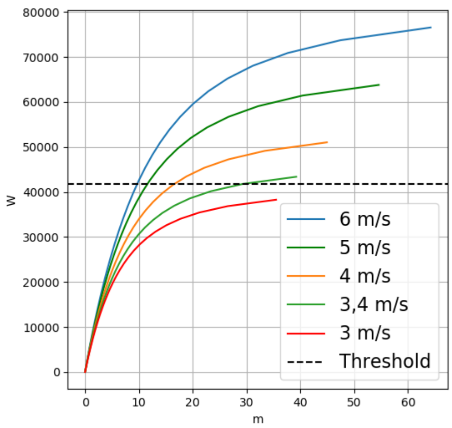


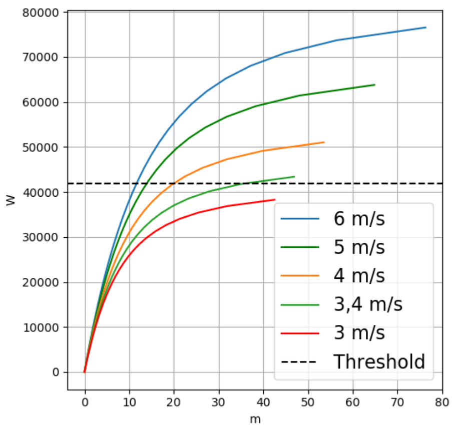
Figure 3. Overall heat transfer coefficient plotted over pipe length

4.1. Flooded evaporator several pipes

When dividing the pipe into either 2 or 3 pipes the pipe length increases to deliver the same amount of heat as a single coil. The velocity remains the same while the mass flow rate decreases to avoid fouling inside the pipe. The diameters are now 0,0069 m for 3 pipes and 0,0085 m for 2 pipes. The convection coefficient for water and CO_2 increases, but because of the reduction in the cross-sectional area, the length increases to compensate.



(a) Power plotted over length for a 2 coil configuration



(b) Power plotted over length for a 3 coil configuration

Figure 4. A comparison of the necessary pipe length for several pipe configuration

5. Discussion

The heat flux calculated in the model is dependent on the Rohsenow coefficient which is assumed. A material fluid combination of n-pentane-copper was chosen because a coefficient for CO_2 and stainless steel was not found since it has to be found empirically through experiments. This can impact the calculations for the CO_2 convection coefficient and should be considered when reviewing the results. Both the heat flux and the convection coefficient significantly drops in the last section of the pipe due to the low-temperature difference and heat flux, and this is the reason that the water can not realistically reach a lower temperature than $3°C$. The Roshesnow correlation has an error of up 20% [2].

6. Conclusion

With a total greywater production of 24645 kg/day the maximum velocity for 20 hours of operation is $3m/s$. However, the velocity needed to reach the evaporator threshold of 41,8 kW needs at least 3,4m/s. This indicates that the apartments does not produce enough greywater to supply the heat pump for all the 20 hours. A model was developed to see how the heat would transfer from greywater to a flooded evaporator through a stainless steel coil. The evaporator is designed to supply a heat pump that makes hot water for an apartment complex and has a gascooler capacity of 60 kW. The model shows that greywater can be used as a source for a flooded evaporator in a heat pump with a gascooler capacity of 60 kW. Using several pipes of smaller diameter will increase the length of pipe needed which means a single coil is a more efficient solution. The water temperature can be cooled to $3°C$ until the pipe length must be increased significantly.

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